Anatomy of Parity-Violating Trispectra in Galaxy Surveys

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Outline

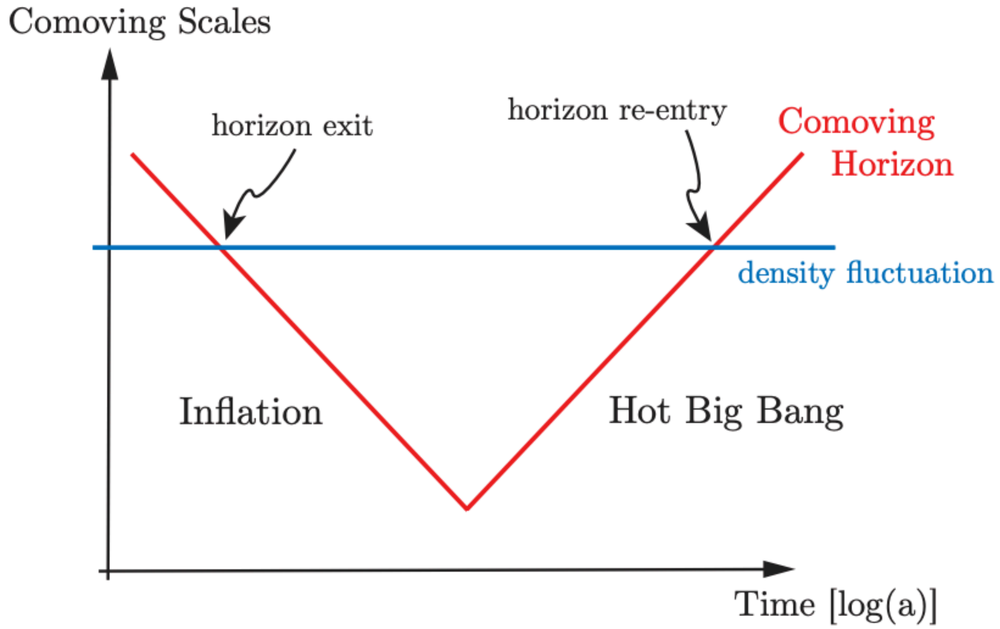
- Introduction
- Formalism
- Parity violating processes:
  - Toy models
  - Full models
- Results
- Summary

Inflation

- The leading paradigm to solve the flatness problem and the horizon problem.
- Quantum fluctuations got stretched and imprinted at superhorizon scales.

⇓

Explaining the origin of the inhomogeneity and anisotropy of our universe.

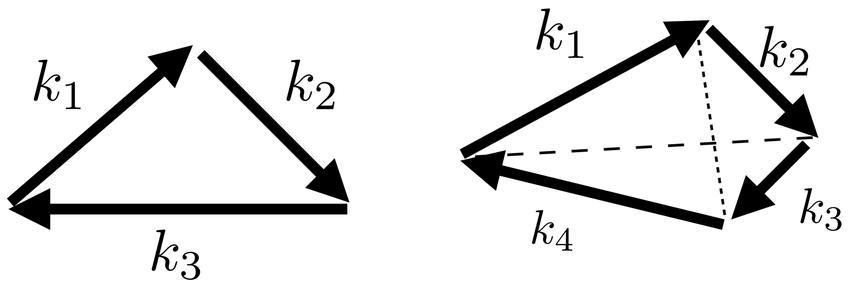


Baumann '12

Primordial non-Gaussianity (PNG)

- The simplest *single-field slow-roll* inflation predicts a **Gaussian** primordial spectra ⇒ nothing new in higher-pt correlators

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle' = 0$$



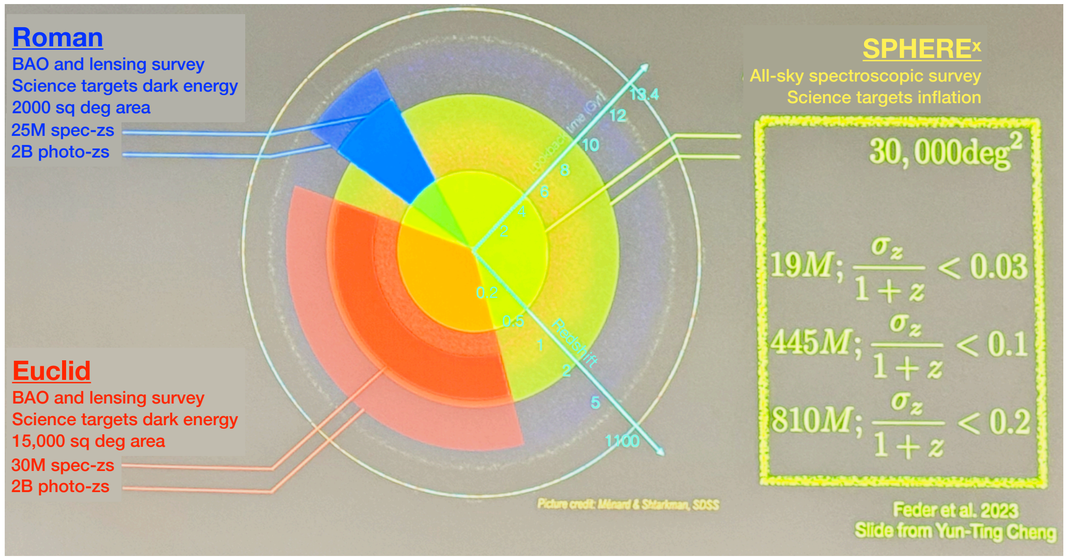
- Many inflation models predict non-trivial higher-pt correlators

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle' = ?$$

- break degeneracies of inflation models
- probe physics at very high energy scales

Searching for PNGs in galaxy surveys

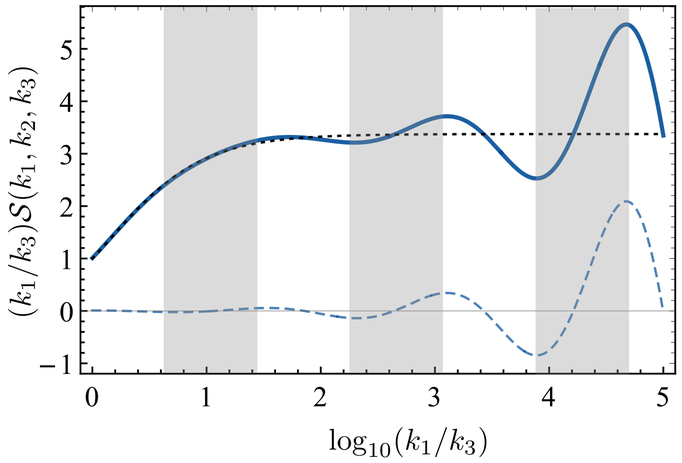
- PNGs can be searched in CMB / Large-Scale Structure surveys.
- 😄 There are abundant galaxy data from current and upcoming galaxy surveys.
- 😔 Observed NG = PNG + late-universe NG induced by gravity and baryonic physics.



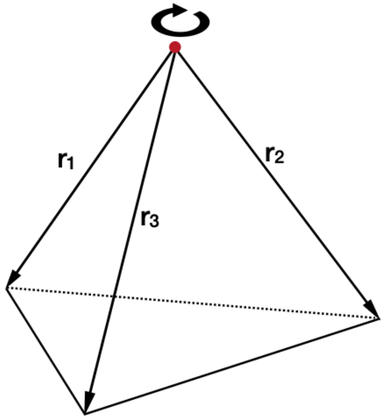
From Howard Hui's talk

What to do?

- A better understanding of late-time effects via dedicated modeling and simulations.
- Looking for targets that are robust against late-time effects.
  - **Cosmological collider signals.**
    (Gravity & EM unlikely to induce the log oscillations.)
  - **Parity violation signals.**
    (Gravity & EM should preserve parity.)

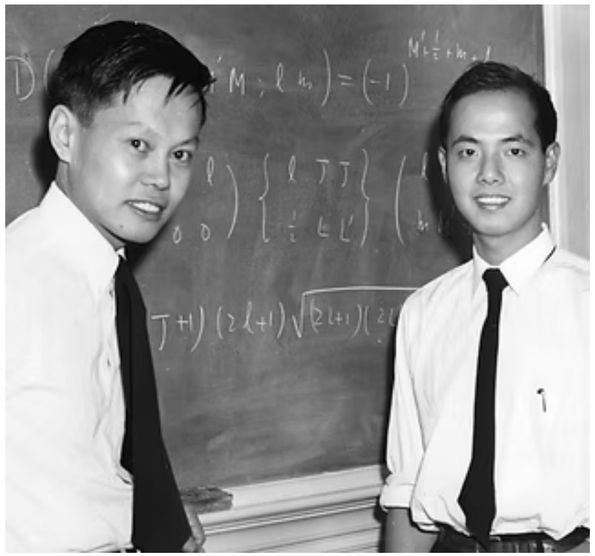


See Xianyu's talk



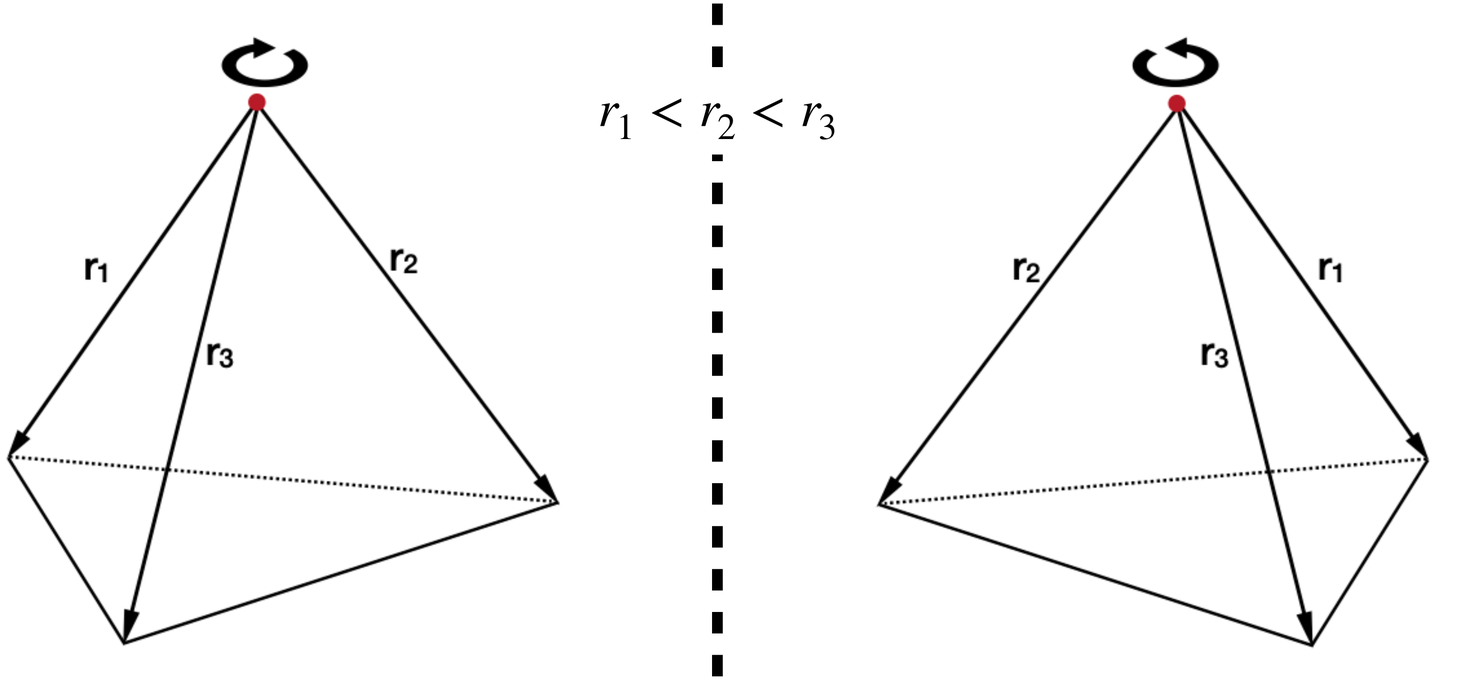
Parity-violation in large-scale structures

