中山大學天琴中心
TIANQIN CENTER FOR GRAVITATIONAL PHYSICS, SYSU

New sources of gravitational waves from the early universe

Fa Peng Huang (黄发朋)

Sun Yat-sen University, TianQin center

The 2025 Beijing Particle Physics and Cosmology Symposium (BPCS 2025): Early Universe, Gravitational-Wave Templates, Collider Phenomenology
@ Beijing, 2025.09.26

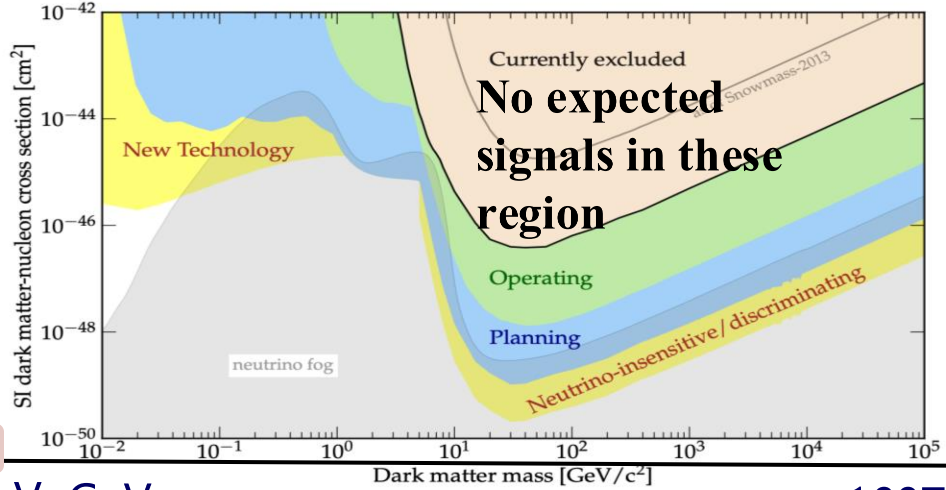
Outline

1. Motivation for new dark matter (DM) mechanism
2. Heavy DM from first-order phase transition (FOPT) and GW
   - Case I: Q-ball and gauged Q-ball DM
   - Case II: filtered DM
3. New gravitational wave (GW) source
4. Summary and outlook

Motivation **DM research status**

What is the microscopic nature of DM?
How DM relic density is produced?

arXiv: 1904:07915
Snowmass 2021,
arXiv: 2209.07426



Ultralight DM:
Axion/ALP

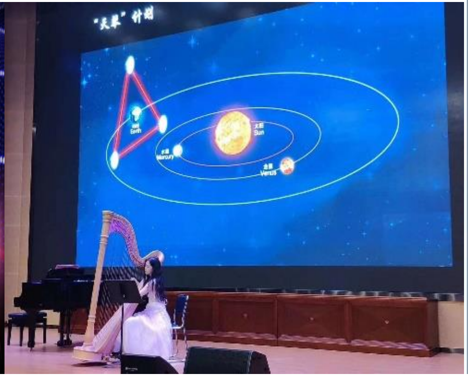
Heavy DM:
Q-ball/filtered/
PBH

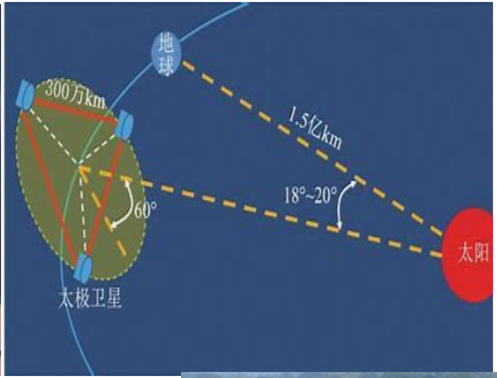
10^{-22}eV keV GeV WIMP 100TeV

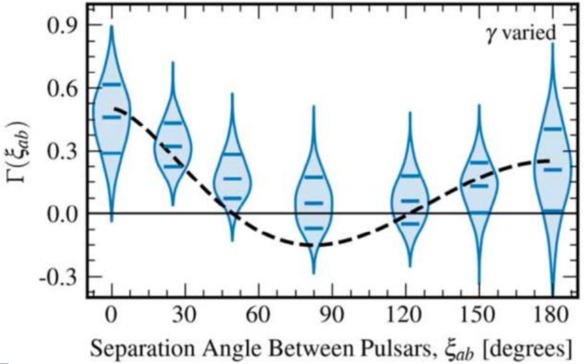
- **new DM mechanism beyond thermal freeze out: cosmic phase transition, Hawking radiation, superradiance…**
- **new detection method: LISA, TianQin, aLIGO, SKA, NanoGrav, Cosmic Explorer, Einstein telescope**

GW experiments

LISA/TianQin/Taiji ~2034







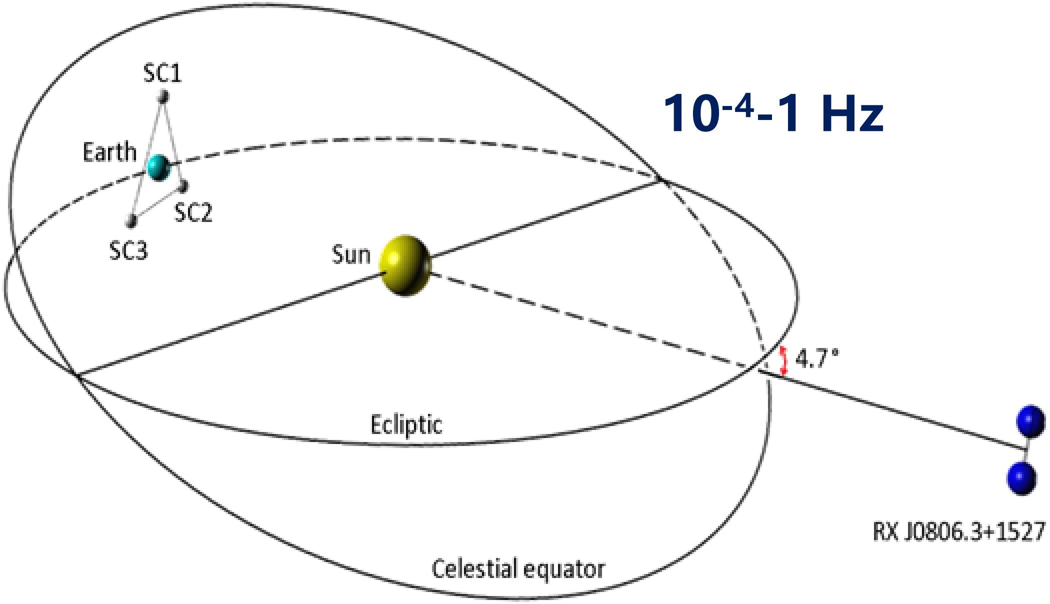
"TianQin"
"Harpe in space"

2023 June 29th： NANOGrav, EPTA, InPTA, Parkes PTA, CPTA

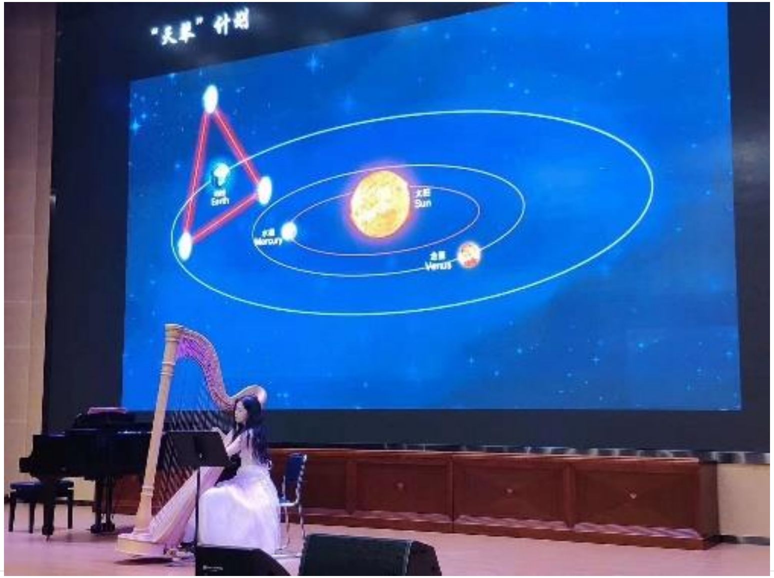
FAST

SKA

What is TianQin ?



- **Expected in 2035**
- **Geocentric orbit, normal triangle constellation, radius ~10^5 km**
- **Unique frequency band, easier for deployment, tracking, control, and communication**



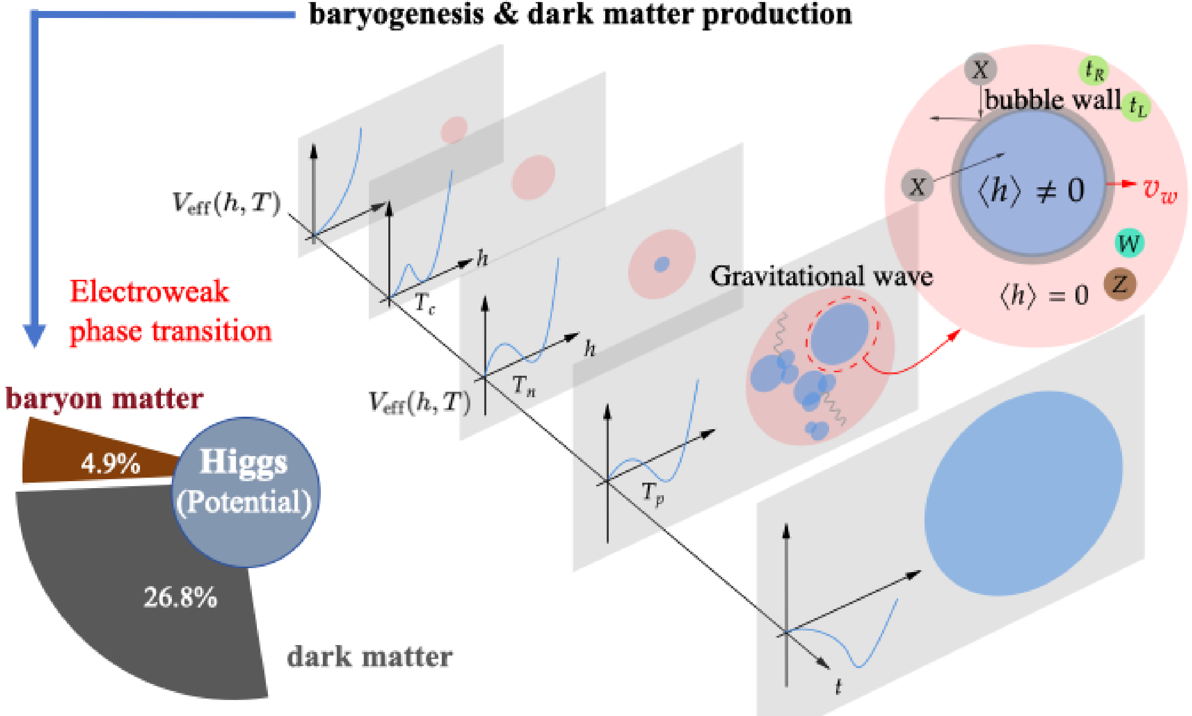
“天琴” (TianQin) “Harp in space”

J. Luo et al. TianQin: a space-borne gravitational wave detector, Class. Quant. Grav. 33 (2016) no.3, 035010.

Motivation DM in post-Higgs and GW Era

The observation of Higgs@LHC and GW@LIGO initiates new era of exploring DM by GW.

