Studying Bell nonlocality of light quarks and new physics effects in fragmentation hadrons at lepton colliders

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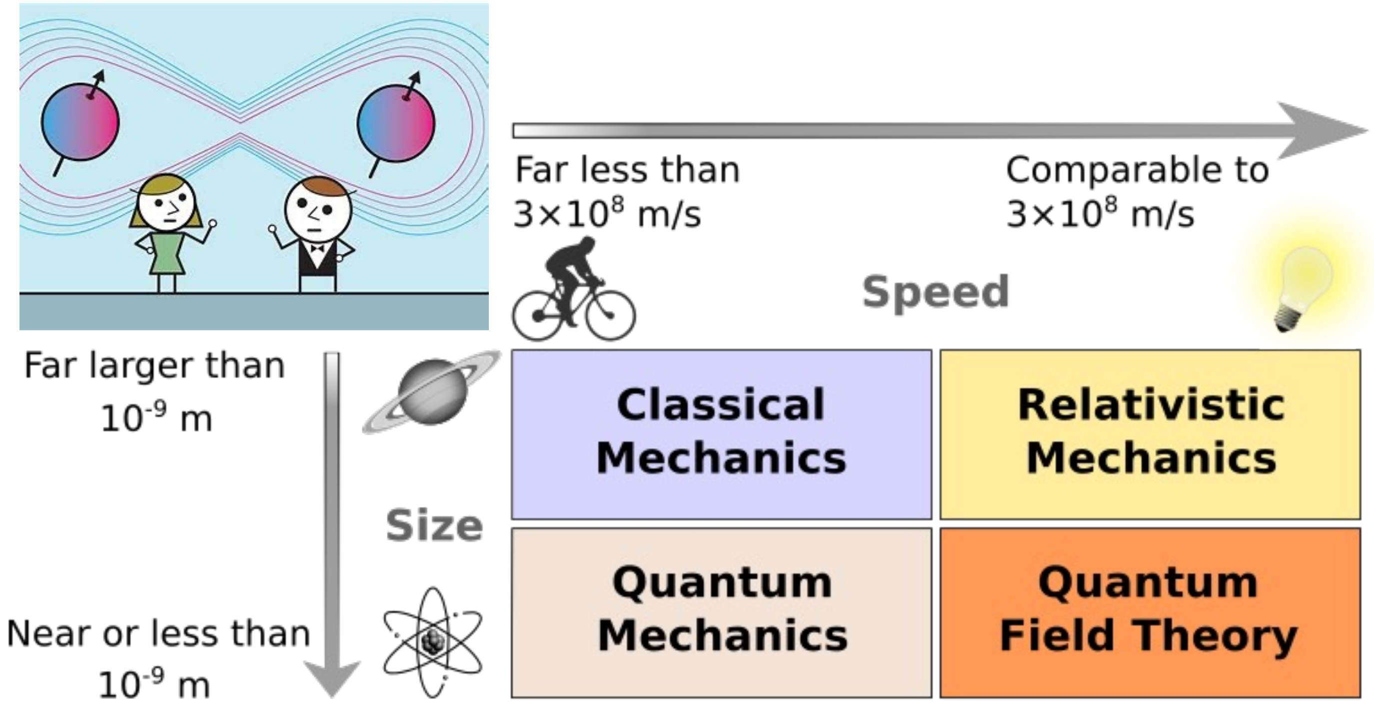
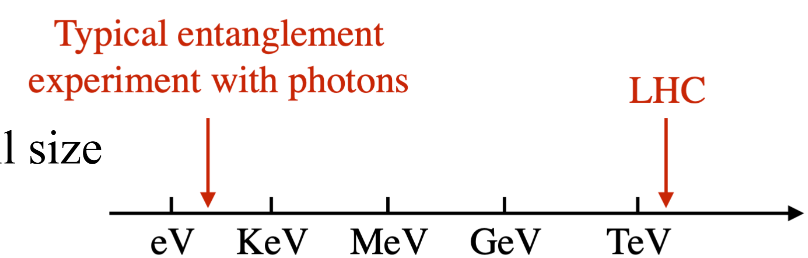
Based on Kun Cheng and Bin Yan, PRL 135 (2025) 011902,
Kun Cheng, Tao Han, Guanghui Li and Bin Yan, 2510.xxxxx
Qing-Hong Cao, Guanghui Li, Xin-Kai Wen and Bin Yan, 2509.18276

Quantum information at collider

➢Quantum entanglement and non-locality are distinctive features of quantum systems

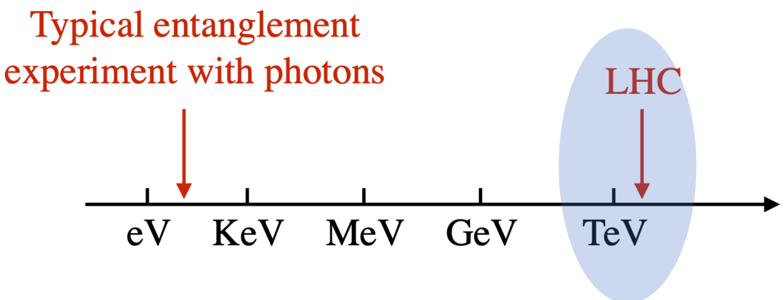




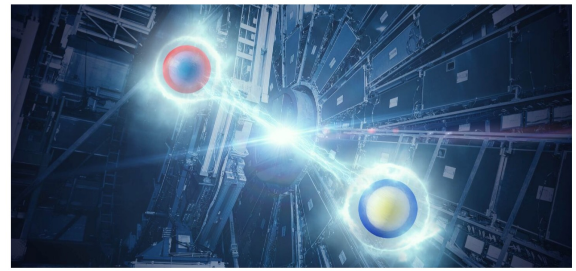
➢Collider physics: high energy and small size

Quantum information at collider

➢High energy: the scattering involves both the QCD and electroweak interactions



The results open up a new perspective on the complex world of quantum physics



ATLAS and CMS has observed the spin entanglement of top quark pair

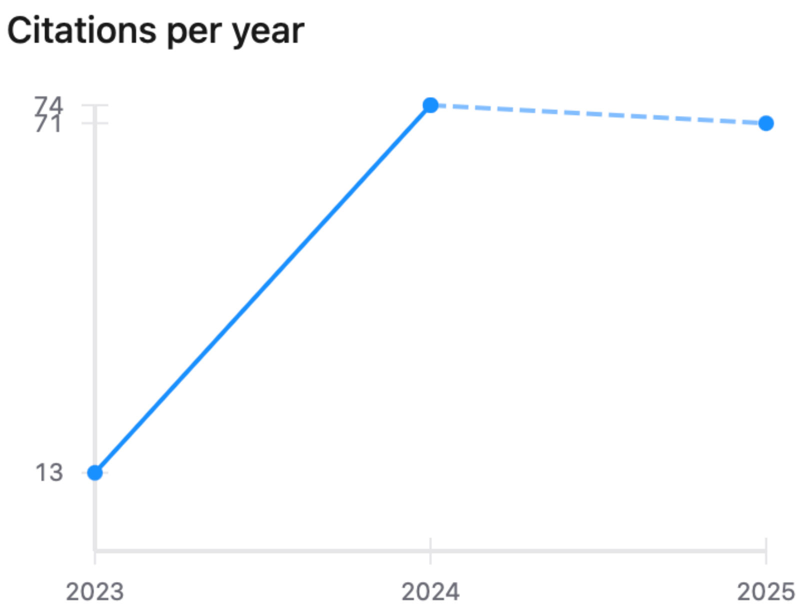
➢New features:

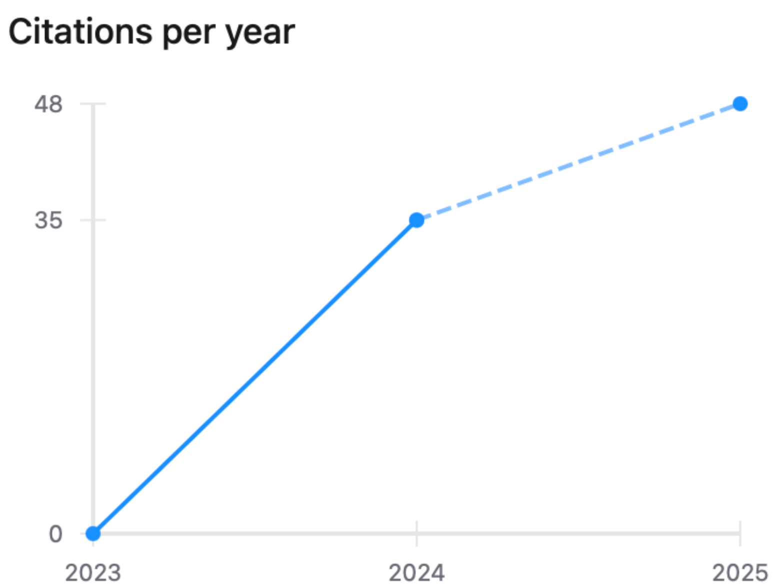
- ❑ Particle spin can not be measured directly
- ❑ New degree of freedom for spin-1 particle (Longitudinal mode)
- ❑ New features from interactions: parity violation, QCD confinement
- ❑ Entanglement beyond the spin space: flavor
- ❑ Quantum information and QCD confinement
- ❑ ……

