Detection of Phase Transition Gravitational Waves

Qingyuan Liang, Supervisor: Huaike Guo

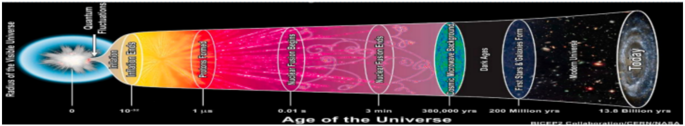
University of Chinese Academy of Sciences

September 28, 2025

Outline

Introduction

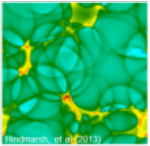
SWs dominated GWs production : Nucleation, Expansion, Percolation



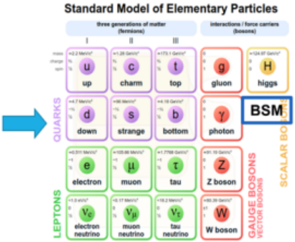
Energy density Spectrum

$$\Omega_{\rm GW}(f) = \frac{d\rho_{\rm GW}}{\rho_c d\log f}$$

sound waves

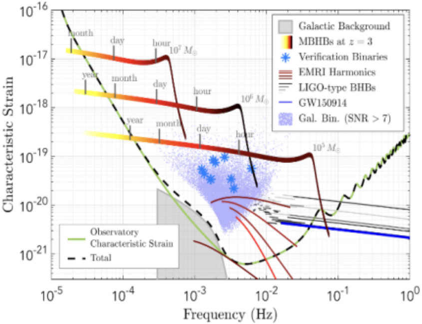


$$\Omega_{\rm sw}(f)h^2 = 2.65\times 10^{-6}\left(\frac{H_{\rm pt}}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} \times v_w\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}\Upsilon(\tau_{\rm sw}),$$



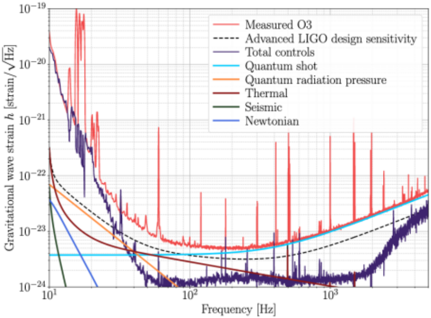
Introduction

Space-based GWs detectors
Miliherze range



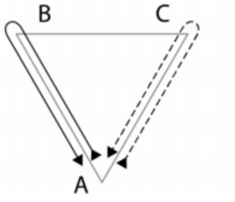
John Baker et al., arXiv:1907.06482

Ground-based GWs detectors
Handred Hertz range



Craig Cahillane arXiv:2202.00847

Time delay interferometry (TDI)



$$\Phi_{A_{BC}}(t) = \Delta\varphi_{A_{BC}} + n_{A_{BC}}(t)$$

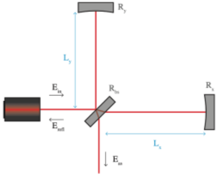
$\Delta\varphi_{A_{BC}}$: phase difference at vertex A

$n_{A_{BC}}(t)$: noise at vertex A

Arm Length $L = 2.5 \times 10^9 m$

TDI (cancel laser frequency noise)

Tristan L. Smith arXiv: 1908.00546



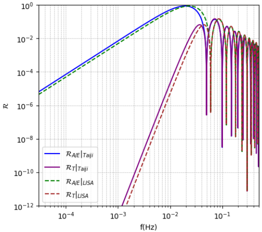
Arm Length $L = 4km$

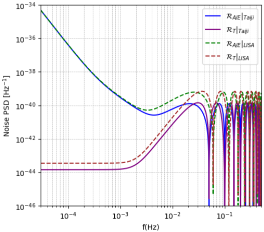
Cross correlation method for LIGO

Craig Cahillane arXiv: 2202.00847

AET channels

XYZ channels	AET channels	Power spectral densities
Vertex A: X	$A = \frac{1}{\sqrt{2}}(Z - X)$	$\mathrm{PSD}_A(f) = S_h(f)\,\mathcal{R}_A(f) + N_A(f)$
Vertex B: Y	$E = \frac{1}{\sqrt{6}}(X - 2Y + Z)$	$\mathrm{PSD}_E(f) = S_h(f)\,\mathcal{R}_E(f) + N_E(f)$
Vertex C: Z	$T = \frac{1}{\sqrt{3}}(X + Y + Z)$	$\mathrm{PSD}_T(f) = S_h(f)\,\mathcal{R}_T(f) + N_T(f)$





