Wess-Zumino-Witten Interactions of Axions with Three-Flavor

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Outlines

- Axion models and general searches
- Problems in axion-meson interactions
  - Consistency of the interaction model
  - Wess-Zumino-Witten interactions of QCD
- Wess-Zumino-Witten interactions of Axions in 3 flavor
- Summary

The Strong CP problem

$$\mathscr{L} \supset -\frac{\theta g_s^2}{32\pi^2} G\tilde{G} - \left(\bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \mathrm{h.c.}\right)$$

A total derivative, but is allowed by non-trivial QCD vacuum

$$G^a_{\mu\nu}\tilde{G}^{a\mu\nu} = \partial_\mu K^\mu,$$

$$K^\mu = \epsilon^{\mu\nu\alpha\beta}\left(G^a_\nu \partial_\alpha G^a_\beta + \frac{g}{3} f^{abc} G^a_\nu G^b_\alpha G^c_\beta\right)$$

- The CKM matrix from $M_{u,d}$
  - CP violating phase θ_{CP} ~ 1.2 radian
- QCD induced CP violating phase, $\bar{\theta}$

$$\bar{\theta} = \theta + \arg\left[\det\left[M_u M_d\right]\right]$$

- $\bar{\theta}$ is invariant under quark chiral rotation
- According to neutron EDM experiment

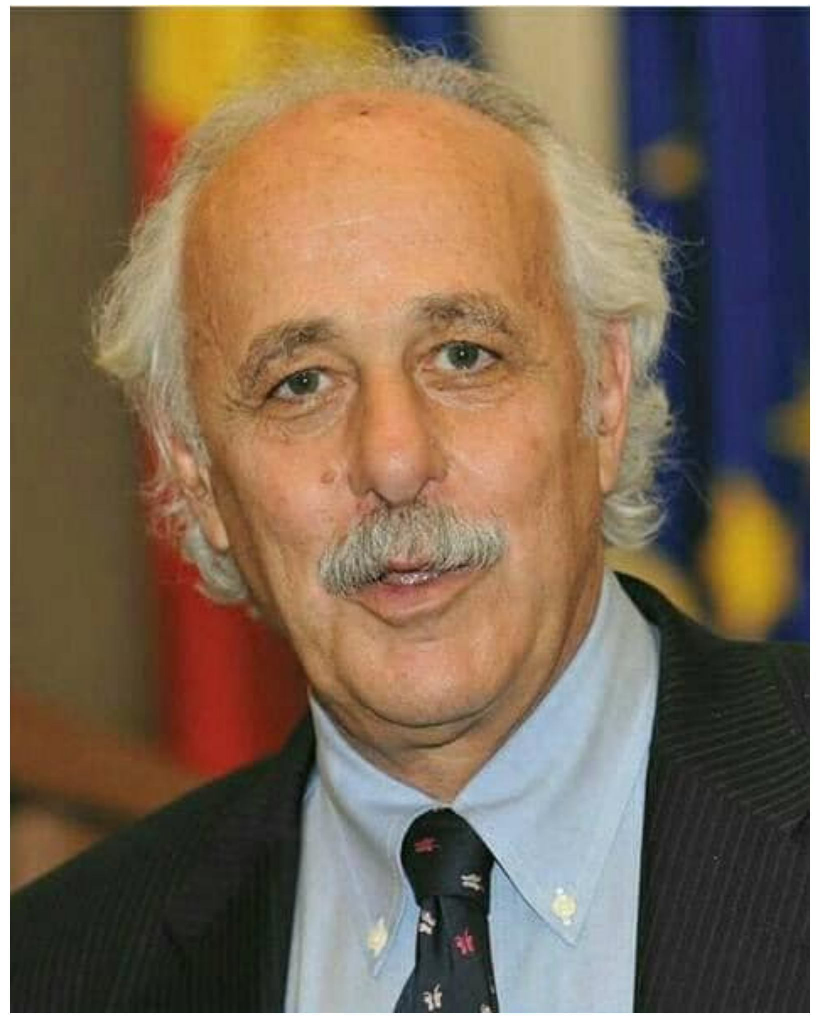
$$\bar{\theta} \lesssim 1.3 \times 10^{-10} \text{ radian}$$

$$d^n_{\mathrm{EDM}} \sim \bar{\theta} \times 10^{-16} \text{ e cm}$$

$$d^n_{\exp} < 10^{-26} \text{ e cm}$$

为什么实验没看到该类CP破坏?

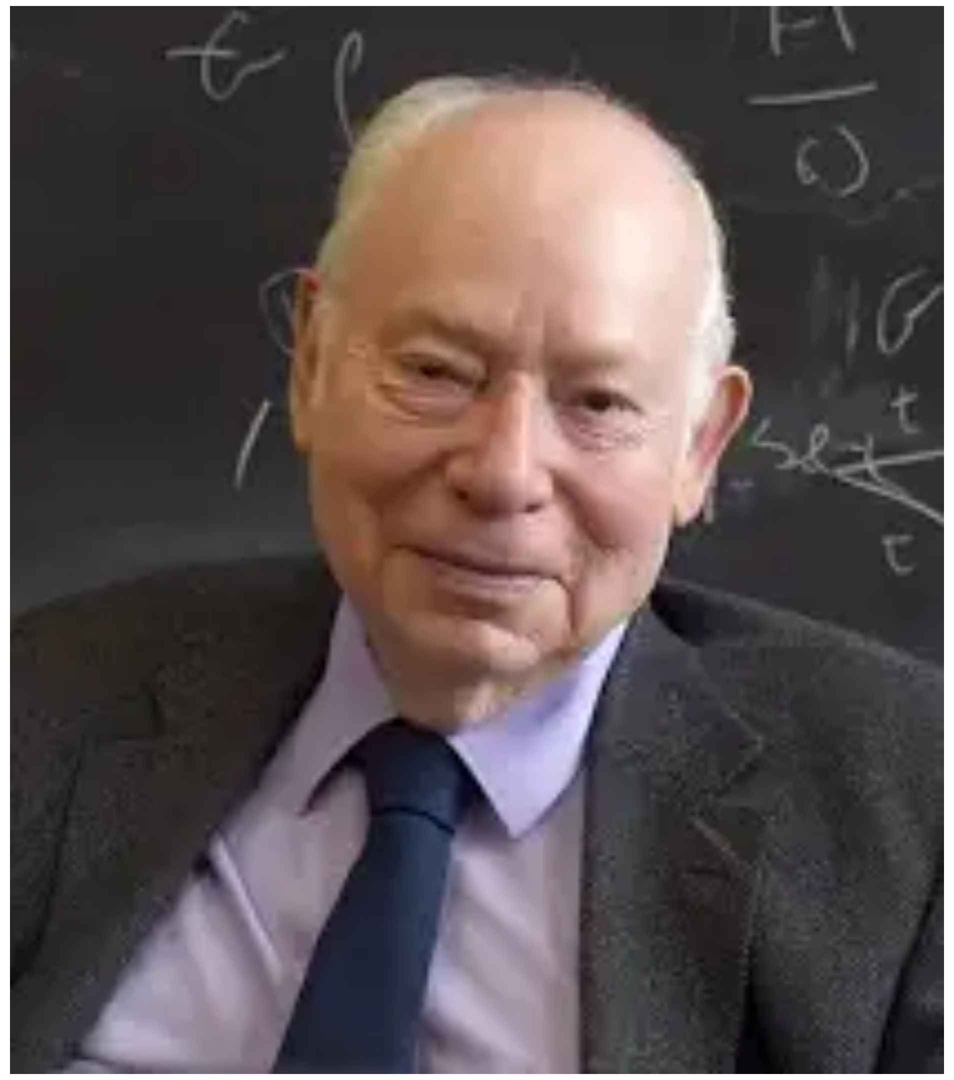
The Peccei-Quinn solution to Strong CP problem

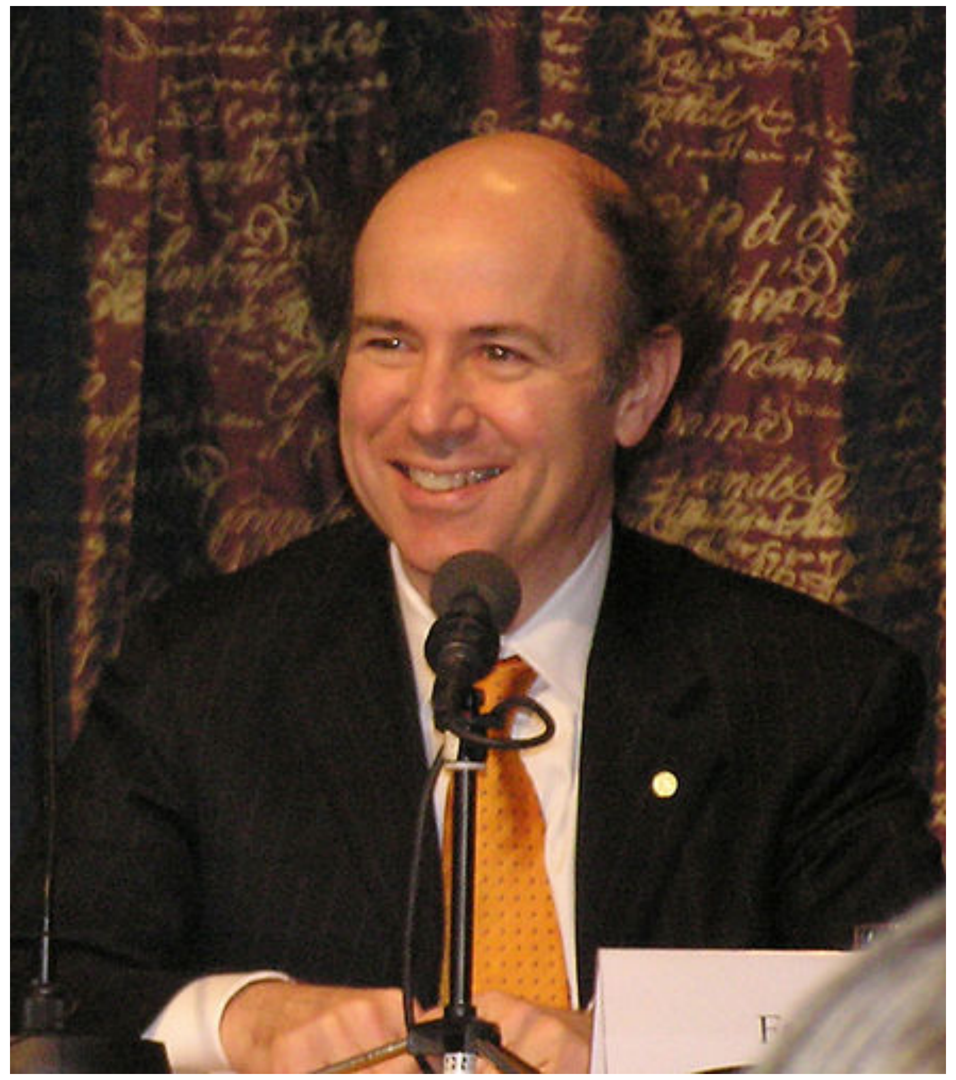


- Experiment requires $\bar{\theta} = \theta + \arg\left[\det\left[M_u M_d\right]\right] \lesssim 10^{-1}\mathrm{rad}$
- PQ: promote the constant $\bar{\theta}$ to a dynamical field, a(x)
- Introduce a *global* PQ-symmetry $U(1)_{\mathrm{PQ}}$, *anomalous* under the QCD
  - $a \to a + \kappa f_a \;\; \Rightarrow \mathcal{S} \to \mathcal{S} + \dfrac{\kappa}{32\pi^2}\displaystyle\int d^4x G\tilde{G}$, cancels $\bar{\theta}$
- Vafa-Witten theorem: vector-like theory (QCD) has ground state $\langle\bar{\theta}\rangle = 0$

PQWW Axion

- In 1978, Weinberg and Wilzeck realize there is an light particle





Axion!!!

- It can wash out the unwanted strong CP phase

- QCD axion : $m_a^2 f_a^2 \approx \Lambda_{\rm QCD}^4$; Neutrino Seesaw: $m_\nu m_{N_R} \approx (y v_h)^2$

- Light particles can probe high scale physics!

