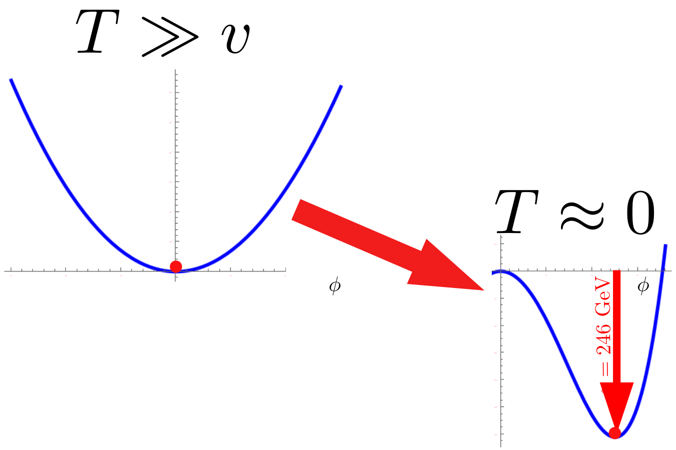
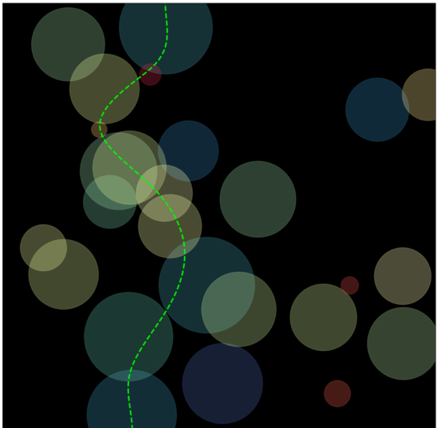


Hints of an Electroweak Phase Transitions?

Peter Athron
(Nanjing Normal University)



Beijing: BPCS2025

As the temperature cools down

the Universe may undergo

cosmological phase transitions

The electroweak phase transition
is a special example of a
cosmological phase transition

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- Connected to known physics - Higgs mechanism (2013 Nobel prize)

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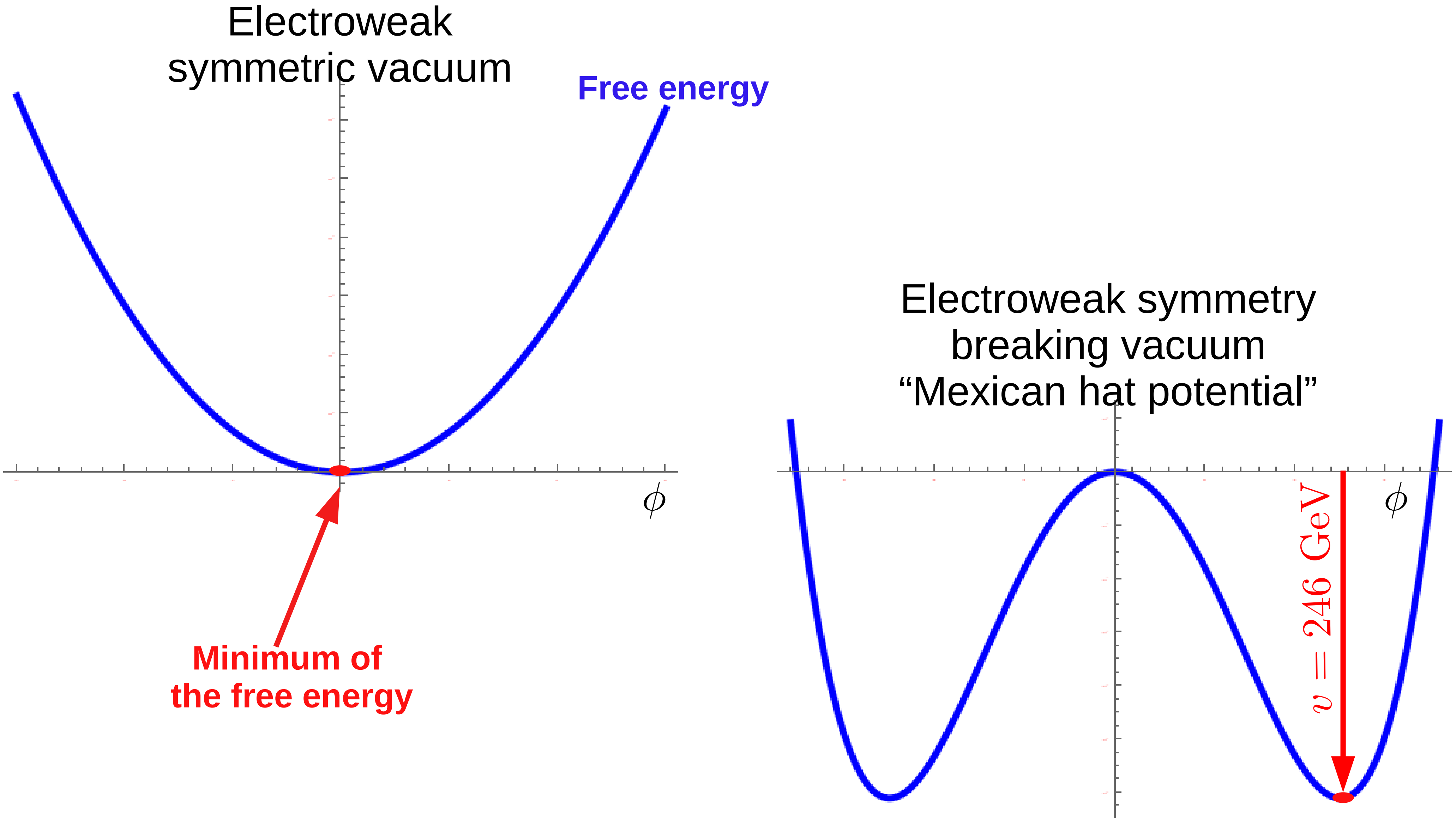
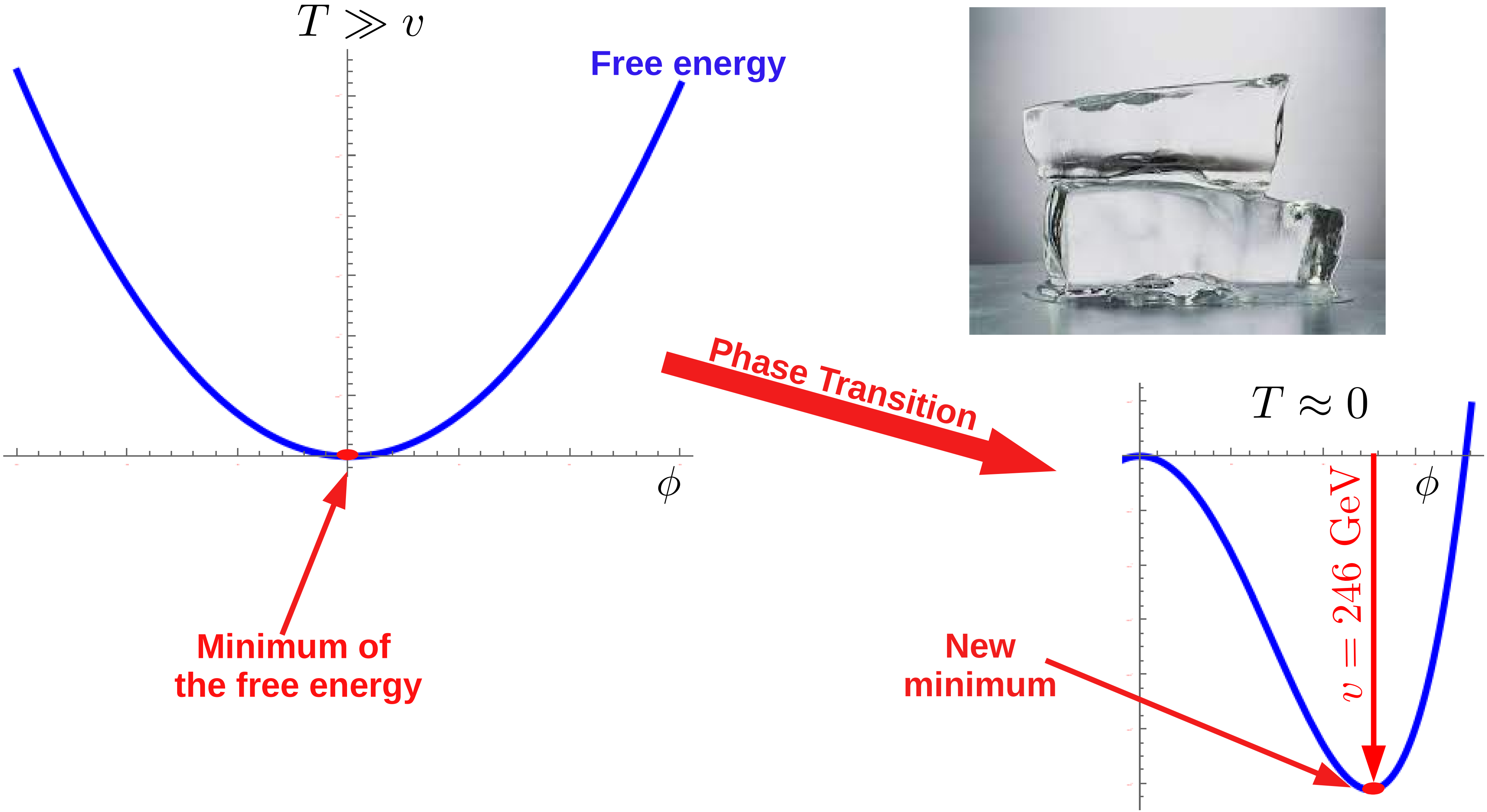
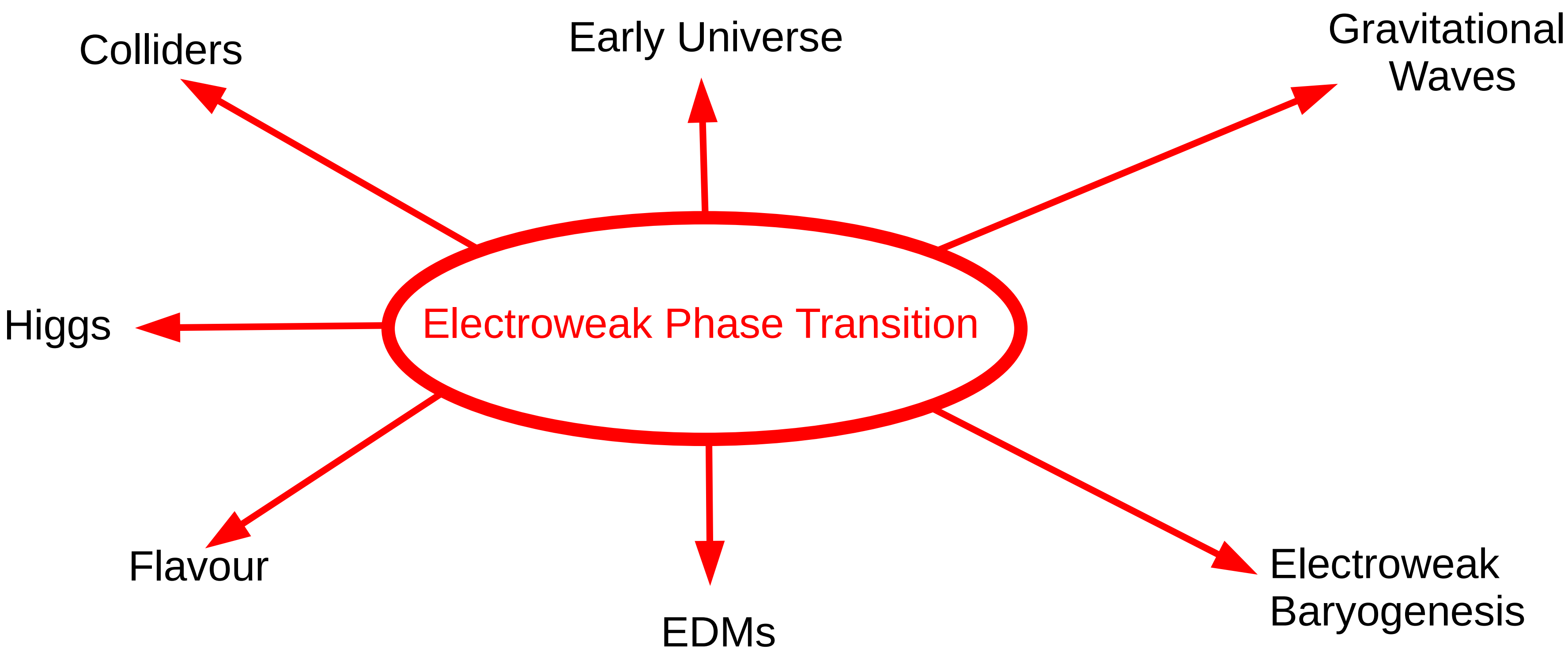
- Connected to known physics - Higgs mechanism (2013 Nobel prize)
- Dramatic impact: Fundamental particles massless $\longrightarrow$ massive
- Matter anti-matter asymmetry: If first order it may have an electroweak baryogenesis explanation of the observed baryon asymmetry
- Freedom: More freedom for modifying SM to make this a first order phase transition (c.f. QCD phase transitions)


Electroweak
symmetric vacuum
Free energy
ϕ
Minimum of
the free energy
Electroweak symmetry
breaking vacuum
“Mexican hat potential”
ϕ
v = 246 GeV

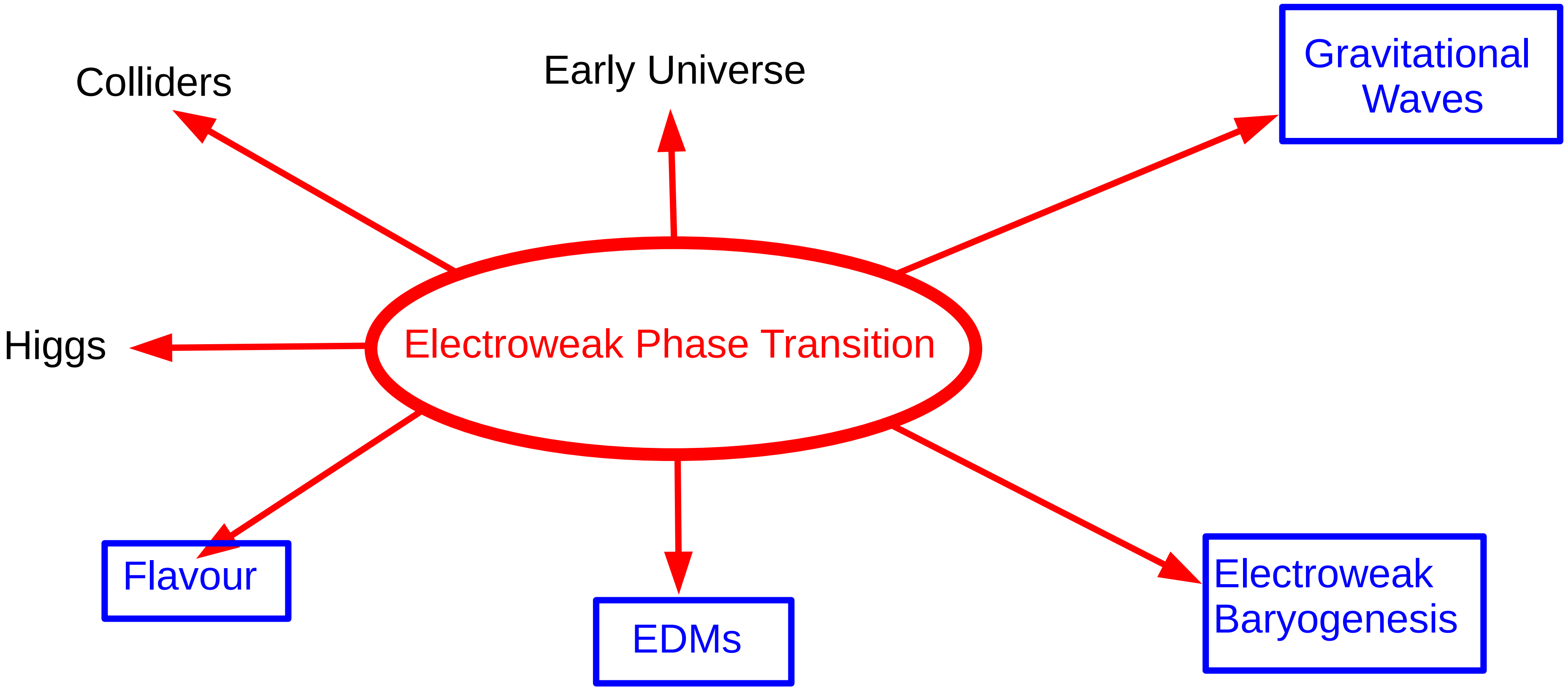


$T \gg v$
Free energy
ϕ
Minimum of
the free energy
Phase Transition
$T \approx 0$
ϕ
$v = 246$ GeV
New
minimum

The EWPT connects many different areas of physics



The things I will talk about here are most related to these



PTA anomaly:

A stochastic gravitational wave background has been observed

by multiple Pulsar Timing Arrays experiments

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Magazine | Latest | People | Impact |

ASTRONOMY AND SPACE | RESEARCH UPDATE

Pulsar timing irregularities reveals hidden gravitational-wave background

29 Jun 2023

Cosmic claims: researchers have used radiotelescopes around the world to hunt for gravitational waves using the subtle variations in the timing of pulsars. (Courtesy: Aurore Simonnet for the NANOGrav Collaboration)

Pulsar Timing Array Signal

Pulsars - highly magnetized and rapidly rotating neutron stars emiiting radiaton from poles

Very stable rotation → regular ‘pulses’ of radiation → cosmic clocks

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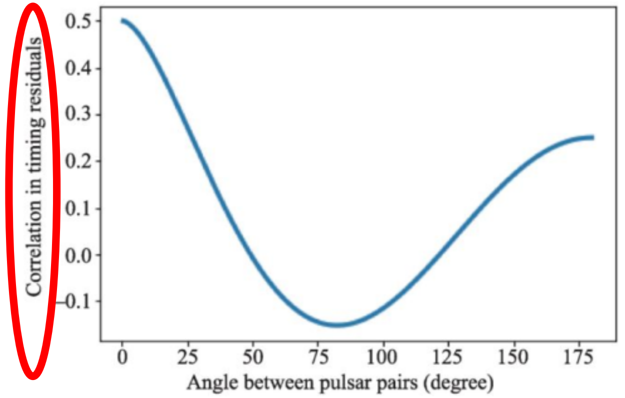
A stochastic gravitational wave background Gives a particular pattern of correlations

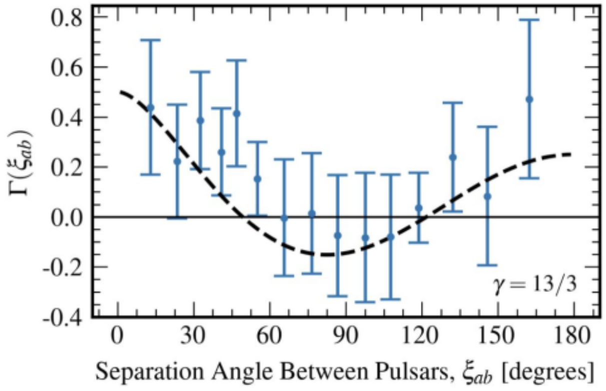
– the Hellings-Downs curve –



Hellings-Downs curve

NANOGrav 15 yr Data Set

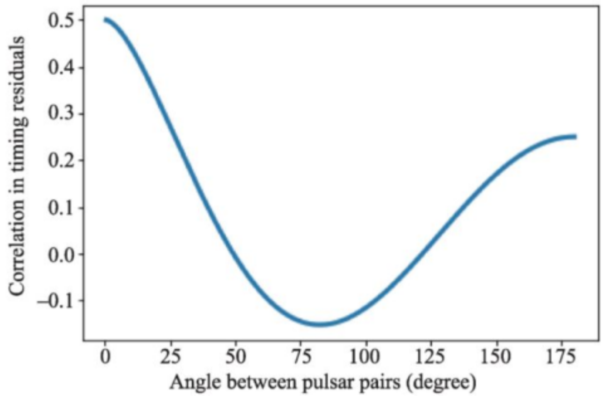
Astrophys.J.Lett. 951 (2023) 1, L8



Pulsar Timing Array Signal

Cosmological phase transition interpretations are possible

Conservative interpretation:
"just" a population of
supermassive black holes binaries



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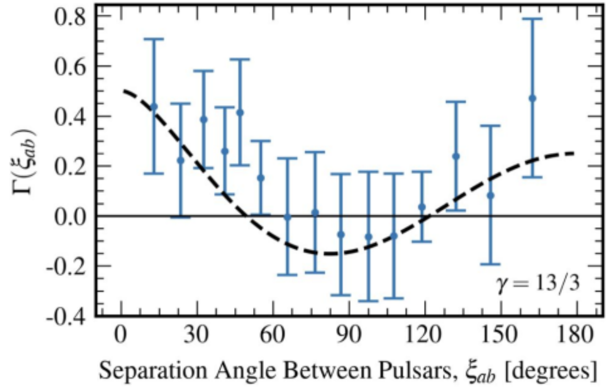


DOUBLE WARNING

For specific models these predictions require great care!

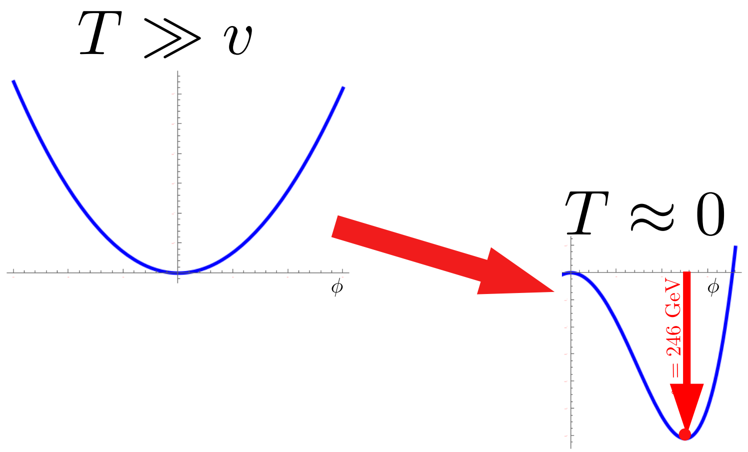
We looked at one model

prominantly cited by NANOGRAV

as able to explain nHz signals from PTAs...

