Glueball Dark Matter: from Gravitational Waves to Production Mechanism

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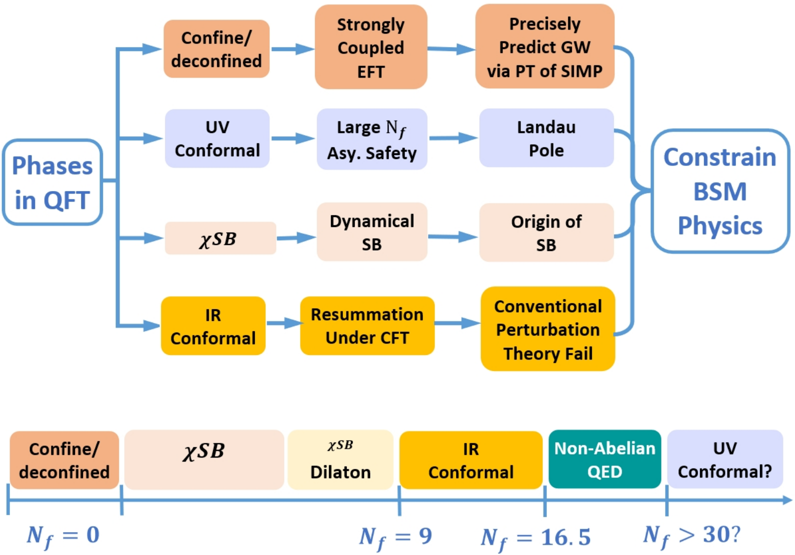
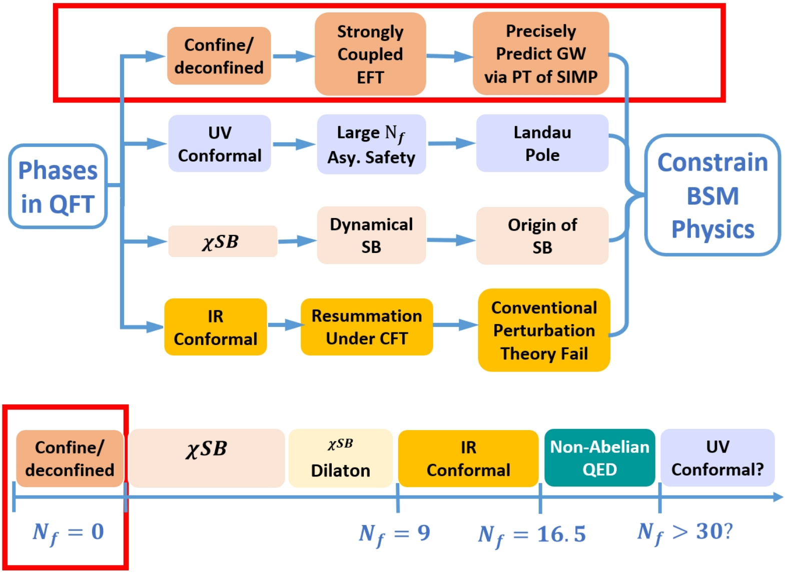
A Landscape of Phases in QFT (QCD-like Theory) and its Relation to BSM Physics

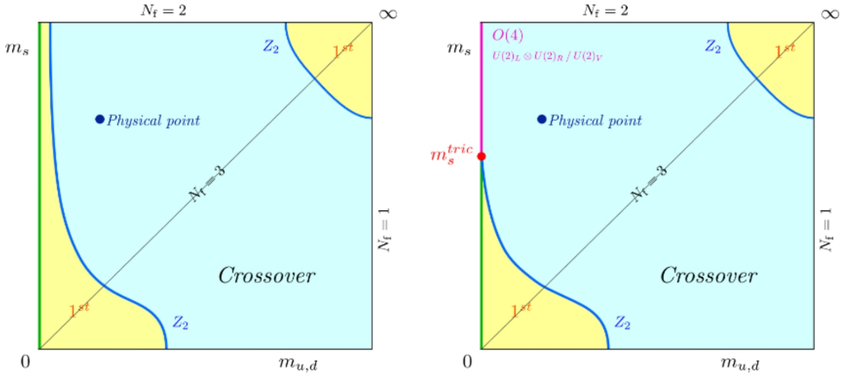


Confine/Deconfined Phase



What composes the strongly coupled sector?

- Dark Yang-Mills theories
- Pure gluons ⇒ confinement-deconfinement phase transition
- Gluons + Fermions
  - Fermions in fundamental representation ⇒ chiral phase transition
  - Fermions in adjoint rep. ⇒ confinement & chiral phase transition
  - Fermions in 2-index symmetric rep. ⇒ confinement & chiral phase transition
- Gluons + Fermions + Scalars (not explored yet)

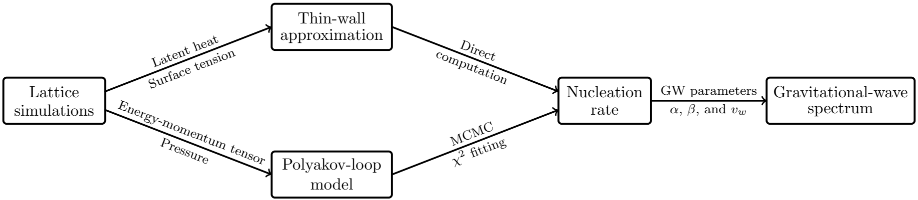


How to describe the strongly coupled sector?

- Pure gluons
  - Polyakov loop model (Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005; Kang, Zhu, Matsuzaki, JHEP 09 (2021) 060; Gao, Sun and White, arXiv:2405.00490.)
  - Matrix Model (Halverson, Long, Maiti, Nelson, Salinas, JHEP **05** (2021) 154)
  - Holographic QCD model (Ares, Henriksson, Hindmarsh, Hoyos, Jokela, PRD **105** (2022) 066020; Ares, Henriksson, Hindmarsh, Hoyos, Jokela, PRL **128** (2022) 131101)
- Gluons + Fermions
  - Polyakov loop improved Nambu-Jona-Lasinio model (Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003; Helmboldt, Kubo, Woude, PRD **100** (2019) 055025)
  - Linear sigma model (Helmboldt, Kubo, Woude, PRD **100** (2019) 055025)
  - Polyakov Quark Meson model (Pasechnik, Reichert, Sannino, Z-W W, JHEP **02** (2024) 159)

Procedure of pure gluon case

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005



Polyakov Loop Model for Pure Gluons: I

- Pisarski first proposed the Polyakov-loop Model as an effective field theory to describe the confinement-deconfinement phase transition of $SU(N)$ gauge theory (Pisarski, PRD **62** (2000) 111501).
- In a local $SU(N)$ gauge theory, a global center symmetry $Z(N)$ is used to distinguish confinement phase (unbroken phase) and deconfinement phase (broken phase)
- An order parameter for the $Z(N)$ symmetry is constructed using the Polyakov Loop (thermal Wilson line) (Polyakov, PLB 72 (1978) 477)

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp\left[i \int_0^{1/T} A_4(\vec{x},\tau)\,\mathrm{d}\tau \right]$$

  The symbol $\mathcal{P}$ denotes path ordering and A_4 is the Euclidean temporal component of the gauge field
- The Polyakov Loop transforms like an adjoint field under local $SU(N)$ gauge transformations

Polyakov Loop Model for Pure Gluons: II

- Convenient to define the trace of the Polyakov loop as an order parameter for the $Z(N)$ symmetry

$$\ell(\vec{x}) = \frac{1}{N}\mathrm{Tr}_c[\mathbf{L}],$$

where Tr_c denotes the trace in the colour space.

