Anatomy of Family Trees in Cosmological Correlators

Zhong-Zhi Xianyu (鲜于中之)
Department of physics, Tsinghua University

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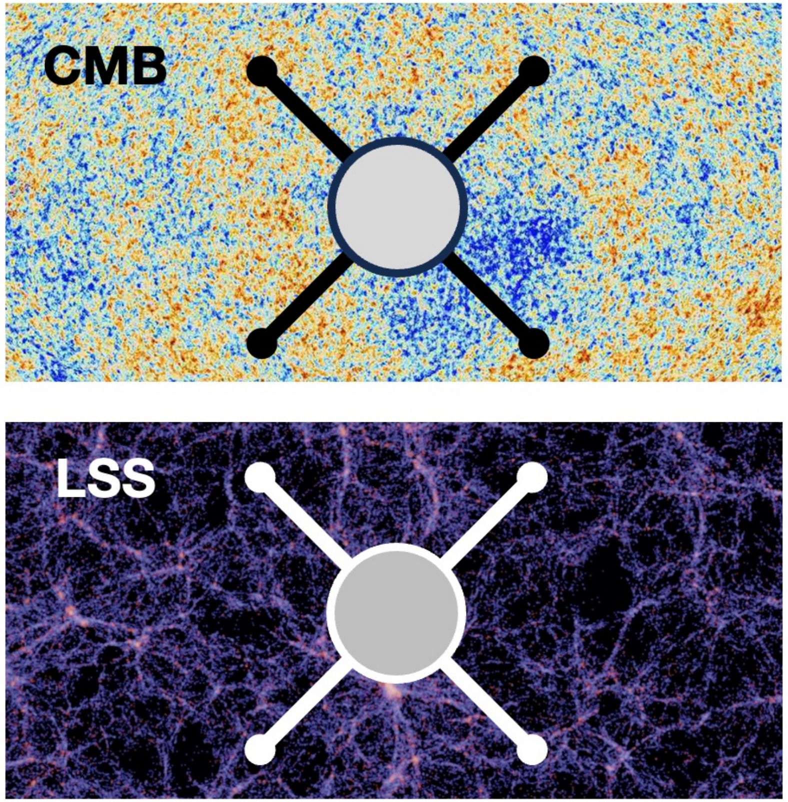
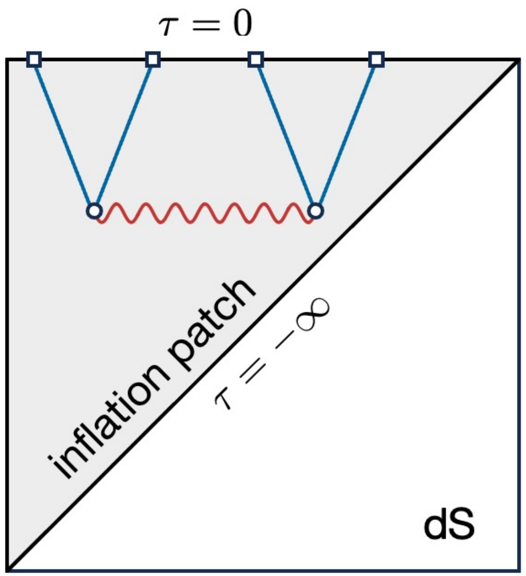
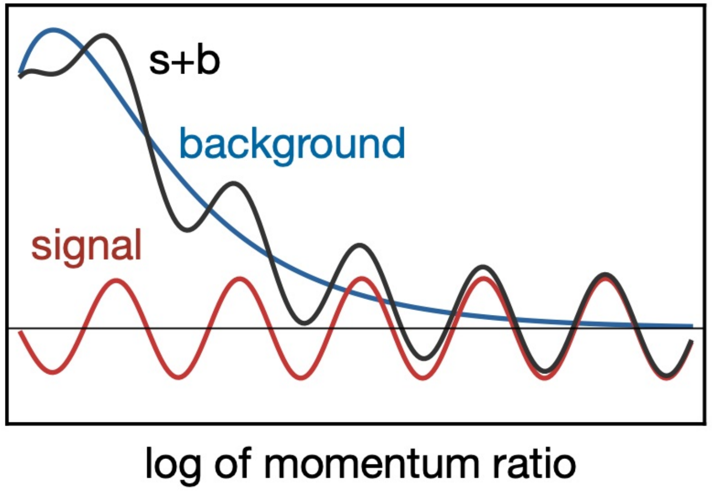
based on Bingchu Fan, ZX, 2509.02684

A Cosmological collider program

[Chen, Wang, 0911.3380; Arkani-Hamed, Maldacena, 1503.08043 and many more]







Inflation ~ dS

particle production

mass ~ 10^{14} GeV

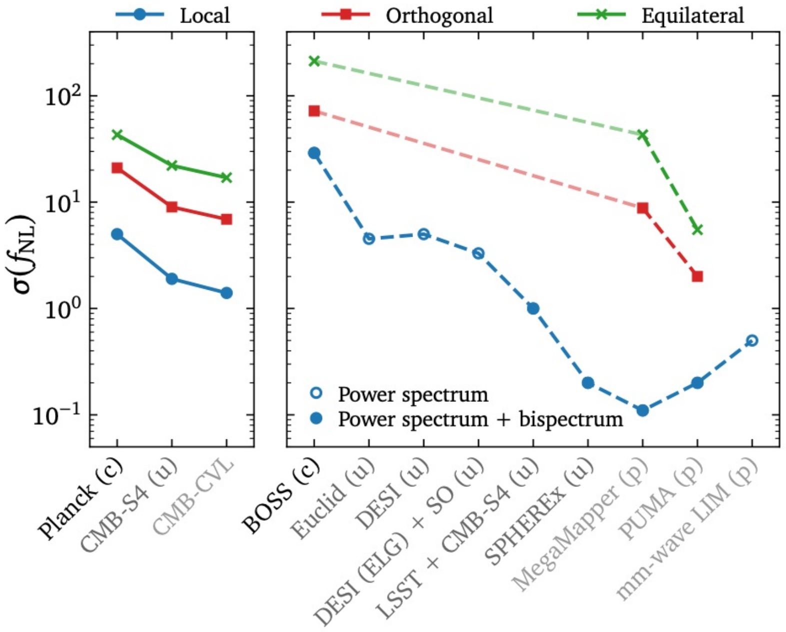
superhorizon resonance

mass, spin, coupling, etc

amplitude nonanalyticity

Data are coming in!

