On-shell Approaches for Gravitational Wave Physics

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BPCS 2025

Gravitational wave: new window to probe our Universe

New physics!

- Probe dynamics of black holes
- Test general relativity
- Black hole formation
- Early universe

Future ground based observatories

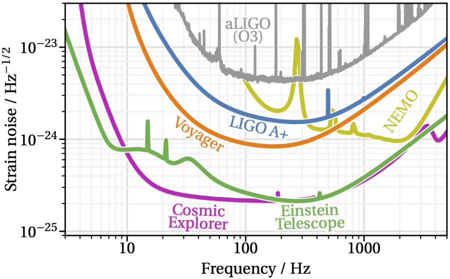
- Advanced LIGO
- Einstein Telescope
- Cosmic Explorer

Future space based observatories

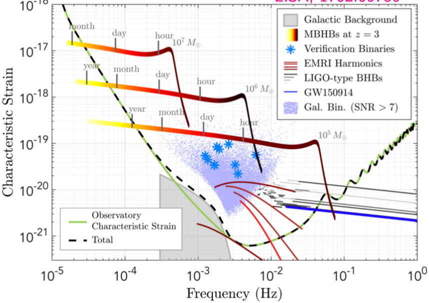
- LISA
- TaiJi
- TianQin

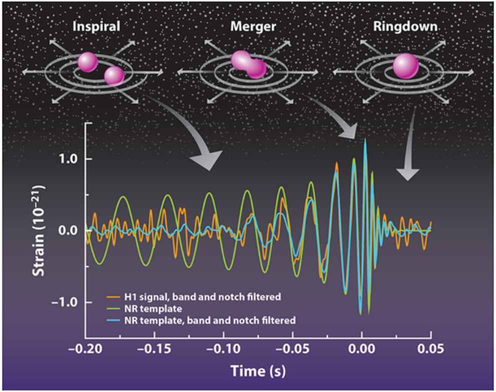
Require accurate theoretical prediction

cosmicexplorer.org



LISA, 1702.00786





Accurate theoretical prediction of the GW production puts challenges on the understanding of its source

$$H = \frac{\mathbf{p}_1^2}{2m_1} + \frac{\mathbf{p}_2^2}{2m_2} - \frac{Gm_1m_2}{|\mathbf{r}|} + \text{(corrections from general relativity)}$$

How to organize perturbations?

- Post-Newtonian (PN) expansion

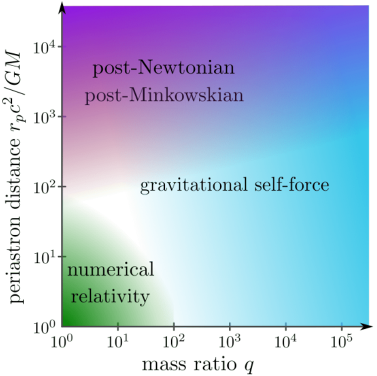
$$v^2 \sim \frac{Gm}{r} \ll 1$$

- Post-Minkowskian (PM) expansion

$$\frac{Gm}{r} \ll v^2 \sim 1$$

- Self-force expansion

$$\frac{Gm}{r} \sim v^2 \sim 1\,, \quad \frac{m_1}{m_2} \ll 1$$

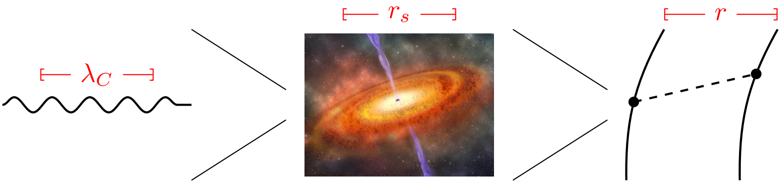


Khalil, Buonanno, Steinhoff, Vines, 2204.05047

Amplitude-based methods naturally lead to PM expansion

PM expansion is relevant to bound orbits with large eccentricity and scattering process

EFT perspective



Full theory: $S_{\text{full}} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R - \frac{1}{2}\int d^4x \sum_{i=1,2}\left(g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i - m_i^2\phi_i^2\right) + \mathcal{O}(R^2\phi^2)$

KMOC

Classical limit $(q,\ell,G)\to(\hbar q,\hbar\ell,\hbar^{-1}G)$

Integrate out soft gravitons

Observables ← Eikonal formula ← $\mathcal{M}_{\text{QFT}} = \mathcal{M}_{\text{EFT}}$