- Under a global $Z(N)$ transformation, the Polyakov loop ℓ transforms as a field with charge one

$$\ell \to e^{i\phi}\ell, \qquad \phi = \frac{2\pi j}{N}, \qquad j = 0, 1, \cdots, (N-1)$$

- The expectation value of ℓ i.e. $< \ell >$ has the important property:

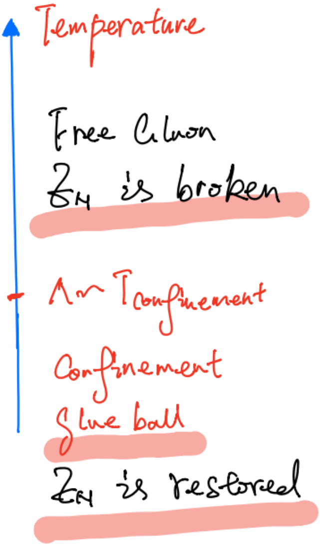
$$\langle\ell\rangle = 0 \quad (T < T_c, \ \mathrm{Confined}); \qquad \langle\ell\rangle > 0 \quad (T > T_c, \ \mathrm{Deconfined})$$

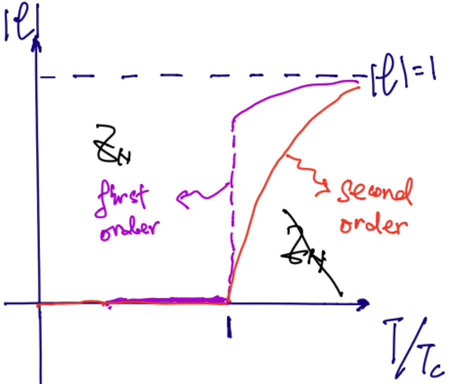
- At very high temperature, the vacua exhibit a $N-$fold degeneracy:

$$\langle\ell\rangle = \exp\left(i\frac{2\pi j}{N}\right)\ell_0, \qquad j = 0, 1, \cdots, (N-1)$$

where ℓ_0 is defined to be real and $\ell_0 \to 1$ as $T \to \infty$

Summary of Pure Gluon Facts





Second Order for $SU(2)$

First Order $SU(N)$ $(N\geq 3)$

Polyakov Loop Model

Effective Potential of the Polyakov Loop Model: I

- The simplest effective potential preserving the Z_N symmetry in the polynomial form is given by (Pisarski, PRD **62** (2000) 111501)

$$V_{\rm PLM}^{(\rm poly)} = T^4\left(-\frac{b_2(T)}{2}|\ell|^2 + b_4|\ell|^4 + \cdots - b_3\left(\ell^N + \ell^{*N}\right)\right)$$

$$\text{where}\quad b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 + a_4\left(\frac{T_0}{T}\right)^4$$

"$\cdots$" represent any required lower dimension operator than ℓ^N i.e. $(\ell\ell^*)^k = |\ell|^{2k}$ with $2k < N$.

- For the $SU(3)$ case, there is also an alternative logarithmic form

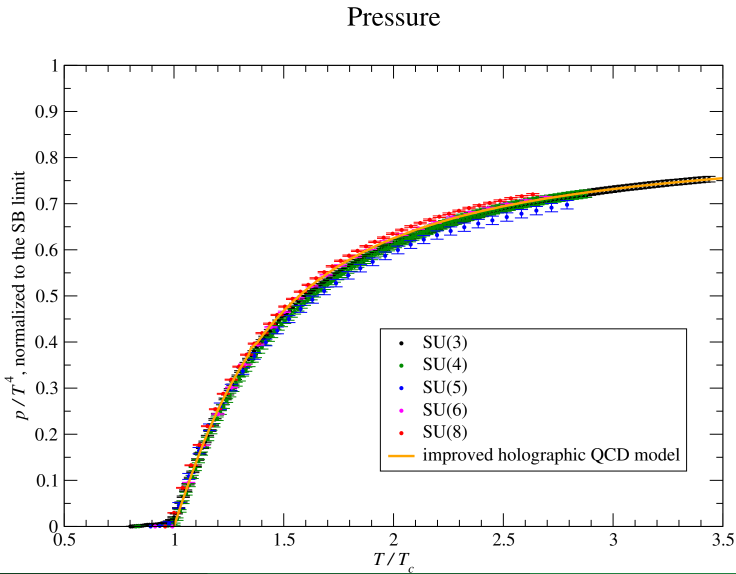
$$V_{\rm PLM}^{(3\log)} = T^4\left(-\frac{a(T)}{2}|\ell|^2 + b(T)\ln\left(1 - 6|\ell|^2 + 4(\ell^{*3} + \ell^3) - 3|\ell|^4\right)\right)$$

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3,\quad b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

- The a_i, b_i coefficients in $V_{\rm PLM}^{(\rm poly)}$ and $V_{\rm PLM}^{(3\log)}$ are determined by fitting the lattice results

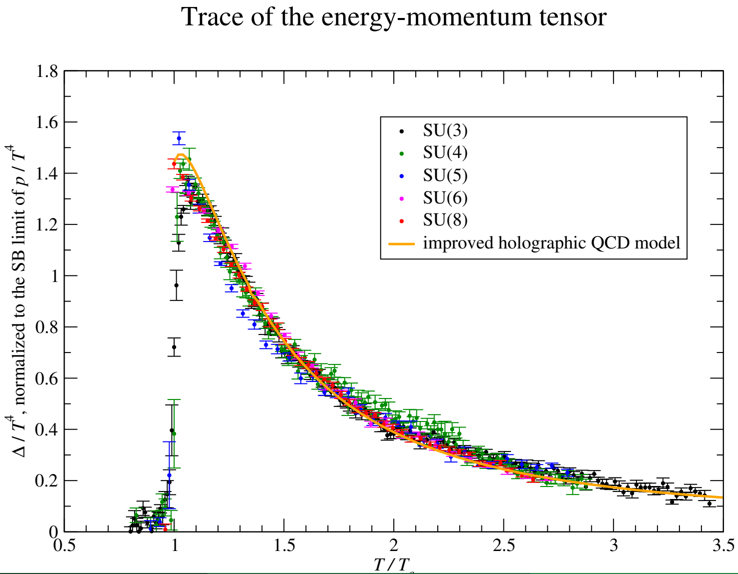
Fitting the Coefficients Using the Lattice Results: I

Marco Panero, Phys.Rev.Lett. 103 (2009) 232001



Fitting the Coefficients Using the Lattice Results: II

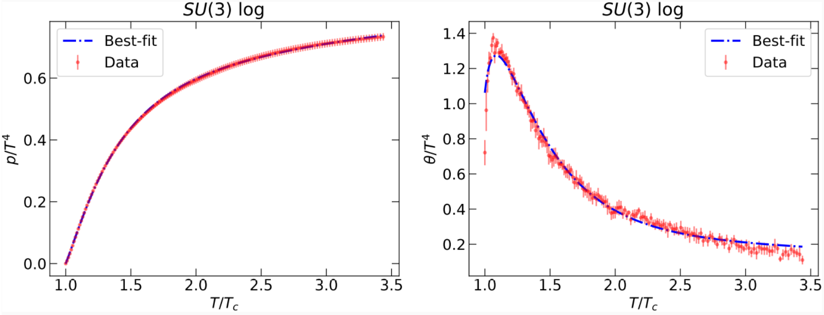
Marco Panero, Phys.Rev.Lett. 103 (2009) 232001



Fitting the Coefficients Using the Lattice Results: III

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

Fitted to lattice data of pressure and the trace of energy momentum tensor.



Fitting the Coefficients Using the Lattice Results: IV

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

表: The parameters for the best-fit points.