- ~ 2 orders in near future; ~ 4 ultimately with 21cm



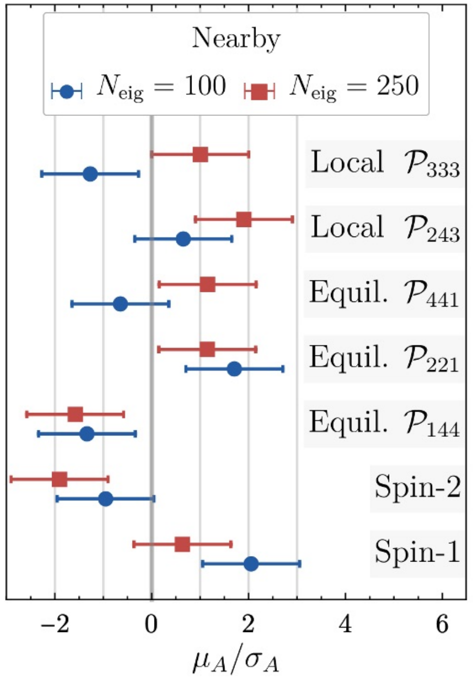
[Snowmass 2021: 2203.08128]

- Searches from CMB [Sohn et al. 2404.07203] and LSS data [Cabass et al. 2404.01894]

- Realistic particle models

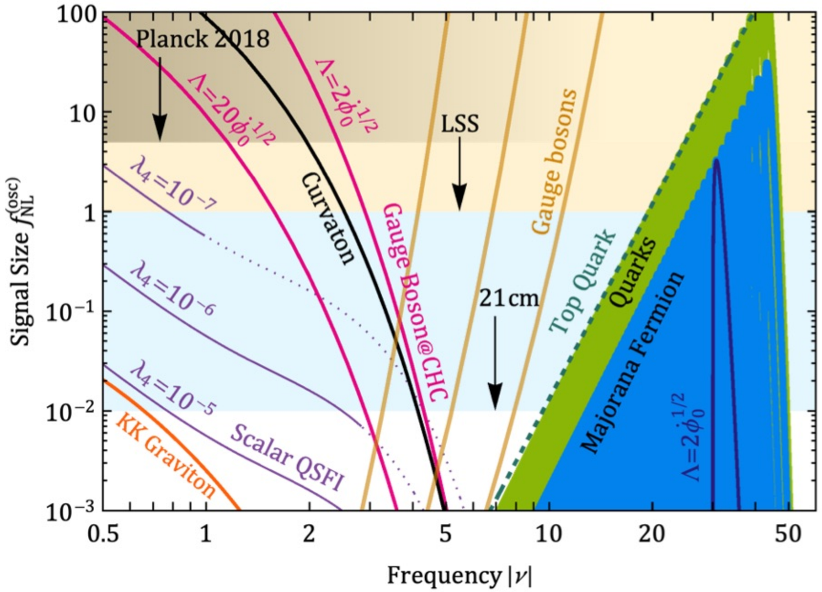
- Parity violation [Bao, Wang, ZX, Zhong, 2504.02931] **[Yi-Ming Zhong's talk]**

- Quasi-single field inflation meets CMB [Kumar, Lu, ZX, Zhang, to appear]



[Bao et al., 2504.02931]

Particle Phenomenology

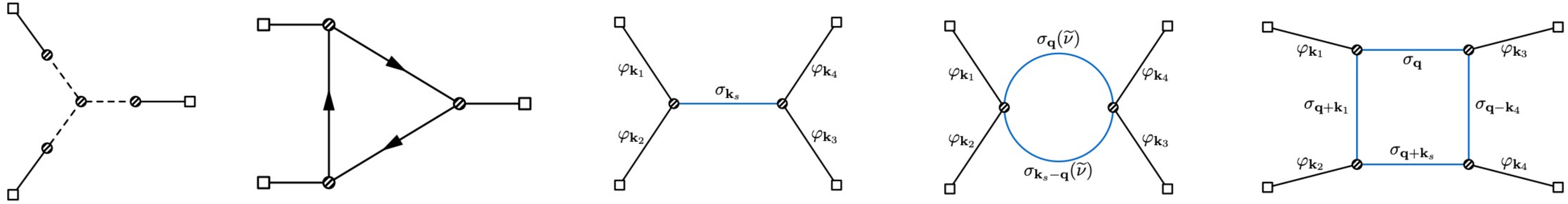


[Lian-Tao Wang, ZX, 1910.12876]

Over the years, many particle models identified in SM/BSM, with naturally large signals
Ongoing! **[Shuntaro Aoki's talk]**

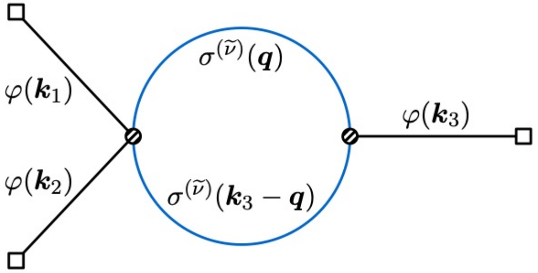
The CC signals can be there, and deserve to be treated seriously

To look for CC signals in real data, we need a template bank --- precision and efficiency



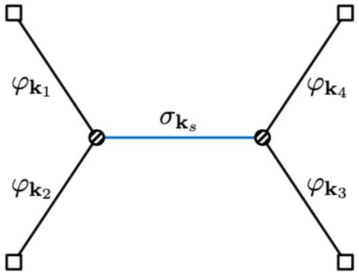
Why analytic?