Idea:

Take the EW phase transition



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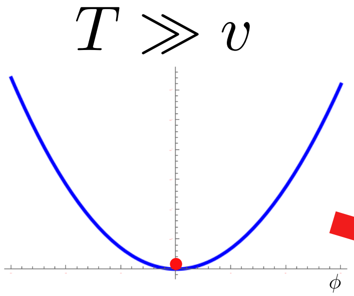

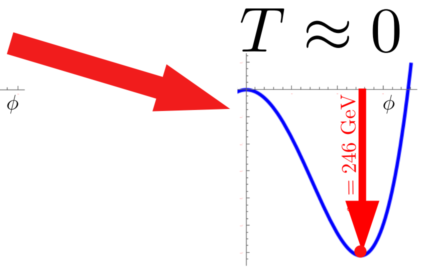

✓ It is one of two phase transitions from known physics

✓ Unlike QCD – plenty of room for new physics to make first order

✗ But the EW scale is at much higher energies –> GWs expected to peak at $\sim 10^{-3}$ Hz

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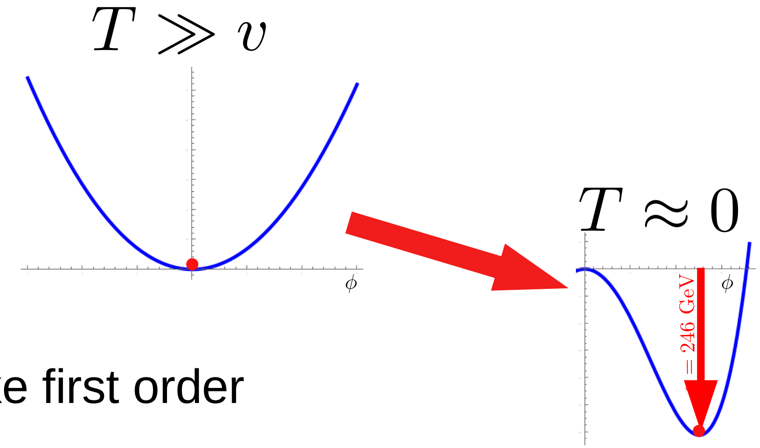

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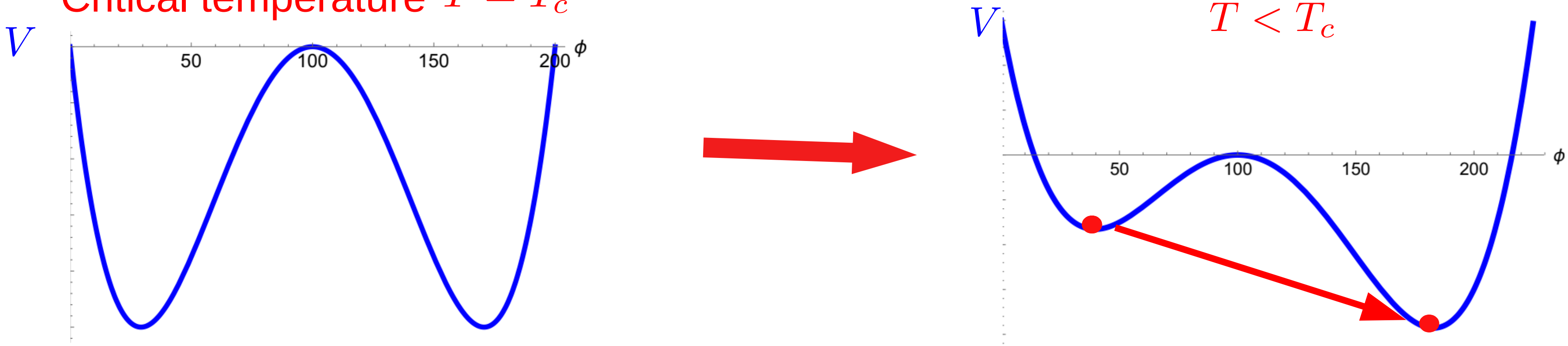
However note: All first order phase transitions exhibit some supercooling

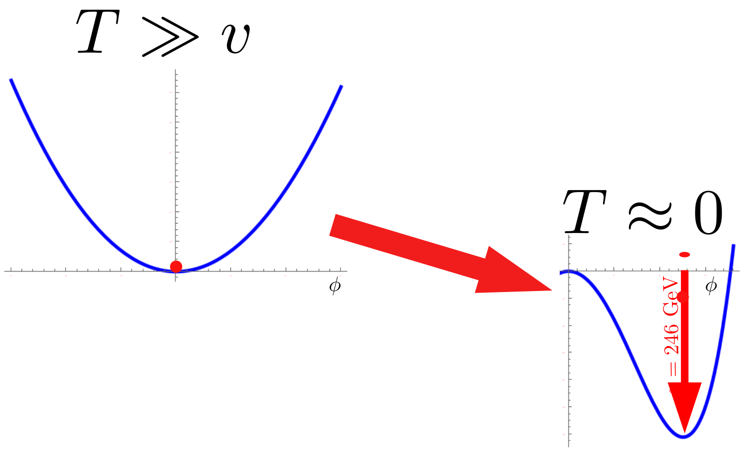
Critical temperature $T = T_c$



Barrier means phase transition happens after critical temperature

Idea:

Take the EW phase transition



Typical EW phase transition occurs at:

$$T_{EW} \sim \mathcal{O}(100\text{GeV}) \longleftrightarrow f^{\text{peak}} \sim 10^{-3}\ \text{Hz}$$

But supercool down to

$$\mathcal{O}(100\ \text{MeV}) \longleftrightarrow f^{\text{peak}} \sim 10^{-9}\ \text{Hz}$$

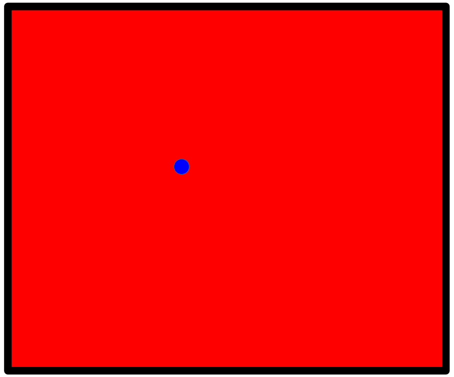
Archetypical example: A. Kobakhidze, C. Lagger, A. Manning and J. Yue, EPJ.C 77 (2017) 570 [1703.06552] cited by NANOGRAV.

But this is a very large degree of supercooling!

Does the Phase transiton complete?

Many studies only check nucleation

Nucleation: one bubble per Hubble volume



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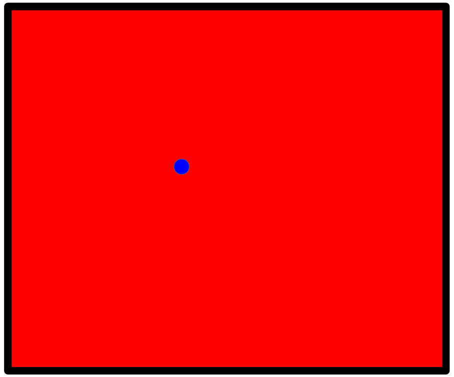

Hubble volume

Often exstimated with simple heuristics

$S(T_n)/T_n = 140$ "bounce action" in $\Gamma(t) = Ae^{-S(t)}$

Or solve $N(T_n) = 1$ $N(T) \approx \int_T^{T_c} dT' \, \frac{\Gamma(T')}{T' H^4(T')}$

If the barrier disolves quickly with temperature

→ Exponential nucleation rate → Bubbles rapidly fill space

"Fast transition" or "low supercooling"

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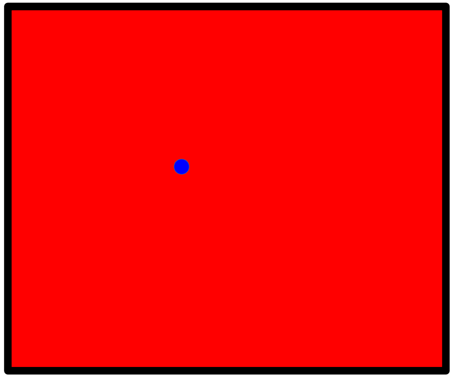
Nucleation: one bubble per Hubble volume

Not sufficient for scenarios with a lot of supercooling,

If the barrier persists to low temperatures,
→ nucleation rate can reach a maximum



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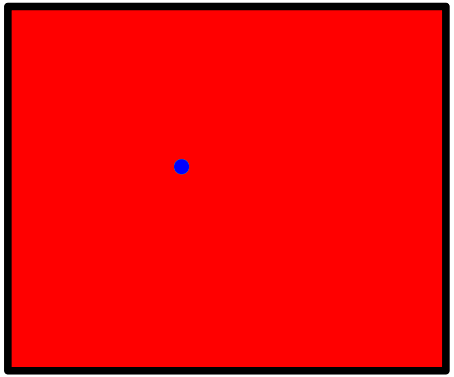

Hubble volume

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For such slow transitions we need the false vacuum fraction $P_f \to 0$

$$P_f(T) = \exp\left[-\frac{4\pi}{3}\int_T^{T_c}\frac{dT'}{T'^4}\frac{\Gamma(T')}{H(T')}\left(\int_T^{T'} dT''\frac{v_w(T'')}{H(T'')}\right)^3\right]$$

Stochastic so actually check:
$P_f < \epsilon$

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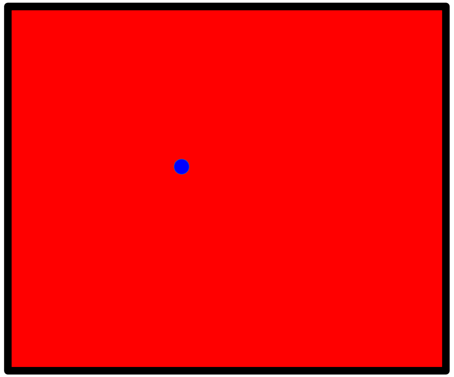

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Warning: even this is not enough because space is expanding

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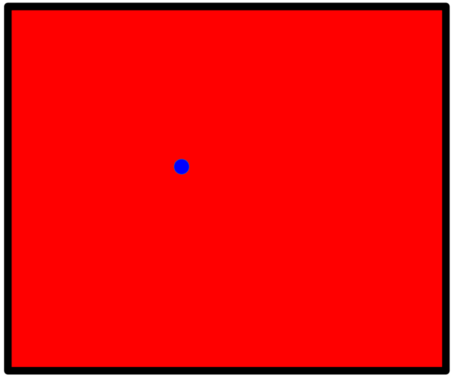

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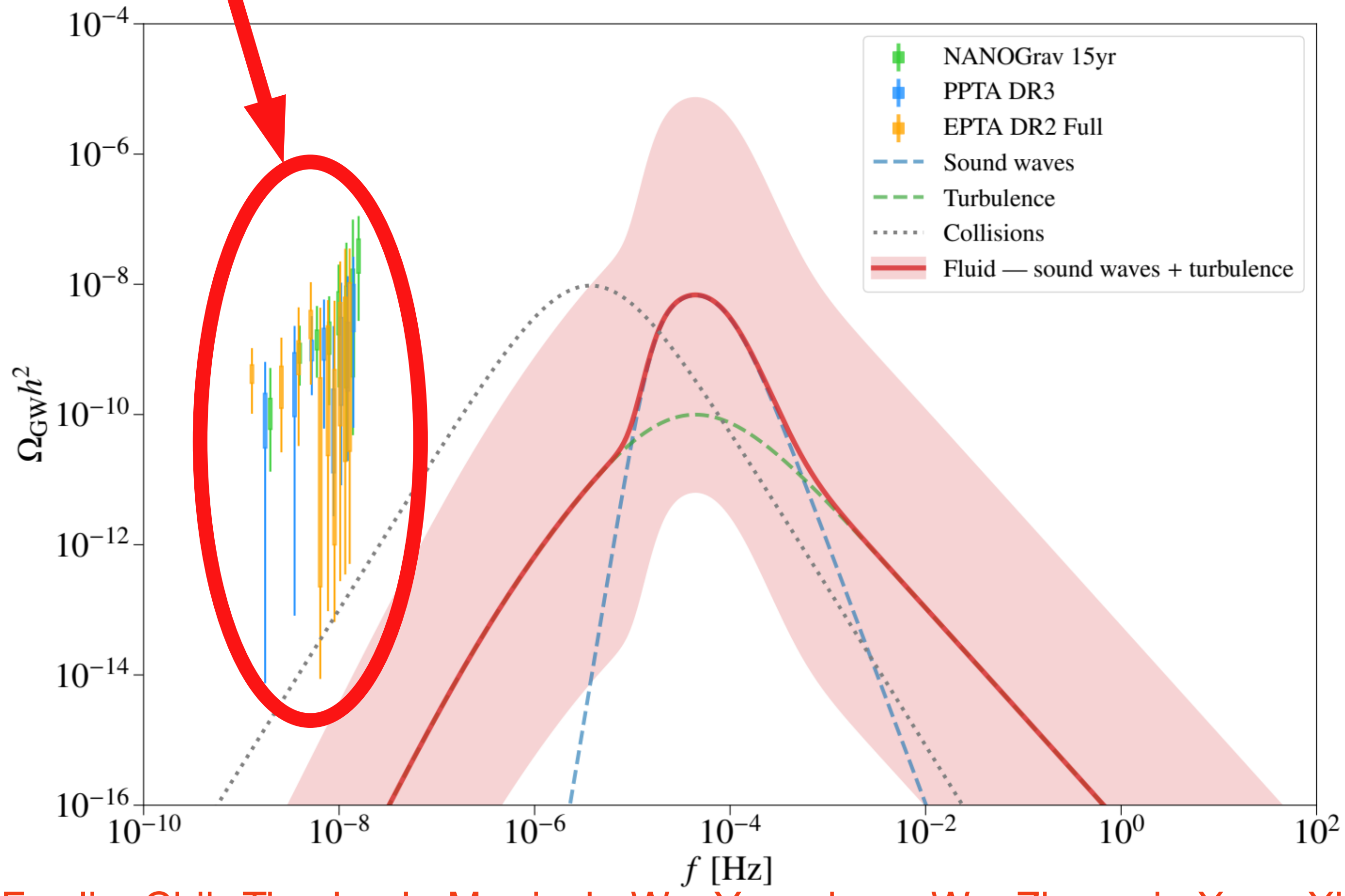
Account for expansion of space-time and check $\frac{\mathrm{d}\mathcal{V}_f^{\mathrm{phys}}}{\mathrm{d}T} < 0$

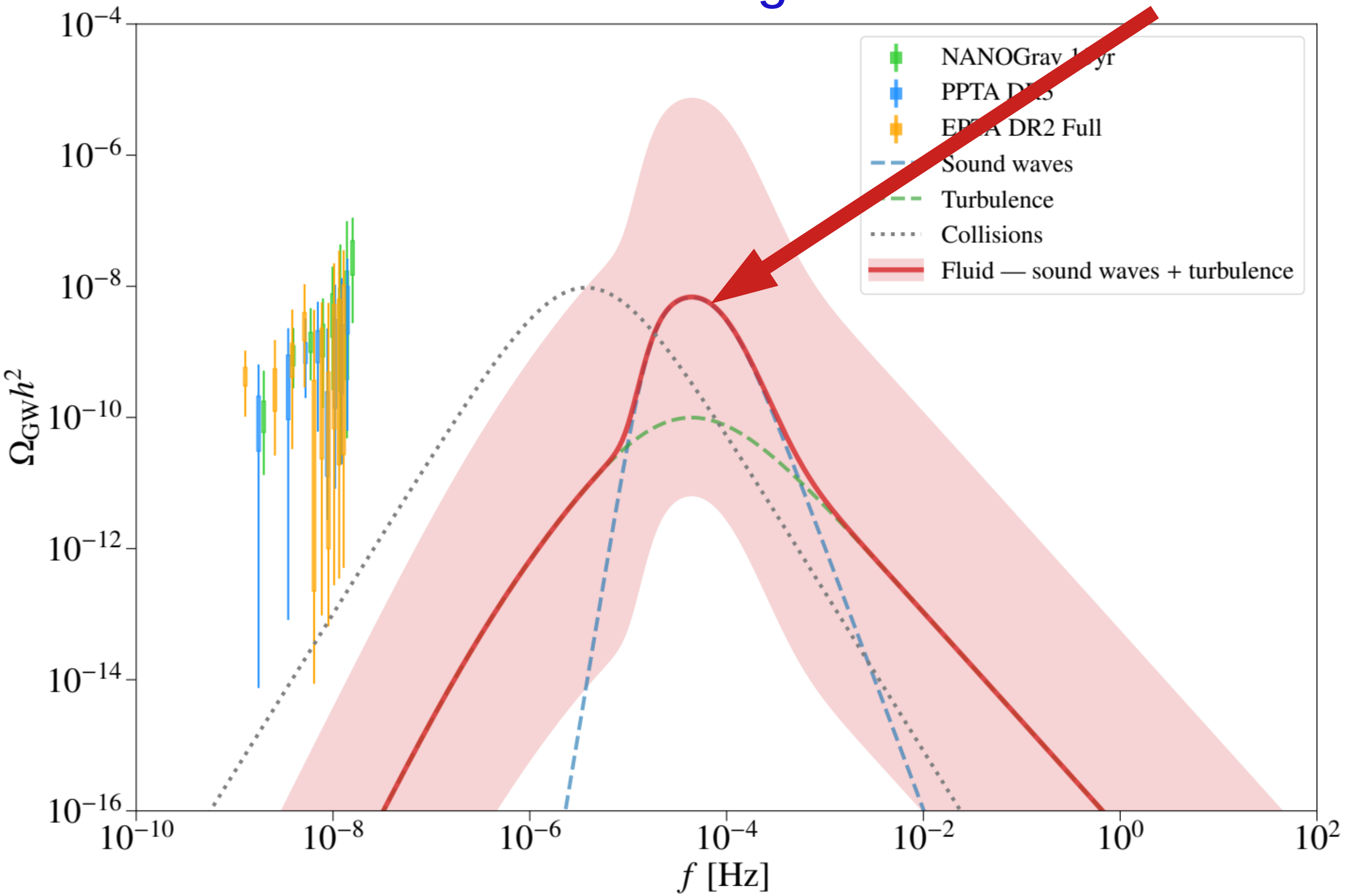
A stochastic gravitational wave background has been observed by multiple Pulsar Timing Arrays experiments



[PA, A. Fowlie, Chih-Ting Lu, L. Morris, L. Wu, Yongcheng Wu, Zhongxiu Xu, arXiv:2306.17239]

But for the protypical model of supercooled PTs cited by NANOgrav as a possible explanation:

GWs can't fit the signal with careful calculation



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A stochastic gravitational wave background has been observed
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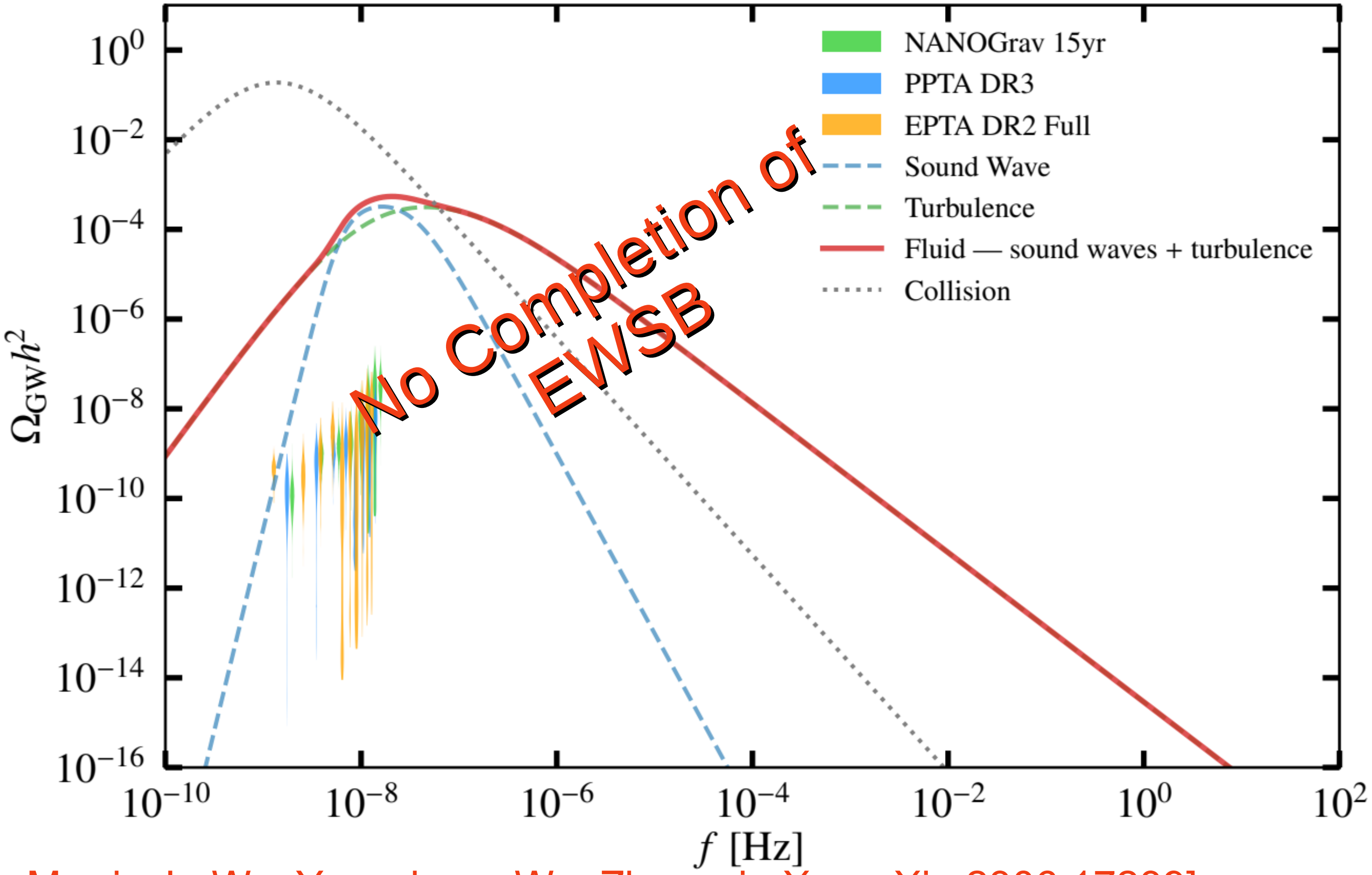
Larger signals are ruled out in this model because the PT does not complete



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- Handling of the perturbative potential (see e.g. Croon, Gould, Schicho, Tenkanen, White, JHEP 04 (2021) 055)

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- Estimating effect of temperature dependent thermal parameters (see PA, Harries, Xu, JCAP 02 (2024))

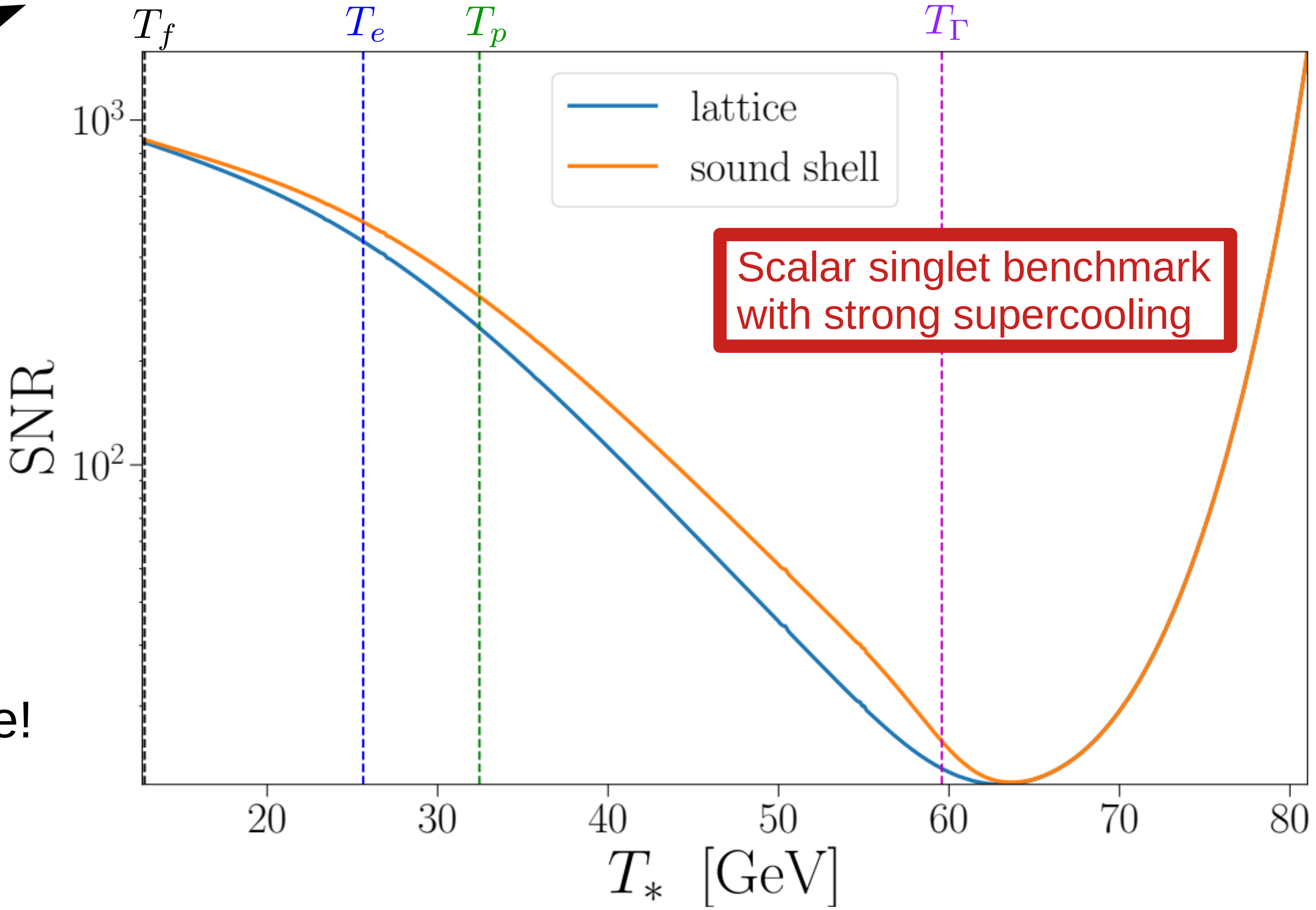
Temperature dependence

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

From here
(but plot simplified)

Slow transition, but percolates and completes *before* nucleation

LISA SNR varies more than an order of magnitude!



