北京師範大學
BEIJING NORMAL UNIVERSITY

Quarkonium Energy Correlator for Heavy Quark Hadronization

Xiaohui Liu

The 2025 Beijing Particle Physics and Cosmology Symposium

ITP CAS, Beijing

Sep 25-29, 2025

Chen, et al., PRL. 133 (2024) 19, 2405.10056

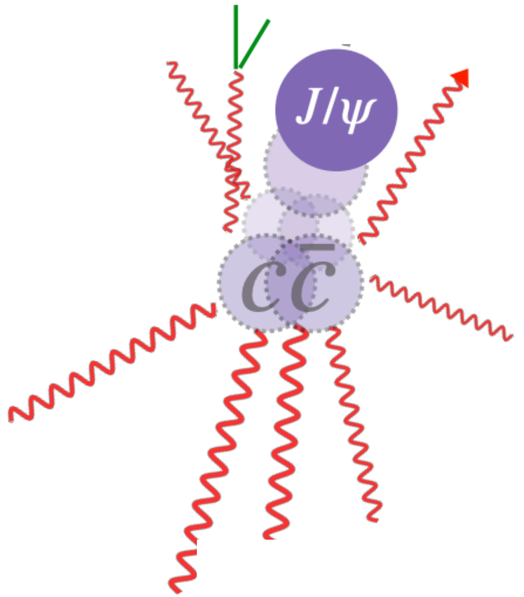
Outline

- The idea of the quarkonium energy correlator
- Connection to energy distribution during heavy quark hadronization
- Conclusion

Quarkonium Energy Correlator and hadronization

Quarkonium Physics

- Regarded as an excellent place to study QCD and non-pert phenomenon
- How $c\bar{c} \to J/\psi$?
  - NRQCD Bodwin et al., PRD 1995
    - Power counting in $v \ll 1$, e.g., $E_{g_s} \sim Mv$
    - Hadronization encoded in $\langle \mathcal{O}_{\mathbf{1}} \rangle$, $\langle \mathcal{O}_{\mathbf{8}} \rangle$
      - Fundamental properties remains largely unknown: amount of energy released? Energy Distribution?



Quarkonium Energy Correlator and hadronization

Quarkonium Physics

Probing Quarkonium Production Mechanisms with Jet Substructure

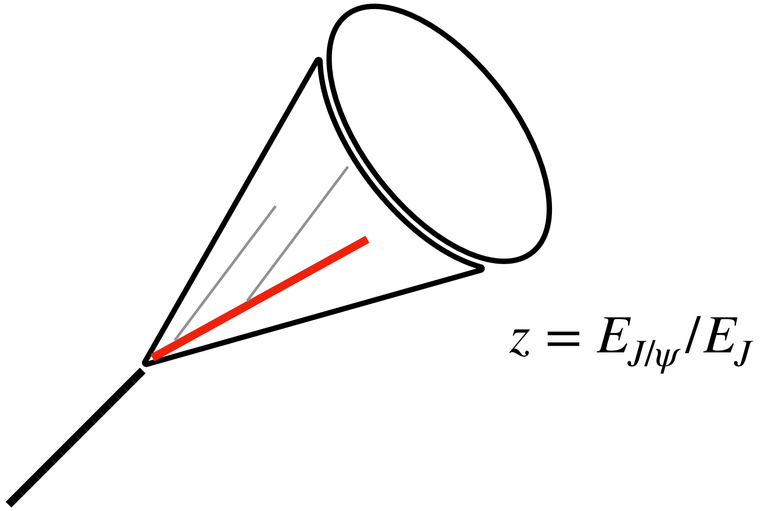
Matthew Baumgart[a],[1] Adam K. Leibovich[b],[2] Thomas Mehen[c],[3] and Ira Z. Rothstein[d1]

[1] *Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213*

[2] *Pittsburgh Particle Physics Astrophysics and Cosmology Center (PITT PACC) Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260*

[3] *Department of Physics, Duke University, Durham, NC 27708*

(Dated: June 27, 2018)



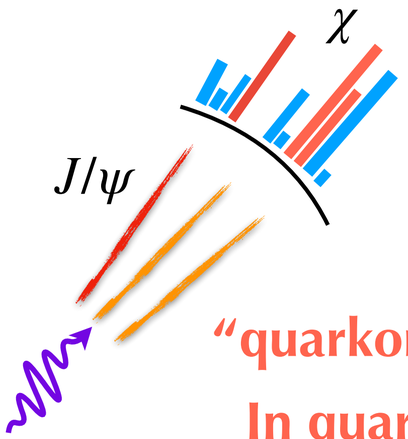
Unlike light hadron fragmentation, $D_{q\to J/\psi}(z)$ is dominated by perturbative radiation (parton shower) from E_J to $2m_c$

$$D_{q\to J/\psi}(z) \sim C_i(z)\langle \mathcal{O}_i \rangle$$

Perturbatively calculable

Quarkonium Energy Correlator and hadronization

Quarkonium Energy Correlator Chen et al., 2405.10056



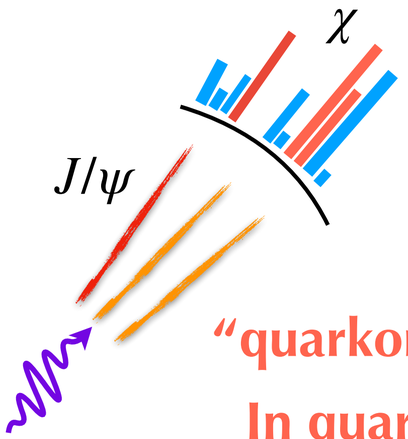
"quarkonim-Jet correlation"

In quarkonium rest frame

$$\Sigma_{\mathrm{QEC}}(\chi) = \frac{1}{\sigma_{J/\psi}} \int d\sigma_{J/\psi} \frac{E_i}{M} \delta(\chi - \chi_i)$$