PQWW Axion

- PQWW axion assumes breaking scale $f_a \sim v_{\rm EW}$
- Axion mass from $100 \text{ keV} \sim 1 \text{ MeV}$, and the coupling strength is large $1/f_a$
- PQWW axion is quickly ruled out by
    - Lab constraints: $K^\pm \to \pi^\pm + a$, $J/\Psi \to \gamma + a$, and $\Upsilon \to \gamma + a$
    - Astrophysical constraints: Red giant and Supernovae
- *(Invisible)* QCD axion: the leading axion models are KSVZ/DFSZ model with $f_a \gg v_{\rm EW}$

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The axion effective Lagrangian at quark-level

- A more detailed effective Lagrangian

$$\mathscr{L}_{\text{eff},0} = \bar{q}_0(iD_\mu\gamma^\mu - \mathbf{m}_{q,0})q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+g_{ag,0}\frac{a}{f_a}G\tilde{G} + g_{a\gamma,0}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_{L,0}\gamma^\mu q_L + \bar{q}_R\mathbf{k}_{R,0}\gamma^\mu q_R + \ldots\right)$$

Bauer et al, PRL 127 (2021), 081803

- Quark mass $\mathbf{m}_{q,0}$ diagonal and real

- Coupling to both left/right fermions $\mathbf{k}_{L,0}$ and $\mathbf{k}_{R,0}$

The axion-dependent chiral rotation

- Use an axion-dependent chiral rotation to eliminate $aG\tilde{G}$ term

Bauer et al, PRL 127 (2021), 081803

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0}\gamma_5)c_{gg}\frac{a(x)}{f_a}\right]q(x) \qquad \mathrm{Tr}(\kappa_{q,0}) = 1$$

- New effective Lagrangian

$$\mathscr{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

The axion-dependent chiral rotation

- Define the chiral rotations (2-flavor for simplicity)

$$\boldsymbol{\theta}_L \equiv \boldsymbol{\delta}_{q,0} - \boldsymbol{\kappa}_{q,0} \qquad U_L \equiv \exp\left[-i\boldsymbol{\theta}_L a/f_a\right]$$

$$\boldsymbol{\theta}_R \equiv \boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0} \qquad U_R \equiv \exp\left[-i\boldsymbol{\theta}_R a/f_a\right]$$

- The relations between parameters

$$\mathbf{m}_q(a) = U_L^\dagger \mathbf{m}_0 U_R \to \begin{pmatrix} m_{u,0}e^{-2i\kappa_{u,0}c_{gg}\frac{a}{f_a}} & 0 \\ 0 & m_{d,0}e^{-2i\kappa_{d,0}c_{gg}\frac{a}{f_a}} \end{pmatrix}$$

$$\mathbf{k}_L(a) = U_L^\dagger[\mathbf{k}_{L,0} + c_{gg}\boldsymbol{\theta}_{L,0}]U_L \to \mathbf{k}_{L,0} + c_{gg}\boldsymbol{\theta}_{L,0}$$

$$\mathbf{k}_R(a) = U_R^\dagger[\mathbf{k}_{R,0} + c_{gg}\boldsymbol{\theta}_{R,0}]U_R \to \mathbf{k}_{R,0} + c_{gg}\boldsymbol{\theta}_{R,0}$$

Anomalous axion contribution

$$g_{a\gamma} = g_{a\gamma_0} - 2N_c c_{gg}\mathrm{Tr}\left[\mathbf{Q}^2\boldsymbol{\kappa}_{q,0}\right]$$

The consistent ChPT axion Lagrangian at meson level

$$\mathscr{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+ g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

- ChPT Lagrangian matching

$$U = \exp[(\sqrt{2}i/f_\pi)\pi^a\boldsymbol{\tau}^a]$$

$$\mathscr{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4}B_0\text{Tr}\left[\mathbf{m}_q(a)U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + g_{a\gamma}\frac{a}{f_a}F\tilde{F}$$

- The axion derivative coupling

Bauer et al, PRL 127 (2021), 081803

$$D^\mu U \to D^\mu U - i\frac{\partial^\mu a}{f_a}\left(\mathbf{k}_L U - U\mathbf{k}_R\right)$$

The importance of consistency

- The physical results should be independent of auxiliary parameters

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0}\gamma_5)c_{gg}\frac{a(x)}{f_a}\right] q(x)$$

- The most important channel BR($K \to \pi a$) is off by a factor of 37 for 35 years

H. Georgi, D. B. Kaplan and L. Randall, Phys. Lett. B 169, 73-78 (1986)

- Model-independent expression for $K \to \pi a$ and $\pi^- \to e^- \bar{\nu}_e a$ have been obtained for all axion couplings, only in 2021

Bauer et al, PRL 127 (2021), 081803

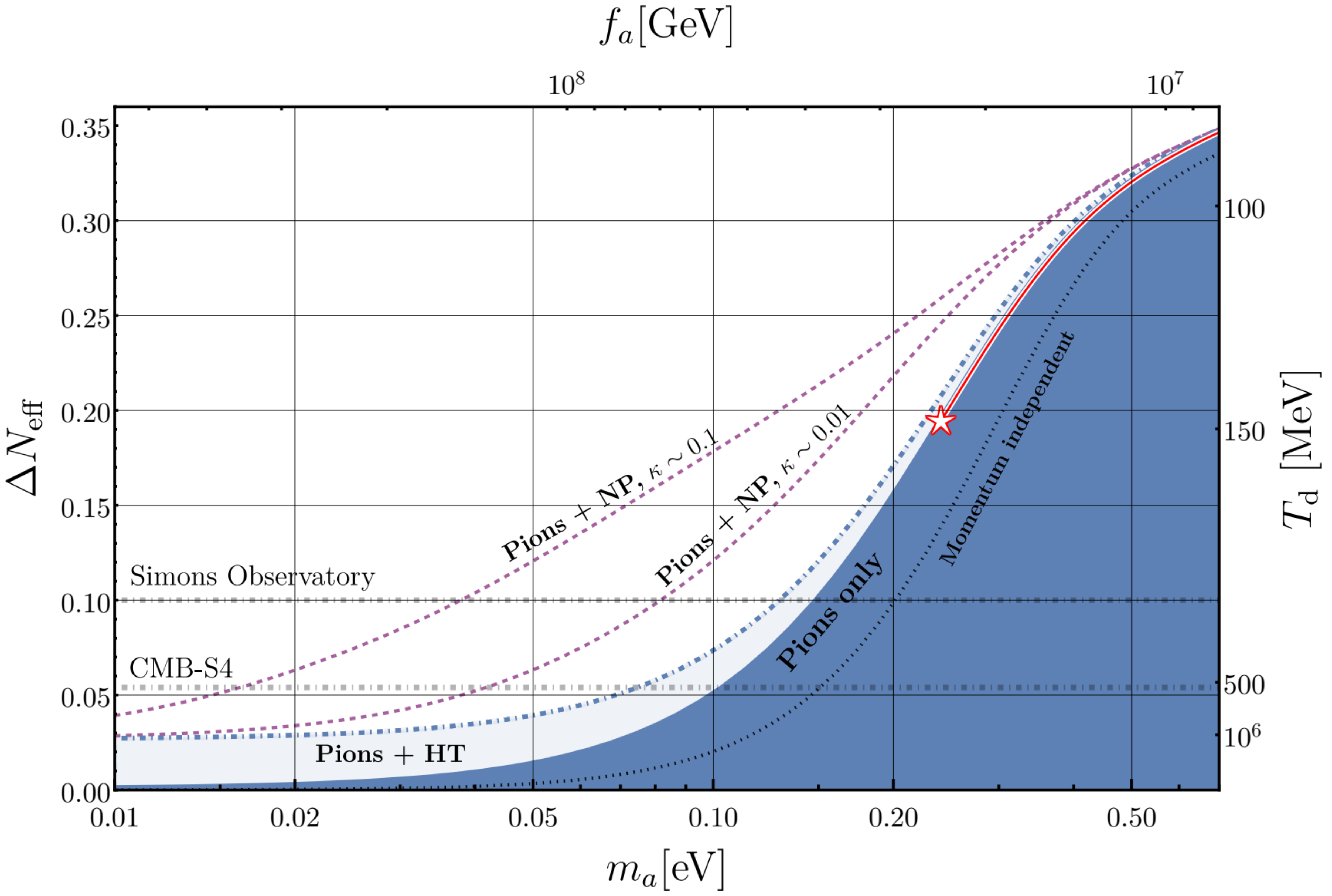
Why accurate interactions are important?

- Prediction for thermal axion and its near future test by CMB observation
  - Thermal axion production (high T): $q\bar{q} \to ga, qg \to qa$

    Ferreira, Notari PRL 120 (2018)191301
  - QCD phase transition:

    D'Eramo, Hajkarim, Yun PRL 128 (2022)152001
  - Improved axion-pion scattering production: $\pi\pi \leftrightarrow \pi a$

    Notari, Rompineve, Villladoro PRL 131 (2023)011004



QCD axion: $m_a < 0.24$ eV

Wess-Zumino-Witten Interactions in QCD

- WZW terms can describe anomalies in QCD, ensuring gauge invariance and completing chiral L

- Low-energy dynamics of mesons:
  e.g. multiple mesons and photons interactions
  $\pi_0/\eta/\eta' \to \gamma\gamma, \eta' \to 4\pi, \gamma^* \to 3\pi, 5\pi$

- Axion should be involved in WZW interactions systematically,
  not only in $a - \gamma - \gamma$ interactions
  - Raised by Harvey Hill Hill in [PRL 99 (2007) 261601], but not solved in previous study

Challenges in axion-meson interactions

- 1. Physics should not depend on the choice of chiral basis
- 2. How to systematically include axion interactions into WZW terms
- 3. Global symmetry (e.g. PQ, $U(1)_B$) will induce mixed anomaly to be dealt with