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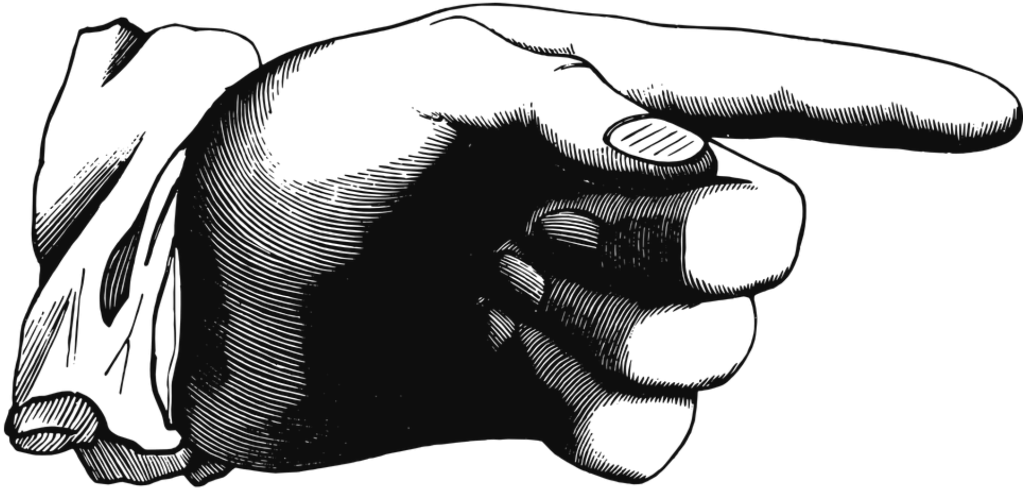
- Estimating effect of temperature dependent thermal parameters (see PA, Harries, Xu, JCAP 02 (2024))
- Uncertainties from using fits to simulations:
  extrapolations beyond region of validty (e.g. often for small alpha) and/or uncertainties associated with models or semi-analytic approaches etc

Gravitational Waves may reveal the EWPT in the future

But could we have some hints now from other data?

Flavour Anomalies

There are long standing anomalies in flavour physics



May seem a weird segue,

but keep listening ;)

Flavour Anomalies

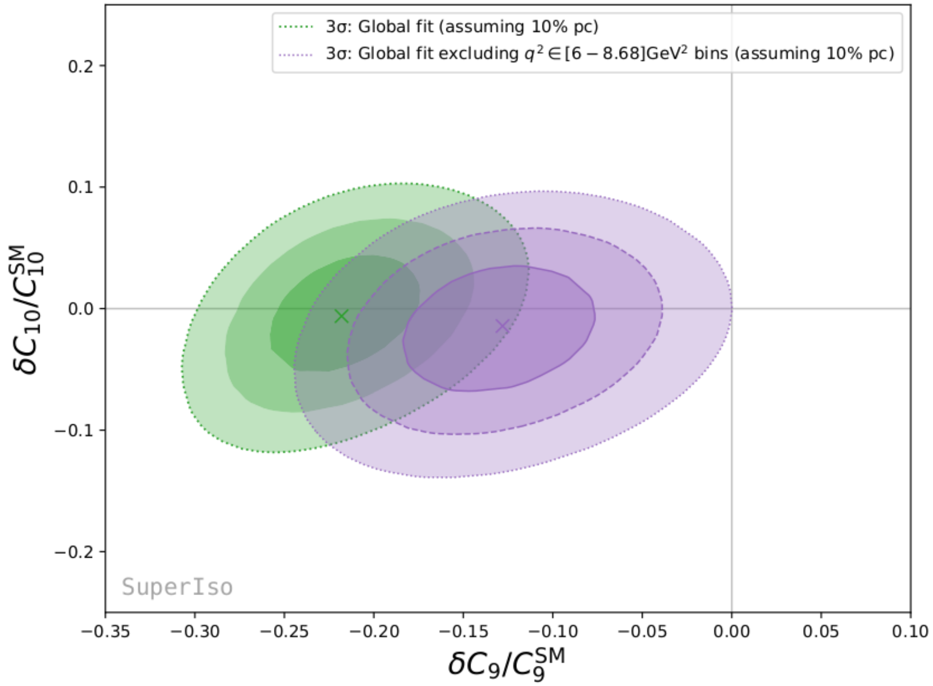
There are long standing anomalies in flavour physics

$b \to s l^+ l^-$ transitions – angular observables and branching ratios have many anomlies that combine to large significance

Many global fits – see e.g. very recent:

Large signifincances, but….

depends on estimate of unknown non-factorisable power corrections



Hurth, Mahmoudi, Monceaux, Neshatpour arXiv:2508.09986

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But not clean

$b \to s l^+ l^-$ transitions – clean test of lepton flavour universality

$$R_{K^{(*)}} = \frac{\Gamma(B \to K^{(*)} \mu^+ \mu^-)}{\Gamma(B \to K^{(*)} e^+ e_-)}$$

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And they did see a deviation (~3σ) ...

Supports angular and BR anomalies

Lots of people got very excited

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And they did see a devaition ($\sim 3\sigma$) 2013-2022

$$0.1 < q^2 < 1.1 \begin{cases} R_K & = 0.994 \ ^{+0.090}_{-0.082}(\text{stat})^{+0.029}_{-0.027}(\text{syst}), \\ R_{K^*} & = 0.927 \ ^{+0.093}_{-0.087}(\text{stat})^{+0.036}_{-0.035}(\text{syst}), \end{cases}$$

$$1.1 < q^2 < 6.0 \begin{cases} R_K & = 0.949 \ ^{+0.042}_{-0.041}(\text{stat})^{+0.022}_{-0.022}(\text{syst}), \\ R_{K^*} & = 1.027 \ ^{+0.072}_{-0.068}(\text{stat})^{+0.027}_{-0.026}(\text{syst}), \end{cases}$$

Supports angular and BR anomalies

Lots of people got very excited

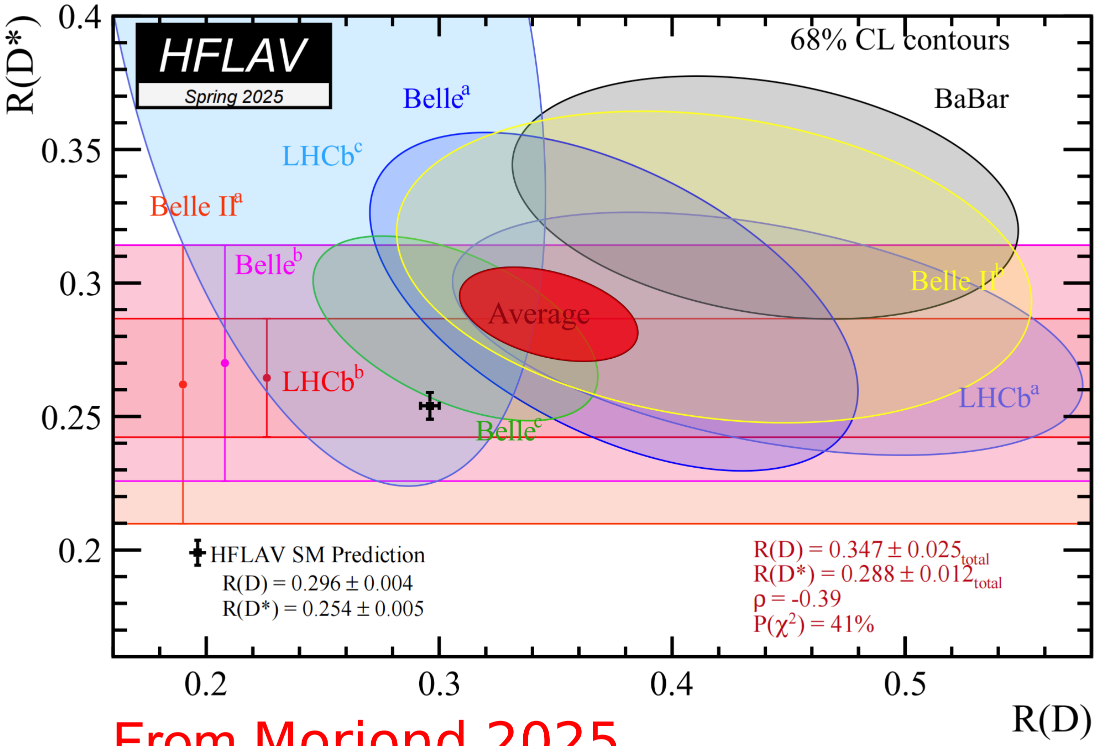
Until… the deviation went away

LHCb, Phys. Rev. Lett. 131 (2023), no. 5 051803

LHCb, Phys. Rev. D 108 (2023), no. 3 032002,

Charged current B-anomalies

There are also anomalies related to $b \to c l \overline{\nu_l}$



From Moriond 2025

$$R_{D^{(*)}} = \frac{\Gamma(B \to D^{(*)} \tau \overline{\nu})}{\Gamma(B \to D^{(*)} l \overline{\nu})}$$

$$R^{\text{exp}}_{D^{(*)}} > R^{\text{SM}}_{D^{(*)}}$$ At $3.8\ \sigma$

https://hflav-eos.web.cern.ch/hflav-eos/semi/spring25/html/RDsDsstar/RDRDs.html

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Now consistent with the Standard Model

$b \to c l \overline{\nu_l}$ transitions – test of lepton flavour universality

$$R_{D^{(*)}} = \frac{\Gamma(B \to D^{(*)} \tau \overline{\nu})}{\Gamma(B \to D^{(*)} l \overline{\nu})}$$

$$R_D^{\rm exp} = 0.347 \pm 0.025 \qquad R_{D^*}^{\rm exp} = 0.288 \pm 0.012$$

$$R_D^{\rm SM} = 0.3296 \pm 0.004 \qquad R_{D^*}^{\rm SM} = 0.254 \pm 0.005$$

→ $3.8\ \sigma$ deviation

[HFLAV spring 2025]

Flavour Anomalies

The combination of the neutral $b \to s l^+ l^-$ angular and BR anomalies and the charged current $R_{D^{(*)}}$ anomalies are still very intersting

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To date it has been unclear why we should expect new physics here, reducing the plausibility of the new physics explanation

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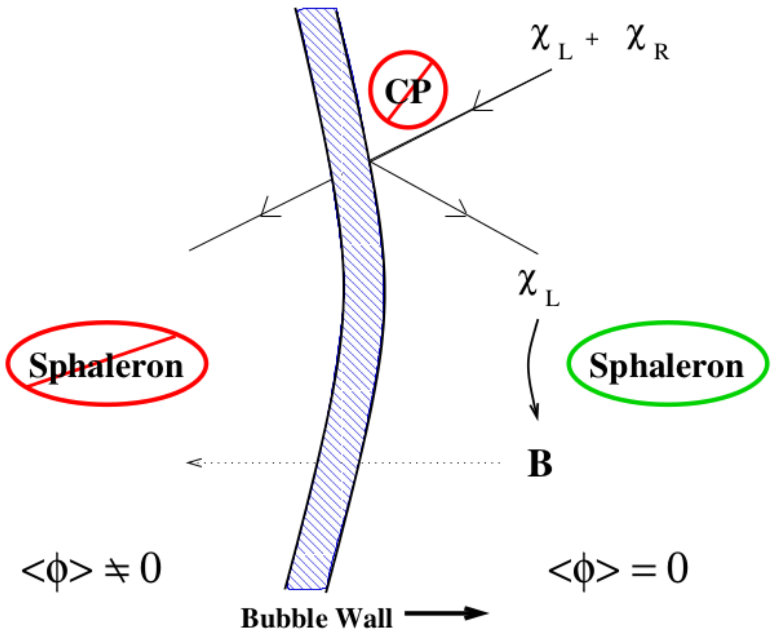
Here I will address this point

A new theoretical reason for taking these anomalies seriously - EWBG

Electroweak baryogenesis

Departure from thermal equilibrium via the abrupt first order phase transition

The generation of the baryon asymmetry is strongly tied to the EWPT



Morrissey, Ramsey-Musolf, New J.Phys. 14 (2012) 125003

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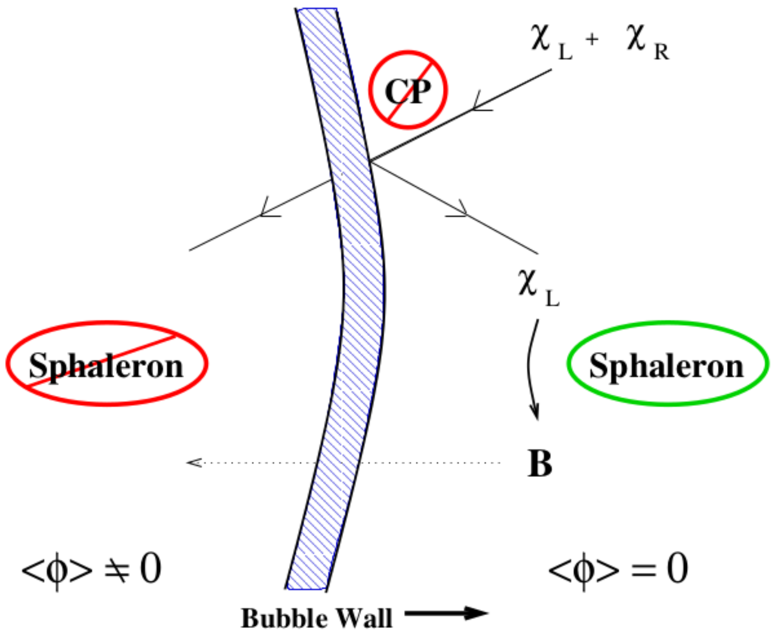
CP violation → scattering generates CP asymmetries



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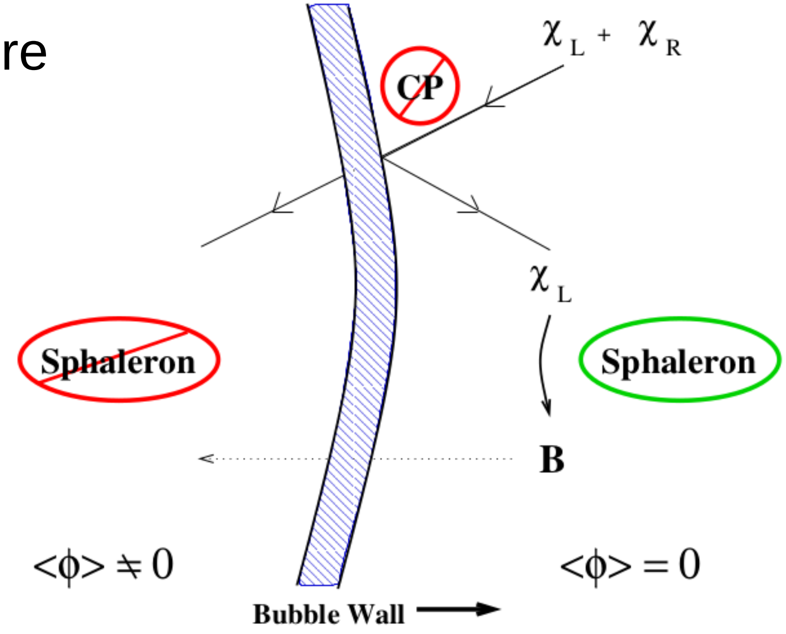
CP violation → scattering generates CP asymmetries

CP asymmetry with sphaleron process generates more baryon than anti-baryons in symmetric phase



Morrissey, Ramsey-Musolf, New J.Phys. 14 (2012) 125003

Electroweak baryogenesis

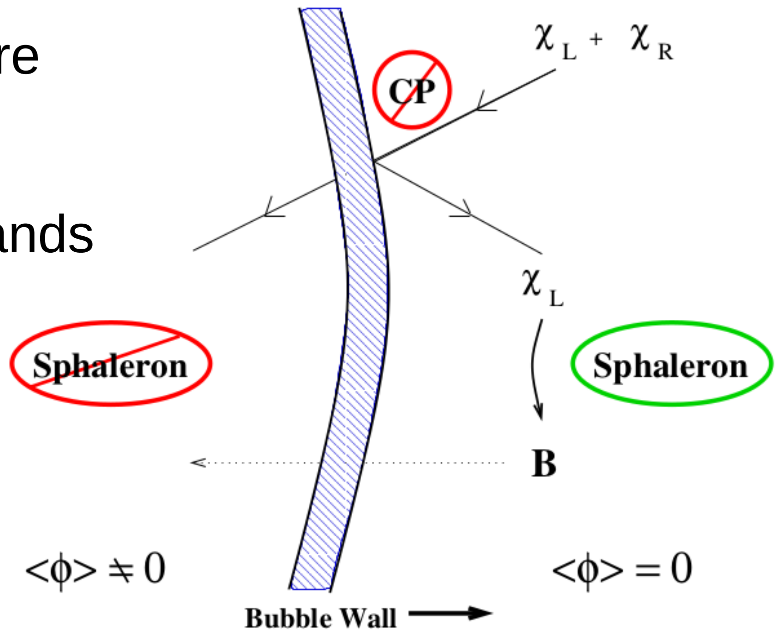
Departure from thermal equilibrium via the abrupt first order phase transition

The generation of the baryon asymmetry is strongly tied to the EWPT

CP violation → scattering generates CP asymmetries

CP asymmetry with sphaleron process generates more baryon than anti-baryons in symmetric phase

Baryons swept into EWSB phase as bubble wall expands



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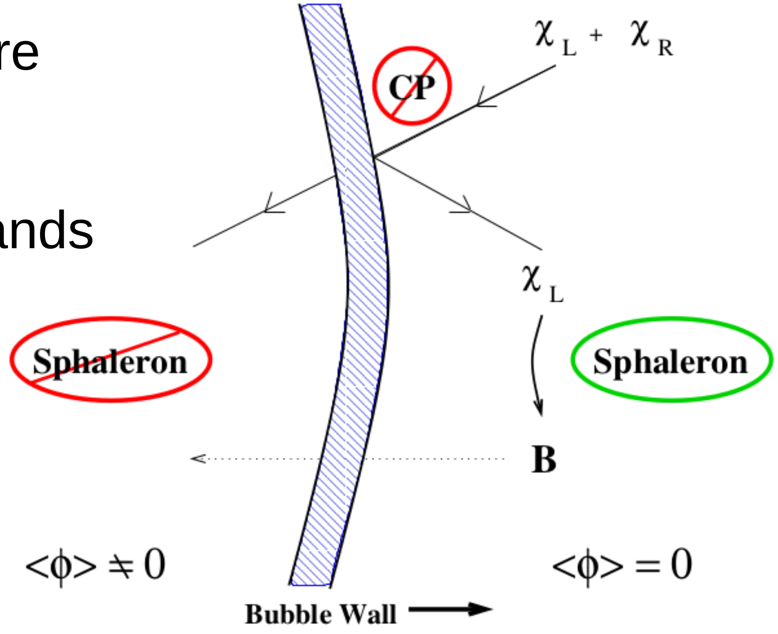

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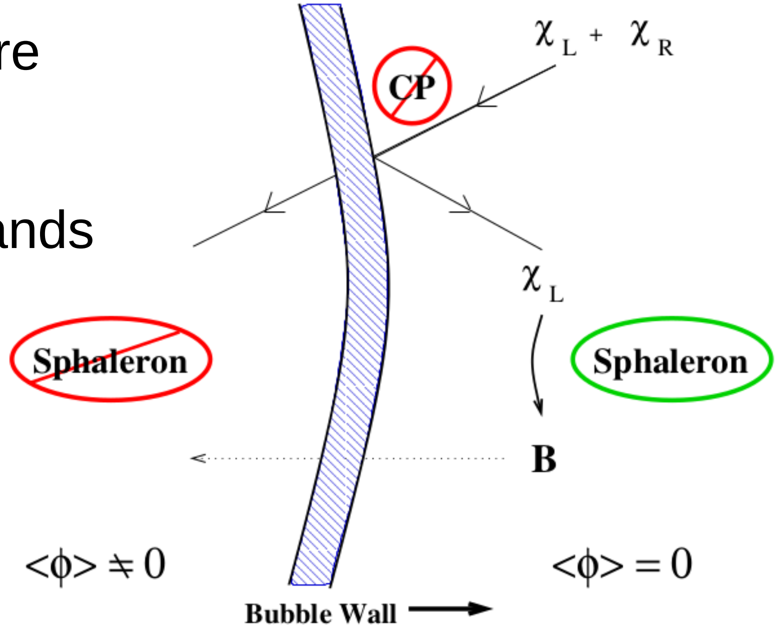
Thus EWBG generates a baryon asymmetry



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EWBG

Recently there’s been interesting deveopments regarding the BAU calculation

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Instead we simply use WKB approach as a conservative estimate for proof of principle

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Instead we simply use WKB approach as a conservative estimate for proof of principle

– if we can fit the BAU using WKB approximation, then it should also be possible with VR

2HDM

2HDM one of simplest extensions of SM – just add an extra Higgs doublet

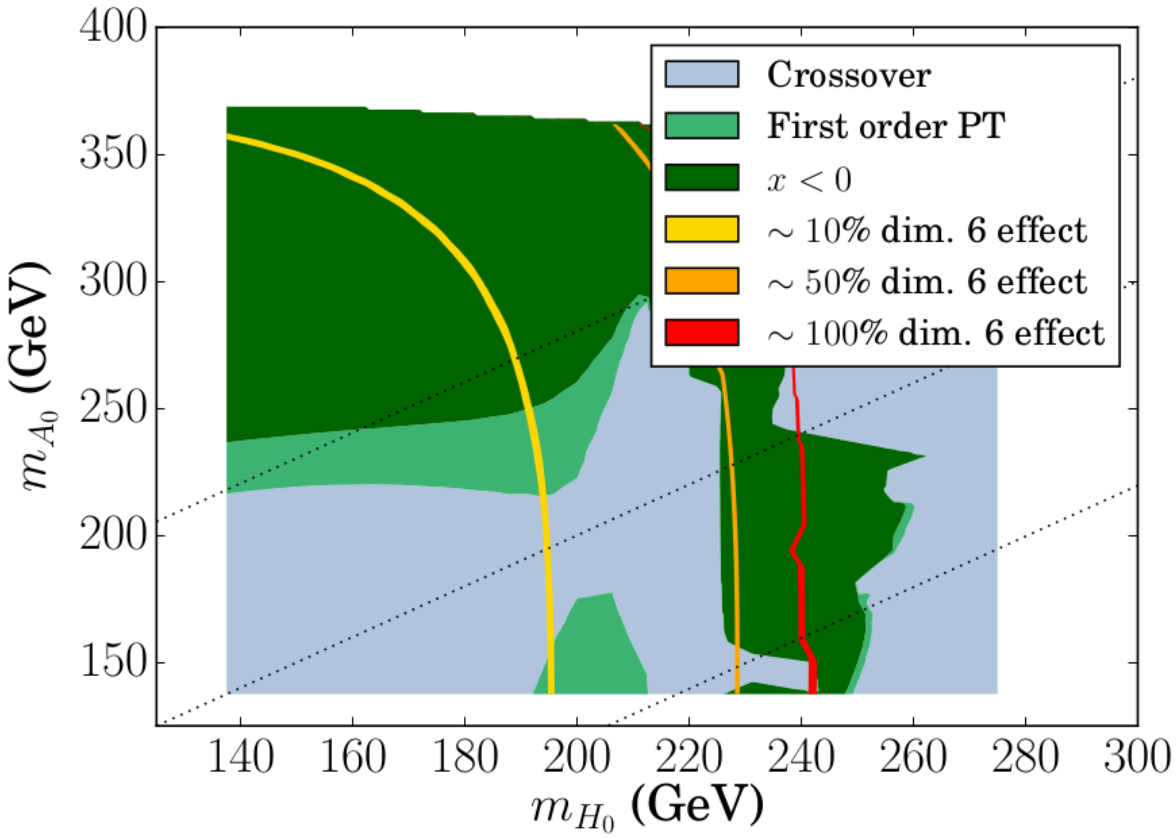
$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \rho_i + i\eta_i) \end{pmatrix}, \quad i = 1, 2.$$

General version without adding discrete symmetries has flavour violation, flavour universality violation and CP violation

$$\begin{aligned} V(\Phi_1, \Phi_2) = & m_{11}^2 (\Phi_1^\dagger \Phi_1) + m_{22}^2 (\Phi_2^\dagger \Phi_2) - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) \\ & + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + \left(\frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left(\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right) (\Phi_1^\dagger \Phi_2) + \text{ h.c.} \right), \end{aligned}$$

$$-\mathcal{L}_{Yukawa} = \bar{Q}^0 \, (Y_u^1 \tilde{\Phi}_1 + Y_u^2 \tilde{\Phi}_2) u_R^0 + \bar{Q}^0 \, (Y_d^1 \Phi_1 + Y_d^2 \Phi_2) d_R^0 + \bar{L}^0 \, (Y_l^1 \Phi_1 + Y_l^2 \Phi_2) l_R^0 + \text{ h.c.},$$

It has been established that the 2HDM has a first order PT perturbatively and non-perturbatively, e.g.