- Parity-violating processes are well-motivated and ubiquitous.
- They can happen during inflation and generate parity-violating PNGs.
- They will be preserved in the late time.



e.g., Anber & Sorbo ’10, Barnaby+ ’09, Liu+ ’20 , Cabass+ ‘23, Stefanyszyn+ ‘24A, ‘24B, Fujita+ ’24, Niu+ ’23, Creque-Sarbinowski+ ’23, Reinhard+ ’24,……

Parity-violations in four-point correlation function (4PCF)



Left-handed 4PCF

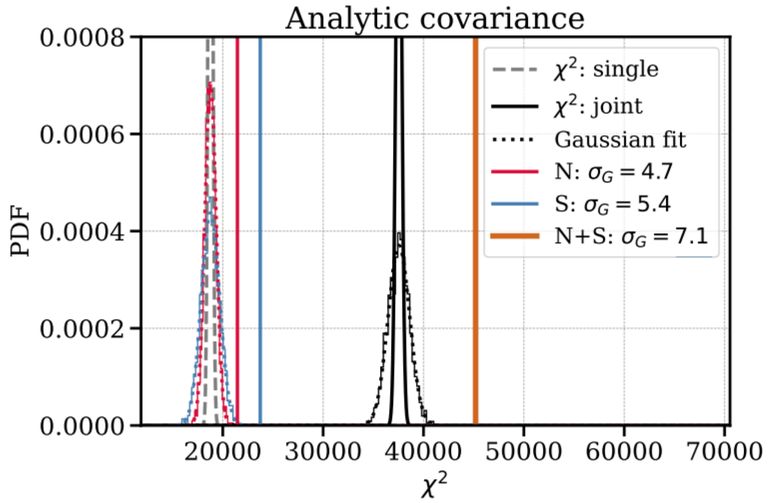
Right-handed 4PCF

Hou+ ‘22

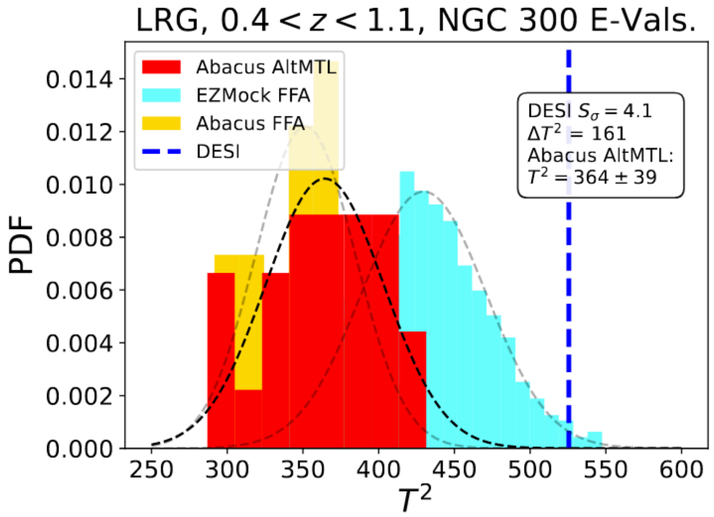
Parity violation = (left-handed 4PCF) — (right-handed 4PCF)

Current status

- Philcox (2022) and Hou, Slepian & Cahn (2022) respectively detected the parity-violating 4PCF at 2-3σ and 4-7σ in BOSS data.
- Slepian et al. (2025) detected the parity-violating 4PCF at 4-10σ in DESI Y1 data.
- Pushbacks over the actual significance. See Krolewski et al. (2024), Philcox & Ereza (2024).



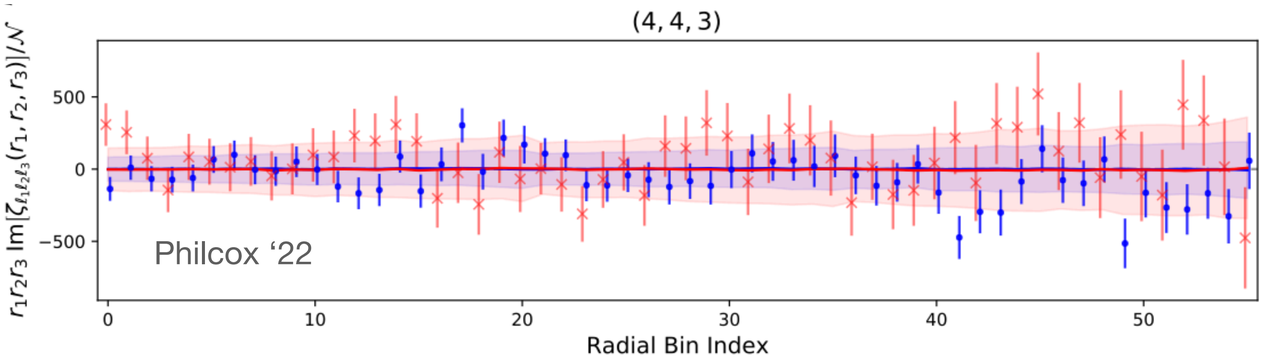
Hou+ ‘22

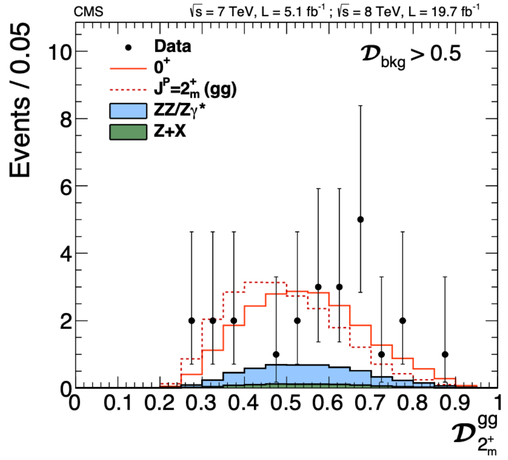


Slepian+ ‘25

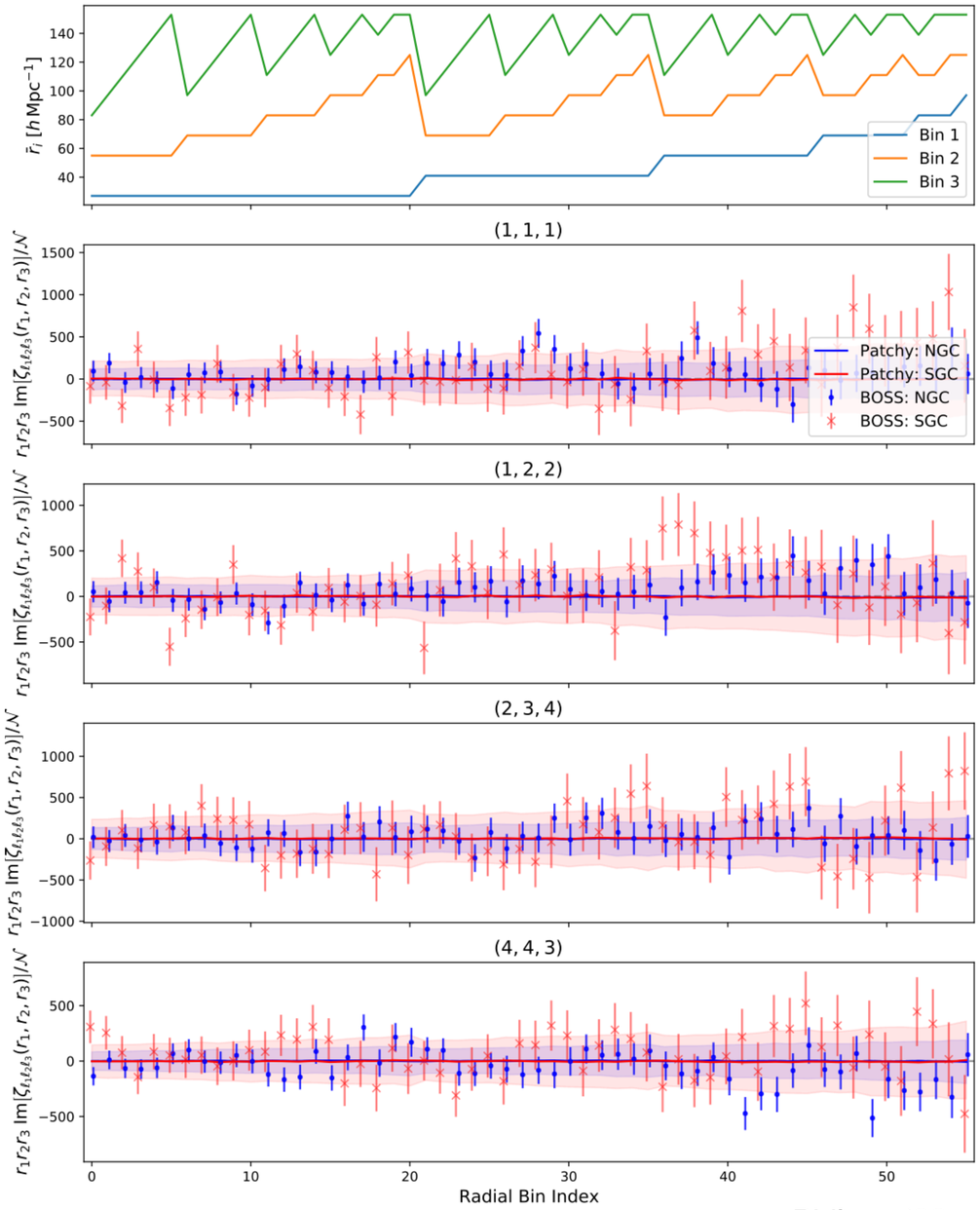
If it is true, what is the next?