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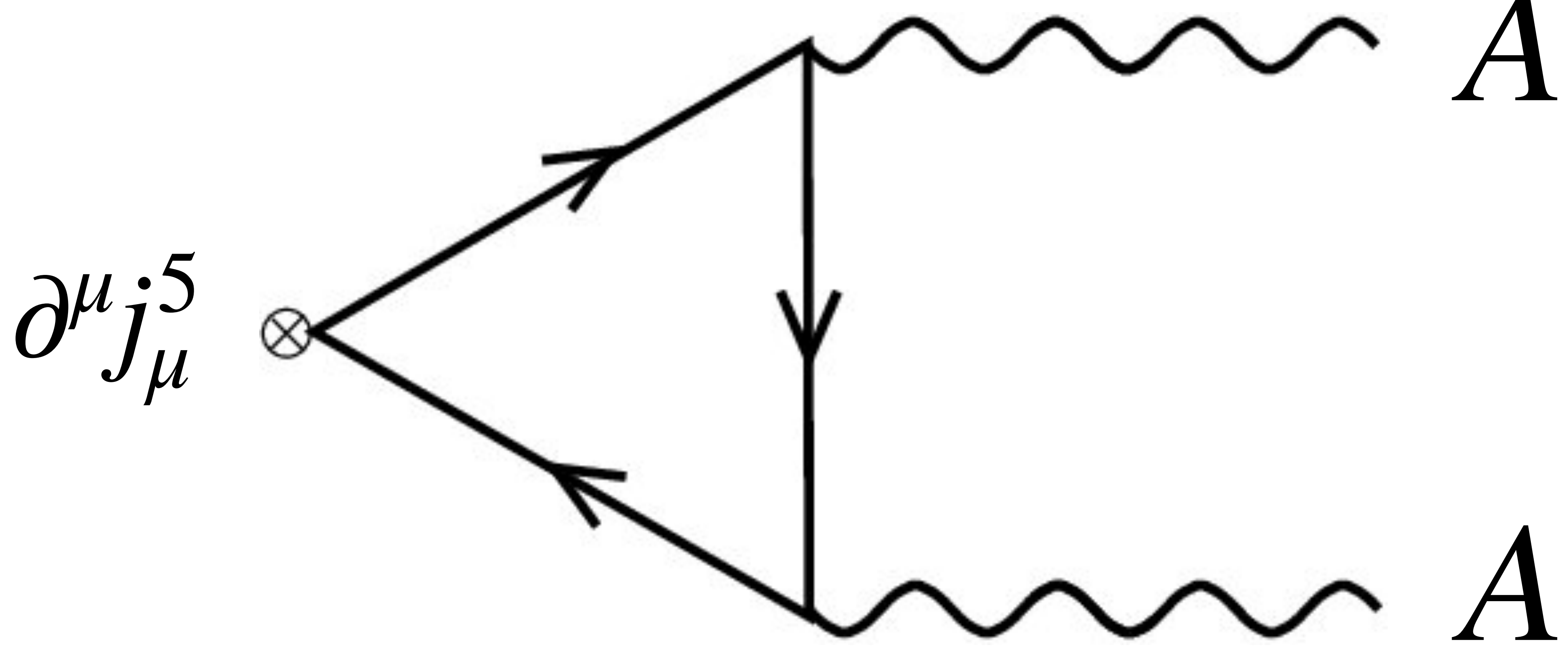
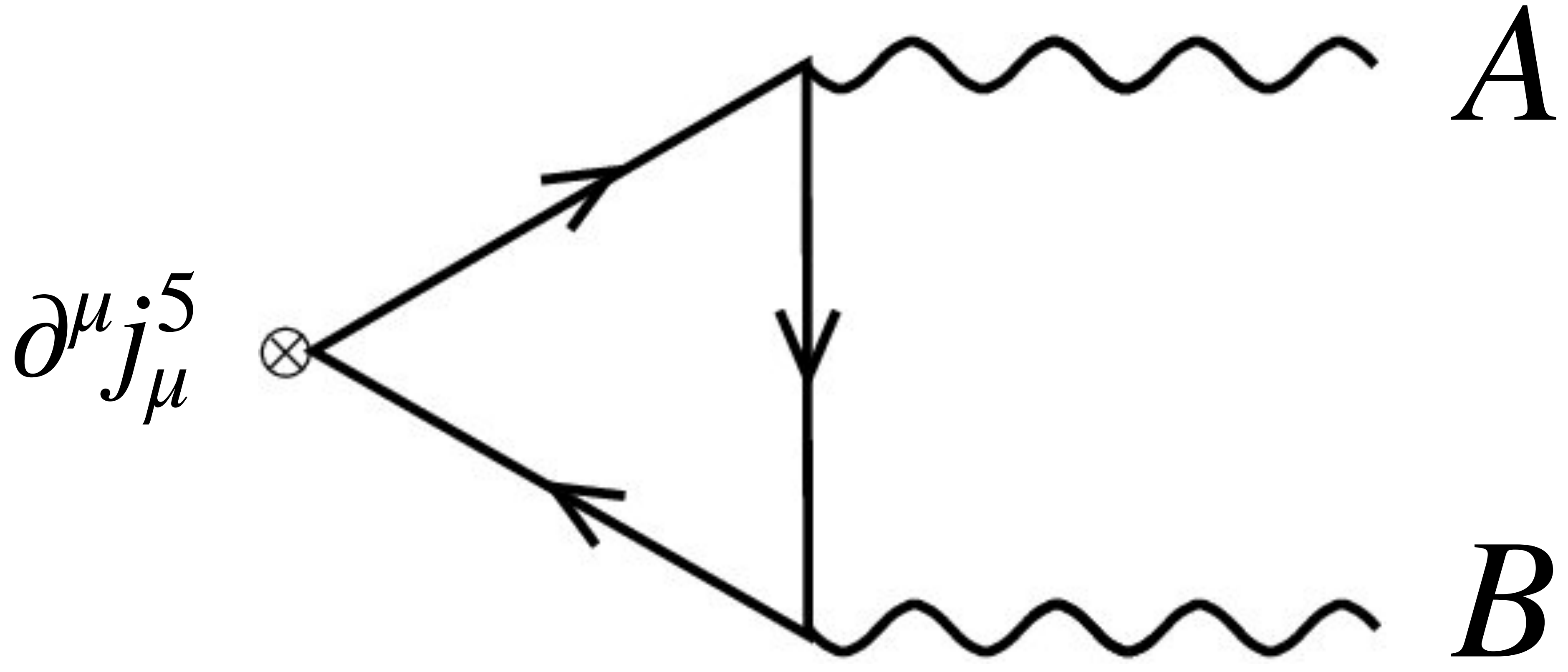
Global currents and background vector fields

- Background fields can couple to currents of $\mathscr{L}_{\chi\mathrm{PT}}$
  - Baryon currents U(1)$_B$ in neutron star, ω meson
  - Z boson vector in neutrino dense environment
- SM gauge invariance needs counter terms

$$\partial^\mu j^5_\mu = \frac{1}{48\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$



$$\delta\partial^\mu j^5_\mu \propto \epsilon_{\mu\nu\rho\sigma}A^{\mu\nu}B^{\rho\sigma}$$



WZW counter terms for global symmetry

J. A. Harvey, C. T. Hill, and R. J. Hill,
PRL 99 (2007) 261601,
PRD 77(2008) 085017

- Generic WZW interactions with counter terms
  - Vector fields in 1-form: $\mathscr{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
    Similar to Hidden Local Symmetry

$$\mathscr{L}^{\text{full}}_{\text{WZW}}(U, \mathscr{A}_{L/R}) = \mathscr{L}_{\text{WZW}}(U, \mathscr{A}_L, \mathscr{A}_R) + \mathscr{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$$

- Counter terms ensures SM invariance

$$\Gamma_c = -2\mathscr{C} \int Tr\left[(\mathbb{A}_L d\mathbb{A}_L + d\mathbb{A}_L \mathbb{A}_L)\mathbb{B}_L + \frac{1}{2}\mathbb{A}_L(\mathbb{B}_L d\mathbb{B}_L + d\mathbb{B}_L \mathbb{B}_L) - \frac{3}{2} i\mathbb{A}_L^3 \mathbb{B}_L - \frac{3}{4} i\mathbb{A}_L \mathbb{B}_L \mathbb{A}_L \mathbb{B}_L - \frac{i}{2}\mathbb{A}_L \mathbb{B}_L^3\right] - (L \leftrightarrow R)$$

- Suitable for chiral gauge fields and background fields

Axion treatment in three flavor

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \bar{q} i \not{D} q - \left(\boxed{\bar{q}_L \boldsymbol{m}_q q_R} + \text{H.c.}\right) + \frac{1}{2}\partial_\mu a\, \partial^\mu a - \frac{m_a^2}{2} a^2$$
$$+ \frac{\partial^\mu a}{f_a}\left(\bar{q}_L \boxed{\gamma^\mu \boldsymbol{k}_L} q_L + \bar{q}_R \boxed{\gamma^\mu \boldsymbol{k}_R} q_R\right) + \boxed{c_{gg}} \frac{\alpha_s}{4\pi}\frac{a}{f_a} G_{\mu\nu}\widetilde{G}^{\mu\nu} + \frac{a}{f_a}\sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1\mathcal{A}_2} F_{\mathcal{A}_1\mu\nu}\widetilde{F}^{\mu\nu}_{\mathcal{A}_2} + \mathcal{L}_c \,,$$

- $D_\mu = \partial_\mu - ig(A_L P_L + A_R P_R)$
- Hints from quark-level L: $D_\mu \to D_\mu + i\dfrac{\partial_\mu a}{f_a}\left(\mathbf{k}_L P_L + \mathbf{k}_R P_R\right)$
- Hints from ChPT L: $D^\mu U \to D^\mu U - i\dfrac{\partial^\mu a}{f_a}\left(\mathbf{k}_L U - U \mathbf{k}_R\right)$

Pseudoscalars in three flavor

- U matrix for mesons in three flavor

$$U = \exp\left[(\sqrt{2}i/f_\pi)\pi^a \boldsymbol{t}^a\right] \equiv \exp\left[(\sqrt{2}i/f_\pi)\boldsymbol{\Phi}\right]$$

$$\boldsymbol{\Phi} = \begin{pmatrix} \pi_0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi_0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\,\overline{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$

- η’ mass from instanton effect $\bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\widetilde{G}^{\mu\nu} \to -\frac{\tau}{2}\left(-i\log\det U - \bar{\theta}\right)^2$

Axionized Lagrangian:

$$\mathcal{L}_{\chi\mathrm{PT}} \supset -\frac{\tau}{2}\left(-i\log\det U - 2c_{gg}\frac{a}{f_a}\right)^2 = -\frac{m_0^2}{2}\left(\eta_0 - \frac{c_{gg}}{\sqrt{3}}\frac{f_\pi}{f_a}a\right)^2$$

Axion treatment in three flavor

- Vector fields in 1-form: $\mathscr{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$ $\quad \mathbb{A}_L = \frac{e}{s_w} W^i \boldsymbol{T_i} + \frac{e}{c_w} W^0 \mathbf{Y}_Q, \ \mathbb{A}_R = \frac{e}{c_w} W^0 \mathbf{Y}_q$
  Similar to Hidden Local Symmetry

- Axion 1-form field can be added into background fields: $\mathbb{B}_{L/R} \to \mathbb{B}_{L/R} + \mathbf{k}_{L/R,0} \frac{da}{f_a}$

- 3-flavor ChPT with SM gauge bosons and background fields

$$\mathbb{B}_V \equiv \mathbb{B}_L + \mathbb{B}_R = g \begin{pmatrix} \rho_0 + \omega & \sqrt{2}\rho^+ & \sqrt{2}K^*_+ \\ \sqrt{2}\rho^- & -\rho_0 + \omega & \sqrt{2}K^*_0 \\ \sqrt{2}K^*_- & \sqrt{2}\,\overline{K}^*_0 & \sqrt{2}\phi \end{pmatrix} + (\boldsymbol{k}_L + \boldsymbol{k}_R)\frac{da}{f_a} \, ,$$

$$\mathbb{B}_A \equiv \mathbb{B}_L - \mathbb{B}_R = g \begin{pmatrix} a_1 + f_1 & \sqrt{2}a^+ & \sqrt{2}K^*_{A+} \\ \sqrt{2}a^- & -a_1 + f_1 & \sqrt{2}K^*_{A0} \\ \sqrt{2}K^*_{A-} & \sqrt{2}\,\overline{K}^*_{A0} & \sqrt{2}f_s \end{pmatrix} + (\boldsymbol{k}_L - \boldsymbol{k}_R)\frac{da}{f_a}$$

The consistent full axion Lagrangian at low-energy

- ChPT:

$$D^\mu = \partial^\mu - i\sum_{\mathcal{A}} \left(\mathcal{A}_L^\mu P_L + \mathcal{A}_R^\mu P_R\right)$$

$$\mathcal{L}_{\chi\mathrm{PT}} = \frac{f_\pi^2}{8}\,\mathrm{Tr}\left[\boxed{(D^\mu U)\,(D_\mu U)^\dagger}\right] + \frac{f_\pi^2}{4} B_0\,\mathrm{Tr}\boxed{\left[\boldsymbol{m}_q U^\dagger\right.}\left. + \mathrm{H.c.}\right] - \boxed{\frac{m_0^2}{2}\left(\eta_0 - \frac{c_{gg}}{\sqrt{3}}\frac{f_\pi}{f_a} a\right)^2}$$

$$+ \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2}{2} a^2 + \frac{a}{f_a}\sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1\mathcal{A}_2} F_{\mathcal{A}_1\mu\nu}\widetilde{F}^{\mu\nu}_{\mathcal{A}_2}\ ,$$

- Full WZW:

$$\mathscr{L}^{\mathrm{full}}_{\mathrm{WZW}}(U, \mathscr{A}_{L/R}) = \mathscr{L}_{\mathrm{WZW}}(U, \boxed{\mathscr{A}_L, \mathscr{A}_R)} + \mathscr{L}_c\boxed{(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})}$$

- Full $\mathscr{L}$:

$$\mathscr{L}^{\mathrm{full}}_{\mathrm{axion}} \equiv \left[\mathscr{L}_{\chi\mathrm{PT}} + \mathscr{L}^{\mathrm{full}}_{\mathrm{WZW}}\right]\left(U, \mathbf{m}_q(a), \mathscr{A}_{L/R} + \mathbf{k}_{L/R}(a)da/f_a\right)$$