EOM

V_{PM} given by an ansatz

Solve V_{PM} by matching amplitudes

Effective theory: $S_{\text{eff}} = -\int dt\left[m_1\sqrt{1-\mathbf{v}_1^2} + m_2\sqrt{1-\mathbf{v}_2^2} + V_{\text{PM}}\right]$

EFT matching Cheung, Rothstein, Solon, 1808.02489

- Full theory: Schwarzschild black hole $\Longrightarrow$ scalar field ϕ

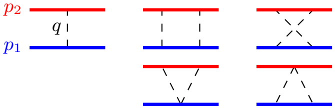
$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{16\pi G} R + \frac{1}{2} \sum_{i=1,2} \left(g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i - m_i^2 \phi_i^2 \right) \right] + \mathcal{O}(R^2 \phi^2)$$

- Effective theory: potential $V(\mathbf{k}, \mathbf{k}')$ given by an ansatz

$$L = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \left[\sum_{i=1,2} a_i^\dagger(-\mathbf{k}) \left(i\partial_t + \sqrt{\mathbf{k}^2 + m_i^2} \right) a_i(\mathbf{k}) - \int \frac{\mathrm{d}^3\mathbf{k}'}{(2\pi)^3} V(\mathbf{k}, \mathbf{k}') a_1^\dagger(\mathbf{k}') a_1(\mathbf{k}) a_2^\dagger(-\mathbf{k}') a_2(\mathbf{k}) \right]$$

- Solve the EFT potential by matching the full theory and EFT amplitudes order-by-order in G in the classical limit

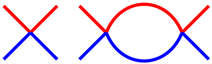
$$\mathcal{M}^{(L)} \xrightarrow{\text{classical limit}} \mathcal{M}^{(L)}_{\text{EFT}}$$



$q \to \hbar q$

$\ell \to \hbar \ell$

$G \to \hbar^{-1} G$



no massless loops (quantum)
no massive contact vertices (delta-function potential)

$$\mathcal{M}^{(L)}_{\text{classical}} \sim \frac{G m^4}{\hbar^3 q^2} (Gmq)^L$$

Cheung, Rothstein, Solon, 1808.02489
Bern, Cheung, Roiban, Solon, Shen, Zeng, 1901.04424
Bern, Parra-Martinez, Roiban, Ruf, Solon, Shen, Zeng, 2112.10750
Bern, Herrmann, Roiban, Ruf, Smirnov, Zeng, 2406.01554

Hamiltonian: $H = E_1 + E_2 + \sum_{n=1}^{\infty} \frac{G^n}{|\mathbf{r}|^n} c_{n\mathrm{PM}}(\mathbf{p}^2)$

$$E_1 = \sqrt{\mathbf{p}^2 + m_1^2} \qquad E_2 = \sqrt{\mathbf{p}^2 + m_2^2}$$
$$\gamma = \frac{E_1 + E_2}{m_1 + m_2} \qquad \xi = \frac{E_1 E_2}{(E_1 + E_2)^2}$$
$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2} \qquad \sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$c_{1\mathrm{PM}} = -\frac{\nu^2 (m_1 + m_2)^2}{\gamma^2 \xi}(2\sigma^2 - 1)$$
Westpfahl and Goller 1979

$$c_{2\mathrm{PM}} = -\frac{\nu^2 (m_1 + m_2)^3}{\gamma^2 \xi}\left[\frac{3(5\sigma^2 - 1)}{4} - \frac{4\nu\sigma(2\sigma^2 - 1)}{\gamma\xi} + \frac{\nu^2(1-\xi)(2\sigma^2 - 1)^2}{2\gamma^3\xi^2}\right]$$
Bel, Damour, Deruelle, Ibanez, Martin, 1981
Westpfahl 1985

$$c_{3\mathrm{PM}} = \frac{\nu^2 m^4}{\gamma^2 \xi}\Bigg[\frac{3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3}{12} - \frac{4\nu(3 + 12\sigma^2 - 4\sigma^4)\,\mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}}$$
$$-\frac{3\nu\gamma(2\sigma^2 - 1)(5\sigma^2 - 1)}{2(1+\gamma)(1+\sigma)} + \frac{3\nu\sigma(20\sigma^2 - 7)}{2\gamma\xi} + \frac{\nu^2(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2)(2\sigma^2 - 1)}{4\gamma^3\xi^2}$$
$$+\frac{2\nu^3(3 - 4\xi)\sigma(2\sigma^2 - 1)^2}{\gamma^4\xi^3} - \frac{\nu^4(1 - 2\xi)(2\sigma^2 - 1)^3}{2\gamma^6\xi^4}\Bigg]$$