- For cosmological colliders, we can learn mass/spin/interactions etc from the oscillation patterns.
- Can we learn something from the parity violating 4PCF?
- Philcox (2022) and Hou, Slepian & Cahn (2022) also give the signal distributions across **bins**. Those distributions can be used to differentiate various parity-violating models.

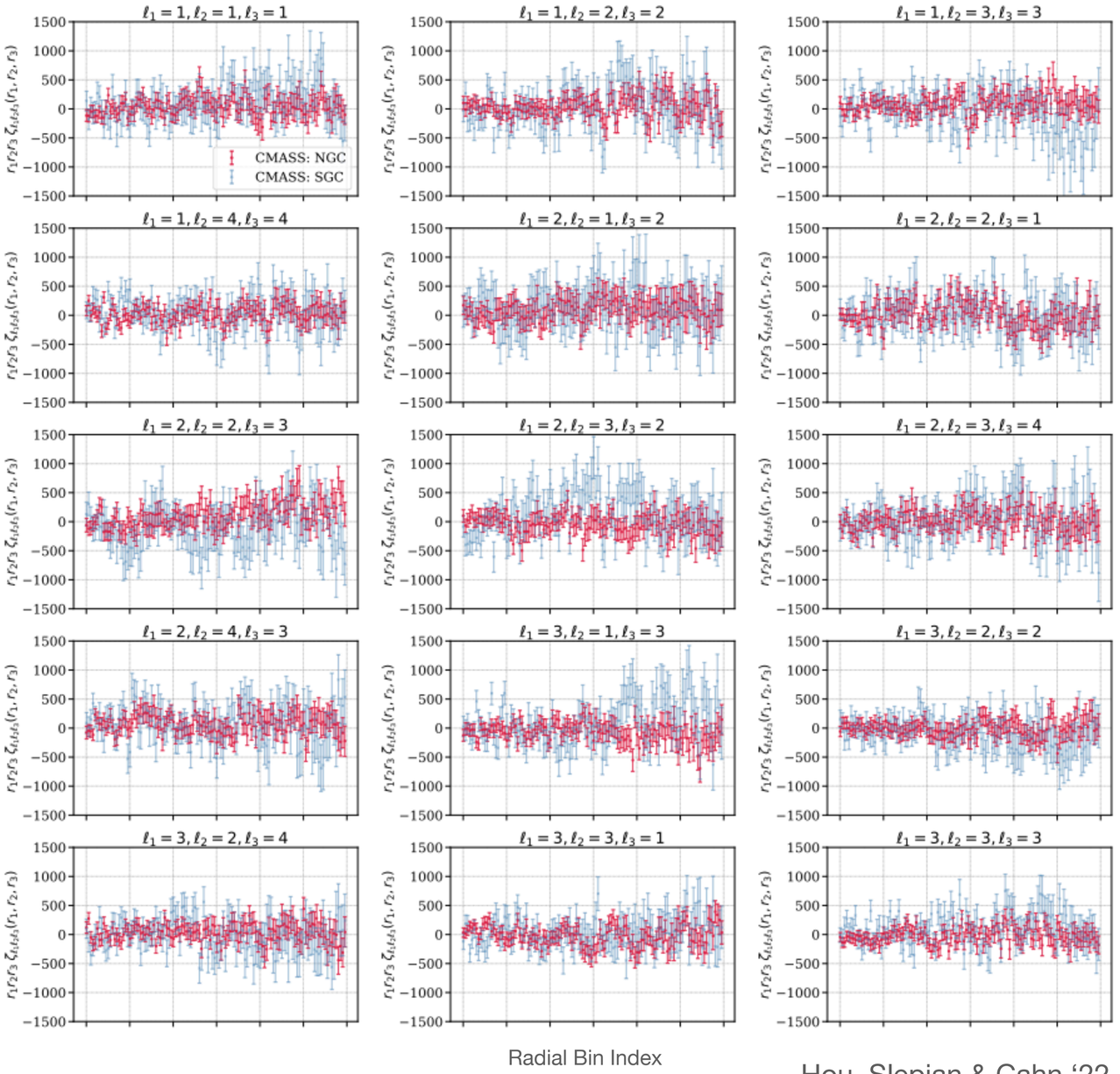




CMS ‘13

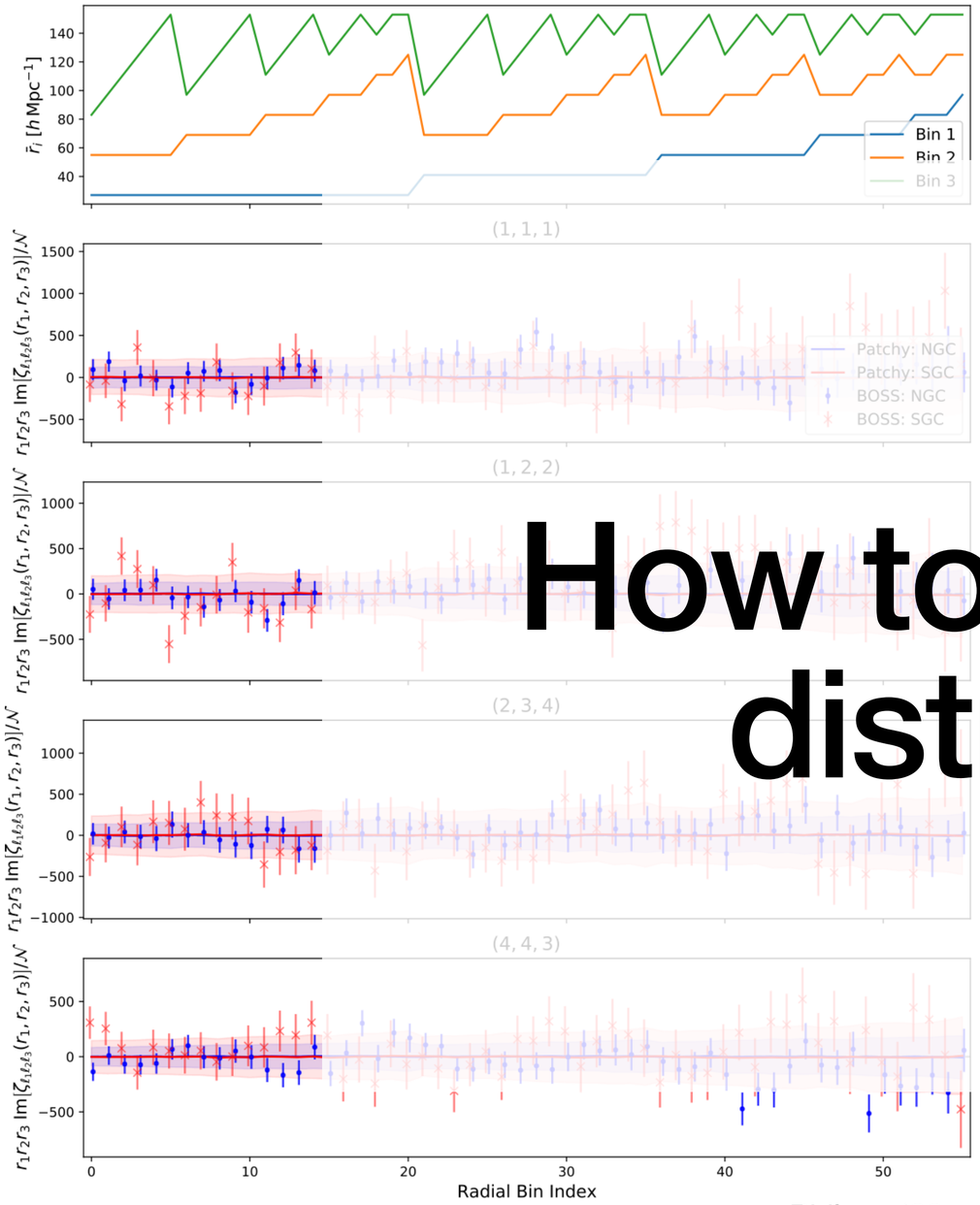


Philcox ‘22

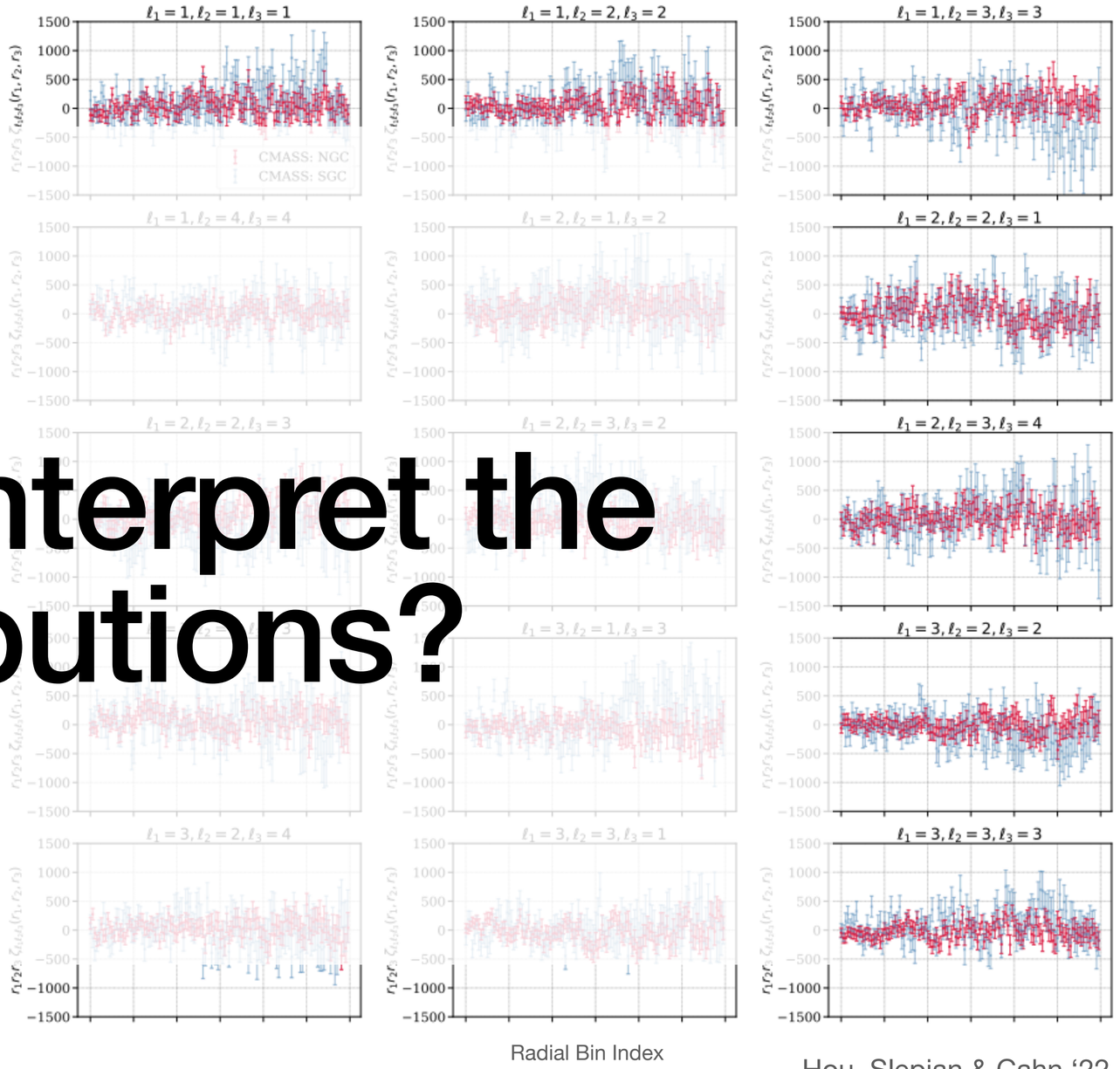


Hou, Slepian & Cahn ‘22

How to interpret the distributions?