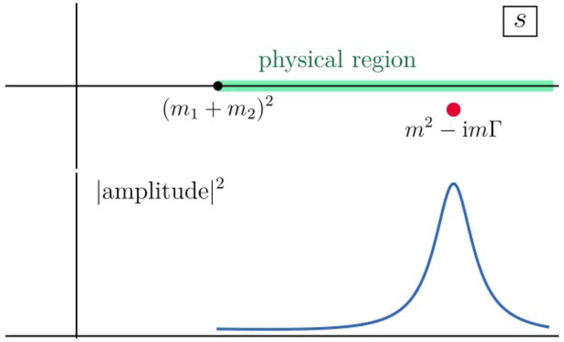
- Data-wise: good analytical strategy speeds up numerical computation
  Example: 3pt massive bubble: numerical [O(10^5) CPU hrs] vs. analytical [O(10s) @ laptop]
  [Wang, ZX, Zhong, 2109.14635] [Liu, Qin, ZX, 2407.12299]

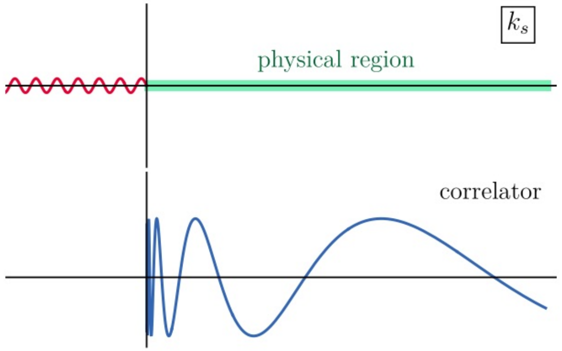


$$\mathcal{J}^{0,-2}(u) = Cu^3 - \frac{u^4}{128\pi\sin(2\pi i\tilde\nu)}\sum_{n=0}^{\infty} \frac{(3+4i\tilde\nu+4n)(1+n)_{\frac12}(1+2i\tilde\nu+n)_{\frac12}}{(\frac12+i\tilde\nu+n)_{\frac12}(\frac32+i\tilde\nu+n)_{\frac12}} \times\left\{ {}_2\mathcal{F}_1\left[\begin{matrix}2+2i\tilde\nu+2n, 4+2i\tilde\nu+2n\\ 4+4i\tilde\nu+4n\end{matrix}\middle| u\right]u^{2n+2i\tilde\nu} - {}_3\mathcal{F}_2\left[\begin{matrix}1,2,4\\ 1-2n-2i\tilde\nu, 4+2n+2i\tilde\nu\end{matrix}\middle| u\right]\right\} + (\tilde\nu\to-\tilde\nu)$$

- Theory-wise: good lessons about QFT in dS from analytical structures of correlators
  Whenever a correlator becomes singular, there is a physical reason





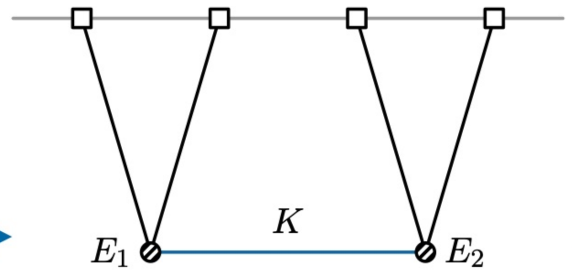


[Qin, ZX, 2308.14802]

Inflationary correlators: general structure

[See Chen, Wang, **ZX**, 1703.10166 for a review]

$$\int_{-\infty}^{0} \mathrm{d}\tau_1 (\mathrm{i}\mathsf{a}_1)(-\tau_1)^{p_1} e^{\mathrm{i}E_1\tau_1}$$



$$\int_{-\infty}^{0} \mathrm{d}\tau_2 (\mathrm{i}\mathsf{a}_2)(-\tau_2)^{p_2} e^{\mathrm{i}E_2\tau_2}$$

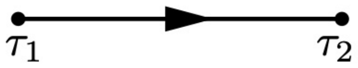
$$D_{\mathsf{ab}}^{(\widetilde{\nu})}(K;\tau_1,\tau_2)$$

$$D_{-+}^{(\widetilde{\nu})}(K;\tau_1,\tau_2) = \frac{\pi}{4} e^{-\pi\widetilde{\nu}} (\tau_1\tau_2)^{3/2} \mathrm{H}_{\mathrm{i}\widetilde{\nu}}^{(1)}(-K\tau_1) \mathrm{H}_{-\mathrm{i}\widetilde{\nu}}^{(2)}(-K\tau_2)$$

$$D_{+-}^{(\widetilde{\nu})}(K;\tau_1,\tau_2) = \frac{\pi}{4} e^{-\pi\widetilde{\nu}} (\tau_1\tau_2)^{3/2} \mathrm{H}_{-\mathrm{i}\widetilde{\nu}}^{(2)}(-K\tau_1) \mathrm{H}_{\mathrm{i}\widetilde{\nu}}^{(1)}(-K\tau_2)$$

$$D_{\pm\pm}^{(\widetilde{\nu})}(K;\tau_1,\tau_2) = D_{\mp\pm}^{(\widetilde{\nu})}(K;\tau_1,\tau_2)\theta(\tau_1-\tau_2) + D_{\pm\mp}^{(\widetilde{\nu})}(K;\tau_1,\tau_2)\theta(\tau_2-\tau_1)$$

Time ordering:



E: external energy

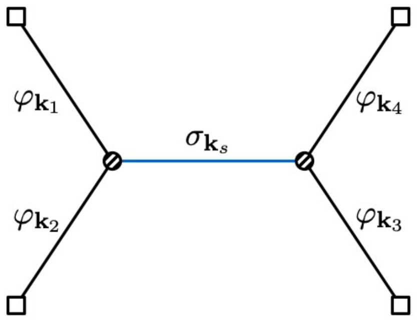
K: line energy (momentum)

p: twist (time dep couplings)

v: mass parameter

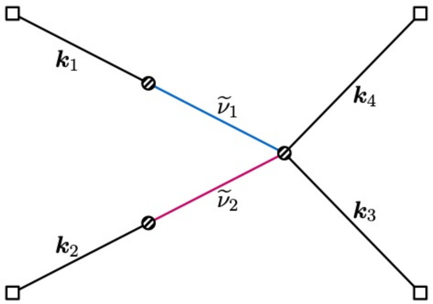
Recent progress

- Arbitrary lines with arbitrary masses: Complexity increases with # of vertices
- Developing fast! Many computations considered impossible a few years ago are now done