State-of-the-art: $c_{4\mathrm{PM}}^{\mathrm{hyp}}$ and $c_{5\mathrm{PM}}^{\mathrm{hyp}\ 1\mathrm{SF}}$

- $c_{3\mathrm{PM}}$ is not known to general relativists before computed this way
- $c_{5\mathrm{PM}}$ for GR is obtained using the amplitude-worldline hybrid method

Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch, 2403.07781
Driesse, Jakobsen, Klemm, Mogull, Nega, Plefka, Sauer, Usovitsch, 2411.11846

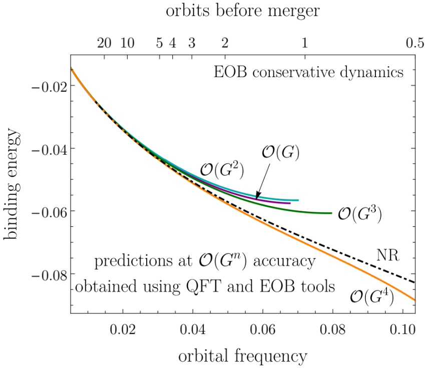
- 5PM 2SF result is partially known for $\mathcal{N} = 8$ supergravity

Bern, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng, 2509.17412

Comparisons with numerical relativity

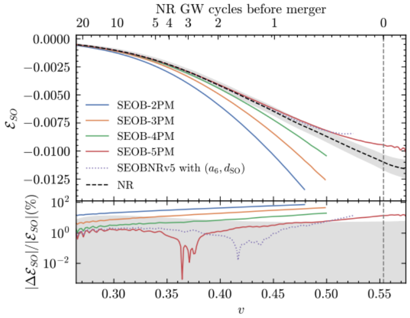
PM-informed two-body effective Hamiltonian

- Local contribution: direct analytic continuation
- Nonlocal (tail) contribution: supplement PN results from GR computations



Khalil, Buonanno, Steinhoff, Vines, 2205.05047

Snowmass white paper, 2204.05194



Buonanno, Mogull, Patil, Pompili, 2405.19181

Eikonalization

Di Vecchia, Heissenberg, Russo, Veneziano, 2306.16488

We have explicitly computed the impact-parameter space amplitudes up to one loop

$$\delta^{(0)} = \mathrm{FT}\left[M^{(0)}_{\text{qft-cl}}\right] = \frac{\kappa^2 \bar{m}_1 \bar{m}_2 (y^2 - 1/2)(-\pi b^2)^{\epsilon}\Gamma(-\epsilon)}{16\pi\sqrt{y^2-1}} = \frac{\kappa^2 \bar{m}_1 \bar{m}_2 (y^2 - 1/2)}{16\pi\sqrt{y^2-1}}\left[-\frac{1}{\epsilon} - \log(-\pi b^2) - \gamma_{\mathrm{E}}\right]$$

$$\delta^{(1)} = \mathrm{FT}\left[M^{(1)}_{\text{qft-cl}}\right] = \frac{3\kappa^4 \bar{m}_1 \bar{m}_2 (\bar{m}_1 + \bar{m}_2)(5y^2 - 1)}{4096\pi\sqrt{y^2-1}\sqrt{-b^2}} \qquad \mathrm{FT}\left[iM^{(1)}_{\text{qft-sc}}\right] = \frac{1}{2}\left(i\delta^{(0)}\right)^2$$

Eikonalization conjecture

$$\mathrm{FT}\left[iM(q)\right] = (1 + i\Delta)e^{i\delta/\hbar} - 1$$

The conjecture is explicitly verified up to two-loop level

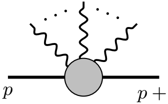
Di Vecchia, Heissenberg, Russo, Veneziano, 2101.05772

- $\delta = \delta^{(0)} + \delta^{(1)} + \ldots$ encodes all the classical physics (generating function for classical conservative observables)

$$\text{scattering angle:} \quad \theta = \frac{\partial \delta}{\partial J} \qquad \text{time delay:} \quad T = \frac{\partial \delta}{\partial E}$$

- $\Delta = \Delta^{(1)} + \Delta^{(2)} + \ldots$ is the quantum reminder

Incorporate spin: higher spin QFT Bern, Luna, Roiban, Shen, Zeng, 2005.03071



- On-shell spin-s states are symmetric traceless and transverse

$$\varepsilon_{a_1 a_2 \ldots a_s} = \varepsilon_{(a_1 a_2 \ldots a_s)} \qquad p^{a_1}\varepsilon_{a_1 a_2 \ldots a_s} = \eta^{a_1 a_2}\varepsilon_{a_1 a_2 \ldots a_s} = 0$$