Quarkonium Energy Correlator and hadronization

Quarkonium Energy Correlator Chen et al., 2405.10056



"quarkonim-Jet correlation"

In quarkonium rest frame

$$\Sigma_{\text{QEC}}(\chi) = \frac{1}{\sigma_{J/\psi}} \int d\sigma_{J/\psi} \frac{E_i}{M} \delta(\chi - \chi_i)$$

~ average energy at the angle χ

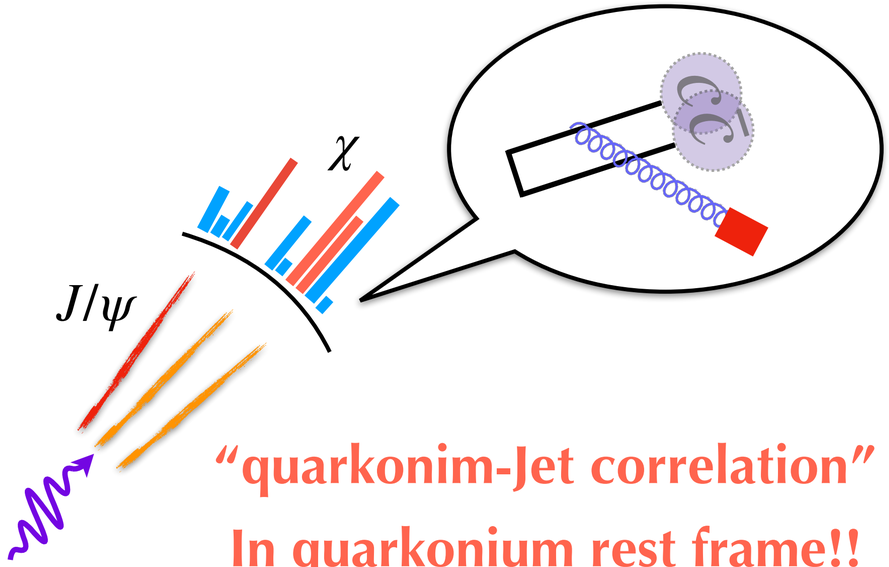
IRC safe, can be calculated perturbatively

Quarkonium Energy Correlator and hadronization

Quarkonium Energy Correlator

Chen et al., 2405.10056

- $\Sigma_{@EC} = \Sigma_{@EC,P.T.} + \Sigma_{@EC,had.}$
  - Hadronization enters as an additive correction, not in the form of convolution
  - $\Sigma_{@EC,had.}$ can be extracted from "Measured - pQCD", If $\Sigma_{@EC,had.}$ is not too small



"quarkonim-Jet correlation"

In quarkonium rest frame!!

$$\Sigma_{@\mathrm{EC}}(\chi) = \frac{1}{\sigma_{J/\psi}} \int d\sigma_{J/\psi} \frac{E_i}{M} \delta(\chi - \chi_i)$$

Quarkonium Energy Correlator and hadronization

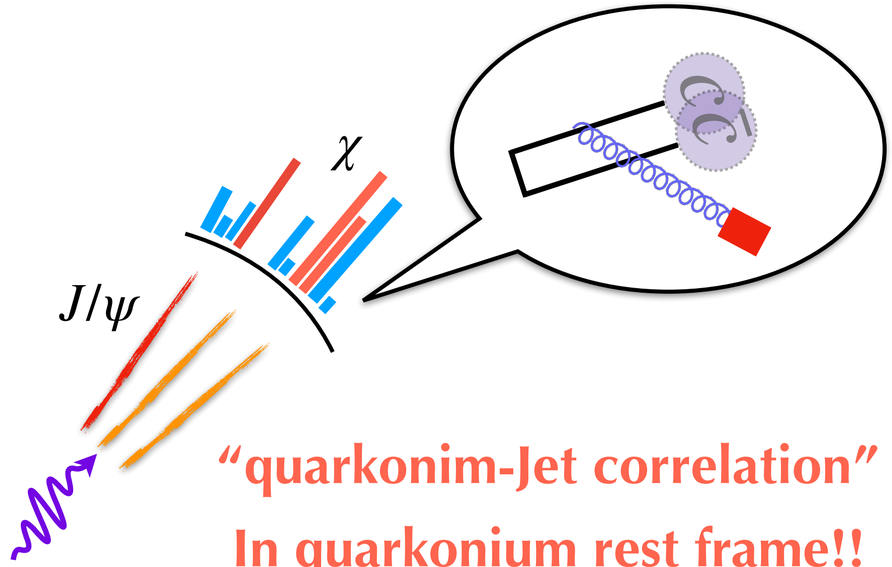
Quarkonium Energy Correlator

Chen et al., 2405.10056

- $\Sigma_{@EC} = \Sigma_{@EC,P.T.} + \Sigma_{@EC,had.}$
  - Hadronization enters as an additive correction, not in the form of convolution

$$\Sigma_{@EC,P.T.} \sim \alpha_s(M)\frac{E(\chi)}{M}E^2(\chi),$$

$$\Sigma_{@EC,had.} \sim 1 \times \frac{Mv}{M}M^2v^2$$



"quarkonim-Jet correlation"

In quarkonium rest frame!!

$$\Sigma_{@\mathrm{EC}}(\chi) = \frac{1}{\sigma_{J/\psi}}\int d\sigma_{J/\psi}\frac{E_i}{M}\delta(\chi-\chi_i)$$

$$\sim \int_{Mv}^{E_{\max}} \frac{E^2dE}{2E}\frac{E}{M}|\mathscr{M}|^2 + \int^{Mv} \frac{E^2dE}{2E}\frac{E}{M}|\mathscr{M}|^2$$

