Swiss National Science Foundation

UNIVERSITÉ DE GENÈVE

Perturbative cosmological phase transitions in a broad temperature range

Philipp Schicho
philipp.schicho@unige.ch
pschicho.github.io

Département de Physique Théorique, Université de Genève

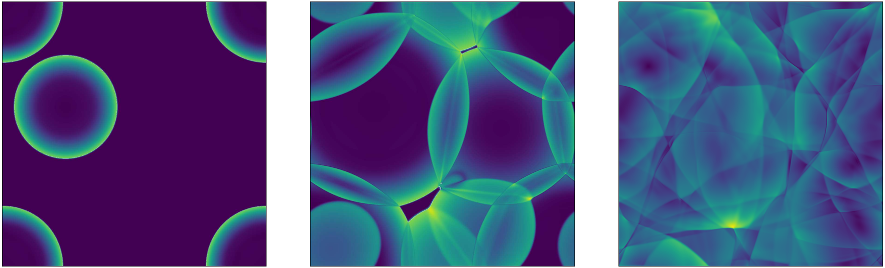
2025年北京粒子物理与宇宙学
早期宇宙、引力波模板、撞机唯象研会、09/2025

Based on collaboration of **F. Bernardo**, P. Klose, P. Schicho, and T. V. I. Tenkanen, *Higher-dimensional operators at finite temperature affect gravitational-wave predictions*, JHEP **08** (2025) 109 [2503.18904]. A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory*, Phys. Rev. D **110** (2024) 096006 [2405.18349]. Supported by the SNSF under grant PZ00P2-215997.

The thermal history of electroweak symmetry breaking

If strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

- Nucleation, growth, and collision of bubbles
- Generation of Baryon asymmetry of the universe

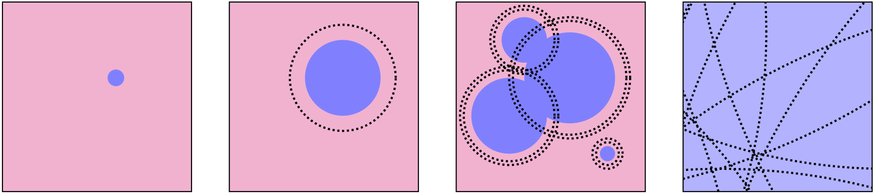


figures by D. Cutting, M. Hindmarsh, and D. J. Weir, *Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions,* Phys. Rev. Lett. **125** (2020) 021302 [1906.00480]

The thermal history of electroweak symmetry breaking

If strong first-order cosmic phase transition at EW scale $T_\mathrm{c} \sim 100$ GeV:

- Nucleation, growth, and collision of bubbles
- Generation of Baryon asymmetry of the universe

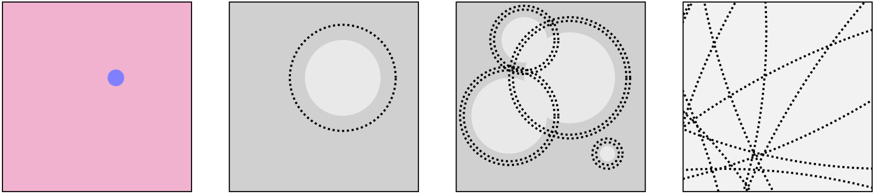


Bubble nucleation | Growth, wall-fluid | Collisions | Shocks, turbulence

The thermal history of electroweak symmetry breaking

If strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

- Nucleation, growth, and collision of bubbles
- Generation of Baryon asymmetry of the universe



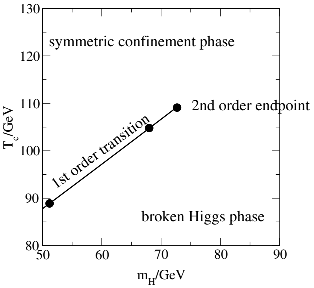
microscopic macroscopic

Extended thermal history of EW symmetry breaking

In Standard Model, EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions.

Study **BSM physics** near EW scale in context of phase transitions:

- Light fields strongly coupled to Higgs
- Collider targets. BSM testing pipeline: Collider phenomenology



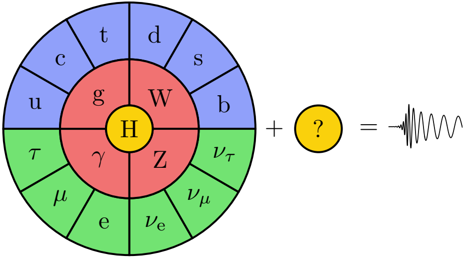


figure by M. Laine, *Electroweak phase transition beyond the standard model*, in *4th International Conference on Strong and Electroweak Matter*, pp. 58–69, 6, 2000 [hep-ph/0010275]

Fingerprinting the GW spectrum

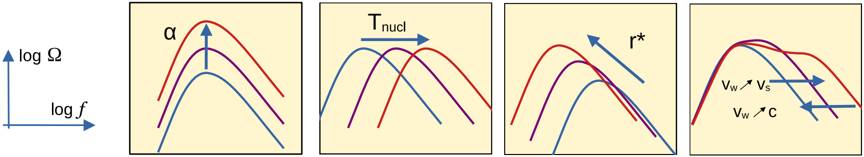
Large dynamic range to resolve $\Omega_{\rm GW}$ at both low and high frequency.

$T_\star$ reference temperature of the transition ($T_\star = T_{\rm nucl}, T_{\rm p}$),

α phase transition strength,

β/H inverse duration of the transition (also $r_\star$),

v_w Iteratively solve coupled fluid, scalar field, Boltzmann equations[1]



[1] Numerical package for v_w: PS, A. Ekstedt, O. Gould, J. Hirvonen, *et al.*, *How fast does the WallGo? A package for computing wall velocities in first-order phase transitions*, JHEP **04** (2025) 101 [2411.04970].

At fixed $\alpha = 0.2$, $r_\star = 0.1$, $T_{\rm n} = 100$ GeV from C. Gowling and M. Hindmarsh, *Observational prospects for phase transitions at LISA: Fisher matrix analysis*, JCAP **10** (2021) 039 [2106.05984]

Fingerprinting the GW spectrum

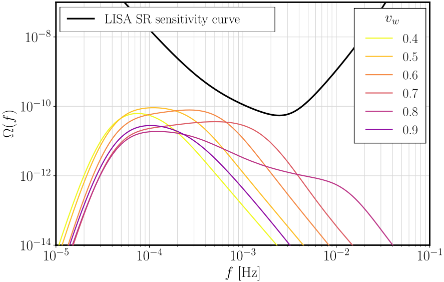
Large dynamic range to resolve $\Omega_{\rm GW}$ at both low and high frequency.

$T_\star$ reference temperature of the transition $(T_\star = T_{\rm nucl}, T_{\rm p})$,

α phase transition strength,

β/H inverse duration of the transition (also $r_\star$),

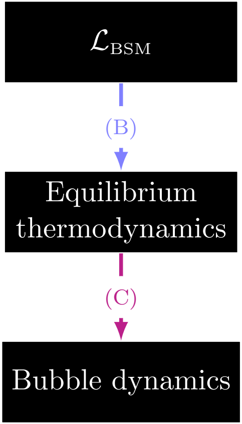
v_w Iteratively solve coupled fluid, scalar field, Boltzmann equations[1]

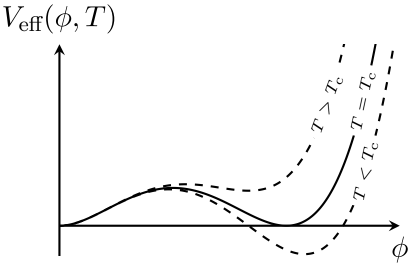


[1] Numerical package for v_w: PS, A. Ekstedt, O. Gould, J. Hirvonen, *et al.*, *How fast does the WallGo? A package for computing wall velocities in first-order phase transitions*, JHEP **04** (2025) 101 [2411.04970].
At fixed $\alpha = 0.2$, $r_\star = 0.1$, $T_{\rm n} = 100$ GeV from C. Gowling and M. Hindmarsh, *Observational prospects for phase transitions at LISA: Fisher matrix analysis*, JCAP **10** (2021) 039 [2106.05984]

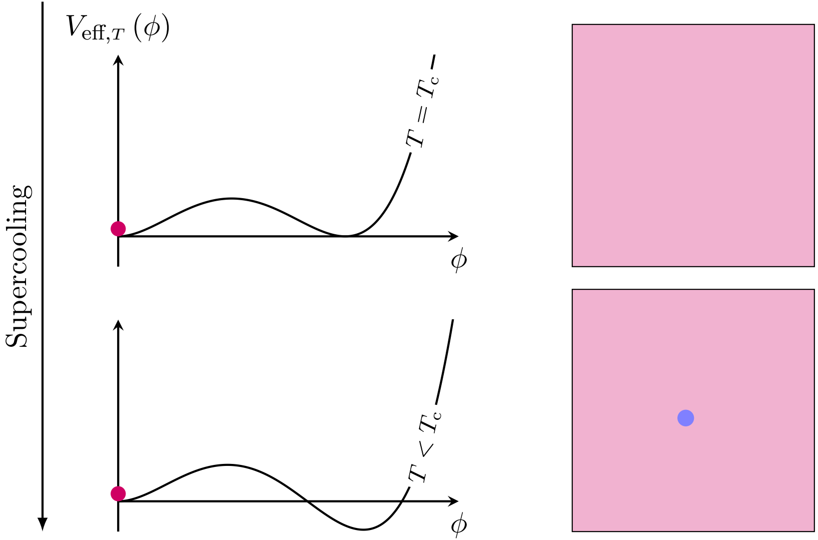
Reliable GW predictions require control at each scale

Uncertainties from microscopic scales manifest in equilibrium thermodynamics (V_{eff}) and bubble dynamics (v_w).