N	3	$3\ \log$	4	5	6	8
a_0	3.72	4.26	9.51	14.3	16.6	28.7
a_1	-5.73	-6.53	-8.79	-14.2	-47.4	-69.8
a_2	8.49	22.8	10.1	6.40	108	134
a_3	-9.29	-4.10	-12.2	1.74	-147	-180
a_4	0.27		0.489	-10.1	51.9	56.1
b_3	2.40	-1.77		-5.61		
b_4	4.53		-2.46	-10.5	-54.8	-90.5
b_6			3.23		97.3	157
b_8					-43.5	-68.9

Include Fermions: the PNJL Model and PQM Model

K. Fukushima, PLB **591** (2004) 277; Ratti, Thaler Weise, PRD **73** (2006) 014019
B. Schaefer, J. Pawlowski, J. Wambach PRD **76** (2007) 074023; B. Schaefer, M. Wagner, PPNP **62** (2009) 391
Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.
Pasechnik, Reichert, Sannino and Z-W W,JHEP **02** (2024) 159.

- The Polyakov-loop-Nambu-Jona-Lasinio (PNJL) model is used to describe phase-transition dynamics in dark gauge-fermion sectors
- The finite-temperature grand potential of the PNJL models can be generically written as

$$V_{\mathrm{PNJL}} = V_{\mathrm{PLM}}[\ell, \ell^*] + V_{\mathrm{cond}}\left[\langle\bar{\psi}\psi\rangle\right] + V_{\mathrm{zero}}\left[\langle\bar{\psi}\psi\rangle\right] + V_{\mathrm{medium}}\left[\langle\bar{\psi}\psi\rangle, \ell, \ell^*\right]$$

- The Polyakov quark meson model (PQM) is widely used as an effective theory to study the first order chiral phase transition
- The Lagrangian of the PQM where mesons couple to a spatially constant temporal background gauge field reads

$$\mathcal{L} = \bar{q}\left(i\not{D} - g\left(\sigma + i\gamma_5 T^a \pi_a\right)\right) q + \frac{1}{2}\left(\partial_\mu \sigma\right)^2 + \frac{1}{2}\left(\partial_\mu \pi_a\right)^2$$
$$- V_{\mathrm{PLM}}^{(\mathrm{poly})} + V_{\mathsf{LSM}} + V_{\mathrm{medium}}\,, \text{where } \not{D} = \gamma_\mu \partial_\mu - i\gamma_0 A_0$$

- The finite temperature contributions to the potential of linear sigma model are typically via CJT resummation.

Second Part: Bubble Nucleation and Gravitatioanl Wave

Bubble Profile of Confinement Phase Transition

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

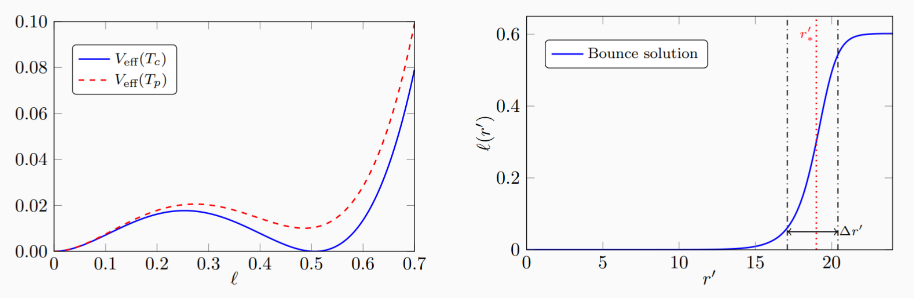


图: The bubble radius is indicated by r'_* and the wall width by $\Delta r'$. Inside of the bubble ($r' \ll r'_*$) lying the confinement phase, the Z_N symmetry is unbroken and $\langle \ell \rangle = 0$, while outside of the bubble ($r \gg r'_*$) lying the deconfinement phase, the Z_N symmetry is broken and $\langle \ell \rangle > 0$.

Gravitational-wave spectrum

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

- Contributions from bubble collision and turbulence are subleading
- The GW spectrum from sound waves is given by

$$h^2\Omega_{\text{GW}}(f) = h^2\Omega_{\text{GW}}^{\text{peak}}\left(\frac{f}{f_{\text{peak}}}\right)^3\left[\frac{4}{7}+\frac{3}{7}\left(\frac{f}{f_{\text{peak}}}\right)^2\right]^{-\frac{7}{2}}$$

- The peak frequency

$$f_{\text{peak}} \simeq 1.9\cdot 10^{-5}\,\text{Hz}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\left(\frac{T}{100\,\text{GeV}}\right)\left(\frac{\tilde\beta}{v_w}\right)$$

- The peak amplitude

$$h^2\Omega_{\text{GW}}^{\text{peak}} \simeq 2.65\cdot 10^{-6}\left(\frac{v_w}{\tilde\beta}\right)\left(\frac{\kappa_{sw}\,\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\Omega_{\text{dark}}^2$$

- The factor Ω_{dark}^2 accounts for the dilution of the GWs by the non-participating SM d.o.f.

$$\Omega_{\text{dark}} = \frac{\rho_{\text{rad,dark}}}{\rho_{\text{rad,tot}}} = \frac{g_{*,\text{dark}}}{g_{*,\text{dark}}+g_{*,\text{SM}}}$$

The Efficiency Factor κ

- The efficiency factor for the sound waves κ_{sw} consist of κ_v as well as an additional suppression due to the length of the sound-wave period τ_{sw}

$$\kappa_{\text{sw}} = \sqrt{\tau_{\text{sw}}}\,\kappa_v$$

- τ_{sw} is dimensionless and measured in units of the Hubble time (H.-K. Guo, Sinha, Vagie and White, JCAP **01** (2021) 001)

$$\tau_{\text{sw}} = 1 - 1/\sqrt{1 + 2\frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta}\,\bar{U}_f}} \Rightarrow \tau_{\text{sw}} \sim \frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta}\,\bar{U}_f} \quad \text{for } \beta >> 1$$

where $\bar{U}_f$ is the root-mean-square fluid velocity

$$\bar{U}_f^2 = \frac{3}{v_w(1+\alpha)} \int_{c_s}^{v_w} \mathrm{d}\xi\, \xi^2 \frac{v(\xi)^2}{1 - v(\xi)^2} \simeq \frac{3}{4}\frac{\alpha}{1+\alpha}\kappa_v$$

- τ_{sw} is suppressed for large β occurring often in strongly coupled sectors
- κ_v was numerically fitted to simulation results depends α and v_w. At the Chapman-Jouguet detonation velocity it reads

$$\kappa_v(v_w = v_J) = \frac{\sqrt{\alpha}}{0.135 + \sqrt{0.98 + \alpha}}$$

GW Signatures for Arbitrary N in the Pure Gluon Case

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

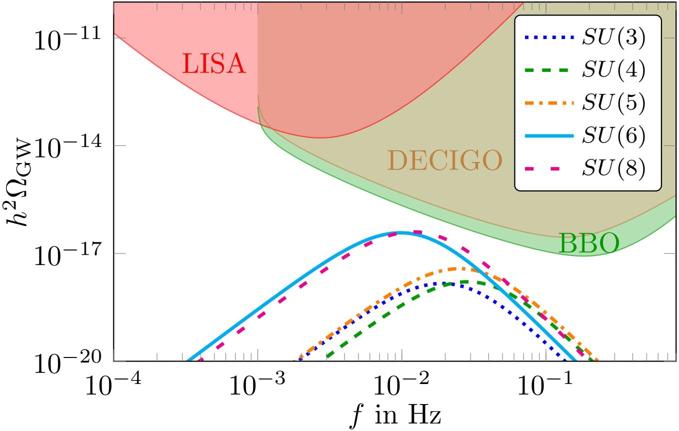


图: The dependence of the GW spectrum on the number of dark colours is shown for the values $N = 3, 4, 5, 6, 8$. All spectra are plotted with the bubble wall velocity set to the Chapman-Jouguet detonation velocity and with Tc$= 1\,\mathrm{GeV}$.