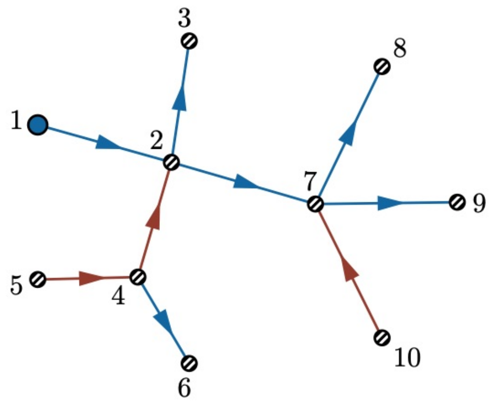
1 exchange (2018)

"Cosmological bootstrap"
[Arkani-Hamed, Baumann, Lee, Pimentel, 1811.00024]



2 exchanges (2024)

[ZX, Zang, 2309.10849]
[Aoki, Pinol, Sano, Yamaguchi, Zhu, 2404.09547]



Arbitrary exchanges (2024)

Partial Mellin-Barnes
[Qin, ZX, 2205.01692, 2208.13790]
Family tree [ZX, Zang, 2309.10849]
Direct solution [Liu, ZX, 2412.07843]

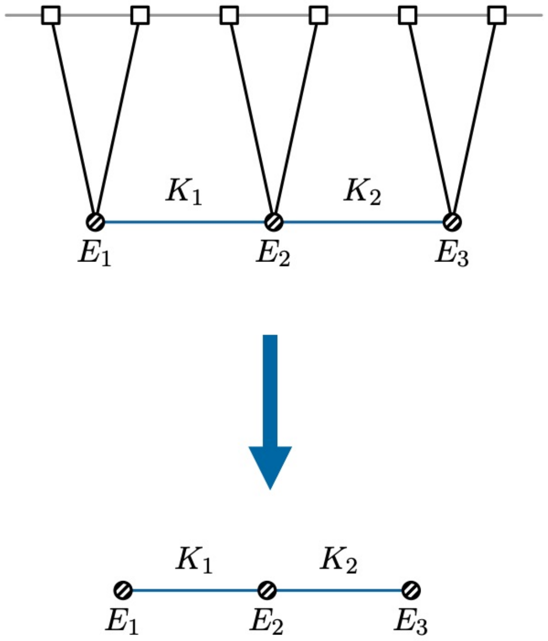
[See also Pimentel, Wang, 2205.00013, Qin, ZX, 2208.13790, 2301.07047, Jazayeri, Renaux-Petel, 2205.10340, Qin, Renaux-Petel, Tong, Werth, Zhu, 2506.01555, etc]

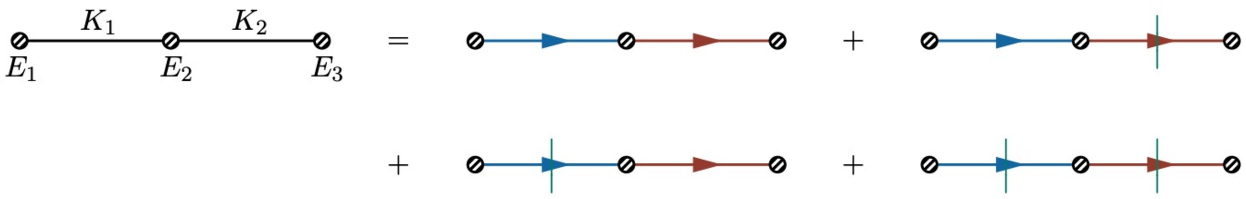
Complete solution

[Liu, ZX, 2412.07843]

$$\mathcal{G} = \sum_{i \in 2^{\{K\}}} \underset{i}{\mathrm{Cut}} \left[\mathcal{G}\right]$$

- The complete solution to arbitrary massive tree is the sum of the CIS (completely inhom sol) and all of its cuts.
- CIS => massive family tree
- Cuts => “tuned” (♯ or ♭) massive family trees





$$\mathcal{G}_3 = [\![123]\!] + [\![1^{\sharp_1}]\!]\Big([\![2^{\sharp_1}3]\!] + [\![2^{\flat_1}3]\!]\Big) + [\![12^{\sharp_2}]\!]\Big([\![3^{\sharp_2}]\!] + [\![3^{\flat_2}]\!]\Big)$$
$$+ [\![1^{\sharp_1}]\!]\Big([\![2^{\sharp_1\sharp_2}]\!] + [\![2^{\flat_1\sharp_2}]\!]\Big)\Big([\![3^{\sharp_2}]\!] + [\![3^{\flat_2}]\!]\Big) + \text{shadows}$$

Where are we now?

- Massive tree graphs: solved; WYSIWYG solutions, in hypergeo series
- Loop level: simple 1-loop graphs (massive bubbles) computed, also in hypergeo series
- Analytical structures largely known for all trees and many loops: only poles / branch points of finite degrees
- Conjecture: Any graphic contribution to a renormalized massive cosmological correlator is a multivariate hypergeometric function with only power-law singularities (finite-deg poles or branch points)
- Most of these hypergeo functions are not yet named, and are like “black boxes”
- Then what does the analytical calculation mean other than giving correlators names?
- Why pFq / Appell / Lauricella look like black boxes to us, but sine and cosine do not?

