Probing Dark Matter with Space-based Gravitational-Wave Laser Interferometers

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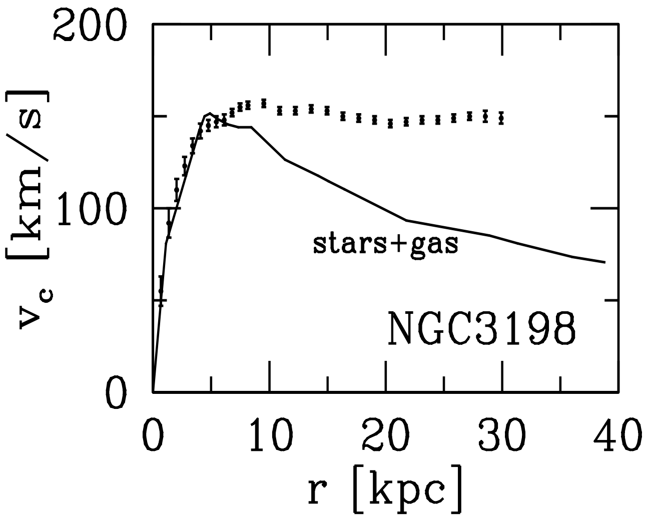
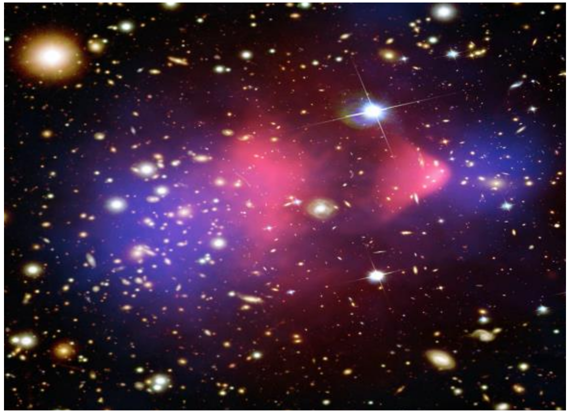
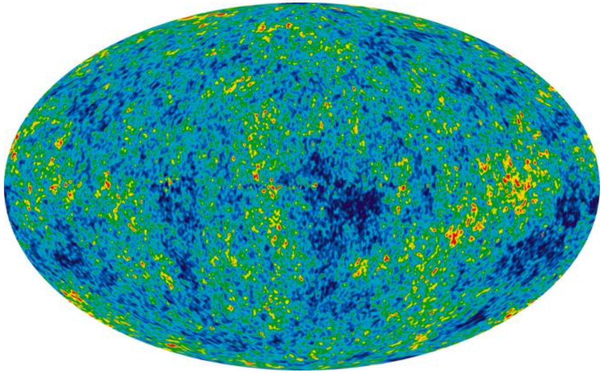
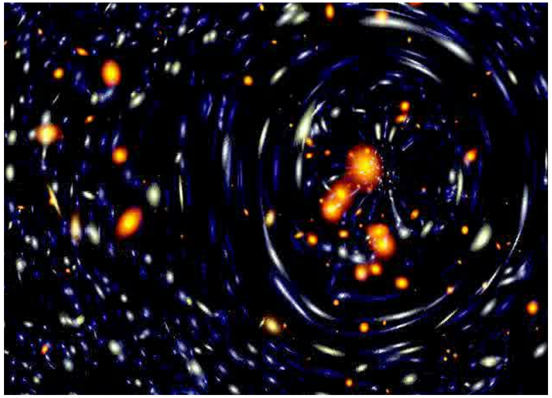
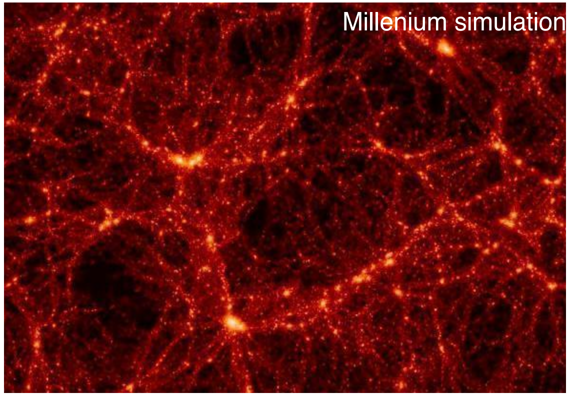




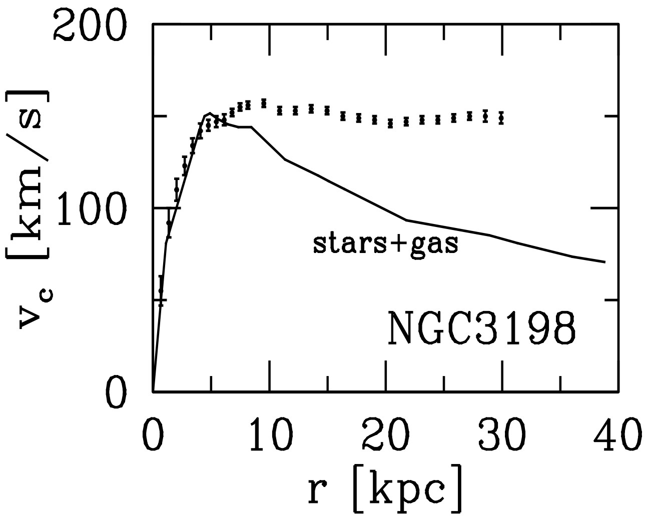


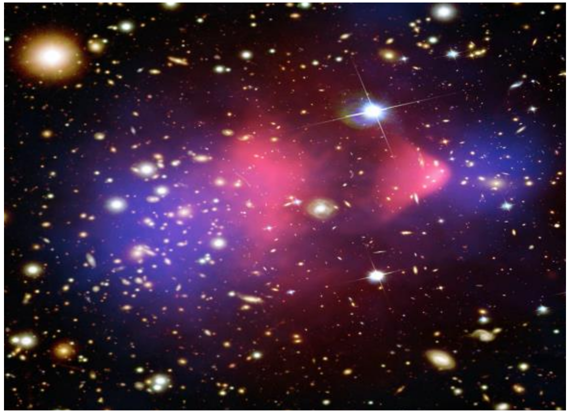


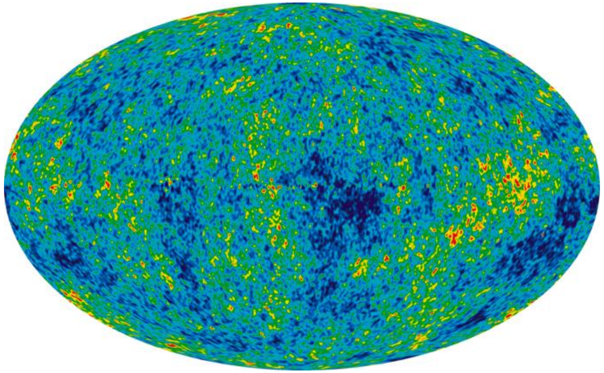


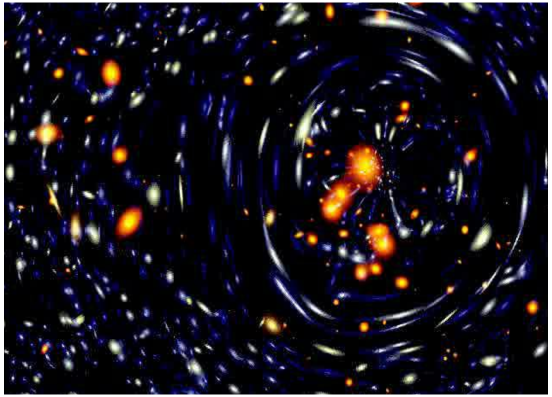
- Rotation curve
- Gravitational lensing
- Bullet cluster
- Large-scale structure
- Anisotropy of CMB