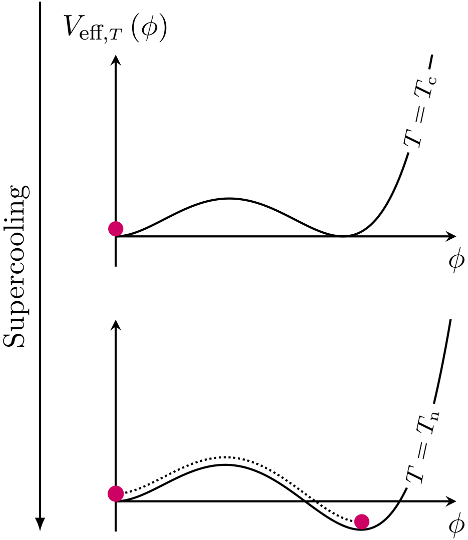


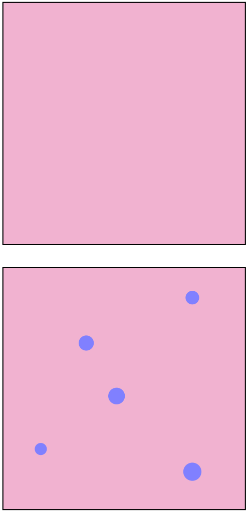
Nucleation rate and transition reference scale



O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions,* JHEP **06** (2021) 069 [2104.04399]

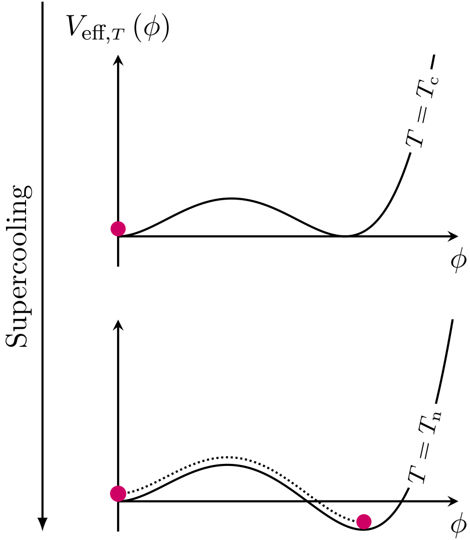
Nucleation rate and transition reference scale

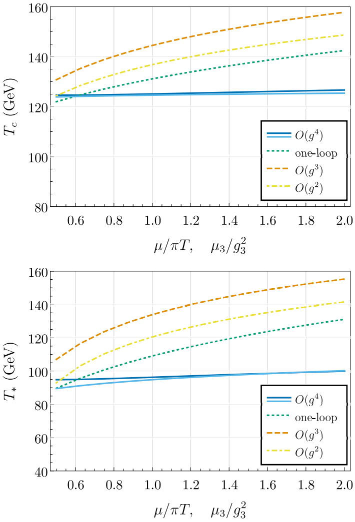




O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

Nucleation rate and transition reference scale





O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

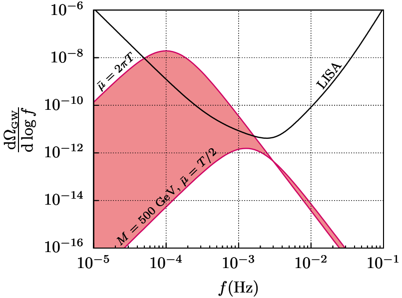
Theoretical predictions are not robust

$\mathcal{O}(10^4)$ uncertainty even for purely perturbative regimes[2] as $\Omega_{\rm GW}$ depends strongly on the transition temperature, $T_\star$, in simulation fits:

$$\Omega_{\rm GW} \propto \frac{(\Delta V_\star)^2}{T_\star^8}\,.$$

Vary RG scale $\bar{\mu}$ in SM extensions:

▷ SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$



[2] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions,* JHEP **04** (2021) 055 [2009.10080], O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions,* JHEP **06** (2021) 069 [2104.04399]

[3] S. Biondini, P. Schicho, and T. V. I. Tenkanen, *Strong electroweak phase transition in t-channel simplified dark matter models,* JCAP **10** (2022) 044 [2207.12207]

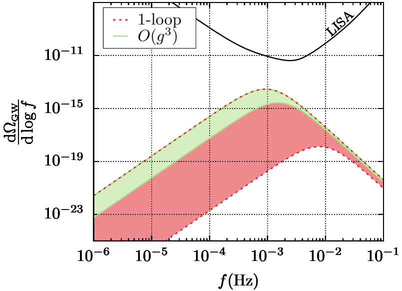
Theoretical predictions are not robust

$\mathcal{O}(10^4)$ uncertainty even for purely perturbative regimes[2] as $\Omega_{\rm GW}$ depends strongly on the transition temperature, $T_\star$, in simulation fits:

$$\Omega_{\rm GW} \propto \frac{(\Delta V_\star)^2}{T_\star^8}\,.$$

Vary RG scale $\bar{\mu}$ in SM extensions:

- SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$
- xSM: SM + singlet
- DM models[3]



[2] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080], O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

[3] S. Biondini, P. Schicho, and T. V. I. Tenkanen, *Strong electroweak phase transition in t-channel simplified dark matter models*, JCAP **10** (2022) 044 [2207.12207]

The effective potential in a broad temperature range

Inspect limiting cases from total potential[4]

$$V_{\text{eff}}^{\text{res}} = V_{\text{eff}}^{\text{res}} - V_{\text{eff}}^{\text{res,soft}} + V_{\text{eff}}^{\text{res,soft}} = \underbrace{\left(V_{\text{eff}}^{\text{naive}} - V_{\text{eff}}^{\text{naive,soft}} \right)}_{\text{UV-modes, IR safe}} + \underbrace{V_{\text{eff}}^{\text{res,soft}}}_{\text{IR safe}} \, .$$

At low-T, approach vacuum Coleman-Weinberg (CW) limit

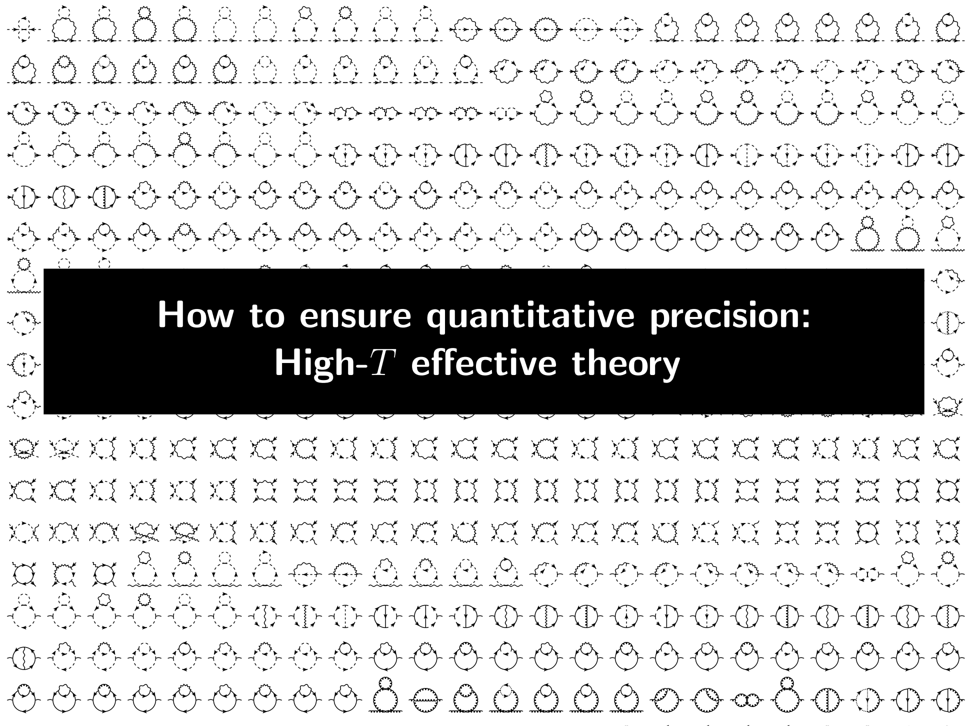
$$V_{\text{eff}}^{\text{res}} = V_{\text{eff}}^{\text{naive}}\Big|_{T\to 0} = V_{\text{eff}}^{\text{CW}} \, .$$

At high-T, can relate potential to EFT for zero-mode

$$V_{\text{eff}}^{\text{res}}\Big|_{M/T\ll 1} = V_{\text{eff}}^{\text{high-}T\text{ EFT}} \, .$$

This talk: construct, test limits, extend validity of high-T EFT.