Quantum information in high-energy physics is an emerging and rapidly growing field at the intersection of particle physics and quantum theory

Quantum information in collider

ATLAS and CMS has observed the spin entanglement of top quark pair





ATLAS: Nature 633 (2024) 8030, 542-547

CMS: Rept. Prog. Phys. 87 (2024) 117801

Quantum spin entanglement

The spin correlation of fermion pair can be described by the general density matrix

$$\rho = \frac{I_2 \otimes I_2 + B_i \sigma_i \otimes I_2 + \bar{B}_i I_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

- $B_i, \bar{B}_i$: the polarization of each particle
- C_{ij} : the spin correlation of top quark pair

A separable state can: $\rho_{AB} = \sum_w p_w \rho_w^A \otimes \rho_w^B$, Entangled is non-separable

Entanglement can be measured by the concurrence observable

$$\mathscr{C}(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

λ_i: eigenvalues of matrix: $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ $\tilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2)$

W. K. Wootters, PRL 80 (1998) 2245

Entanglement and Bell inequality

- Bell (CHSH) inequality: $\hat{A}_i = \pm 1, \; \hat{B}_i = \pm 1$

$$\left| \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle \right| \leq 2$$

[Clauser et al, *PRL* 23, 880 (1969)]

$$\hat{A} = \vec{a} \cdot \hat{\sigma}, \quad \hat{B} = \vec{a} \cdot \hat{\sigma}, \quad \langle \hat{A} \hat{B} \rangle = \vec{a} \cdot C \cdot \vec{b}$$

- Bell inequality is violated iff we can find four directions $\vec{a}_{1,2}, \vec{b}_{1,2}$ so that

$$\left| \vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2) \right| > 2$$

Fix some direction $\vec{a}_{1,2}, \vec{b}_{1,2}$

e.g. $\sqrt{2} |C_{xx} \pm C_{yy}| > 2$

Scan $\vec{a}_{1,2}, \vec{b}_{1,2}$ to maximize

$$\mathcal{B}[\rho] = 2\sqrt{c_1^2 + c_2^2} > 2$$

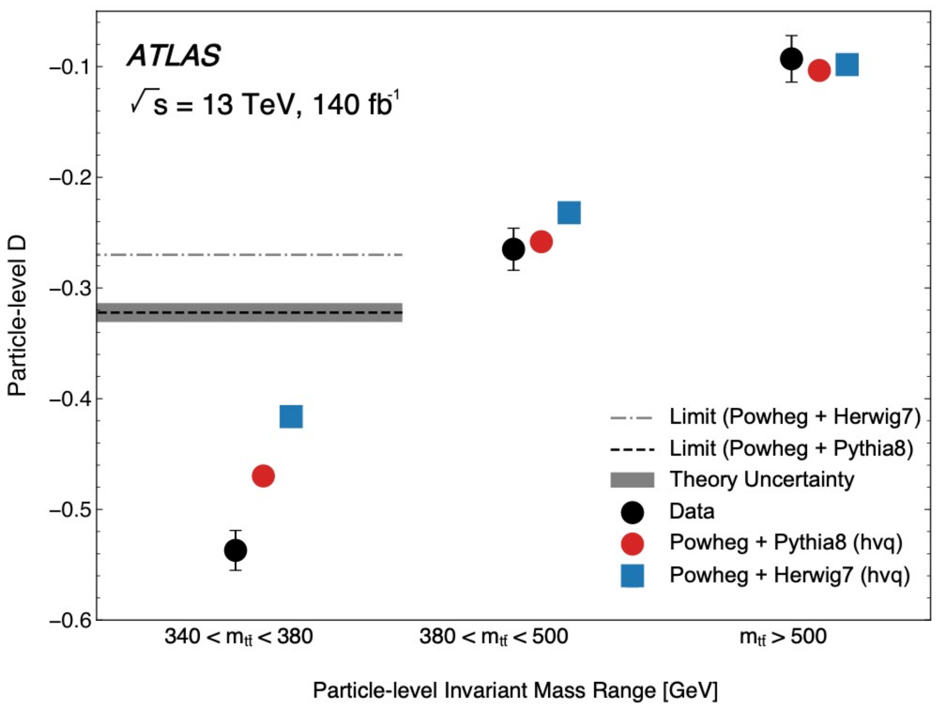
c_1^2, c_2^2 **are the largest two eigenvalue of $C^T C$.**

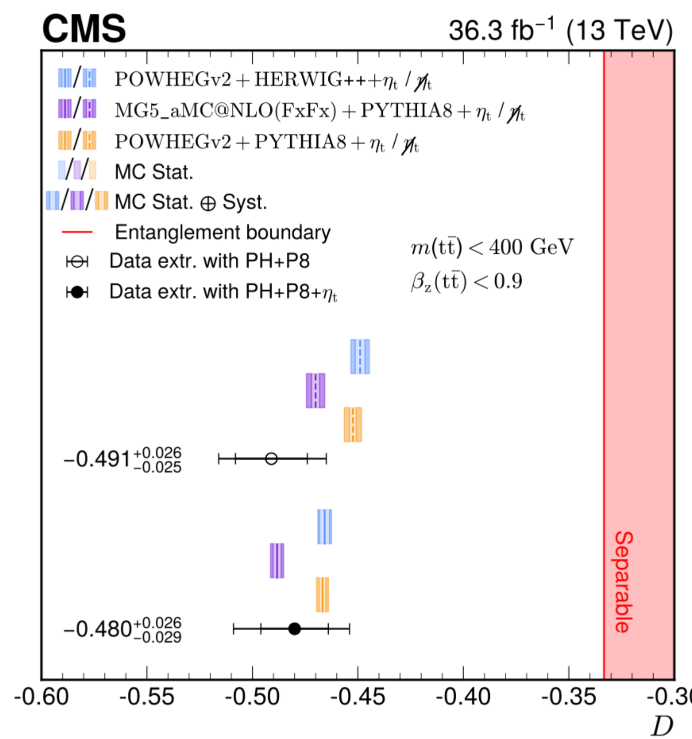
Quantum spin entanglement

Top quark pair: $\mathscr{C}[\rho] = \dfrac{-1-3D}{2}$ $\quad \dfrac{1}{\sigma}\dfrac{d\sigma}{d\cos\varphi} = \dfrac{1}{2}(1 - D\cos\varphi)$

$$D = \min\{\frac{\mathrm{tr}(C)}{3}, \frac{c_1 - c_2 - c_3}{3}, \frac{c_2 - c_1 - c_3}{3}, \frac{c_3 - c_1 - c_2}{3}\}, \qquad c_i = \mathrm{eig}(C)$$

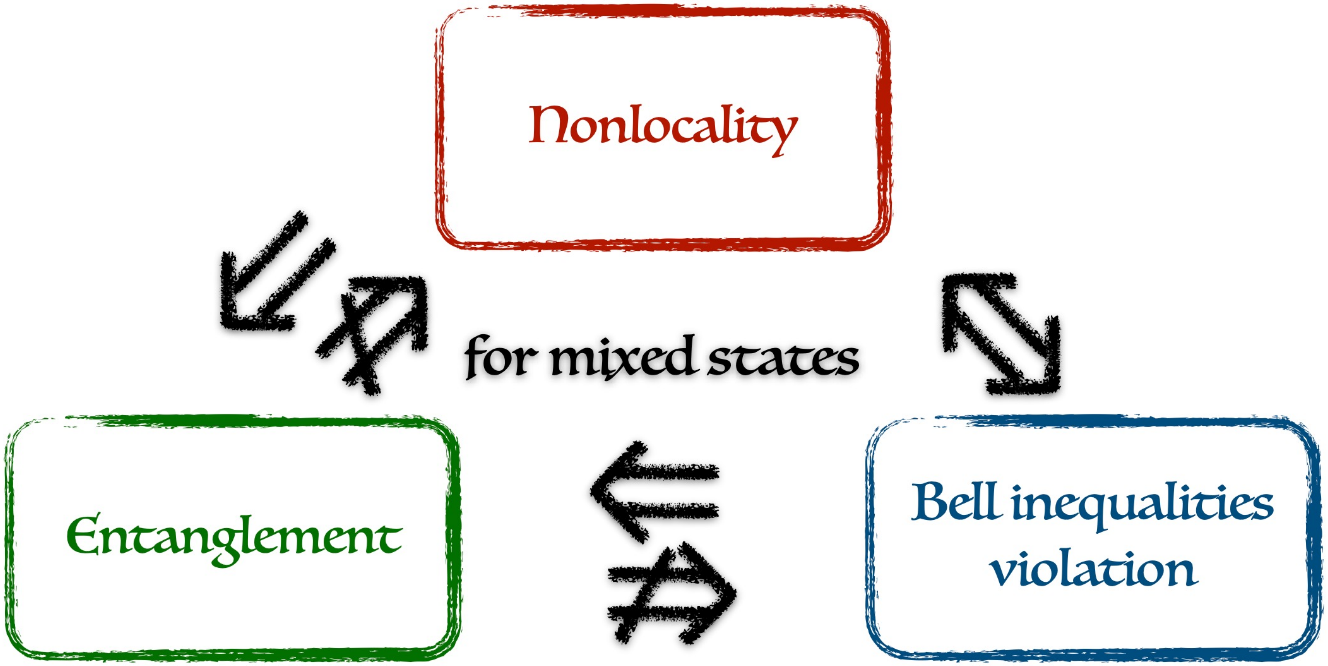
Y. Afik, J. de Nova, EPJC 136 (2021) 907





