



Reaching the Ultimate Quantum Precision Limit at Colliders: Conditions and Case Studies

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Based on arXiv: 2506.10673

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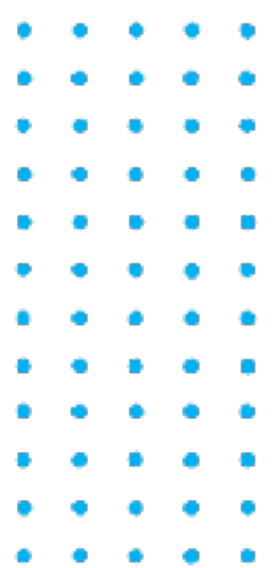
2025-9-28 @ Beijing



Outlines



- Motivation and Background
- General Framework
- Case Studies
- Outlook and Discussions



The Central Question

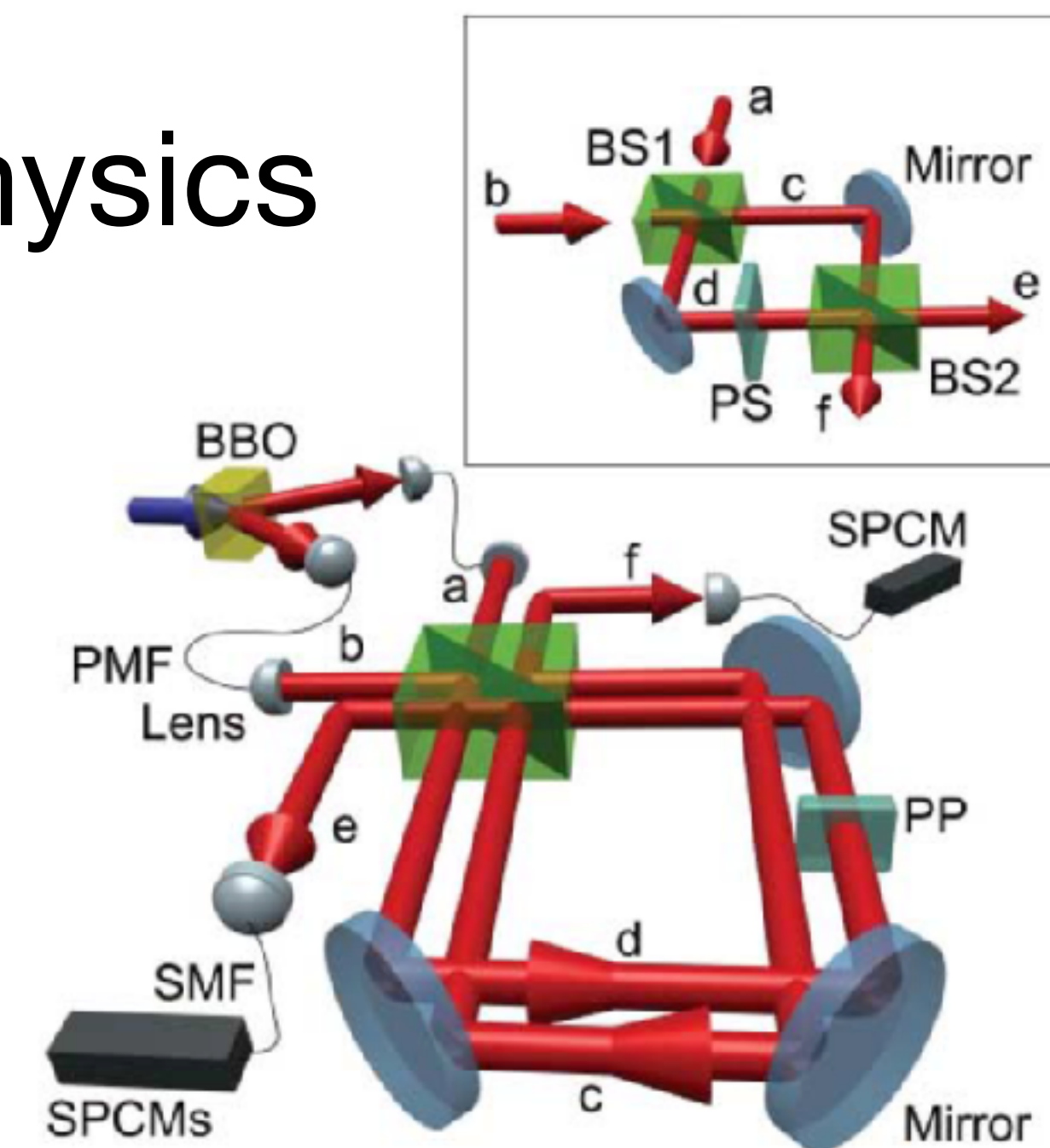
- We have heard news like the following in AMO physics

REPORTS

Beating the Standard Quantum Limit with Four-Entangled Photons

TOMOHISA NAGATA, RYO OKAMOTO, JEREMY L. O'BRIEN, KEIJI SASAKI, AND SHIGEKI TAKEUCHI [Authors Info & Affiliations](#)

SCIENCE • 4 May 2007 • Vol 316, Issue 5825 • pp. 726-729 • DOI: 10.1126/science.1138007



- Can collider experiments reach the Ultimate Quantum Limit?
- **Challenge:** colliders only access to classical observables (momenta)



Start with Classical Fisher Information (CFI)

- CFI: **quantifies** how much **information** an observable random variable carries about an unknown parameter d on which its probability

distribution $f(q, d)$ depends

$$F_c(d) = \int dq f(q | d) \left[\frac{\partial}{\partial d} \log f(q | d) \right]^2$$

- Cramér–Rao Bound**: for any classical observable O_d for parameter d

$$\text{Var}(O_d) \geq \left(\partial_d \langle O_d \rangle \right)^2 / F_c(d)$$

- Only classical optimal observable O_d^{opt} reaches equality



Classical Fisher Information (CFI)



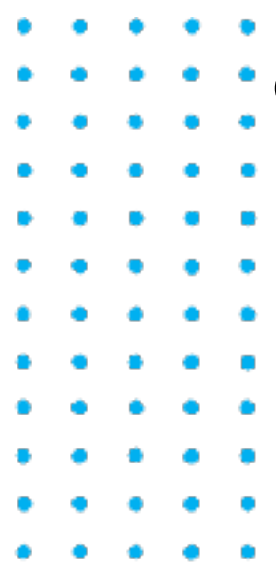
- Classical optimal observable reaching CFI
- In **perturbative** sense: $f(q | d) = f_0 + d \cdot f_1 + \mathcal{O}(d^2)$
- Classical optimal observable $O_d^{\text{opt}} = f_1/f_0$

- Saturating CR bound: $F_c(d) = \int dq f_0 \left(O_d^{\text{opt}} \right)^2$

- E.g. BELLE search of τ Electric Dipole Moments

PLB2003, JHEP04(2022)110

- E.g. machine learning techniques at colliders are optimizing sensitivities towards CFI