A Landscape of GW Signatures with Pure Gluon

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

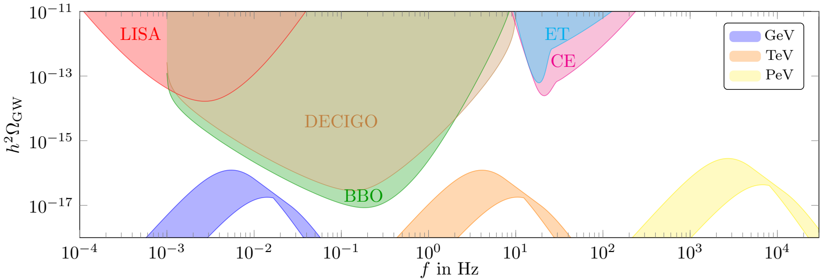


图: We display the GW spectrum of the $SU(6)$ phase transition for different confinement scales including $T_c = 1$ GeV, 1 TeV, and 1 PeV. We compare it to the power-law integrated sensitivity curves of LISA, BBO, DECIGO, CE, and ET.

Landscape of GW spectrum with three Dirac fermions

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

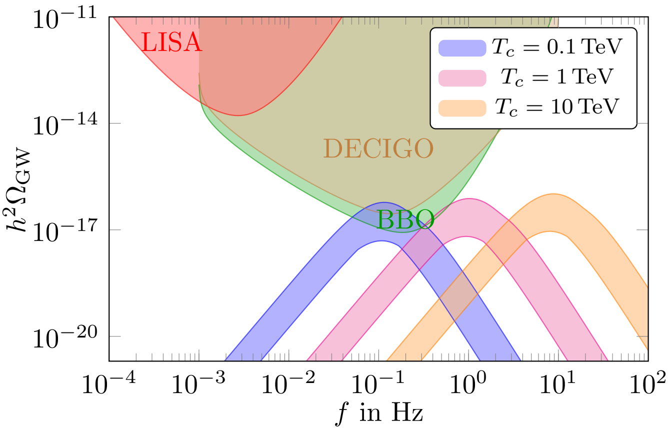


图: Gravitational-wave spectrum with three Dirac fermions in the fundamental representation for different critical temperatures. The band comes from varying wall velocity $c_s \leq v_w \leq 1$.

$\alpha - \beta$ Phase diagram via PQM Model

(Pasechnik, Reichert, Sannino and Z-W W, JHEP **02** (2024) 159.)

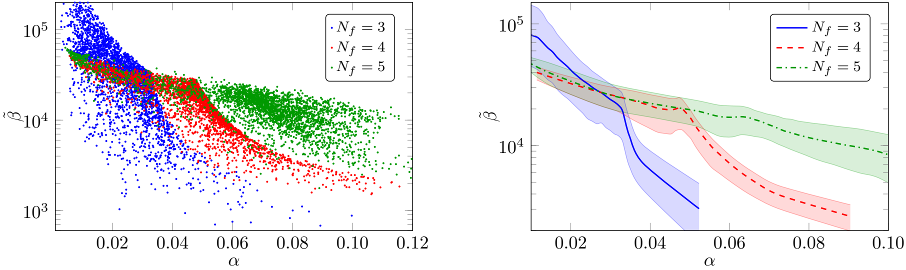


图: We show the range of α and $\tilde{\beta}$ values of the LSM for $N_f = 3, 4, 5$. In the left panel, we show the actual distribution of theory points, while in the right panel, we display the averaged values. On average, the LSM produces stronger GW signals with increasing N_f due to the larger α values. Nonetheless, the strongest GW signals are achieved with the LSM for $N_f = 3$, corresponding to the sparse blue dots at small $\tilde{\beta}$ in the left panel.

Strongest GW Signal at Small $m_\sigma \to$ Near Conformal

(Pasechnik, Reichert, Sannino and Z-W W, JHEP **02** (2024) 159.)

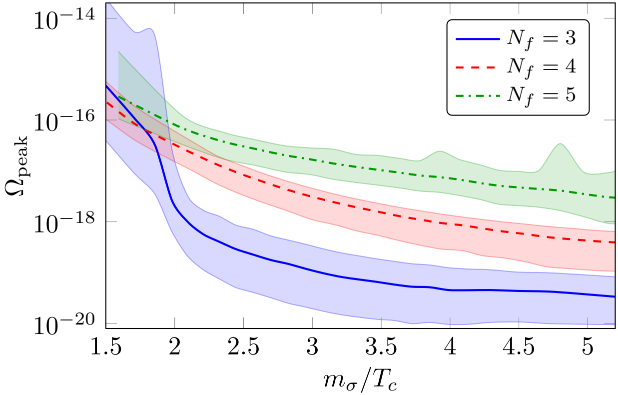


图: We show the averaged values of the peak amplitude Ω_{peak} as a function of physical observables m_σ in units of T_c in the LSM for $N_f = 3, 4, 5$. The sigma meson mass has the strong correlation with the peak amplitude: smaller values of m_σ lead to a larger Ω_{peak}. The strongest signal can almost reach LISA sensitiviity.

Understanding from Thin Wall Approximation

- The three-dimensional Euclidean action S_3 can be written as a function of the latent heat L and the surface tension σ

$$S_3 = \frac{16\pi}{3}\frac{\sigma(T_c)^3}{L(T_c)^2}\frac{T_c^2}{(T_c - T)^2}\,,$$

- The ratio $S_3(T_p)/T_p$ is typically a number $\mathcal{O}(150)$ for phase transitions around the electroweak scale and the inverse duration $\tilde{\beta}$ follows as

$$\tilde{\beta} = T\frac{\mathrm{d}}{\mathrm{d}T}\frac{S_3(T)}{T}\bigg|_{T=T_p} \approx \mathcal{O}(10^3)\frac{T_c^{1/2}L}{\sigma^{3/2}}\,.$$

- $\tilde{\beta}$ stems from the competition between the surface tension and latent heat. $L \sim N^2$ while σ can be either $\sigma \sim N$ or $\sigma \sim N^2$ with limited data up to $SU(8)$
- How to construct models with smaller latent heat and larger surface tension to enhance the gravitational wave signals?

Third Part: Glueball Dark Matter Production Mechanism

Rigorous Dark Gluon-glueball Dynamics

(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)

- In the literature, for glueball dark matter production, only ϕ^5 interaction is considered, making the $3 \to 2$ annihilation the only relevant process for DM formation
- However, since glueball is strongly coupled, this naive calculation is not rigorous. A non-perturbative method is required.
- The dark gluon-glueball dynamics can be effectively described by considering the dimension-4 glueball field $\mathcal{H} \propto$ tr$(G^{\mu\nu}G_{\mu\nu})$:

$$V\left[\mathcal{H}, \ell\right] = \frac{\mathcal{H}}{2} \ln\left[\frac{\mathcal{H}}{\Lambda^4}\right] + T^4\mathcal{V}\left[\ell\right] + \mathcal{H}\mathcal{P}[\ell] + V_T\left[\mathcal{H}\right] .$$

- We keep the lowest order in $\mathcal{P}[\ell]$ to satisfy the symmetry:

$$\mathcal{P}[\ell] = c_1|\ell|^2 ,$$

where c_1 is determined by the lattice results (jumping of gluon condensate).