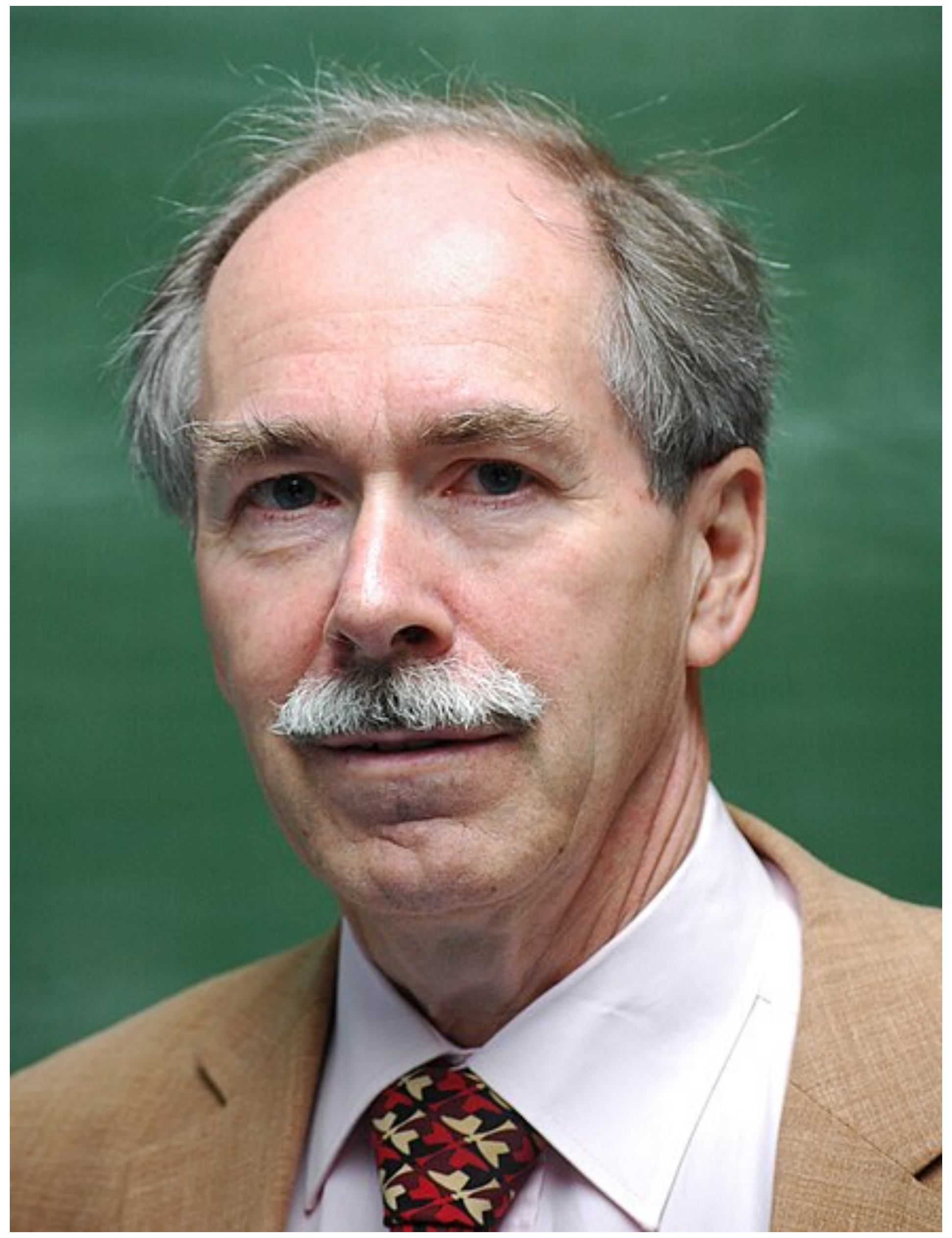
Matching between $\mathscr{L}_{\text{eff}}$ and $\mathscr{L}^{\text{full}}_{\text{axion}}$

$$\begin{tikzcd}
\begin{array}{l} \mathscr{L}_{\text{eff}}\left(q, \mathscr{A}_{L/R} \middle| \boldsymbol{m}_q, \boldsymbol{k}_{L/R}, c_{gg}\right) \\ \supset c_{gg} \dfrac{\alpha_s}{4\pi} \dfrac{a}{f_a} G\tilde{G} \end{array} \arrow[r, "q \to \exp\left[-i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5\right) \frac{c_{gg} a}{f_a}\right] q"'] \arrow[d, "\text{matching}"] & \begin{array}{l} \left\{\mathscr{L}_{\text{eff}}\left(q, \mathscr{A}_{L/R} \middle| \boldsymbol{m}'_q, \boldsymbol{k}'_{L/R}, c'_{gg}\right) + \delta\mathscr{L}^{\text{ano}}_{a}\right\} \\ \supset c_{gg}\left[1 - \mathrm{Tr}(\boldsymbol{\kappa}_q)\right] \dfrac{\alpha_s}{4\pi} \dfrac{a}{f_a} G\tilde{G} \end{array} \arrow[d, "\text{matching}"] \\
\begin{array}{l} \left[\mathscr{L}_{\chi\text{PT}} + \mathscr{L}^{\text{full}}_{\text{WZW}}\right]\left(U, \mathscr{A}_{L/R} \middle| \boldsymbol{m}_q, \boldsymbol{k}_{L/R}, c_{gg}\right) \\ \supset -\dfrac{\tau}{2}\left(-i \log[\det U] - 2 c_{gg} a/f_a\right)^2 \end{array} \arrow[r, "U \to U_L U U_R^\dagger"] & \begin{array}{l} \left\{\left[\mathscr{L}_{\chi\text{PT}} + \mathscr{L}^{\text{full}}_{\text{WZW}}\right]\left(U, \mathscr{A}_{L/R} \middle| \boldsymbol{m}'_q, \boldsymbol{k}'_{L/R}, c'_{gg}\right) + \delta\mathscr{L}^{\text{ano}}_{\text{WZW}}\right\} \\ \supset -\dfrac{\tau}{2}\left(-i \log[\det U] - 2 c_{gg}\left[1 - \mathrm{Tr}(\boldsymbol{\kappa}_q)\right] a/f_a\right)^2 \end{array}
\end{tikzcd}$$

- Important: consistency for any κ_q rotation

Gerard 't Hooft UV-IR anomaly matching condition

- The anomalies of global symmetries must match between the ultraviolet (UV) and infrared (IR) descriptions of a QFT



Gerard 't Hooft

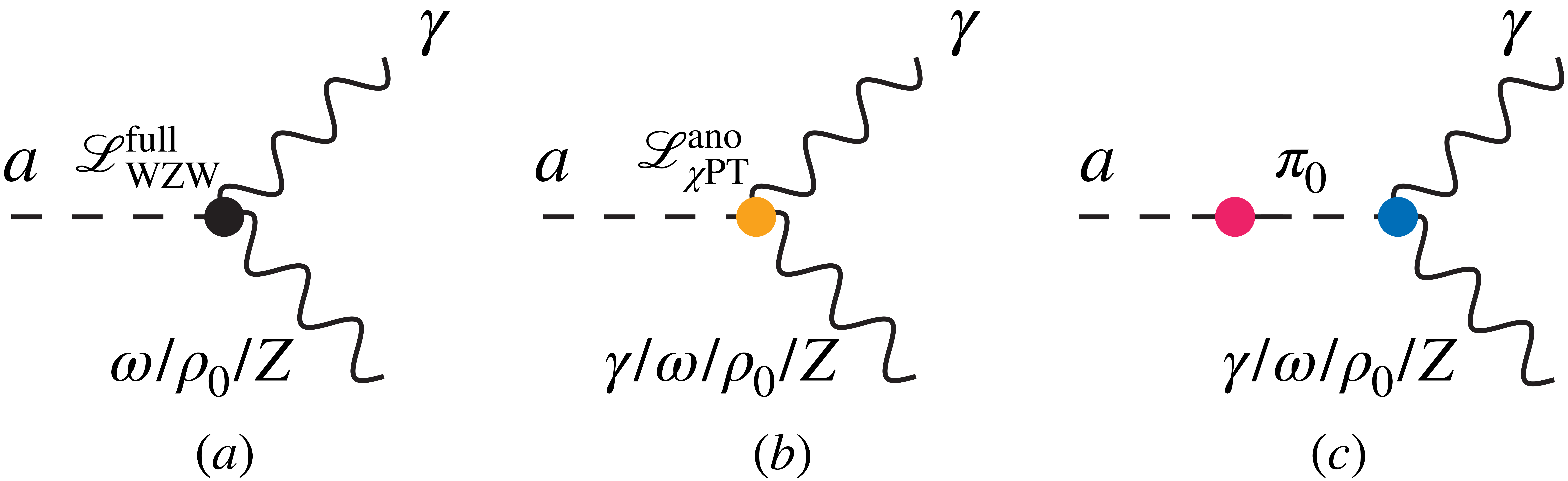
$$\begin{tikzcd}
\begin{array}{l}\mathscr{L}_{\rm eff}\left(q, \mathscr{A}_{L/R}\,\middle|\, \boldsymbol{m}_q, \boldsymbol{k}_{L/R}, c_{gg}\right) \\ \supset c_{gg}\dfrac{\alpha_s}{4\pi}\dfrac{a}{f_a}G\tilde{G}\end{array} \arrow[r, "q \to \exp\left[-i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q\gamma_5\right)\frac{c_{gg}a}{f_a}\right]q"'] \arrow[d, "\text{matching}"] & \begin{array}{l}\left\{\mathscr{L}_{\rm eff}\left(q, \mathscr{A}_{L/R}\,\middle|\, \boldsymbol{m}'_q, \boldsymbol{k}'_{L/R}, c'_{gg}\right) + \boxed{\delta\mathscr{L}_a^{\rm ano}}\right\} \\ \supset c_{gg}\left[1-{\rm Tr}(\boldsymbol{\kappa_q})\right]\dfrac{\alpha_s}{4\pi}\dfrac{a}{f_a}G\tilde{G}\end{array} \arrow[d, "\text{matching}"'] \arrow[d, <->, bend left=60, "\text{Anomaly Matching}"] \\
\begin{array}{l}\left[\mathscr{L}_{\chi\rm PT} + \mathscr{L}_{\rm WZW}^{\rm full}\right]\left(U, \mathscr{A}_{L/R}\,\middle|\, \boldsymbol{m}_q, \boldsymbol{k}_{L/R}, c_{gg}\right) \\ \supset -\dfrac{\tau}{2}\left(-i\log[\det U] - 2c_{gg}a/f_a\right)^2\end{array} \arrow[r, "U \to U_L U U_R^\dagger"] & \begin{array}{l}\left\{\left[\mathscr{L}_{\chi\rm PT} + \mathscr{L}_{\rm WZW}^{\rm full}\right]\left(U, \mathscr{A}_{L/R}\,\middle|\, \boldsymbol{m}'_q, \boldsymbol{k}'_{L/R}, c'_{gg}\right) + \boxed{\delta\mathscr{L}_{\rm WZW}^{\rm ano}}\right\} \\ \supset -\dfrac{\tau}{2}\left(-i\log[\det U] - 2c_{gg}\left[1-{\rm Tr}(\boldsymbol{\kappa_q})\right]a/f_a\right)^2\end{array}
\end{tikzcd}$$

$$\delta\mathscr{L}_a^{\rm ano} = -\delta\left[\mathscr{L}_{\rm WZW} + \mathscr{L}_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = \delta\mathscr{L}_{\rm WZW}^{\rm ano}$$

Consistent physical amplitudes

- A consistent Lagrangian will give physical amplitudes independent of auxiliary rotations

- Full WZW interactions are important for a-A-B amplitudes



Pseudoscalar mixing in three flavor

- Mass/flavor eigenstates

$$a = a_{\rm phys} - \sum_{P^0=\pi^0,\eta,\eta'} h(P^0, m_{P^0}) P^0_{\rm phys} \ ,$$

$$P^0 = P^0_{\rm phys} - \sum_{P^{0\prime}\neq P^0} \frac{M^2_{P^0 P^{0\prime}}}{m^2_{P^0} - m^2_{P^{0\prime}}} P^{0\prime}_{\rm phys} + h(P^0, m_a) a_{\rm phys} \ ,$$

$$h(P^0, m_X) = \frac{1}{m_a^2 - m_{P^0}^2}\left[M^2_{aP^0} - m_X^2 K_{aP^0} + \sum_{P^{0\prime}\neq P^0} M^2_{P^0 P^{0\prime}} \frac{M^2_{aP^{0\prime}} - m_X^2 K_{aP^{0\prime}}}{m_X^2 - m^2_{P^{0\prime}}} \right]$$