Entangled : $\quad D < -1/3, \quad$ separable : $\quad D > -1/3$

Quantum entanglement and nonlocality



e.g. Werner states in quantum information

$$\rho_{\mathrm{Werner}}(\alpha) = \alpha|\Psi_0\rangle\langle\Psi_0| + \frac{1-\alpha}{4}\hat{I}_4$$

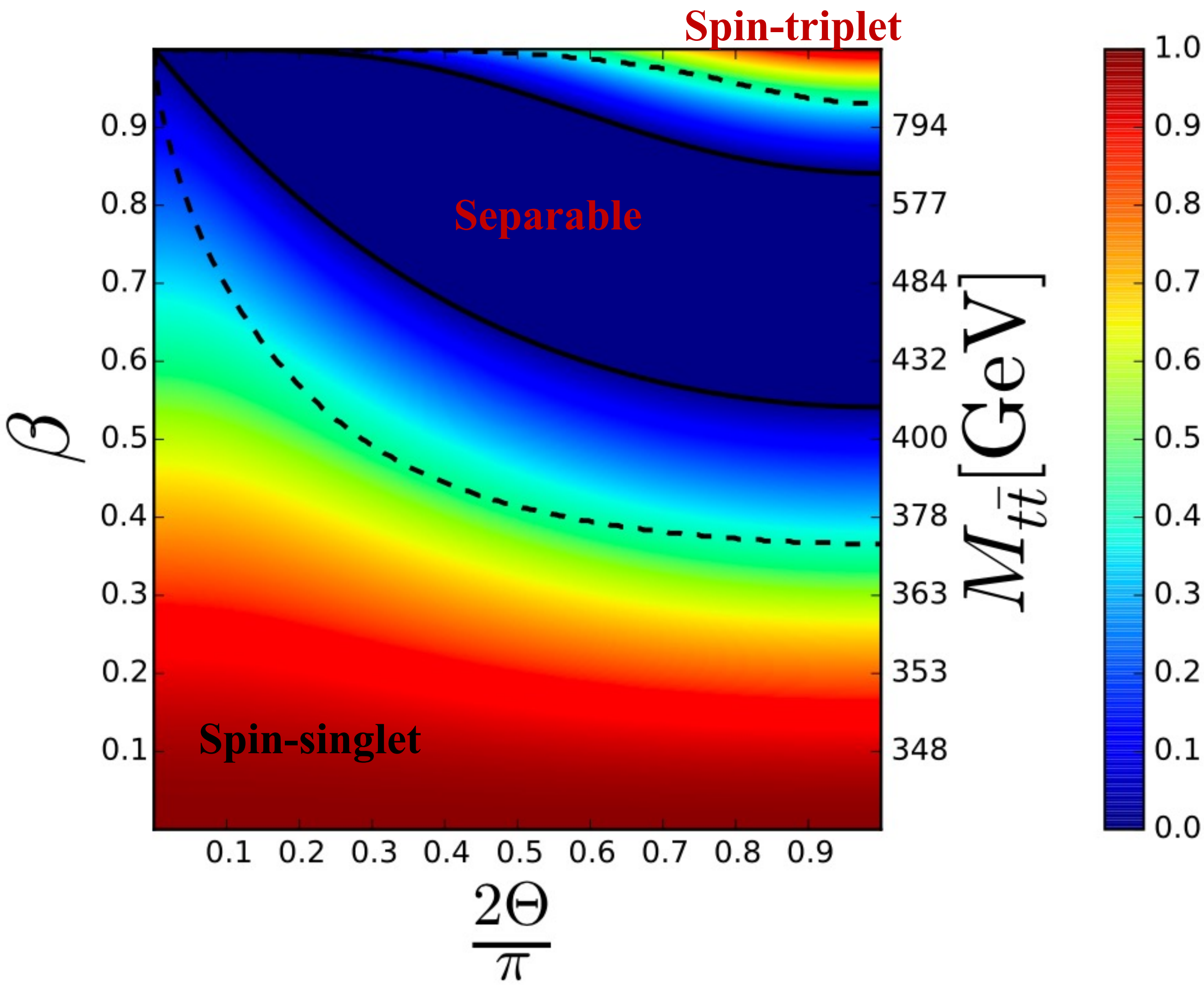
Bell singlet

Testing Bell inequalities violation is much difficult than entanglement

Quantum entanglement of top quark

Solid: Entanglement
Dashed: Belle inequality violation

Y. Afik, J. de Nova, Quantum 6 (2022) 820



Quantum entanglement at colliders

- Top quark pair

Y. Afik, J. R. M. n. de Nova Eur. Phys. J. Plus 136, 907 (2021)
M. Fabbrichesi, R. Floreanini, G. Panizzo, PRL 127, 161801 (2021)
C. Severi. C. D. E. Boschi, F. Maltoni, and M. Sioli, EPJC 82, 285 (2022)
T. Han, M. Low, T. A. Wu, JHEP 07, 192 (2024)
T. Han, M. Low, N. McGinnis, and S. Su, 2412.21158
K. Cheng, T. Han and M. Low, 2410.08303,
…

- Tau lepton pair

M. M. Altakach et al, PRD 107, 093002 (2023)
K. Ehataht et al, PRD 109, 032005 (2024)
Y. Du, X.-G. He, C.-W. Liu and J.-P. Ma, 2409.15418
Y. Zhang et al, 2504.01496
T. Han, M. Low, Y. Su, 2501.04801
…

- Gauge boson pair

A. J. Barr et al, Quantum 7, 1070 (2023)
Q. Bi, Q.-H. Cao, K. Cheng, H. Zhang, PRD 109, 036022 (2024)
R. Ding et al, 2504.09832
…

- Flavor

K. Chen, Z. Xing, R. Zhu, 2407.19242
H. Feng. H. Tang, W. Guo Q. Qin, 2504.15798
K. Chen, T. Han, M. Low, T. Wu, 2507.12513
….

- Entanglement & NP

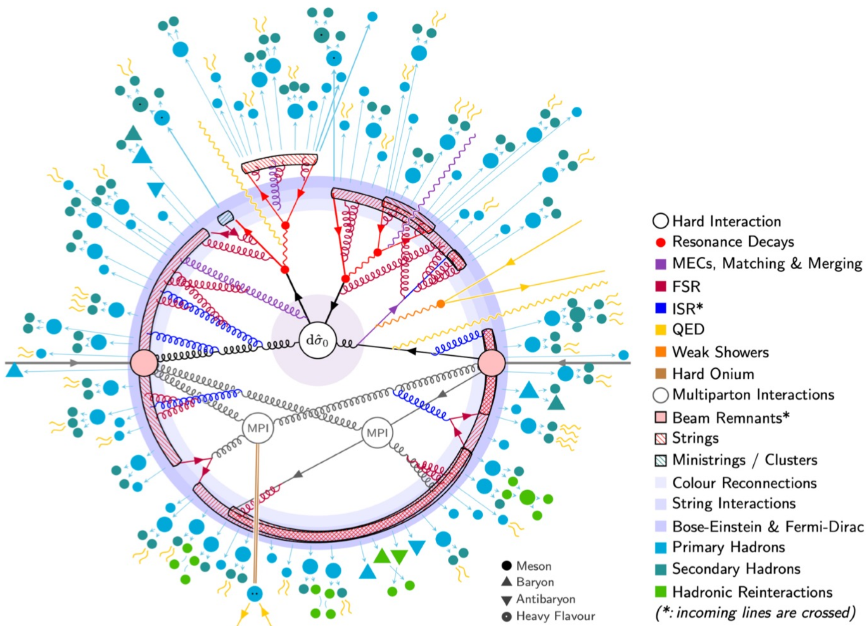
R. Aoude et al, PRD 106 (2022) 055007
M. Fabbrichesi et al, EPJC 83 (2023) 162, JHEP 09 (2023) 195
A. Bernal et al, EPJC 83 (2023) 11, 1050
…

The spin correlation between particles can be measured from its decay products

→ How about the light quarks?