Quarkonium Energy Correlator and hadronization

Quarkonium Energy Correlator

Chen et al., 2405.10056

- $\Sigma_{@EC} = \Sigma_{@EC,P.T.} + \Sigma_{@EC,had.}$
  - Hadronization enters as an additive correction, not in the form of convolution

$$\Sigma_{@EC,P.T.} \sim \alpha_s(M)\frac{E(\chi)}{M}E^2(\chi),$$

$$\Sigma_{@EC,had.} \sim 1 \times \frac{Mv}{M}M^2v^2$$

for J/ψ $\alpha_s(M) \sim v^2, v \sim 0.5$

$$\Sigma_{@EC,had.}/\Sigma_{@EC,P.T.} \sim \frac{Mv}{\alpha_s E}\frac{M^2v^2}{E^2} \sim v \times \frac{v^2}{\alpha_s}\frac{M^3}{E(\chi)^3}$$

If $E(\chi)/M \sim 1$

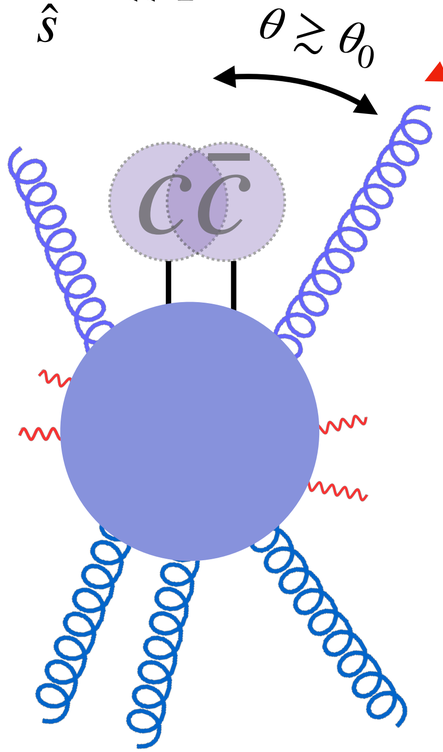
$$\Sigma_{@EC,had.}/\Sigma_{@EC,P.T.} \sim 50\%!$$

Hadronization contribution could be substantial

Quarkonium Energy Correlator and hadronization

Generic configuration of boost J/ψ production in pQCD

COM frame $r \equiv \frac{M^2}{\hat{s}} \ll 1$



$E_s \sim \mathcal{O}(M), E_{J_{near}} \sim E_{J_{away}} \sim \mathcal{O}\left(\frac{\sqrt{\hat{s}}}{2}\right)$

dead-cone effects Dokshitzer et al., J. Phys. G

$$d\sigma_{Q\to Qg} \sim \frac{\alpha_s C_F}{\pi}\frac{dE_g}{E_g}\frac{\theta^2 d\theta^2}{[\theta^2+\theta_0^2]^2}$$

$$\theta_0 \sim \frac{M}{E_{\bar{J}/\psi}} \sim \frac{2M}{\sqrt{\hat{s}}} = 2\sqrt{r}$$

Near-side radiations:

$E_J \sim \sqrt{\hat{s}}/2\,, p_t \sim M$

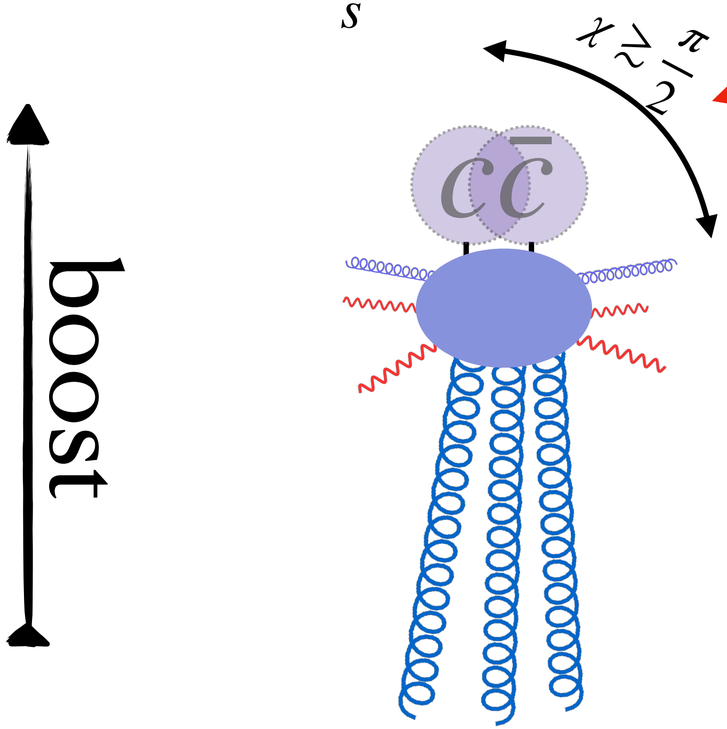
Chen et al., 2405.10056

Quarkonium Energy Correlator and hadronization

Generic configuration of boost J/ψ production in pQCD

J/ψ rest frame $r \equiv \frac{M^2}{\hat{s}} \ll 1$

Near side radiation will be depleted.



$E_{J_{near}} \sim E_s \sim \mathcal{O}(M), E_{J_{away}} \sim \mathcal{O}\left(\frac{\hat{s}}{M}\right)$