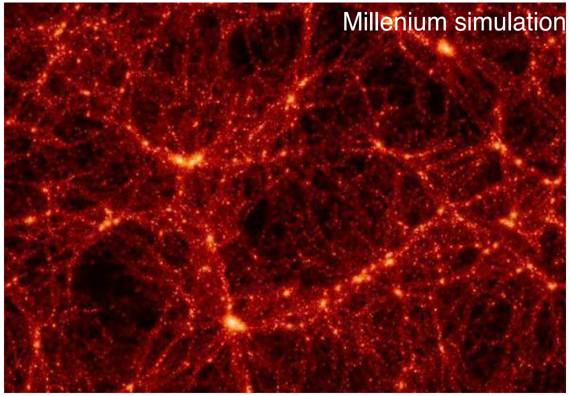
Evidence for Dark Matter

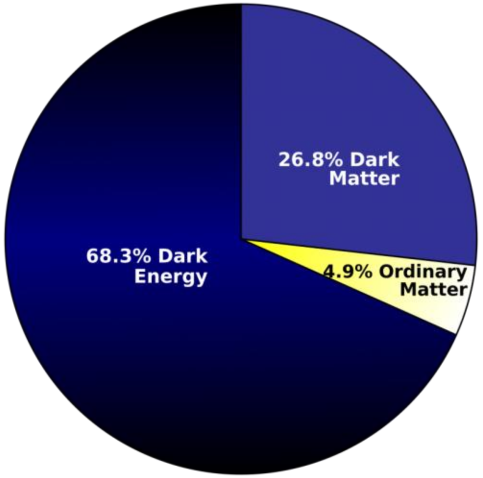






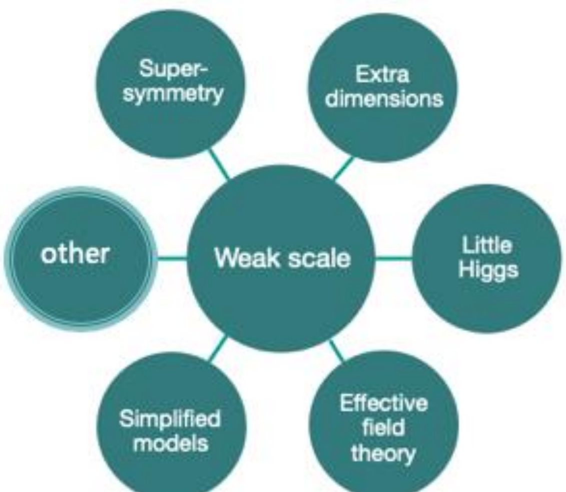


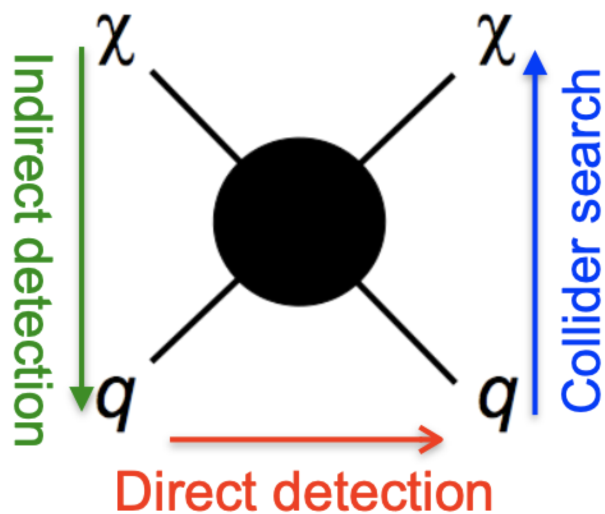


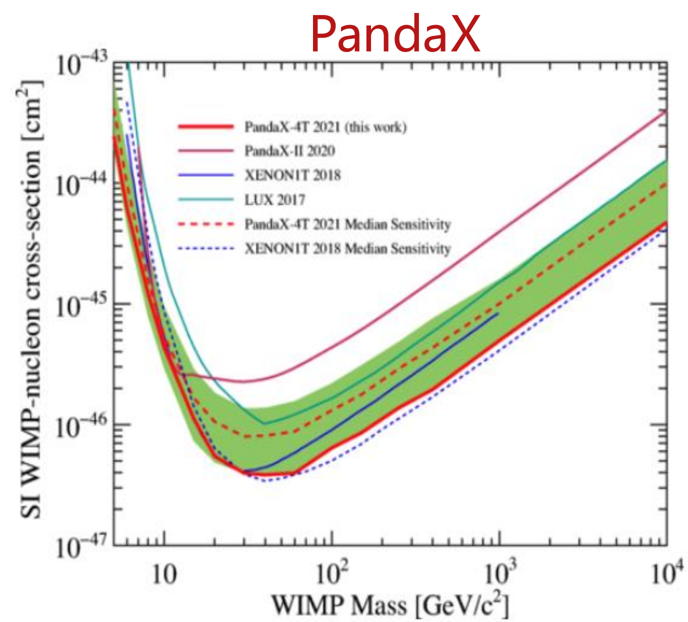


Weakly Interacting Massive Particle (WIMP)

- Theories: supersymmetry, extra dimensions, ……
- Direct, indirect and collider searches
- Impressive progress

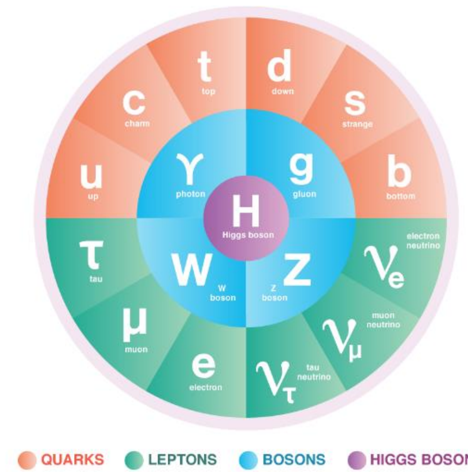


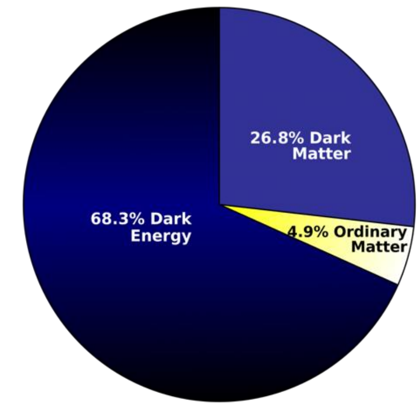


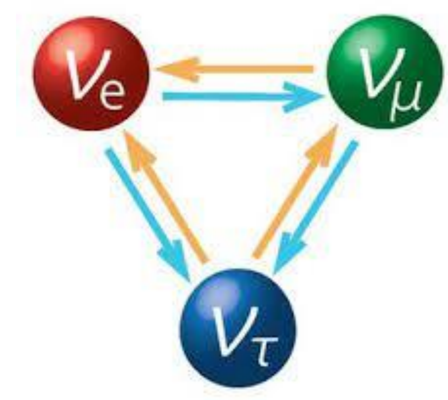


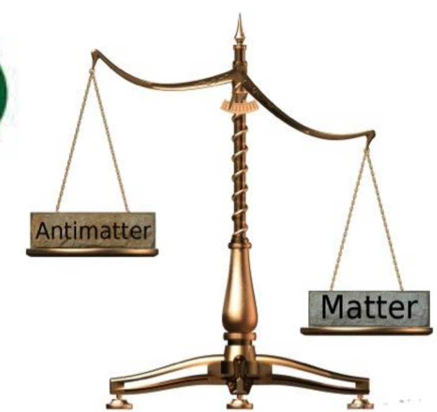
More Motivations

- Standard model is not complete
- Dark Matter and Dark Energy
- Neutrino mass
- Matter-Antimatter asymmetry
- Theoretical Problems
    - Strong CP problem
    - Hierarchy problem
    - Fermion mass hierarchy
    - Unification of forces
    - ……





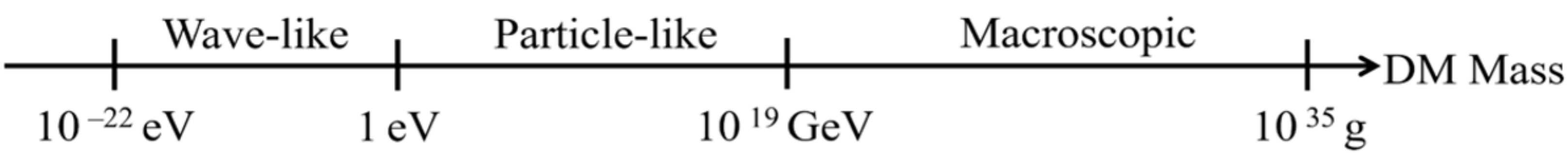




Dark Matter Candidates

- Primordial black holes
- Super heavy particles
- Asymmetric DM
- Hidden sector DM
- ……

- **Weakly-interacting (WIMP)**
- Strongly-interacting (SIMP)
- Sterile neutrino
- **Ultralight DM, Axion (ALP), Dark photon, dilaton etc**