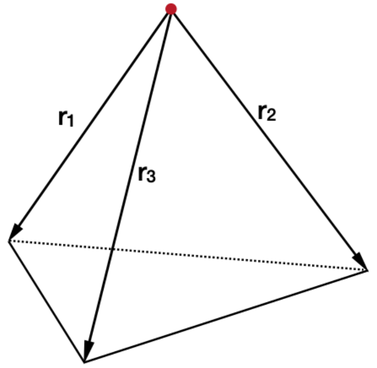
Philcox ‘22



Hou, Slepian & Cahn ‘22

Formalism

Galaxy overdensity 4PCF



$$\zeta_g^4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \langle \delta(\mathbf{r}_1)\delta(\mathbf{r}_2)\delta(\mathbf{r}_3)\delta(\mathbf{r}_4)\rangle$$

$$|||$$

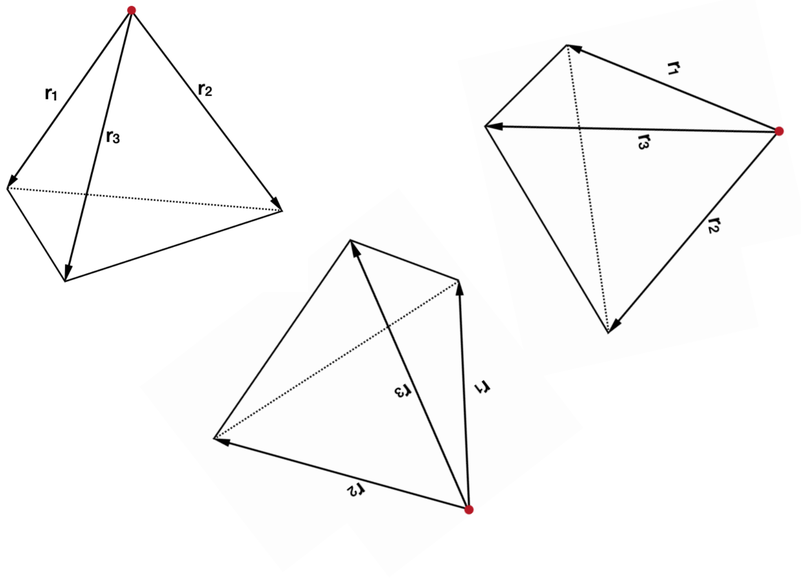
$$\rho(\mathbf{r})/\bar{\rho} - 1 = n(\mathbf{r})/\bar{n} - 1$$

Overdensity

- Homogeneity: remove one coordinate

$$\zeta_g^4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) \to \zeta_g^4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4 = 0)$$

Isotropy



- $\zeta_g^4(\hat{\mathbf{r}}_1, \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_3)$ is a function of the relative orientations.
- Think about Legend polynomials.

$$P_\ell(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2) = \frac{4\pi}{2\ell+1} \sum_{m=-\ell}^{\ell} Y_{\ell_1}^m(\hat{\mathbf{r}}_1) Y_\ell^{*m}(\hat{\mathbf{r}}_2)$$

- The co-rotation invariant function are good basis functions and can be factorized into two spherical harmonics.

Isotropic basis functions

- How about three directions?

Cahn & Slepian ‘10

$$\mathcal{P}_{\ell_1,\ell_2,\ell_3}(\hat{\mathbf{r}}_1,\hat{\mathbf{r}}_2,\hat{\mathbf{r}}_3) \equiv \sum_{m_1,m_2,m_3} (-)^{\ell_1+\ell_2+\ell_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} Y_{\ell_1}^{m_1}(\hat{\mathbf{r}}_1) Y_{\ell_2}^{m_2}(\hat{\mathbf{r}}_2) Y_{\ell_3}^{m_3}(\hat{\mathbf{r}}_3)$$

3j symbol

The isotropic basis functions can be factorized into three spherical harmonics.

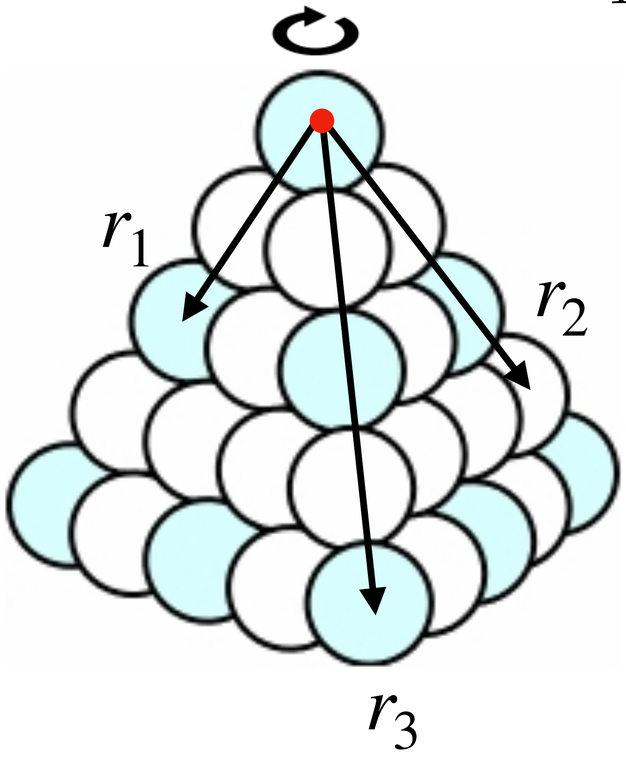
- Examples:

$$\mathcal{P}_{1,1,1}(\hat{\mathbf{r}}_1,\hat{\mathbf{r}}_2,\hat{\mathbf{r}}_3) \sim \mathrm{i}\,\hat{\mathbf{r}}_1\cdot(\hat{\mathbf{r}}_2\times\hat{\mathbf{r}}_3)$$

$$\mathcal{P}_{1,2,2}(\hat{\mathbf{r}}_1,\hat{\mathbf{r}}_2,\hat{\mathbf{r}}_3) \sim \mathrm{i}\,\hat{\mathbf{r}}_1\cdot(\hat{\mathbf{r}}_2\times\hat{\mathbf{r}}_3)(\hat{\mathbf{r}}_2\cdot\hat{\mathbf{r}}_3)$$

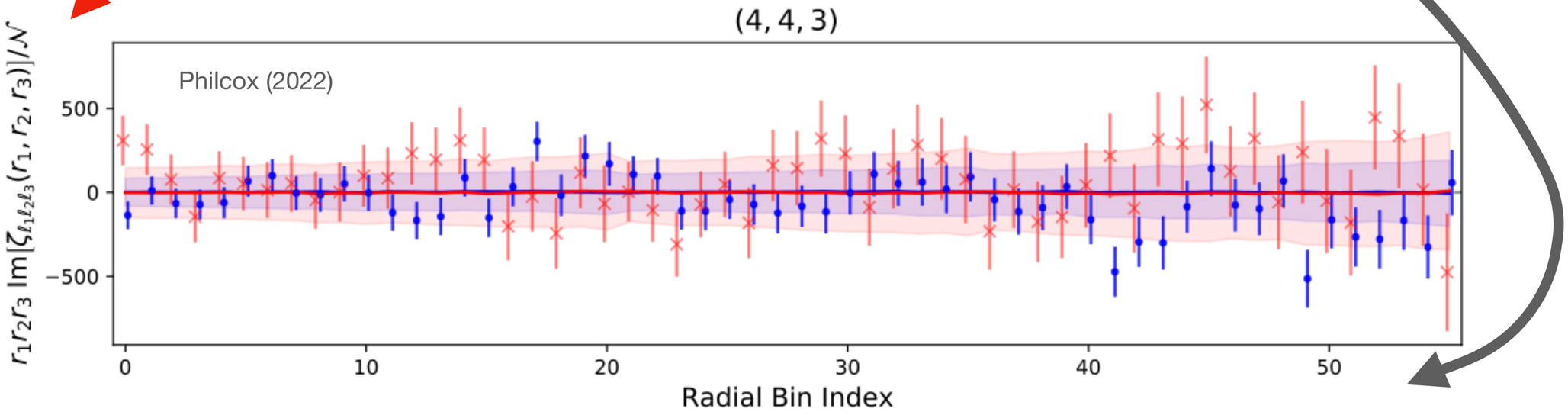
All the information is encoded in coefficients

$$\zeta_g^4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = \sum_{\ell_1,\ell_2,\ell_3} \zeta_{\ell_1,\ell_2,\ell_3}(r_1,r_2,r_3)\mathcal{P}_{\ell_1,\ell_2,\ell_3}(\hat{\mathbf{r}}_1,\hat{\mathbf{r}}_2,\hat{\mathbf{r}}_3)$$



- Six degrees of freedom, $r_1, r_2, r_3, \ell_1, \ell_2, \ell_3$.
- $\ell_1 + \ell_2 + \ell_3 =$ odd means parity violating.
- The radii are discretized into radial bins.

$$\zeta_g^4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \sum_{\ell_1,\ell_2,\ell_3} \zeta_{\ell_1,\ell_2,\ell_3}(r_1, r_2, r_3) P_{\ell_1,\ell_2,\ell_3}(\hat{\mathbf{r}}_1, \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_3)$$



Parity violating processes

From trispectrum to 4PCF

$$\zeta(\mathbf{k}) \xrightarrow{\text{Boltzmann}} \delta(\mathbf{k}) \xrightarrow[\textbf{Projection effect}]{\text{Galaxy bias}} \delta_g(\mathbf{k}) \xrightarrow{\text{Fourier trans.}} \delta_g(\mathbf{r})$$

$$\zeta_g^4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \int \left[\prod_{i=1}^{4} \text{đ}^3 k_i \; Z_1(\hat{\mathbf{k}}_i, z) M(k_i, z)\right] \left\langle \zeta^4 \right\rangle' (\mathbf{k}) (2\pi)^3 \delta^3 \left(\sum_{i=1}^{4} \mathbf{k}_i\right) \exp\left(\sum_{i=1}^{4} \mathrm{i}\mathbf{k}_i \cdot \mathbf{r}_i\right)$$

Kaiser redshift-space distortion | Transfer function | Trispectrum | Momentum conservation | Fourier kernel

A high-dim integral: N grids per dim $\Rightarrow N^{12}$ grids in total

From trispectrum to 4PCF

- ***Factorizability*** is the key in effectively performing the computation.
- A trispectrum needs to be factorized into radial-angular parts.

$$\mathbf{k}_s = \mathbf{k}_1 + \mathbf{k}_2$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle' = \mathcal{J}(k_1, \ldots, k_4, k_s) \mathcal{K}(\hat{\mathbf{k}}_1, \ldots, \hat{\mathbf{k}}_4, \hat{\mathbf{k}}_s) + \text{perms}$$

Better to be further separable.