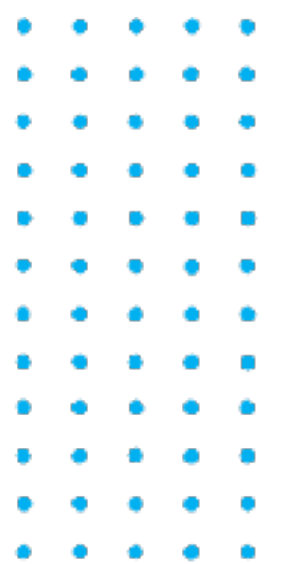




Quantum Fisher Information (QFI)



- **QFI** $F_q(d)$: quantum precision limit for estimating parameters d encoded in quantum density matrix ρ_d
- Consider small parameter d (perturbative sense): $\rho_d = \rho_0 + d \cdot \rho_1 + \mathcal{O}(d^2)$
- There exists **Symmetric Logarithmic Derivative (SLD)** operator $\hat{Q}^{\text{opt}}$:
$$\rho_1 = \frac{1}{2} \left\{ \rho_0, \hat{Q}^{\text{opt}} \right\}$$
- **QFI in a perturbative form**: $F_q(d) = \text{Tr} \left[\rho \left(\hat{Q}^{\text{opt}} \right)^2 \right] \simeq \text{Tr} \left[\rho_0 \left(\hat{Q}^{\text{opt}} \right)^2 \right]$





Ultimate Quantum Limit: Quantum Cramér–Rao bound



- Quantum Cramér–Rao bound: for any quantum operator $\hat{Q}$ for measuring d

- $\text{Var}(\hat{Q}) \geq \left(\partial_d \langle \hat{Q} \rangle \right)^2 / F_q(d)$

- Mean $\langle \hat{Q} \rangle$ and Variance $\text{Var}(\hat{Q})$

- A higher QFI corresponds to a more precise measurement limit

- QFI saturation $\Leftrightarrow$ projective measurements $\hat{\Pi}_i = |\psi_i\rangle\langle\psi_i|$

in SLD $\hat{Q}^{\text{opt}} = \sum_i \lambda_i \hat{\Pi}_i$ eigenbasis

(S. L. Braunstein et al *Phys.Rev.Lett.* 1994)

(J. Liu, H. Yuan, X.-M. Lu, and X. Wang, *J. Phys.A* 2020)



The generalized quantum measurement at colliders



- Colliders perform classical measures on momenta of particles $|\vec{p}\rangle$
- But particle decay can serve as generalized quantum measurement
- Suppose a fermion decay $A \rightarrow BC$, with decay amplitude $M(A_\alpha \rightarrow BC)$
- Spin density matrix: ρ_A
- Generalized measurement : $\hat{D}_{\alpha\alpha'} \equiv N_D^{-1} M(A_\alpha \rightarrow BC) M^\dagger(A_{\alpha'} \rightarrow BC)$

- **Completeness** condition: $\mathbf{I} = \int d\Omega_B \hat{D}(p_B)$

Q. Wang et al, 2402.16574, CPL
C.F. Qiao et al, 2002.04284, PRD
R. Ashby-Pickering et al, 2209.13990, JHEP

- **Normalized differential distribution and conservation:**

$$1 = \text{Tr}[\rho_A] = \int d\Omega_B \text{Tr}[\rho_A \hat{D}(p_B)] = \int d\Omega_B f(p_B) = 1$$



The relation between QFI and CFI



- A classical observable $O(p)$ corresponds to a quantum measurement $\hat{Q}_p$

$$\hat{Q}_O = \int d\Omega_p \hat{D}(p) O(p)$$

where $\hat{D}(p)$ is the generalized quantum measurement operator

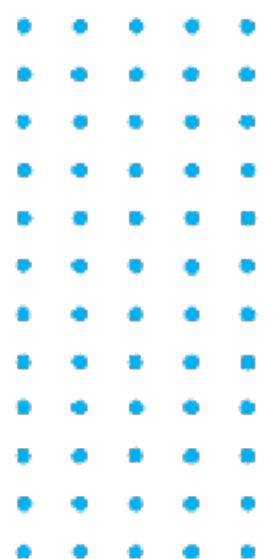
- Same expectation value but different variance

- $\langle \hat{Q}_O \rangle = \text{Tr} [\rho \hat{Q}_O] = \langle O(p) \rangle$

- Using Cauchy inequality: $\text{Var}(O) \geq \text{Var}(\hat{Q}_O)$ (J. Liu, H. Yuan, X.-M. Lu, and X. Wang, *J. Phys.A* 2020)

- Quantum optimal measurement precision is always better than classical optimal precision

- $$F_q(d) \geq \frac{\left(\partial_d \langle \hat{Q}_{O_d^{\text{opt}}} \rangle \right)^2}{\text{Var}(\hat{Q}_{O_d^{\text{opt}}})} \geq \frac{\left(\partial_d \langle O_d^{\text{opt}} \rangle \right)^2}{\text{Var}(O_d^{\text{opt}})} = F_c(d)$$

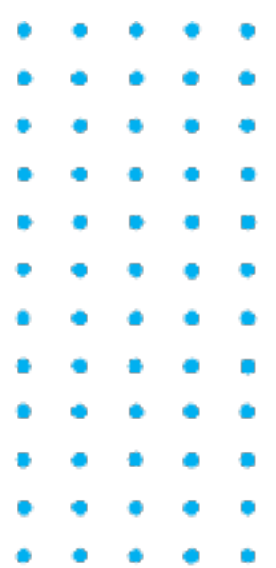




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τ Decay as Quantum Measurement

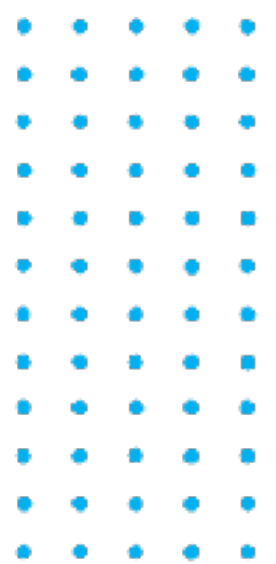


- $\tau^+ \tau^-$ production and **spin density matrix**:

$$\rho_{\alpha\alpha';\beta\beta'}(\hat{k}) = \frac{1}{|\overline{M}|^2} \sum_{\text{initial}} M(\text{initial} \rightarrow \tau_{\alpha}^+(\hat{k})\tau_{\beta}^-(-\hat{k})) M^*(\text{initial} \rightarrow \tau_{\alpha'}^+(\hat{k})\tau_{\beta'}^-(-\hat{k}))$$