- Classical limit $\Longrightarrow$ spin coherent state $\varepsilon^s_{a_1 a_2 \ldots a_s} = \varepsilon^+_{a_1}\varepsilon^+_{a_2}\ldots\varepsilon^+_{a_s}$ with large s

$$\varepsilon^s_p \cdot M^{ab} \cdot \varepsilon^s_{p+q} \sim S^{ab} \qquad (M^{ab})_{c(s)}{}^{d(s)} = -2is\delta^{[a}_{(c_1}\eta^{b](d_1}\delta^{d_2}_{c_2}\ldots\delta^{d_s)}_{c_s)}$$

$$\varepsilon^s_p \cdot \{M^{ab}M^{cd}\} \cdot \varepsilon^s_{p+q} \sim S^{ab}S^{cd} \qquad S^{ab} = (1/m)\varepsilon^{abcd}p_c S_d$$

- The spin tensor satisfy covariant spin supplementary condition (SSC)

$$S^{ab}p_b = 0 \qquad (S^{ab} \text{ is boosted from rest frame } S^{ij})$$

- Transversality and covariant SSC are related
- Spin magnitude is conserved: $S^{ab}S_{ab} \sim S^a S_a \sim \boldsymbol{S}^2 = \text{const}$

Effective field theory for higher-spin particles

$$\nabla_\mu \phi_s = \partial_\mu \phi_s + (i/2)\omega_{\mu ab} M^{ab}\phi_s$$
$$\mathbb{S}^a = (-i/2m)\epsilon^{abcd} M_{cd}\nabla_b$$

Higher spin quantum field theory ($\phi_s \equiv \phi_{a_1 a_2 \ldots a_s}$)

$$\mathcal{L} = -\frac{1}{2}\phi_s(\nabla^2 + m^2)\phi_s + \frac{1}{8}R_{abcd}\phi_s M^{ab} M^{cd}\phi_s - \frac{C_2}{2m^2}R_{af_1bf_2}\nabla^a\phi_s \mathbb{S}^{(f_1}\mathbb{S}^{f_2)}\nabla^b\phi_s$$
$$+ \frac{D_2}{2m^2}R_{abcd}\nabla_i\phi_s\{M^{ai}M^{cd}\}\phi_s + \frac{E_2 - 2D_2}{2m^4}R_{abcd}\nabla^{(a}\nabla^{i)}\phi_s\{M^b{}_i M^d{}_j\}\nabla^{(c}\nabla^{j)}\phi_s + \mathcal{O}(M_{ab}^3)$$

We prefer to use a formalism in which the classical and large spin limit is straightforward

- Contractions of ϕ_s facilitated by M^{ab} only
- Propagator uniform in s: $i\delta_{a(s)}^{b(s)}/(p^2 - m^2)$
- There are additional lower spin ($s' < s$) states in the spectrum

Problematic? Not in the classical limit:

- Ghost nature easily cured by an analytic continuation on classical variables
- We get a more generic non-rigid spinning object (more internal DOFs)
- Conventional rigid spinning objects: $D_2 = E_2 = 0$

Bern, Kosmopoulos, Luna, Roiban, **FT**, 2203.06202
Bern, Kosmopoulos, Luna, Roiban, Scheopner, **FT**, Vines, 2308.14176
Alaverdian, Bern, Kosmopoulos, Luna, Roiban, Scheopner, **FT**, 2407.10928, 2503.03739

order	deflection & spin kick					waveform			
	plain	spin-orbit	spin-spin	$\text{spin}^{>2}$	tidal	plain	spin-orbit spin-spin	tidal	Integration complexity
1PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	X	trivial	trivial	trivial	~ tree-level
2PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	WQFT WEFT Amps	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT	~ 1-loop
3PM cons	WQFT WEFT Amps HEFT	WQFT Amps	WQFT (Amps)		WQFT WEFT	Amps HEFT	Amps		
3PM diss	WQFT WEFT Amps HEFT	WQFT	WQFT		WQFT WEFT				~ 2-loop
4PM cons	WQFT WEFT Amps	WQFT			WQFT				
4PM diss	WQFT WEFT Amps	WQFT			WQFT				~ 3-loop
5PM-1SF cons	WQFT								~ 4-loop