[4] P. Navarrete, R. Paatelainen, K. Seppänen, and T. V. I. Tenkanen, *Cosmological phase transitions without high-temperature expansions*, [2507.07014], A. Kurkela and A. Vuorinen, *Cool Quark Matter*, Phys. Rev. Lett. **117** (2016) 042501 [1603.00750]



How to ensure quantitative precision: High-T effective theory

Perturbative phase transitions need scale hierarchies

for quantum effects ΔV_{fluct} to influence the tree-level potential

$$V_{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}}\ .$$

Assume particle χ couples to the SM via $g^2 \Phi^\dagger \Phi \chi^\dagger \chi$. If $M_\chi \gg m_\Phi$, integrating out χ introduces Higgs-mass corrections of the form:

$$(\Delta m_\Phi^2)\Phi^\dagger\Phi =$$

$$\sim g^2 M_\chi^2 \Phi^\dagger \Phi\,, \qquad \frac{(\Delta m_\Phi^2)}{m_\Phi^2} = g^2 \left[\frac{M_\chi}{m_\Phi}\right]^2 .$$

Relevant operators ($\sigma > 0$) in the IR get large UV contributions and

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 \left[\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}}\right]^\sigma \overset{!}{\sim} 1 \quad \Rightarrow \begin{cases} \text{strong coupling} & g^2 \gtrsim 1 \\ \text{scale hierarchy} & \frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \sim \left[\frac{1}{g^2}\right]^{\frac{1}{\sigma}} \gg 1 \end{cases}$$

Multi-scale hierarchy in hot classicalizing gauge theories

Evaluated **Matsubara sums** yield Bose(Fermi) distribution. Asymptotically high T and weak $g \ll 1$: effective expansion parameter

$$\epsilon_{\rm B} = g^2 n_{\rm B}(E) = \frac{g^2}{e^{E/T} - 1} \approx \frac{g^2 T}{E} \,.$$

$$E \sim \begin{cases} \pi T & \textit{hard}\ \text{scale} \\ gT & \textit{soft}\ \text{scale} \\ g^{3/2} T & \textit{supersoft}\ \text{scale} \\ g^2 T/\pi & \textit{ultrasoft}\ \text{scale} \end{cases}$$

quantum theory

symmetry breaking

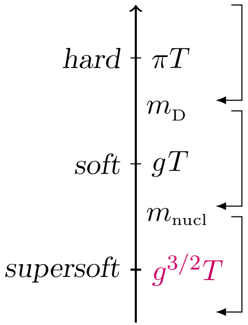
Limit: Confinement-like behavior in ultrasoft sector $g^2 n_{\rm B}(g^2 T) \sim \mathcal{O}(1)$. Ultrasoft bosons are non-perturbative at finite T: Linde IR problem.[5]

[5] A. Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas*, Phys. Lett. B **96** (1980) 289, O. Gould and T. V. I. Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions*, JHEP **01** (2024) 048 [2309.01672]

Effective Field Theory (EFT): Dimensional Reduction (DR)

Integrate out hard modes perturbatively → EFT for static modes.

Precision thermodynamics of non-Abelian gauge theories as QCD and (EW) phase transition[6] using e.g. `DRalgo`.[7] Two `step` procedure:



$\mathcal{L}_{\mathrm{Full}}$, $(d+1)$-dim

`step1`: Dimensional Reduction

$\mathcal{L}_d$, d-dim

`step2`: Integrate out $m_{A_0} \sim m_{\mathrm{D}}$ and $m_{A_i} \sim g\phi$

$\overline{\mathcal{L}}_d$, d-dim, HEFT-like

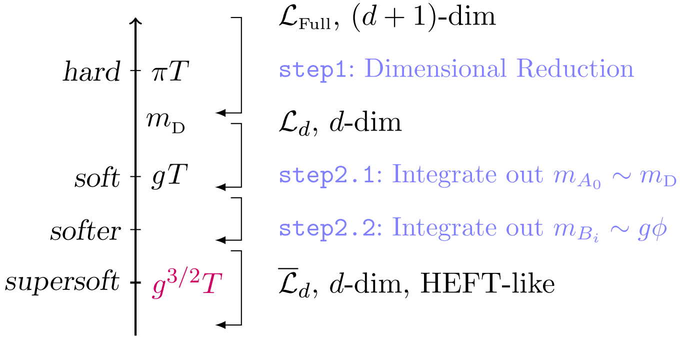
[6] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

[7] A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, `DRalgo`*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

Effective Field Theory (EFT): Dimensional Reduction (DR)

Integrate out hard modes perturbatively → EFT for static modes.

Precision thermodynamics of non-Abelian gauge theories as QCD and (EW) phase transition[6] using e.g. `DRalgo`.[7] Two `step` procedure:



[6] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

[7] A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, `DRalgo`*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

Limits of GW predictions form cosmological phase transitions

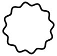
Powercounting the SM-like $3d$ EFT ($\mathrm{SU}(2)$+Higgs)

Describes the thermodynamics[8] of several parent $4d$ theories:

$$\mathcal{L}_{\mathrm{3d}}^{\mathrm{soft}} = \frac{1}{4}F_{ij}^{a}F_{ij}^{a} + (D_i\Phi)^{\dagger}(D_i\Phi) + (\partial_i A_0^a)^2 + V(\Phi) \; ,$$

$$V(\Phi) = m_3^2\Phi^{\dagger}\Phi + m_{\mathrm{D}}^2 A_0^a A_0^a + \lambda_3(\Phi^{\dagger}\Phi)^2 + h_3(\Phi^{\dagger}\Phi)A_0^a A_0^a + \dots \; .$$

If $m_{A_i} \sim g_3\phi \gg m_3$, integrating out vector boson introduces LO barrier

$$V_{\mathrm{LO}}(\Phi) = \quad \bullet \quad +$$  .

[8] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

[9] O. Gould and T. V. I. Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions,* JHEP **01** (2024) 048 [2309.01672], cf. sphaleron EFT and talk by **Y. Wu** on **Mon 11:10**

Powercounting the SM-like $3d$ EFT $(\mathrm{SU}(2)$+Higgs)

Describes the thermodynamics[8] of several parent $4d$ theories:

$$\mathcal{L}_{\rm 3d}^{\rm softer} = \frac{1}{4}F^a_{ij}F^a_{ij} + (D_i\Phi)^\dagger(D_i\Phi) + V(\Phi) \;,$$
$$V(\Phi) = m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 \;.$$

If $m_{A_i} \sim g_3\phi \gg m_3$, integrating out vector boson introduces LO barrier

$$V_{\rm LO}(\Phi) = m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 - \frac{g_3^3}{2\pi}\Big(\frac{\Phi^\dagger\Phi}{2}\Big)^{3/2} \;.$$

---

[8] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

[9] O. Gould and T. V. I. Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions,* JHEP **01** (2024) 048 [2309.01672], cf. sphaleron EFT and talk by **Y. Wu** on **Mon 11:10**

Powercounting the SM-like $3d$ EFT ($\mathrm{SU}(2)$+Higgs)

Describes the thermodynamics[8] of several parent $4d$ theories:

$$\mathcal{L}_{\text{3d}}^{\text{softer}} = \frac{1}{4}F^a_{ij}F^a_{ij} + (D_i\Phi)^\dagger(D_i\Phi) + V(\Phi) \;,$$
$$V(\Phi) = m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 \;.$$

If $m_{A_i} \sim g_3\phi \gg m_3$, integrating out vector boson introduces LO barrier

$$V_{\text{LO}}(\Phi) \to y\Phi^\dagger\Phi + x(\Phi^\dagger\Phi)^2 - \frac{1}{2\pi}\Big(\frac{\Phi^\dagger\Phi}{2}\Big)^{3/2} \;.$$

Since $x \sim \frac{m_3^2}{m_{A_i}^2} \ll 1$, and at the phase transition $y \sim 1/x$, we strictly[9] x-pand the perturbative series using 3d EFT dimensionless couplings

$$x \equiv \frac{\lambda_3}{g_3^2} \;, \qquad y \equiv \frac{m_3^2}{g_3^4} \;.$$

[8] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model*, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

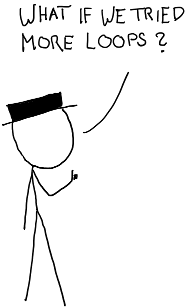
[9] O. Gould and T. V. I. Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions*, JHEP **01** (2024) 048 [2309.01672], cf. sphaleron EFT and talk by **Y. Wu** on **Mon 11:10**

$V_{\text{eff}}^{3\text{d}}$ (soft-to-supersoft matching)

Integrating out vector bosons in two steps up to 2-loops with `DRalgo`

$$m_{A_0}^2\left[\sim (gT)^2\right] \overset{\texttt{step2.1}}{\gg} m_{A_i}^2\left[\sim (g_3\phi)^2\right] \overset{\texttt{step2.2}}{\gg} m_3^2\left[\sim \lambda_3\phi^2\right] .$$

Focus on `step2.2` and add last perturbative orders N^3LO and N^4LO.[10]



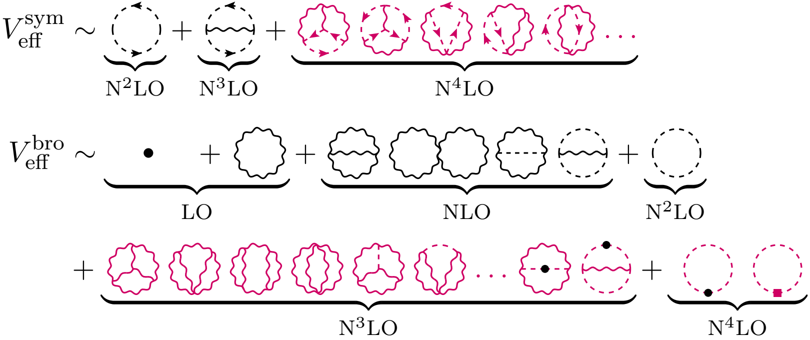
[10] A. Ekstedt, O. Gould, and J. Löfgren, *Radiative first-order phase transitions to next-to-next-to-leading order,* Phys. Rev. D **106** (2022) 036012 [2205.07241], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory,* Phys. Rev. D **110** (2024) 096006 [2405.18349]

$V_{\text{eff}}^{3\text{d}}$ (soft-to-supersoft matching)