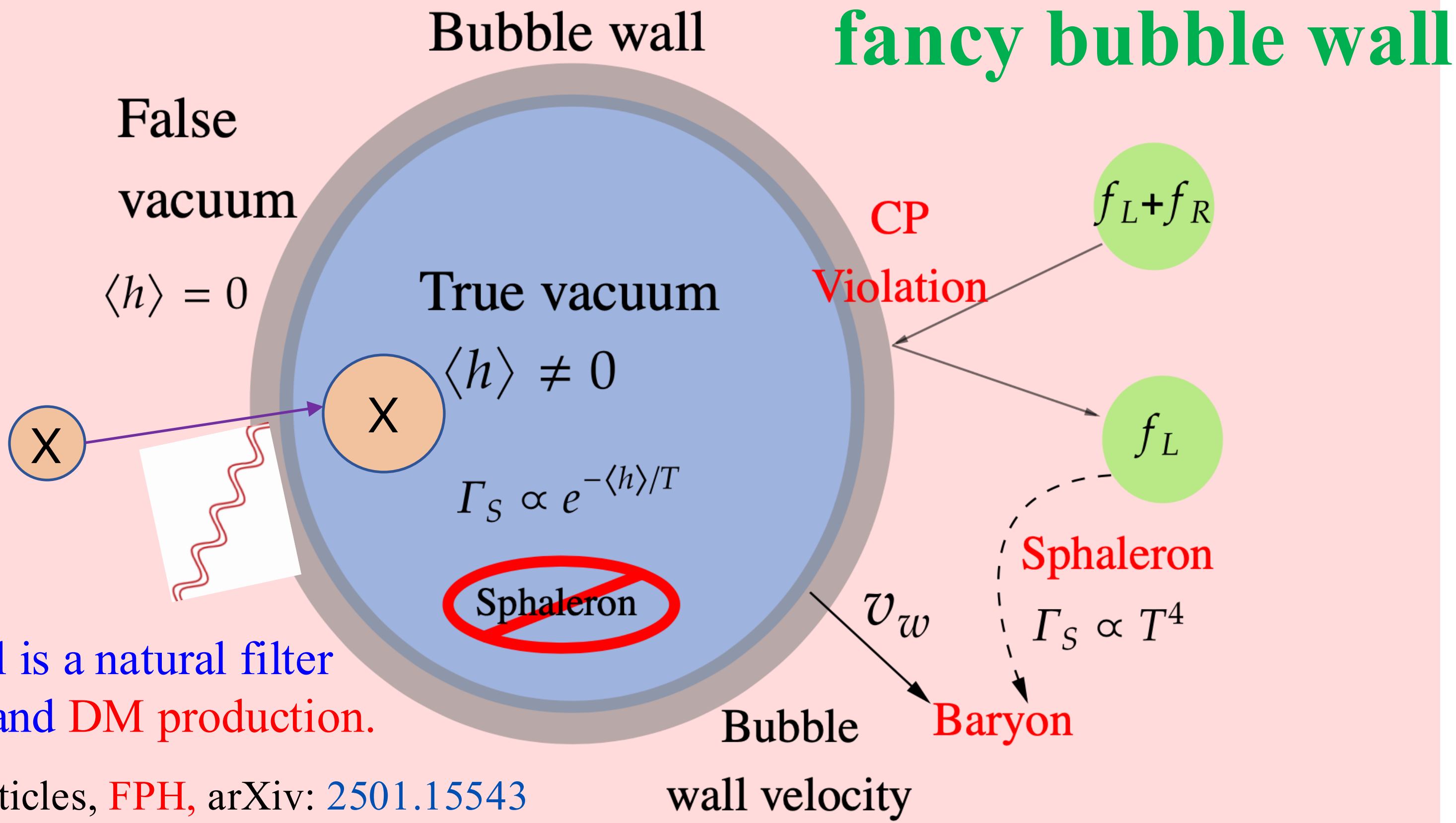
FOPT by Higgs could provide a new approach for DM production.



Bubble wall appears

The First Particles, FPH, arXiv: 2501.15543

DM from cosmic phase transition

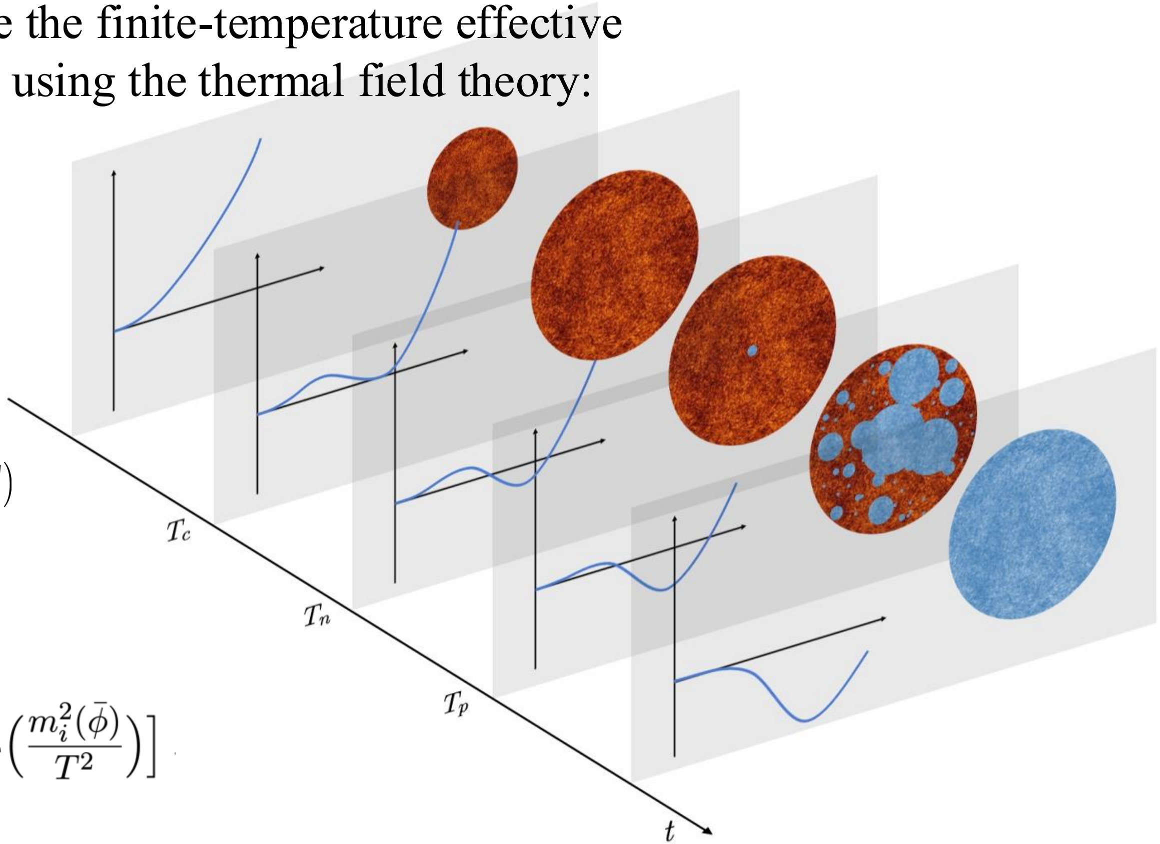


Bubble wall is a natural filter for baryon and DM production.

The First Particles, FPH, arXiv: 2501.15543

Phase transition in a nutshell

Calculate the finite-temperature effective potential using the thermal field theory:



$$\Gamma = \Gamma_0 e^{-S(T)}$$

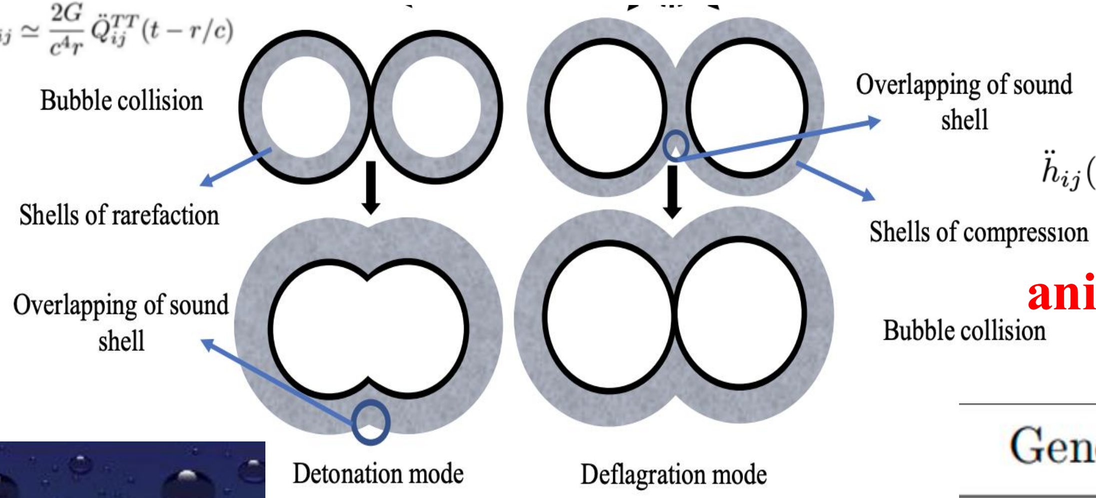
$$S(T) = \int d^4x \left[\frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 + V_{\text{eff}}\left(\phi, T\right)\right]$$

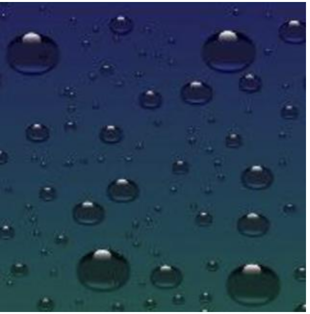
$$V_{\text{eff}}^{(1)}(\bar{\phi}) = \sum_i n_i \Big[\int \frac{\mathrm{d}^D p}{(2\pi)^D} \ln\big(p^2 + m_i^2(\bar{\phi})\big) + J_{\mathrm{B,F}}\Big(\frac{m_i^2(\bar{\phi})}{T^2}\Big)\Big]$$

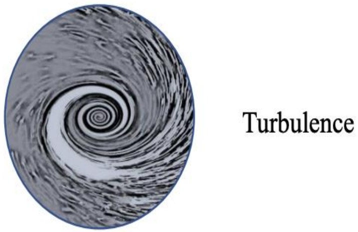
Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Phase transition GW in a nutshell

$$h_{ij} \simeq \frac{2G}{c^4 r}\ddot{Q}_{ij}^{TT}(t - r/c)$$







$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} + \Lambda\,g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$\ddot{h}_{ij}(\mathbf{x},t) + 3\,H\,\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\mathbf{x},t) = 16\pi G\,\Pi_{ij}(\mathbf{x},t)$$

anisotropic stress tenor: source of GW

General form Π_{ij}
$[\partial_i\phi\partial_j\phi]^{TT}$
$[\gamma^2(\rho+p)v_iv_j]^{TT}$
$[-E_iE_j - B_iB_j]^{TT}$
$\partial_i\Psi, \partial_i\Phi$

E. Witten, Phys. Rev. D 30, 272 (1984)
C. J. Hogan, Phys. Lett. B 133, 172 (1983);
M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))
EW phase transition GW becomes more interesting and realistic after the discovery of Higgs by LHC and GW by LIGO.

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

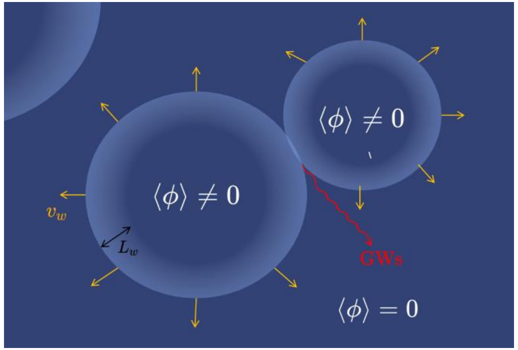
Any new GW sources ?