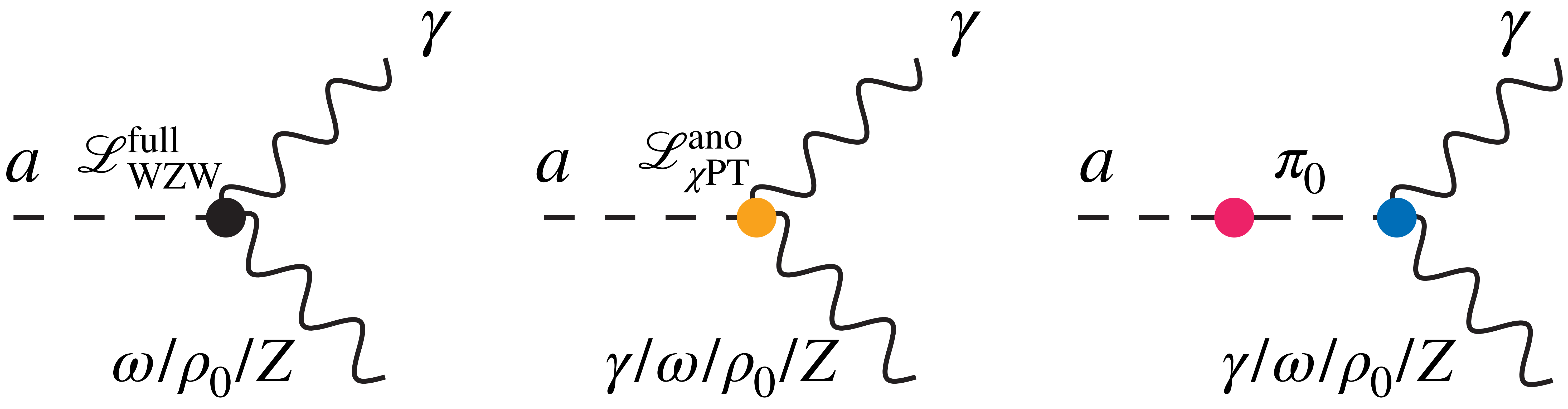
- Mixing angles

$$\begin{aligned}
\theta_{\pi^0 a} = {}& \frac{\epsilon}{2\sqrt{2}(m_\pi^2 - m_a^2)}\Big\{ m_a^2\left[c_d^L - c_u^L - c_d^R + c_u^R - (2\kappa_d - 2\kappa_u)c_{gg}\right] \\
& + 2m_\pi^2 c_{gg}\left[(1+\delta)\kappa_d + (-1+\delta)\kappa_u\right]\Big\} \ , \\
\theta_{\eta a} = {}& -\frac{\epsilon}{2\sqrt{3}(m_\eta^2 - m_a^2)}\left\{ m_a^2\left[c_d^L - c_s^L + c_u^L - c_d^R + c_s^R - c_u^R - (2\kappa_d - 2\kappa_s + 2\kappa_u)c_{gg}\right]\right\} \\
& -\frac{2\epsilon}{2\sqrt{3}(m_\eta^2 - m_a^2)}\left\{ m_\pi^2\left[1 + \delta(\kappa_d - \kappa_u)\right]c_{gg} + m_\eta^2(-1 + \kappa_d - \kappa_s + \kappa_u)c_{gg}\right\} \ , \\
\theta_{\eta' a} = {}& -\frac{\epsilon}{2\sqrt{6}(m_{\eta'}^2 - m_a^2)}\left\{ m_a^2\left[c_d^L + 2c_s^L + c_u^L - c_d^R - 2c_s^R - c_u^R - (2\kappa_d + 4\kappa_s + 2\kappa_u)c_{gg}\right]\right\} \\
& -\frac{2\epsilon}{2\sqrt{6}(m_{\eta'}^2 - m_a^2)}\Big\{ (m_{\eta'}^2 + 3m_\pi^2)(-1 + \kappa_d + 2\kappa_s + \kappa_u)c_{gg} \\
& - m_\pi^2\left[-4 - (-3+\delta)\kappa_d + 6\kappa_s + 3\kappa_u + \delta\kappa_u\right]c_{gg}\Big\} \ .
\end{aligned}$$

Consistent physical amplitudes for $a-\gamma-\omega$

- Auxiliary rotations are cancelled

$$c^{\rm eff}_{\omega\gamma} = c_{\rm WZW} + c_{\rm ano} + \sum_{=\pi^0,\eta,\eta'} c_p \theta_{pa}$$



(a) (b) (c)

$$c_{\rm WZW} = \frac{egN_c(c_d^L Q_d - c_d^R Q_d + c_u^L Q_u - c_u^R Q_u - 2Q_d c_{gg}\kappa_d - 2Q_u c_{gg}\kappa_u)}{16\pi^2 f_a} \,,$$

$$c_{\rm ano} = -\frac{egN_c(Q_d\kappa_d c_{gg} + Q_u\kappa_u c_{gg})}{8\pi^2 f_a} \,,$$

$$c_{\pi^0} = \frac{egN_c(Q_d - Q_u)}{4\sqrt{2} f_\pi \pi^2}, \quad c_{\eta_8} = -\frac{egN_c(Q_d + Q_u)}{4\sqrt{6} f_\pi \pi^2}, \quad c_{\eta_0} = -\frac{egN_c(Q_d + Q_u)}{4\sqrt{3} f_\pi \pi^2} \,,$$

$$c_\eta = \frac{2\sqrt{2}}{3} c_{\eta_8} + \frac{1}{3} c_{\eta_0}, \quad c_{\eta'} = -\frac{1}{3} c_{\eta_8} + \frac{2\sqrt{2}}{3} c_{\eta_0} \,.$$

Consistent physical amplitudes for $a - \gamma - \omega$

- 3-flavor results

$$
\begin{aligned}
c^{\mathrm{eff}}_{\omega\gamma} = {} & \frac{c_{gg} e g N_c (Q_d + Q_u)}{24\pi^2 f_a (m_a^2 - m_\eta^2)(m_a^2 - m_{\eta'}^2)} \Big\{ m_a^2 (2m_\eta^2 + m_{\eta'}^2 - 3m_\pi^2) \\
& \qquad + m_\eta^2 (m_\pi^2 - 3m_{\eta'}^2) + 2m_{\eta'}^2 m_\pi^2 \Big\} \\
& + \frac{c_{gg} e g N_c (Q_d - Q_u) \delta m_\pi^2}{24\pi^2 f_a (m_a^2 - m_\pi^2)(m_a^2 - m_\eta^2)(m_a^2 - m_{\eta'}^2)} \Big\{ m_a^2 (2m_\eta^2 + m_{\eta'}^2 - 3m_\pi^2) \\
& \qquad + m_\eta^2 (m_\pi^2 - 3m_{\eta'}^2) + 2m_{\eta'}^2 m_\pi^2 \Big\} \\
& + \frac{e g N_c}{48\pi^2 f_a} \Big\{ 3Q_d (c_d^L - c_d^R) + 3Q_u (c_u^L - c_u^R) - \frac{3m_a^2 (Q_d - Q_u)(c_d^L - c_d^R - c_u^L + c_u^R)}{m_a^2 - m_\pi^2} \\
& \qquad \frac{2m_a^2 (Q_d + Q_u)(c_u^L - c_u^R + c_d^L - c_d^R + c_s^R - c_s^L)}{m_\eta^2 - m_a^2} \\
& \qquad \frac{m_a^2 (Q_d + Q_u)(c_u^L - c_u^R + c_d^L - c_d^R + 2c_s^L - 2c_s^R)}{m_{\eta'}^2 - m_a^2} \Big\} \\
& - \frac{e g N_c \delta m_a^2 m_\pi^2}{24\pi^2 f_a (m_a^2 - m_\pi^2)(m_a^2 - m_\eta^2)(m_a^2 - m_{\eta'}^2)} \Big\{ (-3m_a^2 + m_\eta^2 + 2m_{\eta'}^2) \left[Q_u (c_u^L - c_u^R) \right. \\
& \qquad \left. - Q_d (c_d^L - c_d^R) \right] - (m_\eta^2 - m_{\eta'}^2)(Q_d - Q_u) \Big\} \, .
\end{aligned}
$$