r-r: Radiation-reaction (...) : partial results

2024 MIAPbP workshop “EFT and Multi-Loop Methods for Advancing Precision in Collider and Gravitational Wave Physics”

Talk by Jan Plefka

KMOC formalism Kosower, Maybee, O'Connell, 1811.10950

Observable in a scattering process as an in-in correlator:

$$\begin{aligned}\Delta O &= \langle \psi_{\text{out}} | \mathbb{O} | \psi_{\text{out}} \rangle - \langle \psi_{\text{in}} | \mathbb{O} | \psi_{\text{in}} \rangle \\ &= \langle \psi_{\text{in}} | \hat{S}^\dagger \mathbb{O} \hat{S} | \psi_{\text{in}} \rangle - \langle \psi_{\text{in}} | \mathbb{O} | \psi_{\text{in}} \rangle \\ &= i \langle \psi_{\text{in}} | [\mathbb{O}, \hat{T}] | \psi_{\text{in}} \rangle + \langle \psi_{\text{in}} | \hat{T}^\dagger [\mathbb{O}, \hat{T}] | \psi_{\text{in}} \rangle\end{aligned}$$

$$\hat{S} = 1 + i\hat{T}$$
$$\hat{T} - \hat{T}^\dagger = i \hat{T}^\dagger \hat{T}$$

Incoming state: $|\psi_{\text{in}}\rangle = \int \mathrm{d}\Phi[p_1] \mathrm{d}\Phi[p_2] \phi(p_1) \phi(p_2) e^{ip_1 \cdot b_1 + ip_2 \cdot b_2} |p_1 p_2\rangle$

Single particle phase-space and wavefunction:

$$\mathrm{d}\Phi[p] = \frac{\mathrm{d}^4 p}{(2\pi)^4} 2\pi \Theta(p^0) \delta(p^2 - m^2) \qquad \int \mathrm{d}\Phi[p] |\phi(p)|^2 = 1$$

We can compute observables by dressing amplitudes and cuts with the corresponding operators

$$\Delta O = \int \mathrm{d}\Phi[p_1] \mathrm{d}\Phi[p_2] \mathrm{d}\Phi[p_1'] \mathrm{d}\Phi[p_2'] \phi(p_1) \phi(p_2) \phi(p_1') \phi(p_2') e^{i(p_1 - p_1') \cdot b_1 + i(p_2 - p_2') \cdot b_2}$$
$$\times \Big[i \langle p_1' p_2' | [\mathbb{O}, \hat{T}] | p_1 p_2 \rangle + \langle p_1' p_2' | \hat{T}^\dagger [\mathbb{O}, \hat{T}] | p_1 p_2 \rangle \Big]$$

Classical observables Kosower, Maybee, O'Connell, 1811.10950

The wave packets are highly localized, while q_i is much smaller than their spread

$$\int \mathrm{d}\Phi[p_1]\mathrm{d}\Phi[p_2]\mathrm{d}\Phi[p_1']\mathrm{d}\Phi[p_2']\phi(p_1)\phi(p_2)\phi^*(p_1')\phi^*(p_2')e^{i(p_1-p_1')\cdot b_1+i(p_2-p_2')\cdot b_2}$$

$$\simeq \int \frac{\mathrm{d}^d q_1}{(2\pi)^d}\frac{\mathrm{d}^d q_2}{(2\pi)^d}\hat{\delta}(2\bar{p}_1\cdot q_1)\hat{\delta}(2\bar{p}_2\cdot q_2)e^{iq_1\cdot b_1+iq_2\cdot b_2} \qquad \hat{\delta}(x)=2\pi\delta(x)$$

Resolution of identity in the two-massive-particle subspace

$$\mathbb{I}=\sum_X\int \mathrm{d}\Phi[r_1]\mathrm{d}\Phi[r_2]|r_1r_2X\rangle\langle r_1r_2X|$$

Expansion in the soft region $(q_i,\ell,k,G)\to(\hbar q_i,\hbar\ell,\hbar k,\hbar^{-1}G)$

Classical observables

$$\Delta O_{\mathrm{cl}}=\int \frac{\mathrm{d}^d q_1}{(2\pi)^d}\frac{\mathrm{d}^d q_2}{(2\pi)^d}\hat{\delta}(2\bar{p}_1\cdot q_1)\hat{\delta}(2\bar{p}_2\cdot q_2)e^{iq_1\cdot b_1+iq_2\cdot b_2}\left[i\langle p_1'p_2'|[\mathbb{O},\hat{T}]|p_1p_2\rangle+\langle p_1'p_2'|\hat{T}^\dagger[\mathbb{O},\hat{T}]|p_1p_2\rangle\right]$$