Quantum entanglement of light quarks

❑ The quark can not be a free particle due to the QCD confinement:

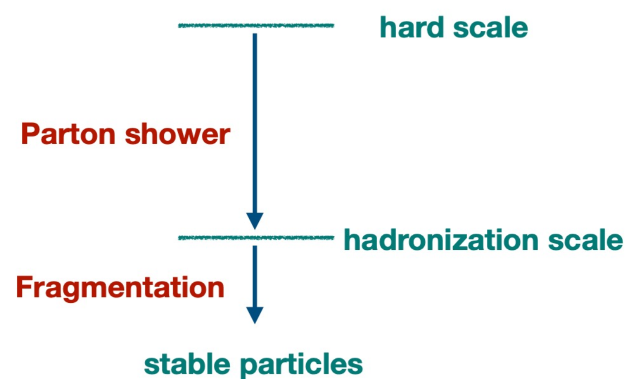


From PYTHIA 8.3

❑ Hard process in high energy

❑ Transition from high energy to low energy —parton shower

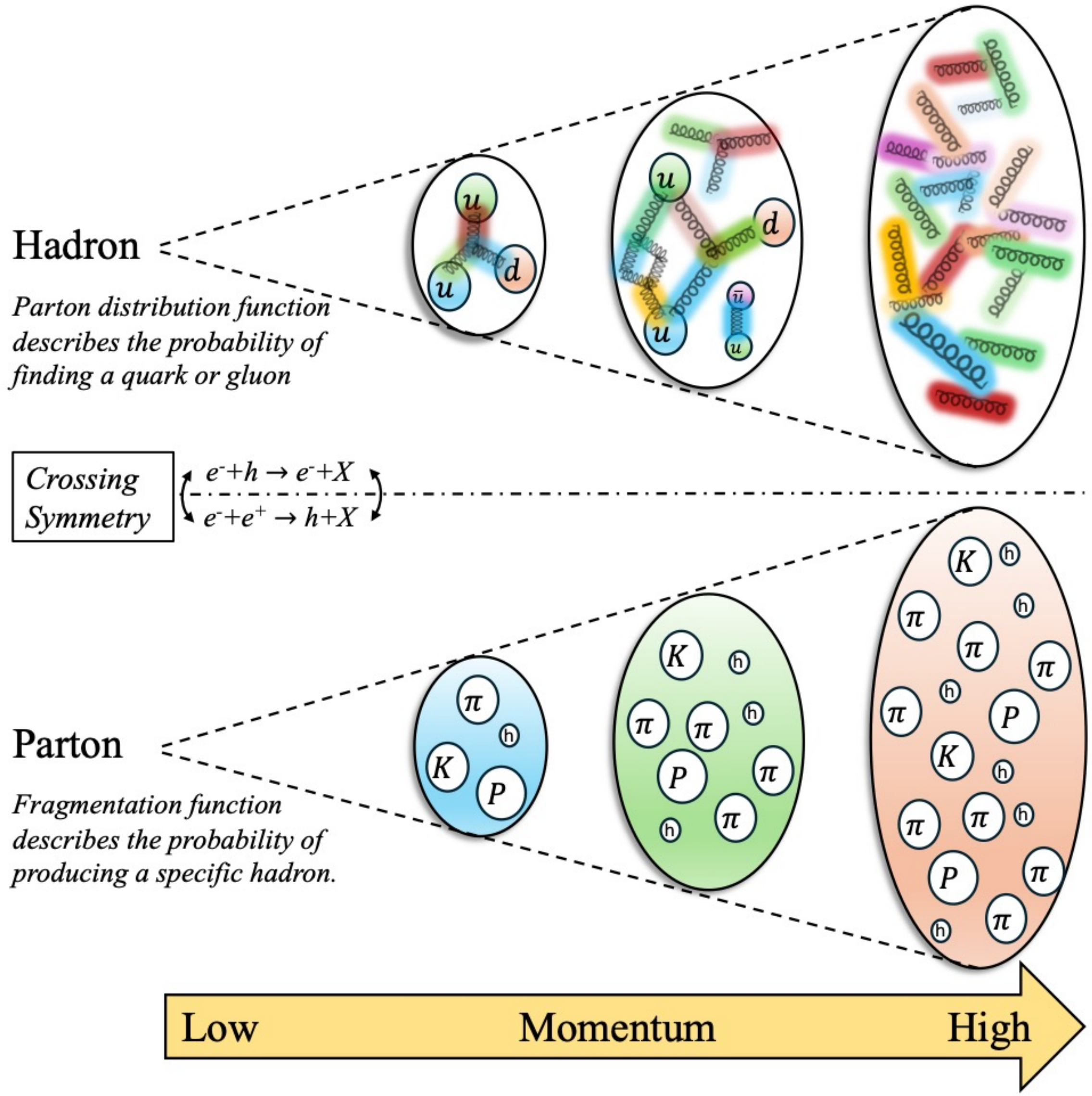
❑ Low energy soft regime —fragmentation



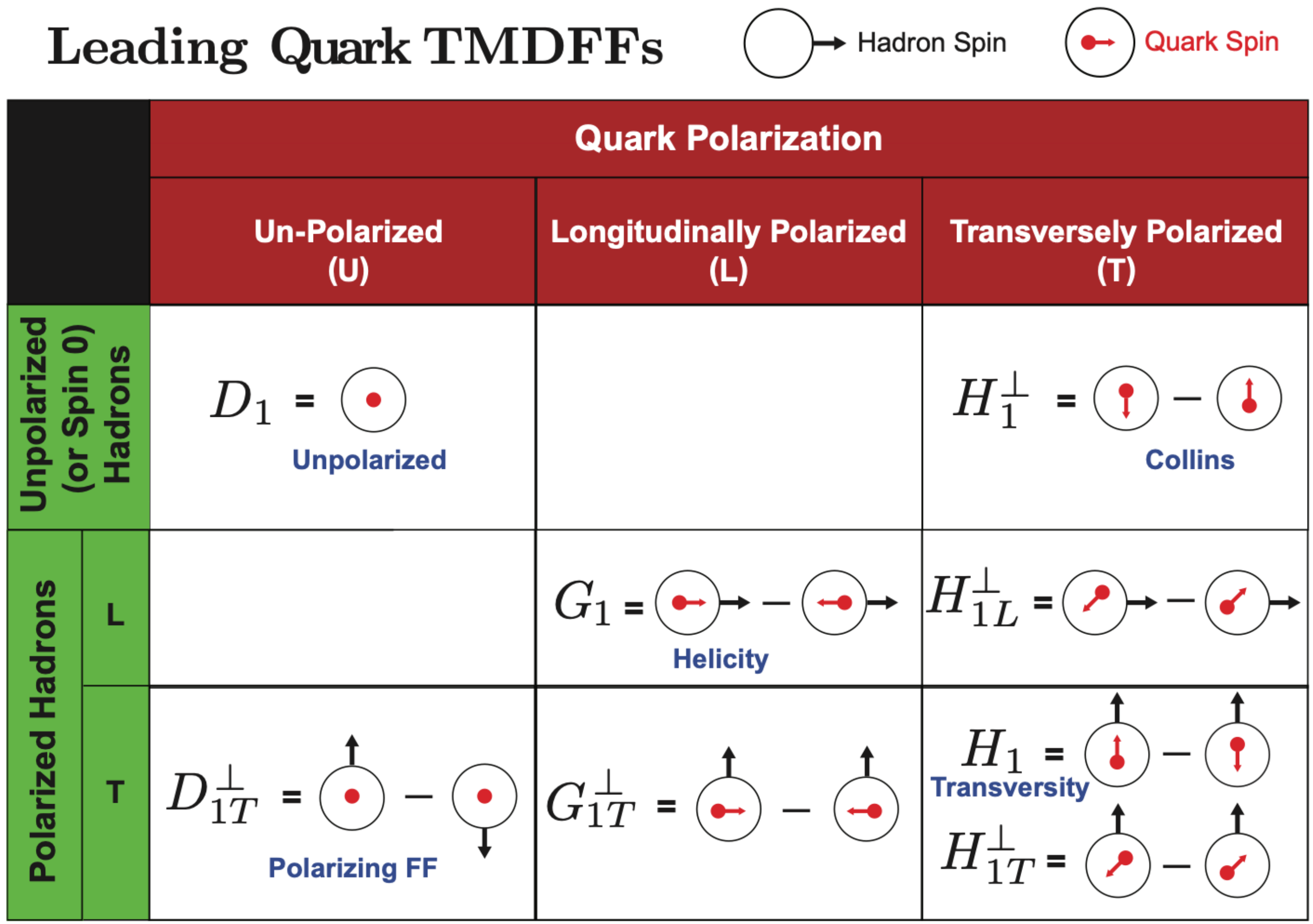
❑ The light quark does not decay but instead fragments into a jet of hadrons after produced from hard scattering

❑ How to probe the spin information of light quarks?
The non-perturbative functions: the fragmentation functions