"Nonperturbative Analysis of the Electroweak Phase Transition in the Two Higgs Doublet Model", Andersen, Gorda, Helset, Niemi,Tenkanen, Tranberg, Vuorinen, Weir Phys. Rev. Lett. 121, 191802 (2018)

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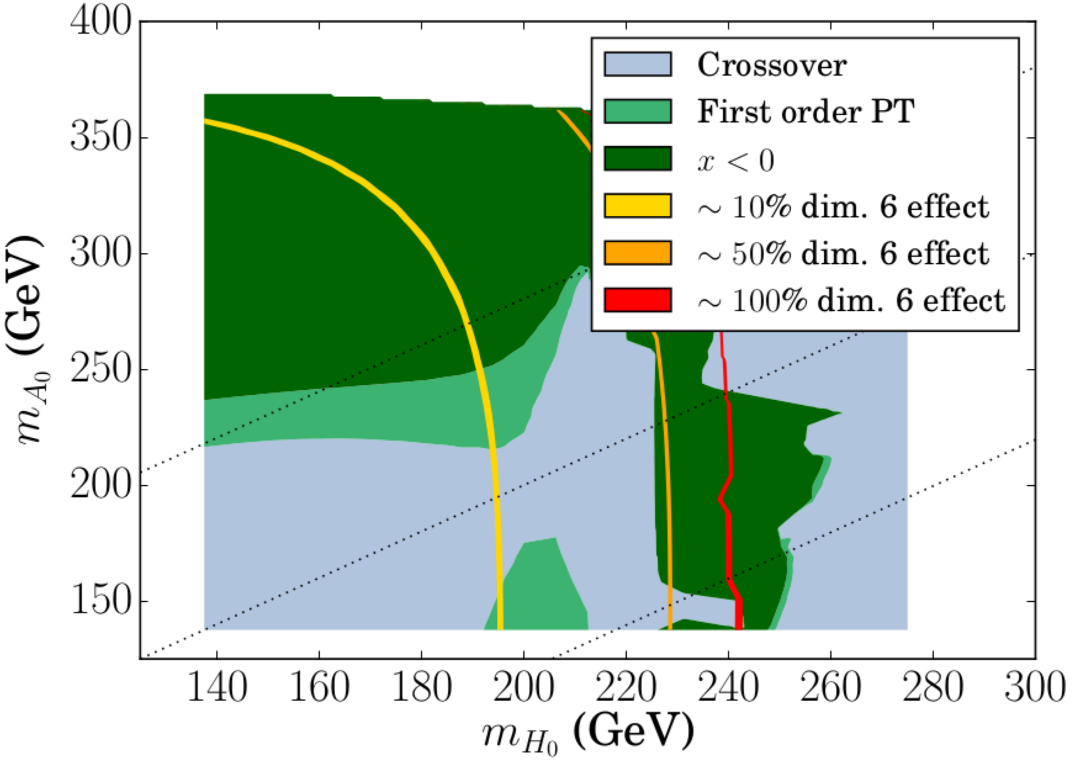
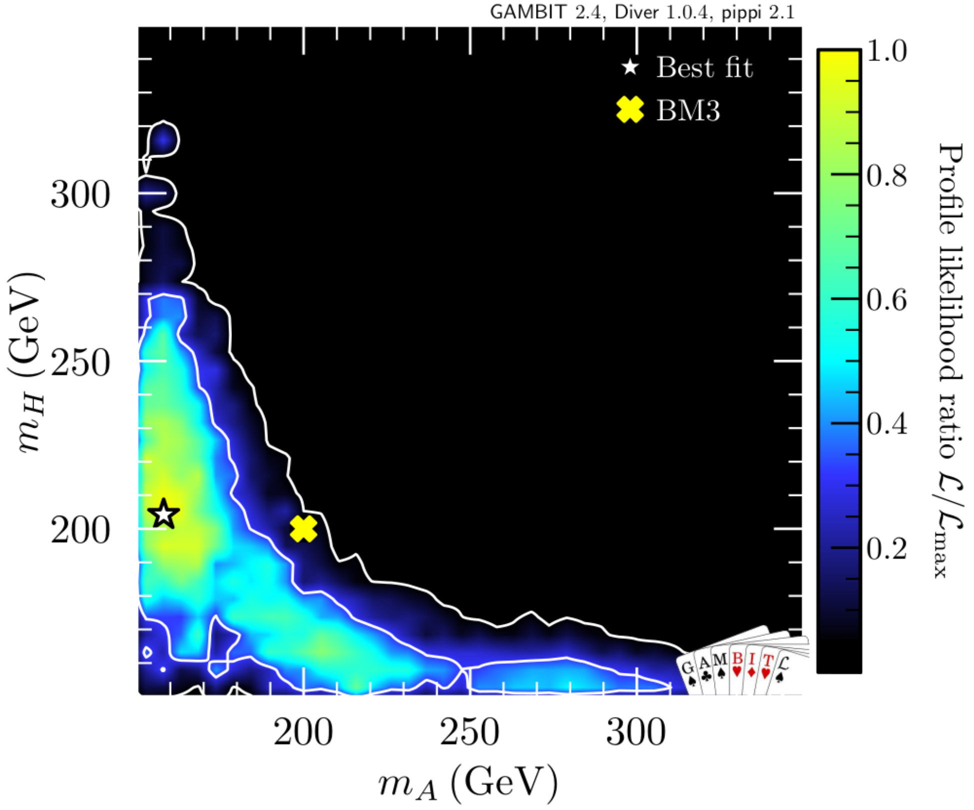


Andersen, Gorda, Helset, Niemi,Tenkanen, Tranberg, Vuorinen, Weır
Phys. Rev. Lett. 121, 191802 (2018)

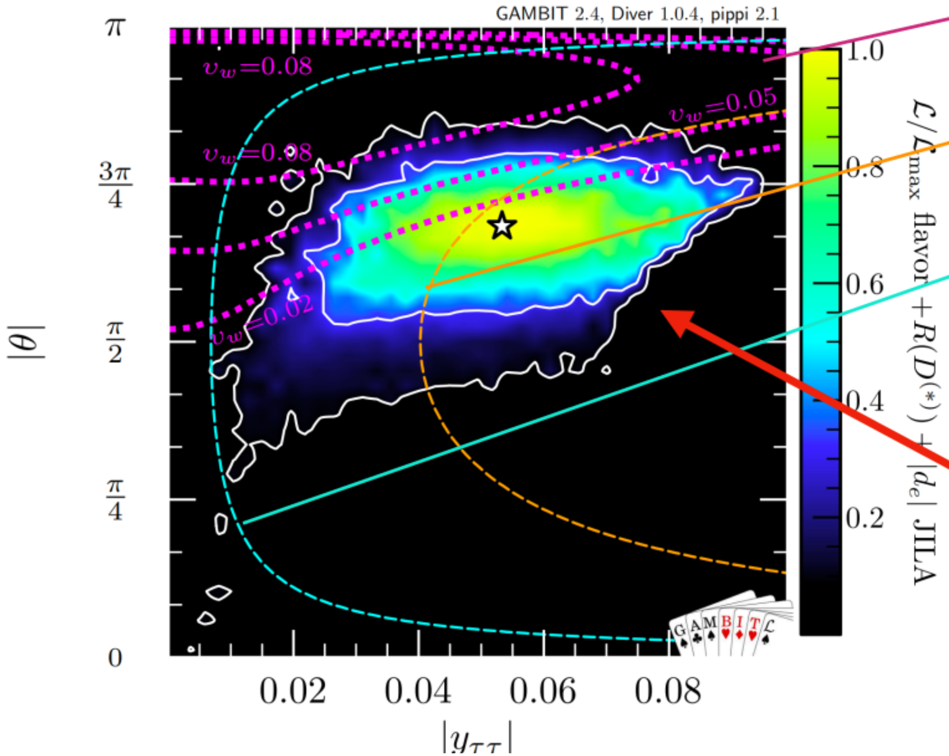
Overlaps with a global fit of flavour anomalies in 2HDM



PA, Crivellin, Gonzalo, Iguro, Sierra,
JHEP11(2024)133

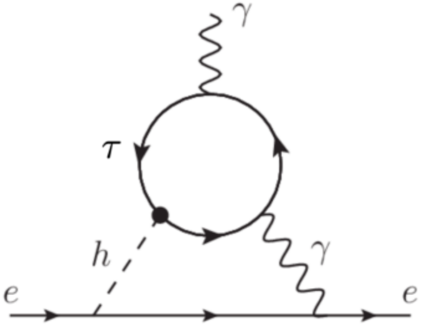
EWBG - results

Projecting the BAU in the parameter space



- Contours of $Y_B = Y_B^{obs}$ for different v_w.
- Sensitivity from CEPC/FCCee.
- Projection from ACME-III.
- eEDM constraints from JILA-NIST cuts a piece of the “liver-like” plot

$|d_e| < 4.1 \times 10^{-30}$ ecm



PA, Ramsey-Musolf, Sierra, Wu arxiv:2502.00445

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Conclusions

- The EWPT is an incredibly rich and facinating phenomena linked to Higgs, colliders, flavour physics, EDMs, BAU and GWs
- The PTA nHz signal for a SGWB does not seem to originate from a supercooled electroweak phase transition
- Fitting the R(D*) anomaly favours CPV violating Yukawa
- Proof of principle that explainations of the R(D*) charged flavour anomalies can be the source of sufficent CPV to explain the BAU, given current EDM constraints
- This works in one of the simplest extensions of the SM – the 2HDM
- Thus the long standing flavour anomlies could be hints for a succesful EWBG mechanism to explain the matter-antimatter asymmetry

… and thus also hints of an EWPT.

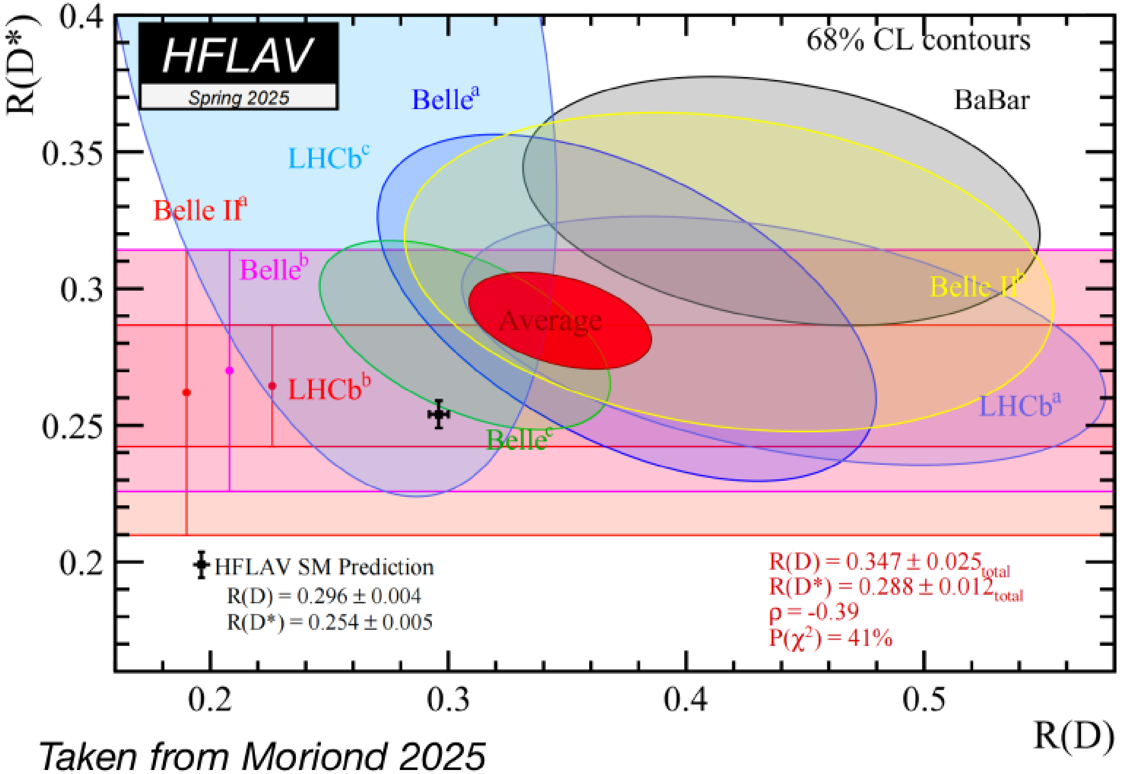
The END

Thanks for listening!

Flavour Anomalies

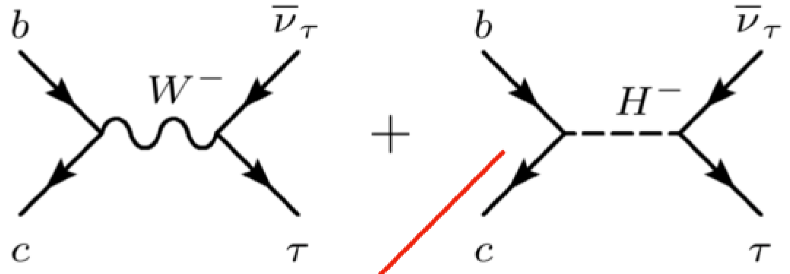
$$R_D = \frac{\Gamma(\overline{B} \to D\tau\overline{\nu})}{\Gamma(\overline{B} \to Dl\overline{\nu})} \qquad R_{D^*} = \frac{\Gamma(\overline{B} \to D^*\tau\overline{\nu})}{\Gamma(\overline{B} \to D^*l\overline{\nu})}$$

$R^{\text{exp}}_{D^{(*)}} > R^{\text{SM}}_{D^{(*)}}$ at $3.8\ \sigma$



Taken from Moriond 2025

Possible interference with NP?



$$C_L^{cb} = \frac{(V_{tb}\rho_u^{tc} + V_{cb}\rho_u^{cc})\rho_\ell^{\tau\tau}}{m^2_{H^\pm}}$$

Non-zero ρ_u^{tc} and $\rho_\ell^{\tau\tau}$ are needed.

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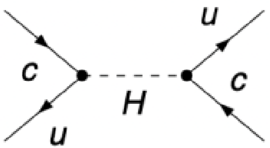
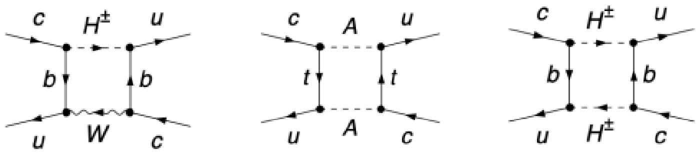
Flavour Physics

$$\rho_u = \begin{pmatrix} \rho_u^{uu} & \rho_u^{uc} & \rho_u^{ut} \\ \rho_u^{cu} & \rho_u^{cc} & \rho_u^{ct} \\ \rho_u^{tu} & \rho_u^{tc} & \rho_u^{tt} \end{pmatrix}$$

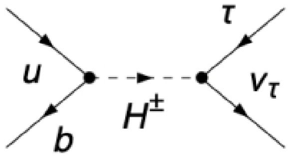
-$D^0 - \bar{D}^0$ mixing at tree level.



-$D^0 - \bar{D}^0$ mixing at one-loop.



-$B_u \to \tau\nu$ at tree level.



$$\rho_u = \begin{pmatrix} 0 & 0 & \rho_u^{ut} \\ 0 & \rho_u^{cc} & 0 \\ 0 & \rho_u^{tc} & \rho_u^{tt} \end{pmatrix}$$

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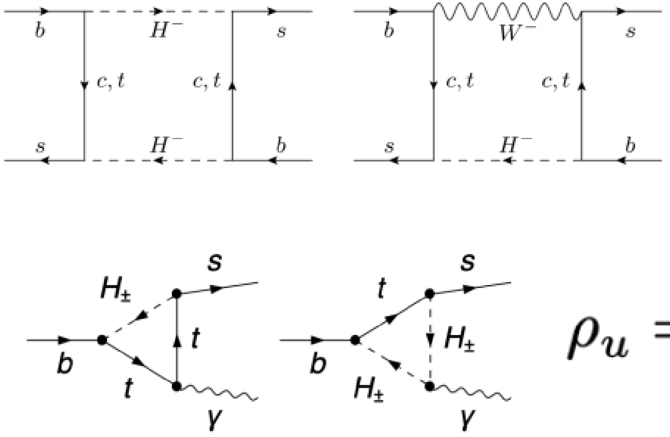
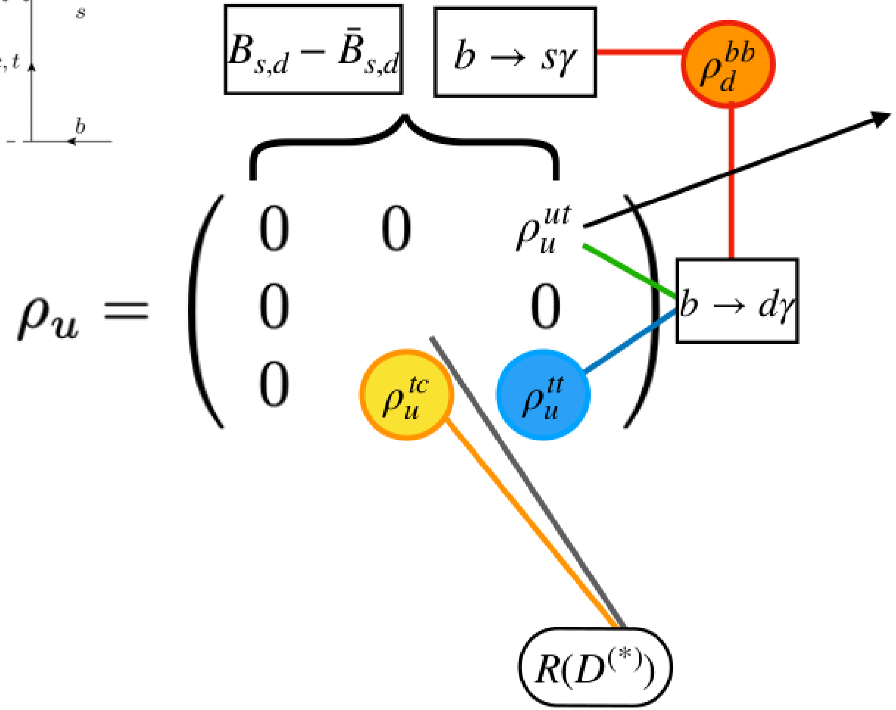
Flavour Physics



(Constrain all four couplings)



- Set to zero for simplicity (affects $B_s - \bar{B}_s$ mainly, large uncertainty in the SM)

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Flavour Physics

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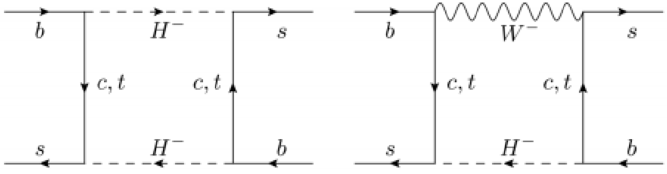
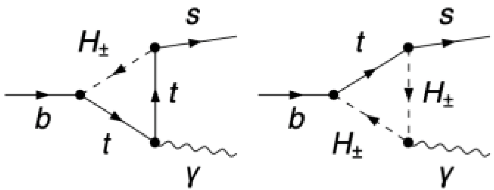
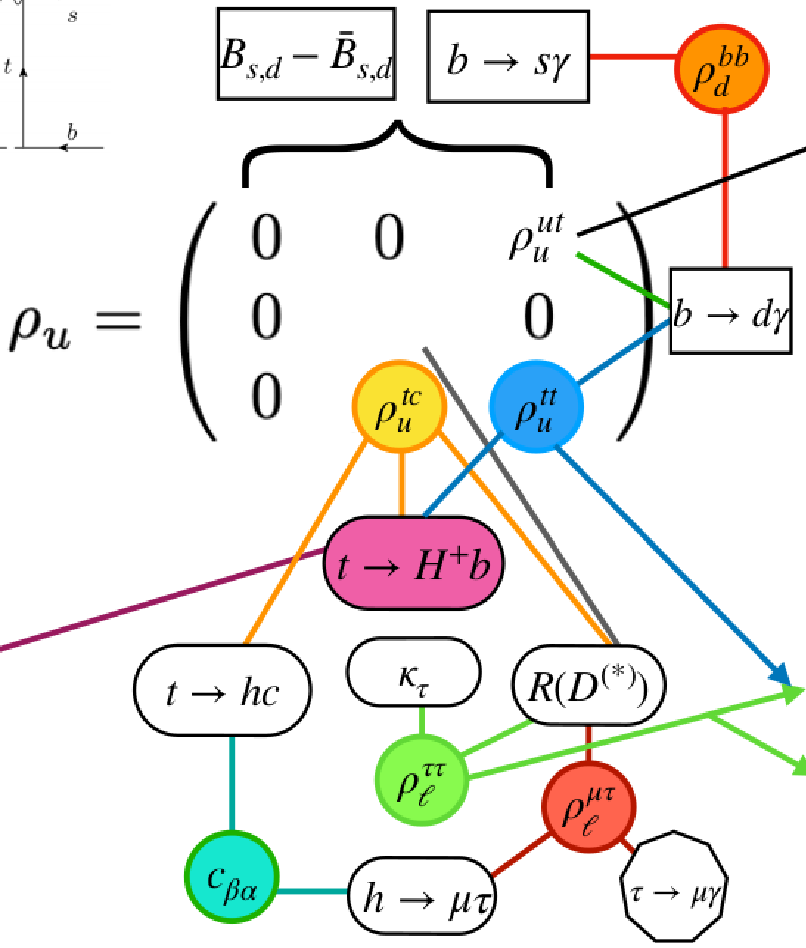






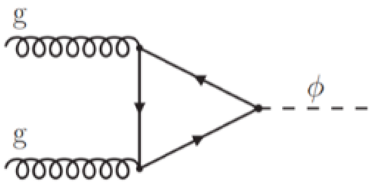
- *Set to zero for simplicity (affects $B_s - \bar{B}_s$ mainly, large uncertainty in the SM)*

$$m_{H^\pm} = 130\,\text{GeV}$$

130 GeV 3σ excess from

ATLAS arXiv:2302.11739 [hep-ex].

Collider constraints



$$\sigma(gg \to \phi \to \tau^+\tau^-) = \sigma(gg \to \phi) \cdot \text{BR}(\phi \to \tau^+\tau^-)$$

- *Multi-tau decays from CMS.*
- *eEDM constrains the imaginary part of* $\rho_\ell^{\tau\tau}$.