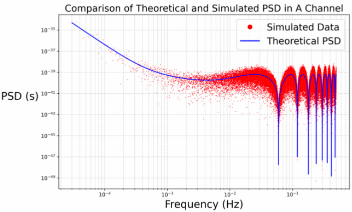
Null channel method
$\mathrm{PSD}_T(f) = N_T(f)$
Tristan L. Smith arXiv: 1908.00546

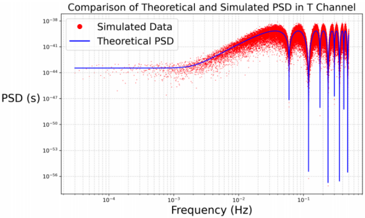
Frequency domain data

Distribution :

$$P\left(\tilde{d}_a(f_k)\right) = \frac{1}{2\pi\sigma_a^2}\exp\left[-\frac{\left|\tilde{d}_a(f_k)\right|^2}{2\sigma_a^2}\right], \qquad \sigma_a^2 = \frac{Tf_s^2}{4}P_a(f_k)$$

Generate in frequency domain





Guillaume Boileau arXiv:2011.05055v3

Likelihood

Logarithmic Likelihood function

$$\ln \mathcal{L} = -\sum_{\kappa=1}^{N_0}\sum_{k=1}^{N/2}\left\{\ln\left(\frac{\pi^2 T_s^3 f_s^6 [S_A(f_k)+N_A(f_k)][S_E(f_k)+N_E(f_k)]N_T(f_k)}{8}\right) + \frac{2}{Tf_s^2}\left[\frac{|\tilde{d}_A^\kappa(f_k)|^2}{S_A(f_k)+N_A(f_k)} + \frac{|\tilde{d}_E^\kappa(f_k)|^2}{S_E(f_k)+N_E(f_k)} + \frac{|\tilde{d}_T^\kappa(f_k)|^2}{N_T(f_k)}\right]\right\}$$

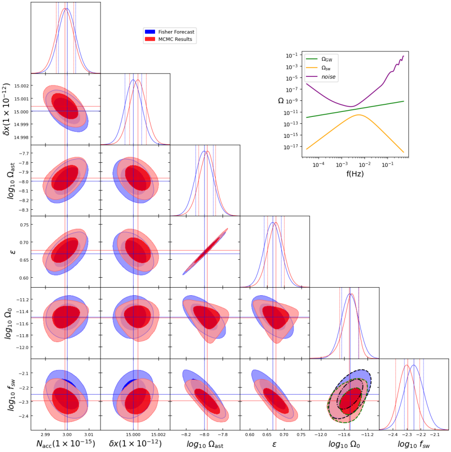
Fisher information matrix $F_{ij} = -E\left(\frac{\partial^2 \ln p(\boldsymbol{\theta})\mathcal{L}(\boldsymbol{\theta})}{\partial\theta_i\partial\theta_j}\right)$

k : index of discrete frequency
$N = 10^6$: data number in one segment
κ : segments index
$N_0 = 126$: number of segments
$T = 4$ years : observation time

$$F_{ij}^{\text{likelihood}} = N_0\sum_{k=0}^{N/2}\left[\frac{2}{[S_A(f_k)+N_A(f_k)]^2}\frac{\partial[S_A(f_k)+N_A(f_k)]}{\partial\theta_i}\frac{\partial[S_A(f_k)+N_A(f_k)]}{\partial\theta_j} + \frac{1}{N_T^2(f_k)}\frac{\partial N_T(f_k)}{\partial\theta_i}\frac{\partial N_T(f_k)}{\partial\theta_j}\right]$$

Provide fast estimates of parameter uncertainties to compare with MCMC

Comparison between FIM and MCMC



Chiara Caprini, arXiv:2403.03723v2

$$S_A = S_E = \frac{3H_0^2}{4\pi^2}\frac{\Omega_{\rm ast}\left(\frac{f}{f_{\rm ref}}\right)^{\epsilon} + \Omega_{\rm sw}(f)}{f^3}\mathcal{R}_A$$

$$\Omega_{\rm sw}(f) = \Omega_0\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$

- Comparison of parameter constraints
- Color coding (red for MCMC, blue for FIM) & Parameters are fully recovered
- Dark for 1σ, light for 2σ

Constraints on xSM parameters

xSM model parameters v_s, m_{h_2}, θ, b_3, b_4

$$V(H,S) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + \frac{a_1}{2} H^\dagger H S + \frac{a_2}{2} H^\dagger H S^2 + \frac{b_2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4$$