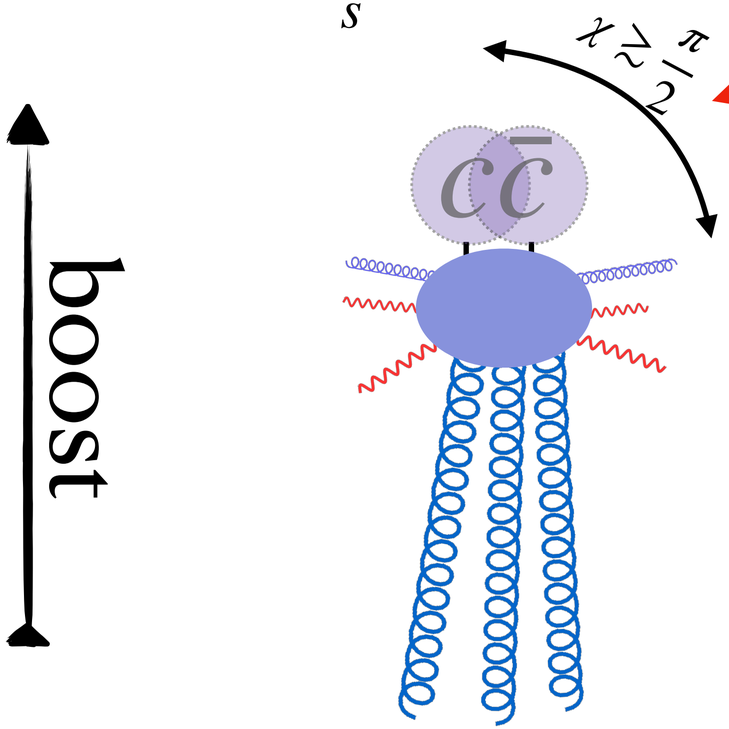
$E_{J_{away}}/E_{J_{near}} \sim \text{boost factor}^2 \sim r^{-1}$

Quarkonium Energy Correlator and hadronization

Generic configuration of boost J/ψ production in pQCD

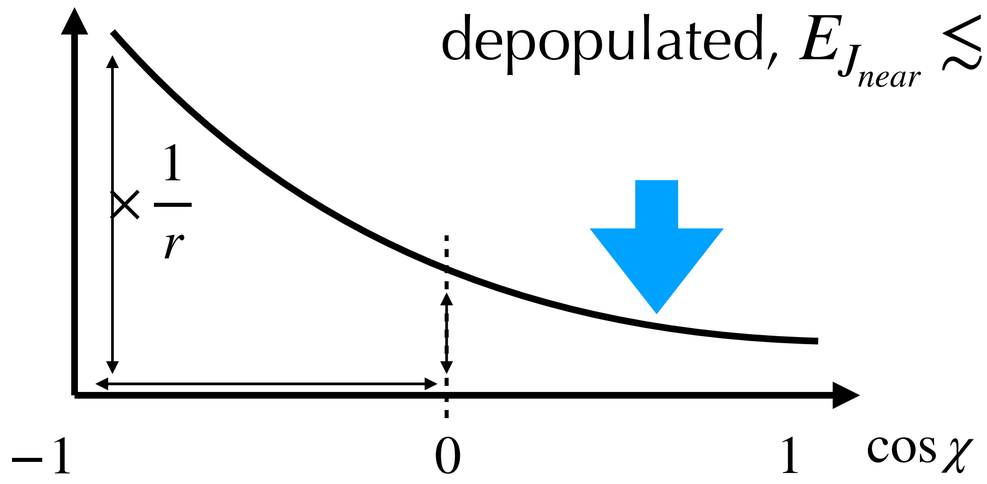
J/ψ rest frame $r \equiv \frac{M^2}{\hat{s}} \ll 1$

Near side radiation will be depleted.



pQCD:

hard radiation depopulated, $E_{J_{near}} \lesssim M$

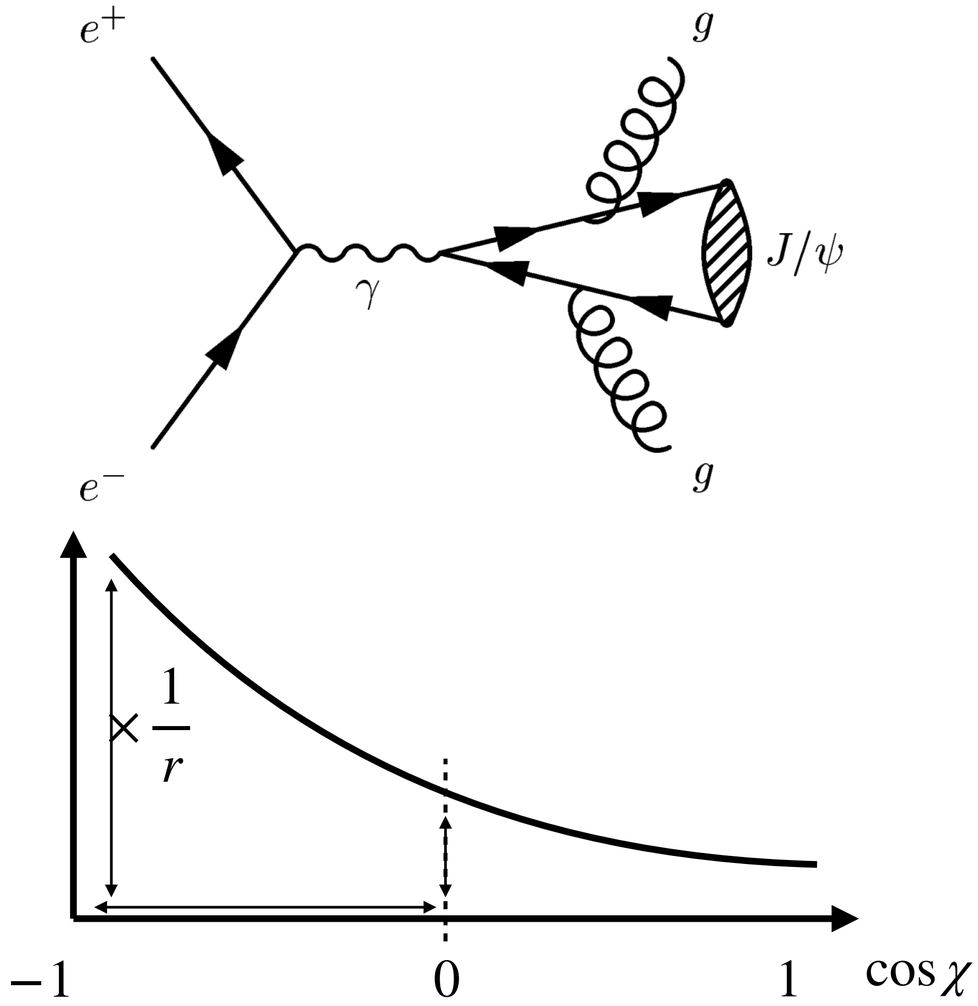


$E_{J_{near}} \sim E_s \sim \mathcal{O}(M), E_{J_{away}} \sim \mathcal{O}\left(\frac{\hat{s}}{M}\right)$