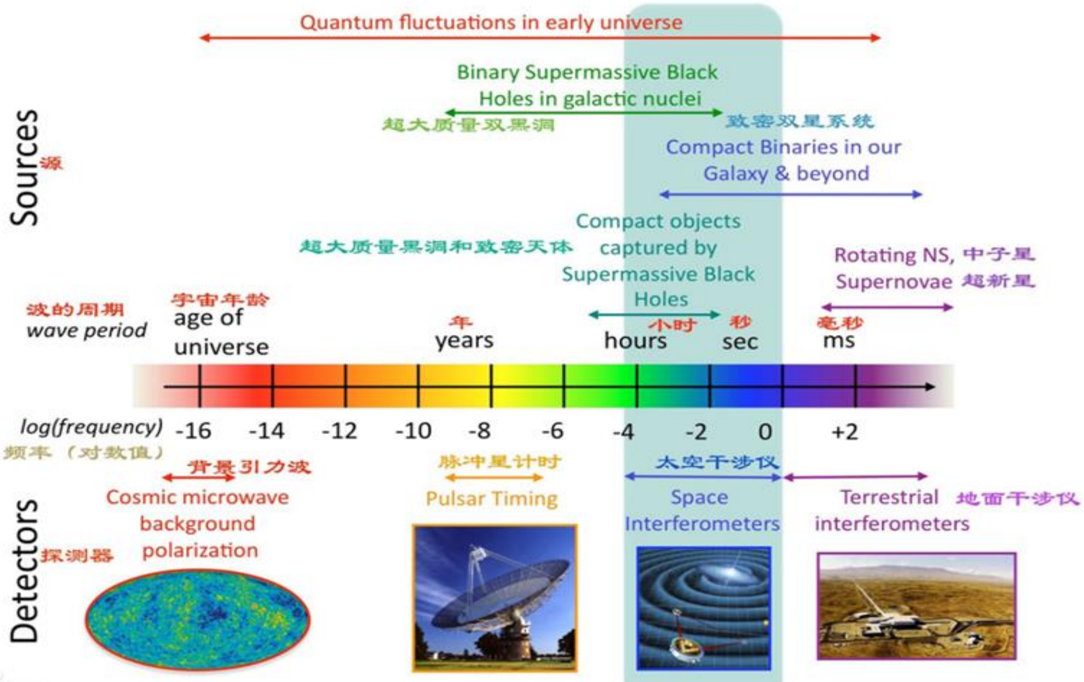
ULDM < 1eV

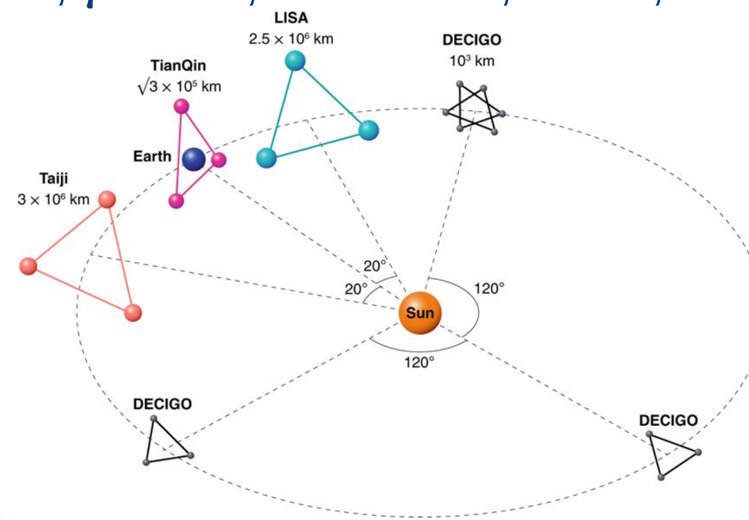
WIMP ~ 10^2 - 10^5 GeV

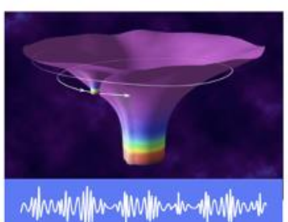
Since we do not know what DM is, we may explore as much as we can!

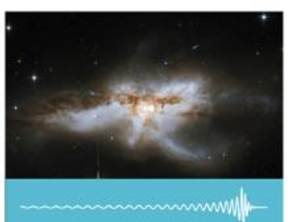
GW Laser Interferometers in Space

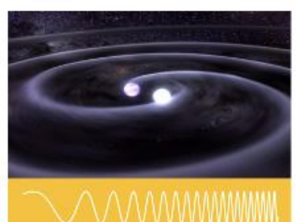
- LISA, Taiji, TianQin, LISAmax, Astrod-GW, μAres, DECIGO, BBO,⋯

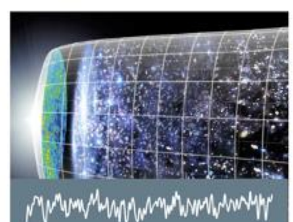


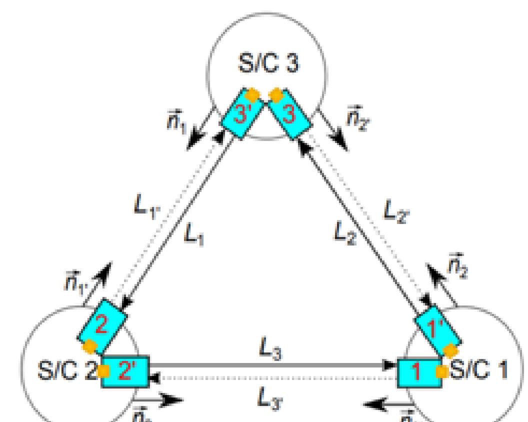






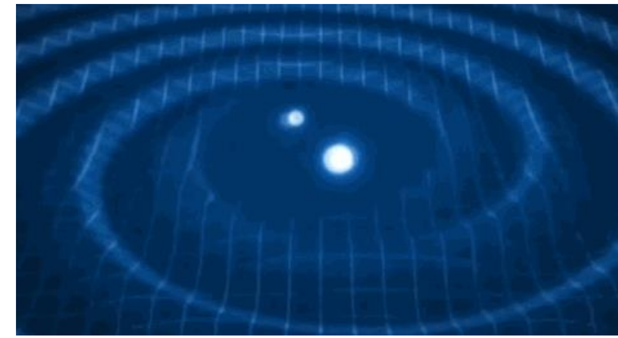




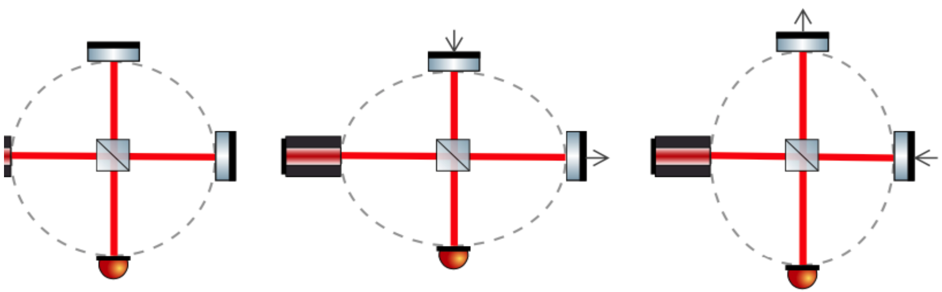


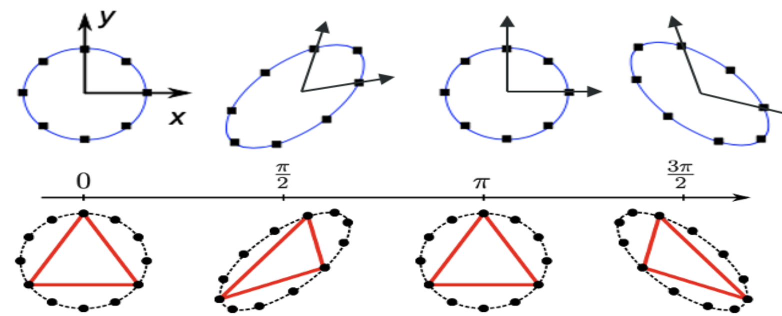
Signal Response

- Gravitational wave can change the structure of spacetime, and the physical distance between objects



- $h \sim \Delta L / L \sim 10^{-12}\mathrm{m}/10^{9}\mathrm{m} \sim 10^{-21}$
- One can measure the length/phase by laser



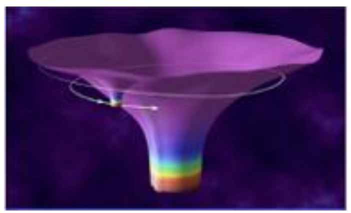


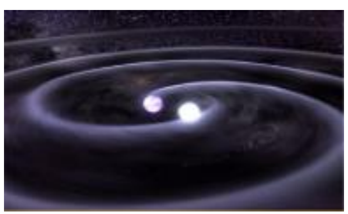
- Doppler shift $\frac{\delta v(t)}{v_0} \equiv y_{BA} = -\frac{1}{2}\frac{n_i n_j}{1+\vec{k}\cdot\vec{n}}\left[h_{ij}\left(t-\frac{\vec{k}\cdot\vec{x}_B}{c}\right) - h_{ij}\left(t-\frac{\vec{k}\cdot\vec{x}_A+L}{c}\right)\right]$