Analytically integrated by using Wigner symbols

Toy models

- Local shape * Parity-odd angular dependence

$$\langle \zeta^4 \rangle'_{\text{toy,local}} \propto P_\zeta(k_1) P_\zeta(k_2) P_\zeta(k_s) \mathcal{P}_{1,1,1}(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_3, \hat{\mathbf{k}}_s) + \text{perms}$$

curvature power spectrum

- Equilateral shape * Parity-odd angular dependence

$$\langle \zeta^4 \rangle'_{\text{toy,eq}} \propto \frac{1}{k_1 k_2 k_3 k_4 k_t^5} \mathcal{P}_{1,1,1}(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_3, \hat{\mathbf{k}}_s) + \text{perms}$$

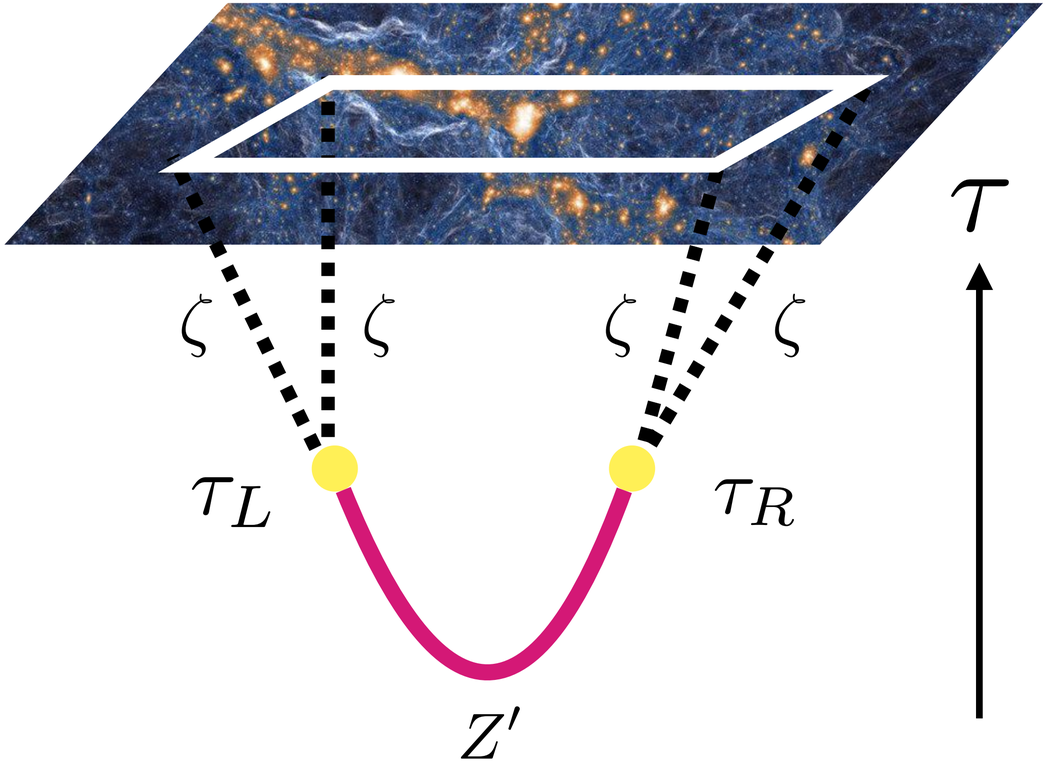
$k_t = \sum_{i=1}^{4} k_i$

Factorizable by Schwinger parametrization

Full model: massive spinning particle exchange

- Spin-1: Higgs model with gauge boson couples to a rolling background $\theta(t)$

$$\mathcal{L} = -\sqrt{-g}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\Sigma^* D^\mu\Sigma - m_\Sigma^2|\Sigma|^2 + \lambda|\Sigma|^4\right] - \frac{c_0\theta(t)}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

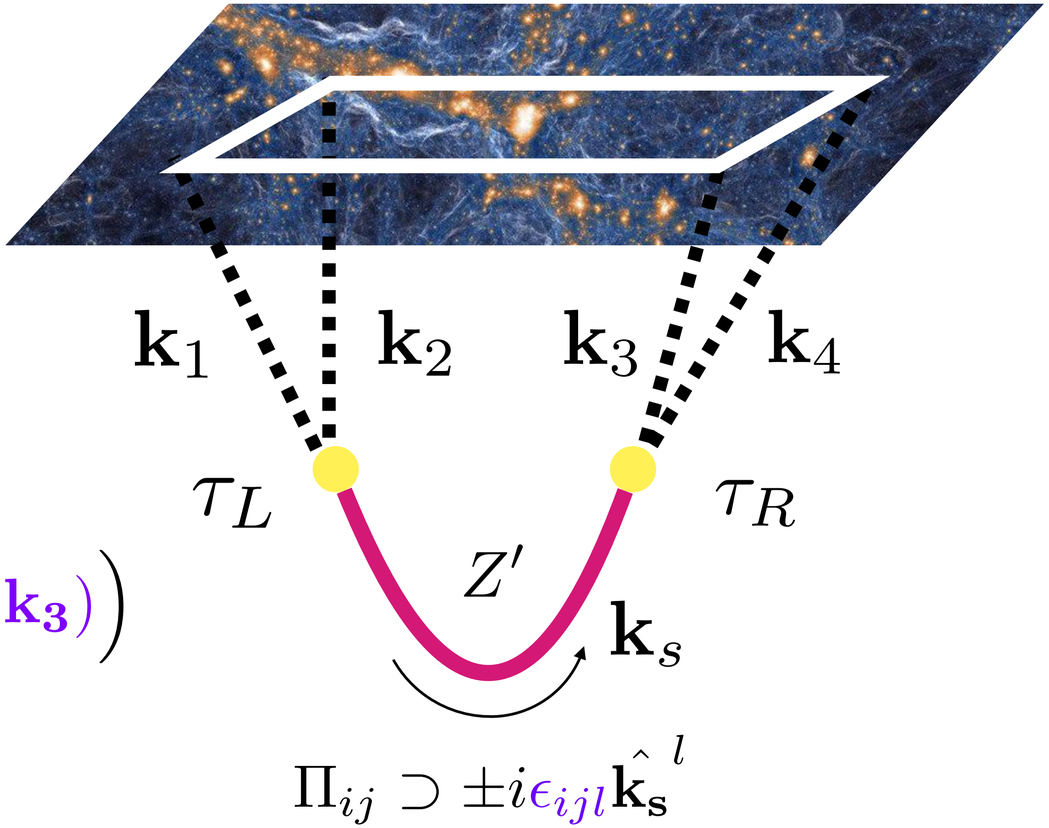


Liu, Tong, Wang, Xianyu ’20, Wang & Xianyu ‘20

Full model: massive spinning particle exchange

- Advantage 1: signal is parity-violating.

$$\Pi \propto (\mathbf{k}_1 \cdot \epsilon^{(-)}(\hat{\mathbf{k}}_s))(\mathbf{k}_3 \cdot \epsilon^{(-)}(\hat{\mathbf{k}}_s))$$
$$= \frac{1}{2}\left(\mathbf{k_1}\cdot\mathbf{k_3} - (\mathbf{k_1}\cdot\hat{\mathbf{k}}_s)(\mathbf{k_3}\cdot\hat{\mathbf{k}}_s) + \mathbf{i}\hat{\mathbf{k}}_s\cdot(\mathbf{k_1}\times\mathbf{k_3})\right)$$

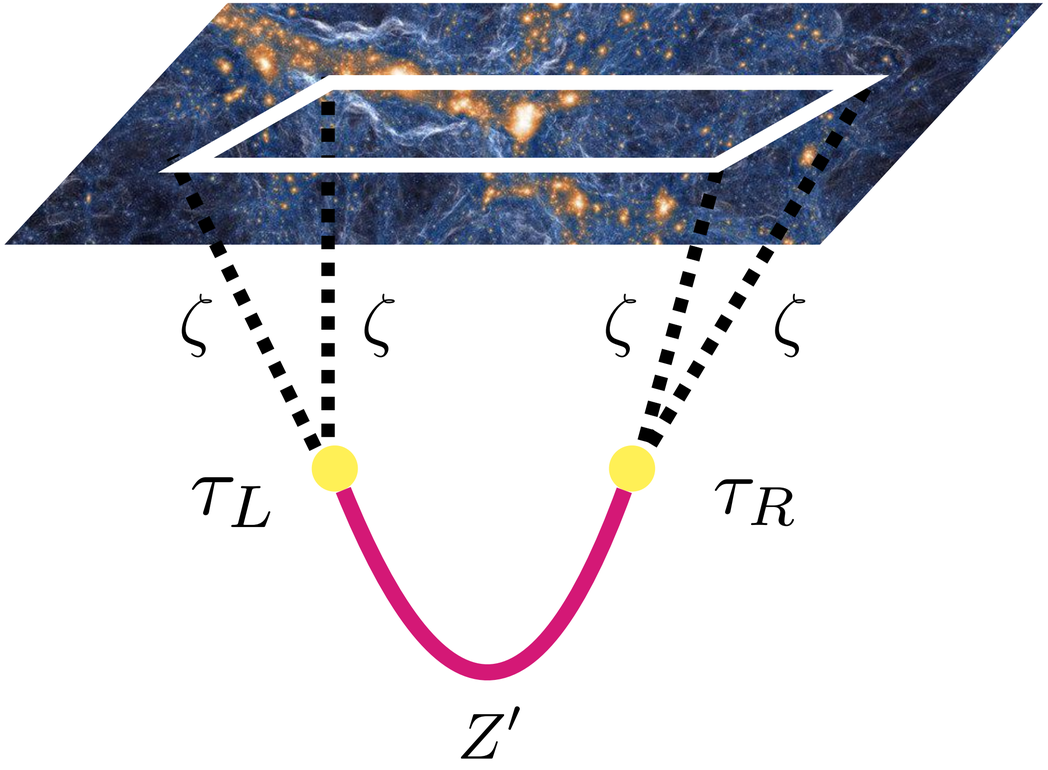


Full model: massive spinning particle exchange

- Advantage 2: signal could be sizable.