Thermodynamics parameters T_{pt}, α, $\frac{\beta}{H_{pt}}$

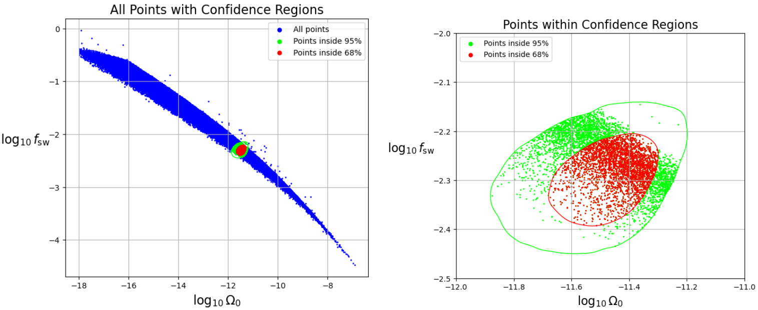
$$\Omega_{\rm sw}(f)h^2 = 2.65 \times 10^{-6} \left(\frac{H_{\rm pt}}{\beta}\right) \left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} \times v_w \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2} \Upsilon(\tau_{\rm sw})$$

Phenomenological parameters Ω_0, f_{sw}

$$\Omega_{\rm sw}(f) = \Omega_0 \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$

Alexandre Alves arXiv: 1812.09333

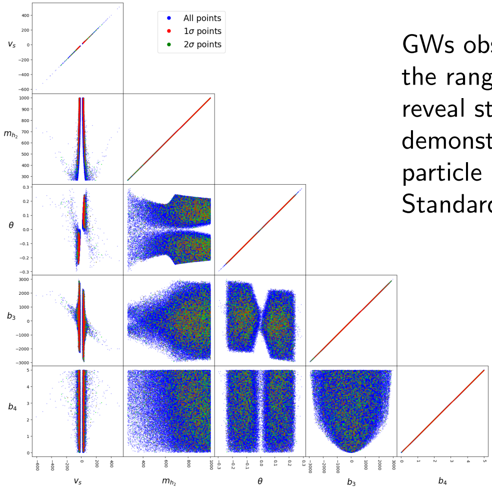
Constraints on xSM parameters



physical constraints (Alexandre Alves arXiv: 1812.09333)

- 1.The Higgs potential must be stable and bounded from below.
- 2.Perturbativity and unitarity must hold at high energies.
- 3.Higgs couplings must remain close to the Standard Model values.

Constraints on xSM parameters

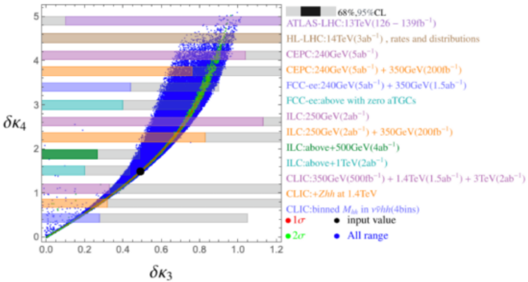


GWs observation significantly restricts the ranges of the xSM parameters and reveal strong parameter correlations, demonstrating it can directly constrain particle physics models beyond the Standard Model.

Constraints on Higgs couping parameters

$$\Delta\mathcal{L} = -\frac{1}{2}\frac{m_{h_1}^2}{v}(1+\delta\kappa_3)h_1^3 - \frac{1}{8}\frac{m_{h_1}^2}{v^2}(1+\delta\kappa_4)h_1^4$$

$$\delta\kappa_3 = \theta^2\left[-\frac{3}{2} + \frac{2m_{h_2}^2 - 2b_3 v_s - 4b_4 v_s^2}{m_{h_1}^2}\right] + \mathcal{O}(\theta^3) \quad \delta\kappa_4 = \theta^2\left[-3 + \frac{5m_{h_2}^2 - 4b_3 v_s - 8b_4 v_s^2}{m_{h_1}^2}\right] + \mathcal{O}(\theta^3)$$



Collider-only measurements leave large uncertainties in here GWs observations from FOPT provide complementary information that helps to narrow the allowed parameter space, highlighting its crucial role in probing the Higgs potential beyond the collider measurements.

Alexandre Alves arXiv: 1812.09333

Future improvement

- 1. Perform simulation in time-domain to capture realistic, non-stationary features.
- 2. Combine the space-based and ground-based detectors for joint observations.
- 3. Implement 2nd-generation TDI to improve laser noise cancellation.
- 4. Apply CLT and CG to reduce computation cost.
- 5.Explore more particle physics for GW predictions.

Thank You!

Questions are welcome.

Qingyuan Liang
University of Chinese Academy of Sciences
qingyuan211045@gmail.com