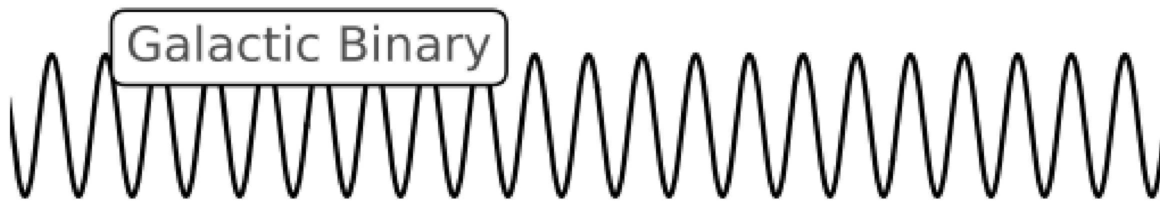
Waveform in time domain $X(t) \leftrightarrow \widetilde{X}(f)$ Power spectrum in frequency domain

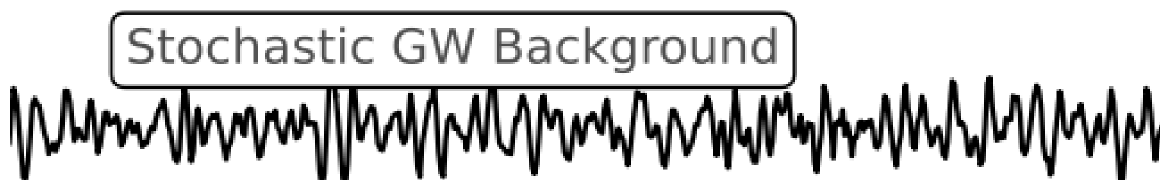
GW sources and waveforms

LISA, 2402.07571









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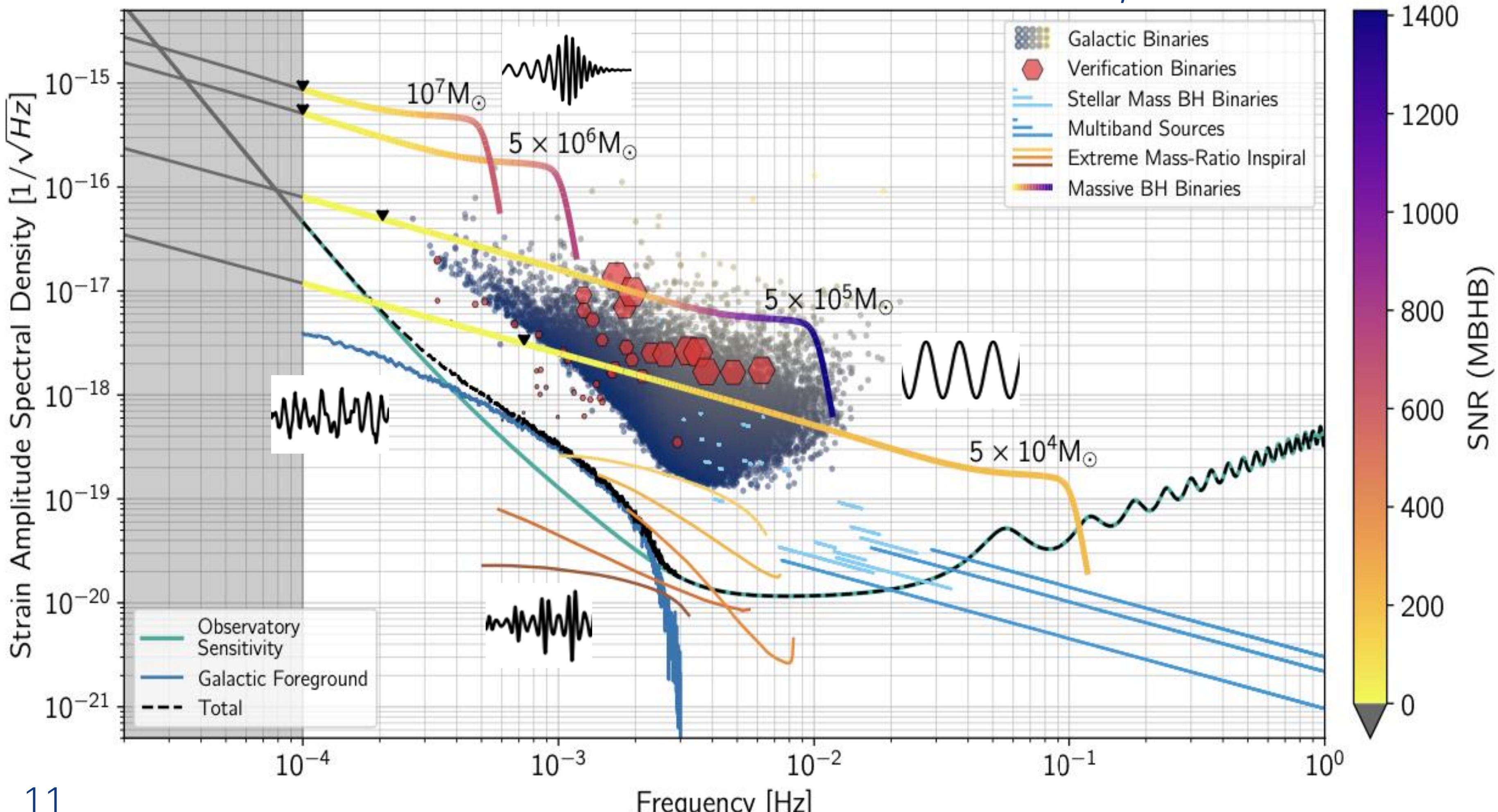
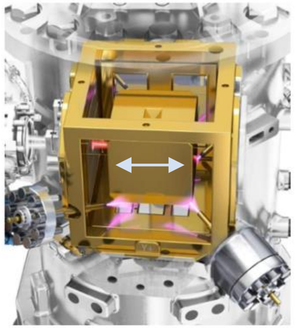


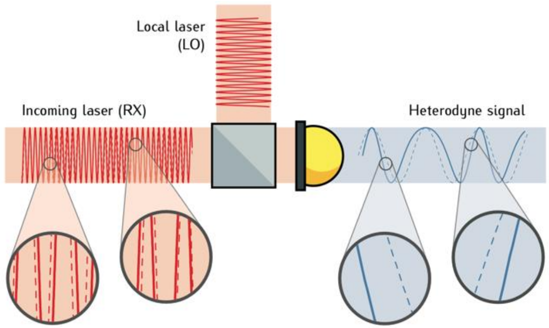
Main Noises

- Laser frequency noise
  - The laser is not perfectly monochromatic, its frequency fluctuates.
  - Need dedicated data analysis - Time-Delay Interferometry
- Secondary noises
  - Acceleration noise of test mass, $s_{acc} \sim 10^{-15} \mathrm{m/s^2}$
    - Residual charges, self-gravity, ⋯
  - Optical metrology noise, $s_{oms} \sim 10^{-12} \mathrm{m}$
    - Shot noise, relative intensity noise,⋯