  $c_0\theta(t)\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$ introduce a chemical potential of $\mu \equiv c_0\dot{\theta}$.

  One of the transverse polarization will be enhanced and could overcome the Boltzmann suppression.



$$Z'_{(-)} : \langle\zeta^4\rangle' \sim e^{-\pi m/H} \Rightarrow e^{\pi(\mu-m)/H}$$

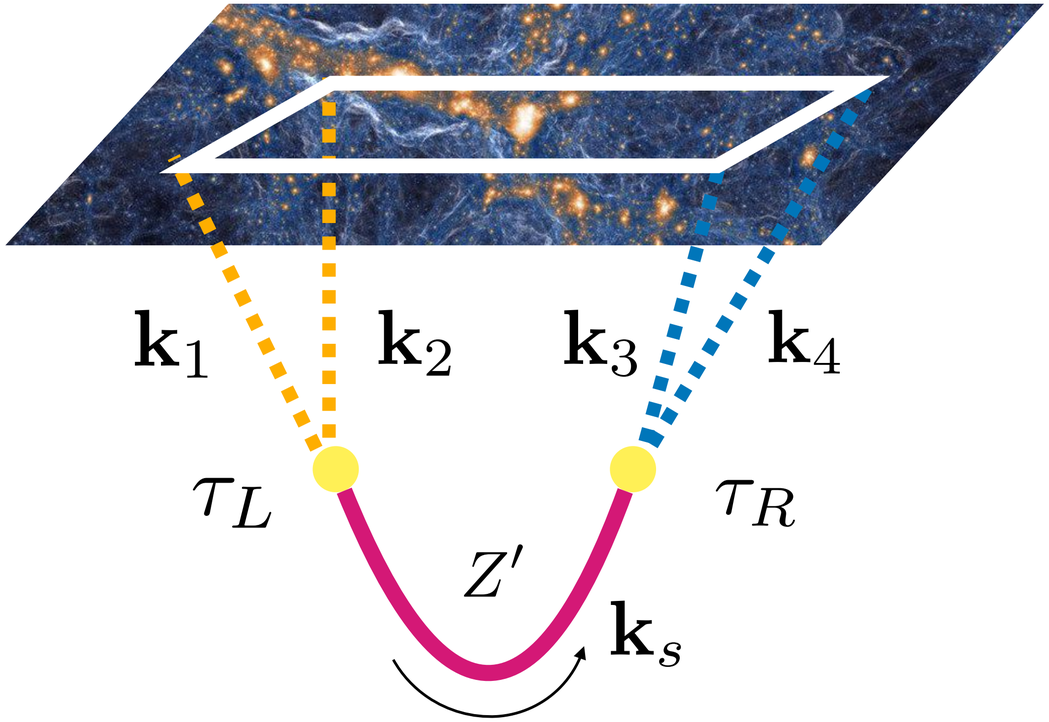
Full model: massive spinning particle exchange

- Advantage 3: computation is factorizable.

The process can be expressed in tree-level SK diagrams

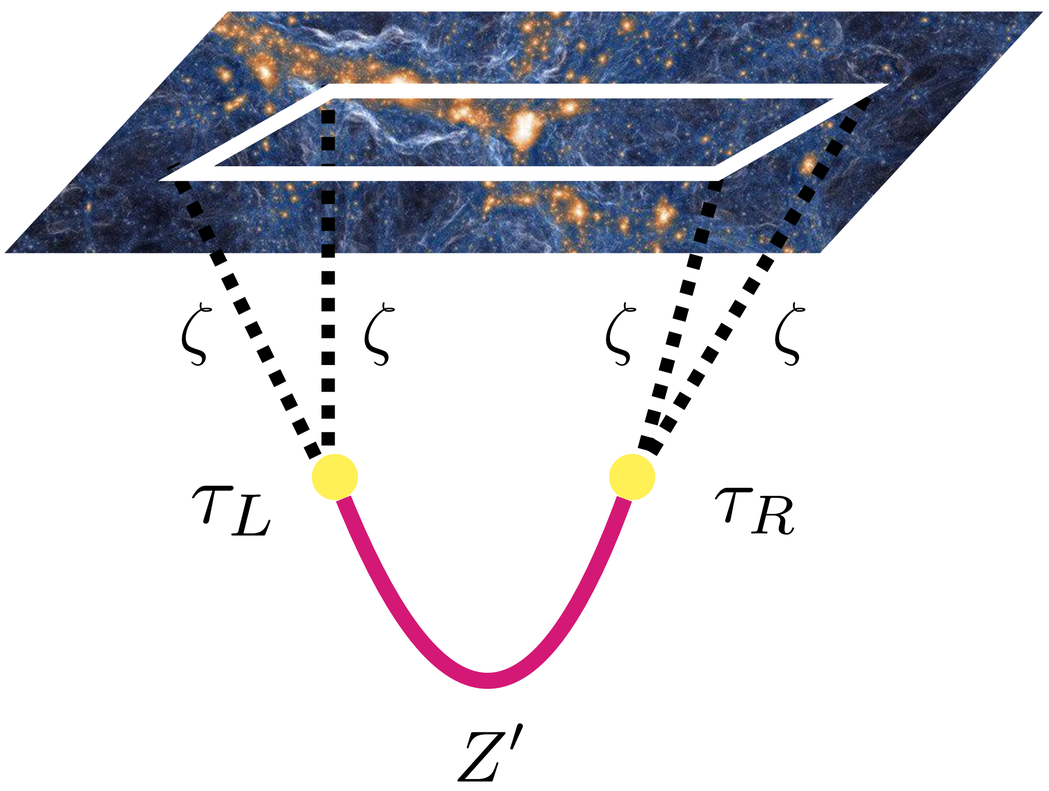
$\Downarrow$

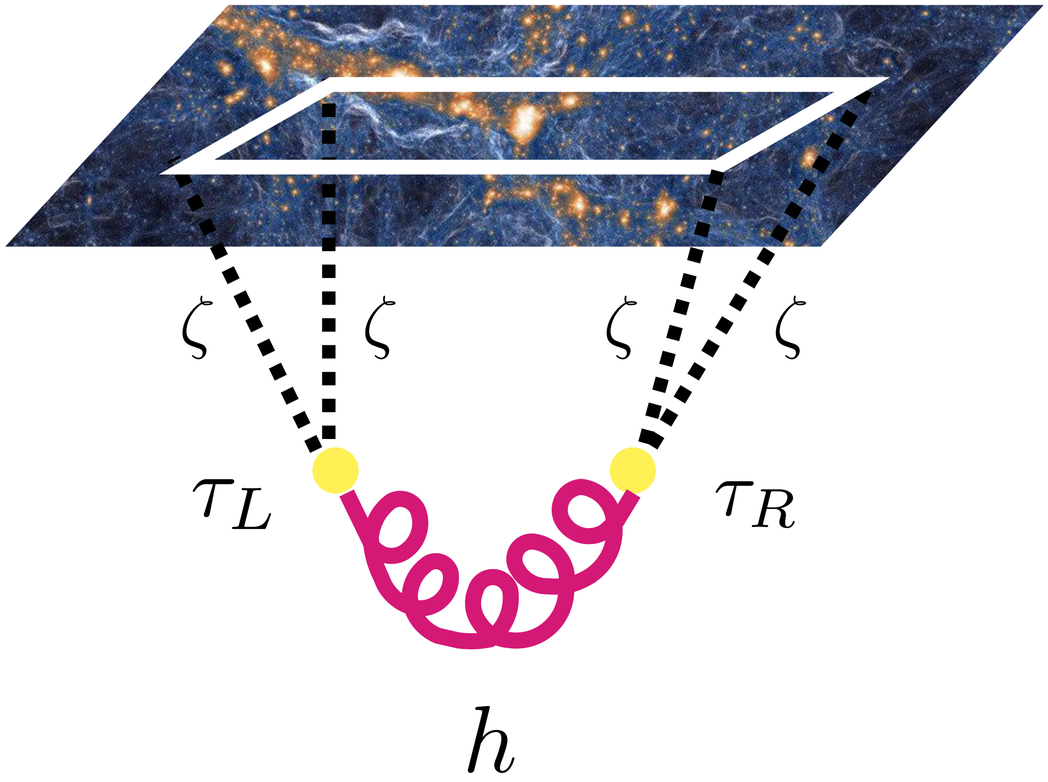
Naturally factorizable



$$\langle\zeta^4\rangle' \sim \int d\tau_L d\tau_R \mathcal{J}_L(k_1,k_2,k_s,\tau_L)\Pi(k_s,\tau_L,\tau_R)\mathcal{J}_R(k_3,k_4,k_s,\tau_R)$$