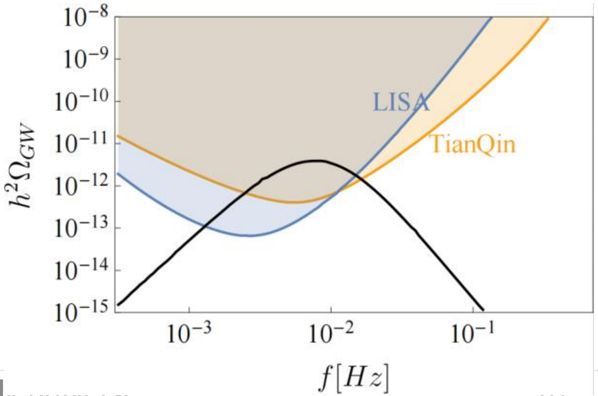
$$\Omega_{GW} = \Omega_{\text{bubble collision}} + \Omega_{\text{sound wave}} + \Omega_{\text{turbulence}}$$ + ...? other sources?

Question:

Besides the well-studied bubble collision, turbulence, and sound wave, are there any new GW sources during a FOPT in the early universe?

Answer: Yes!



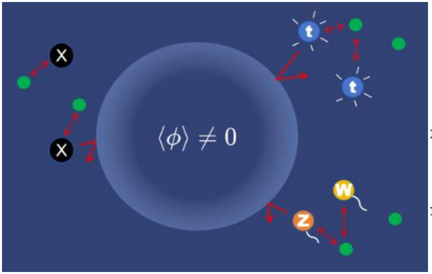


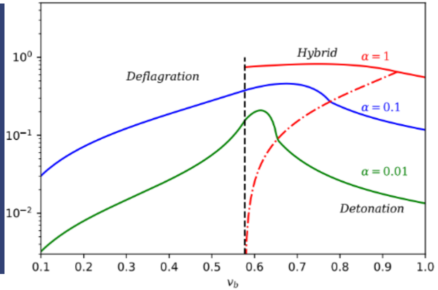
Bubble wall is essential (like a filter)

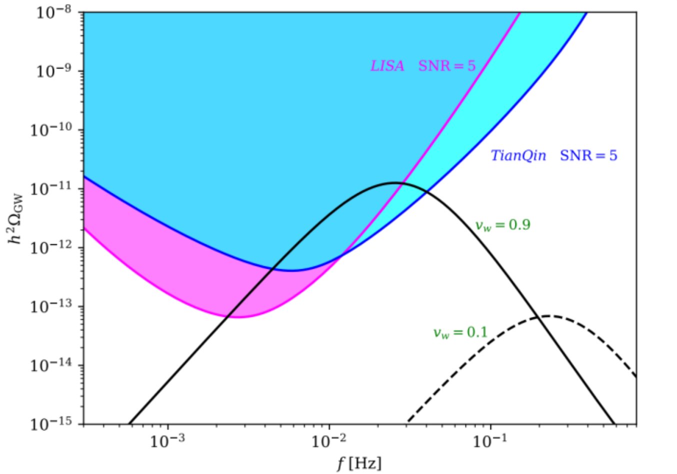
In theory, phase transition GW, phase transition DM, baryogenesis are most sensitive to bubble wall dynamics

GW signals favor lager v_w
EW baryogenesis favor smaller v_w
Dynamical DM is sensitive to v_w

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang, arXiv:2007.10343,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith, arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, **FPH**, xiao wang, Phys.Rev.D 107 (2023) 9, 095005







In experiments, GW experiment is most sensitive to bubble wall velocity v_w Aidi Yang, **FPH**, JCAP 2025

$$\rho_{DM}^4 v_w^{3/4} = 73.5(2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;

Outline

1. Motivation for new dark matter (DM) mechanism
2. Heavy DM from first-order phase transition (FOPT) and GW
   - Case I: Q-ball and gauged Q-ball DM
   - Case II: filtered DM
3. New gravitational wave (GW) source
4. Summary and outlook

Heavy DM from cosmic phase transition

Renaissance of quark nugget DM idea by E. Witten.

Recently, dynamical DM formed by phase transition has became a new idea for heavy DM. Bubble wall in FOPT can be the “filter” to obtain the needed heavy DM when avoiding the unitarity constraints.

FOPT in the early universe	Coffee making process
Bubble wall	filter
Case I:(gauged) Q-ball DM	Large coffee beans
Case II: filtered DM	Coffee
Phase transition GW	Aroma

E. Krylov, A. Levin, V. Rubakov, Phys.Rev.D 87 (2013) 8, 083528

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028

arXiv:1912.04238, Dongjin Chway, Tae Hyun Jung, Chang Sub Shin

Phys.Rev.Lett. 125 (2020) 15, 151102 , M. J. Baker, J. Kopp,and A. J. Long

arXiv:2101.05721, Aleksandr Azatov, Miguel Vanvlasselaer, Wen Yin

arXiv:2103.09827, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov

arXiv:2103.09822, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov

Siyu Jiang, FPH, Chong Sheng Li, arXiv:2305.02218

Siyu Jiang, FPH, Pyungwon Ko, arXiv:2404.16509

more than 100 papers in recent 5 years

Case I: Q-ball DM What is Q-ball?

PHYSICS REPORTS (Review Section of Physics Letters) 221, Nos. 5 & 6 (1992) 251–350. North-Holland PHYSICS REPORTS

Nontopological solitons*

T.D. Lee
Department of Physics, Columbia University, New York, NY 10027, USA

and

Y. Pang
Brookhaven National Laboratory, Upton, NY 11973, USA

Received May 1992; editor: D.N. Schramm

Nuclear Physics B262 (1985) 263–283

Q-BALLS*

Sidney COLEMAN

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Q-ball is the most typical non-topological soliton, initially proposed by Prof. Tsung-Dao Lee and Sidney Coleman. In quantum field theory, a spherically symmetric extended body that forms a non-topological soliton structure with a conserved global quantum number Q is called a Q-ball.

$$\phi = (\phi_R + i\phi_I)/\sqrt{2} \qquad Q = \int j^0 dx = \int \left(\phi_I \dot{\phi}_R - \phi_R \dot{\phi}_I\right) dx. \qquad \delta(E - \omega Q) = 0$$

$$\downarrow$$

$$E = \int \left\{ \frac{1}{2} \left[\dot{\phi}_R^2 + \dot{\phi}_I^2 + (\nabla \phi_R)^2 + (\nabla \phi_I)^2 \right] + U \left[\frac{1}{2} \left(\phi_R^2 + \phi_I^2 \right) \right] \right\} dx \qquad \phi = f(r) e^{-i\omega t}$$

Q-ball production mechanism

Q-ball production：

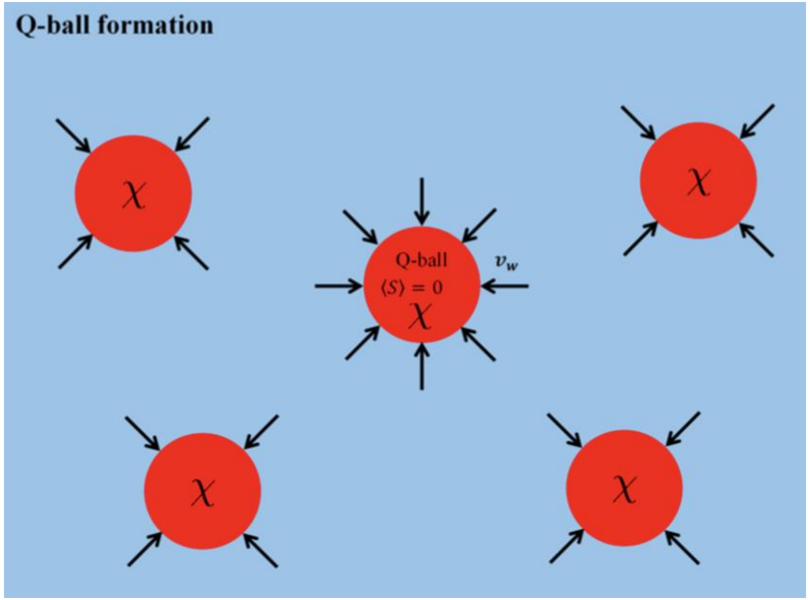
(1) produce the charge asymmetry (i.e.
locally produce lots of particles with the same charge to form Q-ball)

(2) and packet the same sign charge in the small size after overcoming the Coulomb repulsive interaction.

1. Supersymmetry? Affleck-Dine mechanism.

We do not observe the supersymmetry until now!

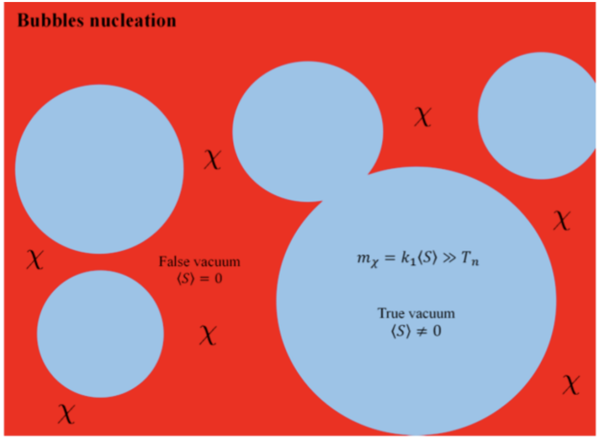
2. Q-ball formation based on FOPT. →



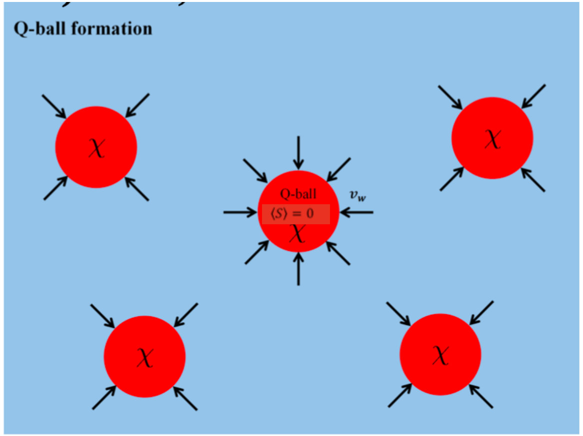
Case I: Q-ball DM

Global Q-ball DM: **The cosmic phase transition with Q-balls production can explain baryogenesis and DM simultaneously.**

$$\rho_{DM}^4 v_w^{3/4} = 73.5(2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$



(a) Bubble nucleation:χ particles trapped in the false vacuum due to Boltzmann suppression



(b) Q-ball formation:After the formation of Q-balls, they should be squeezed by the true vacuum

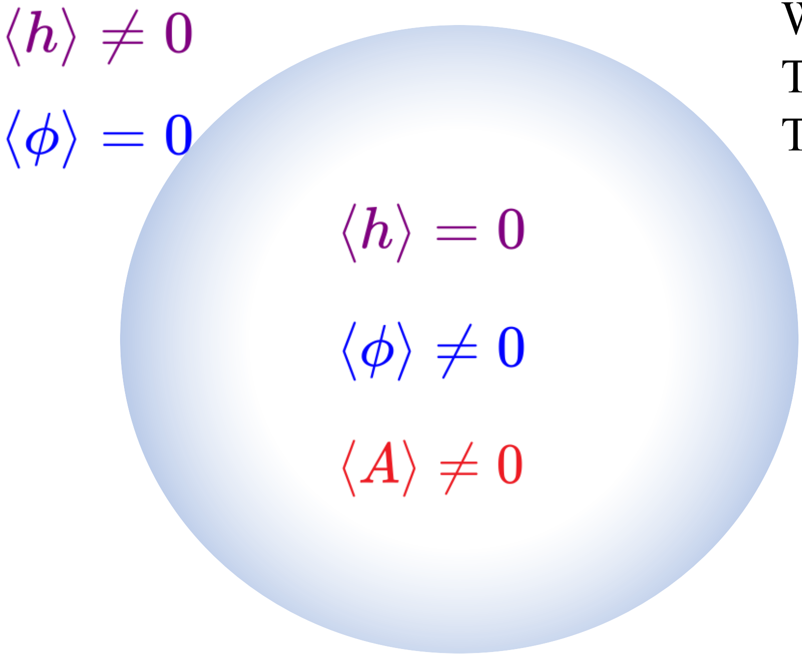
New DM production scenario by the bubbles.
The global Q-ball model proposed by T.D. Lee