PDF and FF

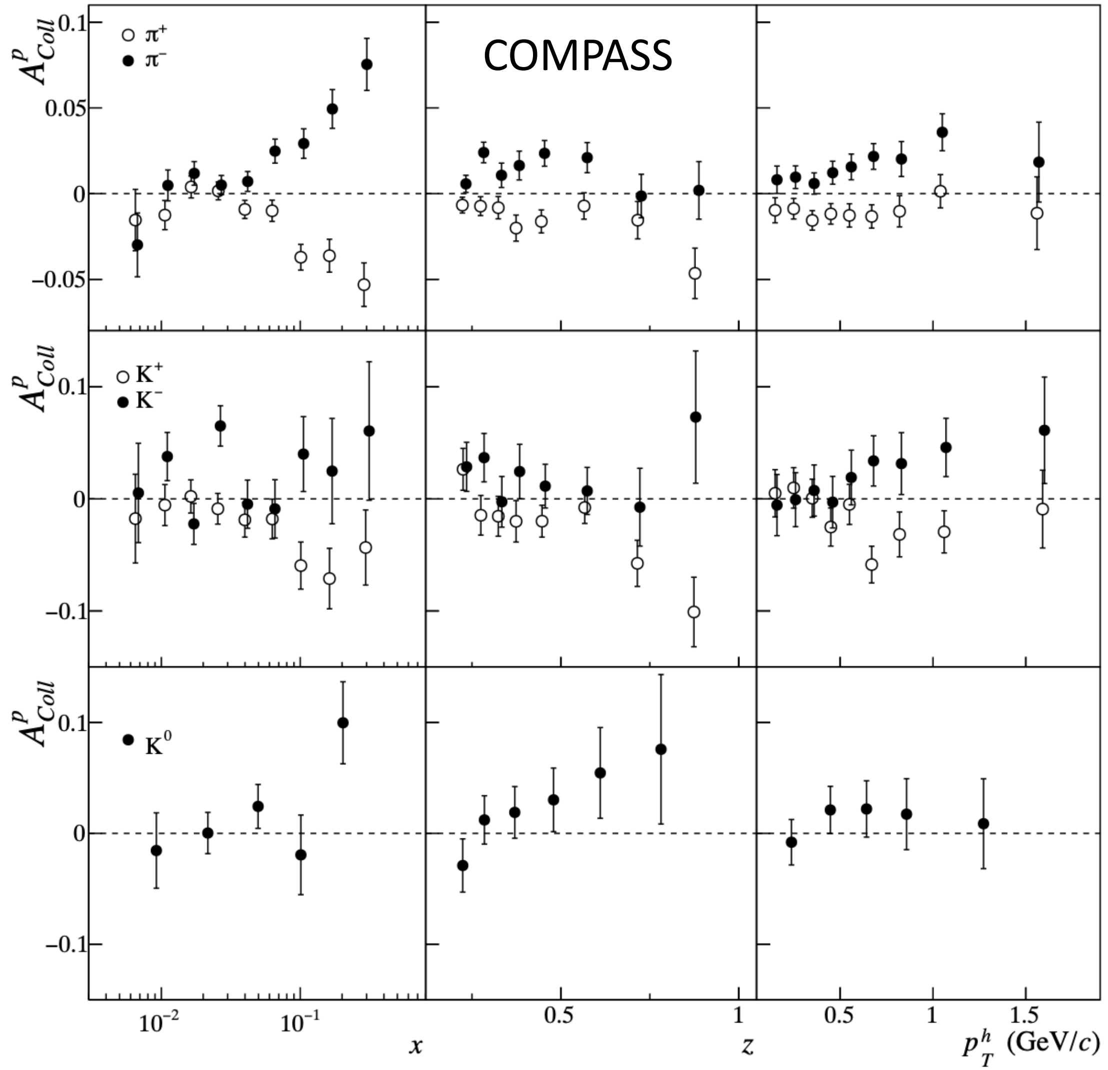

Hadron
Parton distribution function describes the probability of finding a quark or gluon
u
d
ū
Crossing Symmetry
e⁻+h → e⁻+X
e⁻+e⁺ → h+X
Parton
Fragmentation function describes the probability of producing a specific hadron.
π
K
P
h
Low
Momentum
High

Spin information of light quarks



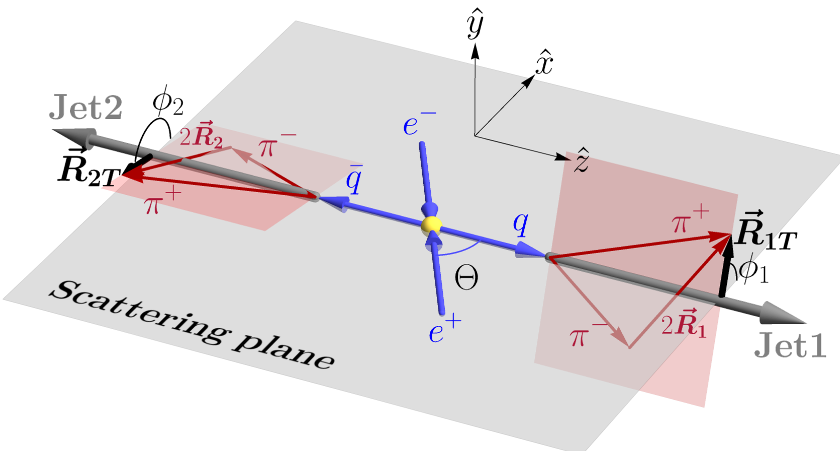
The spin information of light quarks can transfer to the hadrons: e.g. Collins functions

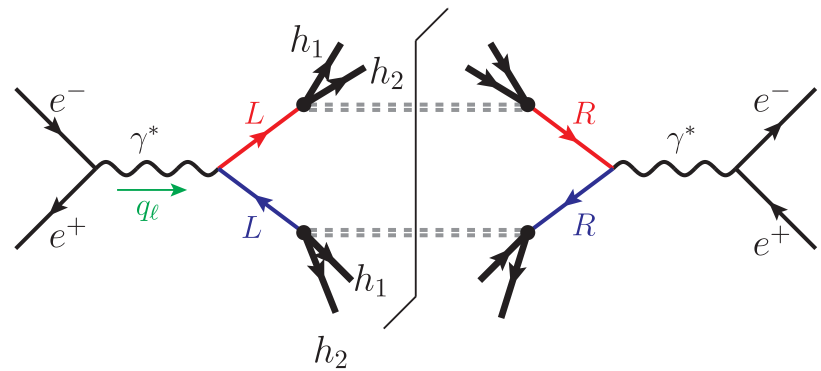
Example: Collins asymmetry



The spin transfer effects have been observed! (TMD)

Dihadron pair production at lepton colldiers





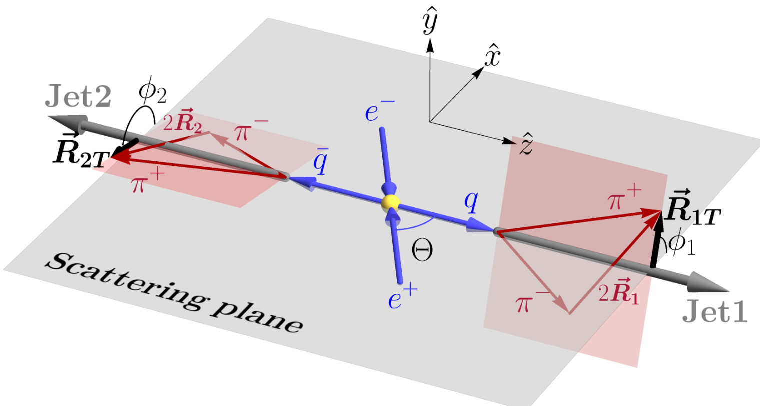
- The transverse spin correlation between light quarks: chiral-odd interference dihadron fragmentations (collinear factorization)

- Light quark pair are 100% correlated in the central scattering region

$$C_{ij} = \mathrm{diag}\left(\frac{\sin^2\Theta}{1+\cos^2\Theta}, -\frac{\sin^2\Theta}{1+\cos^2\Theta}, 1\right)$$

- The maximally entangled Bell state: Bell inequality violation effects

Bell inequality of light quarks



J. C. Collins et al, NPB 420, 565 (1994)

$$\frac{d\sigma}{d\Omega_1 d\Omega_2} = \sigma_{\rm hard} \sum_{ss'\bar{s}\bar{s}'} \bar{\rho}_{ss',\bar{s}\bar{s}'} \mathcal{D}^{ss'}_{\pi^+\pi^-/q} \mathcal{D}^{\bar{s}\bar{s}'}_{\pi^+\pi^-/\bar{q}}$$

$$\frac{1}{2}\mathrm{Tr}(\mathcal{D}_{\pi^+\pi^-/q}) = D_1^q(z_1, M_1),$$

$$\frac{1}{2}\mathrm{Tr}(\sigma_z \mathcal{D}_{\pi^+\pi^-/q}) = 0,$$

$$\frac{1}{2}\mathrm{Tr}(\sigma_i \mathcal{D}_{\pi^+\pi^-/q}) = -\frac{\varepsilon_T^{ij} R^j_{1,T}}{|\vec{R}_{1,T}|} H_1^{\sphericalangle,q}(z_1, M_1),$$

$$\frac{d\sigma}{dz_1 dz_2 dM_1 dM_2 d\phi_1 d\phi_2} = \sigma_{\rm hard}\left[\sum_q e_q^2 D_1^q(z_1,M_1) D_1^{\bar{q}}(z_2,M_2) \right.$$