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$$-\mathcal{L}_{Yukawa} = \bar{u}_b\left(V_{bc}\rho_d^{ca}P_R - V_{ca}\rho_u^{cb*}P_L\right)d_a\,H^+ + \bar{\nu}_b\rho_\ell^{ba}P_R l_a\,H^+ + \text{h.c.}$$

$$+ \sum_{f=u,d,\ell}\sum_{\phi=h,H,A}\bar{f}_b\Gamma_f^{\phi ba}P_R f_a\phi + \text{h.c.},$$

$$\Gamma_f^{hba} \equiv \frac{\bar{M}_f^{ba}}{v}s_{\beta\alpha} + \frac{1}{\sqrt{2}}\rho_f^{ba}c_{\beta\alpha},$$

$$\rho_f^{ba} \equiv \frac{Y_f^{2,ba}}{\cos\beta} - \frac{\sqrt{2}\tan\beta\bar{M}_f^{ba}}{v},$$

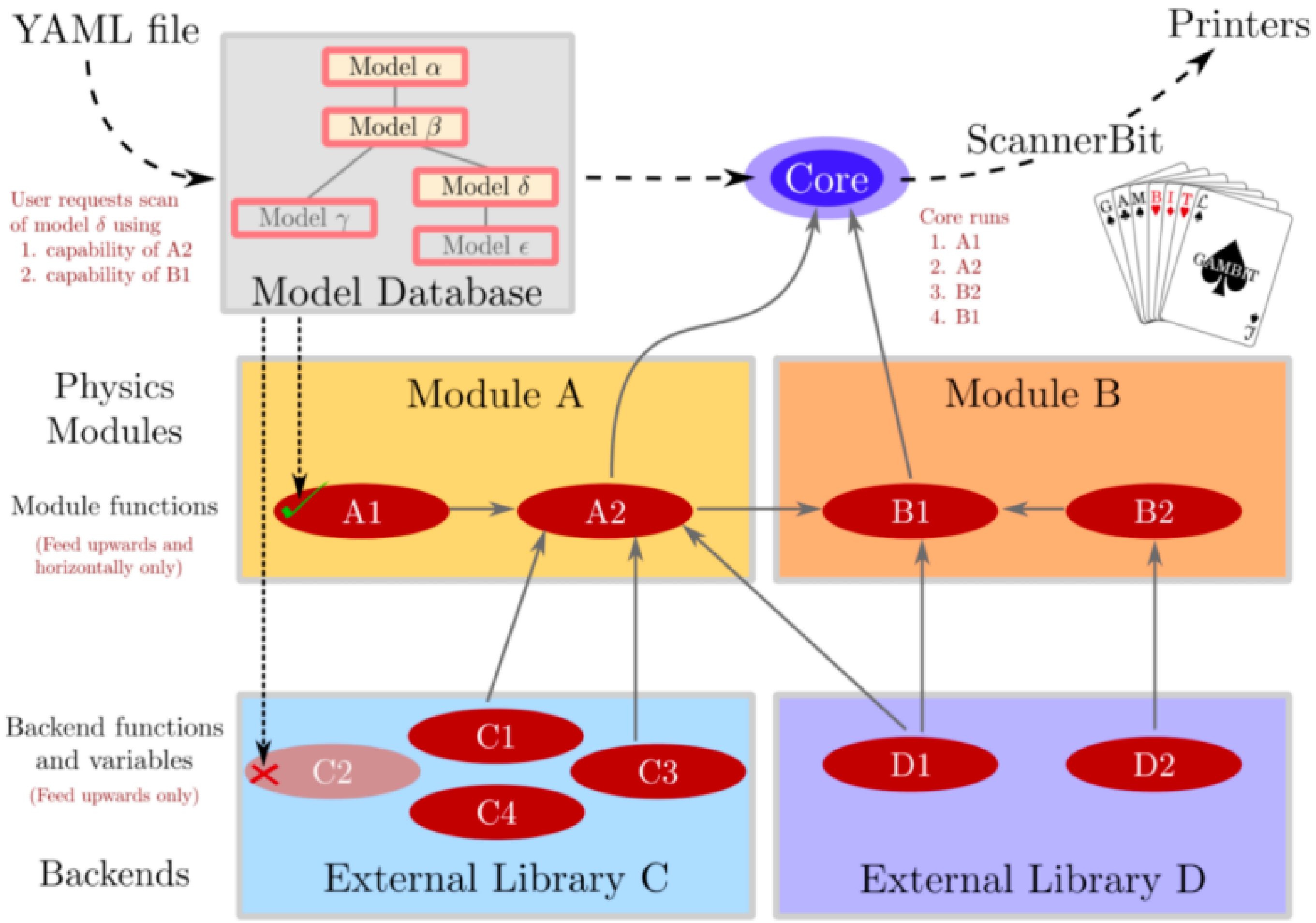
$$\Gamma_f^{Hba} \equiv \frac{\bar{M}_f^{ba}}{v}c_{\beta\alpha} - \frac{1}{\sqrt{2}}\rho_f^{ba}s_{\beta\alpha},$$

$$\Gamma_f^{Aba} \equiv \begin{cases} -\frac{i}{\sqrt{2}}\rho_f^{ba} & \text{if } f = u, \\ \frac{i}{\sqrt{2}}\rho_f^{ba} & \text{if } f = d, \ell\,, \end{cases}$$

$$\rho_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \rho_u^{cc} & 0 \\ 0 & \rho_u^{tc} & \rho_u^{tt} \end{pmatrix}, \qquad \rho_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \rho_d^{bb} \end{pmatrix}, \qquad \rho_\ell = \begin{pmatrix} 0 & 0 & \rho_\ell^{e\tau} \\ 0 & \rho_\ell^{\mu\mu} & \rho_\ell^{\mu\tau} \\ 0 & 0 & \rho_\ell^{\tau\tau} \end{pmatrix},$$

GAMBIT: The Global And Modular BSM Inference Tool

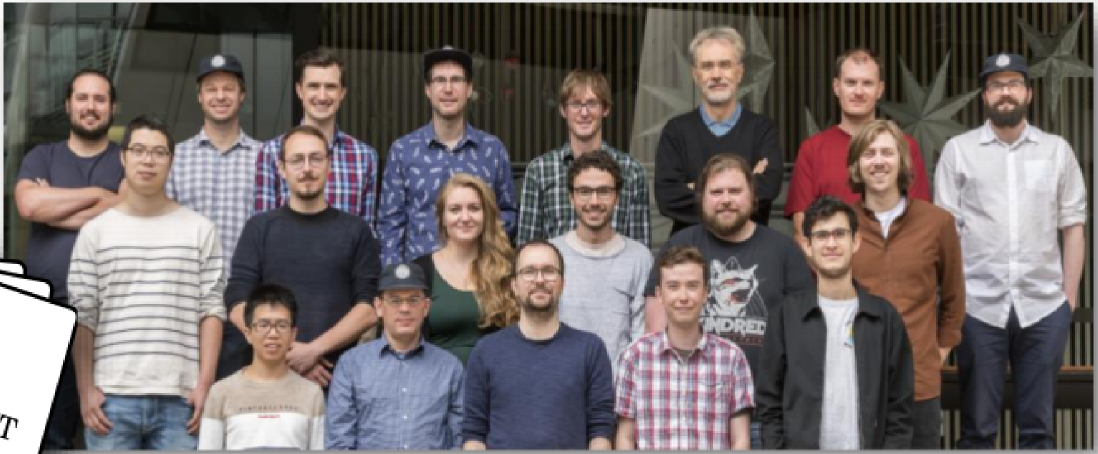
gambit.hepforge.org



GAMBIT: The Global And Modular BSM Inference Tool

gambitbsm.org github.com/GambitBSM EPJC 77 (2017) 784 arXiv:1705.07908

- Extensive model database, beyond SUSY
- Fast definition of new datasets, theories
- Extensive observable/data libraries
- Plug&play scanning/physics/likelihoods
- Various statistical options (frequentist /Bayesian)
- Fast LHC likelihood calculator
- Massively parallel
- Fully open-source



Members of: ATLAS, Belle-II, CLiC, CMS, CTA, Fermi-LAT, DARWIN, IceCube, LHCb, SHiP, XENON

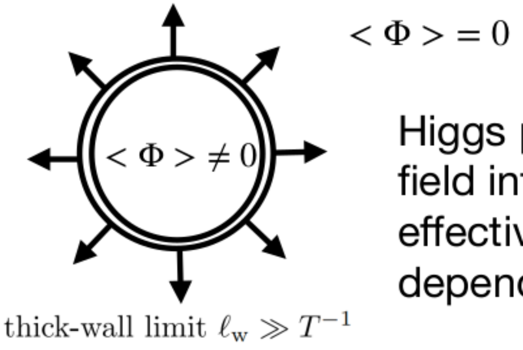
Authors of: BubbleProfiler, Capt'n General, Contur, DarkAges, DarkSUSY, DDCalc, DirectDM, Diver, EasyScanHEP, ExoCLASS, FlexibleSUSY, gamLike, GM2Calc, HEPLike, IsaTools, MARTY, nuLike, PhaseTracer, PolyChord, Rivet, SOFTSUSY, Superlso, SUSY-AI, xsec, Vevacious, WIMPSim

Recent collaborators: Peter Athron, Sowmiya Balan, Csaba Balázs, Torsten Bringmann, Christopher Cappiello, Riccardo Catena, Christopher Chang, Andreas Crivellin, Timon Emken, Tomás Gonzalo, Taylor R Gray, Will Handley, Quan Huynh, Syuhei Iguro, Ida-Marie Johansson, Felix Kahlhoefer, Anders Kvellestad, Michele Lucente, Gregory D Martinez, Marco Palmiotto, Are Raklev, Pat Scott, Cristian Sierra, Patrick Stoecker, Wei Su, Aaron Vincent, Martin White, Yang Zhang

70+ participants in many experiments and numerous major theory codes

EWBG

WKB formalism



Higgs plasma and fermion field interactions generate an effective force from space-dependent mass terms

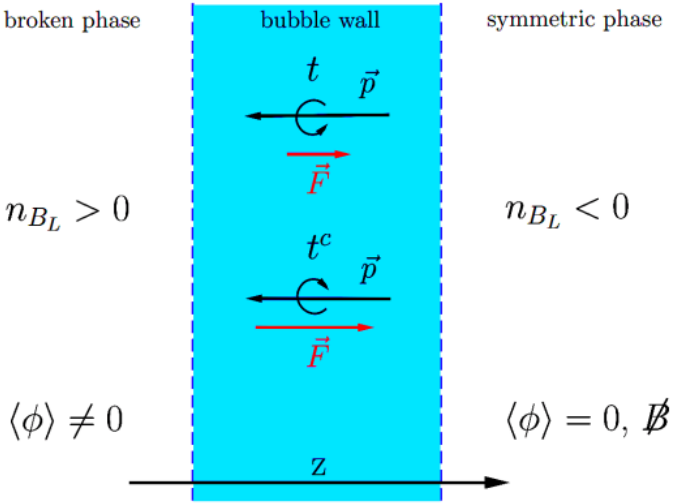
$$F_z = -\frac{(|m|^2)'}{2E} + s\left[\frac{(|m|^2\theta')'}{2EE_z}\right]$$

$$E = (\mathbf{p}^2 + |m|^2)^{1/2},\ E_z = (p_z^2 + |m|^2)^{1/2},\ \text{and } s = \pm 1$$

The effect of this force can be translated into a CPV source $\bar{S}_l$ in a diffusion equation for the lepton number density l:

$$\bar{D}_l\, l''(z) + v_w\, l'(z) + \bar{\Gamma}_l\, l(z) = \bar{S}_l$$

Diffusion coeff. bubble wall vel. Collision term



The solution for l will depend on a convolution of $\bar{S}_l$ which will be a function of the lepton CPV phase

Taken from Dietrich Bodeker, Wilfried Buchmuller, *Rev.Mod.Phys.* 93 (2021) 3, 3 • e-Print: 2009.07294 [hep-ph]

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EWBG

WKB formalism

Lorentz factor

"K" factors from transport equations

$$S_j = -v_w \gamma_w \, Q_j^{8o} \, (\mathrm{Im}(A_{22}))' \,, \quad j = 1, 2$$

Depends on the imaginary part of A

Matrix A borrowed from SUSY solutions for charginos

$$A = U M_l \partial_z M_l^{-1} U^\dagger$$

Matrix U diagonalizing the square of the mass profile

$$U = \frac{\sqrt{2}}{\sqrt{\Lambda(\Lambda + \Delta)}} \begin{pmatrix} \frac{1}{2}(\Lambda + \Delta) & a \\ -a^* & \frac{1}{2}(\Lambda + \Delta) \end{pmatrix}$$

$$\Lambda = \sqrt{\Delta^2 + 4\,|a|^2} \qquad a = (M_l^\dagger M_l)_{12}$$

$$\Delta = (M_l^\dagger M_l)_{11} - (M_l^\dagger M_l)_{22}$$

Lepton mass profile

$$M_l(z) = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} y_{\mu\mu} & y_{\mu\tau} \\ 0 & y_{\tau\tau} \end{pmatrix} h_1 + \begin{pmatrix} y_{\mu\mu} & y_{\mu\tau} \\ 0 & y_{\tau\tau}\, e^{i\theta} \end{pmatrix} h_2 \right]$$

Higgs profiles (kink type)

$$h_1(z) = \frac{v_n \cos\beta}{2} \left[1 + \tanh\left(\frac{z}{L_w} \right) \right],$$

$$h_2(z) = \frac{v_n \sin\beta}{2} \left[1 + \tanh\left(\frac{z}{L_w} - \Delta\beta \right) \right],$$

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EWBG

BAU calculation

Diffusion equation for the baryon number density

$$n_B''(z) - \frac{v_w}{D_q} n_B'(z) = \frac{\Gamma_{ws}}{D_q}\left(\mathcal{R}\, n_B(z) + \frac{3}{2} n_L(z)\right)$$

Weak sphaleron rate

Quarks diffusion coefficient

SM relaxation term (15/4)

Solution with $Y_B \equiv n_B/s$ and $n_L(z) \simeq l(z)$

$$Y_B = -\frac{3\,\Gamma_{ws}}{2\,s}\frac{1}{D_q \lambda_+}\int_{-\infty}^{-L_w} l(z)\, e^{-\lambda_- z} dz,$$

Entropy density

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Simplifications for fast analysis

We simplify the calculation based on a finding from

De Vries, Postma, van de Vis, JHEP 04, 024,

→ $n_L(x)$ dominates density of Left-handed lepttons $l_L(x)$,

→ To order 10% accuracy can consider only a *single* Bolzmanm equation $l_L(x)$

In this treatement:

1) lepton Yukawa interactions assumed slow → inefficient transfer to other species

2) Left handed and right handed diffusion equations approximated as equal

$$D_l \equiv D_L = D_R \simeq 100/T \quad \longrightarrow \quad l \equiv l_L = -\tau_R$$

Shown in De Vries, Postma, van de Vis, JHEP 04, 024, to be within 10% of full treatment

Given orders of magnitude discrepancy to VR treatment is precise enough

From
particle physics theory
to GWs

From particle physics theory to GWs

There is a long chain of steps needed to make GW predictions

Effective Potential

Transition rates

Thermal parameters that influence GW spectrum

$$\hat{V}_{\rm eff} \rightarrow \phi_i^{min}(T) \rightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \rightarrow P_f(T), N(T) \rightarrow \alpha, \beta, v_w \rightarrow h^2\Omega(f)$$

PhaseTracing

Bubble dynamics, nucleation & growth

GW spectrum

At every step there are challenges :

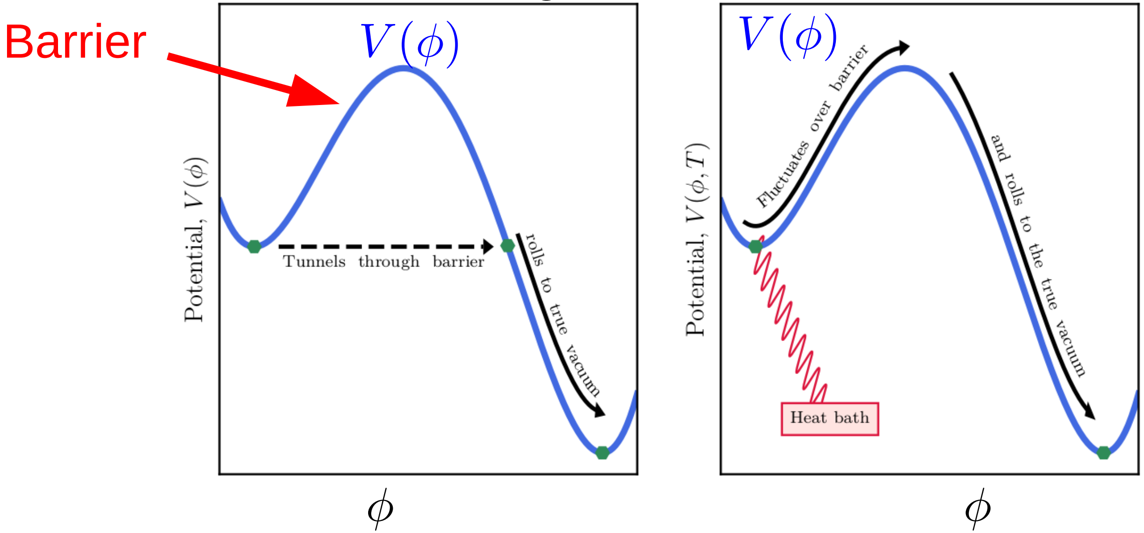
- open questions & active investigation
- Tensions between rigour and feasibility,
- Subtle issues leading to common misunderstandings / mistakes

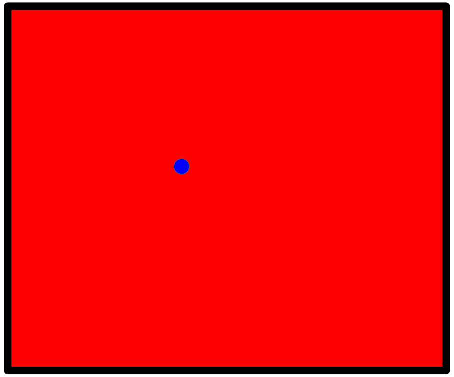
Does the Phase transiton complete?

Many studies only check nucleation

Nucleation: one bubble per Hubble volume

Tunnel through   Fluctuate over





Hubble volume

If the barrier disolves quickly with temperature

→ Exponential nucleation rate → Bubbles rapidly fill space

"Fast transition" or "low supercooling"

Gravitational wave amplitude and frequency

Each component of the amplitude $h^2\Omega_{\text{GW-tot}} = h^2\Omega_{\text{coll}} + h^2\Omega_{\text{sw}} + h^2\Omega_{\text{turb}}$

is defined in terms of the energy density ρ via $\Omega_{\text{GW}}(f) \equiv \frac{1}{\rho_{\text{tot}}}\frac{d\rho_{\text{GW}}}{d\ln f}$

$$\Omega_{\text{GW}}(f) \propto R_\Omega \, K^n L^m$$

redshift factor

Energy fraction

Length scale related to duration

Redshift factor to account for redshifting from the transition time to today

Energy fraction is the energy that can be available to source GWs

Length scale that is sensitive to the lifetime of the source

Implicit dependence of the transition temperature and the velocity the bubble walls expand also influences things

Powers depend on the source and the modelling, coefficients found in simulation/calculations

The temperature choice really matters for gravitational wave signatures

The nucleation temperature is frequently used for evaluating GW signals

$$N(T_n) = 1$$

$$N(T) = \int_T^{T_c} dT' \, \frac{\Gamma(T')}{T' H^4(T')}$$

But it may happen long before collsions or long after or may not even exist…

The nucleation temperature is frequently used for evaluating GW signals

$$N(T_n) = 1$$

$$N(T) \approx \int_T^{T_c} dT' \frac{\Gamma(T')}{T' H^4(T')}$$

But it may happen long before collsions or long after or may not even exist…

False vacuum fraction ⟶ several important milestone temperatures

Completion temperature: T_f: $P_f(T_f) = 0.01$

Percolation temperature: T_p: $P_f(T_p) = 0.71$

T_e: $P_f(T_e) = 1/e$

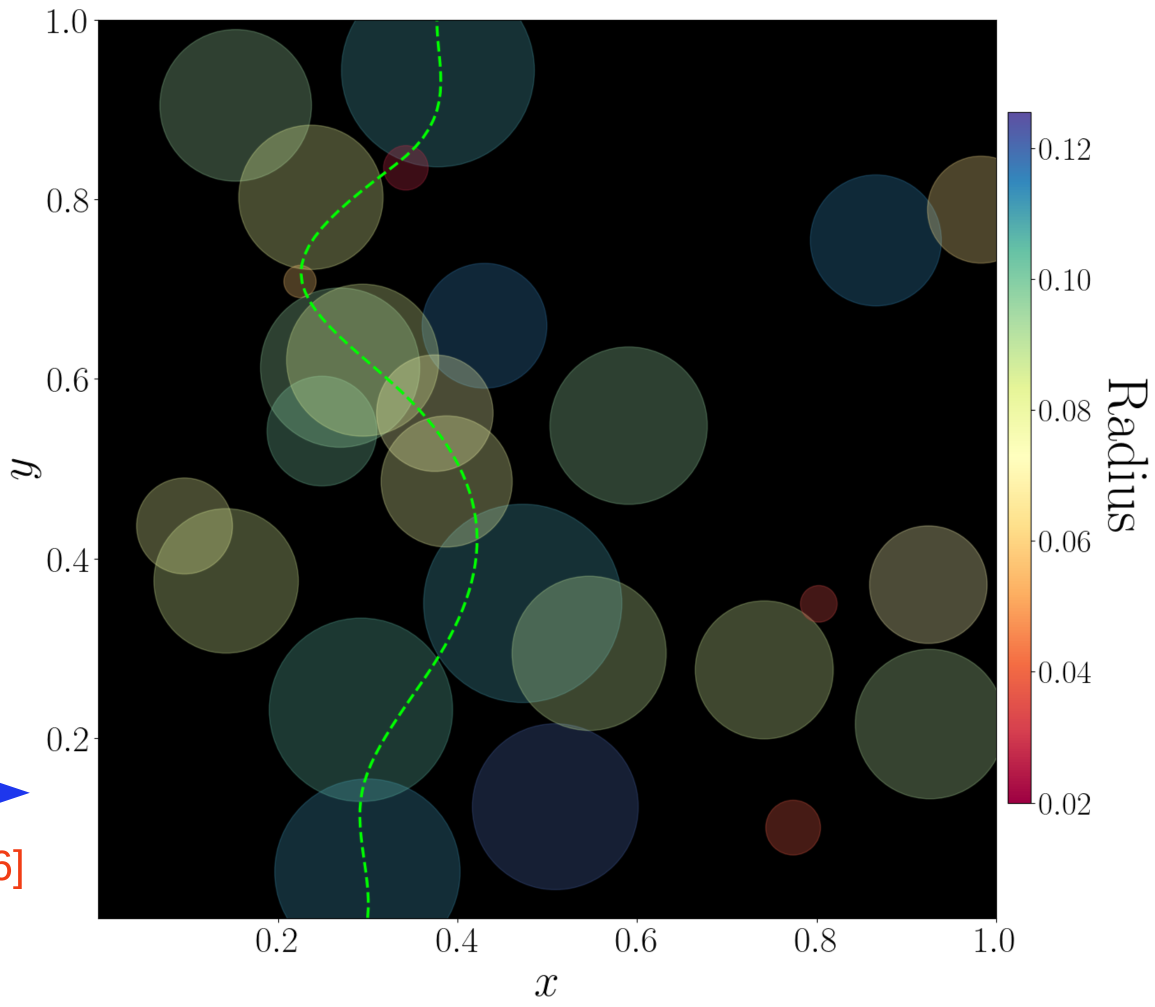
Percolation tempearture

$T_p\colon\ P_f(T_p) = 0.71$

- Percolation is when there is a connected path between bubbles across the space
- Strongly linked to bubble collisions
- Good choice for a temperature at which to evaluate the GWs spectrum

Example from simple simulation

[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]



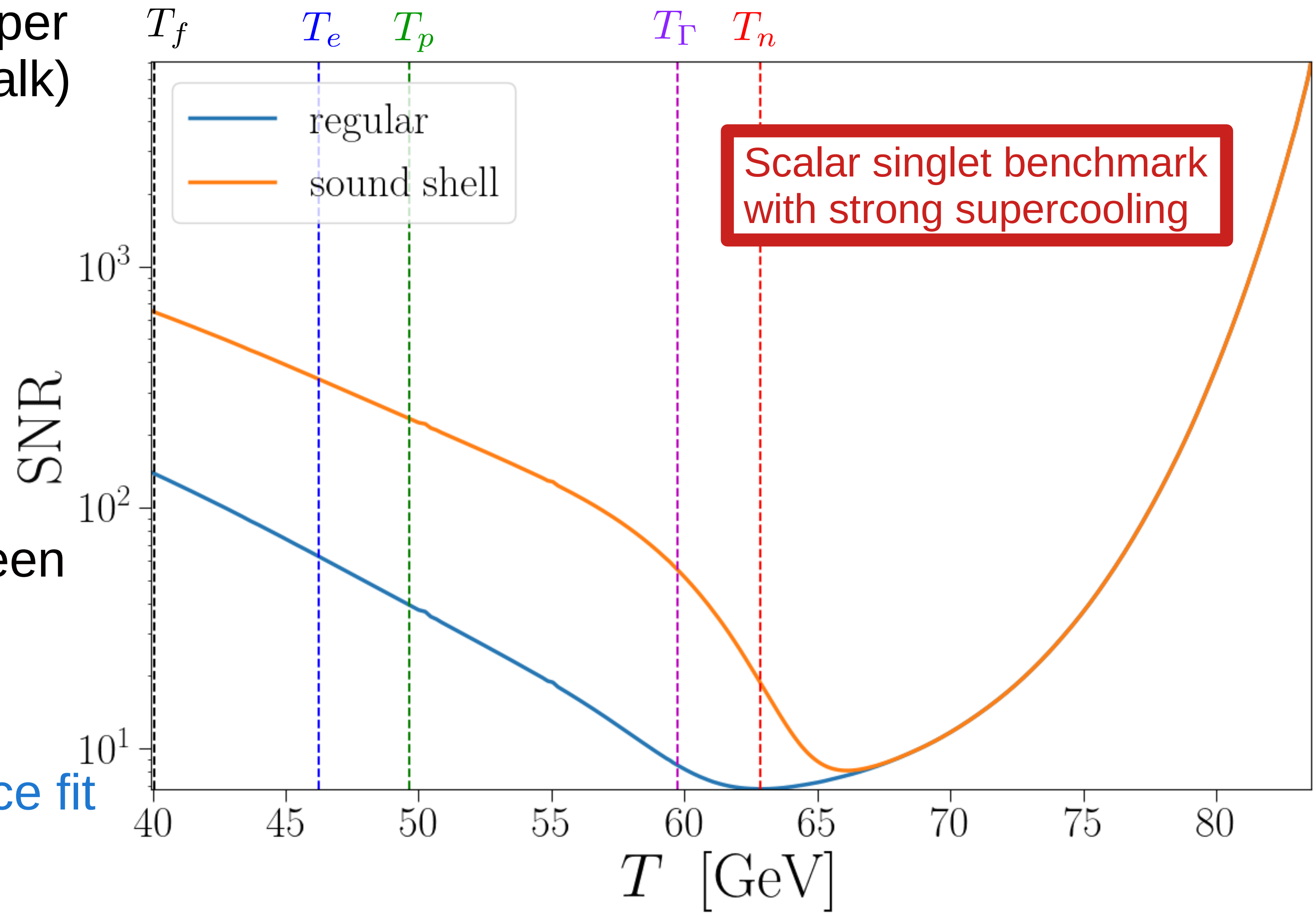
Temperature dependence

Point from same paper (plot made for this talk)

Slow transition, nucleation far earlier than percolation

Detectability (SNR for LISA) very different between percolation vs nucleation!