- Introduce spin density matrix $\rho_d = \rho_0 + d \cdot \rho_1 + \mathcal{O}(d^2)$
- Two-body τ decay: $\tau^{\pm} \rightarrow \pi^{\pm} \bar{\nu}$
- **Single τ spin measurement operator**

$$\hat{D}_{\alpha\alpha'}^{\pm}(\hat{q}_{\pm}) = \frac{f_{\pi}^2}{2m_{\tau}^3 |q|} M(\tau_{\alpha'}^{\pm} \rightarrow \pi^{\pm}(\hat{q}_{\pm})X) M^*(\tau_{\alpha}^{\pm} \rightarrow \pi^{\pm}(\hat{q}_{\pm})X)$$





τ Decay with maximal spin analyzing power

- Single τ spin measurement operator for general decay

$$\hat{D}_{\alpha\alpha'}^{\pm}(\hat{q}_{\pm}) = \frac{1}{2}(\mathbf{I}_2 \mp a\hat{q}_{\pm} \cdot \vec{\sigma})_{\alpha\alpha'}$$

- With $0 \leq |a| \leq 1$, $|a| = 1$ maximal spin analyzing power; $|a| = 0$ no spin analyzing power

- Two-body τ decay: $\tau^{\pm} \rightarrow \pi^{\pm} \bar{\nu}^{(-)}$

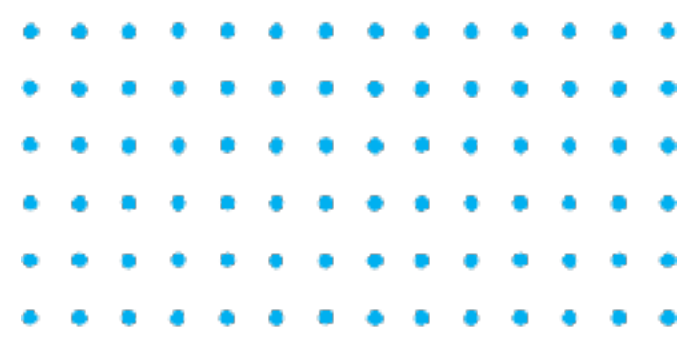
- Maximal spin analyzing power $\rightarrow \text{Rank}[\hat{D}] = 1$

- Projector: $\hat{D} = |\hat{q}\rangle\langle\hat{q}|$, $|\hat{q}\rangle = \{\cos(\theta/2), \sin(\theta/2)e^{i\phi}\}$

- Properties: $\text{Tr}[\hat{D}^{\pm}] = 1$, $(\hat{D}^{\pm})^2 = \hat{D}^{\pm}$, $\int d\Omega_{\pm} \hat{D}^{\pm}(\hat{q}_{\pm}) = 2\pi\mathbf{I}_2$



Collider Reality: Separable (local) Measurements



- $\tau^+ \tau^-$, each decays independently

$$M_{\alpha\beta}^{\text{tot}} \propto M(\text{ini} \rightarrow \tau_{\alpha}^+(k_+) \tau_{\beta}^-(k_-)) \times M(\tau_{\alpha}^+(k_+) \rightarrow \pi^+ \bar{\nu}) M(\tau_{\beta}^-(k_-) \rightarrow \pi^- \nu)$$

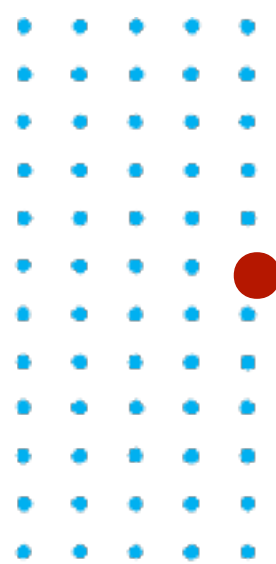
- Their decay forms separable quantum measurements

$$\hat{E}_{\alpha\alpha';\beta\beta'}(\hat{q}_+, \hat{q}_-) = \hat{D}_{\alpha\alpha'}^+(\hat{q}_+) \otimes \hat{D}_{\beta\beta'}^-(\hat{q}_-)$$

- Properties:

$$\text{Tr}[\hat{E}] = 1, \quad \hat{E}^2(\hat{q}_+, \hat{q}_-) = \hat{E}(\hat{q}_+, \hat{q}_-), \quad \frac{1}{4\pi^2} \int d\Omega_+ d\Omega_- \hat{E}(\hat{q}_+, \hat{q}_-) = \mathbf{I}_4.$$

- General collider measurements *cannot* access entangled states

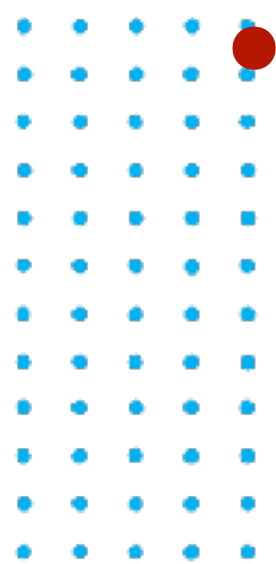




Reaching QFI at collider is very difficult



- Recall to saturate QFI, one should measure with all four optimal projectors $\hat{\Pi}_i = |\psi_i\rangle\langle\psi_i|$, with orthonormal eigenstates $|\Psi_i\rangle$ of SLD $\hat{Q}^{\text{opt}}$
- Collider separable (non-entangled) measurements: $\hat{E} = |\hat{q}_+, \hat{q}_-\rangle\langle\hat{q}_+, \hat{q}_-|$
- Define first measurement: $|E_1\rangle = |\hat{q}_+, \hat{q}_-\rangle \equiv |\hat{q}_+\rangle \otimes |\hat{q}_-\rangle$
- The other orthonormal separable measurements are forced to be $|E_2\rangle = |-\hat{q}_+, \hat{q}_-\rangle$, $|E_3\rangle = |\hat{q}_+, -\hat{q}_-\rangle$, $|E_4\rangle = |-\hat{q}_+, -\hat{q}_-\rangle$
- Conditions for QFI saturation:
 $[\hat{Q}, \hat{E}_j] = 0 \quad \Leftrightarrow \quad \hat{Q} |E_j\rangle = \lambda_j |E_j\rangle$

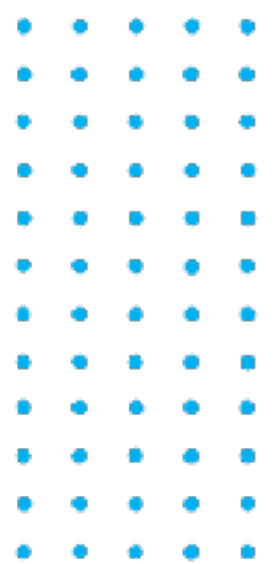




Reaching QFI at collider is very difficult



- Conditions for QFI saturation: $[\hat{Q}, \hat{E}_j] = 0 \quad \Leftrightarrow \quad \hat{Q} |E_j\rangle = \lambda_j |E_j\rangle$
- $\hat{Q}^{\text{opt}}$ from production is independent with the measurement $\hat{E}$
- If eigenstate $|\psi_i\rangle$ is entangled $\rightarrow$ *cannot* reach quantum optimal at colliders
- Schmidt Condition to be separated:
 - $|\psi\rangle = a |\uparrow\uparrow\rangle + b |\uparrow\downarrow\rangle + c |\downarrow\uparrow\rangle + d |\downarrow\downarrow\rangle$
 $|\psi\rangle$ separable $\longleftrightarrow ad - cb = 0$
- Zero measure, very difficult to satisfy