Integrating out vector bosons in two steps up to 2-loops with `DRalgo`

$$m_{A_0}^2\left[\sim (gT)^2\right] \overset{\texttt{step2.1}}{\gg} m_{A_i}^2\left[\sim (g_3\phi)^2\right] \overset{\texttt{step2.2}}{\gg} m_3^2\left[\sim \lambda_3\phi^2\right].$$

Focus on `step2.2` and add last perturbative orders N^{3}LO and N^{4}LO.[10]



[10] A. Ekstedt, O. Gould, and J. Löfgren, *Radiative first-order phase transitions to next-to-next-to-leading order*, Phys. Rev. D **106** (2022) 036012 [2205.07241], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory*, Phys. Rev. D **110** (2024) 096006 [2405.18349]

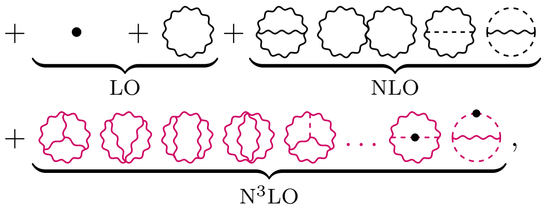
$V_{\text{eff}}^{3\text{d}}$ (soft-to-supersoft matching)

Integrating out vector bosons in two steps up to 2-loops with `DRalgo`

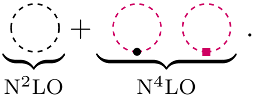
$$m_{A_0}^2\left[\sim (gT)^2\right] \overset{\texttt{step2.1}}{\gg} m_{A_i}^2\left[\sim (g_3\phi)^2\right] \overset{\texttt{step2.2}}{\gg} m_3^2\left[\sim \lambda_3\phi^2\right].$$

Focus on `step2.2` and add last perturbative orders N^{3}LO and N^{4}LO.[10]
In strict EFT expansion this organization can also be understood as:

$$S_{\text{supersoft}}^{\text{tree}} = \int_{\mathbf{x}} \frac{1}{2}(\partial_i s)^2 Z_s +$$



$$S_{\text{supersoft}}^{\text{1-loop}} =$$



[10] A. Ekstedt, O. Gould, and J. Löfgren, *Radiative first-order phase transitions to next-to-next-to-leading order*, Phys. Rev. D **106** (2022) 036012 [2205.07241], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory*, Phys. Rev. D **110** (2024) 096006 [2405.18349]

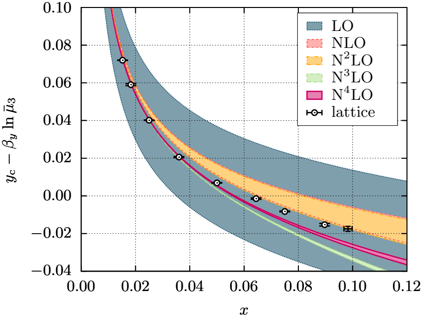
Last perturbative thermodynamic order (for SU(2) + Higgs)

By using $F \sim V_{\text{eff}}(\phi_{\text{min}})$, determine the critical mass y_{c} (or T_{c})

$$\Delta F(y_{\text{c}}(x), x) = [F_{\text{bro}} - F_{\text{sym}}](y_{\text{c}}(x), x) = 0\,,$$

and the scalar *condensates*[11]

$$\Delta\langle\Phi^\dagger\Phi\rangle \equiv \frac{\partial}{\partial y}\Delta F\,, \qquad \Delta\langle(\Phi^\dagger\Phi)^2\rangle \equiv \frac{\partial}{\partial x}\Delta F\,.$$



[11] Lattice data: K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], O. Gould, S. Güyer, and K. Rummukainen, *First-order electroweak phase transitions: a nonperturbative update,* [2205.07238]

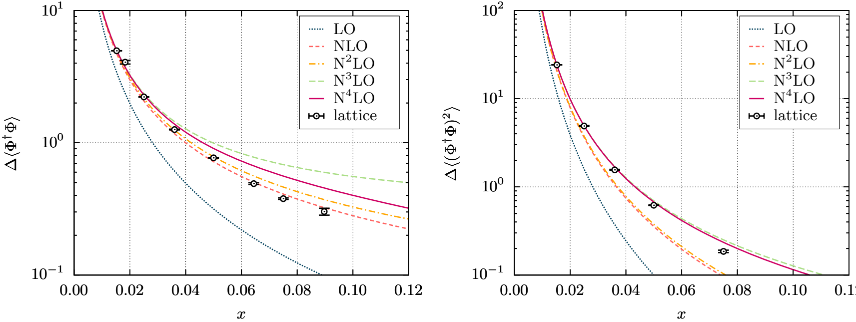
Last perturbative thermodynamic order (for SU(2) + Higgs)

By using $F \sim V_{\text{eff}}(\phi_{\text{min}})$, determine the critical mass y_c (or T_c)

$$\Delta F(y_c(x), x) = [F_{\text{bro}} - F_{\text{sym}}](y_c(x), x) = 0\,,$$

and the scalar *condensates*[11]

$$\Delta\langle\Phi^\dagger\Phi\rangle \equiv \frac{\partial}{\partial y}\Delta F\,, \qquad \Delta\langle(\Phi^\dagger\Phi)^2\rangle \equiv \frac{\partial}{\partial x}\Delta F\,.$$



[11] Lattice data: K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], O. Gould, S. Güyer, and K. Rummukainen, *First-order electroweak phase transitions: a nonperturbative update,* [2205.07238]

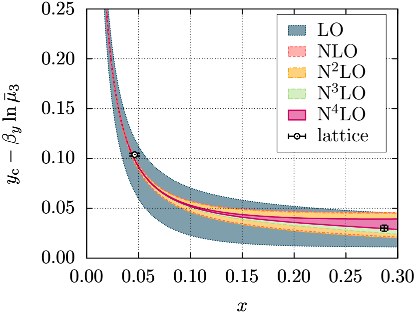
Last perturbative thermodynamic order (for U(1) + Higgs)

By using $F \sim V_{\text{eff}}(\phi_{\text{min}})$, determine the critical mass y_{c} (or T_{c})

$$\Delta F(y_{\text{c}}(x), x) = [F_{\text{bro}} - F_{\text{sym}}](y_{\text{c}}(x), x) = 0\,,$$

and the scalar *condensates*[11]

$$\Delta\langle \Phi^\dagger \Phi \rangle \equiv \frac{\partial}{\partial y} \Delta F\,, \qquad \Delta\langle (\Phi^\dagger \Phi)^2 \rangle \equiv \frac{\partial}{\partial x} \Delta F\,.$$



[11] Lattice data: K. Kajantie, M. Karjalainen, M. Laine, and J. Peisa, *Three-dimensional U(1) gauge + Higgs theory as an effective theory for finite temperature phase transitions,* Nucl. Phys. B **520** (1998) 345 [hep-lat/9711048], S. Mo, J. Hove, and A. Sudbo, *The Order of the metal to superconductor transition,* Phys. Rev. B **65** (2002) 104501 [cond-mat/0109260]

Predicting gravitational waves

Thermodynamics enters the GW spectrum through the strength and inverse duration of the transition, $h^2\Omega_{\rm GW}(f; H_\star, \alpha, \beta, v_w)$:

$$\alpha \sim \frac{\mathrm{d}\,\Delta F(y_c, x)}{\mathrm{d}\ln T} = \left(\frac{\mathrm{d}\,y_c}{\mathrm{d}\ln T}\right)\Delta\langle\Phi^\dagger\Phi\rangle + \left(\frac{\mathrm{d}\,x}{\mathrm{d}\ln T}\right)\Delta\langle(\Phi^\dagger\Phi)^2\rangle\,,$$

$$\frac{\beta}{H} = -\frac{\mathrm{d}\ln\Gamma}{\mathrm{d}\ln T}\,.$$

$\Longrightarrow$ [UV] $\times$ [IR] factorization.[12]

Completed perturbative predictions for: α at N^{4}LO, β/H at N^{2}LO.

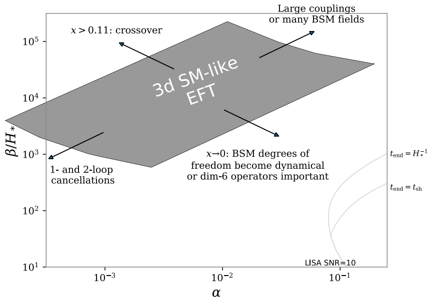
Limit: last perturbative order is N^{4}LO.
Todo: final perturbative correction for thermal bubble nucleation rate.