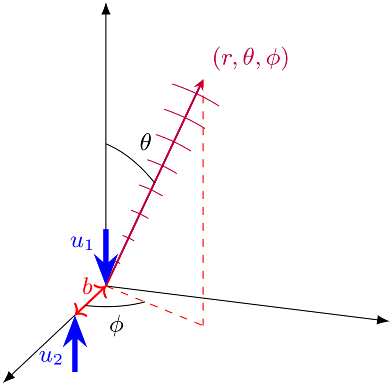
Waveform

Herderschee, Roiban, **FT**, 2303.06112

Bini, Damour, De Angelis, Geralico, Herderschee, Roiban, **FT**, 2402.06604

Metric perturbation: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

Waveform: $W(T_{\mathrm{R}}, \theta, \phi) = \dfrac{1}{4G} \lim_{r\to\infty} r \varepsilon_+^\mu \varepsilon_+^\nu h_{\mu\nu} = \dfrac{1}{4G}(h_+^\infty - i h_\times^\infty)$



Waveform

Cristofoli, Gonzo, Kosower, O'Connell, 2107.10193

Operator: metric perturbation

$$\mathbb{H} = \varepsilon_+^\mu \varepsilon_+^\nu \hat{h}_{\mu\nu}(x) = \int \mathrm{d}\Phi[k] \left[\hat{a}_{--}(k)\, e^{-ik\cdot x} + \mathrm{c.c}\right]$$

Since there is no radiation at $t \to -\infty$,

$$\Delta H(x) = \langle \psi_{\rm in} | \hat{S}^\dagger \mathbb{H} \hat{S} | \psi_{\rm in} \rangle = \int \mathrm{d}\Phi[k] \left[\tilde{J}(k) e^{-ik\cdot x} - \mathrm{c.c}\right] \qquad \tilde{J}(k) = \langle \psi_{\rm in} | \hat{S}^\dagger \hat{a}_{--}(k) \hat{S} | \psi_{\rm in} \rangle$$

Assuming the spatial current $J(y)$ is localized around $y = 0$,

$$\Delta H(x)\Big|_{|\boldsymbol{x}|\to\infty} = \frac{1}{4\pi|\boldsymbol{x}|} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} W(\omega, \boldsymbol{n}) e^{-i\omega\tau} \qquad (\tau = x^0 - |\boldsymbol{x}| : \text{ retarded time})$$

$$\mathcal{W}(\varepsilon, \omega, \boldsymbol{n}, p_1, p_2, b_1, b_2) = (-i) \langle \psi_{\rm in} | \hat{S}^\dagger \hat{a}_{--}(k) \hat{S} | \psi_{\rm in} \rangle \Big|_{k=(\omega, \omega\boldsymbol{n})}^{\text{IR finite}} \qquad \text{(frequency domain waveform)}$$

(The IR divergence can be absorbed in the definition of τ)

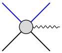
Relevant matrix elements for waveform

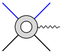
In the space of momentum transfer:

$$(-i)\langle\psi_{\rm in}|\hat{S}^\dagger \hat{a}(k)\hat{S}|\psi_{\rm in}\rangle = \int \frac{\mathrm{d}^d q_1}{(2\pi)^d}\frac{\mathrm{d}^d q_2}{(2\pi)^d}\hat{\delta}(2p_1\cdot q_1)\hat{\delta}(2p_2\cdot q_2)e^{iq_1\cdot b_1 + iq_2\cdot b_2} \times \left[\langle p_1' p_2' k|\hat{T}|p_1 p_2\rangle - i\langle p_1' p_2'|\hat{T}^\dagger \hat{a}(k)\hat{T}|p_1 p_2\rangle\right]$$

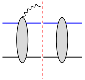
The matrix elements can be classified as follows:

$$\langle p_1' p_2' k|\hat{T}|p_1 p_2\rangle\Big|_{\rm conn} = \mathcal{M}^{\rm tree} + \mathcal{M}^{1\ \rm loop}$$





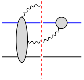
$$i\langle p_1' p_2'|\hat{T}^\dagger \hat{a}(k)\hat{T}|p_1 p_2\rangle\Big|_{\rm conn} = \mathcal{S}^{1\ \rm loop}$$



Contribution from disconnected T matrix elements:

$$\langle p_1' p_2' k|\hat{T}|p_1 p_2\rangle\Big|_{\rm disc} = \mathcal{M}_{\rm const}$$

$$i\langle p_1' p_2'|\hat{T}^\dagger \hat{a}(k)\hat{T}|p_1 p_2\rangle\Big|_{\rm disc} = \mathcal{M}_{\rm disc}$$