Full model: massive spinning particle exchange





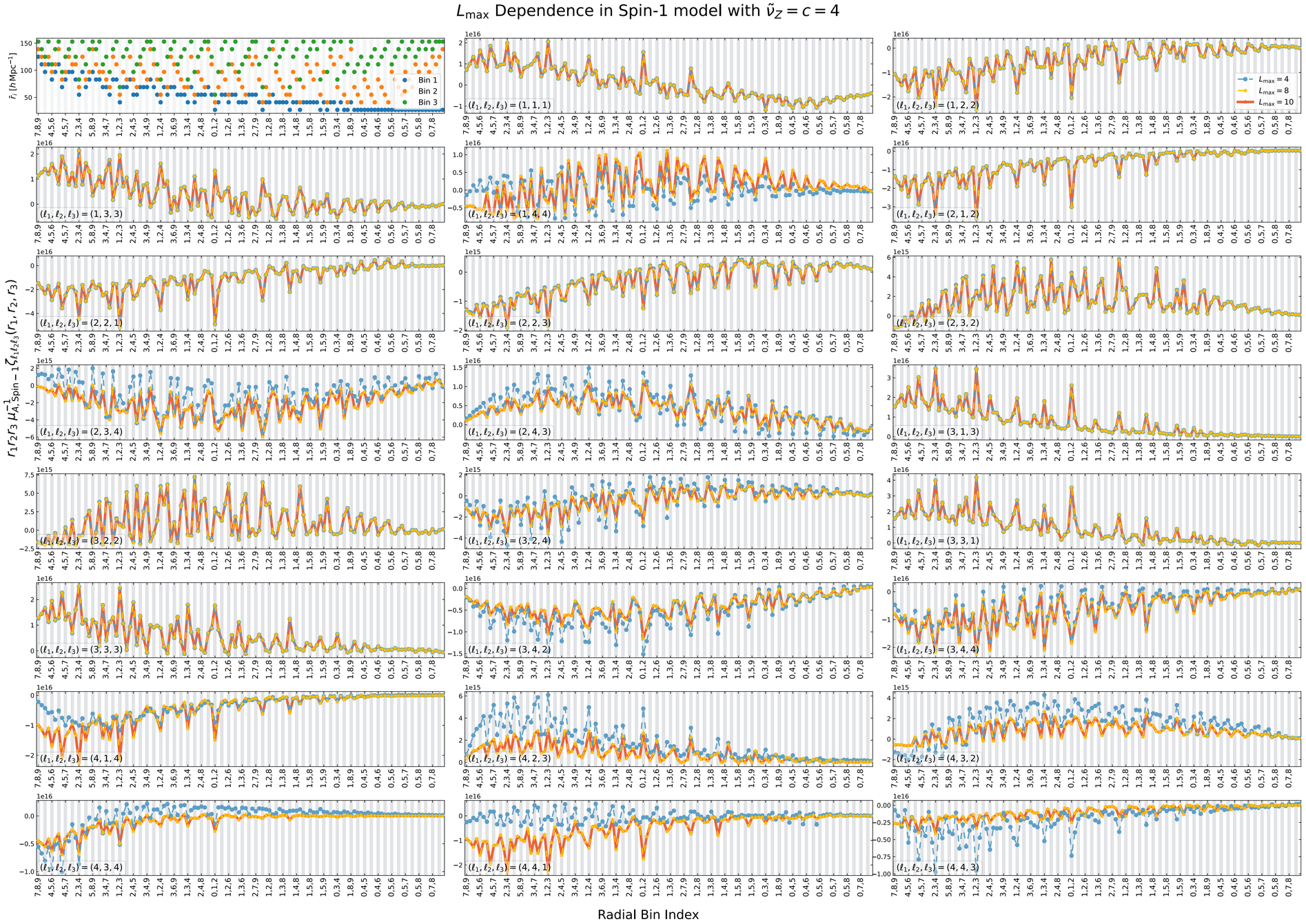
Liu, Tong, Wang, Xianyu ’20, Wang & Xianyu ‘20

Tong & Xianyu ‘22

Result

General result

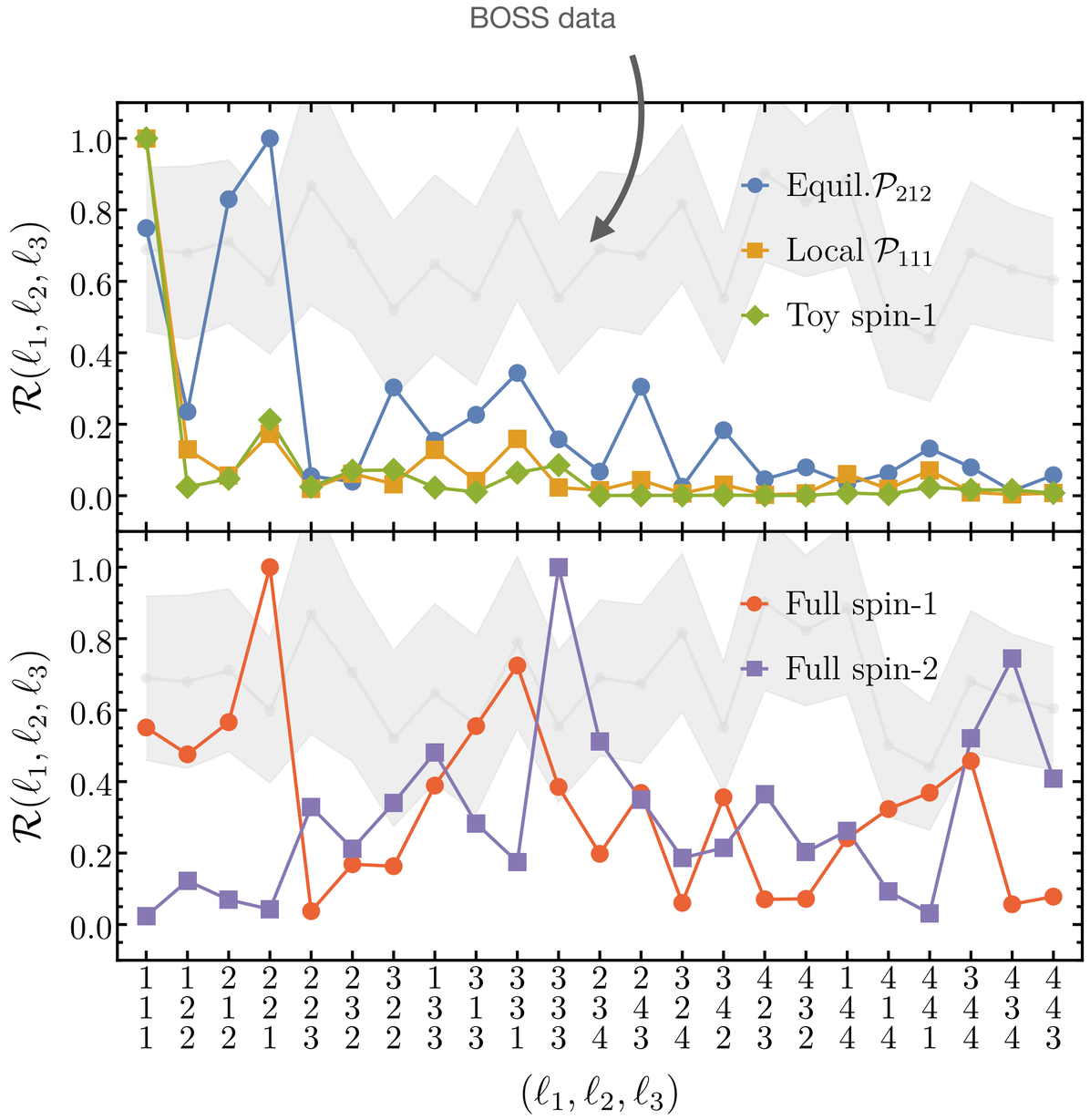
- 49 models in total



Features: Angular part

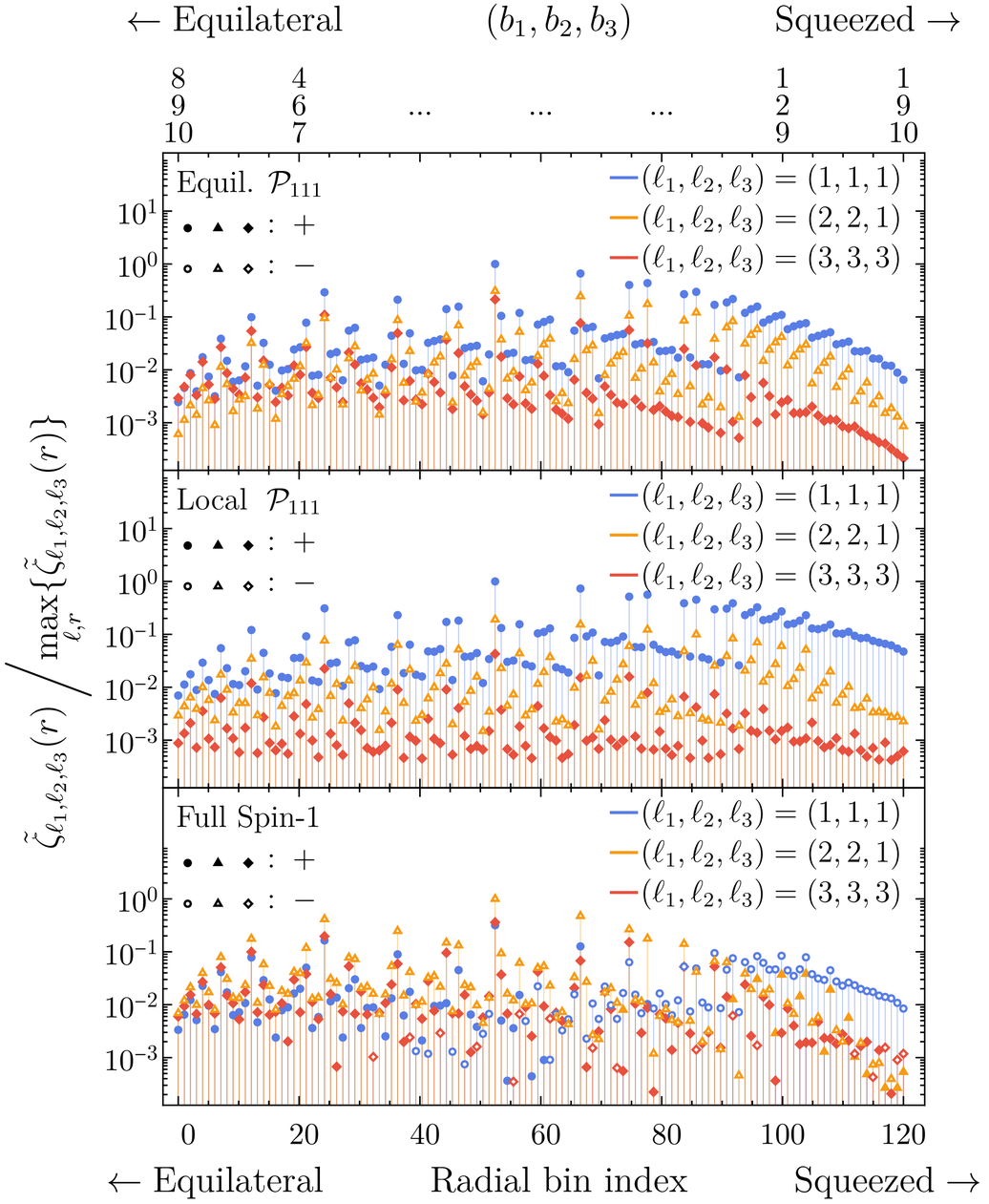
Coefficients marginalized over radii bins

- Trispectra with high-ℓ angular-dependence ⇒ Larger galaxy 4PCF signals in high-ℓ bins.
- Propagating degree carries spatial angular momentum ⇒ Signals spread over a large range of ℓ bins.