LISA, 2402.07571

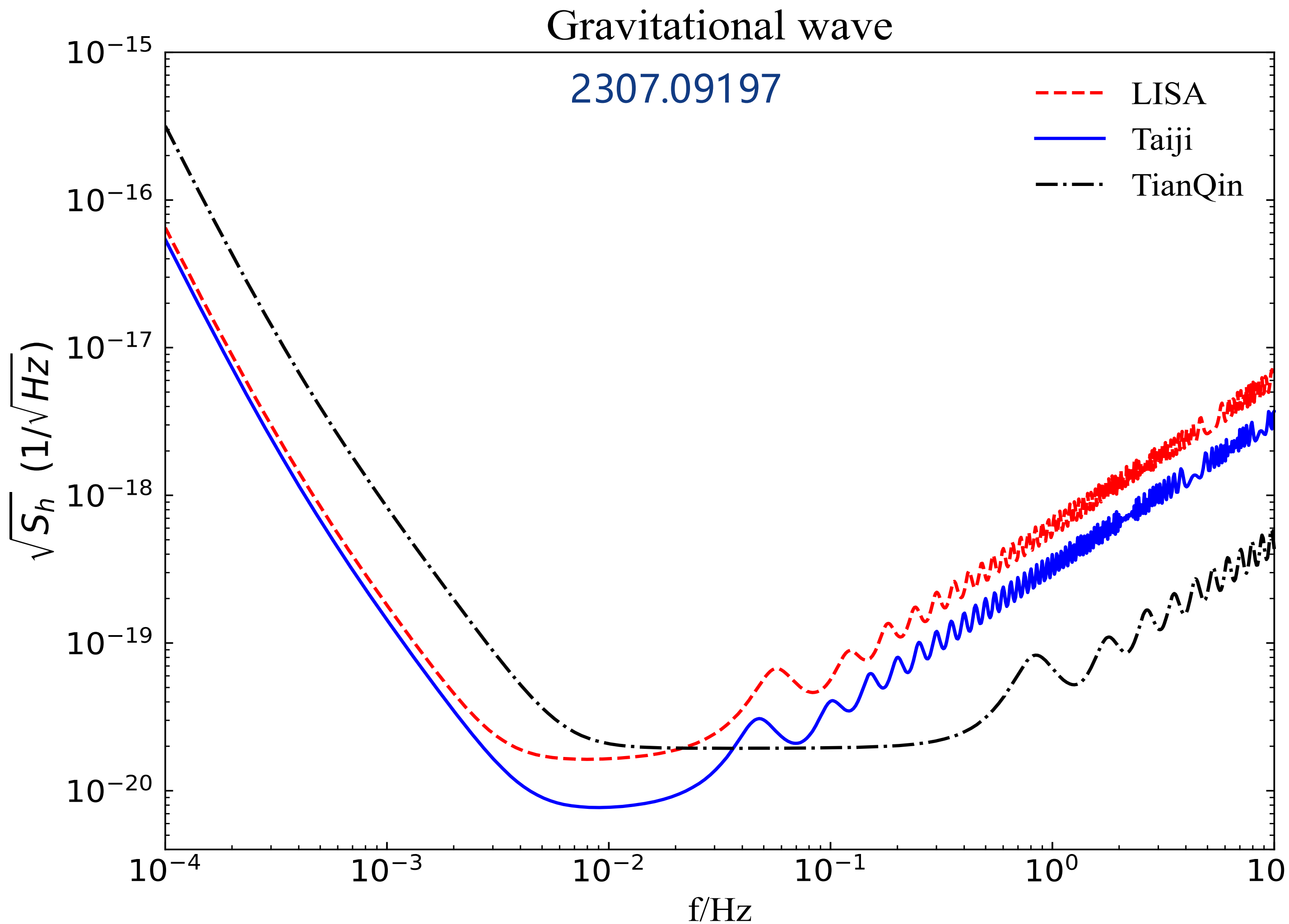
	LISA	Taiji	Tianqin	BBO	DECIGO
L (10^9 m)	2.5	3	0.17	0.05	1×10^{-3}
$s_{\rm acc}$ (10^{-15} $\frac{\rm m/s^2}{\sqrt{\rm Hz}}$)	3	3	1	3×10^{-2}	4×10^{-4}
$s_{\rm oms}$ (10^{-12} $\frac{\rm m}{\sqrt{\rm Hz}}$)	15	8	1	1.4×10^{-5}	2×10^{-6}



Barke 2015

Sensitivity on GW

- General
  - Arm length
  - Noise level
  - Duration
  - TDI
- Particular
  - Direction
  - Frequency
  - Waveform



Time-Delay Interferometry

Review by Tinto & Dhurandhar 2021

- There are multiple combinations
- Michelson channels

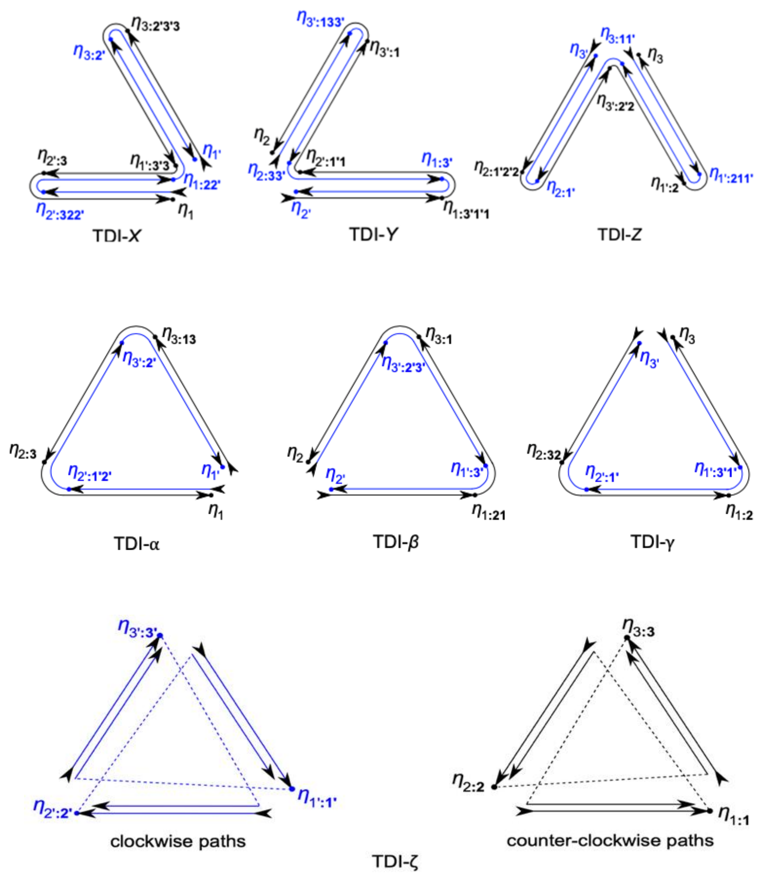
$$X(t) = (\eta_{2':322'} + \eta_{1:22'} + \eta_{3:2'} + \eta_{1'}) - (\eta_{3:2'3'3} + \eta_{1':3'3} + \eta_{2':3} + \eta_{1}),$$
$$Y(t) = (\eta_{3':133'} + \eta_{2:33'} + \eta_{1:3'} + \eta_{2'}) - (\eta_{1:3'1'1} + \eta_{2':1'1} + \eta_{3':1} + \eta_{2}),$$
$$Z(t) = (\eta_{1':211'} + \eta_{3:11'} + \eta_{2:1'} + \eta_{3'}) - (\eta_{2:1'2'2} + \eta_{3':2'2} + \eta_{1':2} + \eta_{3}).$$

- Sagnac channels

$$\alpha(t) = (\eta_{2':1'2'} + \eta_{3':2'} + \eta_{1'}) - (\eta_{3:13} + \eta_{2:3} + \eta_{1}),$$
$$\beta(t) = (\eta_{3':2'3'} + \eta_{1':3'} + \eta_{2'}) - (\eta_{1:21} + \eta_{3:1} + \eta_{2}),$$
$$\gamma(t) = (\eta_{1':3'1'} + \eta_{2':1'} + \eta_{3'}) - (\eta_{2:32} + \eta_{1:2} + \eta_{3}).$$