Full KMOC waveform at NLO [DB, TD, SDA, AG, AH, RR, **FT**, 2402.06604]

$$\mathcal{W}^{\rm KMOC}(\omega, n) = \mathcal{M}_{\rm const}(n) + {\rm FT}\Big[\mathcal{M}^{\rm tree} + \mathcal{M}^{\rm fin}_{1\ {\rm loop}} - \mathcal{S}^{\rm fin}_{1\ {\rm loop}} + \mathcal{M}_{\rm disc}\Big]$$

- Fourier transform to the impact-parameter space can only be done numerically for generic kinematic setup (see Brunello, De Angelis 2403.08009 for improvements)
- Analytic computation is available under the soft graviton limit and/or small relative velocity (PN) expansion

Soft expansion (up to the first non-universal coefficient)

$$\mathcal{W}^{\rm KMOC}(\omega, \boldsymbol{n}) \sim \frac{\mathcal{A}}{\omega} + \mathcal{B}\log\omega + \mathcal{C}\omega(\log\omega)^2 + \mathcal{D}\omega\log\omega$$

All order conjecture in leading log: Alessio, Di Vecchia, Heissenberg, 2407.04128

PN expansion (up to 2.5PN)

$M = m_1 + m_2$
$\nu = m_1 m_2/M^2$
$p_\infty = \sqrt{\sigma^2 - 1}$

$$\mathcal{W}^{\rm KMOC}(\omega, \boldsymbol{n}) \sim 1 + \frac{GM^2\nu}{p_\infty}(1 + p_\infty + p_\infty^2 + p_\infty^3 + p_\infty^4 + p_\infty^5 + \dots)$$
$$+ \frac{GM}{bp_\infty^2}\frac{GM^2\nu}{p_\infty}(1 + p_\infty + p_\infty^2 + p_\infty^3 + p_\infty^4 + p_\infty^5 + \dots)$$

Full KMOC waveform at NLO [DB, TD, SDA, AG, AH, RR, **FT**, 2402.06604]

$$\mathcal{W}^{\rm KMOC}(\omega, n) = \mathcal{M}_{\rm const}(n) + {\rm FT}\Big[\mathcal{M}^{\rm tree} + \mathcal{M}^{\rm fin}_{1\ \rm loop} - \mathcal{S}^{\rm fin}_{1\ \rm loop} + \mathcal{M}_{\rm disc}\Big]$$

- Fouri… …erically for ge…
- Analy… …mall relati…

Both in perfect agreement with
the GR based multipolar post-Minkowskian calculation

Soft expansion (up to the first non-universal coefficient)

$$\mathcal{W}^{\rm KMOC}(\omega, \boldsymbol{n}) \sim \frac{\mathcal{A}}{\omega} + \mathcal{B}\log\omega + \mathcal{C}\omega(\log\omega)^2 + \mathcal{D}\omega\log\omega$$

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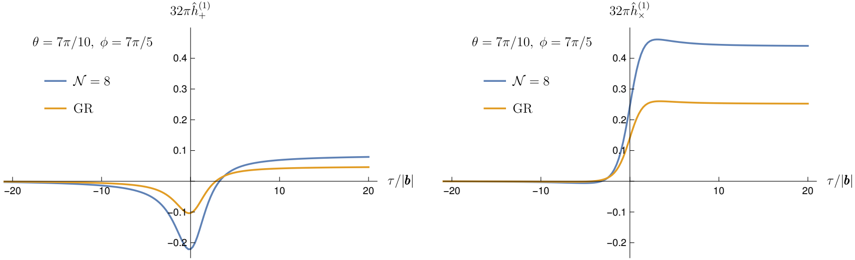
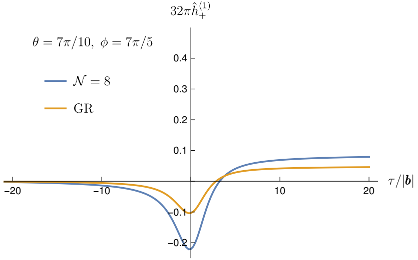
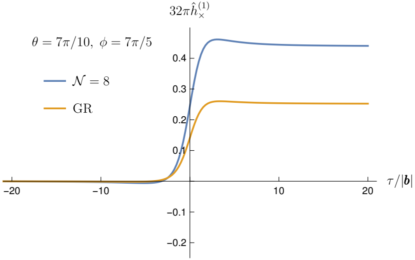
LO waveform