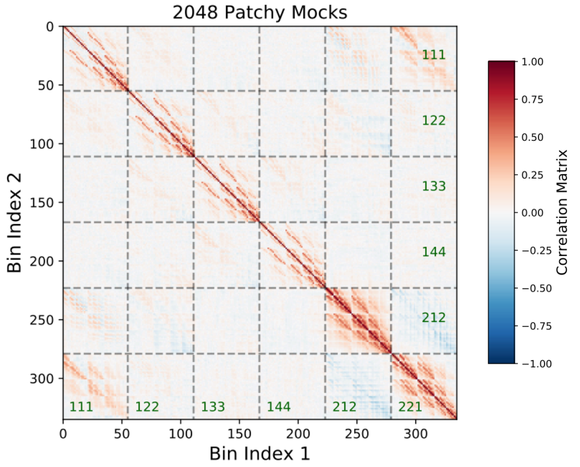
Features: Radial part

- The massive spinning exchange model shows more sign flips of the coefficients than the toy models.
- The coefficients got suppressed for higher-ℓ bins. For Equil. Model and full spin-1 model, the suppression mostly happens at the squeezed limit.

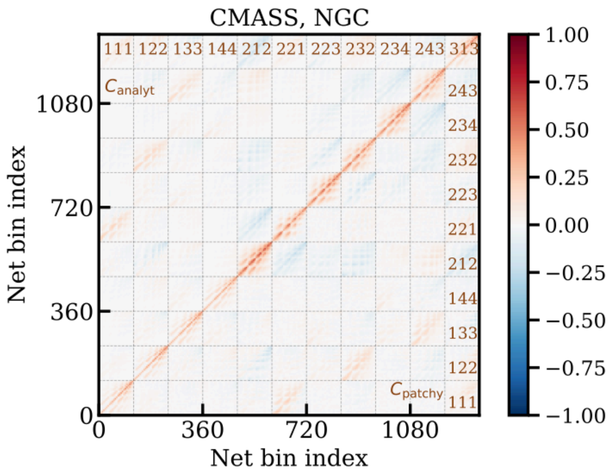


Comparison w/ BOSS

- Philcox (2022) and Hou, Slepian & Cahn (2022) both used the compressed χ^2 analysis pipeline and covariance matrix from the same set of simulations.
- Looking for the most robust linear combination of basis functions ⇒ keeping $N_{\rm eig}$ eigenvectors with smallest eigenvalues of the covariance matrix.
- Remove small-scale effects ⇒ filtering out near-by radii bin combos.



Philcox'22

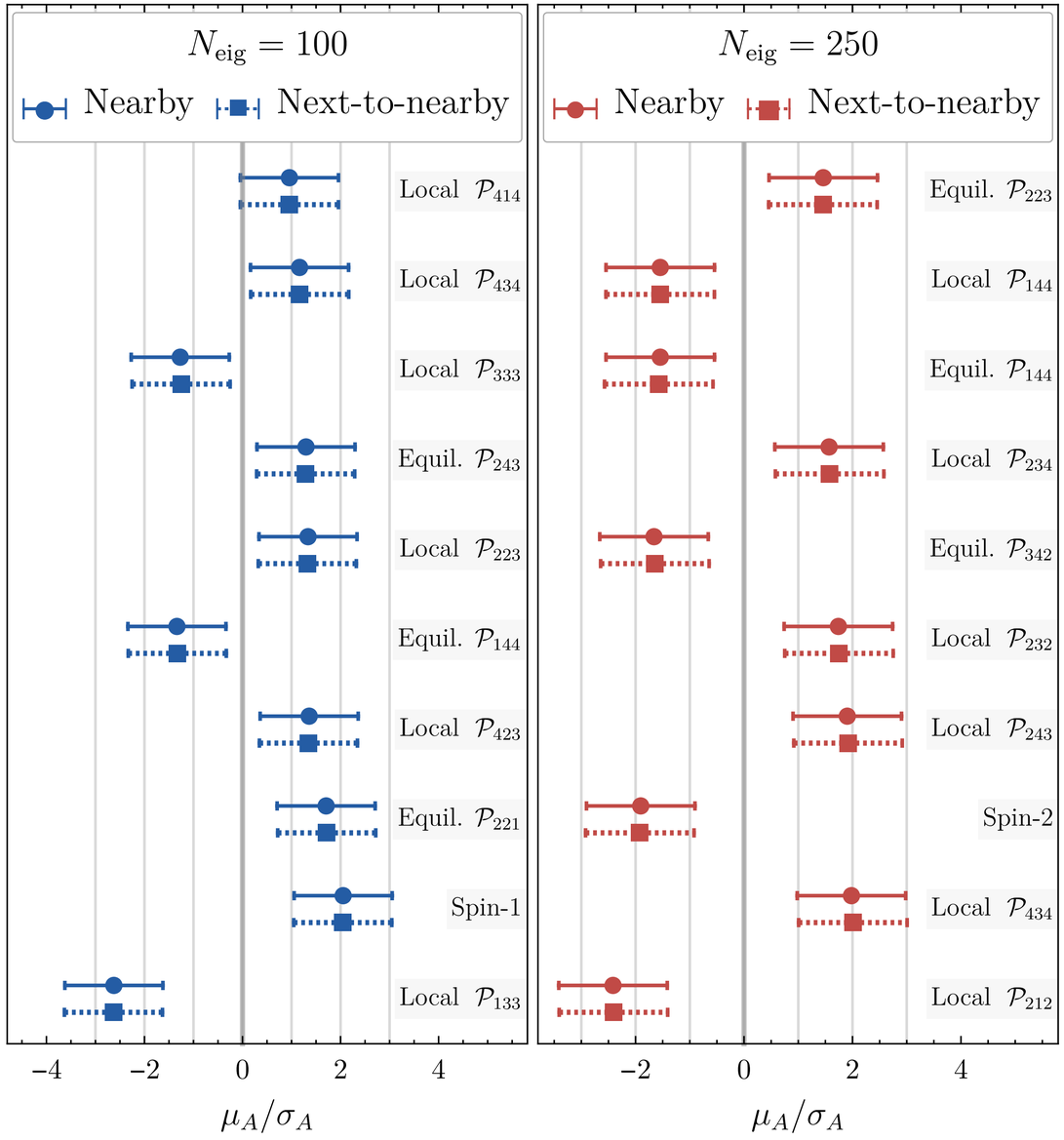


Hou, Slepian & Cahn '22

Comparison w/ BOSS

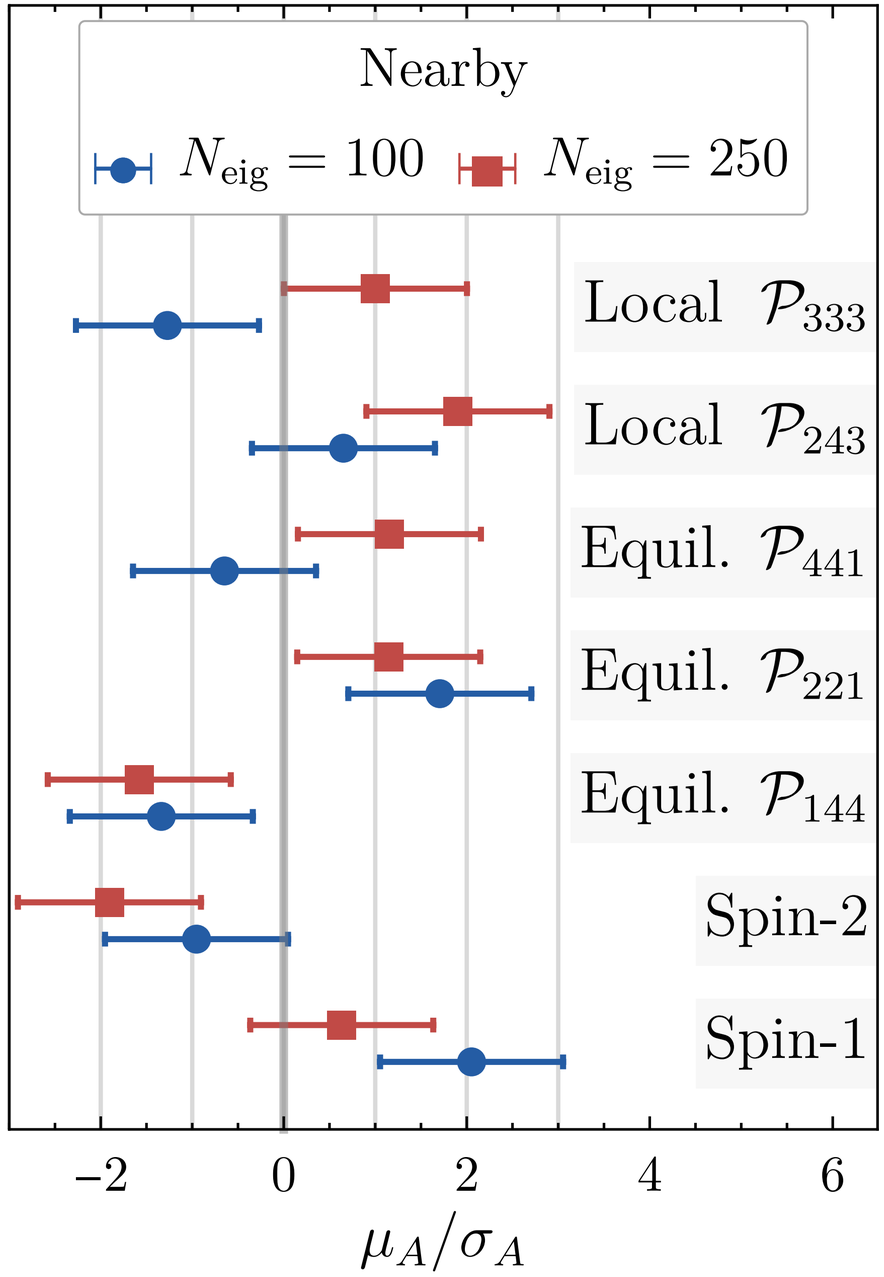
- Top 10 models with highest scaling factor over its standard deviation μ_A/σ_A.
- No models passing detection level of 3σ.
- The radial bin filtering has negligible effects.

For spin-1 and 2 models, $\mu \equiv \sqrt{m^2/H^2 - 9/4} = 4$
Chemical potential enhancement $e^{4\pi}$



Comparison w/ BOSS

- μ_A/σ_A fluctuates for the same model with different $N_{\rm eig}$.
- Questions into the robustness of compressed χ^2 methods.
- Useful information of the parity-violation is likely dropped out due to data compression.



Summary

- Parity violation in correlation functions represents novel opportunities in detecting PNGs.
- High dimensionality poses a significant challenge in converting parity-violating trispectra into galaxy 4PCF. Factorization is necessary.
- Massive spinning particle exchange with chemical-potential enhancement is a type of well-suited models for parity violation detection.