Rank-deficient ρ saves the Day

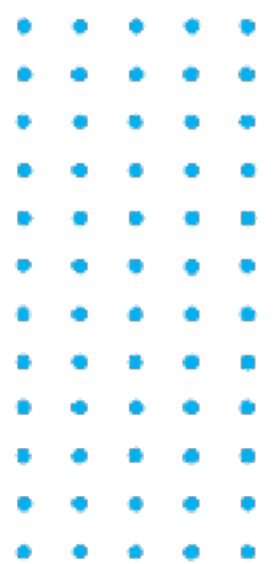


- If ρ_0 is not full-rank, SLD is not unique — — Most general form of SLD:

$$\hat{Q} = \sum_{\substack{i,j \\ p_i + p_j \neq 0}} \frac{\langle p_i | \rho_1 | p_j \rangle}{p_i + p_j} |p_i\rangle \langle p_j| + \sum_{\substack{i,j \\ p_i = p_j = 0}} r_{ij} |p_i\rangle \langle p_j|$$

- $|p_i\rangle$: the eigenstate of ρ_0 with eigenvalue (probability) p_i
- r_{ij} : arbitrary complex numbers satisfying $r_{ij} = r_{ij}^*$
- Null space freedom provides the flexibility of $\hat{Q}$, which provides hope to match $\hat{E}_m(\hat{q}) = \hat{\Pi}_m(r)$

$$F_q(d) \xleftarrow{\hat{Q}^{\text{opt}}} \rho_d \xrightarrow{\hat{E}_m(\hat{q}_+, \hat{q}_-) = \hat{\Pi}_m(r)} \frac{df_m}{d\Omega_+ d\Omega_-} \xrightarrow{O_d^{\text{opt}}(\hat{q}_+, \hat{q}_-)} F_c(d)$$

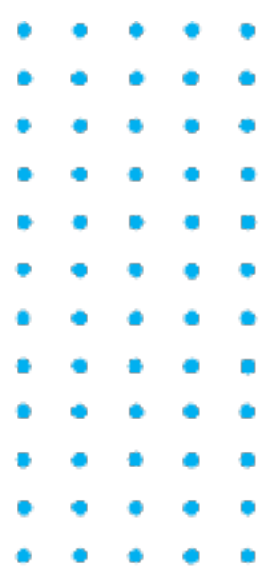




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Three scenarios



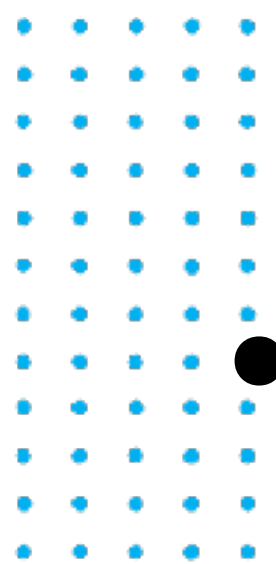
- Higgs decay with CP-violating coupling: $h \rightarrow \tau^+ \tau^-$

- $\mathcal{L}_{\text{CPV}} = -\frac{m_\tau}{v} h \bar{\tau} \left[\cos \delta_h + i\gamma_5 \sin \delta_h \right] \tau$

- τ Magnetic/Electric Dipole Moment: $e^+ e^- \rightarrow \tau^+ \tau^-$

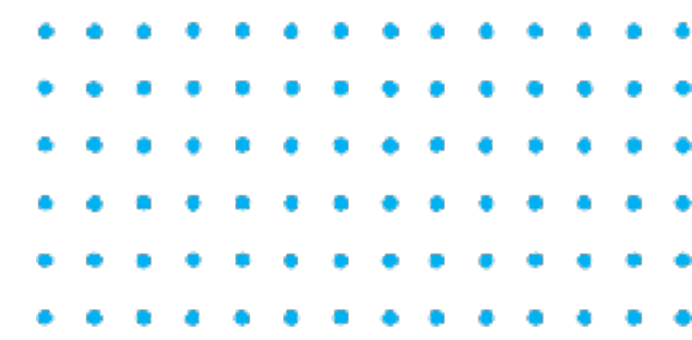
- $\mathcal{L}_{\text{MDM/EDM}} = \frac{1}{2} F_{\mu\nu} \bar{\tau} \sigma^{\mu\nu} \left[-\frac{e}{2m_\tau} a_\tau - i\gamma_5 d_\tau \right] \tau$

- We use helicity basis for spin density matrix





Higgs CP-violating scenario



- Spin density matrix perturbatively: $\rho_0 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $\rho_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
- Most simple and direct SLD: $Q_h = 2\rho_1$,
 - easy to check $\rho_1 = \frac{1}{2}\{\rho_0, Q_h\}$, and QFI: $F_q(\delta_h) = 4$
- Q_h is rank 2, with two entangled eigenstates:
 $\hat{\Pi}_1 = \frac{1}{\sqrt{2}}(i|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$, $\hat{\Pi}_2 = \frac{1}{\sqrt{2}}(-i|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$
- $\hat{\Pi}_{1,2}$ cannot match to collider measurements



Higgs CP-violating scenario



- Most general SLD satisfying $\rho_1 = \frac{1}{2}\{\rho_0, \hat{Q}^{\text{gen}}\}$:

$$\hat{Q}^{\text{gen}}(r) = \begin{pmatrix} r_{44} & -\frac{r_{43}}{\sqrt{2}} & \frac{r_{43}}{\sqrt{2}} & r_{42} \\ -\frac{r_{34}}{\sqrt{2}} & \frac{r_{33}}{2} & -\frac{r_{33}}{2} - 2i & -\frac{r_{32}}{\sqrt{2}} \\ \frac{r_{34}}{\sqrt{2}} & -\frac{r_{33}}{2} + 2i & \frac{r_{33}}{2} & \frac{r_{32}}{\sqrt{2}} \\ r_{24} & -\frac{r_{23}}{\sqrt{2}} & \frac{r_{23}}{\sqrt{2}} & r_{22} \end{pmatrix}$$

- Scan all solutions across the auxiliary variables r 's and particle momentum choices $\hat{q}_+$ and $\hat{q}_-$