Friedberg-Lee-Sirlin model

R. Friedberg, T.D. Lee and A. Sirlin. Rev. D 13 (1976) 2739

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;

Case I: Gauged Q-ball DM



When the conserved U(1) symmetry is local,
This introduces an extra gauge field A.
The minimal model achieving

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) + \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{4}\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} - V(\phi, h)$$

$$V(\phi, h) = \frac{\lambda_{\phi h}}{2} h^2 |\phi|^2 + \frac{\lambda_h}{4}\left(h^2 - v_0^2\right)^2$$

Interestingly, this portal coupling also naturally induces a strong FOPT.

$$J_\mu = i\left(\phi^\dagger \overleftrightarrow{\partial}_\mu \phi + 2i\tilde{g}\tilde{A}_\mu |\phi|^2\right) \qquad Q = \int d^3x J^0$$

Conserved charge

Siyu Jiang, FPH, Pyungwon Ko, JHEP 07 (2024) 053

Gauged Q-ball

$$\mathcal{L} = (D_\mu\phi)^\dagger (D^\mu\phi) + \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{4}\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} - V(\phi, h) \qquad V(\phi,h) = \frac{\lambda_{\phi h}}{2}h^2|\phi|^2 + \frac{\lambda_h}{4}\left(h^2 - v_0^2\right)^2$$

Friedberg-**Lee**-Sirli-Maxwell model

$$\tilde{A}_t(r) = v_0\frac{\tilde{g}}{\sqrt{2\lambda_h}}\mathcal{A}(\rho), \quad \phi(t,r) = \frac{v_0}{\sqrt{2}}\Phi(\rho)e^{-i\omega t}, \quad h(r) = v_0\mathcal{H}(\rho)$$

$$\frac{1}{\rho^2}\partial_\rho\left(\rho^2\partial_\rho\mathcal{A}\right) + (\nu - \alpha^2\mathcal{A})\Phi^2 = 0\,,$$

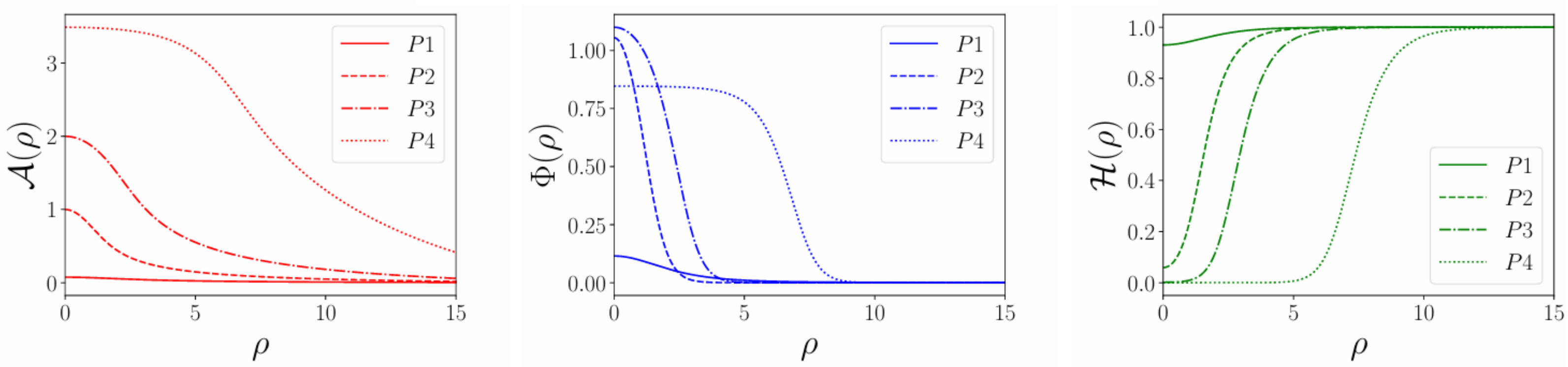
$$\frac{1}{\rho^2}\partial_\rho\left(\rho^2\partial_\rho\Phi\right) + \left[(\nu - \alpha^2\mathcal{A})^2 - k^2\mathcal{H}^2\right]\Phi = 0\,,$$

$$\frac{1}{\rho^2}\partial_\rho\left(\rho^2\partial_\rho\mathcal{H}\right) - k^2\mathcal{H}\Phi^2 - \frac{1}{2}\mathcal{H}\left(\mathcal{H}^2 - 1\right) = 0\,.$$

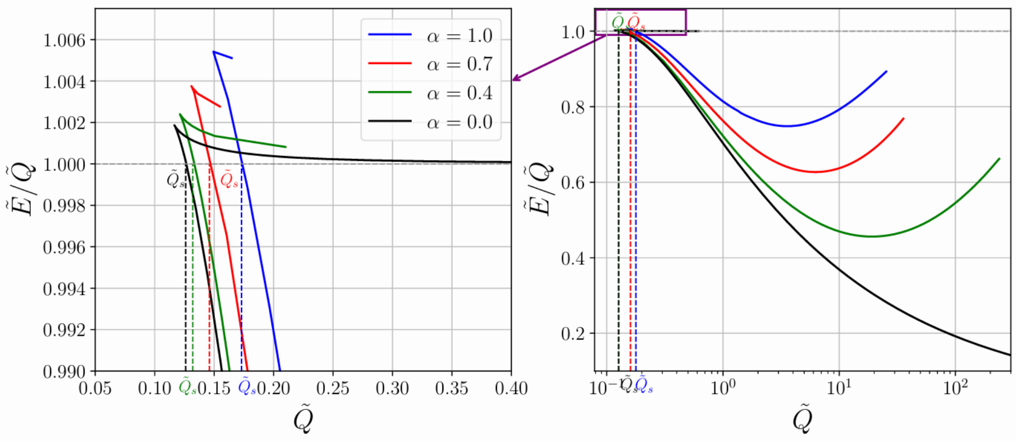
$$\alpha \equiv \frac{|\tilde{g}|}{\sqrt{2\lambda_h}}, k \equiv \frac{\sqrt{\lambda_{\phi h}}}{2\sqrt{\lambda_h}} = \frac{m_\phi}{m_h}$$

$$\nu \equiv \frac{\omega}{\sqrt{2\lambda_h}v_0}$$

relaxation method



Gauged Q-ball stability

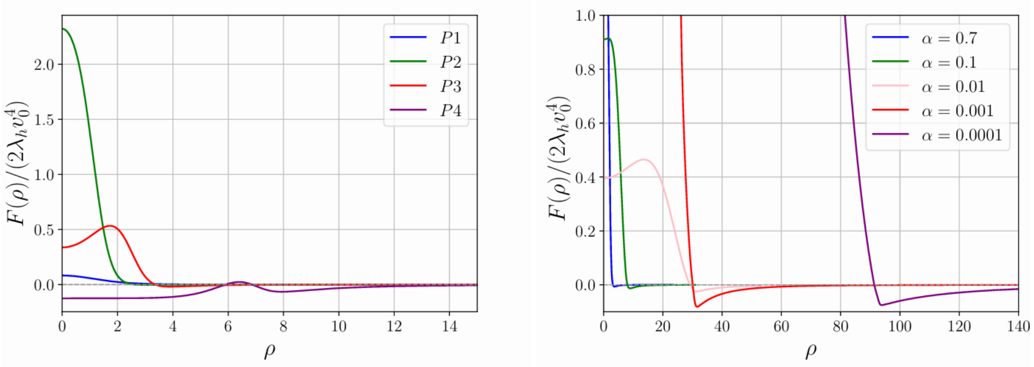


Quantum stability

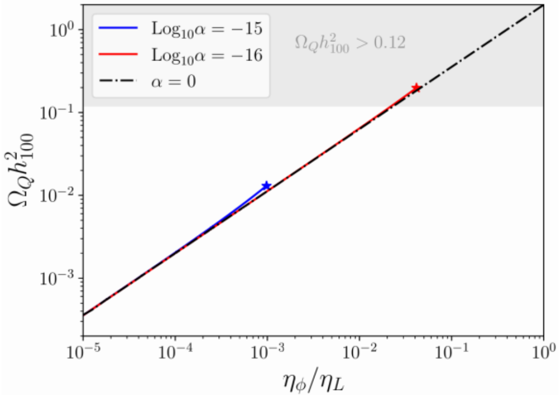
$$E < m_\phi Q \quad \text{or} \quad \tilde{E}/\tilde{Q} < 1$$

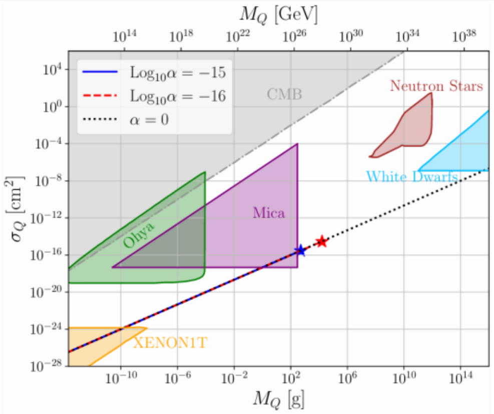
Stress stability

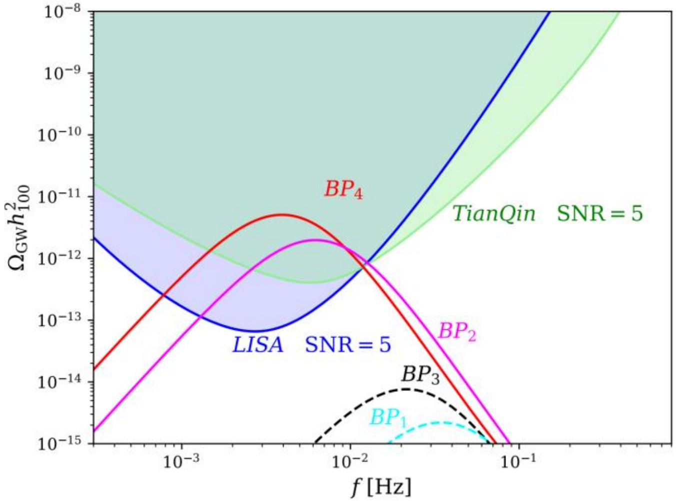
$$F(r) = \frac{2}{3}s(r) + p(r) > 0$$



Gauged Q-ball DM from a FOPT







$$\Omega_Q h_{100}^2 \simeq 2.81 \times \left(\frac{s_0 h_{100}^2}{\rho_c}\right)\left(\frac{\Gamma(T_\star)}{v_w}\right)^{3/16} s_\star^{-1/4}(F_\phi^{\rm trap}\eta_\phi)^{3/4}\lambda_h^{1/4}v_0\left(1+\frac{108^{1/4}\tilde{g}^2 F_\phi^{\rm trap}\eta_\phi s_\star v_w^{3/4}}{5.4\pi^{7/4}\Gamma(T_\star)^{3/4}}\right)$$

	$\lambda_{\phi h}$	T_p [GeV]	α_p	β/H_p	v_w	$F_\phi^{\rm trap}$	η_ϕ/η_L	$\delta\sigma_{Zh}$	GW
BP_1	6.8	69.8	0.12	540	0.1	0.932	0.48	-0.36%	● (cyan)
BP_2	6.8	70.4	0.12	578	0.6	0.805	3.0	-0.36%	● (magenta)
BP_3	7.0	63.0	0.15	372	0.1	0.965	3.4	-0.37%	● (black)
BP_4	7.0	63.9	0.15	403	0.6	0.858	20.8	-0.37%	● (red)