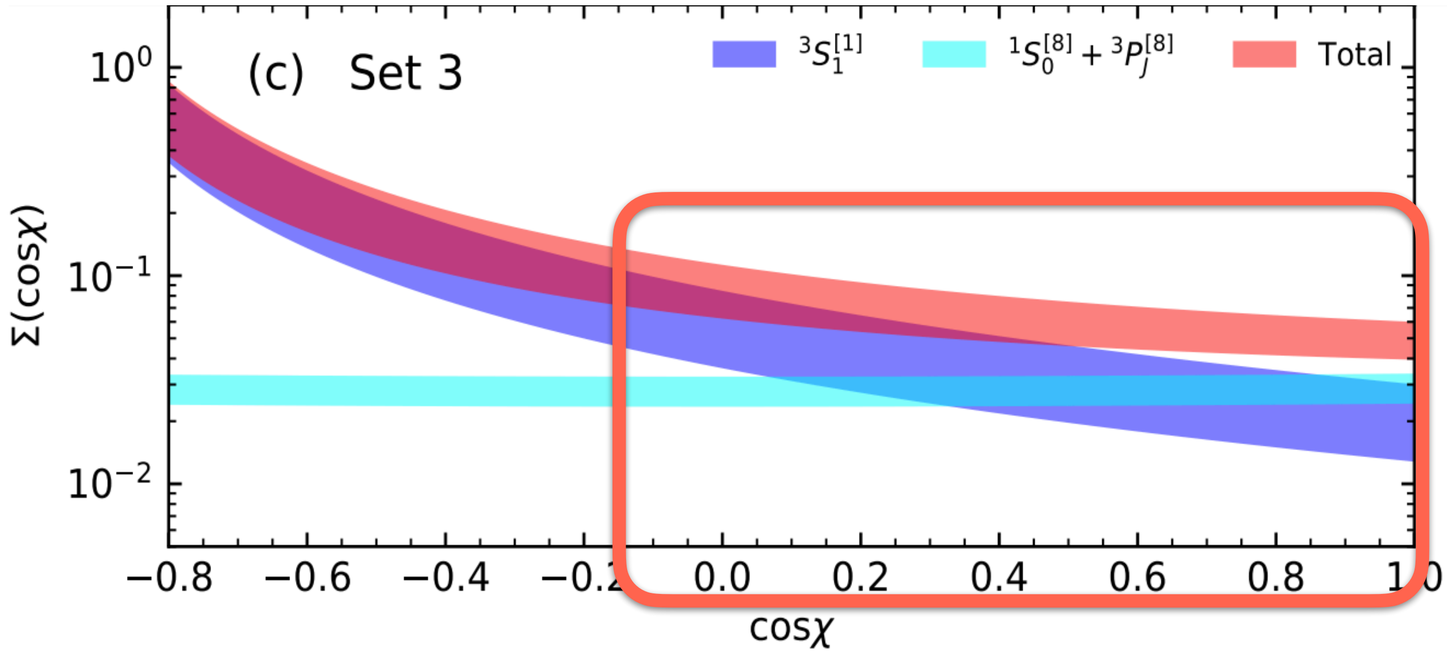
$E_{J_{away}}/E_{J_{near}} \sim \text{boost factor}^2 \sim r^{-1}$

Quarkonium Energy Correlator and hadronization

NRQCD Predictions:



$J/\psi@e^+e^-$ @ $\sqrt{s} = 10.6\,\mathrm{GeV}$



Sizable hadronization effect

"See" the hadronization energy distribution

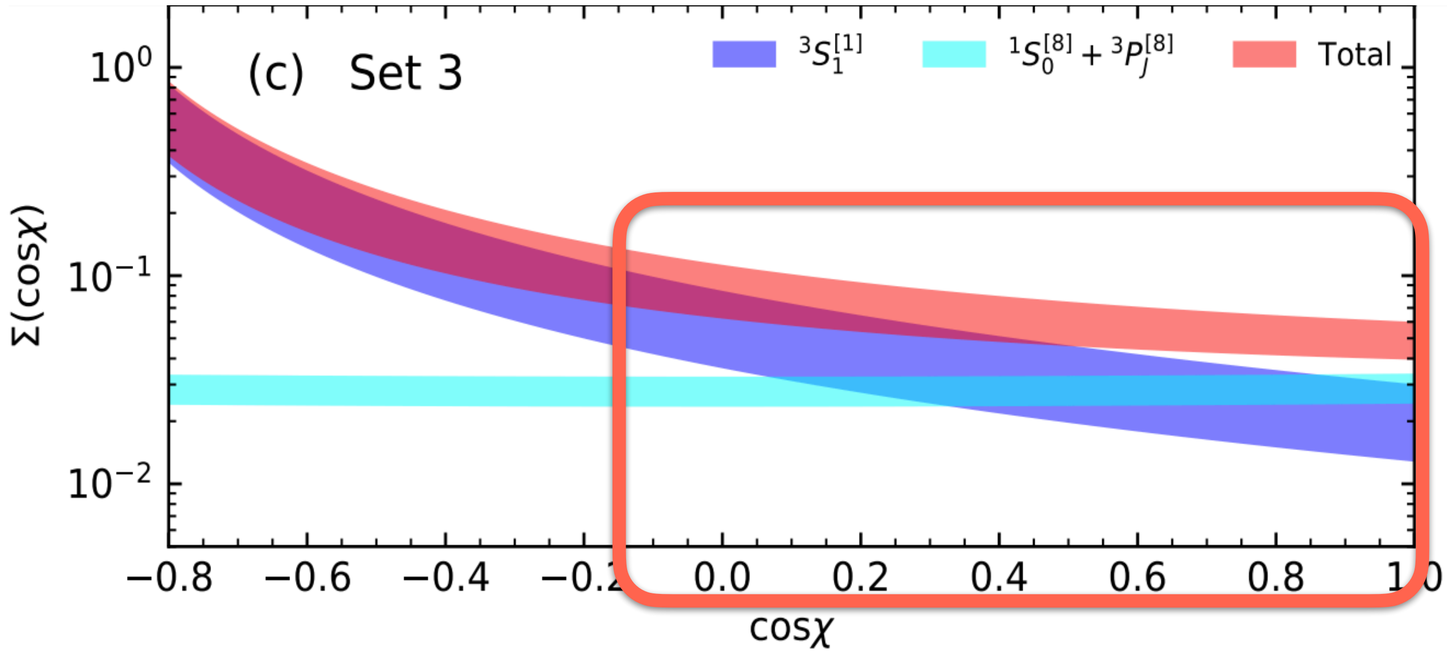
Chen et al., 2405.10056

Quarkonium Energy Correlator and hadronization

NP model:

- $c\bar{c} \to J/\psi + g_s$
- NRQCD matrix element scaling
- HF spin (rotational co-variance) symmetry

$J/\psi @ e^+e^-$ @ $\sqrt{s} = 10.6\,\mathrm{GeV}$



Sizable hadronization effect

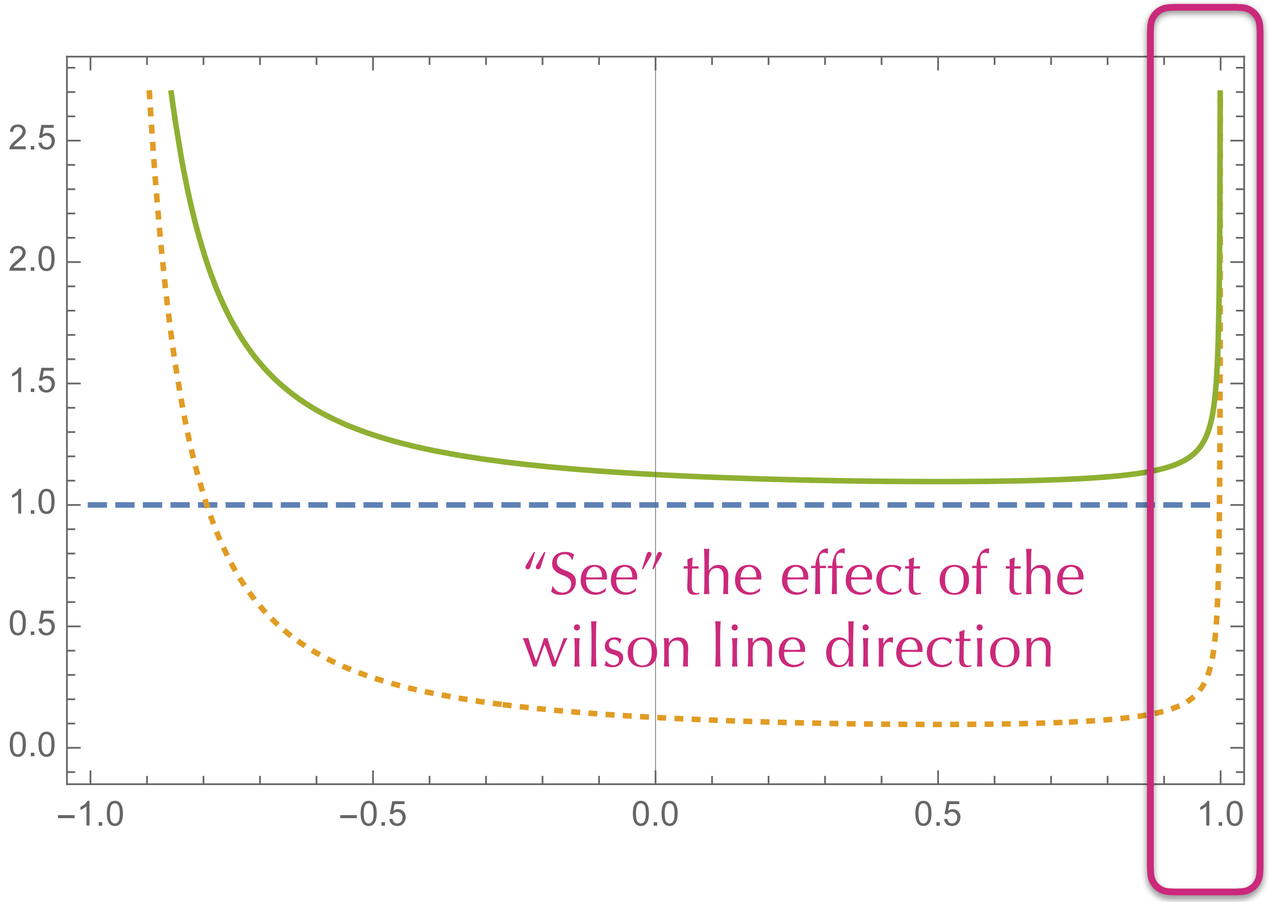
"See" the hadronization energy distribution