Thermal evolution of the dark gluon-glueball system

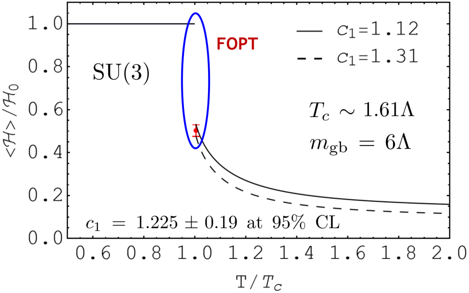
(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)

- Introducing canonically normalized glueball field, we introduce ϕ as $\mathcal{H} = 2^{-8}c^{-2}\phi^4$ and the effective Lagrangian reads:

$$\mathcal{L} = \frac{c}{2}\frac{\partial_\mu \mathcal{H}\partial^\mu \mathcal{H}}{\mathcal{H}^{3/2}} - V[\mathcal{H},\ell] \Rightarrow \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V[\phi,\ell]\,,$$

$$V[\phi,\ell] = \frac{\phi^4}{2^8c^2}\left[2\ln\left(\frac{\phi}{\Lambda}\right) - 4\ln 2 - \ln c\right] + \frac{\phi^4}{2^8c^2}\mathcal{P}[\ell] + T^4\mathcal{V}[\ell] \quad, \mathcal{P}[\ell] = c_1|\ell|^2$$

$$\mathcal{V}[\ell] = T^4\left(-\frac{b_2(T)}{2}|\ell|^2 + b_4|\ell|^4 + \cdots - b_3\left(\ell^N + \ell^{*N}\right)\right),$$



Cosmological evolution of the dark glueball field

(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)

- The glueball field is considered homogeneous and evolves in expanding FLRW universe, with the E.O.M.

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V[\phi, T] = 0\,,$$

- The time variable is found in terms of the photon temperature:

$$t = \frac{1}{2}\sqrt{\frac{45}{4\pi^3 g_{*,\rho}(T_\gamma)}}\frac{m_P}{T_\gamma^2}\,, \qquad T_\gamma = \xi_T T$$

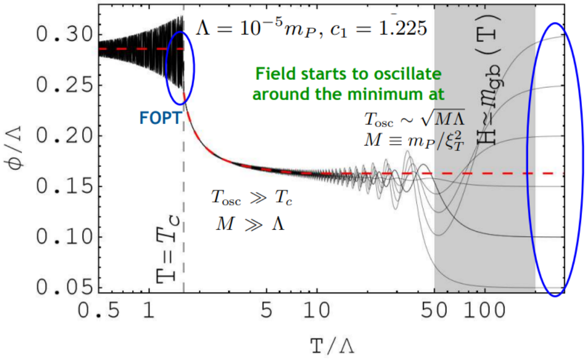
where ξ_T denotes the visible-to-dark sector temperature ratio and $m_P = 1.22 \times 10^{19}$ GeV is the Planck mass and $g_{*,\rho}$ is the number of energy-related degrees of freedom.

- E.O.M. in terms of the dark sector temperature:

$$\frac{4\pi^3 g_{*,\rho}}{45 m_P^2}\xi_T^4 T^6 \frac{d^2\phi}{dT^2} + \frac{2\pi^3}{45 m_P^2}\frac{dg_{*,\rho}}{dT}\xi_T^4 T^6 \frac{d\phi}{dT} + \partial_\phi V[\phi, T] = 0$$

Cosmological Evolution of the Dark Glueball Field

(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)



- Field starts to oscillate around the minimum of the potential when $H \simeq m_{\rm gb}$ with temperature $T_{\rm OSC} \sim \sqrt{M\Lambda}$
- In early times in deconfined regime, for different initial conditions the field evolution follows the minimum (red dashed line).
- First order phase transition washes out any dependence on initial conditions.

Glueball Relic Density

(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)

- Energy stored in these oscillations around $\phi_{\rm min} \approx 0.28\Lambda$ is the relic DM abundance, $\Omega h^2 = \rho/\rho_c$ (critical density $\rho_c = 1.05 \times 10^4\,{\rm eV\,cm^{-3}}$)

$$\rho = \frac{2\pi^3}{45} g_{*,\rho}(T) \frac{T^6}{M^2} \left(\frac{d\phi}{dT}\right)^2 + V[\phi]\,.$$

- Then the relic density today is calculated (f denotes as final):

$$\Omega h^2 = \frac{\Lambda}{\rho_c/h^2} \left\langle \frac{\tilde{\rho}}{\tilde{T}^3} \right\rangle_f T_f^3 \left(\frac{T_{\gamma,0}}{\zeta_T T_f}\right)^3 = 0.12\zeta_T^{-3}\frac{\Lambda}{\Lambda_0}\,,$$

with dilution factor $(T_{\gamma,0}/\zeta_T T_f)^3$ to consider the Universe expansion

- Below freeze-out temperature, the predicted glueball relic density is

$$0.12\zeta_T^{-3}\frac{\Lambda}{137.9\,{\rm eV}} \lesssim \Omega h^2 \lesssim 0.12\zeta_T^{-3}\frac{\Lambda}{82.7\,{\rm eV}}, \quad 1.035 < c_1 < 1.415$$

for $\zeta_T^{-1} = 0.1$, the glueball dark matter mass $M_{\rm gl} \sim 6\Lambda$ is $\sim 0.1{\rm MeV}$

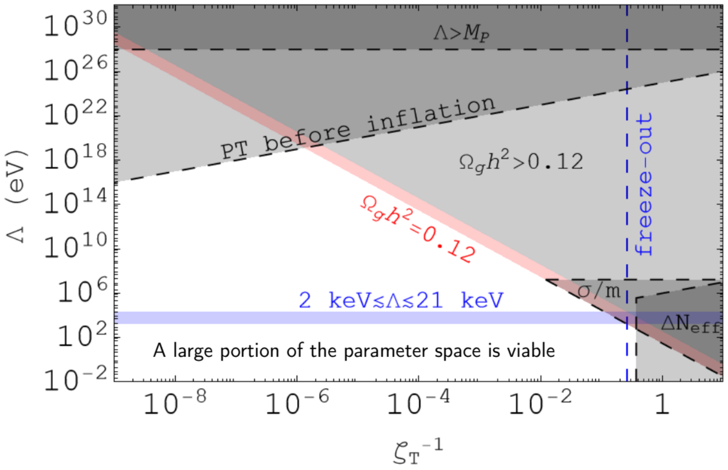
- It is more than a factor of 10 difference compared to the old calculations

$$\Omega h^2 \sim 0.12\,\zeta_T^{-3}\frac{\Lambda}{5.45\,{\rm eV}}$$

Glueball Dark Matter Parameter Space (no portal)

(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26;

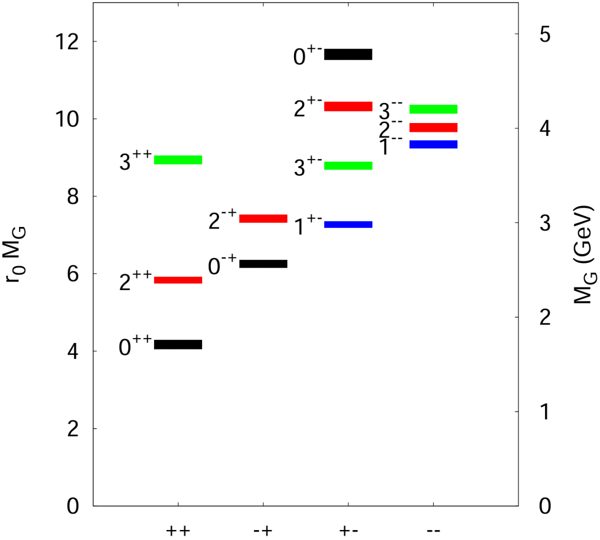
(Carenza, Ferreira, Pasechnik and Z-W W, Phys. Rev. D **108** (2023) no.12, 12)



Fourth Part: Glueball Axion Like Particles (GALPs)