Family tree decomposition

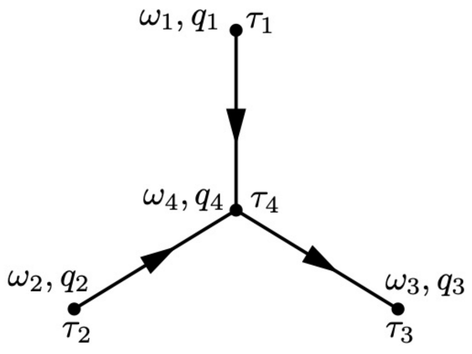
[ZX, Zang, 2309.10849; Fan, ZX, 2403.07050, 2509.02684]

$$\mathcal{T}(\{\boldsymbol{k}\}) \sim \int \mathrm{d}s \times \mathcal{G}(s) \times \left[\int \mathrm{d}^d \boldsymbol{q} K(\boldsymbol{q}, \boldsymbol{k})^\alpha\right] \times \left[\int \mathrm{d}\tau e^{\mathrm{i}E\tau} \times (-\tau)^\beta \times \theta(\tau_i - \tau_j)\right]$$

bulk lines — loop int — nested time int

The most general time integral: $$(-\mathrm{i})^N \int_{-\infty}^{0} \prod_{\ell=1}^{N} \left[\mathrm{d}\tau_\ell\, (-\tau_\ell)^{q_\ell - 1} e^{\mathrm{i}\omega_\ell \tau_\ell}\right] \prod \theta(\tau_j - \tau_i)$$

It naturally acquires a graphic representation [NOT original Feynman diagrams]:



$$= (-\mathrm{i})^4 \int \prod_{\ell=1}^{4} \left[\mathrm{d}\tau_\ell\, (-\tau_\ell)^{q_\ell - 1} e^{\mathrm{i}\omega_\ell \tau_\ell}\right] \theta(\tau_4 - \tau_1)\theta(\tau_4 - \tau_2)\theta(\tau_3 - \tau_4)$$

Family tree decomposition

[ZX, Zang, 2309.10849; Fan, ZX, 2403.07050, 2509.02684]

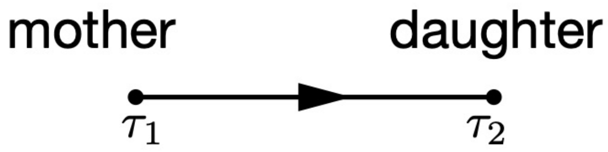
Family tree decomposition: flip the directions such that all graphs are partially ordered

$$\theta(\tau_1 - \tau_2) + \theta(\tau_2 - \tau_1) = 1$$

Partial order:

A mother can have any number of daughters

but a daughter must have only one mother



Every resulting nested graph can be interpreted as a maternal family tree

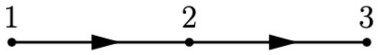
A useful notation for family trees:

$$[12(34\cdots)(5\cdots)]$$

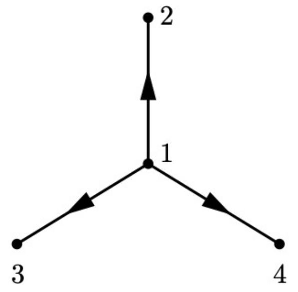
sisters

mother-daughter

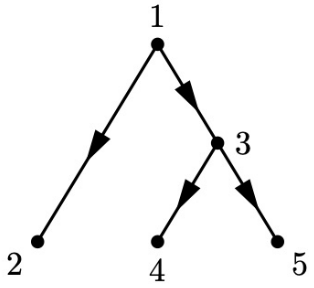
Examples:



$$[123] = (-\mathrm{i})^3 \int \prod_{i=1}^{3} \left[\mathrm{d}\tau_i (-\tau_i)^{q_i - 1} e^{\mathrm{i}\omega_i \tau_i} \right] \theta_{32}\theta_{21}$$



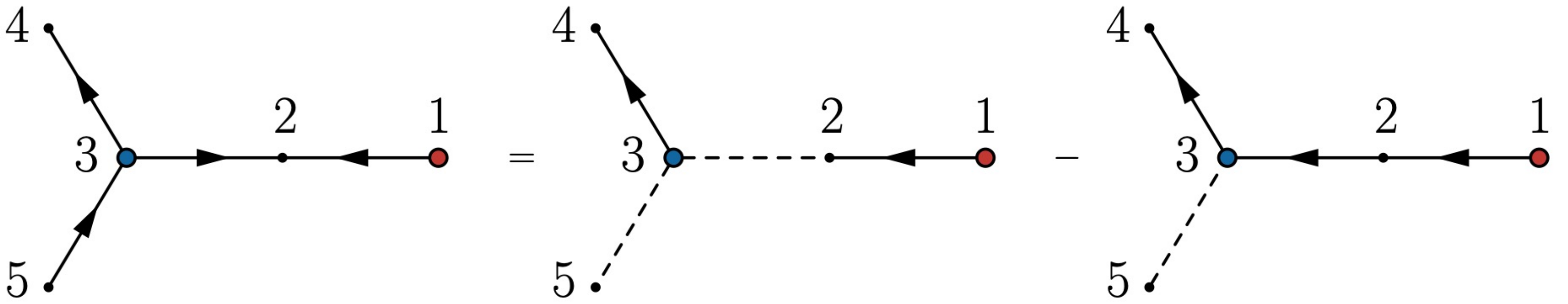
$$[1(2)(3)(4)] = (-\mathrm{i})^4 \int \prod_{i=1}^{4} \left[\mathrm{d}\tau_i (-\tau_i)^{q_i - 1} e^{\mathrm{i}\omega_i \tau_i} \right] \theta_{41}\theta_{31}\theta_{21}$$



$$[1(2)\big(3(4)(5)\big)] = (-\mathrm{i})^5 \int \prod_{i=1}^{5} \left[\mathrm{d}\tau_i (-\tau_i)^{q_i - 1} e^{\mathrm{i}\omega_i \tau_i} \right] \theta_{43}\theta_{53}\theta_{31}\theta_{21}$$