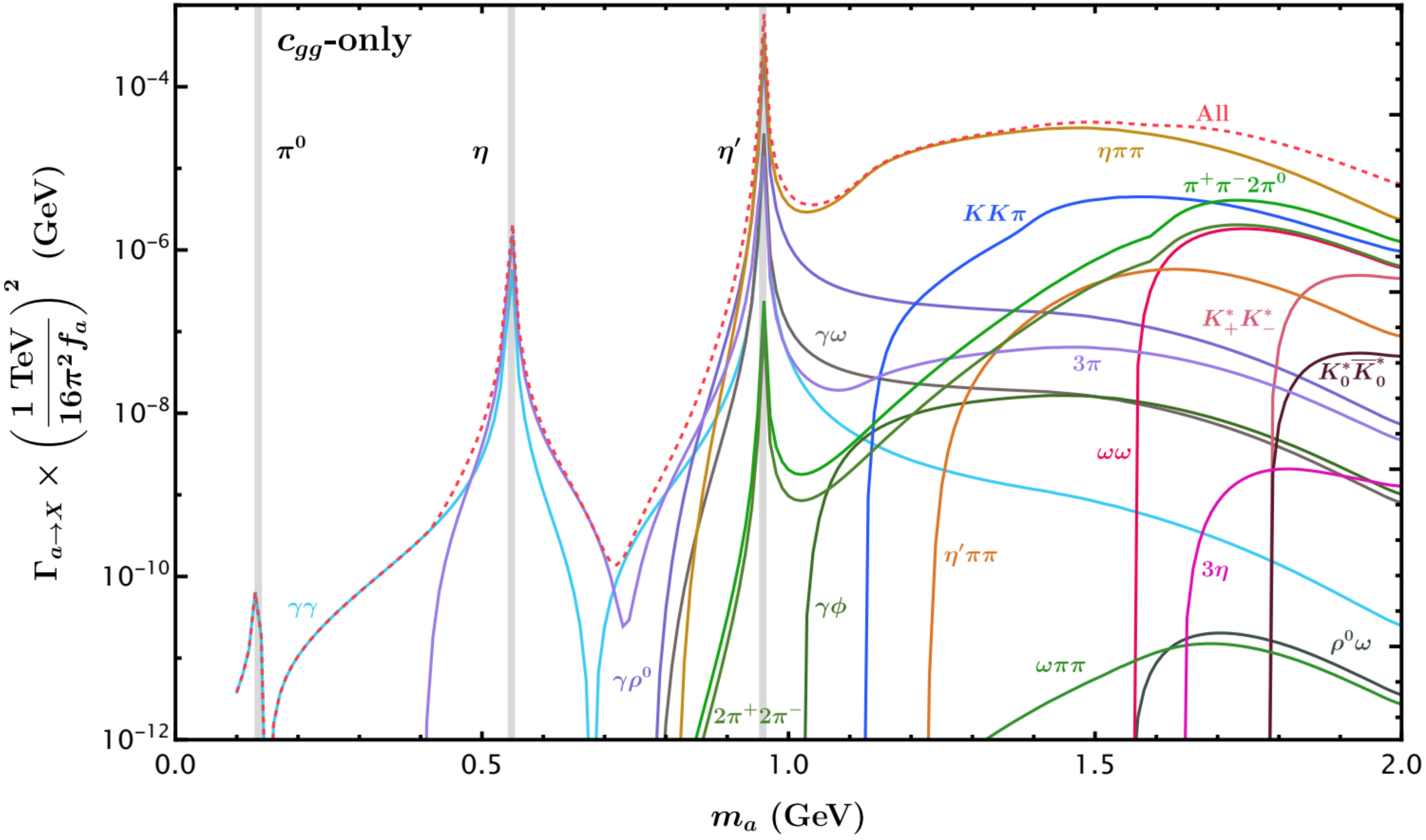
$m_\eta, m_{\eta'} \to \infty$

- 2-flavor results

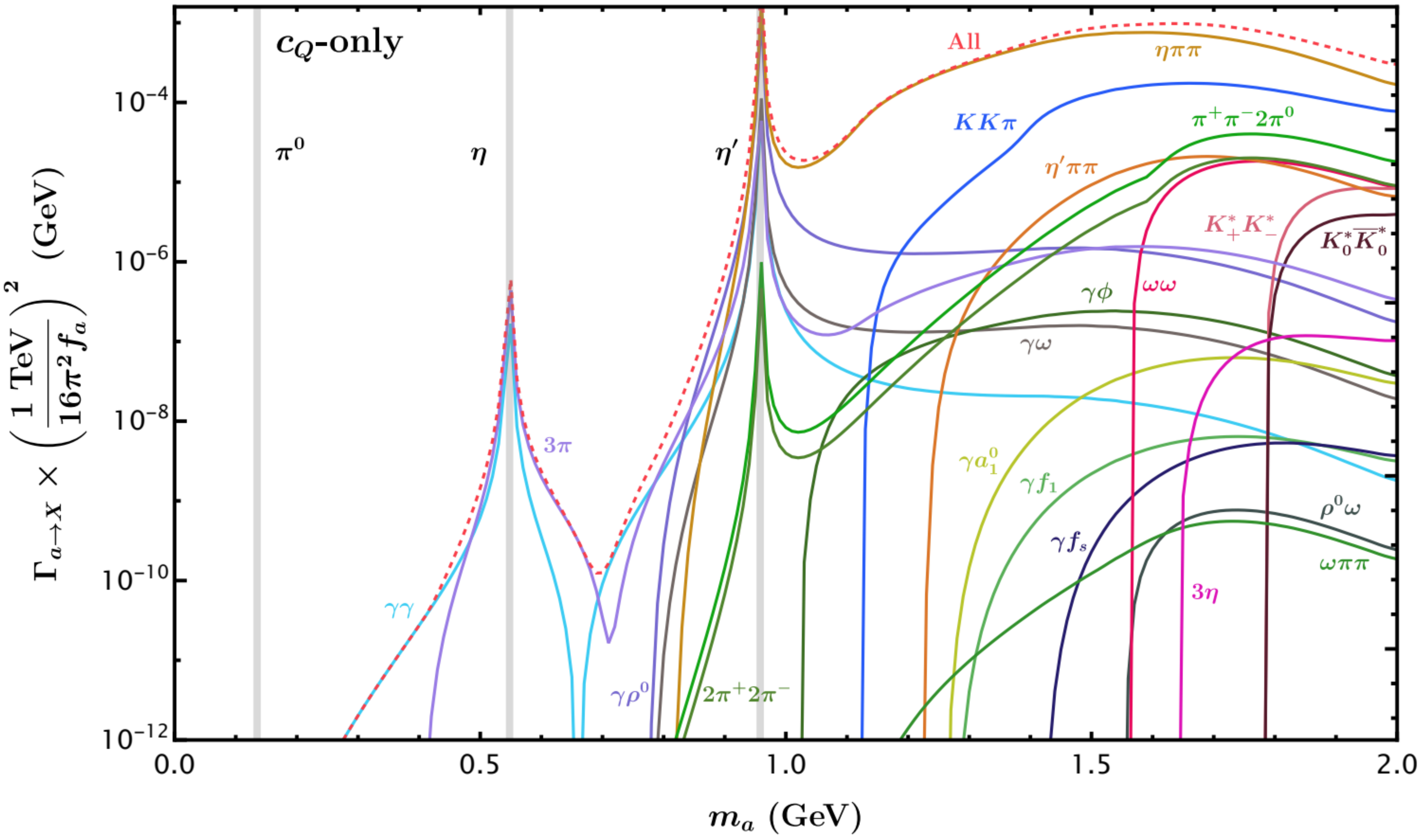
$\longrightarrow$

$$
c^{\mathrm{eff}}_{\omega\gamma} = e g' \left\{ \frac{-c_{gg}}{8\pi^2 f} - \frac{3}{8\pi^2 f} \left[\frac{m_a^2}{m_\pi^2 - m_a^2} \left(\frac{c_u - c_d}{2} \right) + c_{gg} \frac{m_u - m_d}{m_u + m_d} \frac{m_\pi^2}{m_a^2 - m_\pi^2} \right] + \frac{1}{16\pi^2 f} (c_d + c_Q - 2c_u) \right\}
$$

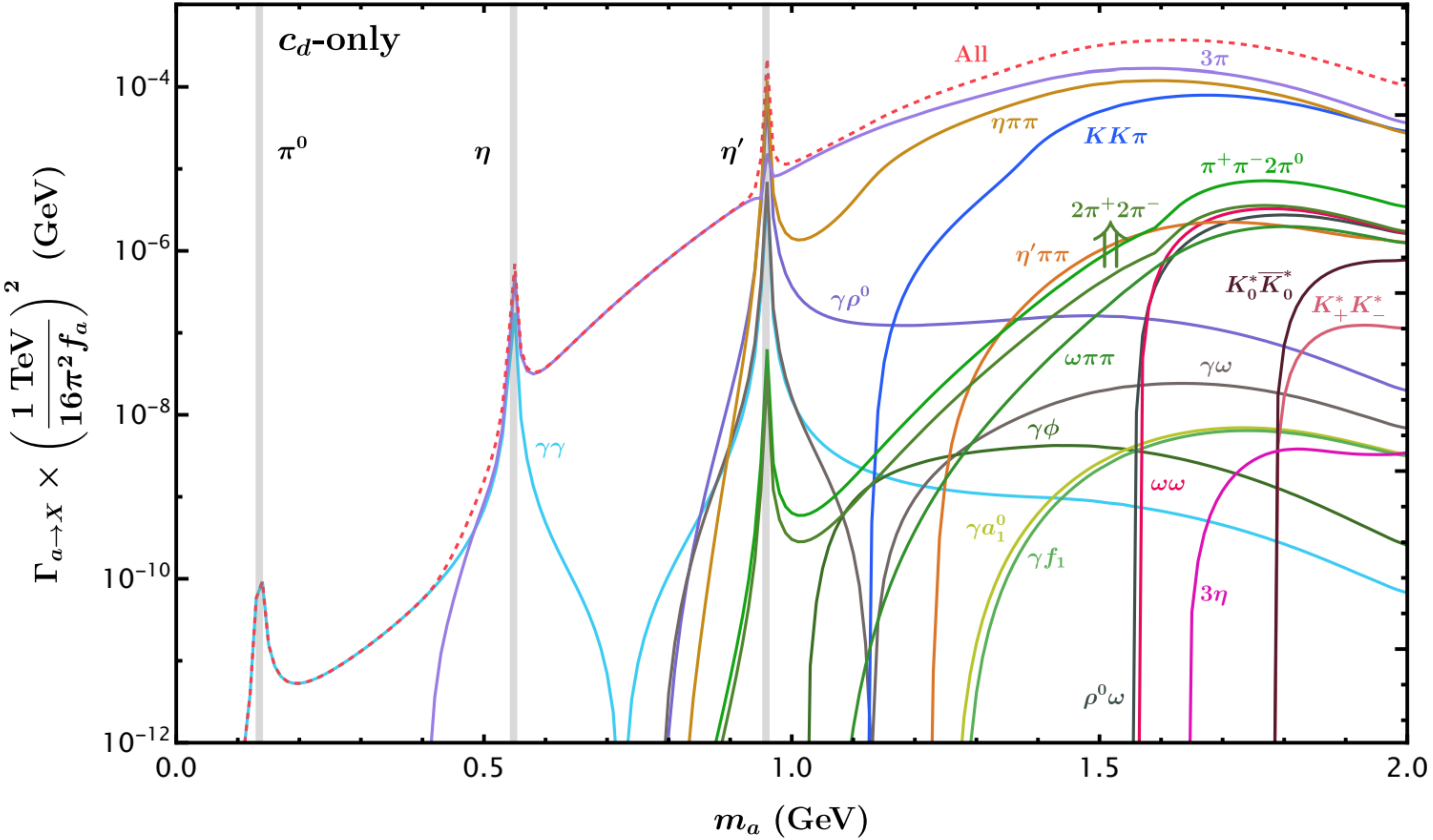
Utilities 1: calculating decay width



Utilities 1: calculating decay width



Utilities 1: calculating decay width

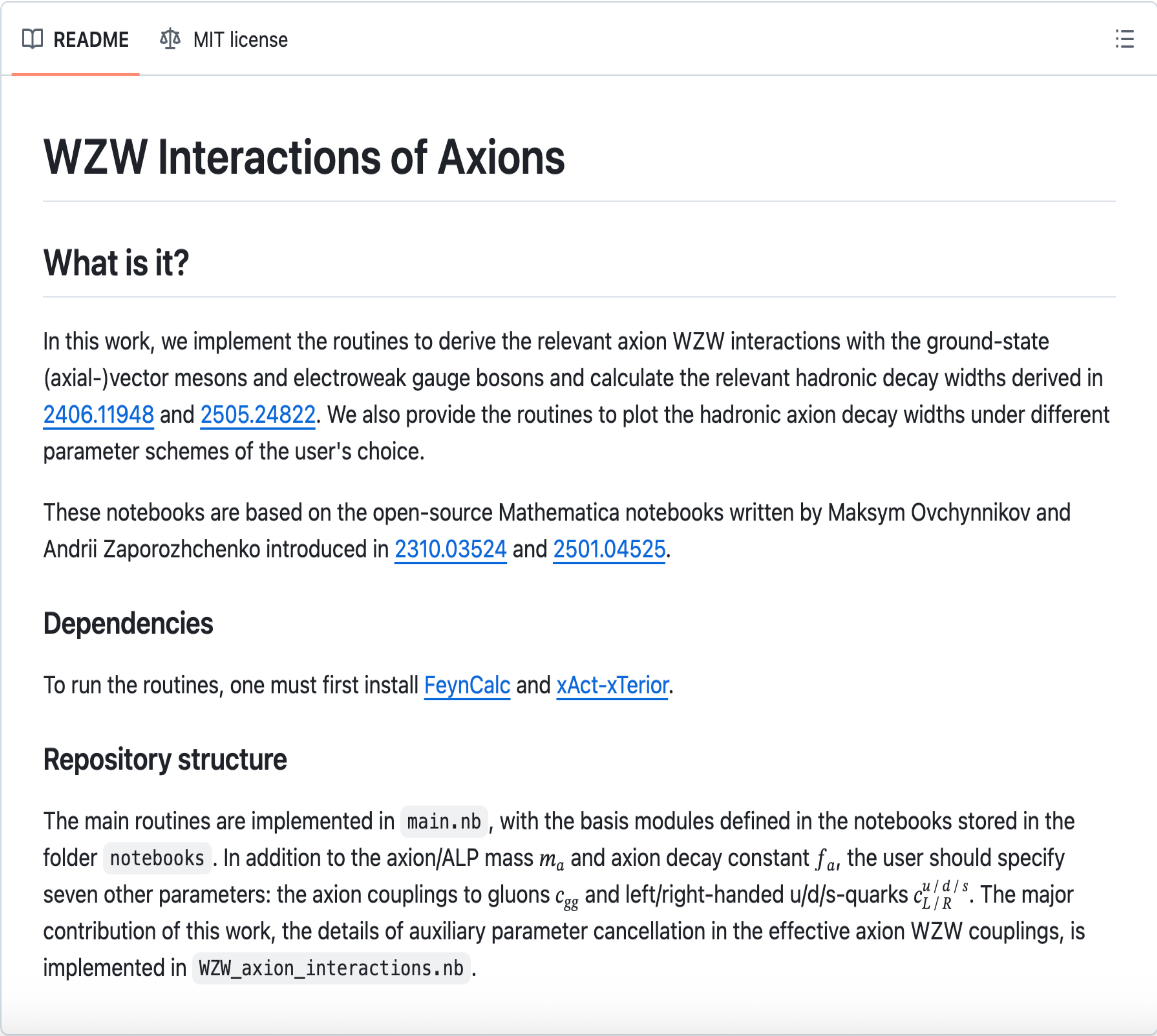


Utilities 1: calculating decay width

$$\mathscr{L}_{\text{eff},0} = \bar{q}_0(iD_\mu\gamma^\mu - \mathbf{m}_{q,0})q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+g_{ag,0}\frac{a}{f_a}G\tilde{G} + g_{a\gamma,0}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_{L,0}\gamma^\mu q_L + \bar{q}_R\mathbf{k}_{R,0}\gamma^\mu q_R + \dots\right)$$

https://github.com/nun3366/Axion-WZW-3



WZW Interactions of Axions

What is it?

In this work, we implement the routines to derive the relevant axion WZW interactions with the ground-state (axial-)vector mesons and electroweak gauge bosons and calculate the relevant hadronic decay widths derived in 2406.11948 and 2505.24822. We also provide the routines to plot the hadronic axion decay widths under different parameter schemes of the user's choice.

These notebooks are based on the open-source Mathematica notebooks written by Maksym Ovchynnikov and Andrii Zaporozhchenko introduced in 2310.03524 and 2501.04525.

Dependencies

To run the routines, one must first install FeynCalc and xAct-xTerior.

Repository structure

The main routines are implemented in `main.nb`, with the basis modules defined in the notebooks stored in the folder `notebooks`. In addition to the axion/ALP mass m_a and axion decay constant f_a, the user should specify seven other parameters: the axion couplings to gluons c_{gg} and left/right-handed u/d/s-quarks $c_{L/R}^{u/d/s}$. The major contribution of this work, the details of auxiliary parameter cancellation in the effective axion WZW couplings, is implemented in `WZW_axion_interactions.nb`.