Glueball Spectrum

(Y. Chen *et al.* "Glueball spectrum and matrix elements on anisotropic lattices," Phys. Rev. D **73** (2006) 014516)



Adding the θ term

(Carenza, Pasechnik, Z-W W, Phys. Rev. Lett. **135** (2025) 021001)

- We can add the θ term to incorporate also the pseudoscalar glueball state $\mathcal{A} \equiv 0^{-+}$

$$\mathcal{L}_{\rm SU(N)} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \frac{\theta}{4} G^a_{\mu\nu} \tilde{G}^{\mu\nu a} \,,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \,.$$

- We obtain the glueball EFT Lagrangian as (here $\mathcal{H}$ is ϕ before):

$$\mathcal{L}_{\rm eff} = \frac{1}{2} \partial_\mu \mathcal{H} \partial^\mu \mathcal{H} + \frac{1}{2} \partial_\mu \mathcal{A} \partial^\mu \mathcal{A} - V_{\rm eff}(\mathcal{H}, \mathcal{A}) \,,$$

  where $\mathcal{H} \equiv 0^{++}$ is the standard scalar glueball state while $\mathcal{A} \equiv 0^{-+}$ denotes the pseudoscalar glueball state.
- Specifically, the 0^{++} and 0^{-+} glueball fields are defined in terms of gauge-invariant operators of the lowest dimension as

$$\mathcal{H}^4 \equiv -\frac{\beta(g)}{2g} G^a_{\mu\nu} G^{\mu\nu a} \,, \quad \mathcal{A}\,\mathcal{H}^3 \equiv G^a_{\mu\nu} \tilde{G}^{\mu\nu a}$$

  in terms of β-function $\beta(g)$.

Glueball Potential

(Carenza, Pasechnik, Z-W W, Phys. Rev. Lett. **135** (2025) 021001)

- The trace anomaly is connected to the non-conservation of the dilatonic current in the dark gluon sector ($\beta_\theta = 0$):

$$\partial_\mu D^\mu = \frac{\beta(g)}{2g} G^a_{\mu\nu} G^{\mu\nu a} = -\mathcal{H}^4 \,.$$

- The dilatation current can be calculated for a field $\varphi \equiv \{\mathcal{H}, \mathcal{A}\}$ that transforms as $\varphi'(x') = \varphi'(\lambda^{-1} x) = \lambda\, \varphi(x)$, leading to

$$\partial_\mu D^\mu = -\Theta^\mu_\mu = 4V_{\rm eff} - \frac{\partial V_{\rm eff}}{\partial \mathcal{H}} \mathcal{H} - \frac{\partial V_{\rm eff}}{\partial \mathcal{A}} \mathcal{A} \,,$$

where $\Theta_{\mu\nu}$ is the canonical energy-momentum tensor.

- The composite EFT potential is restricted to be in the following form

$$V_{\rm eff} \simeq c_0 \mathcal{H}^4 \ln\left(\frac{\mathcal{H}}{\Lambda}\right) + \frac{\theta}{4} \mathcal{A}\, \mathcal{H}^3 + \mathcal{H}^4 f\left(\frac{\mathcal{A}}{\mathcal{H}}\right) + \\ + c_1 \mathcal{H}^4 + c_2 \mathcal{H}^2 \mathcal{A}^2 + c_3 \mathcal{A}^4 + c_4 \mathcal{H}\, \mathcal{A}^3 \,,$$

in terms of an arbitrary continuous function f and c_i should be determined by lattice results.

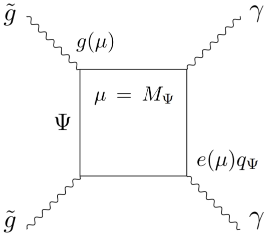
Direct Fermion Portal to the Dark Gauge Sector

(Carenza, Pasechnik, Z-W W, Phys. Rev. Lett. **135** (2025) 021001)

- To connect to the SM, we introduce a heavy Dirac fermion (dark quark): dark $SU(N)$ charged and electrically charged
- Relevant dimension-8 operators below the cutoff scale but above the confinement scale reads

$$\mathcal{L}_{\rm eff} \supset \frac{\tau^2\alpha^2}{M_\Psi^4}\left[c_\gamma\, G^a_{\mu\nu}G^{\mu\nu a}F_{\alpha\beta}F^{\alpha\beta} + \tilde{c}_\gamma\, G^a_{\mu\nu}\tilde{G}^{\mu\nu a}F_{\alpha\beta}\tilde{F}^{\alpha\beta}\right],$$

where $\alpha \equiv \alpha(M_\Psi) = e(M_\Psi)^2/4\pi$ is the fine structure constant, $F^{\alpha\beta}$ and $\tilde{F}^{\alpha\beta}$ are the photon field strength tensor and its dual, respectively.



Effective PQ-like Scale

(Carenza, Pasechnik, Z-W W, Phys. Rev. Lett. **135** (2025) 021001)

- $$\mathcal{L}_{\rm eff} \supset \frac{\tau^2\alpha^2}{M_\Psi^4}\Big[c_\gamma\, G^a_{\mu\nu}G^{\mu\nu a}F_{\alpha\beta}F^{\alpha\beta} + \tilde{c}_\gamma\, G^a_{\mu\nu}\tilde{G}^{\mu\nu a}F_{\alpha\beta}\tilde{F}^{\alpha\beta}\Big].$$
- Linearizing around the VEVs (η_0, a_0) and keeping only terms linear in a, η:

$$G^a_{\mu\nu}G^{\mu\nu a} = -\frac{8g\eta_0^3\,\eta}{\beta(g)}\,, \quad G^a_{\mu\nu}\tilde{G}^{\mu\nu a} = \eta_0^3 a + 3a_0\eta_0^2\eta\,.$$

- Turning to the mass basis $\{\eta, a\} \to \{\phi_1, \phi_2\}$, we obtain:

$$\mathcal{L}_{\rm eff} \supset \frac{1}{4}\sum_{i=1,2}\Big[g_{\phi_i\gamma}\,\phi_i F_{\mu\nu}F^{\mu\nu} + \tilde{g}_{\phi_i\gamma}\,\phi_i F_{\mu\nu}\tilde{F}^{\mu\nu}\Big]\,.$$

- The effective couplings satisfy $g_{\phi_i\gamma} \sim \tilde{g}_{\phi_i\gamma} \equiv g_{{\rm GALP}\gamma}$, where

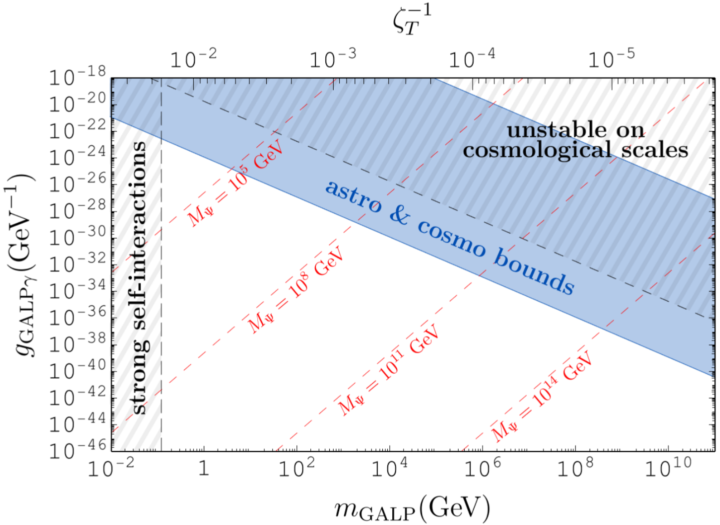
$$g_{{\rm GALP}\gamma} = \kappa\,\alpha^2\Lambda^{-1}\left[\frac{\Lambda}{M_\Psi}\right]^4 = 2.45\times 10^{-7}\,{\rm GeV}^{-1}\,\kappa\left[\frac{m_{\rm GALP}}{\rm GeV}\right]^3\left[\frac{M_\Psi}{\rm GeV}\right]^{-4}.$$