Chen et al., 2405.10056

Quarkonium Energy Correlator and hadronization

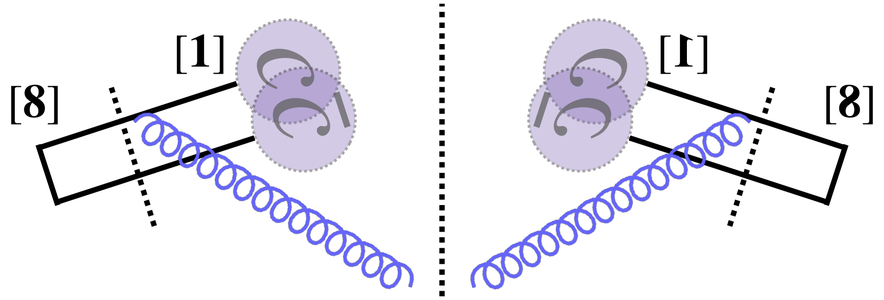
NP model:

Relative size between NRQCD vs interference

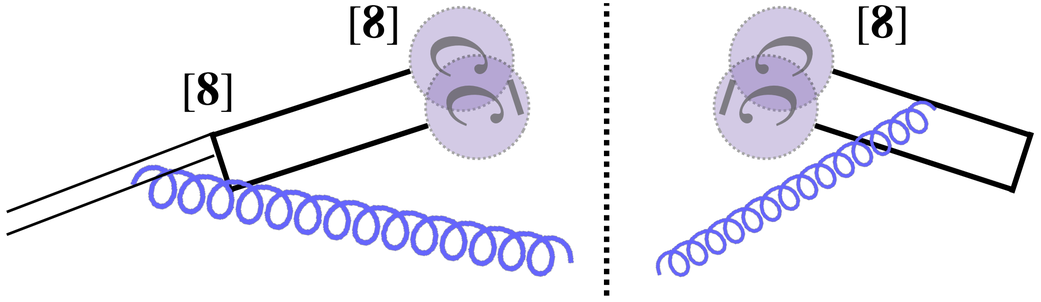


NRQCD: rotational covariant, E is "independent of the angle"

Nayak, Qiu, Sterman,0608066



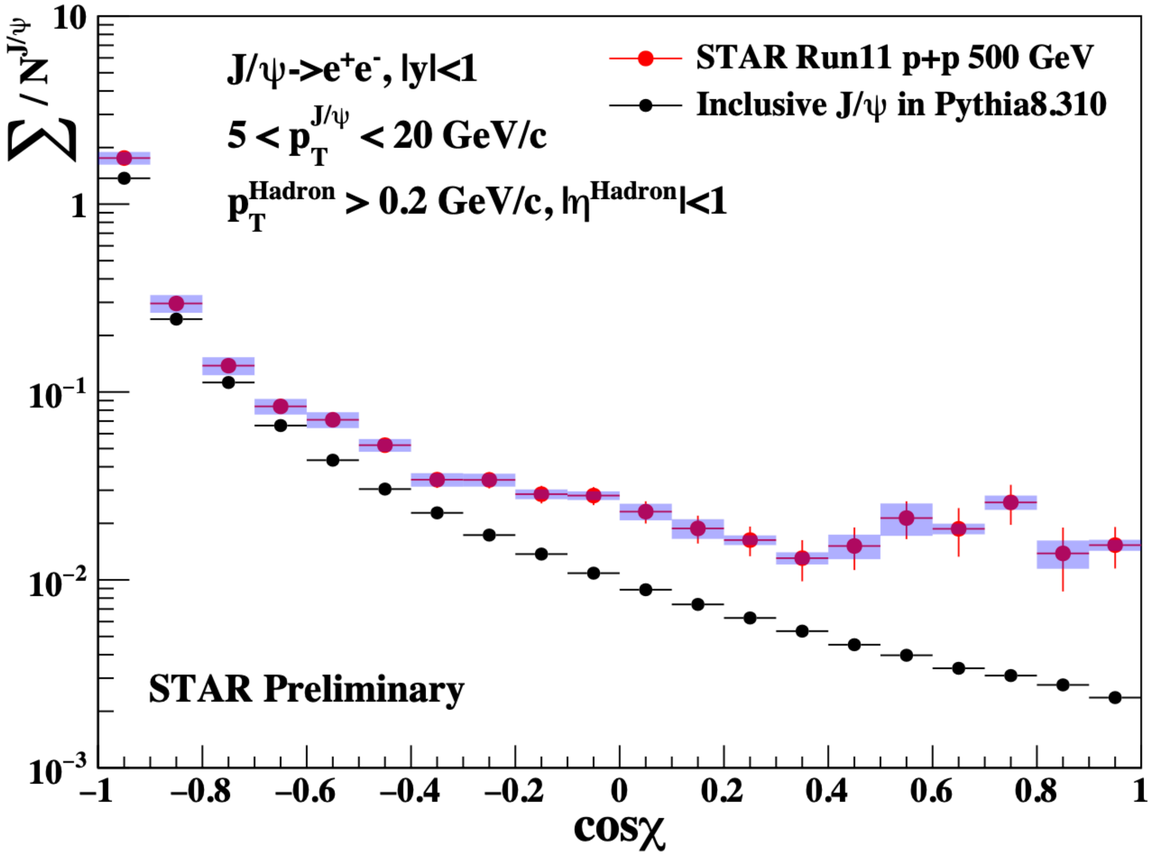
Interference: boost covariant, pt is independent of the angle