Utilities 2: explain other's HLS formalism

- A weird "shift pion" method in Hidden Local Symmetry Lagrangian

$$\Sigma = \exp[2i\mathcal{P}/f_\pi]$$

P = $$\Phi = \begin{pmatrix} \pi_0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi_0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\,\overline{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$

$$\exp\left[-ic_G\kappa_q\frac{a}{f_a}\right]\cdot\Sigma\cdot\exp\left[-ic_G\kappa_q\frac{a}{f_a}\right]$$

Baker-Campbell-Hausdorff formula

$$\to \exp\left[i\frac{2}{f_\pi}\left(\mathcal{P}(x) - \epsilon c_G\kappa_q a\right) + \mathcal{O}(\epsilon^2)\right] \quad (5)$$

To maintain the κ_q invariance, we have to replace (note the sign change)

$$\mathcal{P} \to \mathcal{P} + \epsilon c_G\kappa_q a \quad (6)$$

Why?

Maksym Ovchynnikov, Andrii Zaporozhchenko, 2501.04525

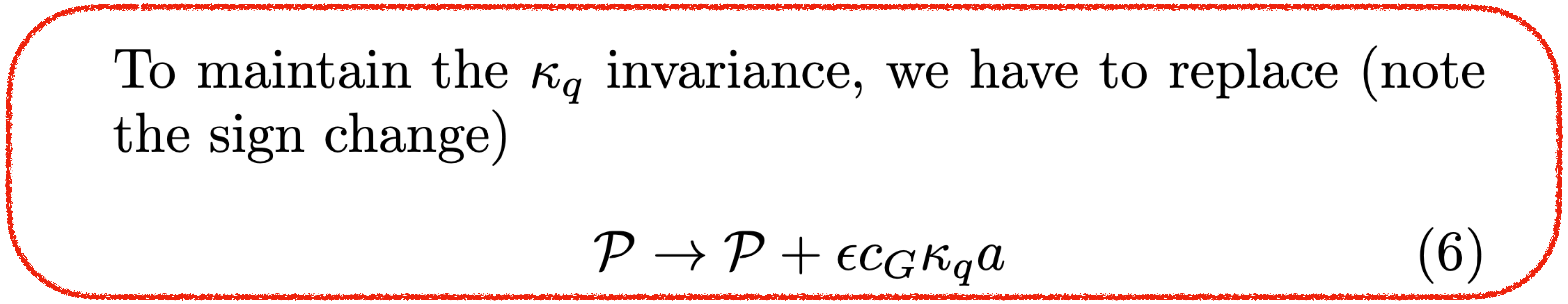
Utilities 2: explain other's HLS formalism

- Vector meson interaction from HLS

$$\mathcal{L}_{\text{vec+an}} = -\frac{3g^2}{8\pi^2 f_\pi}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}[\mathcal{P}(x)\partial_\mu V_\nu(x)\partial_\alpha V_\beta(x)] + \frac{7}{60\pi^2 f_\pi^5}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}[\mathcal{P}(x)\partial_\mu\mathcal{P}\partial_\nu\mathcal{P}\partial_\alpha\mathcal{P}\partial_\beta\mathcal{P}]$$

$$+ 2f_\pi^2\mathrm{Tr}\left|gV_\mu - eA_\mu Q - \frac{i}{2f_\pi^2}[\mathcal{P},\partial_\mu\mathcal{P}]\right|^2$$

- With the replacement



To maintain the κ_q invariance, we have to replace (note the sign change)

$$\mathcal{P} \to \mathcal{P} + \epsilon c_G \kappa_q a \qquad (6)$$

Why?

Maksym Ovchynnikov, Andrii Zaporozhchenko, 2501.04525

Utilities 2: explain other's HLS formalism

q basis

$$\mathscr{L}_{\text{eff}}\left(q, \mathscr{A}_{L/R}\middle|\boldsymbol{m}_q, \boldsymbol{k}_{L/R}, c_{gg}\right)$$

$$\supset c_{gg}\frac{\alpha_s}{4\pi}\frac{a}{f_a}G\tilde{G}$$

$$q \to \exp\left[-i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q\gamma_5\right)\frac{c_{gg}a}{f_a}\right]q$$

$$\left\{\mathscr{L}_{\text{eff}}\left(q, \mathscr{A}_{L/R}\middle|\boldsymbol{m}'_q, \boldsymbol{k}'_{L/R}, c'_{gg}\right) + \delta\mathscr{L}_a^{\text{ano}}\right\}$$

$$\supset c_{gg}\left[1 - \text{Tr}(\boldsymbol{\kappa}_q)\right]\frac{\alpha_s}{4\pi}\frac{a}{f_a}G\tilde{G}$$

q' basis

matching

matching

$$U \to U_L U U_R^\dagger, \; U_{L/R} = e^{-ic_{gg}\boldsymbol{\theta}_{L/R}\frac{a}{f_a}}, \; \mathbb{A}_{L/R} \to \mathbb{A}_{L/R}, \; \mathbb{B}_{L/R} \to \mathbb{B}_{L/R} + c_{gg}\boldsymbol{\theta}_{L/R}\frac{da}{f_a},$$

$$\begin{aligned}&\Gamma_{\text{WZW}}\left(U, \mathcal{A}_L, \mathcal{A}_R\right) + \Gamma_c\left(\mathbb{A}_{L/R}, \mathbb{B}_{L/R}\right)\\ &\quad\to \Gamma_{\text{WZW}}\left(U, \mathcal{A}_L, \mathcal{A}_R\right) + \Gamma_c\left(\mathbb{A}_{L/R}, \mathbb{B}_{L/R}\right) - \delta\left[\Gamma_{\text{WZW}} + \Gamma_c\right]\left(c_{gg}\boldsymbol{\theta}_L, c_{gg}\boldsymbol{\theta}_R\right)\\ &\quad\equiv \Gamma_{\text{WZW}}\left(U, \mathcal{A}_L, \mathcal{A}_R\right) + \Gamma_c\left(\mathbb{A}_{L/R}, \mathbb{B}_{L/R}\right) + \int d^4x\, \delta\mathcal{L}_{\text{WZW}}^{\text{ano}},\end{aligned}$$

ChPT Lag in q > q' basis

+

HLS Lag in q > q' ->(use formula on the left to go back) q basis

Ovchynnikov, Andrii Zaporozhchenko, 2501.04525

Utilities 2: explain other’s HLS formalism

q basis

$$\mathscr{L}_{\rm eff}\left(q,\mathscr{A}_{L/R}\middle|\boldsymbol{m}_q,\boldsymbol{k}_{L/R},c_{gg}\right)$$

$$\supset c_{gg}\frac{\alpha_s}{4\pi}\frac{a}{f_a}G\tilde{G}$$

$$q\to\exp\left[-i\left(\boldsymbol{\delta}_q+\boldsymbol{\kappa}_q\gamma_5\right)\frac{c_{gg}a}{f_a}\right]q$$

$$\left\{\mathscr{L}_{\rm eff}\left(q,\mathscr{A}_{L/R}\middle|\boldsymbol{m}'_q,\boldsymbol{k}'_{L/R},c'_{gg}\right)+\delta\mathscr{L}_a^{\rm ano}\right\}$$

$$\supset c_{gg}\left[1-\mathrm{Tr}(\boldsymbol{\kappa_q})\right]\frac{\alpha_s}{4\pi}\frac{a}{f_a}G\tilde{G}$$

q’ basis

matching

- A simple form for gluon interactions, but the origin of minus sign not explained
- HLS with only 4 counter terms (max 11 terms)
- Work with vector mesons