$$\theta_{ij} \equiv \theta(\tau_i - \tau_j)$$

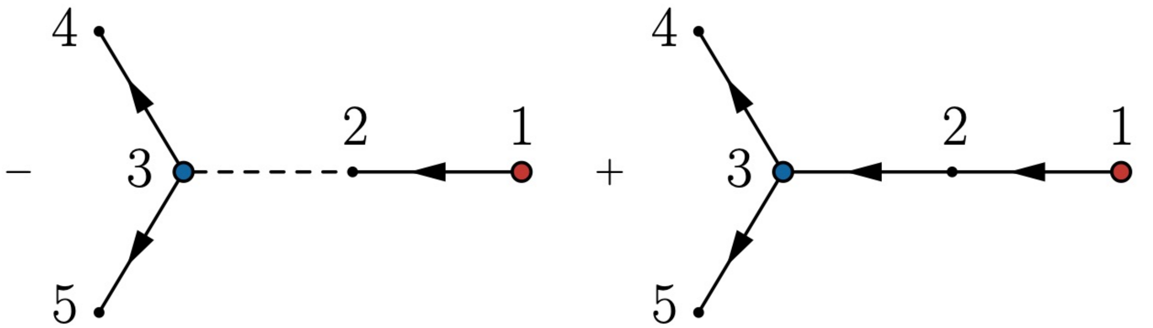
Example: a 5-fold int



Choose Site 1 as the earliest

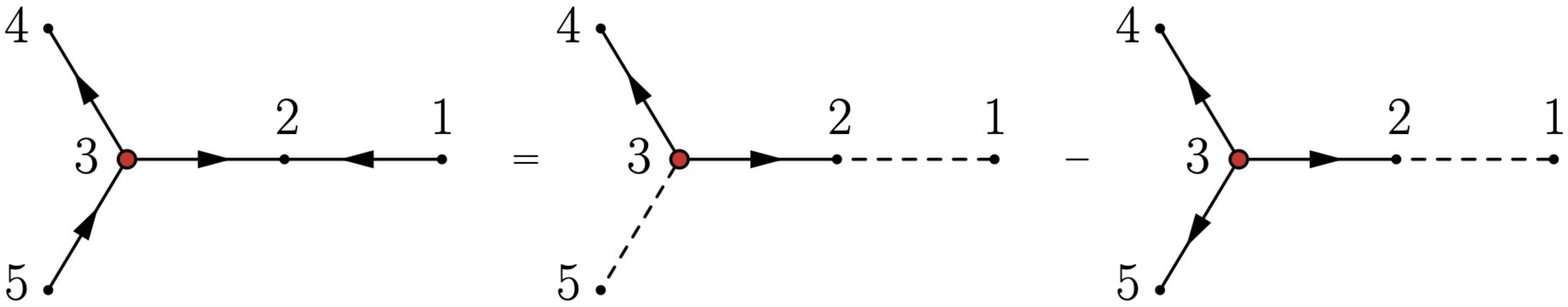
1->2 good 2->3 flip

3->4 good 3->5 flip

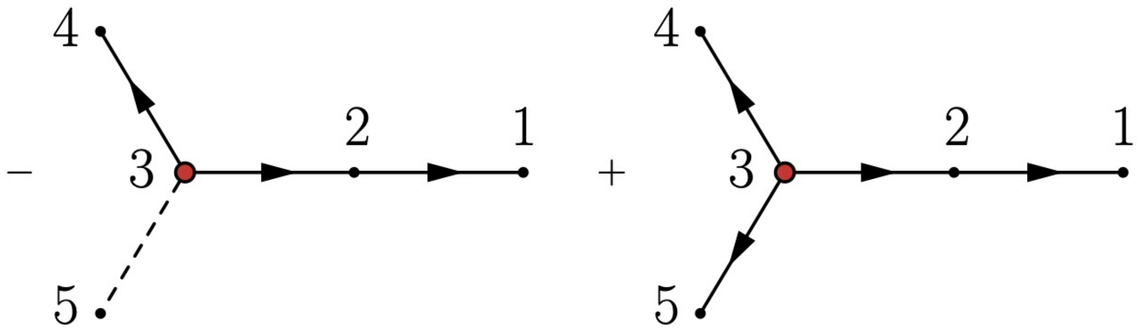


[Also need to decide “locally” earliest site in all nested subgraph, in this case Site 3]

Example: a 5-fold int



We can as well choose a differerent site as the earliest



Say Site 3:

3->4 good 3->2 good

3->5 flip 2->1 flip

For a tree graph: choosing an earliest site fixes the partial order

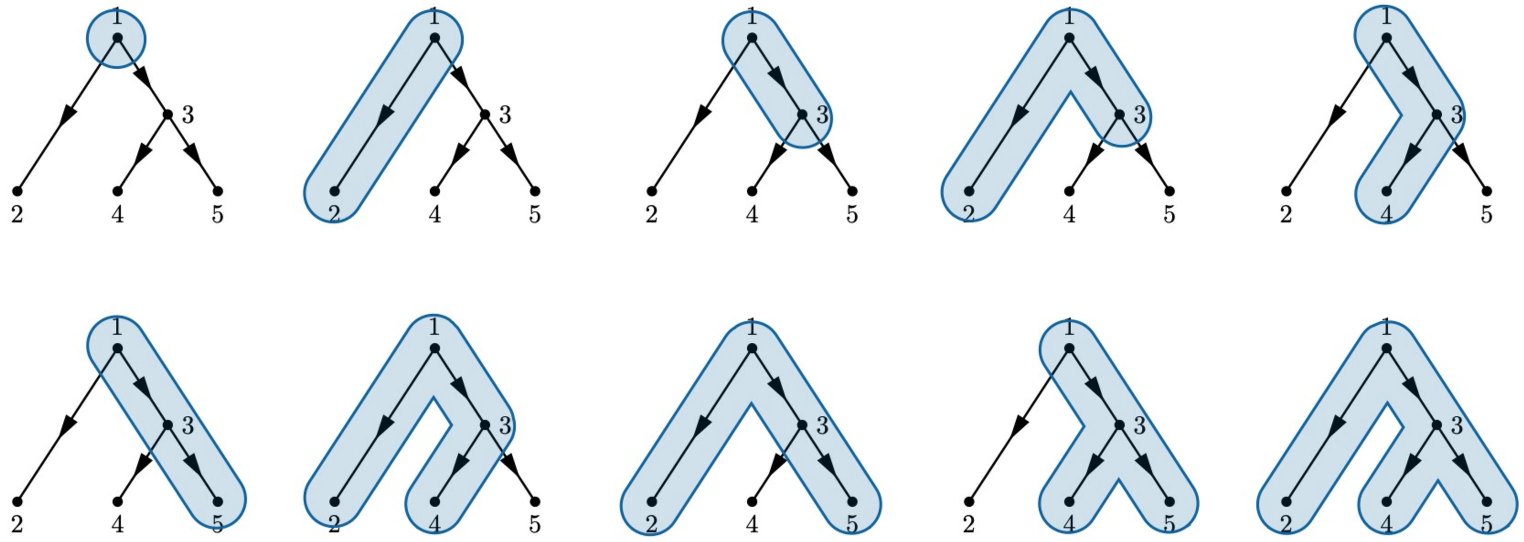
Anatomy of family trees

[Fan, ZX, 2509.02684]

All family trees are multivariate hypergeo functions: Understand their analytical properties!