Sound shell and lattice fit also very different



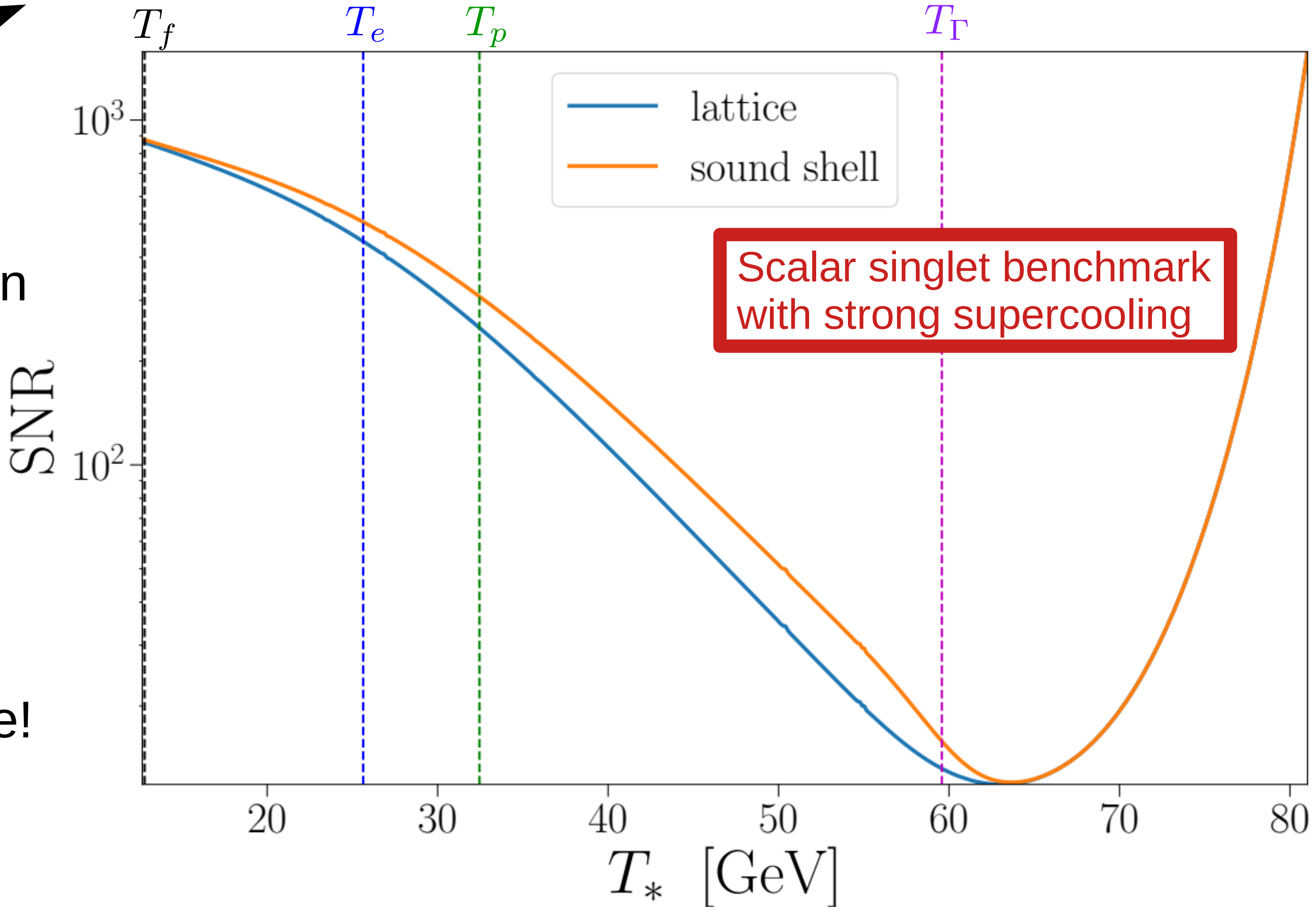
Temperature dependence

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

From here
(but plot simplified)

Another slow transition but percolates and completes *before* nucleation

LISA SNR varies more than an order of magnitude!



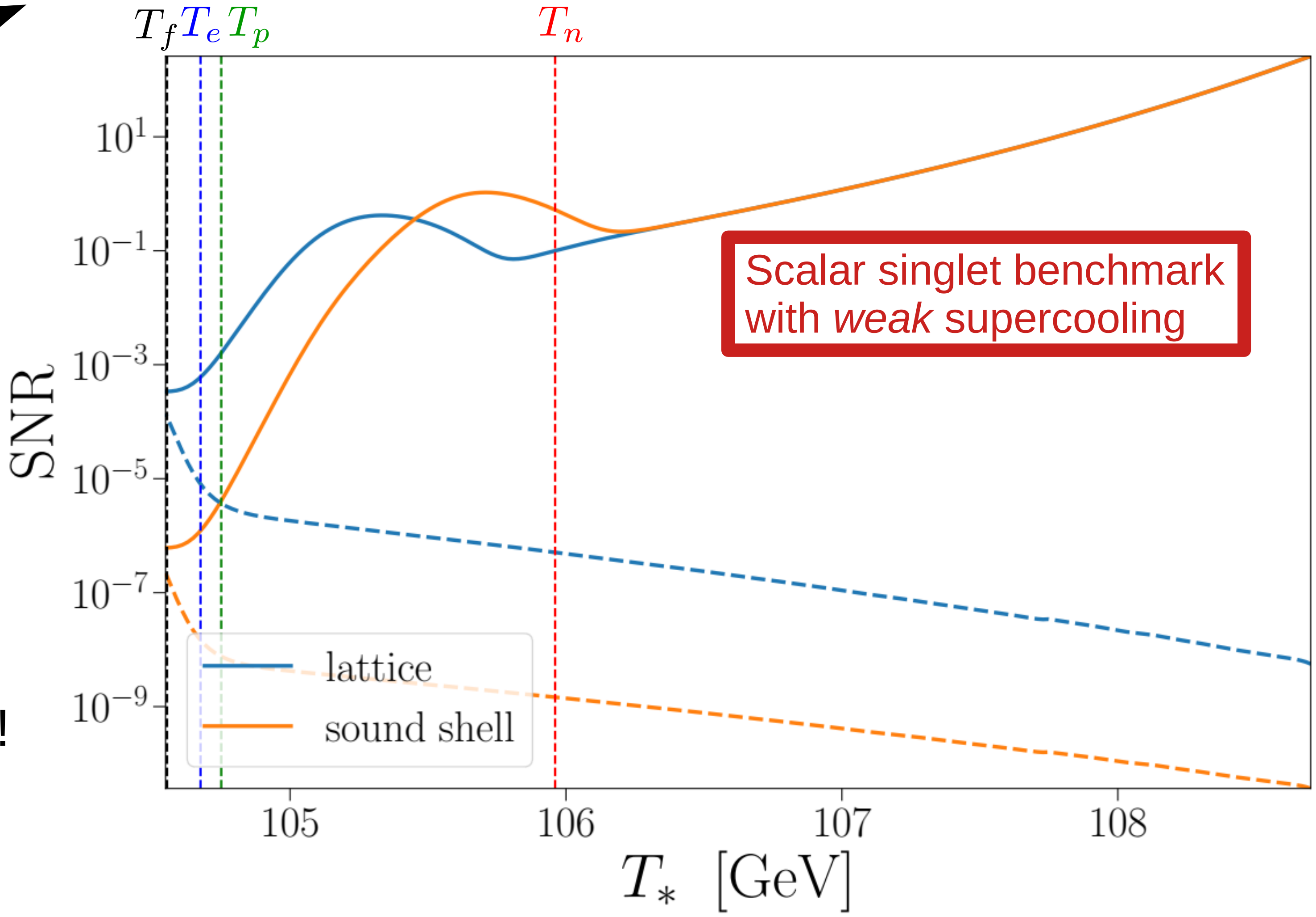
Temperature dependence

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Plot from here

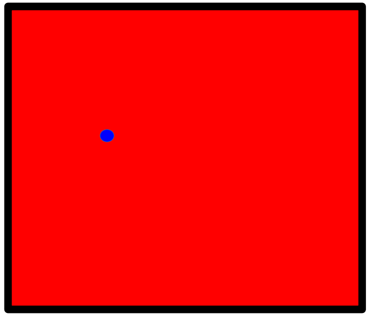
A fast transition still has big variation between T_n & T_p

LISA SNR
Still varies more by orders of magnitude!



Scalar singlet benchmark with *weak* supercooling

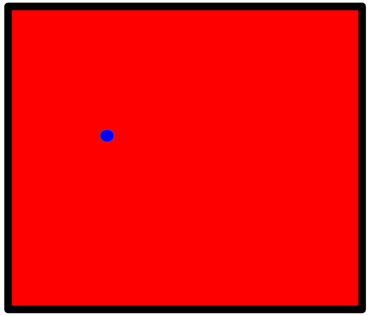
Temperature dependence



Nucleation temperature is a bad temperature to use

- not connceted to bubble collisions

Temperature dependence



Nucleation temperature is a bad temperature to use

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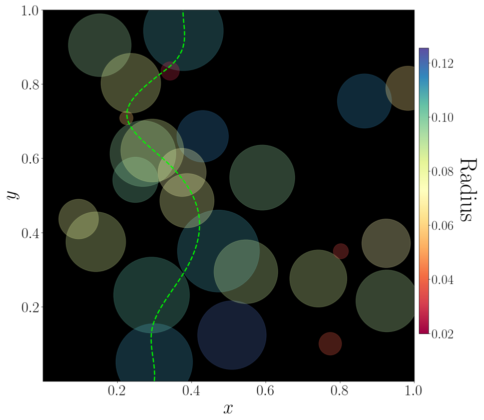
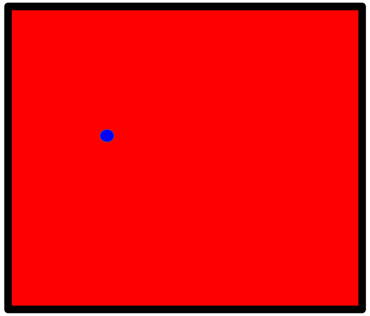
Percolation is directly defined in terms of contact between bubbles



Percolation temperature is much better, but…

Temperature dependence



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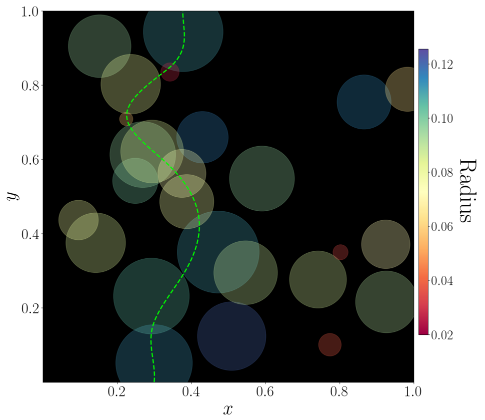
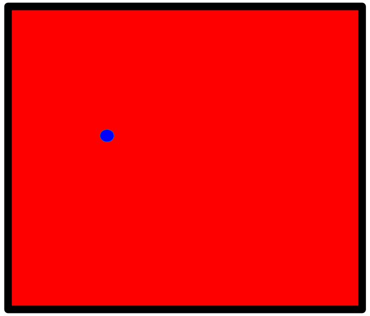


Percolation temperature is much better, but…

We still don’t know exactly correct temperature and...

Temperature dependence



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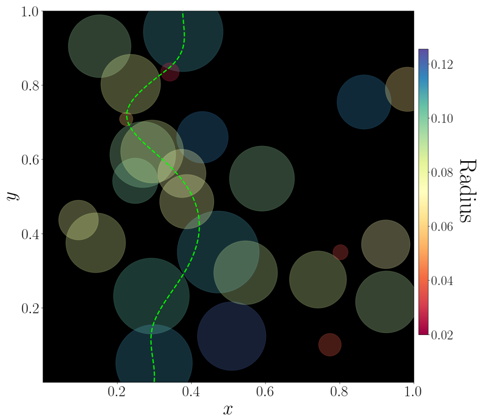


Percolation temperature is much better, but…

We still don’t know exactly correct temperature and...

Percolation criteria $P_f(T_p) = 0.71$ does not account for expanding space time

→ Temperature dependence represents a significant uncertainty

Numerical Packages

[PA, C. Balázs, A. Fowlie, W. Searle, G. White, L. Morris, Y. Xiao and Y. Zhang]

The good news is many of these issues can be avoided with careful numerical implementations

We are developing a set of numerical packages for PhaseTransitions:
PhaseTracer, BubbleProfiler and TransitionSolver

$\hat{V}_{\text{eff}} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$

PhaseTracer: handles this part $\longrightarrow$ mapping out the phases

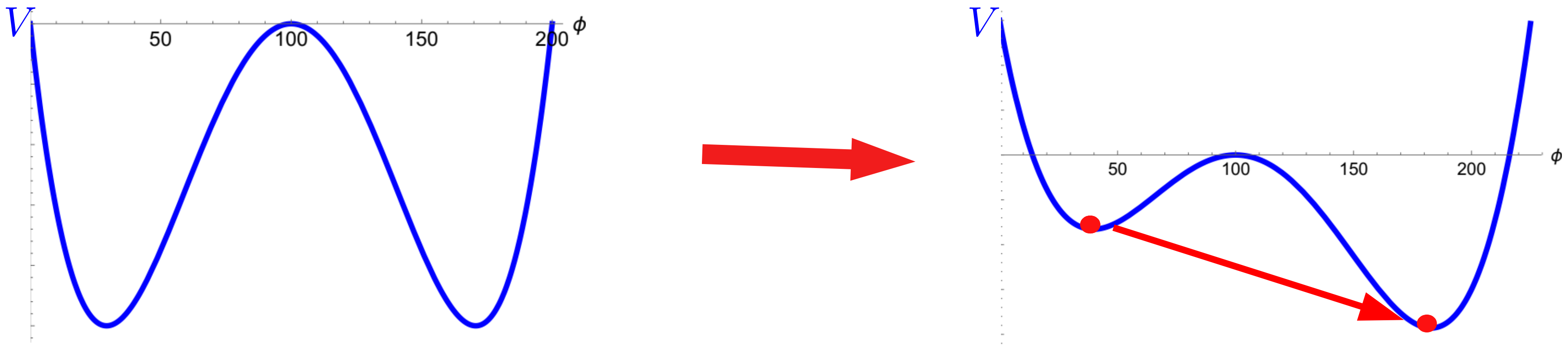
PhaseTracer reveals all potential phase transitions in cosmological history

$$\hat{V}_{\text{eff}} \rightarrow \phi_i^{min}(T) \rightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \rightarrow P_f(T), N(T) \rightarrow \alpha, \beta, v_w \rightarrow h^2\Omega(f)$$

PhaseTracer: handles this part → mapping out the phases

PhaseTracer reveals all potential phase transitions in cosmological history

Critical temperature $T = T_c$ $\qquad\qquad$ $T < T_c$



Barrier means phase transition happens after critical temperature

$$\hat{V}_{\text{eff}} \rightarrow \phi_i^{min}(T) \rightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \rightarrow P_f(T), N(T) \rightarrow \alpha, \beta, v_w \rightarrow h^2\Omega(f)$$

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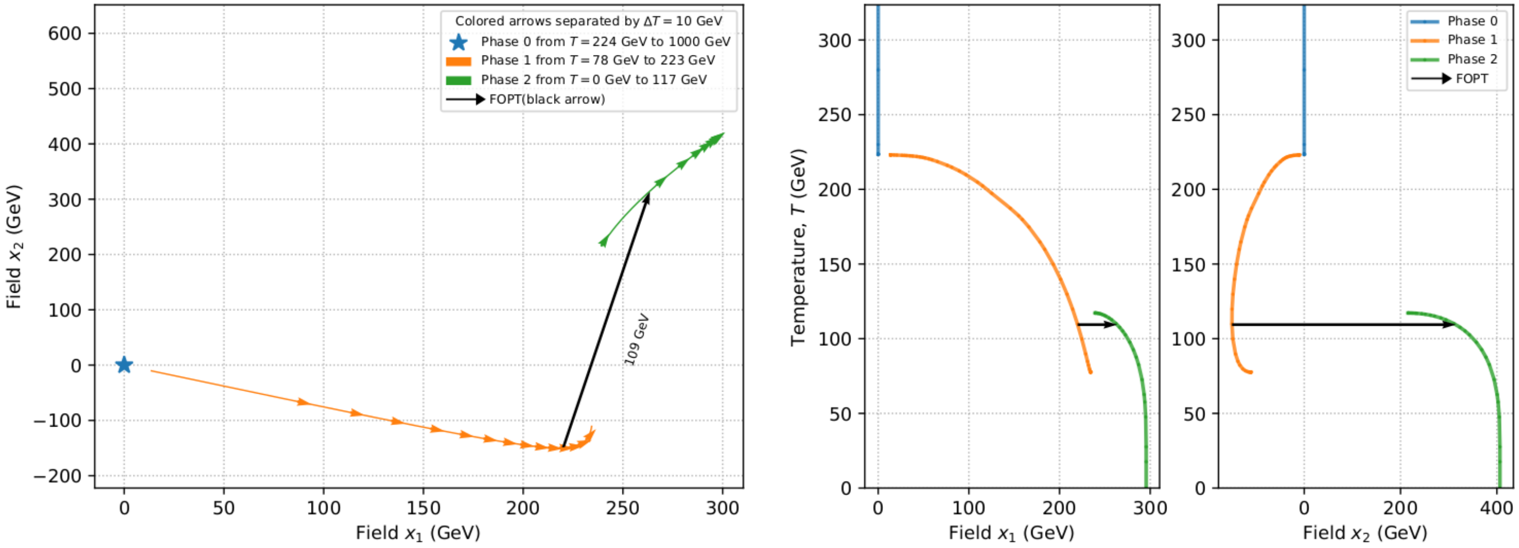


[PhaseTracer, PA, Csaba Balazs, Andrew Fowlie, Yang Zhang, Eur.Phys.J.C 80 (2020) 6, 567]

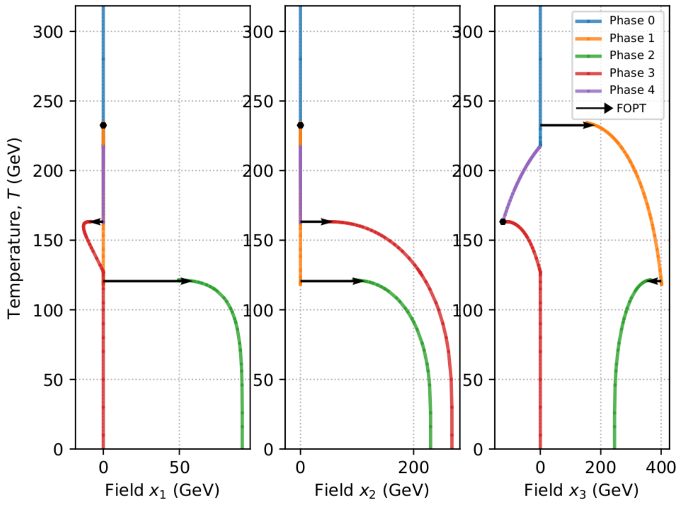
Handles multi-dimensional fields

$$\hat{V}_{\text{eff}} \rightarrow \phi_i^{min}(T) \rightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \rightarrow P_f(T), N(T) \rightarrow \alpha, \beta, v_w \rightarrow h^2\Omega(f)$$

PhaseTracer: handles this part → mapping out the phases

PhaseTracer reveals all potential phase transitions in cosmological history

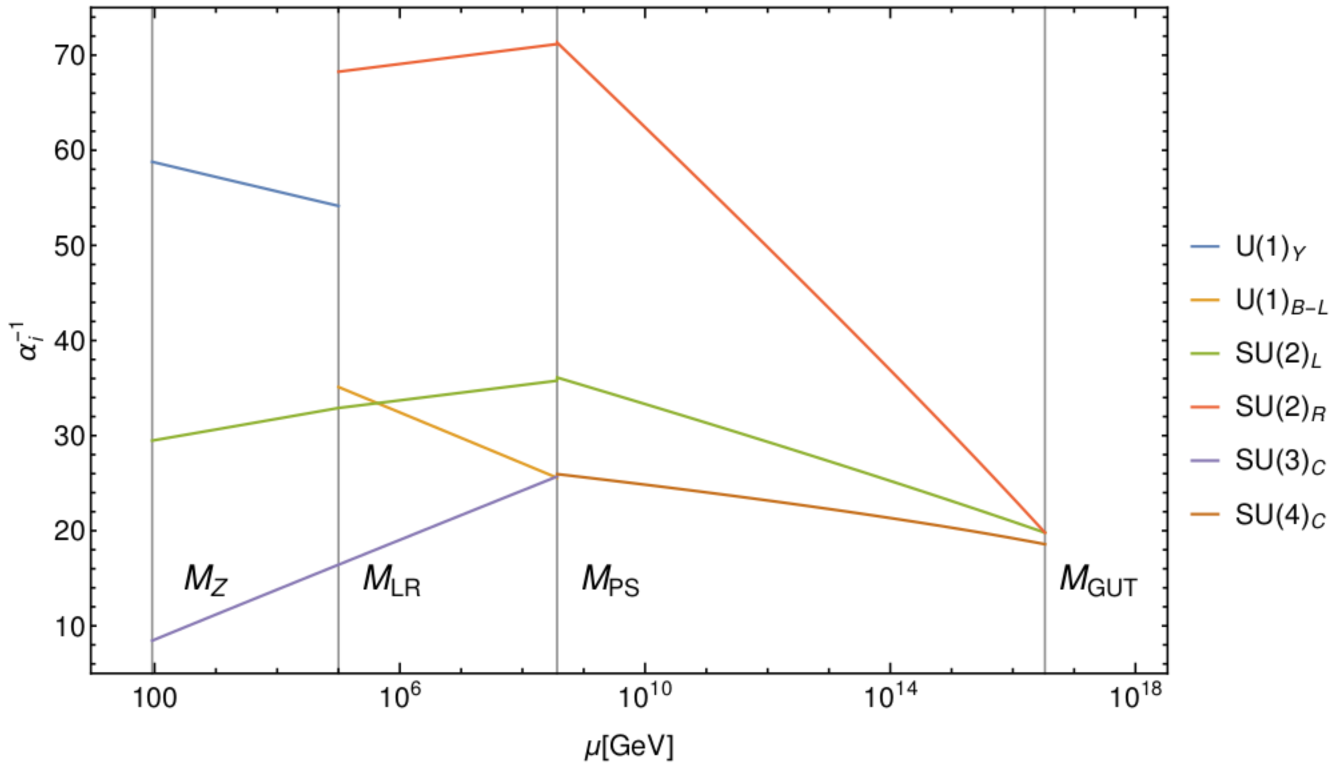
Not easy: multiple FOPTs & possible phase histories are common



PhaseTracer works very well if the effective potential input is reliable

Pati-Salam two step grand unification

$$
\begin{aligned}
SO(10) &\to SU(4) \times SU(2)_L \times SU(2)_R \\
&\to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
&\to SU(3)_C \times SU(2)_L \times U(1)_Y
\end{aligned}
$$



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&\to SU(3)_C \times SU(2)_L \times U(1)_Y
\end{aligned}
$$

Scalar fields at the Pati-Salam scale

Fields	$SU(4)_c$	$SU(2)_L$	$SU(2)_R$	Purpose
ϕ	1	2	2	Breaks SM
Δ_R	$\overline{10}$	1	3	Breaks LR
Δ_L	$\overline{10}$	3	1	Seesaw
Ξ	15	1	1	Breaks PS
Ω_R	15	1	3	Unification

Gravitational waves and thermal parameters

Lattice fit to single broken power law for sound wave source :

[M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, PRD 96 (2017) 103520]

$$h^2\Omega^{\rm lat}_{\rm sw}(f) = 5.3\times 10^{-2}\, R_\Omega K^2 \left(\frac{HL_*}{c_{s,f}}\right)\Upsilon(\tau_{\rm sw})S_{\rm sw}(f),$$

← Shape

Speed of sound in false vacuum

Accounts for finite lifetime of source

Sound shell model:

[Hindmarsh PRL 120 (2018) 071301, (+Hijazi) JCAP 12 (2019) 062, + (C. Gowling, D.C. Hooper and J. Torrado), JCAP 04 (2023) 061]

$$h^2\Omega_{\rm sw}(f) = 0.03 R_\Omega K^2 \left(\frac{H_* L_*}{c_{s,f}}\right)\Upsilon(\tau_{\rm sw})\frac{M(s,r_b,b)}{\mu_f(r_b)}$$

← Shape

Sound shell model is new but very promising

Turbulence also contributes, but not well modeled → Significant uncertainty!