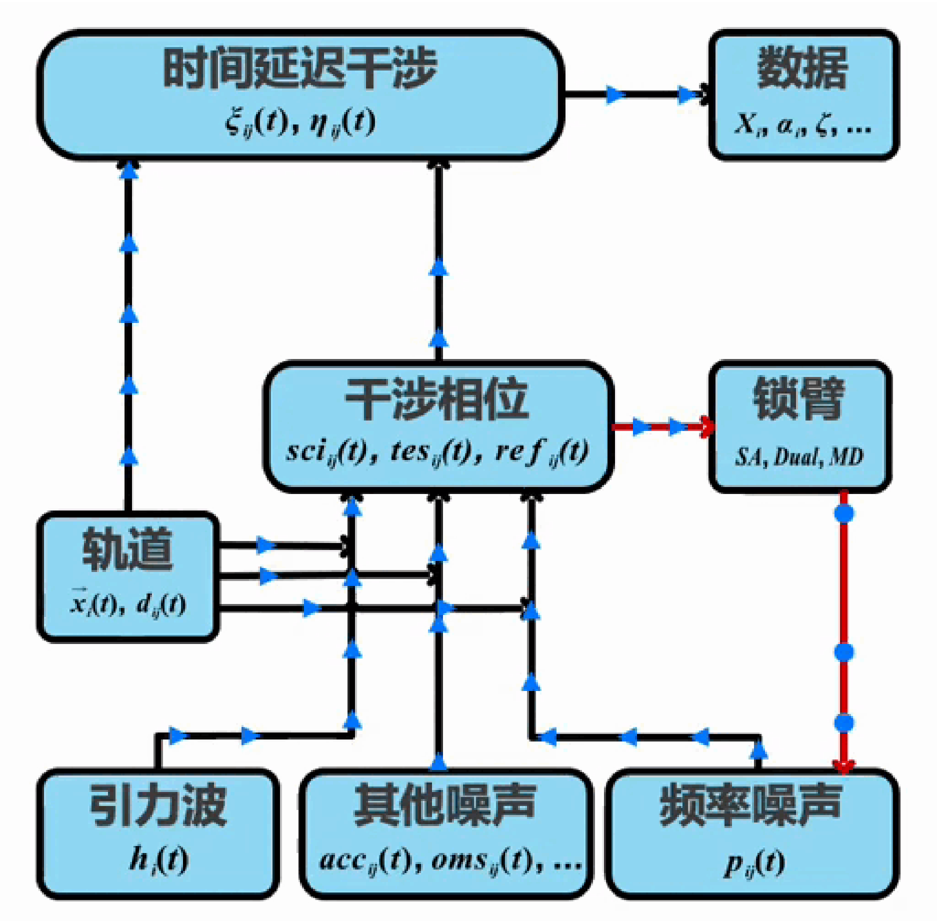
- ζ channel

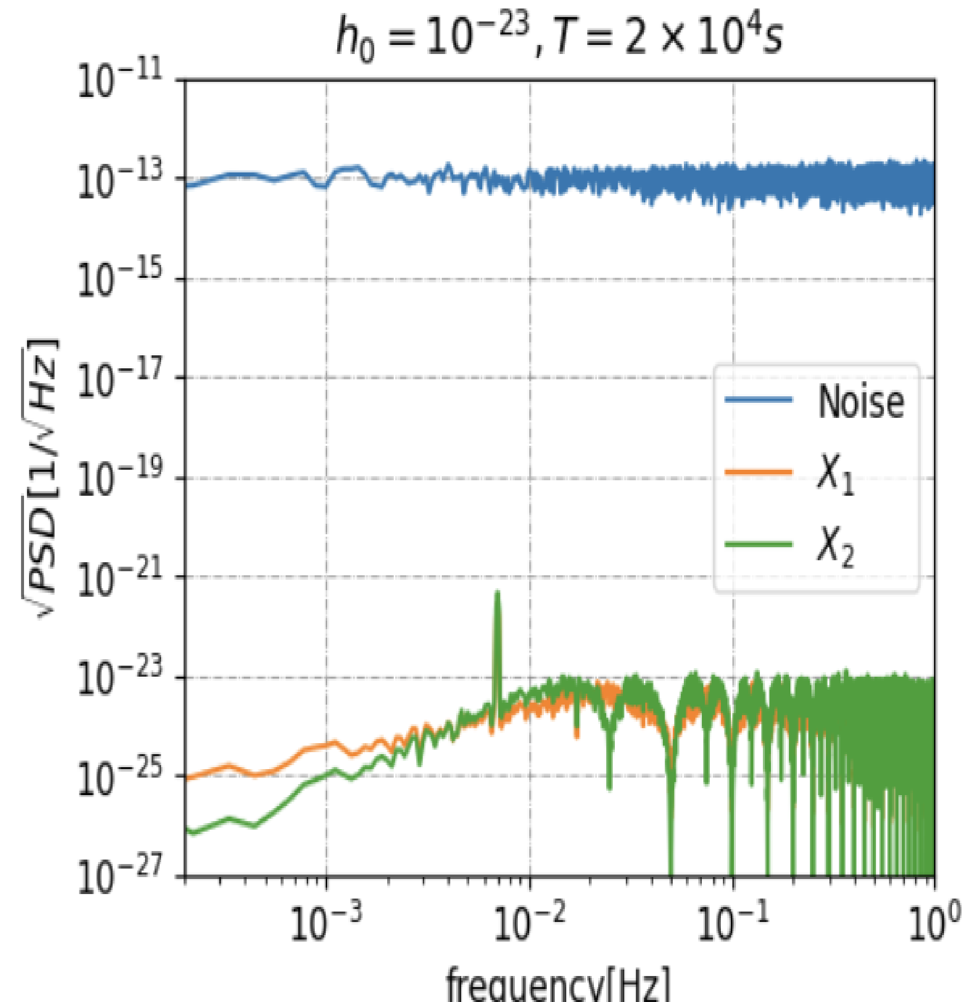
$$\zeta(t) = (\eta_{1':1'} + \eta_{2':2'} + \eta_{3':3'}) - (\eta_{1:1} + \eta_{2:2} + \eta_{3:3}).$$



Credit: Otto 2015

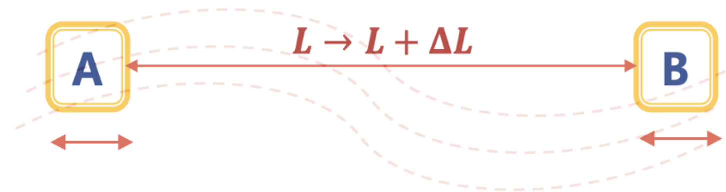
Time-Delay Interferometry





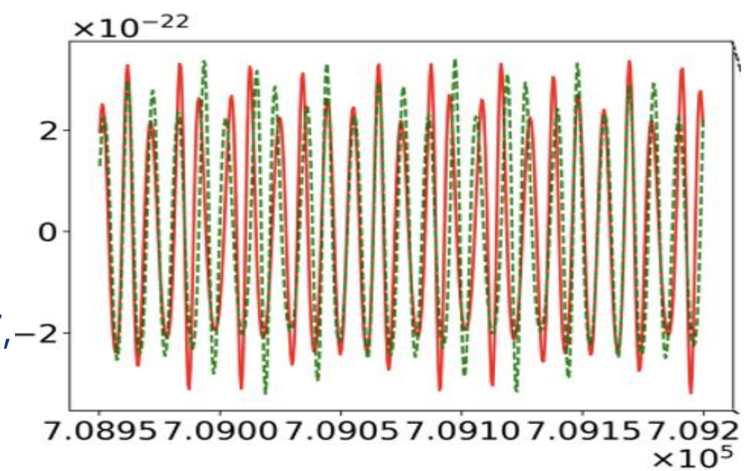
Possible Effects from Dark Matter

- Direct - Interact with the detector directly
  - Induce additional motion of test mass
  - Change the size of test mass, etc
  - Modify the propagation of laser light



Refs. Pierce, Riles & Zhao 2018, Grote & Stadnik 2019, Morisaki & Suyama 2019, ……

- Indirect - Affect the messengers that interact with the detector
  - Modify the GW from astrophysical sources
  - Cosmological phase transition
  - As new GW sources, PBHs, ⋯



Refs. Eda, Itoh, Kuroyanagi & Silk 2013, Yue, Han, Chen 2017, Bertone et al 2020, Cardoso et al 2022, ……

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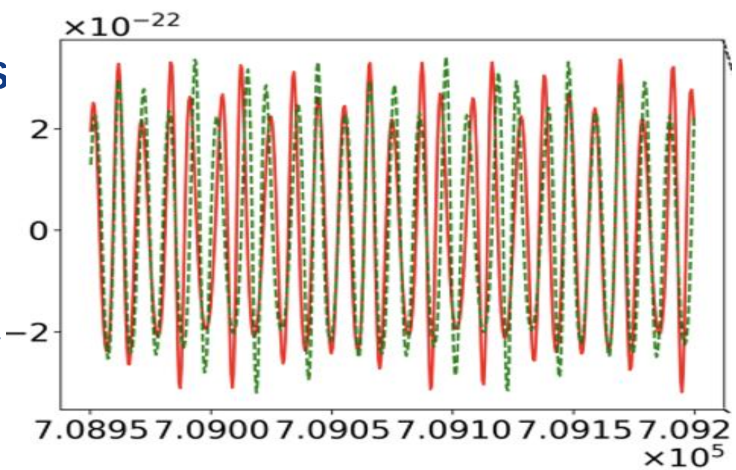


Refs. Eda, Itoh, Kuroyanagi & Silk 2013, Yue, Han, Chen 2017, Bertone et al 2020, Cardoso et al 2022, ……

Ultralight/Wave Dark Matter

- Mass < 1 eV, QCD axion, ALP, Dark Photon, ⋯
- A phenomenological approach
- Scalar $\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - C\frac{\phi}{M_P}\mathcal{O}_{\rm SM},$
  - Light dilaton, $\delta\mathcal{L} = \frac{\phi}{M_P}\left[-\frac{d_g\beta_3}{2g_3}F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=u,d}(d_{m_i} + \gamma_{m_i}d_g)\, m_i\bar{\psi}_i\psi_i\right]$ Damour & Donoghue 2010
  - ALP, $-\frac{g_{a\gamma}}{4}aF_{\mu\nu}\tilde{F}^{\mu\nu},$
- Vector $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_A^2A^\nu A_\nu - \epsilon_D e J_D^\nu A_\nu,$
  - Baryon number, B-L, Dark $U(1)$, ⋯⋯
- Production Mechanism – viable DM candidate

scalar: misalignment, ⋯

vector: Graham, Mardon & Rajendran 2015, Ema, Nakayama &Tang 2019, Kolb & Long 2024