- to match $\hat{\Pi}_{1,2,3,4}(r) = \hat{E}_{1,2,3,4}(\hat{q}_+, \hat{q}_-)$



Two Construction Methods



- **Matrix-based method**

- **Brute force linear algebra:** $[\hat{Q}, \hat{E}_j] = 0 \Leftrightarrow \hat{Q} |E_j\rangle = \lambda_j |E_j\rangle$

- Direct, but computational challenge

- **Work in Pauli basis:**

$$\hat{Q} = a\mathbf{I}_4 + \sum_i b_i^+ (\sigma_i \otimes \mathbf{I}_2) + \sum_j b_j^- (\mathbf{I}_2 \otimes \sigma_j) + \sum_{i,j} c_{ij} \sigma_i \otimes \sigma_j$$

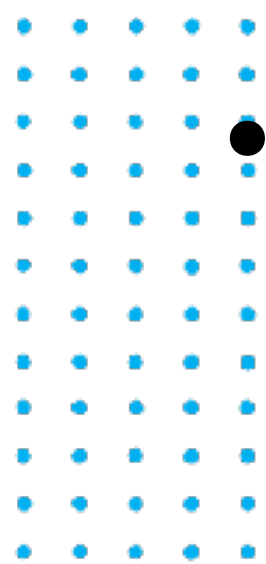
- If $\hat{Q}$ is separable and can match to $\hat{E}(\hat{q}_+, \hat{q}_-)$, if and only if,

$$a = P_1, \quad \vec{b}^\pm = P_{2/3} \hat{q}_\pm, \quad c_{ij} = P_4 \hat{q}_+^i \hat{q}_-^j$$

- **Requiring separable spin-spin correlation matrix:** $\vec{c} \propto \vec{\hat{q}}_+ \vec{\hat{q}}_-$

- **Amplitude-based method** (focus on the transition amplitude M)

- Closer to particle interactions and physics, see Appendix of arXiv:2506.10673





Higgs CP-violating scenario



- We found $\hat{\Pi}_{1,2,3,4}(r) = \hat{E}_{1,2,3,4}(\hat{q}_+, \hat{q}_-)$ for Higgs CPV scenario!

- **Collider accessible SLD:** $Q_h^{\text{opt}} = \frac{2}{\sin \varphi} \begin{pmatrix} -\cos \varphi & 0 & 0 & e^{-i\Phi} \\ 0 & -\cos \varphi & e^{-i\varphi} & 0 \\ 0 & e^{i\varphi} & -\cos \varphi & 0 \\ e^{i\Phi} & 0 & 0 & -\cos \varphi \end{pmatrix} \neq Q_h^{\text{old}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -2i & 0 \\ 0 & 2i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

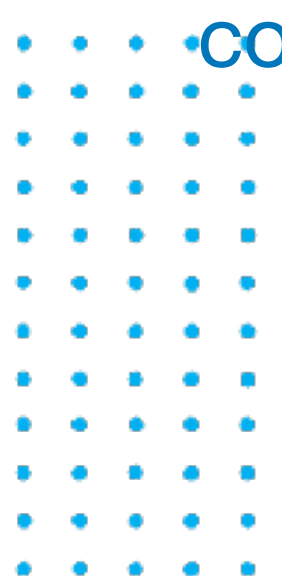
- With $\varphi = \phi_1 - \phi_2$ and $\Phi = \phi_1 + \phi_2$; $\phi_{1,2}$ are free phase parameters

CPV h	θ_+	θ_-	ϕ_+	ϕ_-
$ E_1\rangle$	$\pi/2$	$\pi/2$	ϕ_1	ϕ_2
$ E_2\rangle$	$\pi/2$	$\pi/2$	$\phi_1 + \pi$	ϕ_2
$ E_3\rangle$	$\pi/2$	$\pi/2$	ϕ_1	$\phi_2 + \pi$
$ E_4\rangle$	$\pi/2$	$\pi/2$	$\phi_1 + \pi$	$\phi_2 + \pi$

$\theta_{\pm}$: polar angle of $\hat{q}_{\pm}$

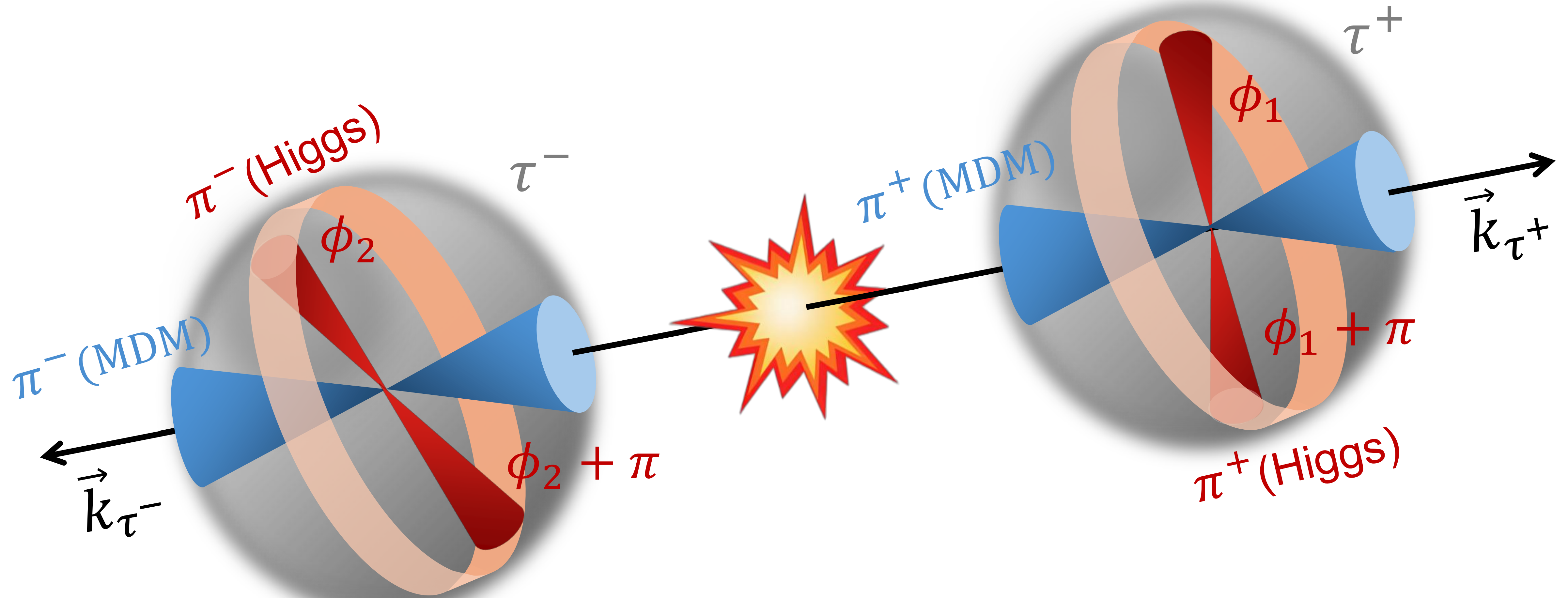
$\phi_{\pm}$: azimuthal angle of $\hat{q}_{\pm}$