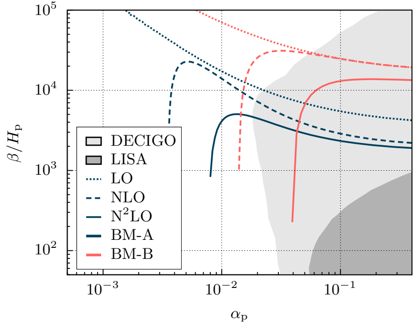
[12] O. Gould, J. Kozaczuk, L. Niemi, M. J. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition,* Phys. Rev. D **100** (2019) 115024 [1903.11604]

Impact on gravitational waves

$\alpha/(\beta/H)$ rhombus of SM-like EFT with no prospect for large SNR.[13]
Better access interesting LISA SNR by increasing loop order:

BM-A xSM with weakly portal-coupled singlet (decoupled)
BM-B xSM with strongly portal-coupled singlet

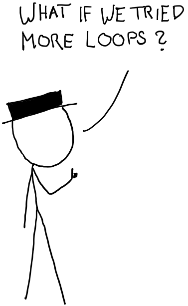




[13] O. Gould, J. Kozaczuk, L. Niemi, M. J. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition*, Phys. Rev. D **100** (2019) 115024 [1903.11604], talk by **M. Ramsey-Musolf** on **Fri 8:00**

Limitations of GW predictions form cosmological phase transitions

Dimension-six operators in U(1) + Higgs

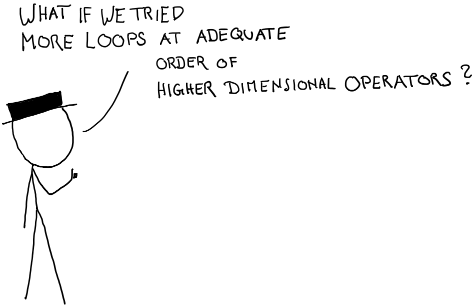


So far truncated operators at high T at dimension 4:

$$S^{3\mathrm{d}}_{\mathrm{soft}} = \frac{1}{T}\int_{\mathbf{x}} \left\{ \mathcal{L}^{3\mathrm{d}}_{\mathrm{soft}} + \sum_{n\geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\},$$

$$S^{3\mathrm{d}}_{\mathrm{softer}} = \frac{1}{T}\int_{\mathbf{x}} \left\{ \mathcal{L}^{3\mathrm{d}}_{\mathrm{softer}} + \sum_{n\geq 5} \frac{\mathcal{O}_n}{(m_{\mathrm{D}})^n} \right\}.$$

Dimension-six operators in U(1) + Higgs



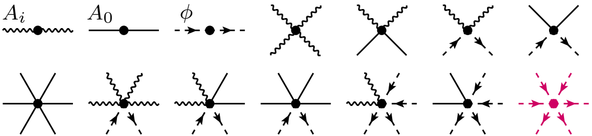
So far truncated operators at high T at dimension 4:

$$S^{\rm 3d}_{\rm soft} = \frac{1}{T}\int_{\mathbf{x}}\left\{\mathcal{L}^{\rm 3d}_{\rm soft} + \sum_{n\geq 5}\frac{\mathcal{O}_n}{(\pi T)^n}\right\},$$

$$S^{\rm 3d}_{\rm softer} = \frac{1}{T}\int_{\mathbf{x}}\left\{\mathcal{L}^{\rm 3d}_{\rm softer} + \sum_{n\geq 5}\frac{\mathcal{O}_n}{(m_{\rm D})^n}\right\}.$$

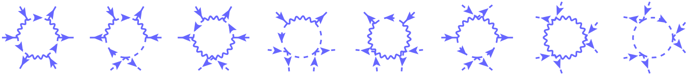
Vertex structures

$\mathcal{L}_{\text{soft}}$ is non-super-renormalizable.



Determine Wilson coefficients $\alpha_i(d)$ in d-dimensions:

- ▷ Evaluate (2–6)-point vertices at one-loop order

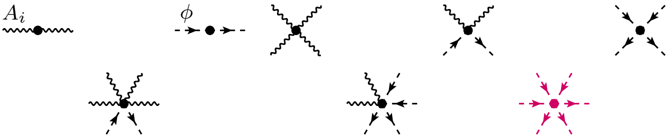


- ▷ Field redefinitions

Wilson coefficients are gauge-parameter (ξ) independent order-by-order.
Now focus on $c_6(\phi^\dagger\phi)^3$ effect.

Vertex structures

$\mathcal{L}_{\text{softer}}$ is non-super-renormalizable.



Determine Wilson coefficients $\alpha_i(d)$ in d-dimensions:

- ▷ Evaluate (2–6)-point vertices at one-loop order

- ▷ Field redefinitions

Wilson coefficients are gauge-parameter (ξ) independent order-by-order.
Now focus on $c_6(\phi^\dagger\phi)^3$ effect.

Marginal operators at dimension six

The soft-scale marginal operator is suppressed at $\mathcal{O}(g^6)$

$$c_6 = \frac{\zeta_3}{32\pi^4}\Big(g^6 - \frac{31}{30}g^4\lambda + 5\,g^2\lambda^2 + \frac{20}{3}\lambda^3\Big) + \mathcal{O}(g^8)\,.$$

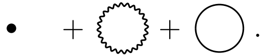
The softer-scale marginal operator is enhanced at $\mathcal{O}(g^3)$

$$\bar{c}_6 = c_6 + \frac{\sqrt{3}\,g^3}{8\pi}\Big(\frac{m_{\rm D}^{\rm LO}}{m_{\rm D}}\Big)^3\big(1 - x_{\rm LO}\big) + \mathcal{O}(g^4)\,.$$

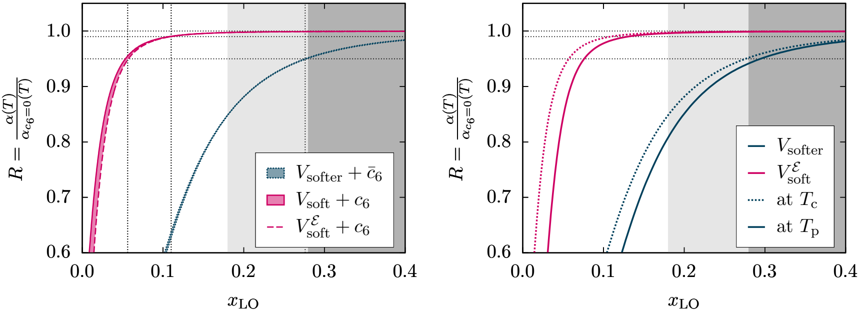
Validity of EFT

Leading-order effective potential given by

$$V^{\mathrm{LO}}_{\mathrm{soft}}(\Phi) =$$



Effect of c_6 shows broad window of high-T validity for soft EFT:[14]



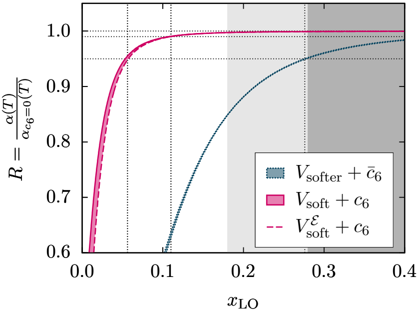
[14] F. Bernardo, P. Klose, P. Schicho, and T. V. I. Tenkanen, *Higher-dimensional operators at finite temperature affect gravitational-wave predictions,* JHEP **08** (2025) 109 [2503.18904]

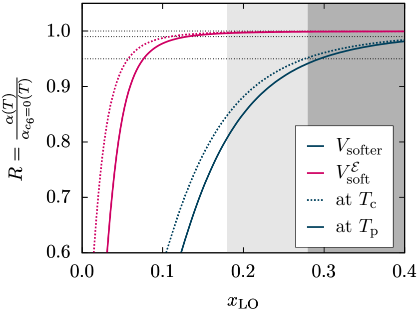
Validity of EFT

Leading-order effective potential given by

$$V_{\rm soft}^{\rm LO}(\Phi) = \frac{1}{2}m_3^2 v_3^2 + \frac{1}{4}\lambda_3 v_3^4 + \frac{1}{8}c_6 v_3^6 - \frac{1}{12\pi}\Big(2m_A^3 + m_{A_0}^3\Big)\,.$$

Effect of c_6 shows broad window of high-T validity for soft EFT:[14]





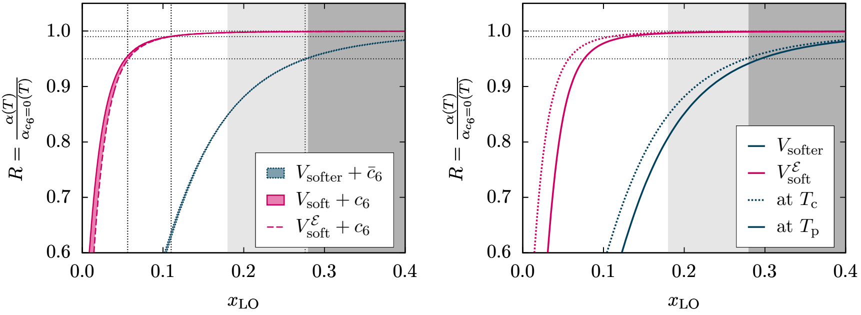
[14] F. Bernardo, P. Klose, P. Schicho, and T. V. I. Tenkanen, *Higher-dimensional operators at finite temperature affect gravitational-wave predictions,* JHEP **08** (2025) 109 [2503.18904]

Validity of EFT

Leading-order effective potential given by

$$V_{\rm soft}^{\rm LO}(\Phi) = \frac{1}{2}m_3^2 v_3^2 + \frac{1}{4}\lambda_3 v_3^4 + \frac{1}{8}c_6 v_3^6 - \frac{1}{12\pi}\Big(2g_3^3 v_3^3 + (m_{\rm D}^2 + h_3 v_3^2)^{\frac{3}{2}}\Big)\,.$$

Effect of c_6 shows broad window of high-T validity for soft EFT:[14]



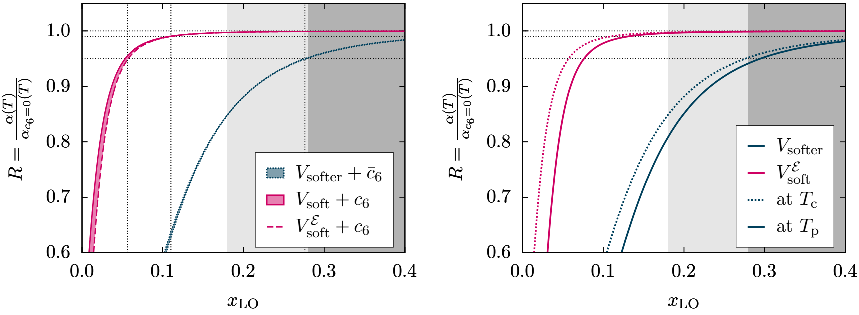
[14] F. Bernardo, P. Klose, P. Schicho, and T. V. I. Tenkanen, *Higher-dimensional operators at finite temperature affect gravitational-wave predictions,* JHEP **08** (2025) 109 [2503.18904]