⋯⋯

Wave Dark Matter Background

Foster, Rodd, Safdi 2018

- ➢ Number density is very large, behaves as classical wave

$$\Phi(x) = \sum_{\mathbf{v}} \frac{\sqrt{2\rho/N}}{m} e^{i(\omega t - \mathbf{k}\cdot\mathbf{x} + \theta_{\mathbf{v}})},$$

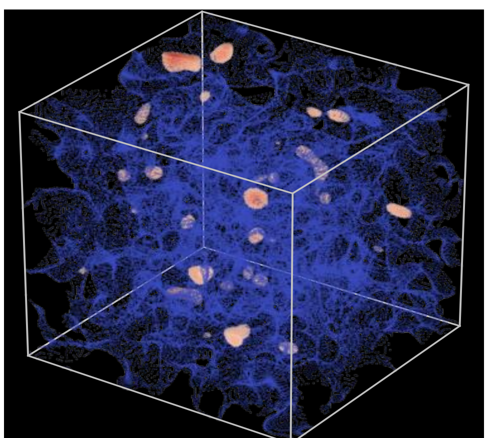
$$\vec{k} = m\vec{v}, \omega = 2\pi f \simeq m,\ v \sim 10^{-3},$$

$$f \approx 2.4 \times \left(\frac{m}{10^{-17}\mathrm{eV}}\right) \mathrm{mHz}$$

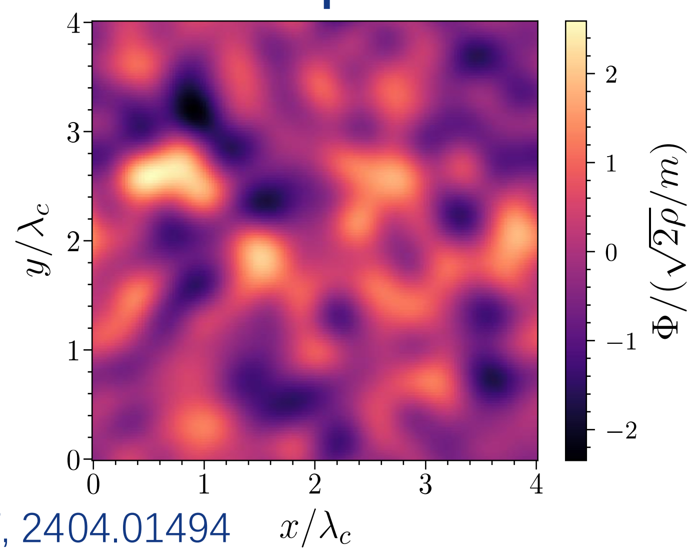
at a specific point ~ monochromatic within τ_c.

$$\lambda_c = \frac{2\pi}{mv} \approx 1.24 \times 10^{11} \times \left(\frac{10^{-17}\mathrm{eV}}{m}\right) \mathrm{km},\ \ \tau_c = \frac{\lambda_c}{v} \approx 4 \times 10^{8} \times \left(\frac{10^{-17}\mathrm{eV}}{m}\right) \mathrm{s},$$

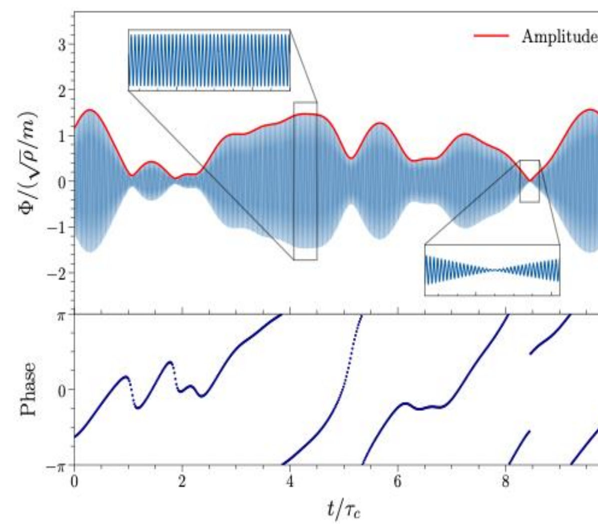
3D snapshot



2D snapshot



at some point

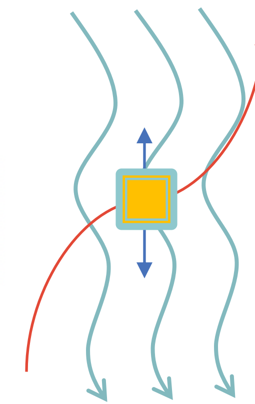


姚跃辉, YT, 2404.01494

Physical Effects

- Plane wave within τ_c, $\phi(t,\vec{x}) = \phi_{\vec{k}} e^{i(\omega t - \vec{k}\cdot\vec{x} + \theta_0)}$,
- Scalar ϕ, $\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 - C\frac{\phi}{M_P}\mathcal{O}_{\rm SM}$,
- Interaction depending on the underlying theory, e.g.

$$C\frac{\phi}{M_P} m_\psi \bar{\psi}\psi \Rightarrow m_\psi \to \left(1 + C\frac{\phi}{M_P}\right) m_\psi, \; S = -\int m(\phi)\sqrt{-\eta_{\mu\nu}dx^\mu dx^\nu}.$$



- Additional gradient or force

$$\delta x^i(t,\vec{x}) = \mathcal{M}_s \hat{k}^i e^{i m_\phi (t - v\hat{k}\cdot\vec{x})}, \mathcal{M}_s \propto \phi_{\vec{k}} |\vec{k}| / m_\phi^2$$

- Vector $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_A^2 A^\nu A_\nu - \epsilon_D e J_D^\nu A_\nu$, $\vec{A}(t,\vec{x}) = |\vec{A}|\hat{e}_A e^{i(\omega t - \vec{k}\cdot\vec{x})}$,

$$\delta x^i(t,\vec{x}) = \mathcal{M}_v \hat{e}_A^i e^{i m_A (t - v\hat{k}\cdot\vec{x})}, \mathcal{M}_v \propto \epsilon_D e q_{D,j} |\vec{A}| / m_A M_j$$

- Axion – laser propagation

Ultralight/Wave DM - Signal Response

- DM couples to SM particles, inducing oscillations of test mass, effectively changing the length

- One-way Doppler shift

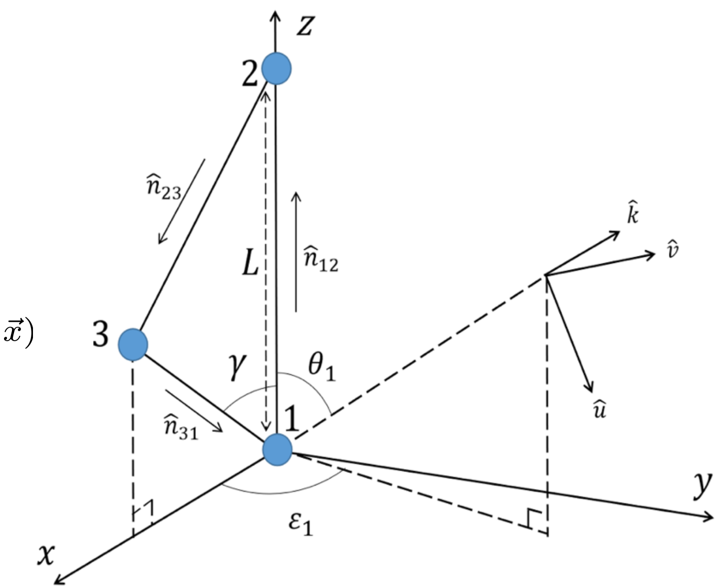
$$\delta t_{rs} = -\hat{n}_{rs} \cdot [\delta \vec{x}(t, \vec{x}_r) - \delta \vec{x}(t - L, \vec{x}_s)] \,,$$

$$\frac{\delta \nu_{rs}}{\nu_0} = \frac{\nu_{rs} - \nu_0}{\nu_0} = -\frac{d\,\delta t_{rs}}{dt} \,.$$

- Fractional frequency change

$$y_{rs}(t) \equiv \frac{\delta \nu_{rs}}{\nu_0} = \mu_{rs} \, [h(t, \vec{x}_r) - h(t - L, \vec{x}_s)], \; h(t, \vec{x}) \propto e^{im(t - v\hat{k} \cdot \vec{x})}$$

$$\mu_{rs} = \begin{cases} \hat{k} \cdot \hat{n}_{rs} & \text{for scalar field,} \\ \hat{e}_A \cdot \hat{n}_{rs} & \text{for vector field,} \\ \dfrac{\hat{n}^i_{rs} \hat{n}^j_{rs} e_{ij}(\hat{k}, \psi)}{2(1 + \hat{n}_{rs} \cdot \hat{k})} & \text{for gravitational wave,} \end{cases}$$