Quantum optimal
collider measurements
 $\hat{E}_{1,2,3,4}$



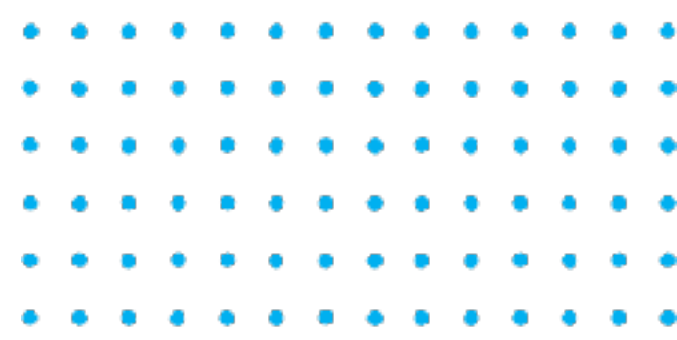


Optimal collider measurements: four sets of directions





MDM scenario



- The spin density matrix for MDM:

$$\rho_0 = c_0 \begin{pmatrix} \frac{1}{2}(\cos 2\theta + 3) & -\frac{im}{\sqrt{s}} \sin 2\theta & -\frac{im}{\sqrt{s}} \sin 2\theta & -\sin^2 \theta \\ \frac{im}{\sqrt{s}} \sin 2\theta & \frac{4m^2}{s} \sin^2 \theta & \frac{4m^2}{s} \sin^2 \theta & \frac{im}{\sqrt{s}} \sin 2\theta \\ \frac{im}{\sqrt{s}} \sin 2\theta & \frac{4m^2}{s} \sin^2 \theta & \frac{4m^2}{s} \sin^2 \theta & \frac{im}{\sqrt{s}} \sin 2\theta \\ -\sin^2 \theta & -\frac{im}{\sqrt{s}} \sin 2\theta & -\frac{im}{\sqrt{s}} \sin 2\theta & \frac{1}{2}(\cos 2\theta + 3) \end{pmatrix}$$

$$\rho_1 = \left(1 - \frac{4m^2}{s}\right) c_0^2 \begin{pmatrix} -2 \sin^2 \theta (\cos 2\theta + 3) & -\frac{ic_1 \sqrt{s}}{4m} \sin 2\theta & -\frac{ic_1 \sqrt{s}}{4m} \sin 2\theta & 4 \sin^4 \theta \\ \frac{ic_1 \sqrt{s}}{4m} \sin 2\theta & 2 \sin^2 \theta (\cos 2\theta + 3) & 2 \sin^2 \theta (\cos 2\theta + 3) & \frac{ic_1 \sqrt{s}}{4m} \sin 2\theta \\ \frac{ic_1 \sqrt{s}}{4m} \sin 2\theta & 2 \sin^2 \theta (\cos 2\theta + 3) & 2 \sin^2 \theta (\cos 2\theta + 3) & \frac{ic_1 \sqrt{s}}{4m} \sin 2\theta \\ 4 \sin^4 \theta & -\frac{ic_1 \sqrt{s}}{4m} \sin 2\theta & -\frac{ic_1 \sqrt{s}}{4m} \sin 2\theta & -2 \sin^2 \theta (\cos 2\theta + 3) \end{pmatrix}$$



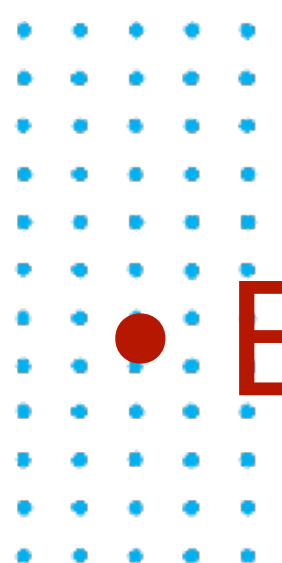
- Quantum optimal collider-accessible SLD for MDM:

$$\hat{Q} = 2 \left(\frac{s}{4m^2} - 1 \right) c_0 \begin{pmatrix} -\frac{8m^2}{s} \sin^2 \theta & 0 & 0 & 0 \\ 0 & \cos 2\theta + 3 & 0 & 0 \\ 0 & 0 & \cos 2\theta + 3 & 0 \\ 0 & 0 & 0 & -\frac{8m^2}{s} \sin^2 \theta \end{pmatrix} \quad F_q^{\text{MDM}}(\theta) = \frac{2s \sin^2 \theta (\cos 2\theta + 3) (s - 4m^2)^2}{m^2 (8m^2 \sin^2 \theta + s \cos 2\theta + 3s)^2}$$

- Corresponding classical optimal collider measurements:

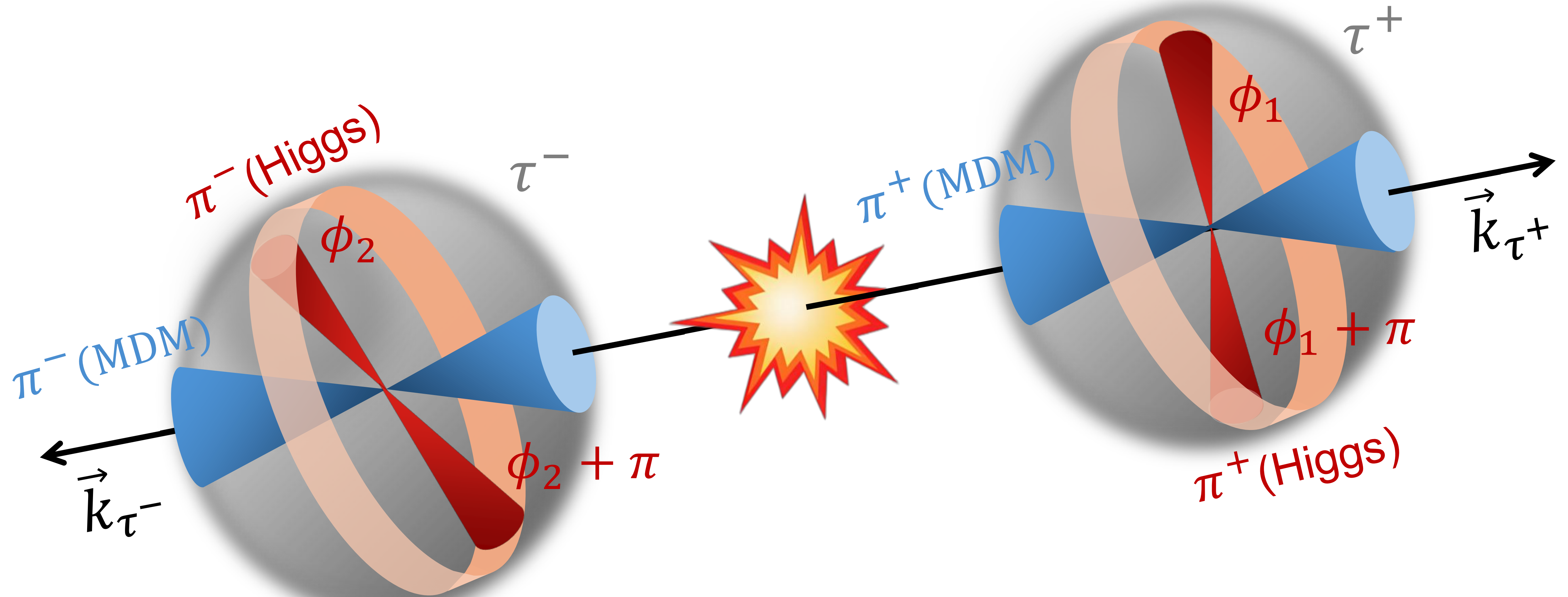
MDM	θ_+	θ_-	ϕ_+	ϕ_-
$ E_1\rangle = \downarrow\downarrow\rangle$	π	π	—	—
$ E_2\rangle = \downarrow\uparrow\rangle$	π	0	—	—
$ E_3\rangle = \uparrow\downarrow\rangle$	0	π	—	—
$ E_4\rangle = \uparrow\uparrow\rangle$	0	0	—	—