matching

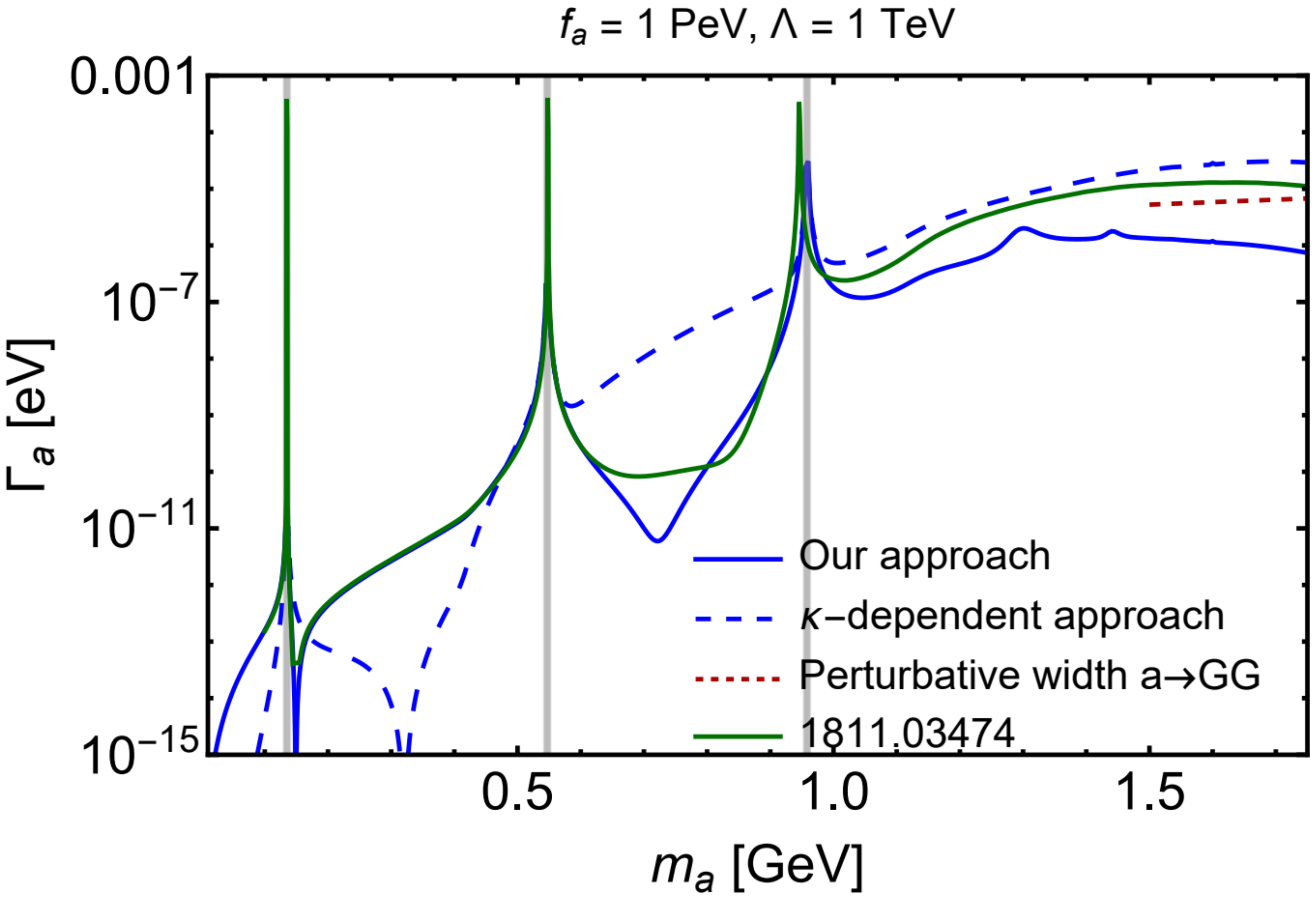
ChPT Lagrangian in q’ basis

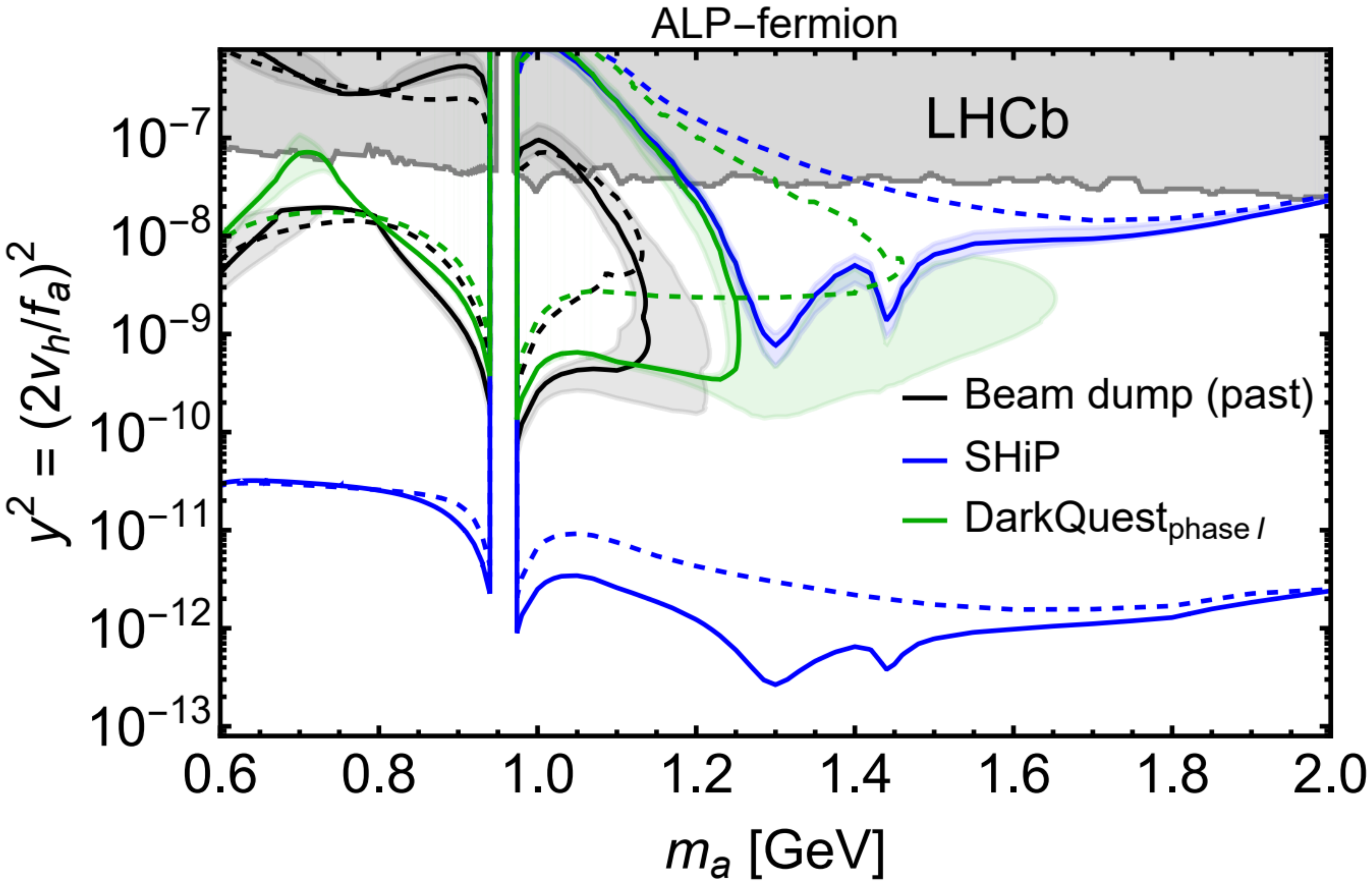
+

HLS Lagrangian in q basis

Maksym Ovchynnikov, Andrii Zaporozhchenko, 2501.04525

Utilities 3: change the interpretation of ALP exp

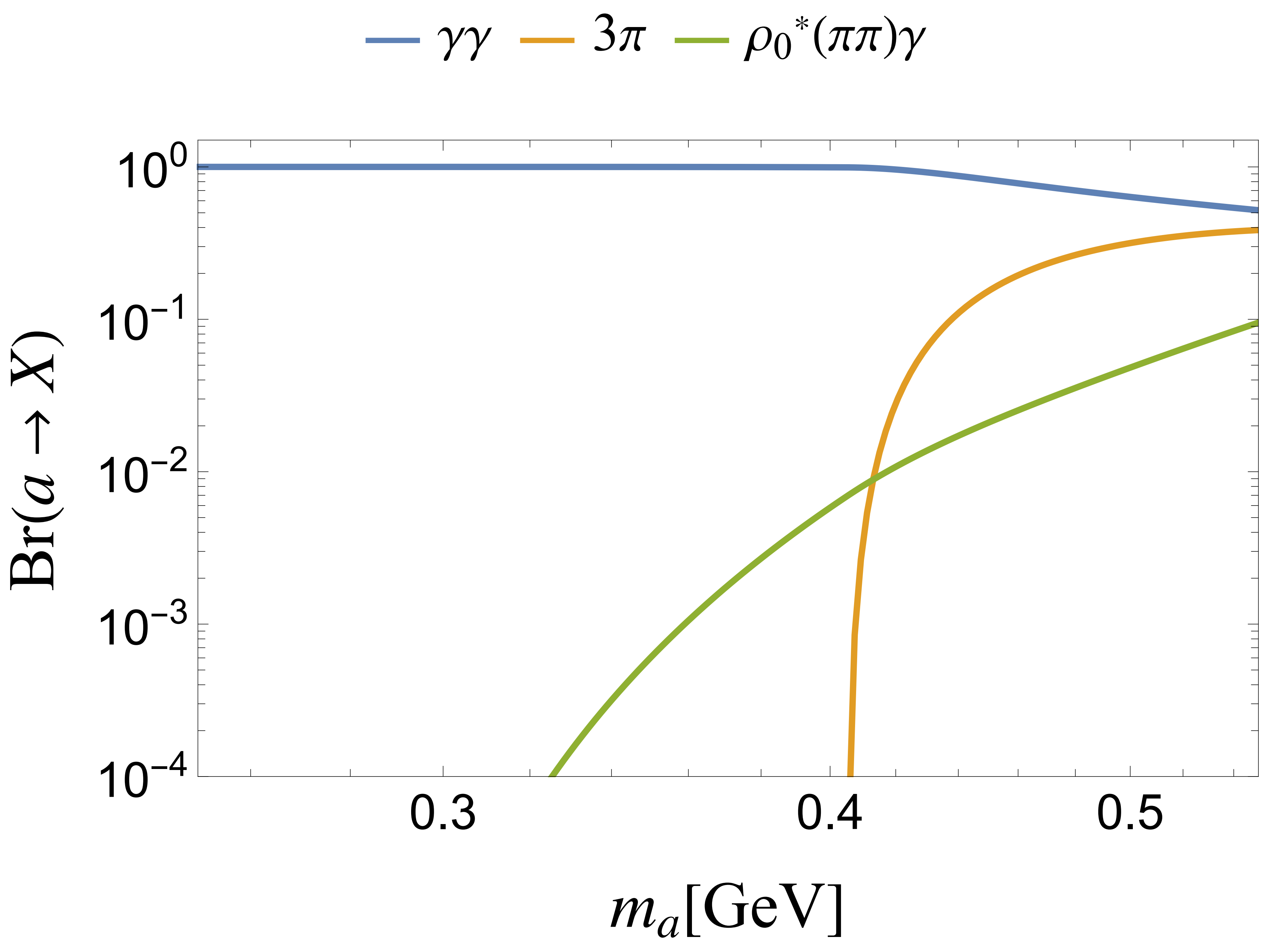




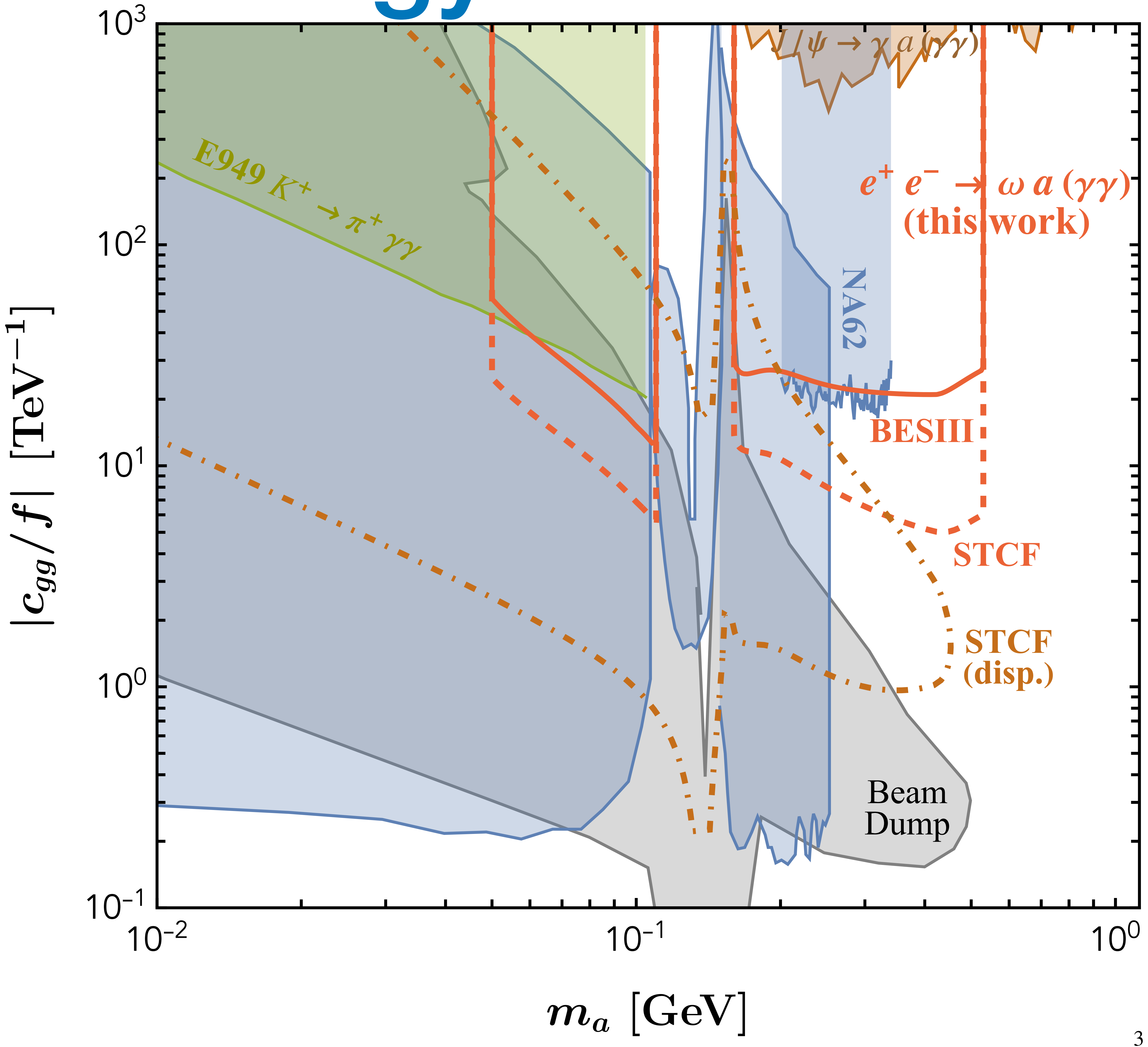
- Updated consistent axion partial widths changes exp results

Maksym Ovchynnikov, Andrii Zaporozhchenko, 2501.04525

Utilities 4: searches at low energy ee collider



- Production $e^+e^- \to \gamma^*(J/\Psi) \to \omega a$
- Prompt decay: $a \to \gamma\gamma$
- Displaced decay of a



$$\frac{\mathrm{BR}(J/\psi \to \omega a)}{\mathrm{BR}(J/\psi \to ee)} = \frac{m_{J/\psi}^2}{32\pi\alpha}\left|c^{\mathrm{eff}}_{\omega\gamma}(q^2 = m^2_{J/\psi})\right|^2 \left[\left(1 - \frac{(m_a + m_\omega)^2}{m^2_{J/\psi}}\right)\left(1 - \frac{(m_a - m_\omega)^2}{m^2_{J/\psi}}\right)\right]^{\frac{3}{2}}$$

Summary

- A full chiral axion Lagrangian for axion and pseudoscalar/vector mesons
- $\mathscr{L}^{\text{full}}_{\text{axion}} \equiv \left[\mathscr{L}_{\chi\text{PT}} + \mathscr{L}^{\text{full}}_{\text{WZW}}\right]\left(U, \mathbf{m}_q(a), \mathscr{A}_{L/R} + \mathbf{k}_{L/R}(a)da/f_a\right)$
  - 1. Wess-Zumino-Witten counter term is included for gauge invariance
  - 2. t'Hooft UV-IR anomaly matching is achieved
  - 3. Consistent physical amplitudes without auxiliary rotation parameters
- Three light quarks scheme: consistent treatment with η'
  - 4. Demonstrates two ways of resolving $aG\tilde{G}$ are consistent. Work for any ch-rotation.
- Important for Axion/ALP searches via mesons.
- All calculation machinery provided as Mathematica code in GitHub.

Thank you!