Wilson line

$$E \sim \frac{p_T}{\sin\chi} \Rightarrow \frac{1}{(1+\cos\chi)\sin\chi}$$

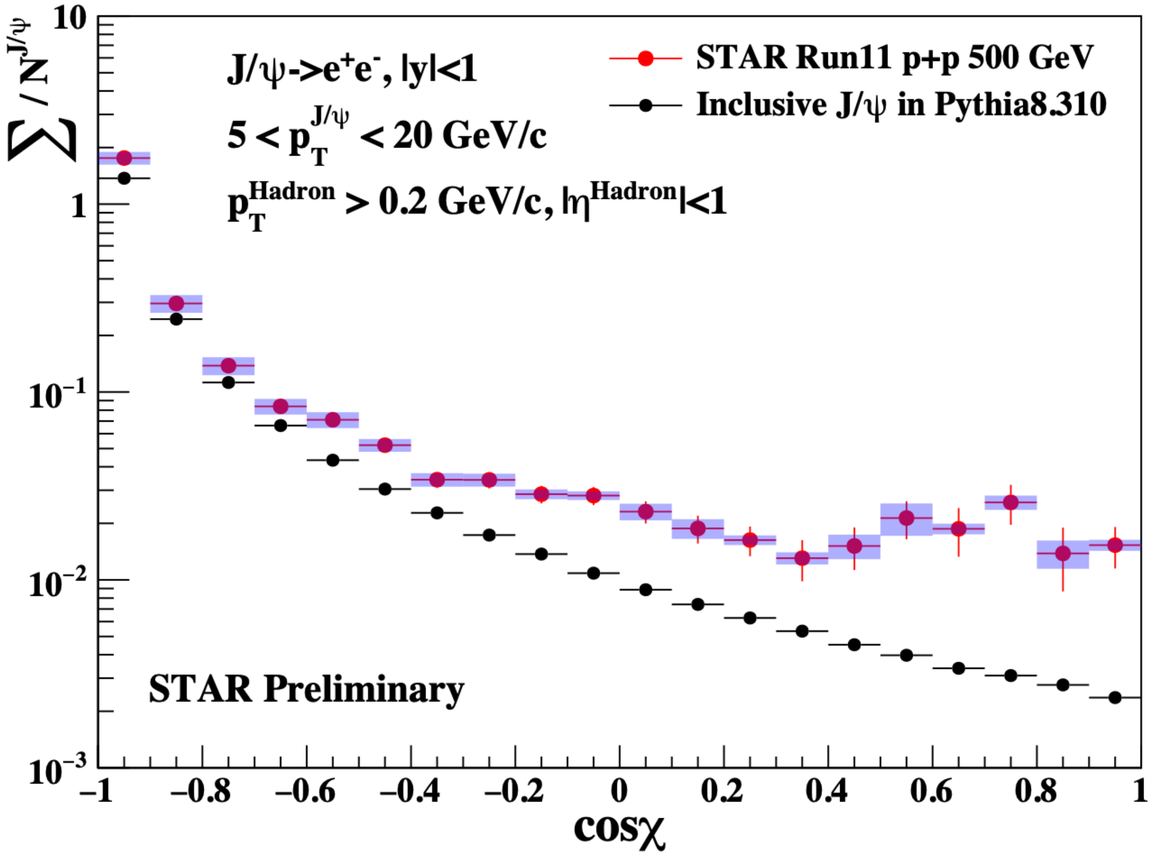
Quarkonium Energy Correlator and hadronization



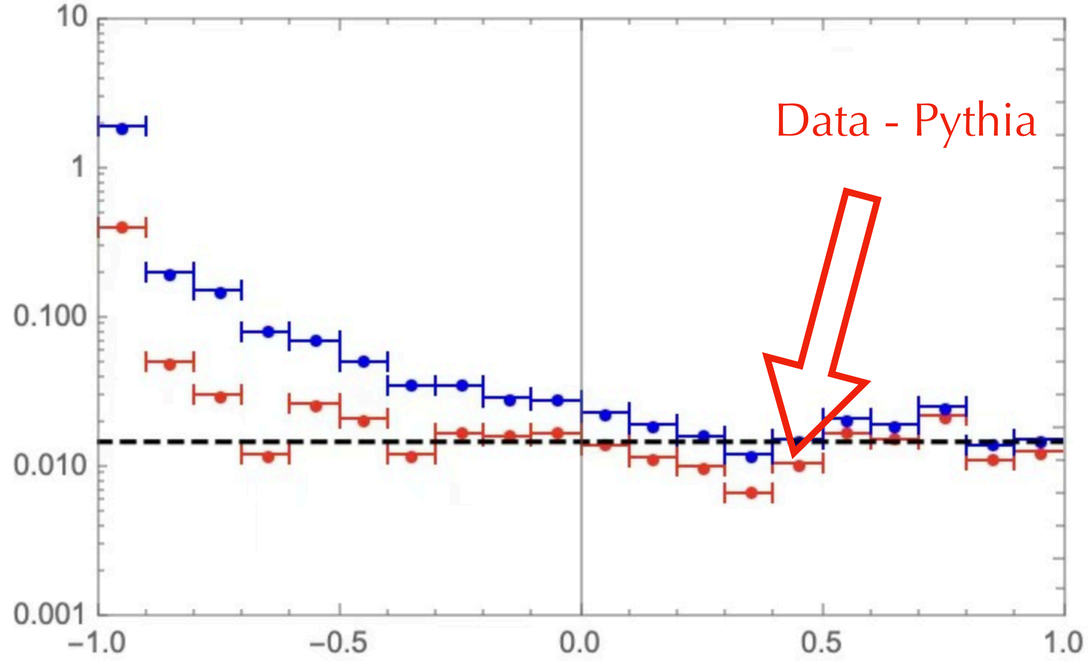
Dandan Shen's Poster @ QM 2025

- Pythia does not capture correctly the J/ψ EC when $\chi < \pi/2$ (radiations around J/ψ)
- Pythia shape consistent with pQCD expectation
  - Pythia models pQCD radiations reasonably well?

Quarkonium Energy Correlator and hadronization



Dandan Shen's Poster @ QM 2025



- Data - Pythia consistent with the NRQCD NP model
  - Have we seen the energy from hadronization? NEED precision pQCD INPUT!

Conclusion

- The Quarkonium Energy Correlator can provide us new insights into heavy quark hadronization
- High chance to identify the energy distribution during hadronization at current available facilities
- Calls for inputs from precision calculation

Thanks