- EDM has no solution





Optimal collider measurements: four sets of directions





Collider Strategy: Phase-Space Restricted PS

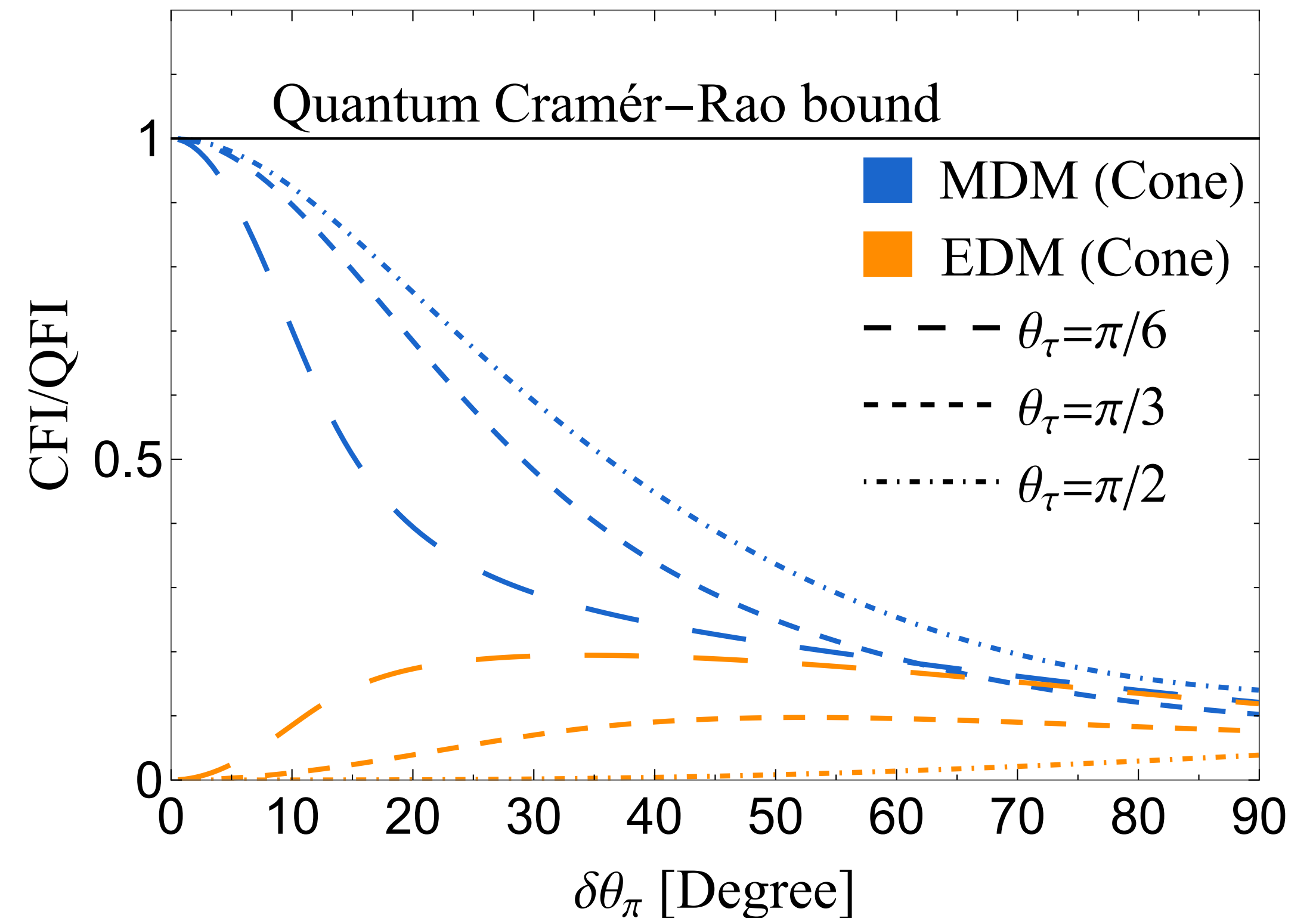
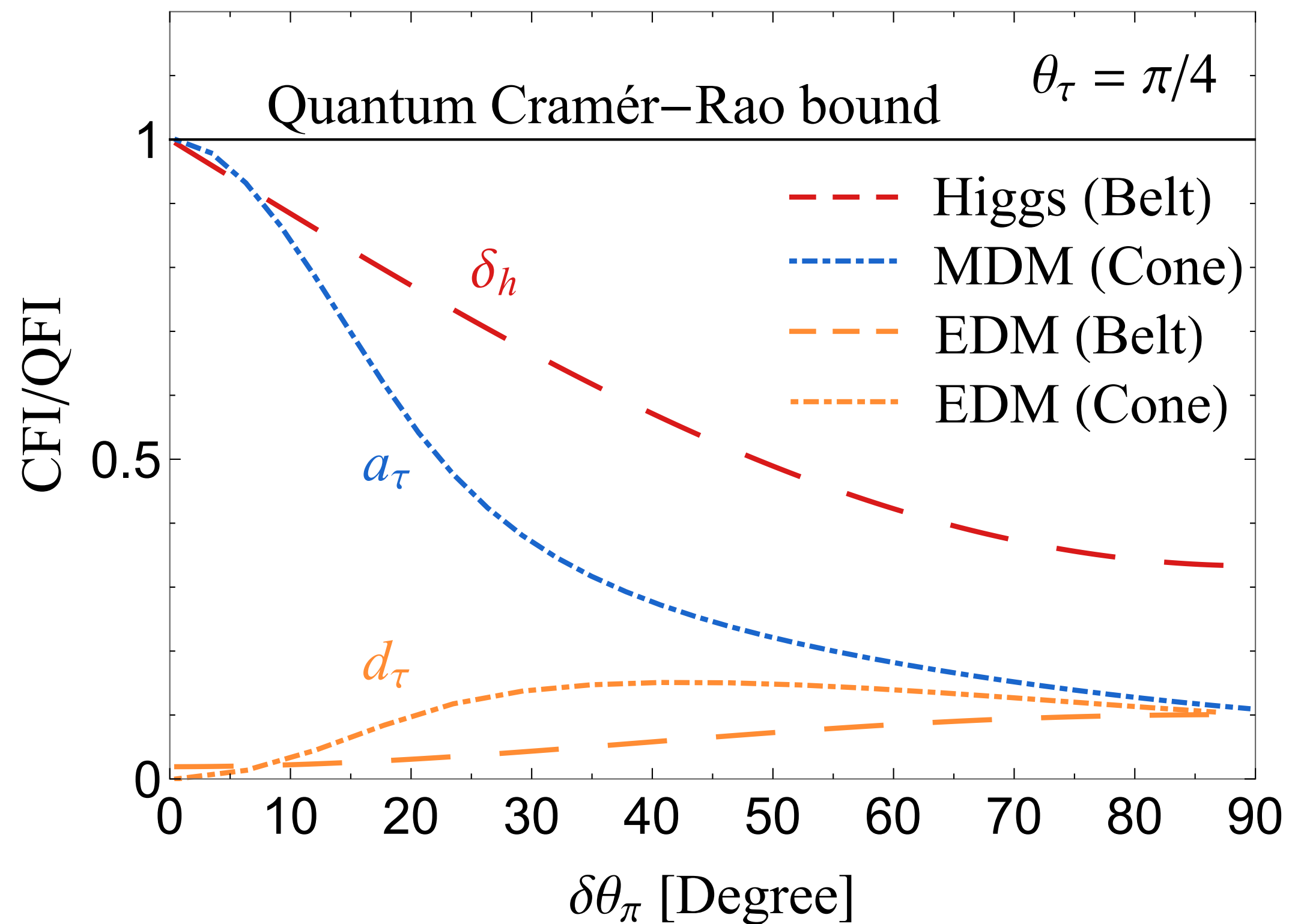