An N-site family tree lives in a compact kinematic space CP^N

Theorem: *all possible singularities of a family tree come from root-bearing partial energies going to zero or infinity* [proved by Landau analysis]



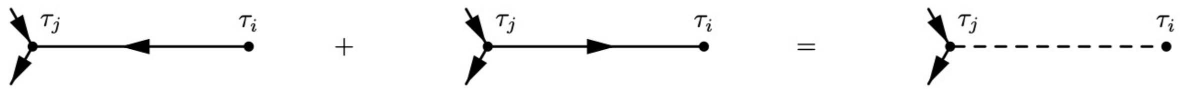
Hypeogeo series rep at all singularities

With appropriate MB reps, we have derived hypergeometric series represenations of an arbitrary family tree around all of its singularities

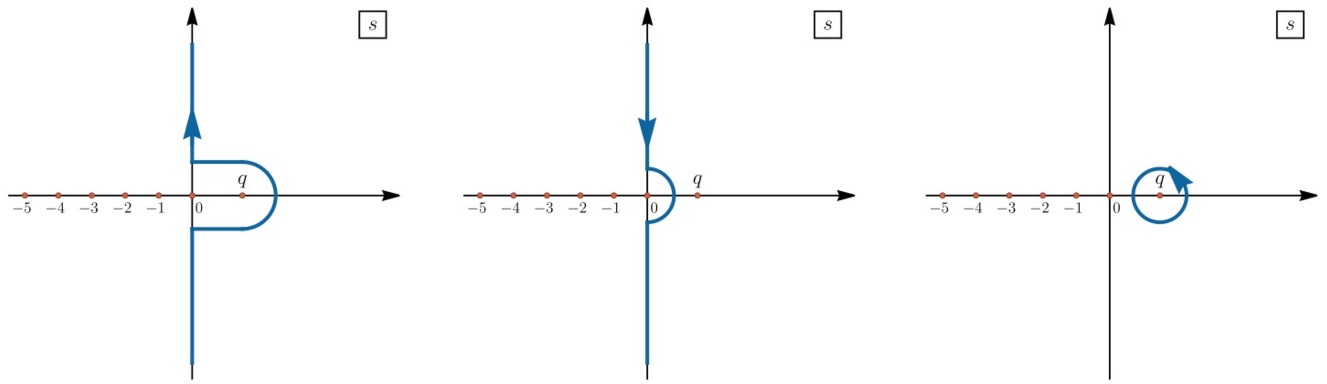
Example: series in a large partial-energy limit:

$$[\mathscr{P}(\widehat{1}2\cdots N)] = \frac{(-\mathrm{i})^N}{(\mathrm{i}\Omega_1)^{\widetilde{q}_1}} \sum_{n_2,\cdots,n_N=0}^{\infty} \Gamma(\widetilde{q}_1+\widehat{n}_1) \prod_{j=2}^{M} \left[\frac{(\Omega_j/\Omega_1)^{n_j}}{(\widetilde{q}_j+\widehat{n}_j)_{n_j+1}}\right] \prod_{k=M+1}^{N} \left[\frac{(-\omega_k/\Omega_1)^{n_k}}{(\widetilde{q}_k+\widetilde{n}_k)n_k!}\right] .$$

A key insight from MB rep:



A flip of time ordering =>
flip of MB poles =>
analytical continuation



For simple family trees, the series sum to named hypergeometric functions [all dressed]

$$[1] = \frac{-\mathrm{i}}{(\mathrm{i}\omega_1)^{q_1}}\Gamma[q_1]$$ Euler Gamma function

$$[12] = \frac{-1}{(\mathrm{i}\omega_1)^{q_{12}}}\ {}_2\mathcal{F}_1\left[\begin{matrix} q_2, q_{12} \\ q_2+1 \end{matrix}\middle| -\frac{\omega_2}{\omega_1}\right]$$ Gauss hypergeometric function

$$[2(1)(3)] = \frac{\mathrm{i}}{(\mathrm{i}\omega_2)^{q_{123}}}\mathcal{F}_2\left[q_{123}\middle|\begin{matrix} q_1, q_3 \\ q_1+1, q_3+1 \end{matrix}\middle| -\frac{\omega_1}{\omega_2}, -\frac{\omega_3}{\omega_2}\right]$$ Appell function

$$[123] = \frac{\mathrm{i}}{(\mathrm{i}\omega_1)^{q_{123}}}\ {}^{2+1}\mathcal{F}_{1+1}\left[\begin{matrix} q_{123}, q_{23} \\ q_{23}+1 \end{matrix}\middle|\begin{matrix} \text{-}\ , q_3 \\ \text{-}\ , q_3+1 \end{matrix}\middle| -\frac{\omega_2}{\omega_1}, -\frac{\omega_3}{\omega_1}\right]$$ Kampé de Fériet function

$$[1(2)\cdots(N)] = \frac{(-\mathrm{i})^N}{(\mathrm{i}\omega_1)^{q_{1\cdots N}}}\mathcal{F}_A\left[q_{1\cdots N}\middle|\begin{matrix} q_2, \cdots, q_N \\ q_2+1, \cdots, q_N+1 \end{matrix}\middle| -\frac{\omega_2}{\omega_1}, \cdots, -\frac{\omega_N}{\omega_1}\right]$$ Lauricella function

… while more complicated family trees are not yet named

What is analytical computation?

- Using series solutions to define, identify, and represent family trees (hypergeo functions)
- Using the flexibility of FTD to link different reps of family trees => Analytical continuation!