Comparison of predictions for a weakly supercooled point in SSM

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Differences in sound wave SNR: latent heat (and pressure) variants give substanial differences

Variation	$h^2\Omega_{\rm sw}^{\rm lat}$ ($\times 10^{-13}$)	$h^2\Omega_{\rm sw}^{\rm ss}$ ($\times 10^{-14}$)	$f_{\rm sw}^{\rm lat}$ ($\times 10^{-5}$)	$f_{\rm sw}^{\rm ss}$ ($\times 10^{-4}$)	$h^2\Omega_{\rm turb}$ ($\times 10^{-16}$)	$f_{\rm turb}$ ($\times 10^{-5}$)	$\rm SNR_{lat}$	$\rm SNR_{ss}$	α ($\times 10^{-2}$)	κ	K ($\times 10^{-3}$)
None	3.590	5.673	129.6	60.20	3.898	287.0	10.08	2.031	5.450	0.2074	10.64
$T_* = T_e$	2.552	4.042	159.9	73.75	2.662	354.2	8.763	1.204	5.575	0.2096	10.99
$T_* = T_f$	2.146	3.410	181.7	82.91	2.187	402.5	8.110	0.8892	5.771	0.2129	11.54
$T_* = T_n$	676.5	1046	2.189	1.098	8968	4.849	1.310	5.142	4.297	0.1857	7.597
$R_{\rm sep}(\beta)$	2.019	3.191	177.5	82.45	2.078	393.1	7.510	0.8449			
$K(\alpha(\theta))$	3.372	5.329			3.676		9.469	1.908	5.362	0.2011	10.23
$K(\alpha(p))$	1.428	2.256			1.649		4.010	0.8081	3.698	0.1682	5.997
$K(\alpha(\rho))$	14.45	22.84			14.61		40.59	8.172	10.35	0.2736	25.68
ϵ_2					11.03		10.11	2.064			
ϵ_3					290.2		11.21	3.406			
ϵ_4					301.7		11.26	3.462			

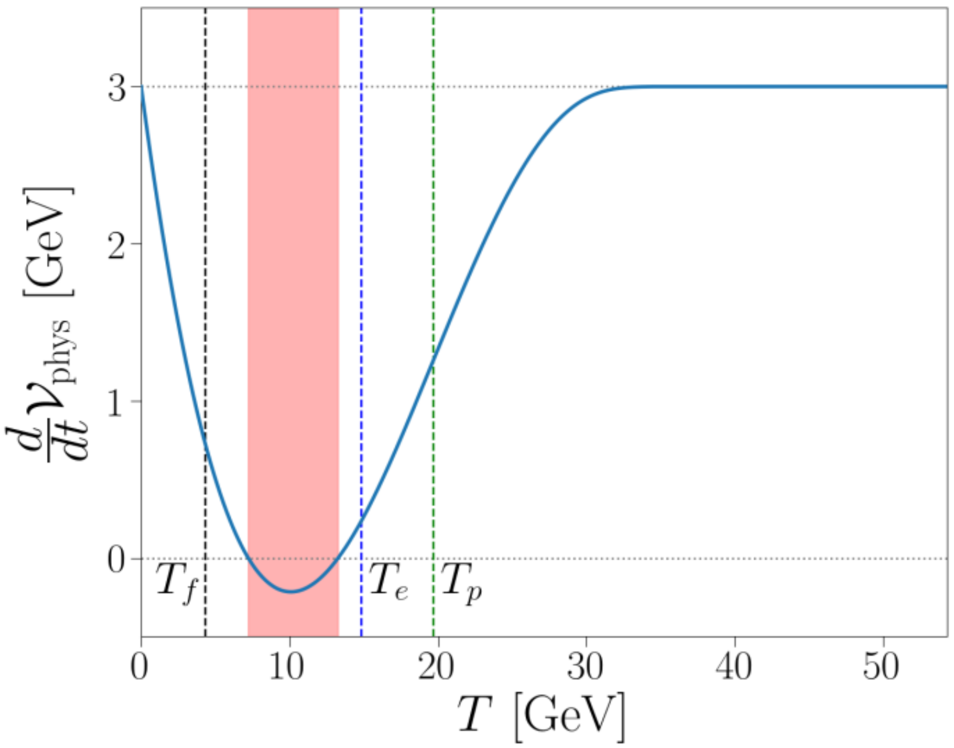
Comparison of predictions for a strongly supercooled point

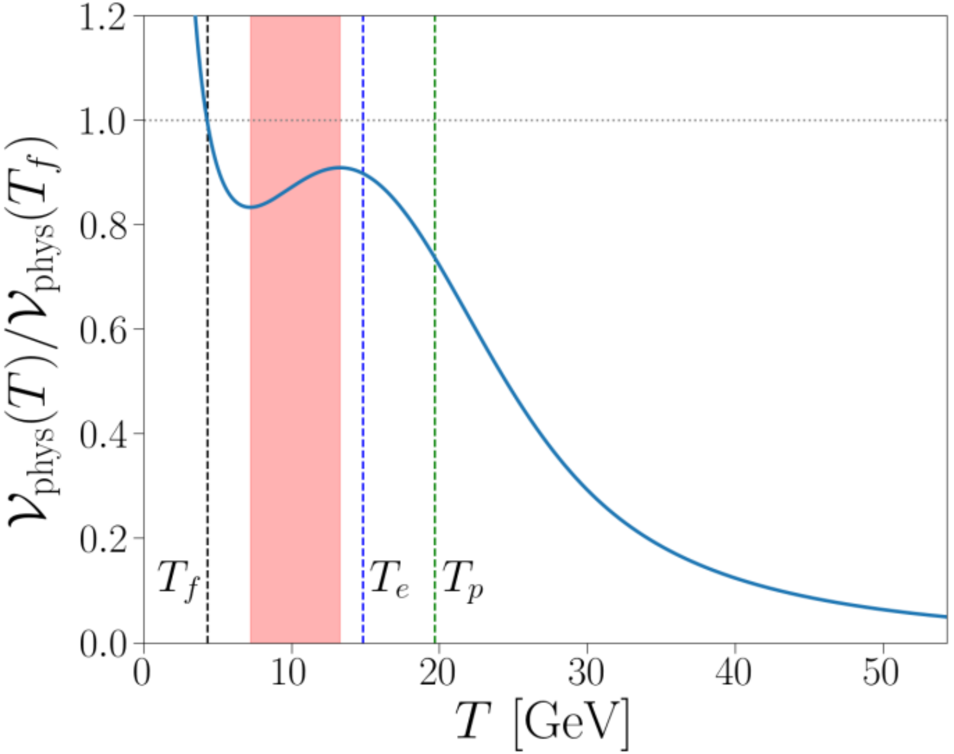
[PA, L. Morris, Z. Xu, arXiv:2309.05474]

However the variation in K estimates is much smaller for strongly supercooled scenarios

Variation	$h^2\Omega_{\rm sw}^{\rm lat}$ ($\times10^{-7}$)	$h^2\Omega_{\rm sw}^{\rm ss}$ ($\times10^{-8}$)	$f_{\rm sw}^{\rm lat}$ ($\times10^{-6}$)	$f_{\rm sw}^{\rm ss}$ ($\times10^{-6}$)	$h^2\Omega_{\rm turb}$ ($\times10^{-10}$)	$f_{\rm turb}$ ($\times10^{-6}$)	$\mathrm{SNR_{lat}}$	$\mathrm{SNR_{ss}}$	α	κ	K
None	1.861	3.748	9.345	23.48	6.348	20.70	249.6	307.7	1.651	0.7175	0.4536
$T_* = T_e$	4.318	8.872	7.908	19.12	14.74	17.52	443.7	498.2	4.257	0.8422	0.6950
$T_* = T_f$	17.04	35.42	4.111	9.722	81.84	9.106	864.5	876.4	71.06	0.9831	0.9803
$R_{\rm sep}(\beta_V)$	1.193	2.402	12.80	32.17	3.394	28.36	222.6	356.9			
$K(\alpha(\theta))$	1.819	3.663			6.227		244.9	301.5	1.605	0.7269	0.4478
$K(\alpha(p))$	1.768	3.560			6.083		239.2	294.2	1.564	0.7269	0.4409
$K(\alpha(\rho))$	1.967	3.962			6.646		261.4	323.0	1.728	0.7383	0.4677
ϵ_2					17.95		700.0	742.2			
ϵ_3					0		18.36	130.9			
ϵ_4					288.4		11210	11230			

Addional check for Percolation / completion





[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]

To ensure it really completes, also require: $\frac{\mathrm{d}\mathcal{V}_f^{\rm phys}}{\mathrm{d}T} < 0$

Non-trivial because whole volume is expanding

The duration affects the of the source of gravitational waves affects the GW signal a lot

This depends on the particle physics model

The duration can be related to a length scale and in hydrodynamical simulations of sound waves contributions the mean bubble separation is used:

$$R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')}$$

bubble number density

Best treatement

This can also be estimated by taylor expanding the bounce action

$$S(t) \approx S(t_*) + \left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*}(t-t_*) + \frac{1}{2}\left.\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}\right|_{t=t_*}(t-t_*)^2 + \cdots ,$$

1st order → explonential nucleation rate $\Gamma(t) = \Gamma(t_*)\exp(\beta(t-t_*)),$

$$\beta = -\left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*} = HT_* \left.\frac{\mathrm{d}S}{\mathrm{d}T}\right|_{T=T_*}$$

Widely used to replace mean bubble separation

$$R_{\rm sep} = (8\pi)^{\frac{1}{3}} \frac{v_w}{\beta}$$

Rough approximation

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2nd order → Gaussian nucleation rate $\Gamma(t) = \Gamma(t_*)\exp\left(\frac{\beta_{\rm V}^2}{2}(t-t_*)^2\right),$

$$\beta_{\rm V} = \sqrt{\left.\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}\right|_{t=t_\Gamma}}$$

Can be used to replace mean bubble separation

$$R_{\rm sep} = \left(\sqrt{2\pi}\frac{\Gamma(T_\Gamma)}{\beta_{\rm V}}\right)^{-\frac{1}{3}}$$

Rough approximation

The mean bubble separation varies a lot with temperature

Should not be used until $T \approx T_p$

For fast transitions

Estimating this with $\beta(T_p)$ GW amp. falls by factor 2 (larger variation in SNR)
Worse if using $\beta(T_n)$ as is standard practise

Mean bubble radius is more stable and $\beta(T)$ tracks this better.

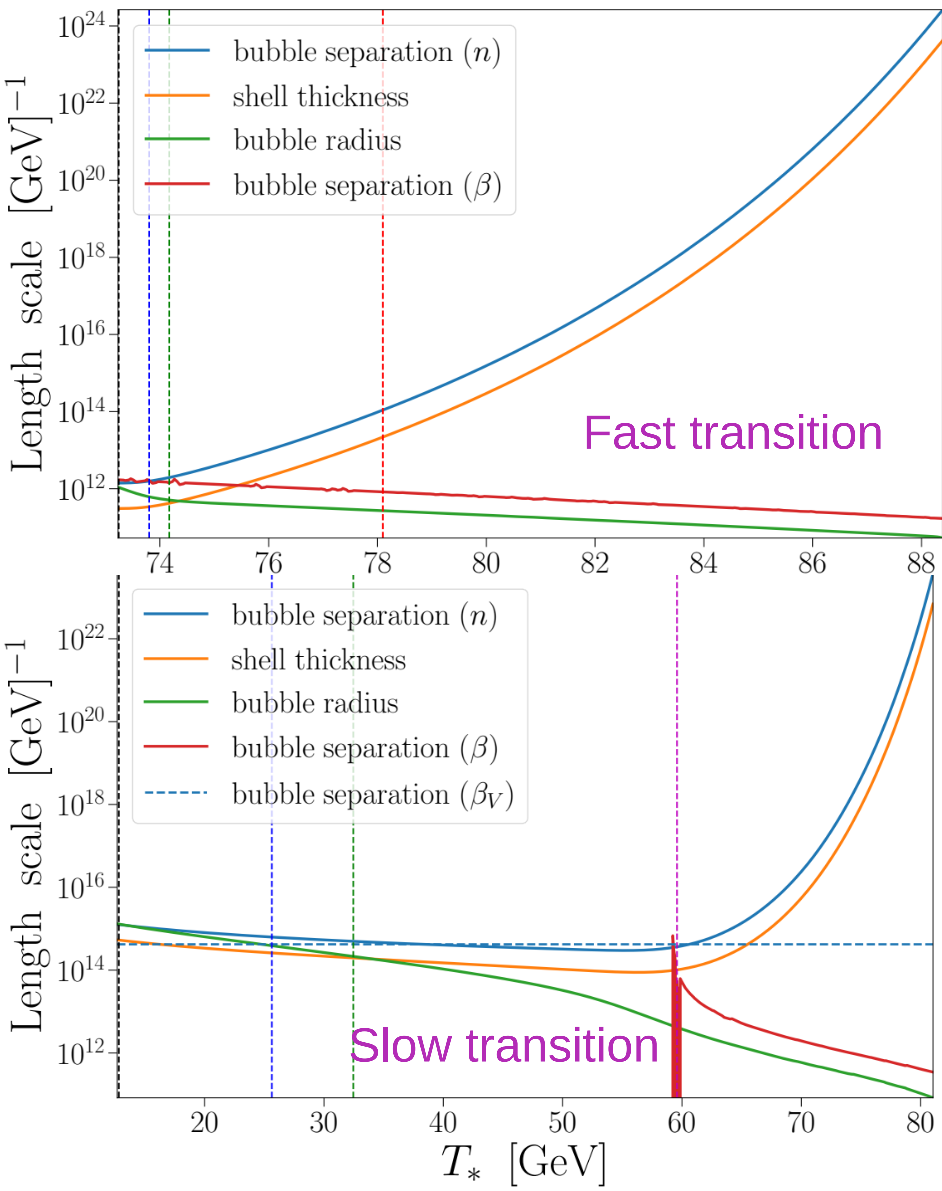
For slow transitions

Mean bubble radius varies more as bubbles have longer to grow.

Using $\beta(T_p)$ makes no sense below T_Γ orders of magnitude errors above

β_V gives a factor 1.5 drop in GW amplitide

[PA, L. Morris, Z. Xu, arXiv:2309.05474]



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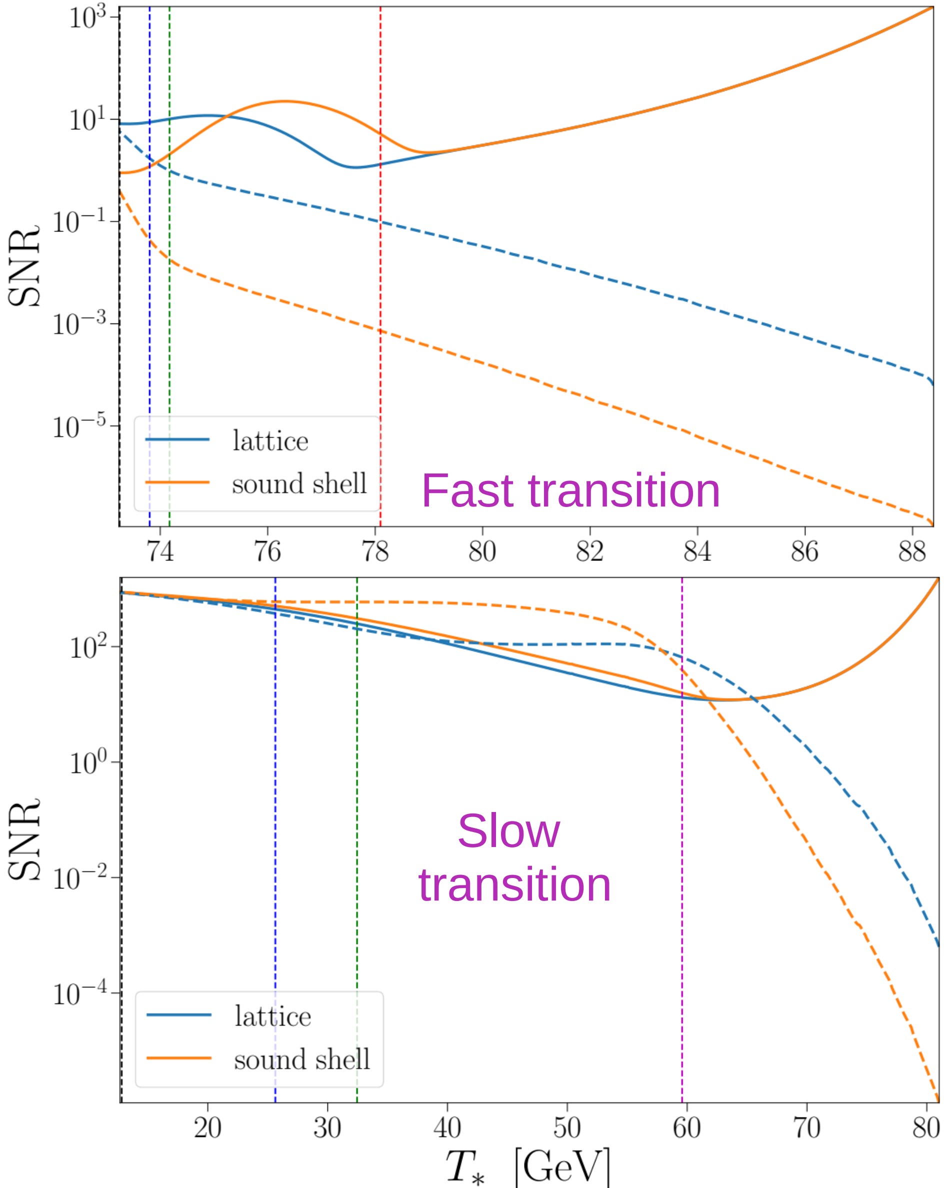


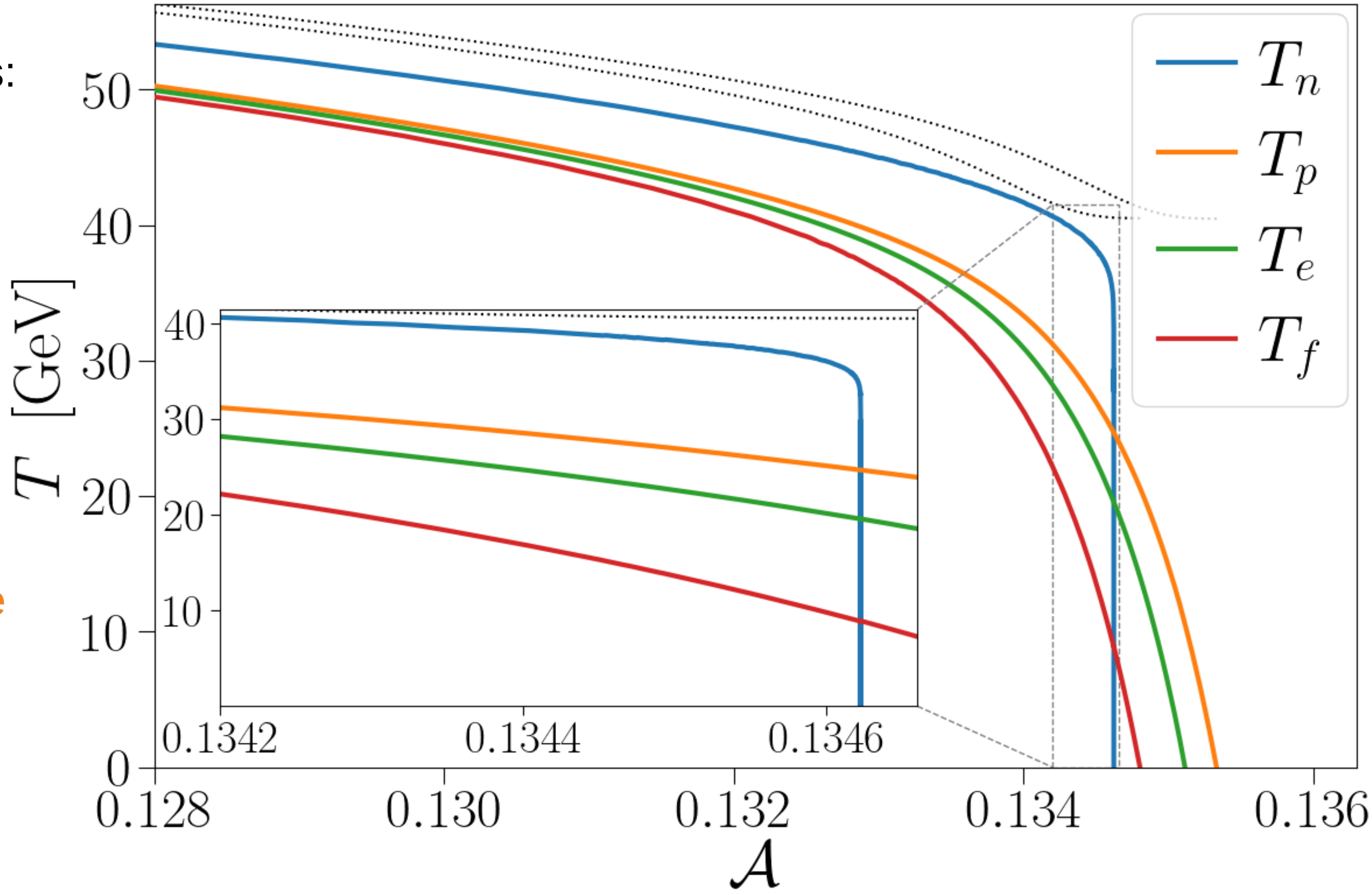
Milestone temperatures

[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]

Nucleation temperature is:

- Not related to bubble collisions
- Not related to other temperatures
- May not even exist