Validity of EFT

Leading-order effective potential given by assuming $m_{\mathrm{D}}^2 \gg h_3 v_3^2$

$$V^{\mathrm{LO}}_{\mathrm{supersoft}}(\Phi) = \frac{1}{2}\bar{m}_3^2 v_3^2 + \frac{1}{4}\bar{\lambda}_3 v_3^4 + \frac{1}{8}\bar{c}_6 v_3^6 - \frac{1}{6\pi}\bar{g}_3^3 v_3^3\,.$$

Effect of c_6 shows broad window of high-T validity for soft EFT:[14]



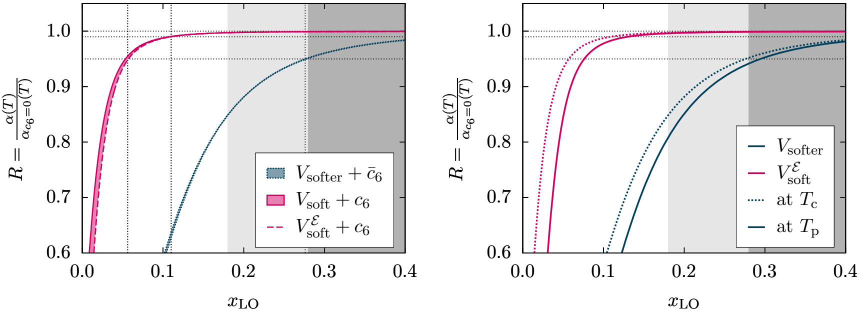
[14] F. Bernardo, P. Klose, P. Schicho, and T. V. I. Tenkanen, *Higher-dimensional operators at finite temperature affect gravitational-wave predictions*, JHEP **08** (2025) 109 [2503.18904]

Validity of EFT

Leading-order effective potential given by assuming $m_{\rm D}^2 \ll h_3 v_3^2$

$$V^{\rm LO}_{\rm supersoft}(\Phi) = \frac{1}{2}m_3^2 v_3^2 + \frac{1}{4}\lambda_3 v_3^4 + \frac{1}{8}c_6 v_3^6 - \frac{v_3^3}{6\pi}\Big(g_3^3 + \frac{1}{2}h_3^{\frac{3}{2}}\Big)\,.$$

Effect of c_6 shows broad window of high-T validity for soft EFT:[14]



---

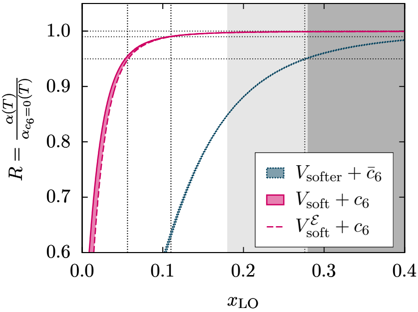
[14] F. Bernardo, P. Klose, P. Schicho, and T. V. I. Tenkanen, *Higher-dimensional operators at finite temperature affect gravitational-wave predictions*, JHEP **08** (2025) 109 [2503.18904]

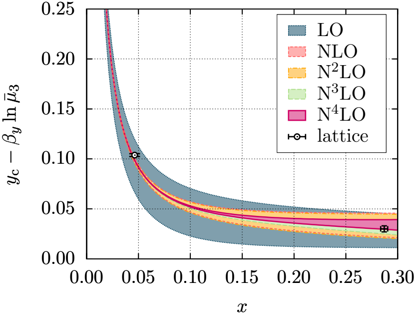
Validity of EFT

Leading-order effective potential given by assuming $m_{\mathrm{D}}^2 \ll h_3 v_3^2$

$$V^{\mathrm{LO}}_{\mathrm{supersoft}}(\Phi) = \frac{1}{2} m_3^2 v_3^2 + \frac{1}{4}\lambda_3 v_3^4 + \frac{1}{8} c_6 v_3^6 - \frac{v_3^3}{6\pi}\Big(g_3^3 + \frac{1}{2} h_3^{\frac{3}{2}}\Big)\,.$$

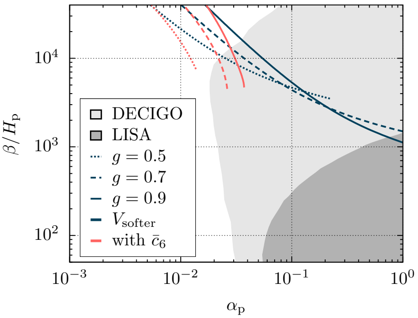
Effect of c_6 shows broad window of high-T validity for soft EFT:[14]





[14] F. Bernardo, P. Klose, P. Schicho, and T. V. I. Tenkanen, *Higher-dimensional operators at finite temperature affect gravitational-wave predictions,* JHEP **08** (2025) 109 [2503.18904]

Soft scale enhances phase-transition strength, $\alpha(T)$.
High-T expansion is compromised for regime relevant for LISA:



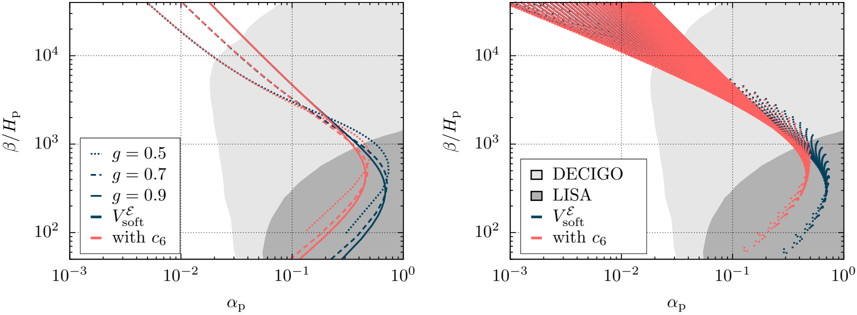
Limitation: conventional lattice results of (dim-4) super-renormalizable 3d EFT do not describe hard/soft-scale driven transitions.[15]

Need 4d full theory lattice simulations for a reliable description?

[15] M. Laine, *Comparison of 4-D and 3-D lattice results for the electroweak phase transition*, Phys. Lett. B **385** (1996) 249 [hep-lat/9604011]

Gravitational wave prospects

Soft scale enhances phase-transition strength, $\alpha(T)$.
High-T expansion is compromised for regime relevant for LISA:



Limitation: conventional lattice results of (dim-4) super-renormalizable 3d EFT do not describe hard/soft-scale driven transitions.[15]

Need 4d full theory lattice simulations for a reliable description?

[15] M. Laine, *Comparison of 4-D and 3-D lattice results for the electroweak phase transition*, Phys. Lett. B **385** (1996) 249 [hep-lat/9604011]

Phase transitions beyond high-T

Supercooling in EFT: Classical scale-invariant models

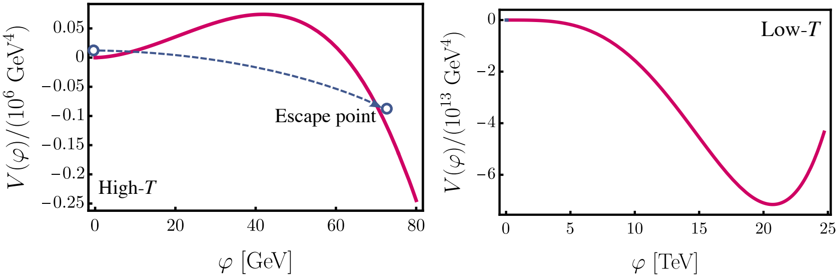
exhibit strong supercooling and phase transition. Barrier until low-T

$$m_\varphi^2(T) = \left[\mu_0^2\right] + m_T^2\,.$$

Trapped field φ in false vacuum φ_F until $T_p \ll T_c$. Split computation:[16]

High-T: Small field regime $M(\varphi) < T$ use 3D EFT

Low-T: Large field regime $M(\varphi) > T$ use vacuum potential



[16] M. Kierkla, P. Schicho, B. Swiezewska, T. V. I. Tenkanen, and J. van de Vis, *Finite-temperature bubble nucleation with shifting scale hierarchies,* JHEP **07** (2025) 153 [2503.13597], M. Kierkla, B. Swiezewska, T. V. I. Tenkanen, and J. van de Vis, *Gravitational waves from supercooled phase transitions: dimensional transmutation meets dimensional reduction,* JHEP **02** (2024) 234 [2312.12413], cf. PBH and talk by **R. Jinno** on **Sat 8:30**

The formalism ($\texttt{3DEFT}^+$) to extend V_{eff}

requires the full effective potential[17]

$$V_{\text{eff}}^{\text{res}} = V_{\text{eff}}^{\text{res}} - V_{\text{eff}}^{\text{res,soft}} + V_{\text{eff}}^{\text{res,soft}} = \left(V_{\text{eff}}^{\text{naive}} - V_{\text{eff}}^{\text{naive,soft}}\right) + V_{\text{eff}}^{\text{res,soft}} \,.$$

Use (hot) Loop-Tree Duality ($\texttt{hotLTD}$)[18] to evaluate IR safe ($m^2 \to 0$)

$$\Delta V_{\text{eff}}^{\text{hard}} = V_{\text{eff}}^{\text{naive}} - V_{\text{eff}}^{\text{naive,soft}} \,.$$

---

[17] P. Navarrete, R. Paatelainen, K. Seppänen, and T. V. I. Tenkanen, *Cosmological phase transitions without high-temperature expansions*, [2507.07014]

[18] (i) Subtract UV divergences, (ii) Matsubara sums $\to$ 3D integrand, (iii) Monte Carlo spatial integration.