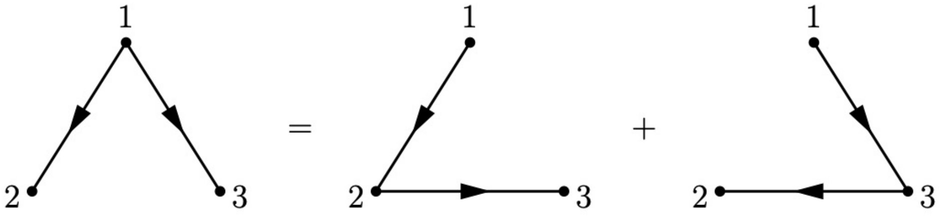
$[12] = [12]$ $$\frac{1}{\omega_1^{q_{12}}}\ {}_2\mathcal{F}_1\left[\begin{matrix} q_2, q_{12} \\ q_2+1 \end{matrix}\middle| -\frac{\omega_2}{\omega_1}\right] = \frac{\Gamma[q_2]}{\omega_{12}^{q_{12}}}\ {}_2\mathcal{F}_1\left[\begin{matrix} 1, q_{12} \\ q_2+1 \end{matrix}\middle| \frac{\omega_2}{\omega_{12}}\right]$$

$[12] + [21] = [1]\,[2]$ $$\frac{1}{\omega_1^{q_{12}}}\ {}_2\mathcal{F}_1\left[\begin{matrix} q_2, q_{12} \\ q_2+1 \end{matrix}\middle| -\frac{\omega_2}{\omega_1}\right] + \frac{1}{\omega_2^{q_{12}}}\ {}_2\mathcal{F}_1\left[\begin{matrix} q_1, q_{12} \\ q_1+1 \end{matrix}\middle| -\frac{\omega_1}{\omega_2}\right] = \frac{\Gamma[q_1, q_2]}{\omega_1^{q_1}\omega_2^{q_2}}$$

$[123] + [2(1)(3)] = [1]\,[23]$ $$\frac{1}{\omega_1^{q_{123}}}\ {}^{2+1}\mathcal{F}_{1+1}\left[\begin{matrix} q_{123}, q_{23} \\ q_{23}+1 \end{matrix}\middle| \begin{matrix} \text{-}\ , q_3 \\ \text{-}\ , q_3+1 \end{matrix}\middle| -\frac{\omega_2}{\omega_1}, -\frac{\omega_3}{\omega_1}\right] + \frac{1}{\omega_2^{q_{123}}}\mathcal{F}_2\left[q_{123}\middle| \begin{matrix} q_1, q_3 \\ q_1+1, q_3+1 \end{matrix}\middle| -\frac{\omega_1}{\omega_2}, -\frac{\omega_3}{\omega_2}\right]$$
$$= \frac{\Gamma[q_1]}{\omega_1^{q_1}\omega_2^{q_{23}}}\ {}_2\mathcal{F}_1\left[\begin{matrix} q_3, q_{32} \\ q_3+1 \end{matrix}\middle| -\frac{\omega_3}{\omega_2}\right]$$

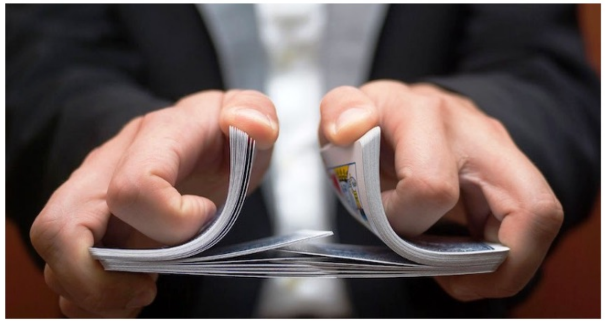
Family trees are further decomposible into chains [Fan, ZX, 2403.07050; 2509.02684]

$$\theta_{21}\theta_{31}(\theta_{32}+\theta_{23}) = \theta_{32}\theta_{21} + \theta_{23}\theta_{31}$$



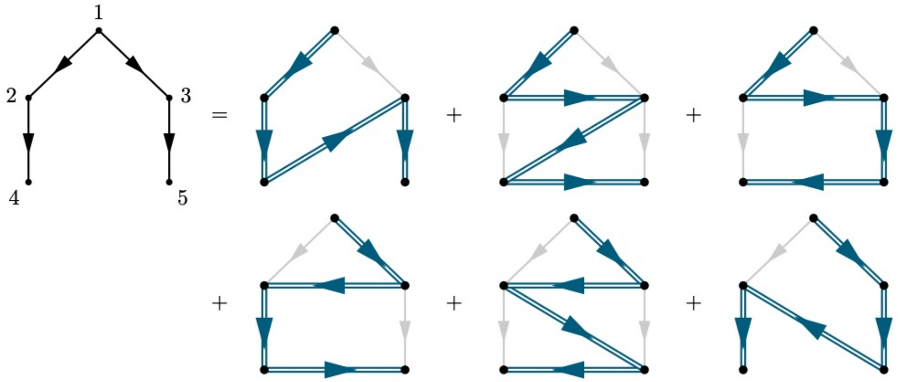
Shuffle product:

$$\begin{aligned} ab \sqcup\!\sqcup\, cd = &\; abcd + acbd + acdb \\ &+ cabd + cadb + cdab \end{aligned}$$



Practically: taking shuffle product recursively among all subfamilies

$$\begin{aligned} [1(24)(35)] &= \{1(24) \sqcup\!\sqcup\, (35)\} \\ &= \{12435\} + \{12345\} + \{12354\} \\ &+ \{13245\} + \{13254\} + \{13524\} \end{aligned}$$



Family chain: standard iterated integrals; Hopf algebra; transcendental weight; Higher weight functions cannot be fully reduced to lower weight functions

Final thoughts and outlooks

- We have found simple rules to identify & write down all hypergeo sols for any tree graphs
- We discovered the canonical structure of family trees in cosmo correlators, and found hypergeo series at all of their singular points
- Next step: using the singular series as boundary conditions to (at least numerically) determine the family trees for all kinematics
- In the meantime, many classic pheno examples remain challenging (triple exchange / strong mixing / chemical potential loops), even numerically. We should work harder
- Thinking pheno-wise: all computations must be initiated analytically and finished numerically, the only question being where to execute the analytical-to-numerical transition
- We hope that some of the analytical progress can provide new insights and better answers!

Thank you!