- The PQ-like scale is thus obtained ($f_a \propto g^{-1}_{{\rm GALP}\gamma}$):

$$f_a \simeq 4.1\times 10^3\ {\rm GeV}\,\kappa^{-1}\left[\frac{\Lambda}{\rm GeV}\right]^{-3}\left[\frac{M_\Psi}{\rm GeV}\right]^4.$$

Dark Glueball Parameter Space (with fermion Portal)

(Carenza, Pasechnik, Z-W W, Phys. Rev. Lett. **135** (2025) 021001)



GALPs vs ALPs

(Carenza, Pasechnik, Z-W W, arXiv:2408.14245 [hep-ph])

- We refer to it as GALP because its production mechanism resembles the misalignment mechanism, and its low-energy effective theory shares similarities with that of an axion-like particle (ALP)
- Not a composite axion. The proposed GALP model does not lead to a solution of the strong-CP problem, as done by composite axion models
- DM more massive than usual axion models. The natural prediction for GALP DM with a mass $m_{\rm GALP} \geq 180{\rm MeV}$ lies in between the light axion DM paradigm and the GeV-scale traditional WIMP particles.

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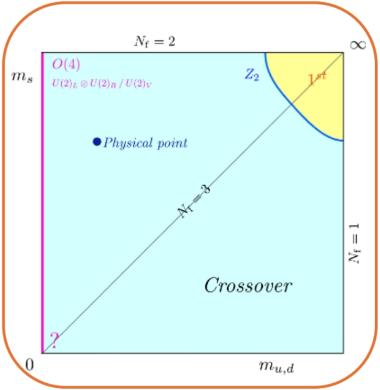
Thank you for your attention!

New Progresses in Columbia Plot



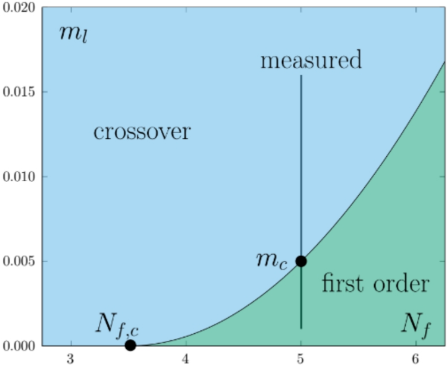


图: Left Fig. Columbia plot from JHEP 11 (2021) 141. Staggered fermions used. Right Fig. from PoS **LATTICE2022** (2023) 027.

About Center Symmetry and Confinement

- The standard physical interpretation is that it is related to the free energy of adding an external static color source in the fundamental representation.

$$\ell\left(\vec{x}\right) = \exp\left(-\mathrm{F}\beta\right)$$

- In the confinement phase, Polyakov loop is zero corresponds to infinity free energy to add a color source and the same time center symmetry is unbroken.

Center Symmetry $Z(N)$ at Nonzero Temperature

- The boundary conditions in imaginary time τ the fields must satisfy are:

$$A_\mu(\vec{x},\beta) = +A_\mu(\vec{x},0), \qquad q(\vec{x},\beta) = -q(\vec{x},0)\ ,$$

  where gluons as bosons must be periodic in τ while quarks as fermions must be anti-periodic.
- ’t Hooft first noticed that one can consider more general gauge transformations which are only periodic up to Ω_c

$$\Omega(\vec{x},\beta) = \Omega_c, \quad \Omega(\vec{x},0) = 1 \qquad \left(\text{here, } \Omega_c = e^{i\phi} I,\ \phi = \frac{2\pi j}{N}\right).$$

- Color adjoint fields are invariant under this transformation, while those in the fundamental representation are not:

$$A^\Omega(\vec{x},\beta) = \Omega_c^\dagger A_\mu(\vec{x},\beta)\,\Omega_c = A_\mu(\vec{x},\beta) = +A_\mu(\vec{x},0)\ ,$$
$$q^\Omega(\vec{x},\beta) = \Omega_c^\dagger q(\vec{x},\beta) = e^{-i\phi} q(\vec{x},\beta) \neq -q(\vec{x},0)\ .$$

- Thermal Wilson line transforms like an adjoint field under local $SU(N)$ gauge transformations:

$$L(x) \to \Omega^\dagger(\vec{x},\beta)\, L(\vec{x}) \Omega^\dagger(\vec{x},0)\ .$$

The PNJL model Lagrangian (Fukushima, Skokov, PPNP **96** (2017) 154)

- The PNJL Lagrangian can be generically written as:

$$\mathcal{L}_{\mathrm{PNJL}} = \mathcal{L}_{\text{pure-gauge}} + \mathcal{L}_{4\mathrm{F}} + \mathcal{L}_{6\mathrm{F}} + \mathcal{L}_k$$

- Without losing generality, we consider below massless 3-flavour case in fundamental representation of $SU(3)$ gauge symmetry
- Here, $\mathcal{L}_{4\mathrm{F}}$ is the four-quark interaction which reads:

$$\mathcal{L}_{4\mathrm{F}} = G_S \sum_{a=0}^{8} [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma^5\lambda^a\psi)^2], \qquad \psi = (u, d, s)^T$$

- Six-fermion interaction $\mathcal{L}_{6\mathrm{F}}$ denotes the Kobayashi-Maskawa-'t Hooft (KMT) term breaking $U(1)_A$ down to Z_3 (generically Z_{N_f} for N_f flavours)

$$\mathcal{L}_{6\mathrm{F}} = G_D[\det(\bar{\psi}_{Li}\psi_{Rj}) + \det(\bar{\psi}_{Ri}\psi_{Lj})]$$

Medium Potential: Finite Temperature Contribution

- In the standard NJL model, the medium effect (finite temperature contribution) is implemented by the grand canonical partition function
- In the PNJL model, we can simply do the following replacement to include the contribution from Polyakov loop

$$V_{\text{medium}} = -2N_c T \sum_{u,d,s} \int \frac{d^3p}{(2\pi)^3} \left(\ln\left[1 + e^{-\beta(E-\mu)}\right] + \ln\left[1 + e^{-\beta(E+\mu)}\right] \right)$$

$$\to -2T \sum_{u,d,s} \int \frac{d^3p}{(2\pi)^3} \,\mathrm{Tr}_c \left\{ \left(\ln\left[1 + \mathbf{L}\, e^{-\beta(E-\mu)}\right] + \ln\left[1 + \mathbf{L}^\dagger e^{-\beta(E+\mu)}\right] \right) \right\}$$

- $\mathbf{L}$ is the Polyakov loop:

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp\left[i \int_0^{1/T} A_4(\vec{x}, \tau)\, \mathrm{d}\tau \right]$$

- In this work, we consider chemical potential $\mu = 0$.