Percolation temperature is a better choice for GWs



From particle physics theory to GWs

$$\hat{V}_{\rm eff} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$$

Effective Potential: can be computed perturbatively with finite temperature quantum field theory

$$\hat{V}_{\rm eff} = V_0 + V_1^{T=0} + V_{1T}$$

Tree-level

Zero temperature Coleman-Weinberg corrections

Finite temperature corrections

Effective Potential: can be computed perturbatively with finite temperature quantum field theory

$$\hat{V}_{\rm eff} = V_0 + V_{1,T=0} + V_{1T}$$

$$V_{1,\,T=0}^{R_\xi} = \frac{1}{4(4\pi)^2}\left[\sum_\phi n_\phi m_\phi^4(\{\phi_j\},\xi)\left(\ln\left(\frac{m_\phi^2(\{\phi_j\},\xi)}{Q^2}\right) - k_s\right)\right.$$
$$+\sum_V n_V m_V^4(\{\phi_j\})\left(\ln\left(\frac{m_V^2(\{\phi_j\})}{Q^2}\right) - k_V\right) - \sum_V (\xi m_V^2(\{\phi_j\}))^2\left(\ln\left(\frac{\xi m_V^2(\{\phi_j\})}{Q^2}\right) - k_V\right)$$
$$\left. - \sum_f n_f m_f^4(\{\phi_j\})\left(\ln\left(\frac{m_f(\{\phi_j\})^2}{Q^2}\right) - k_f\right)\right],$$

$$V_{1T}^{R_\xi} = \frac{T^4}{2\pi^2}\left[\sum_i n_\phi J_B\left(\frac{m_{\phi_i}^2(\xi)}{T^2}\right) + \sum_j n_V J_B\left(\frac{m_{V_j}^2}{T^2}\right) - \frac{1}{3}\sum_j n_V J_B\left(\frac{\xi m_{V_j}^2}{T^2}\right) + \sum_l n_f J_F\left(\frac{m_{f_l}^2}{T^2}\right)\right]$$

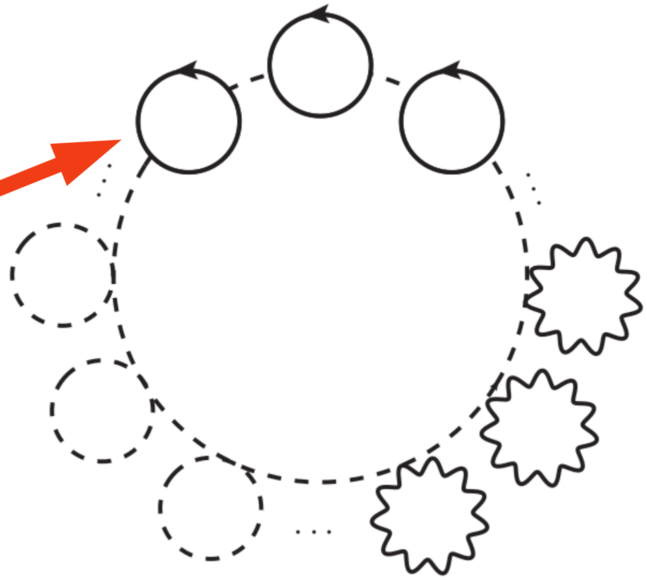
$$J_B(y^2) = \int_0^\infty dk\ k^2 \log\left[1 - e^{-\sqrt{k^2+y^2}}\right] \quad J_F(y^2) = \int_0^\infty dk\ k^2 \log\left[1 + e^{-\sqrt{k^2+y^2}}\right]$$

Effective Potential

Perturbative estimates of the effective potential can be tricky

Resummation needed to to deal with high temperatures spoiling perturbativity

Daisy diagram with N-loops:



Individual petals are inserted one-loop corrections

Resum daisy diagrams for leading order $\frac{T^2}{m^2}$

From particle physics theory to GWs

$$\hat{V}_{\rm eff} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$$

Effective Potential: can be computed perturbatively with finite temperature quantum field theory

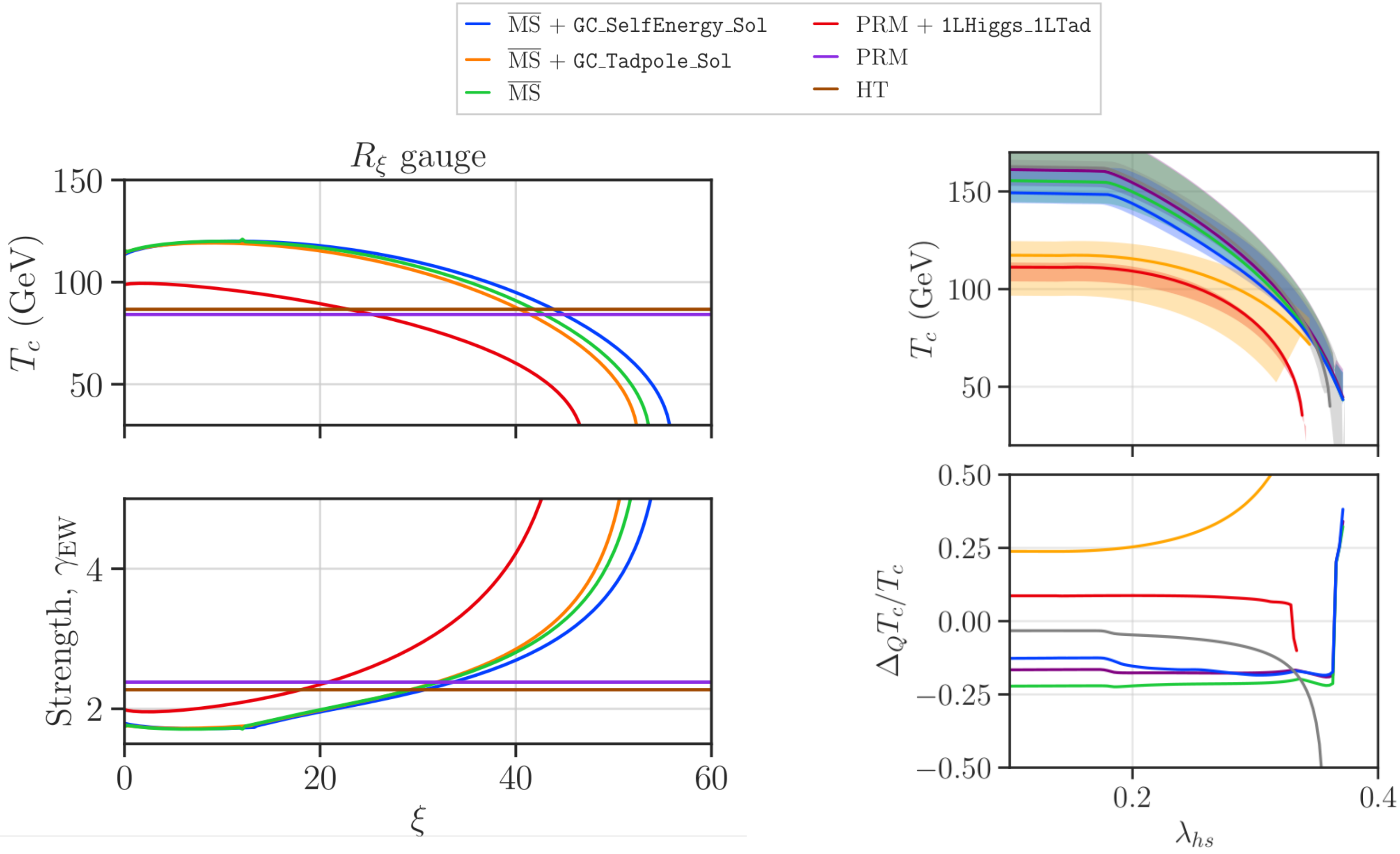
However there are problems appling this for phase transitons at finite temp

- Unphysical Gauge dependence
- Infrared divergences / problems with perturbativity for large T^2/m^2
- Many different scales in the problem
- thus large dependence on the renormalisation scale

Effective Potential

[PA, C. Balazs, A. Fowlie, L. Morris, G. White and Y.~Zhang, JHEP 01 (2023) 050]

Significant variance from gauge and renormalisation scale



Effective Potential

[PA, C. Balazs, A. Fowlie, L. Morris, G. White and Y.~Zhang, JHEP 01 (2023) 050]

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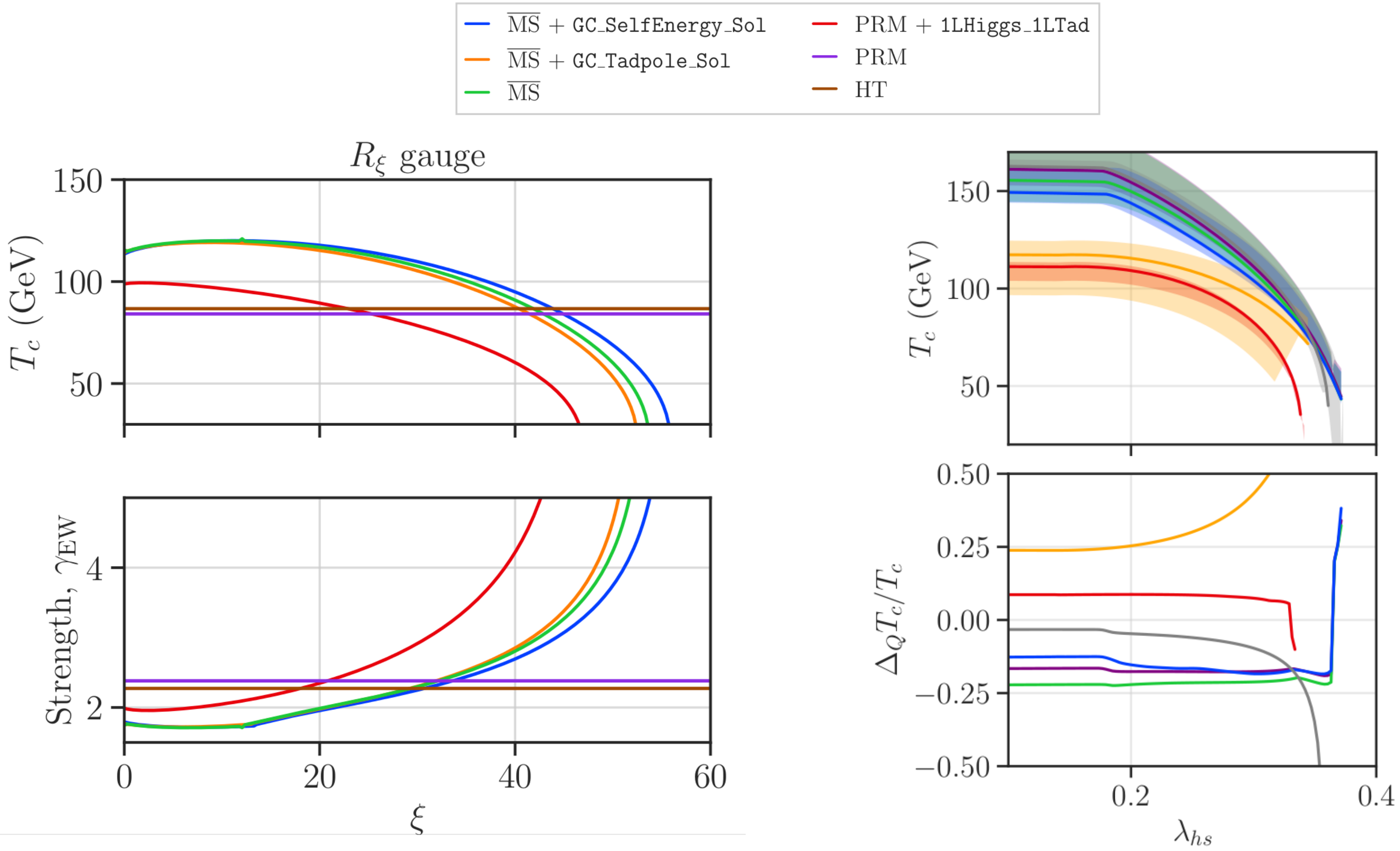


Effective Potential

These issues have substantial impact on uncertainties in GW predictions

[Djuna Croon, Oliver Gould, Philipp Schicho, Tuomas V. I. Tenkanen, Graham White, JHEP 04 (2021) 055]

$\Delta\Omega_{GW}/\Omega_{GW}$	4d approach	3d approach
RG scale dependence	$\mathcal{O}(10^2 - 10^3)$	$\mathcal{O}(10^0 - 10^1)$
Gauge dependence	$\mathcal{O}(10^1)$	$\mathcal{O}(10^{-3})$
High-T approximation	$\mathcal{O}(10^{-1} - 10^0)$	$\mathcal{O}(10^0 - 10^2)$
Higher loop orders	unknown	$\mathcal{O}(10^0 - 10^1)$
Nucleation corrections	unknown	$\mathcal{O}(10^{-1} - 10^0)$
Nonperturbative corrections	unknown	unknown

High temperature effects can be resummed by effective field theory techniques

But non-perurbative effects may cause problems

Effective Potential

Most rigorous approach is to do this non-perturbatively on lattice

This is how we know SM EW and QCD transtions are smooth cross-overs

[K. Kajantie, M. Laine, J. Peisa, K. Rummukainen, M. Shaposhnikov, PRL 77 (1996) 2887-2890,
Y. Aoki, G. Endrodi*, Z. Fodor*, S. D. Katz*, and K. K. Szabo, Nature, 443:675–678, 2006]
[*Eötvös affiliation]

Downside: Very time consuming to do this on the lattice

Not feasible in general for new physics, we have:

- many models
- many transitions in specific models
- huge parameter spaces

⟶ Tension between rigour and feasability

Effective Potential

- Standard: 4D Perturbative approach with “Daisy resummation”

  Easy to implement
  Feasible for scans

- Better: 3D EFT Perturbative calculation

  Hard to implement*
  Feasible for scans

- Gold standard: non-perturbative lattice

  Hard to implement
  Not feasible for scans

* Very recently DRalgo code was developed to make this easier!

[Andreas Ekstedt, Philipp Schicho, Tuomas V. I. Tenkanen, Comp.Phys.Comm. 288 (2023) 108725]

State of the art: match to 3DEFT models with lattice results where possible,
use 3DEFT where not available (or create new lattice results...)

See e.g. [PRD 100 (2019) 11, 115024, Phys.Rev.Lett. 126 (2021) 17, 171802]

From particle physics theory to GWs

$\hat{V}_{\text{eff}} \rightarrow \phi_i^{min}(T) \rightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \rightarrow P_f(T), N(T) \rightarrow \alpha, \beta, v_w \rightarrow h^2\Omega(f)$

PhaseTracing

So cubic terms are generated at finite temperature

Tree-level cubic terms can also be introduced in SM extensions

These may or may not lead to first order phase transitions

Depends on detailed calculation, e.g. SM is a smooth cross-over for the measured Higgs mass..

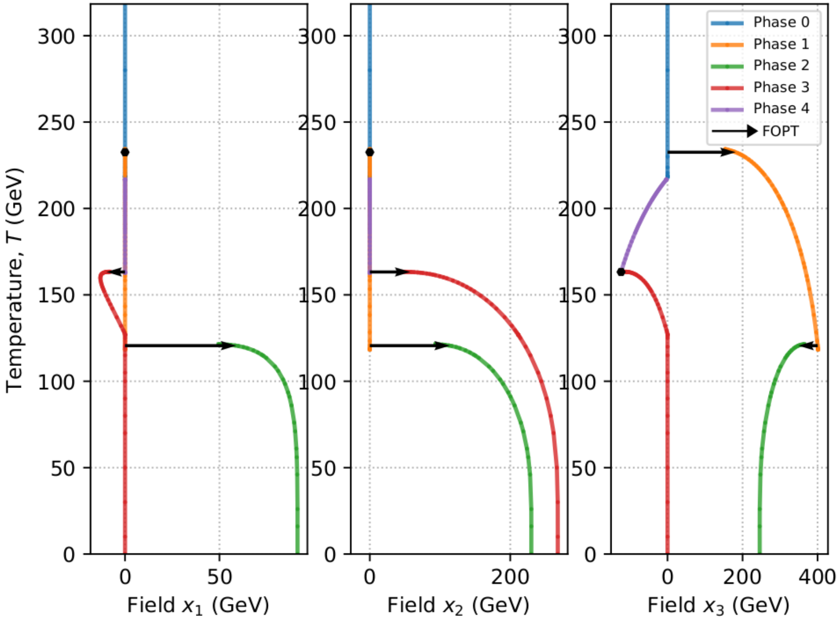
...but could have been first order if the Higgs mass was much lighter.

From particle physics theory to GWs

$$\hat{V}_{\text{eff}} \rightarrow \phi_i^{min}(T) \rightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \rightarrow P_f(T), N(T) \rightarrow \alpha, \beta, v_w \rightarrow h^2\Omega(f)$$

PhaseTracing

This is not straightforward: multiple FOPTs and possible paths common in realistic models



Careful algorithims needed to handle this, e.g.

- PhaseTracer ← *My own code, but I do recommend this one*
- Cosmotransitions ← *Tricky to use, often just hangs or exits*
- BSMPT ← *Simple and fast but won't get complicated patterns, mutiple PTs*

[PhaseTracer, PA, Csaba Balazs, Andrew Fowlie, Yang Zhang, Eur.Phys.J.C 80 (2020) 6, 567]

From particle physics theory to GWs

$\hat{V}_{\rm eff} \rightarrow \phi_i^{min}(T) \rightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \rightarrow P_f(T), N(T) \rightarrow \alpha, \beta, v_w \rightarrow h^2\Omega(f)$

Transition rates Semi-classical approx $\Gamma \approx Ae^{-B}$

Action at saddle point

Fluctuations around saddle point

B solved by finding a “bounce” instanton solution numerically

Tricky numerical problem, many public bounce solvers

CosmoTransitions [C. L. Wainwright, CPC 183 (2012) 2006–2013,],

AnyBubble [A. Masoumi, K. D. Olum and B. Shlaer, JCAP 1701 (2017) 051],

BubbleProfiler [PA, Balazs, Bardsley, Fowlie, Harries & White CPC 244 (2019) 448-468]

SimpleBounce [Ryosuke Sato, CPC 258 (2021) 107566]

All bounce solvers to date have some signifcant drawbacks

(numerical stability, reliability, noise/precision, speed, number of fields)

From particle physics theory to GWs

$$\hat{V}_{\rm eff} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$$

Transition rates   Semi-classical approx $\Gamma \approx A e^{-B}$ ← Action at saddle point

Fluctuations around saddle point → A

A usually assumed less important,
Often estimated on dimensional grounds

$$A \approx T^4$$

$$A \approx T^4 \left(B/(2\pi T)^{3/2}\right)$$

Problem: what if A has exponential dependence?

→ Calculate it directly → BubbleDet

[Ekstedt, Gould, and Hirvonen, arXiv:2308.15652]

Bubble nucleation

Bubbles of the new phase
form at random locations

The bubbles that already formed
grow in size

while more bubbles nucleate

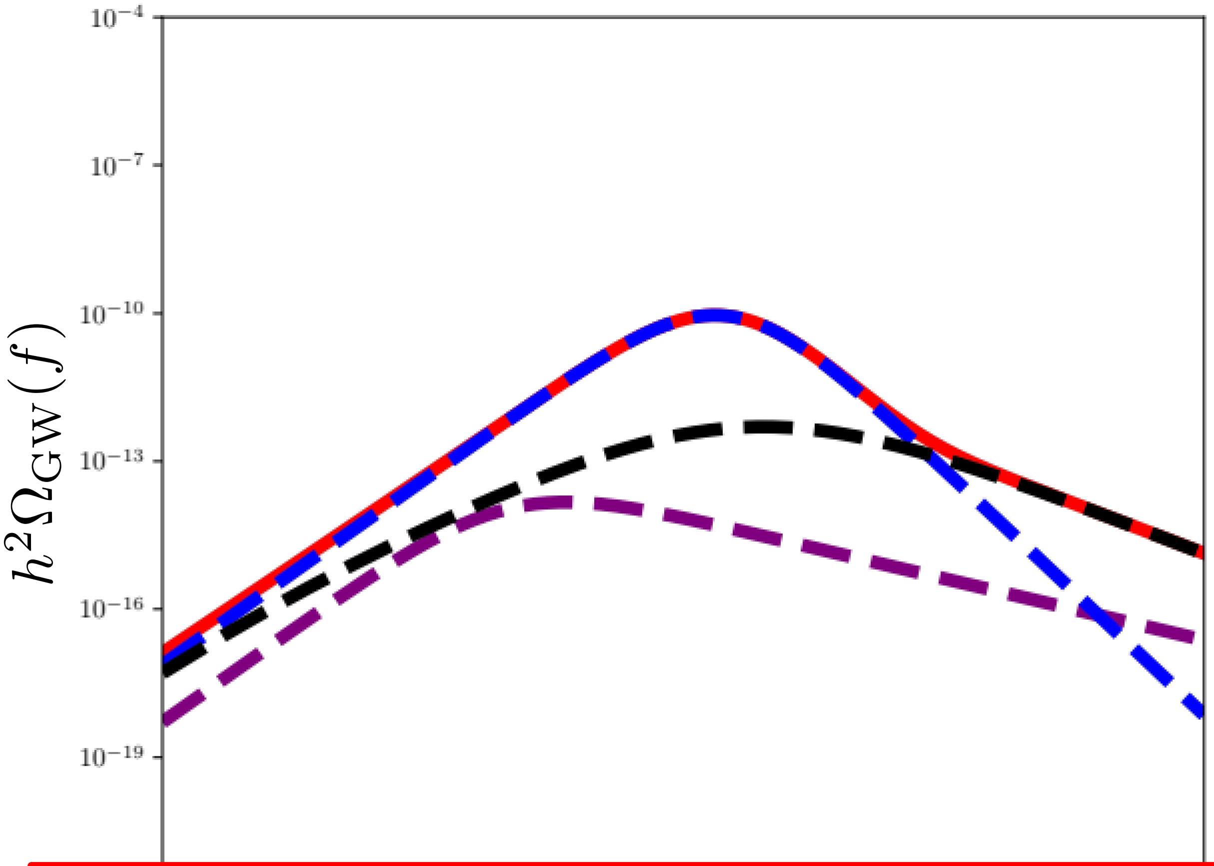
As the bubbles grow,
and the number increases,
collisions become more likely

And more and more of the space is
converted to the true vacuum

[image: from Lachlan Morris]

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_{\rm tot}} \frac{d\rho_{\rm GW}}{d\ln f}$$

$$h^2\Omega_{\rm GW} = h^2\Omega_{\rm coll} + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb}$$



The peak amplitide varies with the frequency

The signal has several contributions:

1) the collision of bubbles – which breaks their spherical symmetry.

2) waves of plasma accelerated. by the bubble wall.

3) shocks in the fluid leading to turbulence

Understanding this quantitatively requires hyrdodynamical simulations and/or clever modeling of how it happens

Times scales for sources gravitational waves affect the GWs signal

Depends on the particle physics model

Can be related to a length scale, mean bubble separation used in hydrodynamical simulations of sound:

$$R_{\text{sep}}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')}$$

bubble number density

Best treatment

Often estimated by taylor expanding the bounce action $\Gamma(t) = Ae^{-S(t)}$

$$S(t) \approx S(t_*) + \left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*} (t - t_*) + \frac{1}{2}\left.\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}\right|_{t=t_*} (t - t_*)^2 + \cdots ,$$

2nd order ⟶ Gaussian nucleation rate $\Gamma(t) = \Gamma(t_*) \exp\left(-\frac{\beta_\mathrm{V}^2}{2}(t - t_*)^2\right),$

$$\beta_\mathrm{V} = \sqrt{\left.\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}\right|_{t=t_\Gamma}}$$

Can be used to replace mean bubble separation

$$R_{\text{sep}} = \left(\sqrt{2\pi}\frac{\Gamma(T_\Gamma)}{\beta_\mathrm{V}}\right)^{-\frac{1}{3}}$$

Rough approximation

Times scales for sources gravitational waves affect the GWs signal

Depends on the particle physics model

Can be related to a length scale, mean bubble separation used in hydrodynamical simulations of sound:

$$R_{\text{sep}}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')}$$

bubble number density

One more thing:

Alternative length scale - mean bubble radius

$$\bar{R}(T) = \frac{T^2}{n_B(T)} \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')} \int_T^{T'} dT'' \frac{v_w(T'')}{H(T'')}.$$

This has been proposed in the literature but not used in simulations