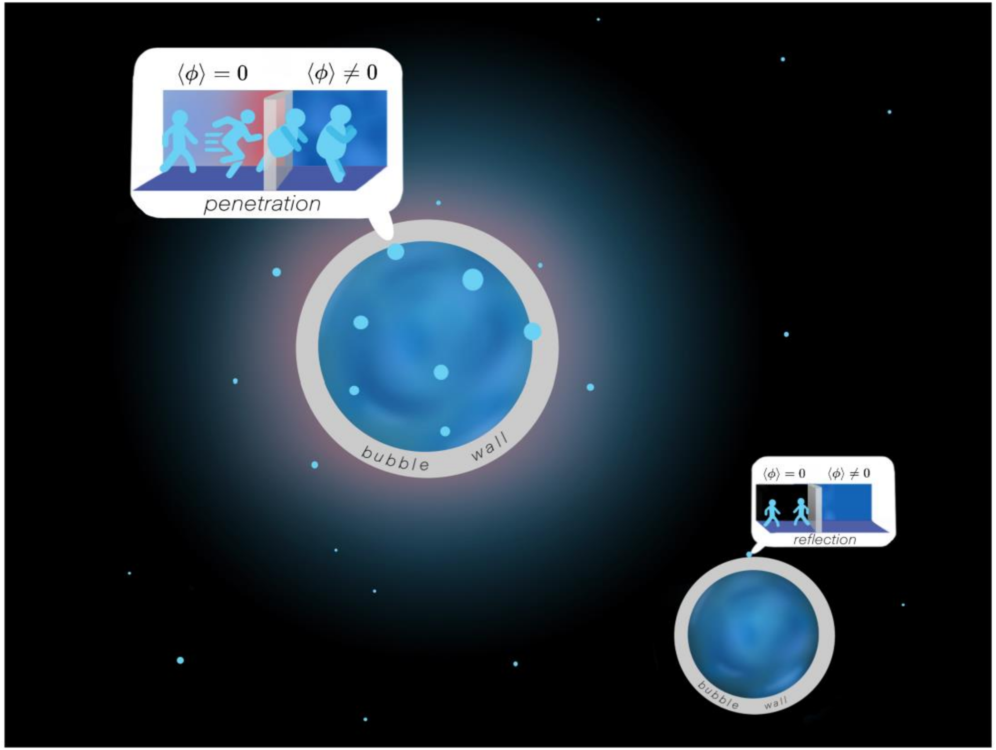
F_ϕ^{trap}:The fraction of particles trapped into the false vacuum. It is determined by the phase transition dynamics.

Siyu Jiang, FPH,
Pyungwon Ko, JHEP 07 (2024) 053

Case II: filtered DM from a FOPT

Bubble wall plays an essential role in the filtered DM mechanism.

DM



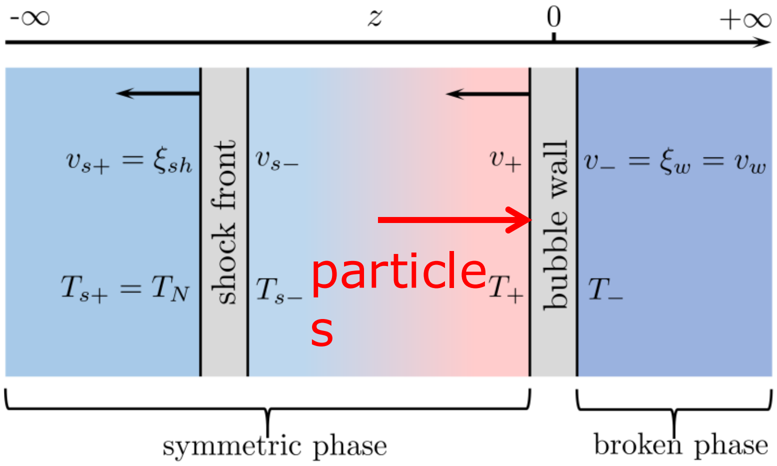
Siyu Jiang, FPH, Chong Sheng Li,
Phys.Rev.D 108 (2023) 6, 063508

Case II: filtered DM

Original work:

$$\tilde{v}_{\rm pl} = v_w, \quad T = T' = T_n$$

Phys.Rev.Lett. 125 (2020) 15, 151102 , M. J. Baker, J. Kopp, and A. J. Long



$$\tilde{v}_{\rm pl} = \tilde{v}_+, \quad T = T_+, \quad T' = T_- \qquad \text{(this work with hydrodynamic effects)}\ .$$

$$J_w^{\rm in} = \frac{g_\chi}{(2\pi)^2} \int_0^{-1} d\cos\theta \cos\theta \int_{-\frac{m_\chi^{\rm in}}{\cos\theta}}^{\infty} dp \frac{p^2}{e^{\tilde{\gamma}_+(1+\tilde{v}_+\cos\theta)p/T_+}} = \frac{g_\chi T_+^3 \left(1 + \tilde{\gamma}_+ m_\chi^{\rm in}(1-\tilde{v}_+)/T_+\right)}{4\pi^2 \tilde{\gamma}_+^3 (1-\tilde{v}_+)^2} e^{-\tilde{\gamma}_+ m_\chi^{\rm in}(1-\tilde{v}_+)/T_+}.$$

$$n_\chi^{\rm in} = \frac{J_w^{\rm in}}{\gamma_w v_w} \qquad \Omega_{\rm DM}^{\rm (hy)} h^2 = \frac{m_\chi^{\rm in}(n_\chi^{\rm in} + n_{\bar{\chi}}^{\rm in})}{\rho_c/h^2} \frac{g_{\star 0} T_0^3}{g_\star(T_-) T_-^3} \simeq 6.29 \times 10^8 \frac{m_\chi^{\rm in}}{\rm GeV} \frac{(n_\chi^{\rm in} + n_{\bar{\chi}}^{\rm in})}{g_\star(T_-) T_-^3}$$

Case II: filtered DM

$$T_\phi^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 - V_{T=0}(\phi)\right]$$

Energy-momentum tensor of scalar field

$$T_{\rm pl}^{\mu\nu} = \sum_i \int \frac{d^3k}{(2\pi)^3 E_i} k^\mu k^\nu f_i^{\rm eq}(k)$$

Energy-momentum tensor of fluid

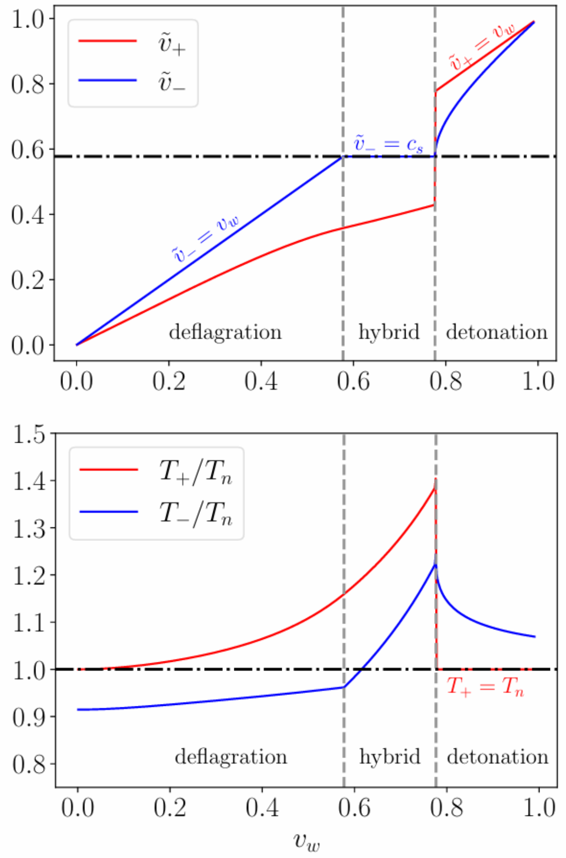
$$T_{\rm fl}^{\mu\nu} = T_\phi^{\mu\nu} + T_{\rm pl}^{\mu\nu} = \omega u^\mu u^\nu - p g^{\mu\nu}$$

Energy-momentum conservation

$$\omega_+ \tilde{v}_+^2 \tilde{\gamma}_+^2 + p_+ = \omega_- \tilde{v}_-^2 \tilde{\gamma}_-^2 + p_-, \quad \omega_+ \tilde{v}_+ \tilde{\gamma}_+^2 = \omega_- \tilde{v}_- \tilde{\gamma}_-^2$$

$$\alpha_+ \equiv \epsilon / \left(a_+ T_+^4\right) \qquad r_\omega = \omega_+/\omega_- = (a_+T_+^4)/(a_-T_-^4)$$

$$\nabla_\mu T^{\mu\nu} = 0 \quad \longleftrightarrow \quad \begin{aligned} j\frac{v}{\xi} &= \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v \\ \frac{\partial_\xi \omega}{\omega} &= \left(1 + \frac{1}{c_s^2}\right)\gamma^2 \mu \partial_\xi v\ . \end{aligned}$$



Case II: filtered DM

Boltzmann equation

$$\mathbf{L}[f_\chi] = \mathbf{C}[f_\chi]$$

$$f_\chi = \mathcal{A}(z,p_z)\, f^{\rm eq}_{\chi,+} = \mathcal{A}(z,p_z)\exp\left(-\frac{\tilde\gamma_+(E-\tilde v_+ p_z)}{T_+}\right)$$

$$\mathbf{L}[f_\chi] = \frac{p_z}{E}\frac{\partial f_\chi}{\partial z} - \frac{m_\chi}{E}\frac{\partial m_\chi}{\partial z}\frac{\partial f_\chi}{\partial p_z} \qquad m_\chi(z) \equiv \frac{m_\chi^{\rm in}(\phi_-)}{2}\left(1+\tanh\frac{2z}{L_w}\right)$$

$$g_\chi\int\frac{dp_x dp_y}{(2\pi)^2}\mathbf{L}[f_\chi] \approx \left[\left(\frac{p_z}{m_\chi}\frac{\partial}{\partial z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\partial}{\partial p_z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\tilde\gamma_+\tilde v_+}{T_+}\right)\mathcal{A}(z,p_z)\right]\frac{g_\chi m_\chi T_+}{2\pi\tilde\gamma_+}e^{\tilde\gamma_+\left(\tilde v_+ p_z - \sqrt{m_\chi^2+p_z^2}\right)/T_+}$$

including $\chi\bar\chi \leftrightarrow \phi\phi, \chi\phi \leftrightarrow \chi\phi, \chi\chi \leftrightarrow \chi\chi, \chi\bar\chi \leftrightarrow \chi\bar\chi, \cdots$

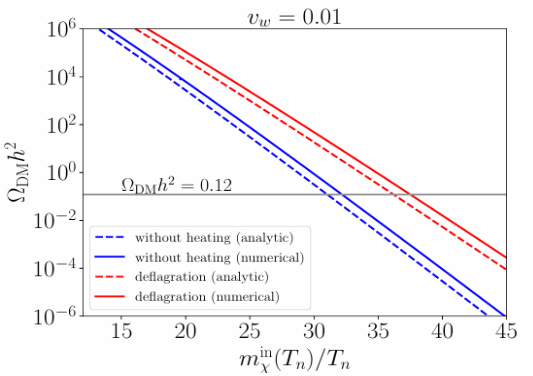
$$g_\chi\int\frac{dp_x dp_y}{(2\pi)^2}\mathbf{C}[f_\chi] = -g_\chi g_{\bar\chi}\int\frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}}d\Pi_{q^{\mathcal{P}}}4F\sigma_{\chi\bar\chi\to\phi\phi}\left[f_{\chi_p}f^{\rm eq}_{\bar\chi_q,+} - f^{\rm eq}_{\chi_p}f^{\rm eq}_{\bar\chi_q}\right]$$