The Constituent Quark Mass and Zero Point Energy: I

(Fukushima, Skokov, PPNP **96** (2017) 154)

- In $\mathcal{L}_{6\mathrm{F}}$, there is also $\langle \bar{u}u \rangle^2 \bar{u}u$ term contributes to the constituent quark mass of u
- The total constituent quark mass from $\mathcal{L}_{4\mathrm{F}}$ and $\mathcal{L}_{6\mathrm{F}}$ is:

$$M = -4G_S\sigma - \frac{1}{4}G_D\sigma^2$$

- The expression for the zero-point energy is given by:

$$V_{\mathrm{zero}}\left[\langle \bar{\psi}\psi \rangle\right] = -\dim(\mathrm{R})\, 2N_f \int \frac{\mathrm{d}^3 p}{(2\pi)^3} E_p\,, \qquad E_p = \sqrt{\vec{p}^{\,2} + M^2}$$

  E_p is the energy of a free quark with constituent mass M and three-momentum $\vec{p}$
- The above integration diverges and a regularization is required. We choose a sharp three-momentum cutoff Λ entering the expression for observables and thus also a parameter of the theory.
- Parameters: $G_S,\, G_D,\, \Lambda$; Observables: $M,\, f_\pi,\, m_\sigma$

The Constituent Quark Mass and Zero Point Energy: II

(Fukushima, Skokov, PPNP **96** (2017) 154)

- The integration can be carried analytically and the result is:

$$V_{\text{zero}}\left[\langle\bar{\psi}\psi\rangle\right] = -\frac{\dim(\mathrm{R}) N_f \Lambda^4}{8\pi^2}\Bigg[(2+\xi^2)\sqrt{1+\xi^2} + \frac{\xi^4}{2}\ln\frac{\sqrt{1+\xi^2}-1}{\sqrt{1+\xi^2}+1}\Bigg],$$

in which $\xi \equiv \frac{M}{\Lambda}$.

The Condensate Energy (Fukushima, Skokov, PPNP **96** (2017) 154)

- In $\mathcal{L}_{4\mathrm{F}}$, the condensate energy then comes from the combination

$$(\bar{\psi}\lambda^0\psi)^2 + (\bar{\psi}\lambda^3\psi)^2 + (\bar{\psi}\lambda^8\psi)^2 = 2(\bar{u}u)^2 + 2(\bar{d}d)^2 + 2(\bar{s}s)^2$$

- We use the trick is to rewrite $(\bar{u}u)^2$ as

$$\begin{aligned}(\bar{u}u)^2 &= [(\bar{u}u - \langle\bar{u}u\rangle) + \langle\bar{u}u\rangle]^2 = (\bar{u}u - \langle\bar{u}u\rangle)^2 + 2\langle\bar{u}u\rangle\,(\bar{u}u - \langle\bar{u}u\rangle) + \langle\bar{u}u\rangle^2 \\ &\simeq -\langle\bar{u}u\rangle^2 + 2\langle\bar{u}u\rangle\bar{u}u\,,\end{aligned}$$

  where the $(\bar{u}u - \langle\bar{u}u\rangle)^2$ term is dropped in the spirit of the mean-field approximation.
- The $2\langle\bar{u}u\rangle\bar{u}u$ term contributes to the constituent quark mass of u
- The $-\langle\bar{u}u\rangle^2$ term leads to a contribution to the condensate energy
- Similar procedures can be applied to $(\bar{d}d)^2$ and $(\bar{s}s)^2$, and to $\mathcal{L}_{6\mathrm{F}}$ gives $\langle\bar{u}u\rangle^3$ and we obtain the total condensate energy:

$$V_{\mathrm{cond}} = 6G_S\sigma^2 + \frac{1}{2}G_D\sigma^3\,, \qquad \sigma \equiv \langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{s}s\rangle = \frac{1}{3}\langle\bar{\psi}\psi\rangle$$

Bubble Nucleation: Chiral Phase Transition (PNJL)

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552)

- Chiral phase transition occurs when including fermions
- $\bar{\sigma}$ is classically nonpropagating in PNJL and it's kinetic term is induced only via quantum fluctuations
- We thus include its wave-function renormalization Z_σ with

$$Z_\sigma^{-1} = -\frac{\mathrm{d}\Gamma_{\sigma\sigma}(q^0, \mathbf{q}, \bar{\sigma})}{\mathrm{d}\mathbf{q}^2}\bigg|_{q^0=0, \mathbf{q}^2=0}, \qquad \Gamma_{\sigma\sigma} = -i \sum 2\,\text{point 1PI}\,\sigma\sigma\,\text{graph}$$

- The three-dimensional Euclidean action and E.O.M. are modified to:

$$S_3(T) = 4\pi \int_0^\infty \mathrm{d}r\, r^2 \left[\frac{Z_\sigma^{-1}}{2} \left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r} \right)^2 + V_{\text{eff}}(\bar{\sigma}, T) \right]$$

$$\frac{\mathrm{d}^2\bar{\sigma}}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r} - \frac{1}{2}\frac{\partial \log Z_\sigma}{\partial \bar{\sigma}} \left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r} \right)^2 = Z_\sigma \frac{\partial V_{\text{eff}}}{\partial \bar{\sigma}}$$

- The associated boundary conditions:

$$\frac{\mathrm{d}\bar{\sigma}(r=0, T)}{\mathrm{d}r} = 0\,, \qquad \lim_{r\to\infty} \bar{\sigma}(r, T) = 0$$

GW parameters α, β and PNJL observables

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

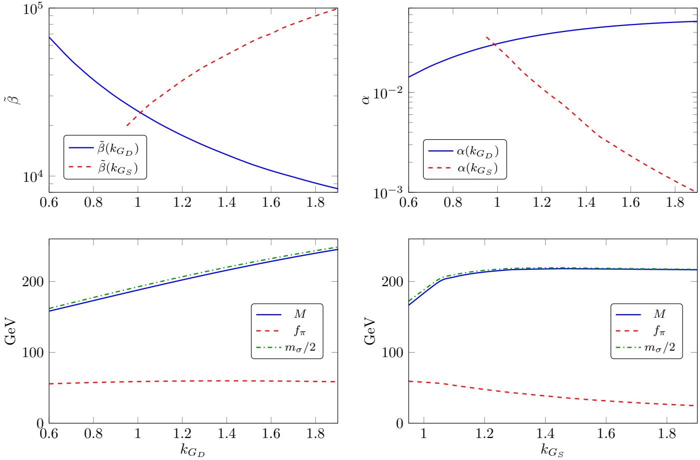


图: The GW parameters $\tilde{\beta}$, α with the observables M, f_π, and m_σ as a function of $G_S = k_{G_S} \cdot 4.6\,\text{GeV}^{-2}$ and $G_D = k_{G_D} \cdot (-743\,\text{GeV}^{-5})$. We use $T_c = 100\,\text{GeV}$, the ratio $\Lambda/T_0 = 3.54$. Below $k_{G_S,\text{crit}} = 0.882$, no chiral symmetry breaking occurs.

Signal to Noise Ratio

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

$$\mathrm{SNR} = \sqrt{\frac{3\mathrm{year}}{s}\int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} \mathrm{d}f \left(\frac{h^2\Omega_{\mathrm{GW}}}{h^2\Omega_{\mathrm{det}}}\right)^2}$$

$h^2\Omega_{\mathrm{GW}}$ is the GW spectrum while Ω_{det} is the sensitivity curve of the detector.

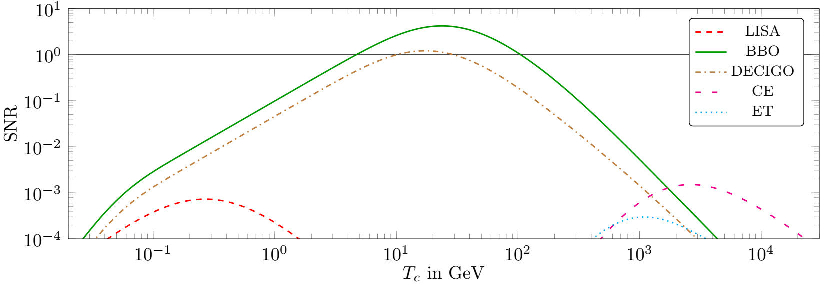


图: We display the SNR for the phase transition in a dark $SU(6)$ sector as a function of the confinement temperature Tc from experiments of LISA, BBO, DECIGO, CE, and ET. We assume an observation time of three years.