- Expand each optimal direction into a cone of angular size $\delta\theta_\pi$

- Allowing non-zero events

- CFI asymptotic saturates QFI

$$F_c(d) = \int d\text{PS}_{\text{Belt/Cone}} \times \frac{\Sigma_1^2}{\Sigma_0}$$

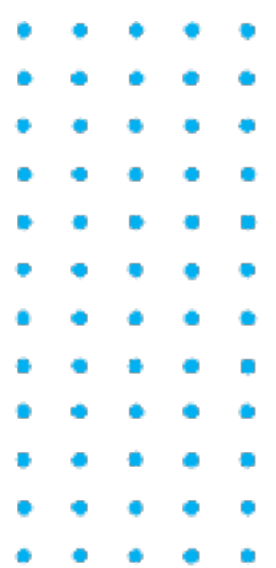




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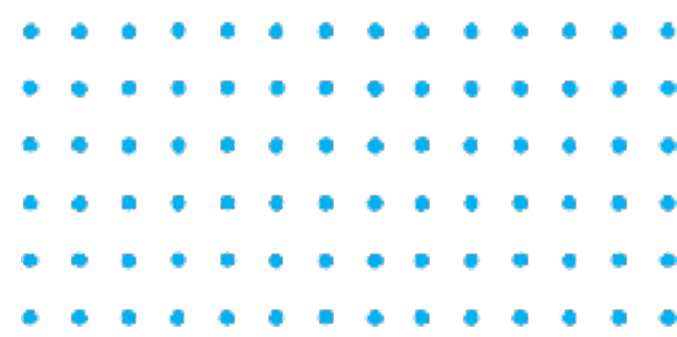


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- Case Studies
- Outlook and Discussions





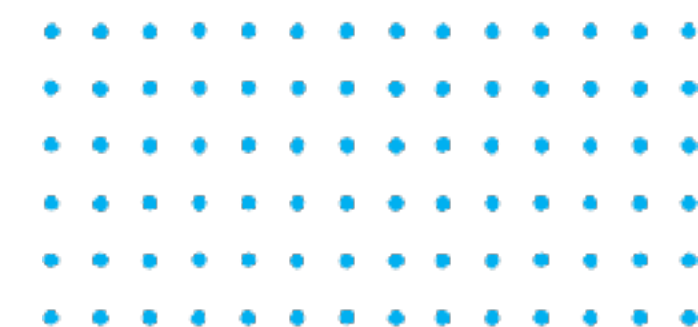
A comparison with AMO measurements



Aspect	Colliders	AMO systems
Type of Quantum Measurement	Generalized quantum measurements arising naturally from particle decays	Controlled projective or generalized quantum measurements
Measurement Structure	Forced to be separable measurements — no entanglement between unstable particle decays	Prefer local (separable) measurements to conserve quantum resources; typically do not exploit entanglement unless intentionally designed
Control over Measurement	Quantum measurements occur "spontaneously" via decay dynamics but lack experimental control	Quantum measurements are deliberately prepared and precisely controlled
QFI Saturation	Achievable for certain interactions; occurs passively at specific regions of phase space	Achieved by preparing specific quantum states (e.g., squeezed or entangled) tailored to maximize sensitivity



Summary and Outlook



- **QFI** represents the ultimate precision limit in parameter estimating, $\text{QFI} \geq \text{CFI}$
- Quantum measurement connect quantum states to classical distribution
Particle decay serves as generalized quantum measurements
- Collider measurements **can only access separable projectors** $\hat{E} = \hat{D}(q_+)\hat{D}(q_-)$

- **Condition for QFI saturation:** (specific directions)

$$F_q(d) \xleftarrow{\hat{Q}^{\text{opt}}} \rho_d \xrightarrow{\hat{E}_m(\hat{q}_+, \hat{q}_-) = \hat{\Pi}_m(r)} \frac{df_m}{d\Omega_+ d\Omega_-} \xrightarrow{O_d^{\text{opt}}(\hat{q}_+, \hat{q}_-)} F_c(d)$$

- Rank deficiency of $\rho \longrightarrow$ Flexibility of SLD
- **QFI saturation is achievable** for Higgs CPV decay and MDM cases, but not for EDM
- **Framework easily applicable to other entangled biparticle systems:**
 $t\bar{t}$ from ee, $t\bar{t}$ pseudo-scalar resonance, baryon pairs ($\Delta, \Lambda, \Lambda_b$), gauge boson pairs

Thank you!