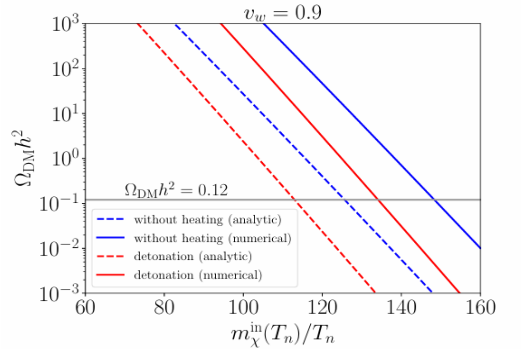
$$= -g_\chi g_{\bar\chi}\int\frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}}d\Pi_{q^{\mathcal{P}}}4F\sigma_{\chi\bar\chi\to\phi\phi}\left[\mathcal{A}f^{\rm eq}_{\chi_p,+}f^{\rm eq}_{\bar\chi_q,+} - f^{\rm eq}_{\chi_p}f^{\rm eq}_{\bar\chi_q}\right]$$

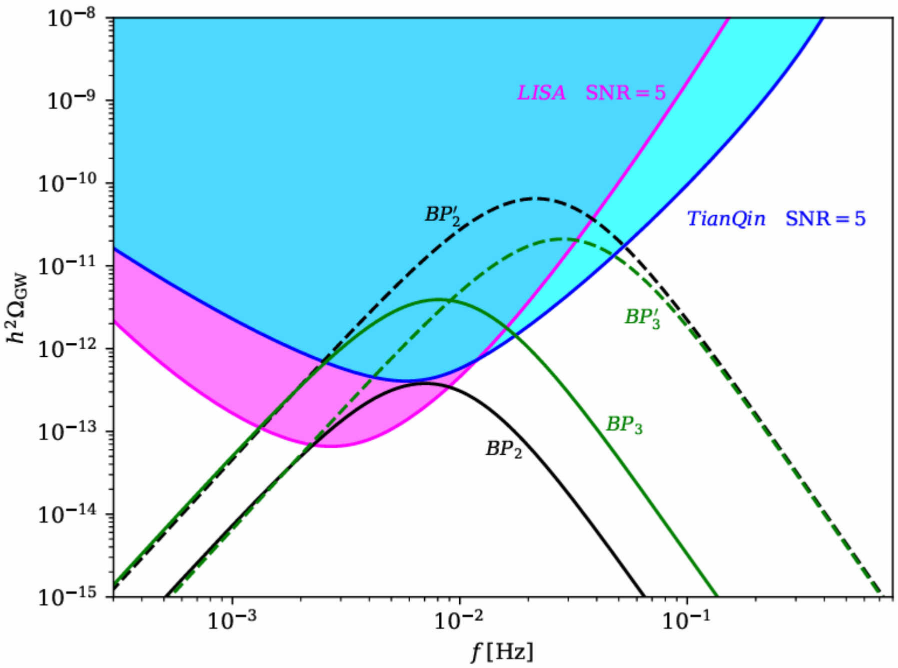
$$\equiv \Gamma_{\rm P}(z,p_z)\mathcal{A}(z,p_z) - \Gamma_{\rm I}(z,p_z)\ ,$$

Case II: filtered DM

$$n_\chi^{\rm in} = \frac{T_+}{\gamma_w \tilde{\gamma}_+} \int_0^\infty \frac{dp_z}{(2\pi)^2} \mathcal{A}(z \gg L_w, p_z) \exp\left[\tilde{\gamma}_+ \left(\tilde{v}_+ p_z - \sqrt{p_z^2 + (m_\chi^{\rm in})^2}\right)/T_+\right] \left(\sqrt{p_z^2 + (m_\chi^{\rm in})^2} + \frac{T_+}{\tilde{\gamma}_+}\right)$$







The missing GW source ?

Outline

1. Motivation for new dark matter (DM) mechanism
2. Heavy DM from first-order phase transition (FOPT) and GW

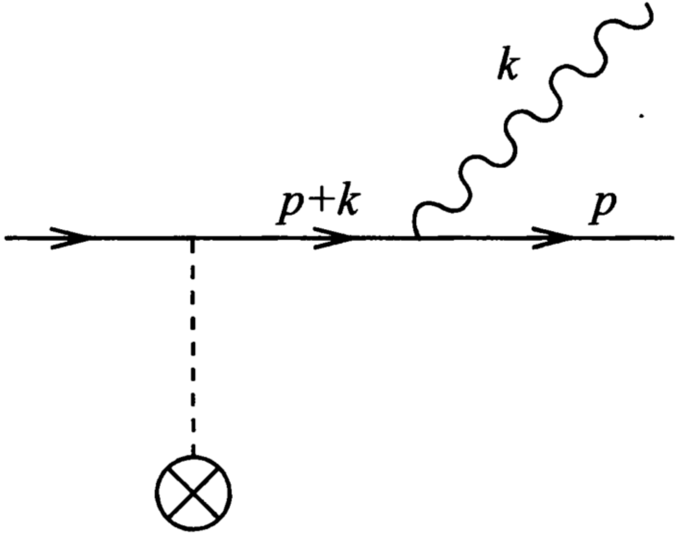
    Case I: Q-ball and gauged Q-ball DM

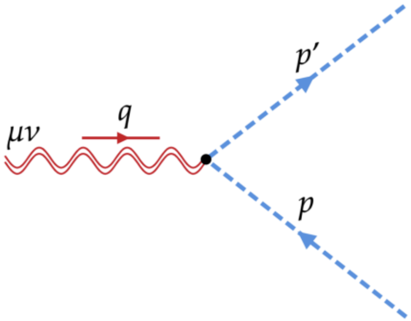
    Case II: filtered DM

3. New gravitational wave (GW) source
4. Summary and outlook

Recall from the textbook

Photon/Graviton emission by an accelerated charge/mass





$$= \tau_1^{\mu\nu}(p, p', m_a)$$

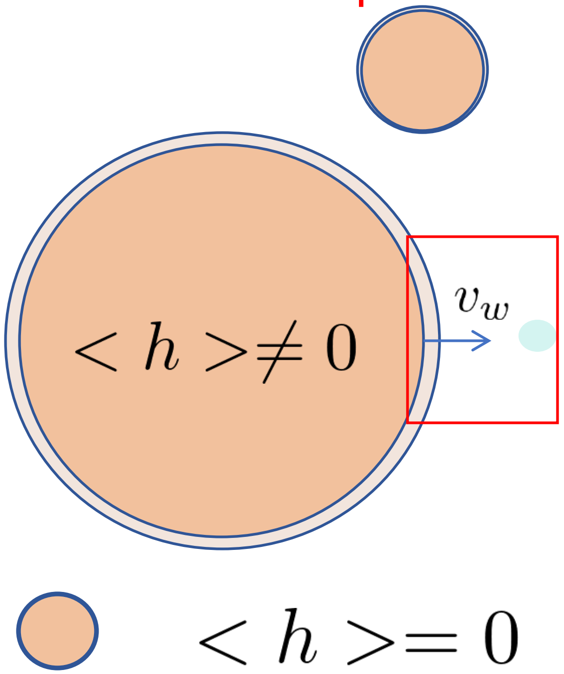
where

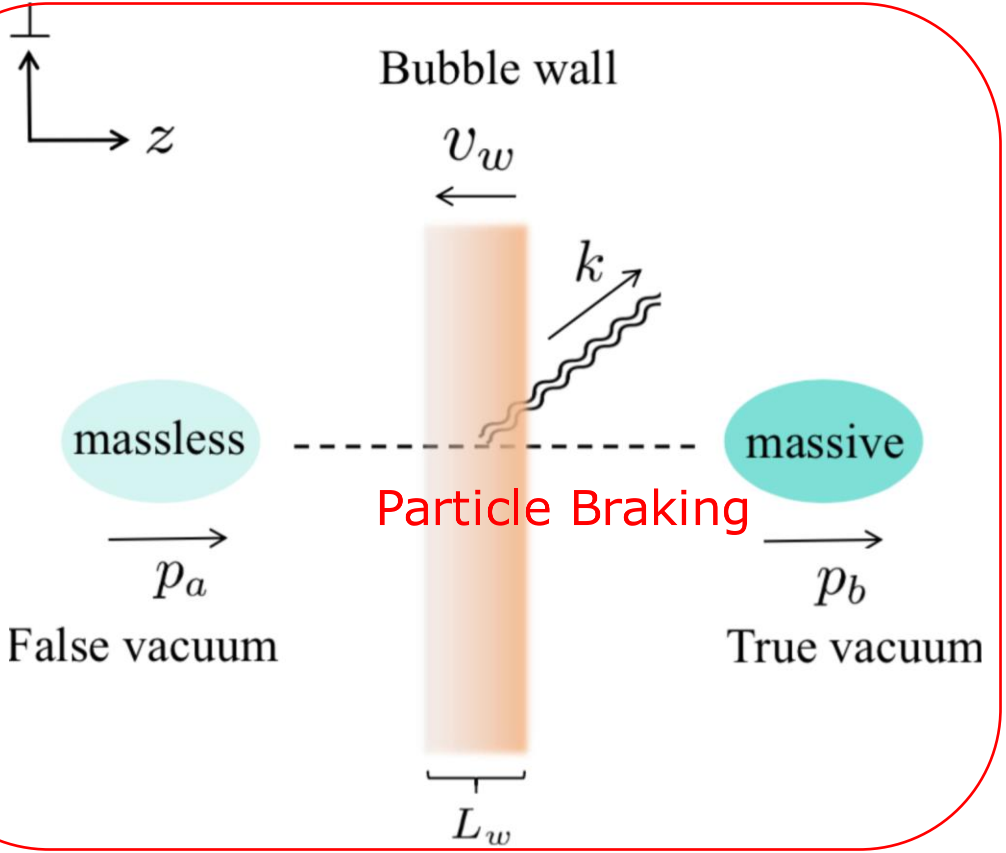
$$\tau_1^{\mu\nu}(p, p', m_a) = -\frac{i\kappa}{2}\left(p^\mu p'^\nu + p^\nu p'^\mu - \eta^{\mu\nu}\left(p \cdot p' - m_a^2\right)\right).$$

Feynmann diagram
for graviton radiation

Braking GW from phase transition

The missing GW radiation during phase transition process





arXiv: 2508.04314, Dayun Qiu, Siyu Jiang, FPH

Calculation on the Braking GW

$$\rho_{\rm GW} = \int \frac{{\rm d}^3 p_a}{(2\pi)^3} f_a(p_a) \int {\rm d}P_{s\to sg}\, E_k$$

the distribution function of the thermal plasma

bremsstrahlung probability

Bodeker–Moore method
JCAP 05, 025

Bremsstrahlung probability

For the process $a(p_a) \to b_1(p_1) b_2(p_2) \dots b_n(p_n)$, the splitting probability after integration over the final states reads

$$\int dP_{1\to n} \equiv \left(\prod_{j=1}^{n} \tilde{V} \int \frac{d^3 p_j}{(2\pi)^3} \right) \frac{|\langle \vec{p}_1, \dots, \vec{p}_n | \mathcal{T} | \phi_a \rangle|^2}{\langle \phi_a | \phi_a \rangle \prod_{j=1}^{n} \langle \vec{p}_j | \vec{p}_j \rangle},$$

the volume of the spatial integration range

How to calculate the interaction matrix element？

$$|\phi_a\rangle \equiv \int \frac{d^3 p_a}{(2\pi)^3} \frac{\phi(\vec{p}_a)}{2E_a} |\vec{p}_a\rangle, \quad \int \frac{d^3 p_a}{(2\pi)^3} \frac{|\phi(\vec{p}_a)|^2}{2E_a} = 1, \quad |\vec{p}_i\rangle = \sqrt{2E_i} a_i^\dagger |0\rangle.$$

$$\langle \phi_a | \phi_a \rangle = 1, \quad \langle \vec{p}_j | \vec{p}_j \rangle = 2E_{\vec{p}_j} (2\pi)^3 \delta^{(3)}(\vec{p}_j - \vec{p}_j) = 2E_{\vec{p}_j} \int d^3x \, e^{i(\vec{p}_j - \vec{p}_j)\cdot \vec{x}} = 2E_{\vec{p}_j} \tilde{V}.$$

Bremsstrahlung probability

- Quantization of scalar fields in the presence of bubble walls

JHEP 05, 294, arXiv:2310.06972

equation of motion
$$(\partial^2 + m_0^2 + \Delta m^2(z))\phi = 0,$$

$$\phi = e^{-i(p^0 t - p^1 x - p^2 y)}\chi(z),$$

$$\chi'' + (p_s^z)^2\chi = \Delta m^2(z)\chi.$$

$$p_s^z = \sqrt{(p^0)^2 - (p^1)^2 - (p^2)^2 - m_0^2},$$

the longitudinal momentum of the particle in the symmetric phase

Bremsstrahlung probability

- Quantization of scalar fields in the presence of bubble walls

1. $p_s^z \gg L_w^{-1}$, WKB approximation:

$$\chi(z) = \sqrt{\frac{p_s^z}{p^z(z)}} \exp\left(i \int_0^z p^z(z') dz' + \dots\right) \approx e^{\pm i \int_0^z p^z(z') \mathrm{d}z'}, \quad p^z(z) = \sqrt{(p_s^z)^2 - \Delta m^2(z)}.$$

2. $p_s^z \ll L_w^{-1}$, step-like bubble wall profile: $\Delta m^2(z) \simeq (\tilde{m}^2 - m_0^2) \cdot \Theta(z)$,

mass of the field ϕ in the broken phase

$$\chi(z, p_s^z) = \begin{cases} C_1 e^{i p_s^z z} + C_2 e^{-i p_s^z z}, & z < 0 \\ C_3 e^{i p_b^z z} + C_4 e^{-i p_b^z z}, & z \geq 0 \end{cases}, \quad p_b^z = \sqrt{(p_s^z)^2 + m_0^2 - \tilde{m}^2}$$

the longitudinal momentum of the particle in the broken phase.