The formalism ($\mathtt{3DEFT}^{+}$) to extend V_{eff}

requires the full effective potential[17]

$$V_{\text{eff}}^{\text{res}} = V_{\text{eff}}^{\text{res}} - V_{\text{eff}}^{\text{res,soft}} + V_{\text{eff}}^{\text{res,soft}} = \Big(V_{\text{eff}}^{\text{naive}} - V_{\text{eff}}^{\text{naive,soft}}\Big) + V_{\text{eff}}^{\text{res,soft}} \,.$$

Use (hot) Loop-Tree Duality (`hotLTD`)[18] to evaluate IR safe $(m^2 \to 0)$

$$\Delta V_{\text{eff}}^{\text{hard}} = \Big(\frac{1}{2}\SumInt_P \ln\big(P^2 + M_\phi^2\big) - V_{\text{eff}}^{\text{naive,soft}}\Big) \,.$$

$V_{\text{eff}}^{\text{naive,soft}}$ is a local IR counterterm.

---

[17] P. Navarrete, R. Paatelainen, K. Seppänen, and T. V. I. Tenkanen, *Cosmological phase transitions without high-temperature expansions,* [2507.07014]

[18] (i) Subtract UV divergences, (ii) Matsubara sums → 3D integrand, (iii) Monte Carlo spatial integration.

The formalism ($\texttt{3DEFT}^+$) to extend V_{eff}

requires the full effective potential[17]

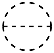
$$V_{\text{eff}}^{\text{res}} = V_{\text{eff}}^{\text{res}} - V_{\text{eff}}^{\text{res,soft}} + V_{\text{eff}}^{\text{res,soft}} = \Big(V_{\text{eff}}^{\text{naive}} - V_{\text{eff}}^{\text{naive,soft}}\Big) + V_{\text{eff}}^{\text{res,soft}} \,.$$

Use (hot) Loop-Tree Duality ($\texttt{hotLTD}$)[18] to evaluate IR safe ($m^2 \to 0$)

$$\Delta V_{\text{eff}}^{\text{hard}} = \Big(\frac{1}{2}\sum\!\!\!\!\!\!\!\int_P \ln\big(P^2 + M_\phi^2\big) - V_{\text{eff}}^{\text{naive,soft}}\Big) \,.$$

Evaluate soft part $V_{\text{eff}}^{\text{naive,soft}}$ in 3D EFT with coupling and IR expansion of masses, using unresummed propagators with $X_3 = X + \delta X_3$:

$V_{\text{eff}}^{(1)\text{naive,soft}} \supset$  $= \dfrac{T}{2}\displaystyle\int_{\mathbf{p}} \ln\left(p^2 + M_\phi^2\right) \,,$

$V_{\text{eff}}^{(2)\text{naive,soft}} \supset$  $= \dfrac{g^2}{12} S_{3d}(M_\phi) + \delta g_3^2[\ldots] + \delta M_3^2[\ldots] + \ldots \,.$

[17] P. Navarrete, R. Paatelainen, K. Seppänen, and T. V. I. Tenkanen, *Cosmological phase transitions without high-temperature expansions*, [2507.07014]

[18] (i) Subtract UV divergences, (ii) Matsubara sums $\to$ 3D integrand, (iii) Monte Carlo spatial integration.

The formalism ($\texttt{3DEFT}^{+}$) to extend V_{eff}

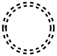
requires the full effective potential[17]

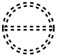
$$V_{\text{eff}}^{\text{res}} = V_{\text{eff}}^{\text{res}} - V_{\text{eff}}^{\text{res,soft}} + V_{\text{eff}}^{\text{res,soft}} = \Big(V_{\text{eff}}^{\text{naive}} - V_{\text{eff}}^{\text{naive,soft}} \Big) + V_{\text{eff}}^{\text{res,soft}} \,.$$

Use (hot) Loop-Tree Duality ($\texttt{hotLTD}$)[18] to evaluate IR safe ($m^2 \to 0$)

$$\Delta V_{\text{eff}}^{\text{hard}} = \Big(\frac{1}{2} \sum\!\!\!\!\!\!\int_P \ln\big(P^2 + M_\phi^2\big) - V_{\text{eff}}^{\text{naive,soft}} \Big) \,.$$

Evaluate soft part $V_{\text{eff}}^{\text{res,soft}}$ in 3D EFT without coupling nor IR expansion of masses, using resummed propagators with $X_3 = X + \delta X_3$:

$$V_{\text{eff}}^{(1)\text{res,soft}} \supset$$  $$= \frac{T}{2} \int_{\mathbf{p}} \ln\left(p^2 + M_3^2\right) \,,$$

$$V_{\text{eff}}^{(2)\text{res,soft}} \supset$$  $$= \frac{g_3^2}{12T} S_{3d}(M_3) \,.$$

---

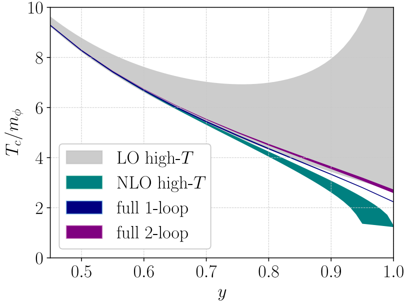
[17] P. Navarrete, R. Paatelainen, K. Seppänen, and T. V. I. Tenkanen, *Cosmological phase transitions without high-temperature expansions*, [2507.07014]

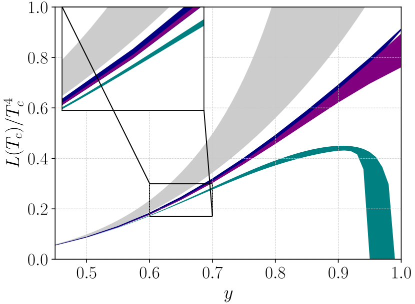
[18] (i) Subtract UV divergences, (ii) Matsubara sums → 3D integrand, (iii) Monte Carlo spatial integration.

Proof of principle example: Higgs-Yukawa model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \sigma\phi + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{3!}g\phi^3 + \frac{1}{4!}\lambda\phi^4 + \bar{\psi}(\not{\partial} + m_\psi)\psi + y\phi\bar{\psi}\psi\,.$$

For small T/m_ϕ high-T breaks down but the full result remains robust.





Alternatives: full thermal integrals,[19] tadpole resummation,[20] ...

[19] M. Laine, M. Meyer, and G. Nardini, *Thermal phase transition with full 2-loop effective potential,* Nucl. Phys. B **920** (2017) 565 [1702.07479]

[20] D. Curtin, J. Roy, and G. White, *Gravitational waves and tadpole resummation: Efficient and easy convergence of finite temperature QFT,* Phys. Rev. D **109** (2024) 116001 [2211.08218]

Conclusions

Precision thermodynamics of BSM theories:

- reliably describe cosmological FOPT and GW production,
- practical approach: **Effective Theories** + **universality**.

Reaching perturbative limits and overcoming limitations:

- higher dimensional operators at $\mathcal{O}(g^6)$ in the $3d$ EFT,
- purely perturbative $\mathcal{O}(g^6)$ contributions from hard scale πT,
- final perturbative corrections for bubble nucleation rate,

Going beyond high-T:

3DEFT$^+$ New frameworks with large range of validity,

4d lattice simulations to test reliable description.[a]

[a] Work in progress . . . .

Overfull hbox (badness 270925)

A minimal extension of the EW sector

Real singlet (S) extension of the SM (xSM)

$$\mathcal{L}_{\rm xSM} = \mathcal{L}_{\rm SM} + \mathcal{L}_S - V(\phi, S) \; ,$$

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu S)^2 \; ,$$

$$V(\phi, S) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 + \frac{1}{2}\mu_S^2 \, S^2 + \frac{1}{4}\lambda_s \, S^4 + \frac{1}{2}\lambda_{hs} \, S^2 \phi^\dagger \phi \; ,$$

with portal coupling λ_{hs}. Phases at finite T:

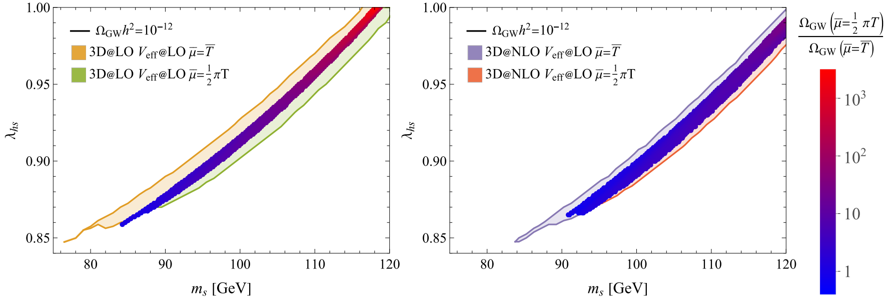
- ϕ condenses, SM-like Higgs regime
- S condenses
- No direct Dark Matter candidate. See complex singlet with parameterizaton $S \to v_S + S + iA$.

❶ $\mathcal{L}_{3d}$ (hard-to-soft matching)

$3d$ EFT at NLO (gauge-invariant) with `DRalgo`. Todo: NNLO.