Unpolarized diFF

$$\left. + \frac{1}{2}\sum_q e_q^2 H_1^{\sphericalangle,q}(z_1,M_1) H_1^{\sphericalangle,\bar{q}}(z_2,M_2)\Big(\mathcal{B}_- \cos(\phi_1+\phi_2) - \mathcal{B}_+ \cos(\phi_1-\phi_2)\Big)\right]$$

Transverse polarized diFF

$$\mathcal{B}_\pm \equiv C_{xx} \pm C_{yy}$$

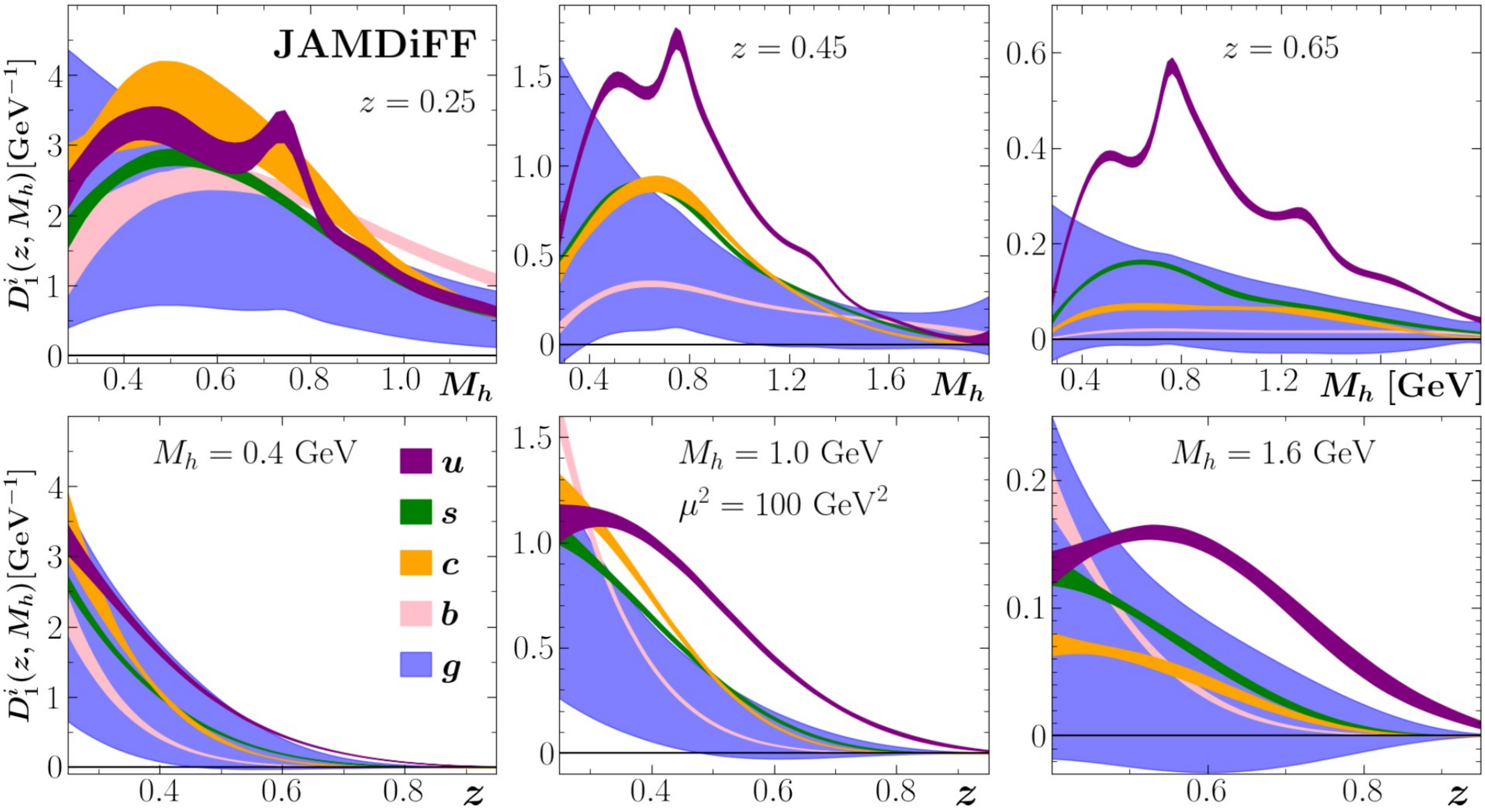
$$\mathcal{B}_- = \frac{2\langle\cos(\phi_1+\phi_2)\rangle}{\alpha^{z_1,z_2}_{M_1,M_2}} = \frac{A_{12}}{\alpha^{z_1,z_2}_{M_1,M_2}}$$

$$\mathcal{B}_+ = 0, \quad \mathcal{B}_- = \frac{2\sin^2\Theta}{1+\cos^2\Theta}.$$

$$\alpha^{z_1,z_2}_{M_1,M_2} = \frac{1}{2}\frac{\sum_q e_q^2 H_1^{\sphericalangle,q}(z_1,M_1) H_1^{\sphericalangle,\bar{q}}(z_2,M_2)}{\sum_q e_q^2 D_1^q(z_1,M_1) D_1^{\bar{q}}(z_2,M_2)}.$$

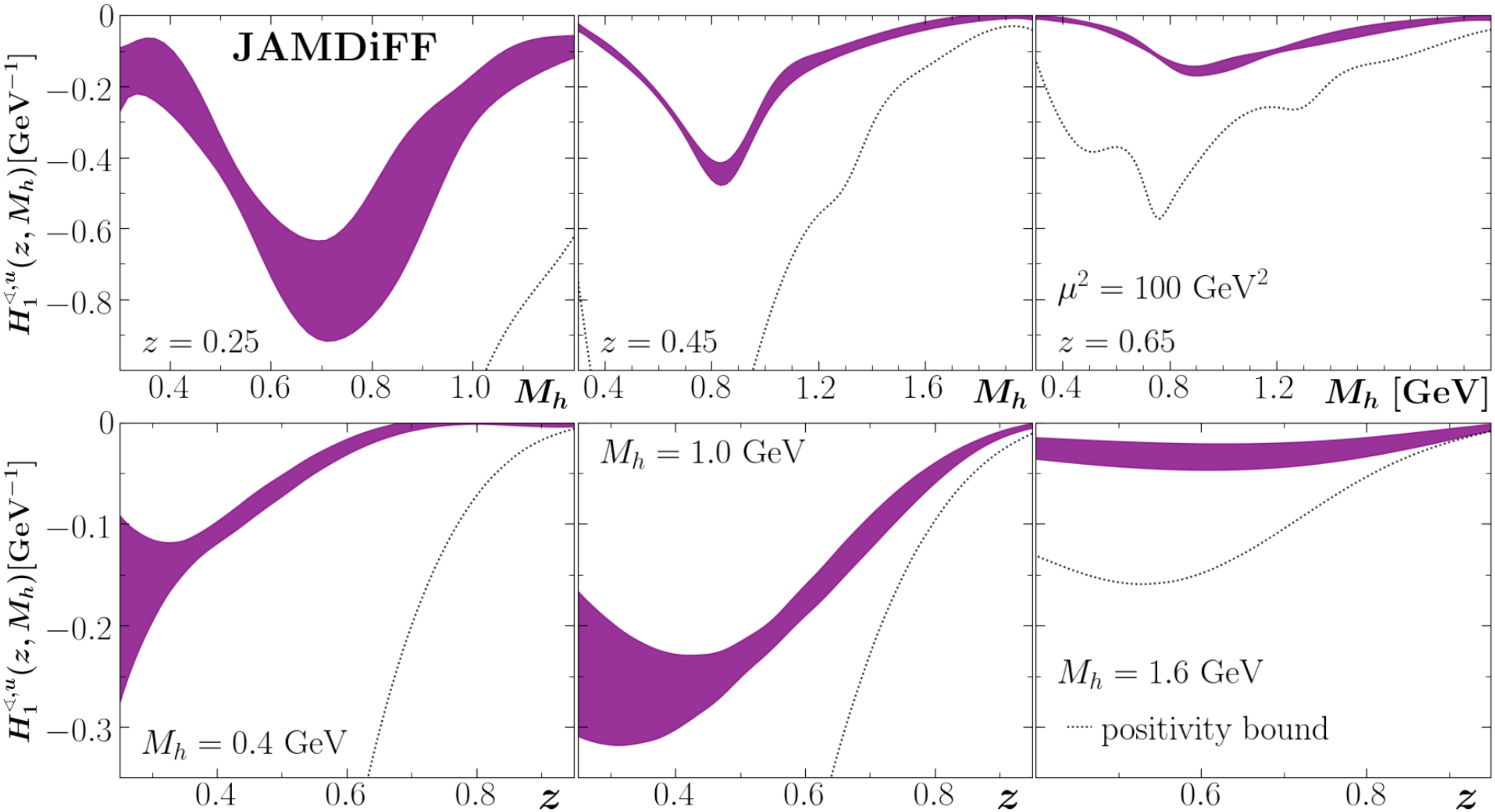
CHSH type Bell inequality $\quad |\mathcal{B}| > \sqrt{2}$