Bremsstrahlung probability

- Quantization of Scalar Fields in the Presence of Bubble Walls

To facilitate quantization, we adopt a basis consisting of "right-moving waves" and "left-moving waves":

normalization coefficients

$$\chi_R(z,p_s^z) = N_R \begin{cases} e^{ip_s^z z} + r_R e^{-ip_s^z z}, & z < 0, \\ t_R e^{ip_b^z z}, & z \geq 0, \end{cases},$$

$$\chi_L(z,p_s^z) = N_L \begin{cases} t_L e^{-ip_s^z z}, & z < 0, \\ r_L e^{ip_b^z z} + e^{-ip_b^z z}, & z \geq 0, \end{cases}.$$

The transmission and reflection coefficients can be determined by imposing the continuity of the mode function and its derivative at the interface $z=0$.

Bremsstrahlung probability

- Quantization of scalar fields in the presence of bubble walls

The basis is also applicable to the WKB region.

$$p_s^z \sim p_b^z \gg L_w^{-1} \sim \sqrt{\tilde{m}^2 - m_0^2},$$

the momentum $p^z(z)$ can be expanded near $z = \pm\infty$ using a Taylor series.

$$\chi^{\text{WKB}}(z, p_s^z) \approx \begin{cases} \xi_{<0}(z) e^{\pm i p_s^z z}, & z < 0, \\ \xi_{>0}(z) e^{\pm i p_b^z z}, & z \geq 0, \end{cases}$$

Bremsstrahlung probability

- Quantization of scalar fields in the presence of bubble walls

Therefore, by incorporating the transverse plane wave components, we obtain the “plane wave solution” that satisfies the Klein–Gordon equation,

$$\phi_R(p) = e^{-ip_n x^n}\chi_R(z, p_s^z), \quad p_n x^n = p^0 t - \vec{p}_\perp \cdot \vec{x}_\perp,\ p^0 > m_0,$$

$$\phi_L(p) = e^{-ip_n x^n}\chi_L(z, p_s^z), \quad p_n x^n = p^0 t - \vec{p}_\perp \cdot \vec{x}_\perp,\ p^0 > \tilde{m}.$$

$$\phi(x) = \sum_{I=R,L} \int \frac{\mathrm{d}^3 p}{(2\pi)^3 \sqrt{2p^0}} \left(a_{I,p}\phi_I(p) + a_{I,p}^\dagger \phi_I^*(p) \right),$$

$$\left[a_{I,p}, a_{J,q}^\dagger\right] = (2\pi)^3 \delta^{(2)}(\vec{p}_\perp - \vec{q}_\perp)\delta(p_s^z - q_s^z)\delta_{IJ},$$

$$\left[a_{I,p}, a_{J,q}\right] = \left[a_{I,p}^\dagger, a_{J,q}^\dagger\right] = 0, \quad I, J \in \{R, L\}.$$

The single particle states are defined by

$$|p^R\rangle \equiv \sqrt{2p^0} a_{R,p}^\dagger |0\rangle,$$

$$|p^L\rangle \equiv \sqrt{2p^0} a_{L,p}^\dagger |0\rangle.$$

the incident state!

Bremsstrahlung probability

- Quantization of scalar fields in the presence of bubble walls

By using the time reversal, we can get another set of orthogonal bases,

(outgoing state basis)

$$\phi_L^{\text{out}}(p) = e^{-ip_n x^n}\zeta_L(z,p_s^z) = e^{-ip_n x^n}\chi_R^*(z,p_s^z)$$
$$= e^{-ip_n x^n}\left(r_{R,p}^*\chi_R(z,p_s^z) + t_{R,p}^*\sqrt{\frac{p_b^z}{p_s^z}}\chi_L(z,p_s^z)\right),$$
$$\phi_R^{\text{out}}(p) = e^{-ip_n x^n}\zeta_R(z,p_s^z) = e^{-ip_n x^n}\chi_L^*(z,p_s^z)$$
$$= e^{-ip_n x^n}\left(r_{L,p}^*\chi_L(z,p_s^z) + t_{L,p}^*\sqrt{\frac{p_s^z}{p_b^z}}\chi_R(z,p_s^z)\right).$$

These bases correspond to the outgoing particle states.

$$|p^{L,\text{out}}\rangle = r_{R,p}^*|p^R\rangle + t_{R,p}^*\sqrt{\frac{p_b^z}{p_s^z}}|p^L\rangle\,,\quad |p^{R,\text{out}}\rangle = t_{L,p}^*\sqrt{\frac{p_s^z}{p_b^z}}|p^R\rangle + r_{L,p}^*|p^L\rangle\ .$$

Bremsstrahlung probability

Now, we can calculate the interaction matrix element.

Feynman amplitude

$$\langle \vec{p}_b^{I,\text{out}}, \vec{k}| \mathcal{T} |\vec{p}_a^{R}\rangle = \int \mathrm{d}^4x \, \langle \vec{p}_b^{I,\text{out}}, \vec{k}| \mathcal{H}_{\text{int}} |\vec{p}_a^{R}\rangle$$

$$= \int \mathrm{d}z \int \frac{\mathrm{d}^3p'_a}{(2\pi)^3} \int \frac{\mathrm{d}^3p'_b}{(2\pi)^3} \int \frac{\mathrm{d}^3k'}{(2\pi)^3} \boxed{V^\dagger(z)} \chi_R(z, p_a'^z) \zeta_I^*(z, p_b'^z) \chi^*(z, k'^z)$$

$$\times (2\pi)^3 \delta(E'_a - E'_b - E'_k) \delta^{(2)}(\vec{p}'_{a,\perp} - \vec{p}'_{b,\perp} - \vec{k}'_\perp) \langle \vec{p}_b^{I,\text{out}}, \vec{k}| a_k^\dagger a_{I,b}^\dagger a_{R,a} |\vec{p}_a^{R}\rangle$$

$$= (2\pi)^3 \delta\left(\sum E\right) \delta^{(2)}\left(\sum \vec{p}_\perp\right) \mathcal{M}_I,$$

$$\mathcal{M}_I = \int_{-\infty}^{+\infty} \mathrm{d}z \, V^\dagger(z) \chi_R(z, p_a^z) \zeta_I^*(z, p_b^z) \chi^*(z, k^z).$$

Bremsstrahlung probability

Thus, the bremsstrahlung probability becomes

$$\int \mathrm{d}P_{s\to sg} = \int \frac{\mathrm{d}^3 p_b}{(2\pi)^3 2E_b} \int \frac{\mathrm{d}^3 k}{(2\pi)^3 2E_k} \int \frac{\mathrm{d}^3 p'_a}{(2\pi)^3} \frac{|\phi(\vec{p}'_a)|^2}{2E'_a} \frac{1}{2p'^z_a} \times (2\pi)^3 \delta^{(2)}\left(\sum \vec{p}'_\perp\right) \delta\left(\sum E'\right) \left(|\mathcal{M}_R|^2 + |\mathcal{M}_L|^2\right).$$

Assume that $\phi(\vec{p})$ is highly localized around $\vec{p} = \vec{p}_a$, we have finally

$$\int \mathrm{d}P_{s\to sg} = \int \frac{\mathrm{d}^3 p_b}{(2\pi)^3 2E_b} \int \frac{\mathrm{d}^3 k}{(2\pi)^3 2E_k} \frac{1}{2p^z_{a,s}} (2\pi)^3 \delta^{(2)}\left(\sum \vec{p}_\perp\right) \delta\left(\sum E\right) \times \left(|\mathcal{M}_R|^2 + |\mathcal{M}_L|^2\right).$$

Calculation on the Braking GW

$$\rho_{\rm GW} = \int \frac{d^3 p_a}{(2\pi)^3} f_a(p_a) \int dP_{s\to sg}\, E_k$$

the distribution function of the thermal plasma

bremsstrahlung probability

Bodeker–Moore method
JCAP 05, 025

Under the ultra-relativistic limit, it is appropriate to employ the Wenzel–WKB approximation for evaluating the matrix element.

$$\chi(z, p_s^z) \simeq \exp\left[i \int_0^z dz'\, p^z(z') \right].$$

$$\mathcal{M}_L \simeq 0,$$

$$\mathcal{M}_R \simeq \mathcal{M}^{\rm WKB} = \int_{-\infty}^{\infty} dz\, \chi(z, p_{a,s}^z)\chi^*(z, p_{b,s}^z)\chi^*(z, k^z)V(z).$$

$$\mathcal{M}^{\rm WKB} \simeq \frac{V_s}{i\Delta p_s^z} - \frac{V_b}{i\Delta p_b^z}$$

Calculation on the Braking GW

$$\rho_{\rm GW} = \int \frac{{\rm d}^3 p_a}{(2\pi)^3} f_a(p_a) \int {\rm d}P_{s\to sg}\, E_k$$

the distribution function of the thermal plasma

bremsstrahlung probability

Bodeker–Moore method
JCAP 05, 025

In wall frame,

$$\int {\rm d}P_{s\to sg} \simeq \int \frac{{\rm d}^3 k}{(2\pi)^3 2E_k} \mathcal{P}(k)\Theta\left(p_b^z - L_w^{-1}\right)\Theta\left(L_w^{-1} - \Delta p^z\right)\Theta\left(k^z\right)$$

$$\mathcal{P}(k) \equiv \frac{\kappa^2 m^4 E_a^2 E_k^2}{2(E_a^2 k_\perp^2 + m^2 E_k^2)^2},$$

collinear gravitons dominate

WKB condition

non-adiabatic condition

Calculation on the Braking GW

In plasma frame,

$$\int \mathrm{d}P_{s\to sg}\,\tilde{E}_k$$

||

Lorentz transformation

$$\rho_{\mathrm{GW}} = \int \mathrm{d}^3\tilde{p}_a\ f_a(\tilde{p}_a)\,\langle\tilde{E}_k\rangle\,,$$

average energy of the gravition

$$\tilde{E}_k = \gamma(E_k + v_w k^z),\quad \tilde{k}^z = \gamma(k^z + v_w E_k),\quad \tilde{k}_\perp = k_\perp,$$

Change the order of integration

heavily suppressed

$\tilde{E}_k < \gamma L_w^{-1}$ $\qquad$ $\tilde{E}_k > \gamma L_w^{-1}$

$$\rho_{\mathrm{GW}} = \frac{\kappa^2 m^4 T}{64\pi^4}\left[\int_{\mathrm{low}} \mathrm{d}\tilde{E}_k\ I_{\mathrm{low}}(\tilde{E}_k) + \int_{\mathrm{high}} \mathrm{d}\tilde{E}_k\ I_{\mathrm{high}}(\tilde{E}_k)\right],$$