Agree with Jakobsen, Mogull, Plefka, Steinhoff, 2101.12688
From spinning binaries: De Angelis, Novichkov, Gonzo, 2309.17429
Brandhuber, Brown, Chen, Gowdy, Travaglini, 2310.04405
Aoude, Haddad, Heissenberg, Helset, 2310.05832

LO waveform





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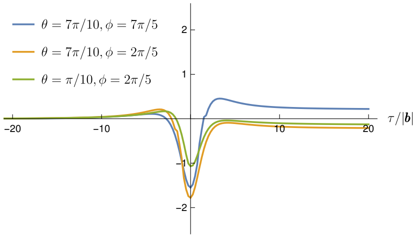
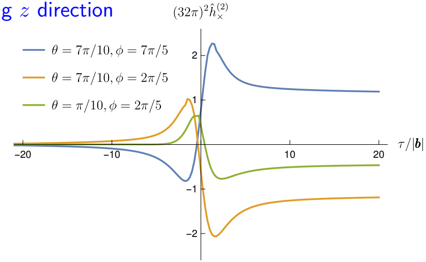
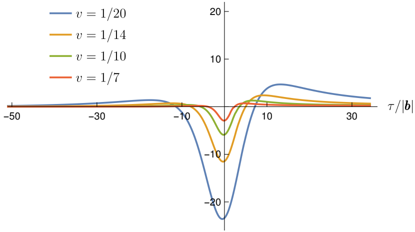
NLO waveform (GR)

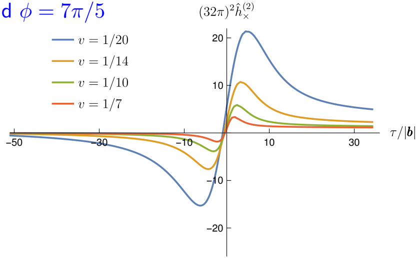
$v = 1/5$ along z direction





$\theta = 7\pi/10$ and $\phi = 7\pi/5$





Synergy between amplitude and GR

Bini, Damour, De Angelis, Geralico, Herderschee, Roiban, **FT**, 2402.06604

One of the important takeaway messages of our new results is that, after having sorted out the subtleties that were hidden in the EFT one-loop waveform, we obtained a remarkable confirmation that the classical limit of an amplitude-based waveform does correctly incorporate the many subtle classical effects that were included in the $O(G^2\eta^5)$-accurate MPM waveform such as (i) radiation-reaction effects on the worldlines, (ii) high-multipolarity tail effects in the wave-zone, and (iii) cubically nonlinear multipole couplings in the exterior zone. The fact that the road leading to the present successful EFT/MPM comparison had some bumps, which taught us interesting lessons, is another example of the useful synergy between amplitude-based, and classical perturbation-theory-based, approaches to gravitational physics.

Outlook

Explicit higher order spinning and spinless calculation

- Challenge: IBP reduction and DE for loop integrals; higher rank tensor reduction
- What do we mean by analytic computation?

Systematic inclusion of tidal operators and absorption

- Need to understand the running and mixing of WCs

How to describe generic spinning bodies?

- Perhaps relevant to future higher precision GW observations

Outlook

Re-summation in observables

- SCET for gravity

Beneke, Hager, Szafron, 2112. 04983; Rothstein, Saavedra, 2412.04428

- Self-force effective theory

Cheung, Parra-Martinez, Rothstein, Shah, Wilson-Gerow, 2308.14832; Kosmopolous, Solon, 2308.15304
Akpinar, del Duca, Gonzo, 2504.02025

How to directly compute bound-state observables?

- Phenomenological prescription exists and works very well in matching NR

Buonanno, Mogull, Patil, Pompili, 2405.19181

- Remain as a theoretical challenge (Bethe-Salpeter equation, quantum spectrum method, etc)

Adamo, Gonzo, Ilderton, 2402.00124; Khalaf, Shen, Telem, 2503.23317

Thanks for listening!

Backup slides

Why don't we just use numerical relativity?

Numerical relativity provides the most accurate observables (waveform, scattering angle, etc) for binaries with mass ratio $q \sim 1$

- Works for the entire binary merger process
- Numerical error under very good control (can be considered as the TRUTH)
- Computationally expansive: need days of computation time on a super-computer to produce one waveform template
- Matched filtering analysis requires a dense sampling of the parameter space
- In light of future observatories, we need $\sim 10^6$ waveform templates

We need analytic perturbative computations to efficiently generate observables