$\pi^+\pi^-$ Dihadron fragmentation functions



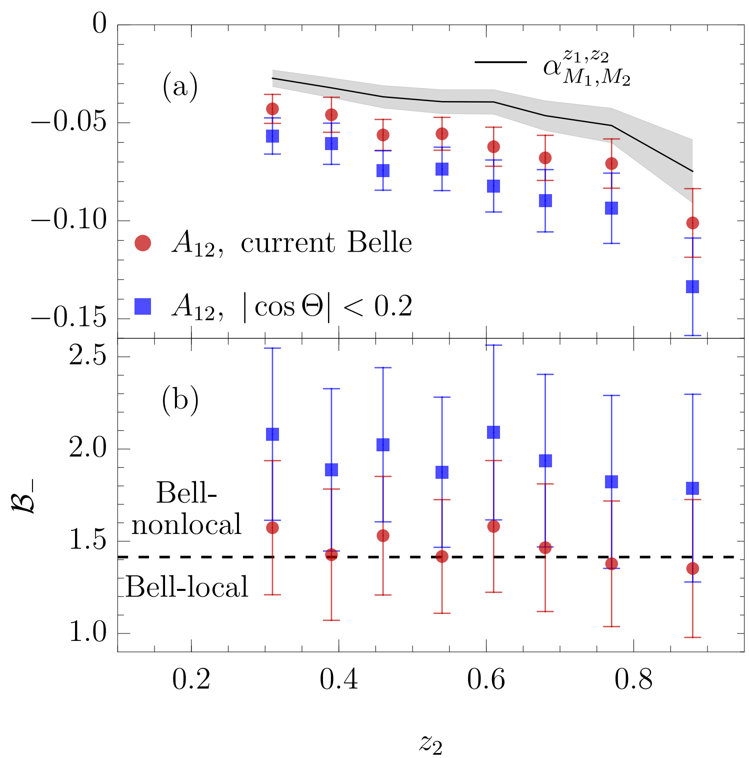
JAM Collaboration, PRL 132 (2024) 091901 , PRD 109 (2024) 034024

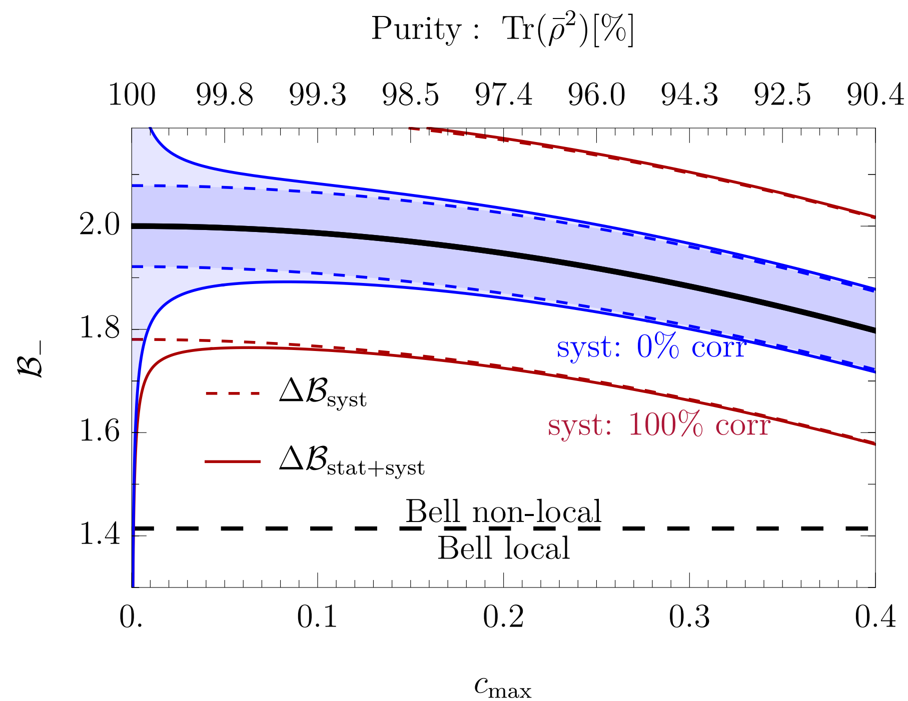
$\pi^+\pi^-$ Dihadron fragmentation functions



JAM Collaboration, PRL 132 (2024) 091901 , PRD 109 (2024) 034024

Dihadron pair production





Kun Cheng and Bin Yan, PRL 135 (2025) 011902

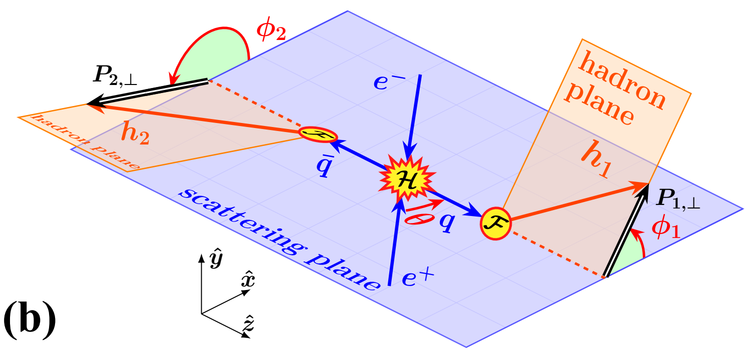
- Current data exhibiting no significant evidence of Bell inequality violation
- The optimal cuts on scattering angle will significantly improve the results
- The light quark pair would be a highly pure spin Bell state
- Combined results: 2.5 σ for 100% correlated systematic uncertainties and 6.7 σ for the uncorrelated case

$$\mathcal{B}_- = \frac{2\sin^2\Theta}{1+\cos^2\Theta}.$$

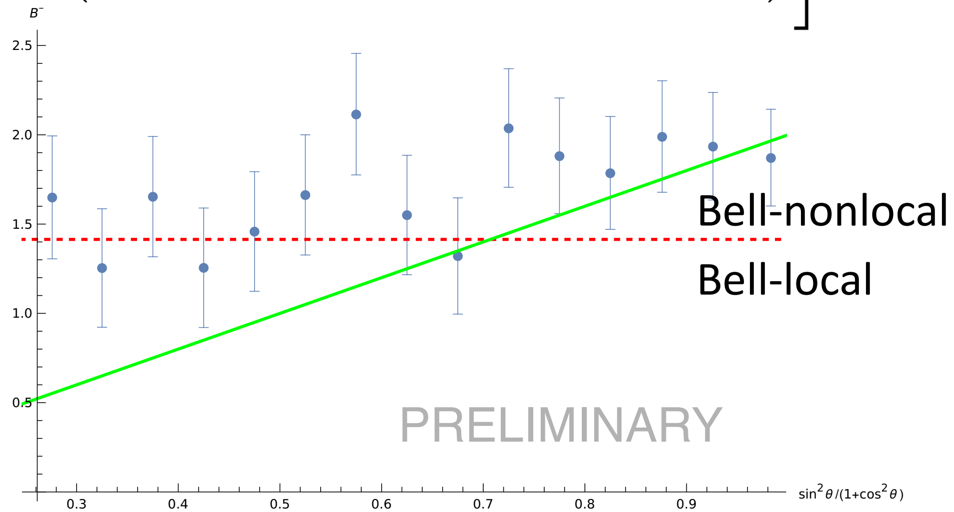
TMD hadron pair production

➢ The transverse spin correlation between light quarks can also be described by the Collins functions under the TMD framework

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma(e^+e^-\to h_1h_2X)}{p_{1,\perp}p_{2,\perp}\mathrm{d}z_1\mathrm{d}z_2\mathrm{d}p_{1,\perp}\mathrm{d}p_{2,\perp}\mathrm{d}\phi_1\mathrm{d}\phi_2\mathrm{d}\cos\theta}$$
$$=(1+\cos^2\theta)\Bigg[\sum_q Q_q^2 D_1^q(z_1,p_{1,\perp})D_1^{\bar q}(z_2,p_{2,\perp})$$
$$+\frac{1}{2}\sum_q Q_q^2\mathcal{F}_h H_1^{\perp,q}(z_1,p_{1,\perp})H_1^{\perp,\bar q}(z_2,p_{2,\perp})\Big(\mathcal{B}_-\cos(\phi_1+\phi_2)-\mathcal{B}_+\cos(\phi_1-\phi_2)\Big)\Bigg],$$