GW spectrum

$$h^2\Omega^{\rm brakes}_{\rm GW,0}(f_0) = \frac{h^2}{\rho_{c,0}}\frac{d\rho_{\rm GW,0}}{d\ln f_0} = \frac{h^2}{\rho_{c,0}}\frac{d\rho_{\rm GW,0}}{d\ln \tilde{E}_k}$$

$$\simeq 6.91\times 10^{-20}\left(\frac{3.94}{g_{*,s}}\right)\left(\frac{m}{T}\right)^2\left(\frac{m}{10^{13}\ \rm GeV}\right)^2\left(\frac{f_0}{10^{10}\ \rm Hz}\right)$$

(amplitude) $\times I_{\rm low}\left(\tilde{E}_k\right)\Theta\left(\gamma L_w^{-1}-\tilde{E}_k\right),$ cutoff point (peak)

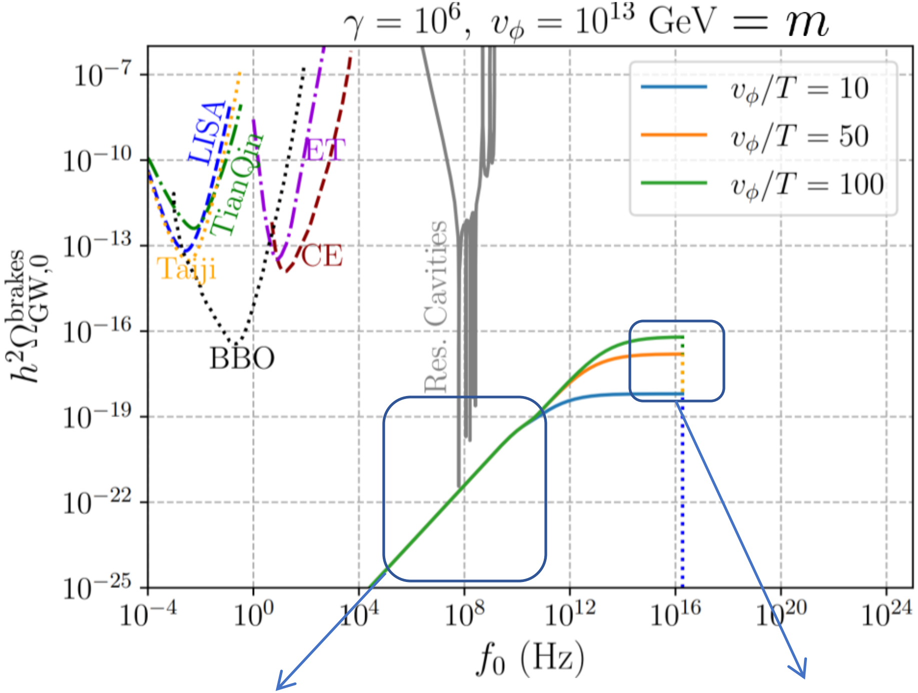
$$\tilde{E}_k \ll T,\ I_{\rm low}\simeq 2\zeta_3 T^2/m^2,$$

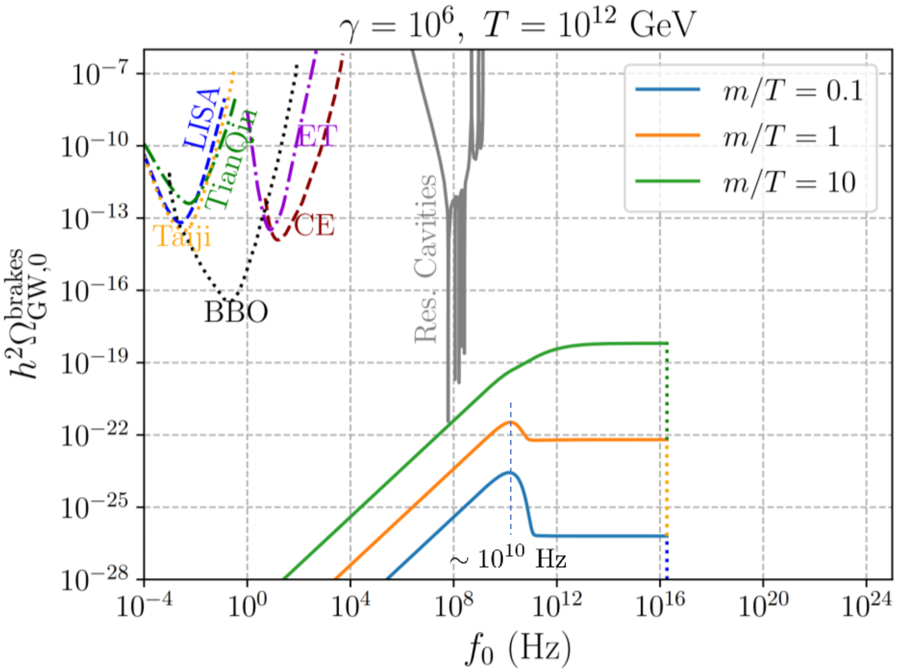
$$\tilde{E}_k \gg m,\ I_{\rm low}\simeq \pi^2 T/(48\tilde{E}_k),$$

$$\tilde{E}_k = 2.71\times 10^{24}\left(\frac{g_{*,s}}{3.94}\right)^{1/3}\left(\frac{T}{10^{11}\ \rm GeV}\right) f_0,$$

GW spectrum

arXiv: 2508.04314, Dayun Qiu, Siyu Jiang, FPH





$$h^2\Omega^{\rm brakes}_{\rm GW,0}(f_0) \propto m^2 f_0, \quad h^2\Omega^{\rm brakes}_{\rm GW,0}(f_{\rm peak}) \propto m^4/T^2. \quad \text{when } T \gtrsim m,$$

collinear gravitons non-collinear gravitons double-peaked structure

GW spectrum

Specific model: $V(S,\Phi) = \lambda_s|S|^4 + \lambda_\phi|\Phi|^4 + \lambda_{\phi s}|S|^2|\Phi|^2\ ,$

$\Phi = (v_\phi + \phi + i\varphi)/\sqrt{2}$ $\qquad V_{\rm eff} = V_0(\phi) + V_T(\phi,T) + V_{\rm daisy}(\phi,T).$

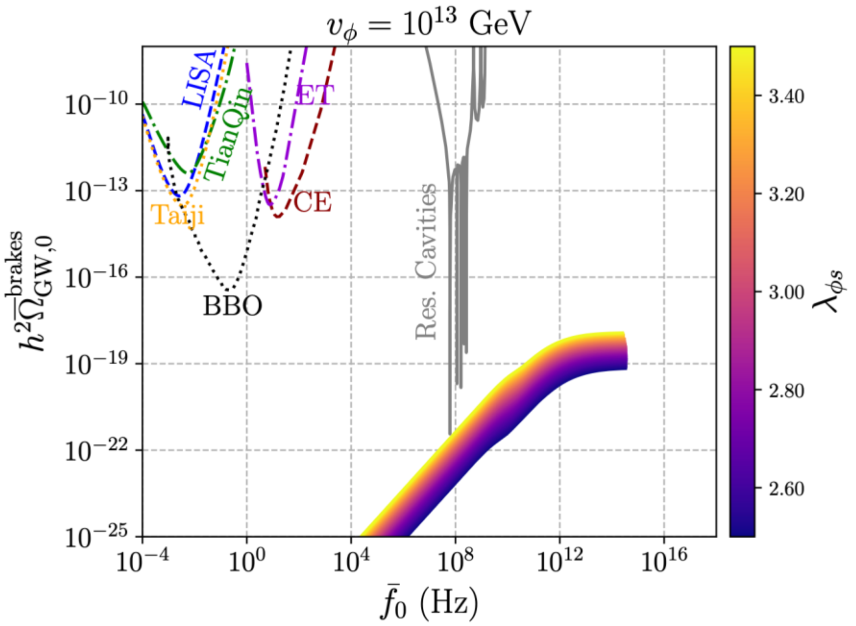
$S = (s_1 + is_2)/\sqrt{2}$

$$V_T(\phi,T) = \sum_{i=\rm bosons} \frac{g_i T^4}{2\pi^2} J_B\left(\frac{m_i^2(\phi)}{T^2}\right) - \sum_{i=\rm fermions} \frac{g_i T^4}{2\pi^2} J_F\left(\frac{m_i^2(\phi)}{T^2}\right),$$

$$V_0(\phi) = B_1\phi^4\left(\ln\frac{\phi}{v_\phi} - \frac{1}{4}\right), \qquad B_1 = \frac{3}{2\pi^2}\left(\frac{\lambda_{\phi s}^2}{96} - \sum_i \frac{y_{R,i}^4}{96}\right).$$

$$V_{\rm daisy}(\phi,T) = -\frac{T}{12\pi}\sum_{i=\rm bosons} g_i\left[\left(m_i^2(\phi) + \Pi_i(T)\right)^{\frac{3}{2}} - m_i^3(\phi)\right],$$

- The mass of the scalar particle m
- the temperature of the plasma T
- the thickness of the bubble wall L_w
- the Lorentz factor of the bubble wall γ



arXiv: 2508.04314, Dayun Qiu, Siyu Jiang, FPH

New GW spectrum recap

The GW power spectrum exhibits two distinct behaviors across different frequency regimes.

arXiv: 2508.04314, Dayun Qiu, Siyu Jiang, FPH

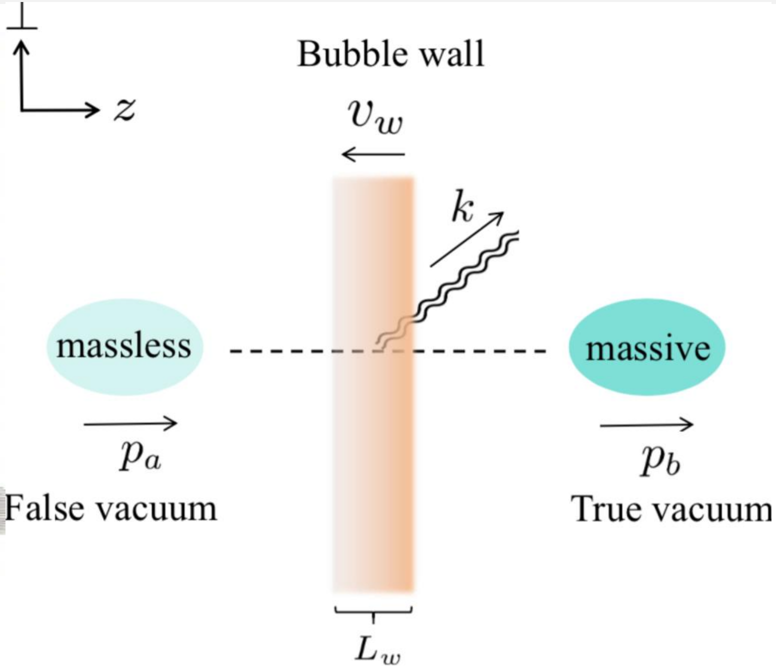
- In the low-frequency regime, the spectrum scales linearly with frequency and is proportional to the square of the mass, primarily sourced from ultra-collinear radiation emitted as particles traverse the bubble wall.

- In contrast, the high-frequency regime displays an approximately flat spectrum up to a cutoff frequency and the amplitude scales with the fourth power of the mass, dominated by non-collinear gravitons.

proportional to the Lorentz factor of the bubble wall

These distinct behaviors may help to more directly to extract the new particle information. However the detection of high frequency GW is challenge now.

Summary and outlook



- **Bubbles walls from FOPT have lots of fancy effects, eg. naturally production of heavy DM.**
- **Particles braking across the bubble walls can radiate GW.**
- **Various GW sources provide new approaches to explore DM.**

Thanks!

Comments and collaborations are welcome!