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \to x + s$. 2-loop corrections significantly affect on GW signal.[21]

Parameters	m_s λ_{hs} $\lambda_s = 1$	$\alpha > 1$ $\mathrm{SNR}_{\mathrm{LISA}} > 10$

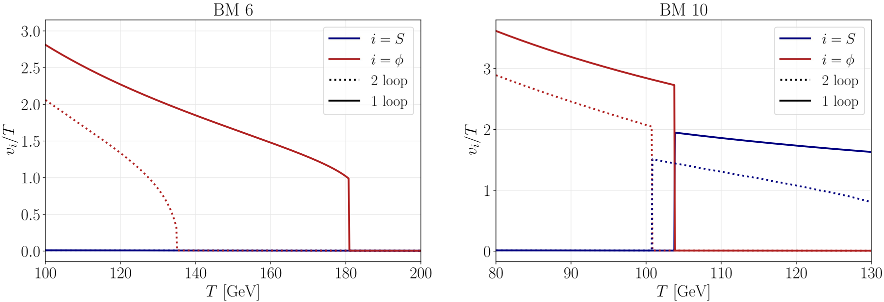


[21] M. Lewicki, M. Merchand, L. Sagunski, P. Schicho, and D. Schmitt, *Impact of theoretical uncertainties on model parameter reconstruction from GW signals sourced by cosmological phase transitions*, Phys. Rev. D **110** (2024) 023538 [2403.03769], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]

Transitions in the xSM (real singlet)

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \to x + s$. 2-loop corrections are significant.[22]

Benchmark	m_s	λ_{hs}	λ_s
BM6	350 GeV	3.5	0.3
BM10	325 GeV	3.5	0.3

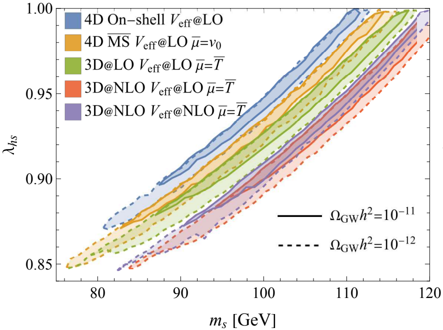


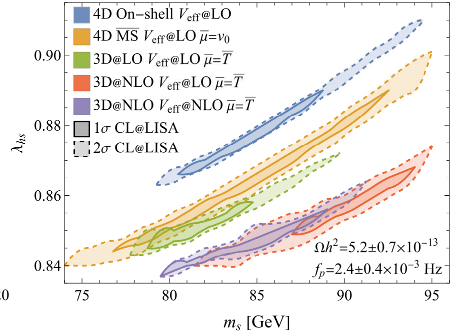
[22] L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]

Computational diligence at $\mathcal{O}(g^4)$

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \to x + s$.
2-loop corrections move "Bananas": significant effect on GW signal.[23]

Parameters	m_s	λ_{hs}	$\lambda_s = 1$	$\alpha > 1$	$\mathrm{SNR}_{\mathrm{LISA}} > 10$



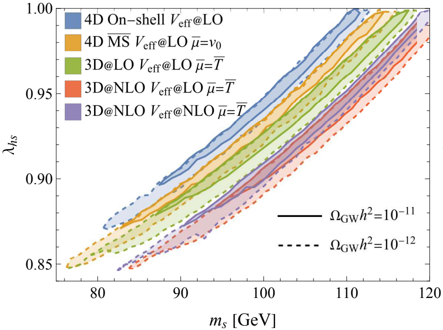


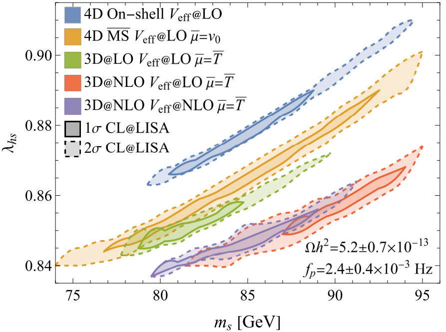
[23] M. Lewicki, M. Merchand, L. Sagunski, P. Schicho, and D. Schmitt, *Impact of theoretical uncertainties on model parameter reconstruction from GW signals sourced by cosmological phase transitions*, Phys. Rev. D **110** (2024) 023538 [2403.03769], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]

Does the banana move?

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \to x + s$.
2-loop corrections move "Bananas": significant effect on GW signal.[23]

Parameters	m_s	λ_{hs}	$\lambda_s = 1$	$\alpha > 1$	$\mathrm{SNR}_{\mathrm{LISA}} > 10$





[23] M. Lewicki, M. Merchand, L. Sagunski, P. Schicho, and D. Schmitt, *Impact of theoretical uncertainties on model parameter reconstruction from GW signals sourced by cosmological phase transitions*, Phys. Rev. D **110** (2024) 023538 [2403.03769], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]

Renormalization scale (in)dependence at finite T

At zero temperature

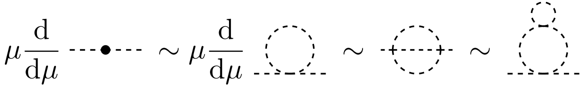
$$V_{\rm eff}(\phi,\bar{\mu}) = \underbrace{V_{\rm eff}^{\rm tree}}_{\mathcal{O}(g^2)} + \underbrace{V_{{\rm CW},1\ell}}_{\mathcal{O}(g^4)}\,, \quad \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\Big(V_{\rm eff}^{\rm tree} + V_{{\rm CW},1\ell}\Big) = 0\,.$$

At finite temperature[24]

$$V_{\rm eff}^{\rm res.}(\phi,T,\bar{\mu}) = \underbrace{V_{\rm eff}^{\rm tree}}_{\mathcal{O}(g^2)} + \underbrace{V_{\rm res.,soft}}_{\mathcal{O}(g^3)} + \underbrace{V_{\rm hard}}_{\mathcal{O}(g^2T^2)+\mathcal{O}(g^4)}\,,$$

running of 1-loop thermal masses is of the same order as 2-loop thermal-mass logarithms.

Automatically included in dimensionally reduced 3d EFT:



$$\sim \mathcal{O}(g^4T^2)$$

[24] O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

Constructing the supersoft EFT

Two approaches to a *final* (nucleation) EFT

$$\mathcal{L}_{4\text{d}} \to \mathcal{L}_{3\text{d}}^{\text{soft}} \to \mathcal{L}_{3\text{d}}^{\text{softer}} \to V_{\text{eff}}^{\text{supersoft}} \,,$$

In the broken phase, utilize different hierarchies among Lorentz scalars, temporal scalars, and vectors:

$$m_{A_0}^2 \big[\sim (gT)^2\big] \overset{\texttt{step2.1}}{\gg} m_{A_i}^2 \big[\sim (g_3\phi)^2\big] \overset{\texttt{step2.2}}{\gg} m_3^2 \big[\sim \lambda_3\phi^2\big] \,,$$

Constructing the supersoft EFT

Two approaches to a *final* (nucleation) EFT

$$\mathcal{L}_{4\mathrm{d}} \to \mathcal{L}_{3\mathrm{d}}^{\mathrm{soft}} \to \mathcal{L}_{3\mathrm{d}}^{\mathrm{softer}} \to V_{\mathrm{eff}}^{\mathrm{supersoft}} \,,$$
$$\mathcal{L}_{4\mathrm{d}} \to \mathcal{L}_{3\mathrm{d}}^{\mathrm{soft}} \to V_{\mathrm{eff}}^{\mathrm{supersoft}} \,.$$

In the broken phase, utilize different hierarchies among Lorentz scalars, temporal scalars, and vectors:

$$m_{A_0}^2 \big[\sim (gT)^2\big] \overset{\texttt{step2.1}}{\gg} m_{A_i}^2 \big[\sim (g_3\phi)^2\big] \overset{\texttt{step2.2}}{\gg} m_3^2 \big[\sim \lambda_3\phi^2\big] \,,$$
$$m_{A_0}^2 \big[\sim (gT)^2\big] = m_{A_i}^2 \big[\sim (g_3\phi)^2\big] \overset{\texttt{step2}}{\gg} m_3^2 \big[\sim \lambda_3\phi^2\big] \,.$$

Constructing the supersoft EFT

Two approaches to a *final* (nucleation) EFT

$$\mathcal{L}_{4\text{d}} \to \mathcal{L}_{3\text{d}}^{\text{soft}} \to \mathcal{L}_{3\text{d}}^{\text{softer}} \to V_{\text{eff}}^{\text{supersoft}} \,,$$
$$\mathcal{L}_{4\text{d}} \to \mathcal{L}_{3\text{d}}^{\text{soft}} \to V_{\text{eff}}^{\text{supersoft}} \,.$$

In the broken phase, utilize different hierarchies among Lorentz scalars, temporal scalars, and vectors:

$$m_{A_0}^2 \big[\sim (gT)^2\big] \overset{\texttt{step2.1}}{\gg} m_{A_i}^2 \big[\sim (g_3\phi)^2\big] \overset{\texttt{step2.2}}{\gg} m_3^2 \big[\sim \lambda_3\phi^2\big] \,,$$
$$m_{A_0}^2 \big[\sim (gT)^2\big] = m_{A_i}^2 \big[\sim (g_3\phi)^2\big] \overset{\texttt{step2}}{\gg} m_3^2 \big[\sim \lambda_3\phi^2\big] \,.$$

In the symmetric phase, the hierarchy is flipped

$$m_{A_0}^2 \big[\sim (gT)^2\big] \gg m_3^2 \big[\sim (g^{3/2}T)^2\big] \overset{\texttt{step2}}{\gg} m_{A_i}^2 \big[\sim (g^2T)^2\big] \,.$$