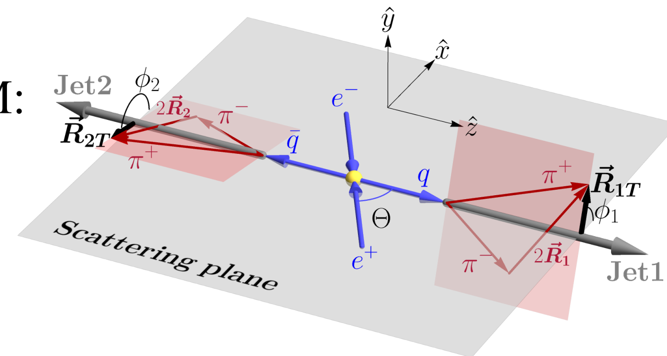
(b)



Kun Cheng, Tao Han, Guanghui Li and Bin Yan, 2510.xxxxx

New physics and entangled quarks

- The entangled triplet nature of the quark pair in SM:



$$\frac{d\sigma}{dz_1 dz_2 dM_1 dM_2 d\phi_1 d\phi_2} = \sigma_{\text{hard}} \left[\sum_q e_q^2 D_1^q(z_1, M_1) D_1^{\bar{q}}(z_2, M_2) + \frac{1}{2} \sum_q e_q^2 H_1^{\sphericalangle,q}(z_1, M_1) H_1^{\sphericalangle,\bar{q}}(z_2, M_2) \Big(\mathcal{B}_- \cos(\phi_1+\phi_2) - \mathcal{B}_+ \cos(\phi_1-\phi_2) \Big) \right]$$

$$\mathcal{B}_\pm \equiv C_{xx} \pm C_{yy}$$

$$\mathcal{B}_+ = 0, \quad \mathcal{B}_- = \frac{2\sin^2\Theta}{1+\cos^2\Theta}. \qquad C_{ij} = \text{diag}\left(\frac{\sin^2\Theta}{1+\cos^2\Theta}, -\frac{\sin^2\Theta}{1+\cos^2\Theta}, 1 \right)$$

- $\mathcal{B}_+ = 0$ is a result of the chiral symmetry of SM (massless quarks)

- New spin structure from chiral symmetry breaking interactions: dipole couplings

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New physics and entangled quarks

- Chiral symmetry breaking effects:

$$\mathcal{L}_{\text{eff}} \supset \frac{v}{\sqrt{2}\Lambda^2}\left(C_\gamma^u \bar{u}_L \sigma^{\mu\nu} u_R + C_\gamma^d \bar{d}_L \sigma^{\mu\nu} d_R\right) A_{\mu\nu} + \text{h.c.}$$

- Entangled states for quark pair $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_z\downarrow_z\rangle + e^{i\phi}|\downarrow_z\uparrow_z\rangle\right)$

- CP transformation interchanges the spins of quark and antiquark: $\phi = \begin{cases} 0 & \text{CP even} \\ \pi & \text{CP odd} \end{cases}$

$$C_{xx} + C_{yy} = \frac{\sin^2\theta}{1+\cos^2\theta}\frac{sv^2}{\pi\alpha\Lambda^4 Q_q^2}\left([\text{Re}C_\gamma^q]^2 - [\text{Im}C_\gamma^q]^2\right),$$

New spin structures

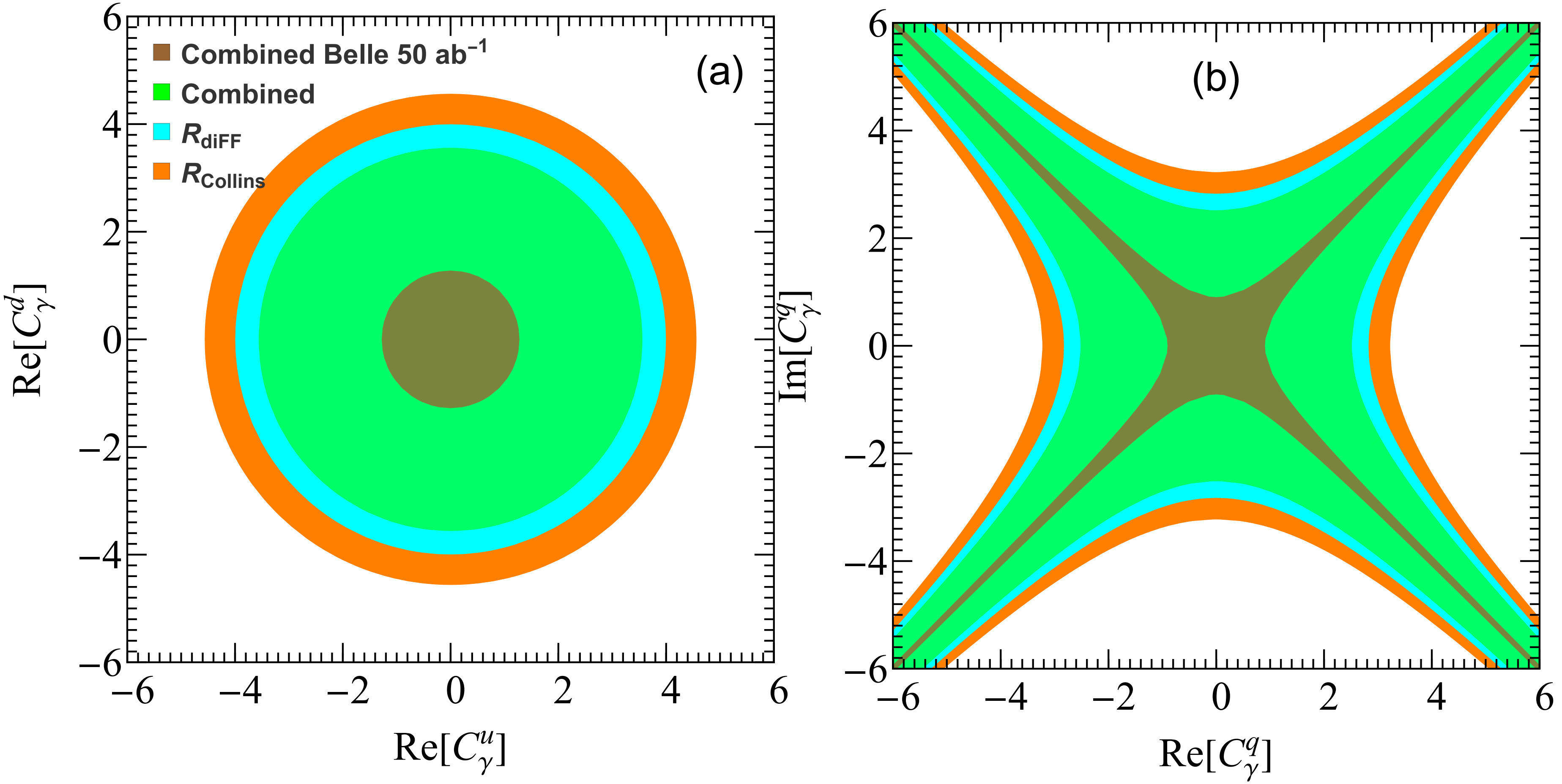
$$C_{xx} - C_{yy} = \frac{\sin^2\theta}{1+\cos^2\theta}\left(2 + \frac{1}{\Lambda^2}\mathcal{F}_1 + \frac{1}{\Lambda^4}\mathcal{F}_2\right),$$

$$C_{xy} = -C_{yx} = -\frac{\sin^2\theta}{1+\cos^2\theta}\frac{sv^2}{\pi\alpha\Lambda^4 Q_q^2}[\text{Re}C_\gamma^q][\text{Im}C_\gamma^q],$$

Qing-Hong Cao, Guanghui Li, Xin-Kai Wen and Bin Yan, 2509.18276

New physics and fragmentation hadrons

- The chiral symmetry breaking effects: $R = \frac{A_-}{A_+} = \frac{\langle \cos(\phi_1 - \phi_2) \rangle}{\langle \cos(\phi_1 + \phi_2) \rangle}$



The non-perturbaitve fragmenetaion functions are cancelled!

Summary

- The era of quantum information study in colliders has just began
- Prior work focused only on massive particles with perturbative decays, leaving massless particles unexplored
- We proposed studying entanglement and Bell inequalities in massless quark pair via the hadron final states by the fragmentation mechanism
- The azimuthal correlations in Belle's $\pi^+\pi^-$ dihadron pair could probe Bell inequality for massless quarks, with $> 5\sigma$ significance when considering uncorrelated systematic uncertainties
- The chiral symmetry breaking effects from dipole interactions can induce new spin structures and can be measured in fragmentation hadrons

Thank you!