Yu, 姚跃辉, Tang, Wu, 2307.09197

Transfer Functions

- Fourier transform

$$h(t) = \frac{\sqrt{T}}{2\pi}\int_0^{\infty} \tilde{h}(\omega) e^{i\omega t} d\omega$$

- One-way single link

$$y_{rs}(t) = \mu_{rs}\,\frac{\sqrt{T}}{2\pi}\int_0^{\infty} d\omega\; \tilde{h}(\omega) e^{i\omega t}\left[e^{-i\vec{k}\cdot\vec{x}_r} - e^{-i(\tau+\vec{k}\cdot\vec{x}_s)}\right],$$

$$\tilde{y}_{rs}(\omega) = \mu_{rs}\,\tilde{h}(\omega)\left[e^{-i(\vec{k}\cdot\vec{x}_r)} - e^{-i(\tau+\vec{k}\cdot\vec{x}_s)}\right]. \qquad \tau = 2\pi f L$$

- Transfer function, sky and polarization averaged

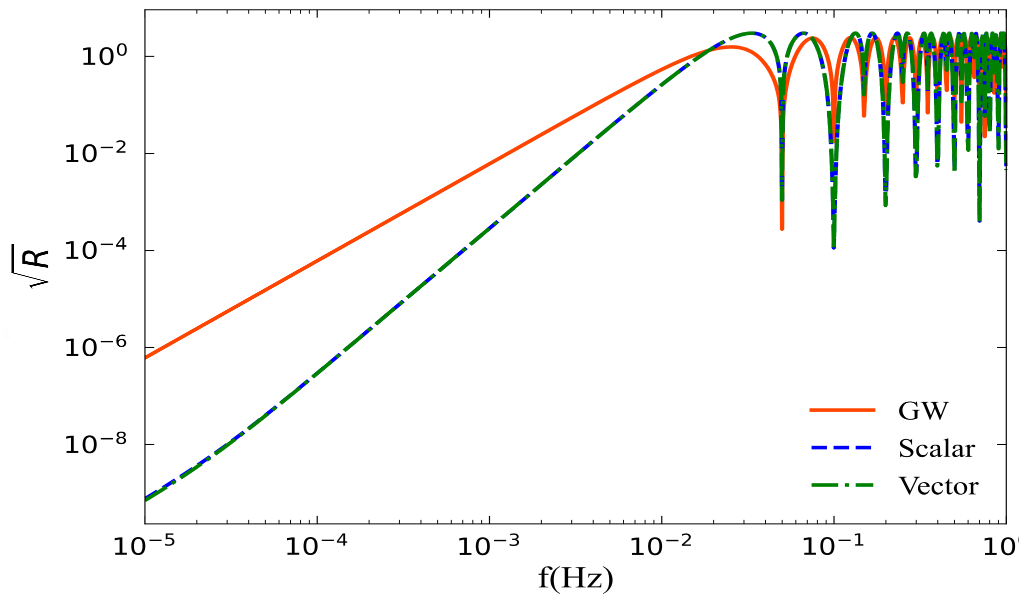
$$R(\omega) = \overline{\left|\frac{\tilde{y}_{rs}(\omega)}{\tilde{h}(\omega)}\right|^2},$$

$$I_s \equiv \frac{1}{4\pi}\int_{-1}^{1} d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \cdots,$$

$$I_v \equiv \frac{1}{16\pi^2}\int_{-1}^{1} d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \int_{-1}^{1} d\cos\theta_2 \int_0^{2\pi} d\epsilon_2 \cdots$$

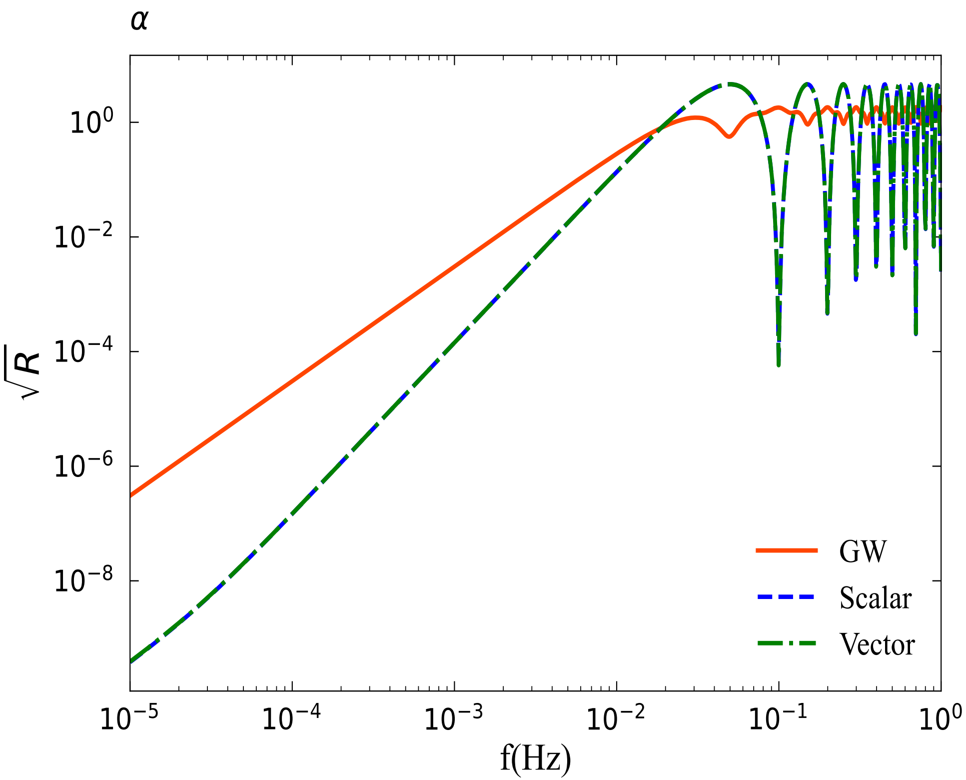
$$I_{GW} \equiv \frac{1}{8\pi^2}\int_{-1}^{1} d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \int_0^{2\pi} d\psi \cdots.$$

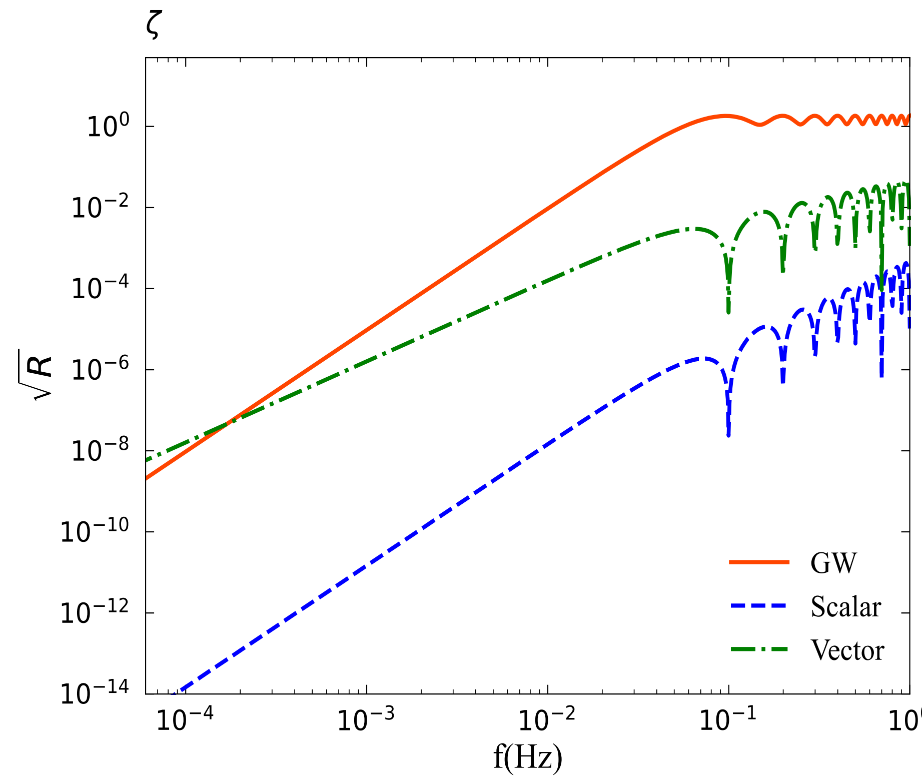
x



Transfer Functions

- Different channels have different transfer functions
- DM is also different from gravitational wave, velocity effect, ⋯





Sensitivity – Michelson Interferometer

- Defined by $S_O(f) = \dfrac{N_O(f)}{R_O(f)}$, $N_X = 16\sin^2(\tau)\left\{[3+\cos(2\tau)]\,S_{acc} + S_{oms}\right\}$, $\tau = 2\pi f L$

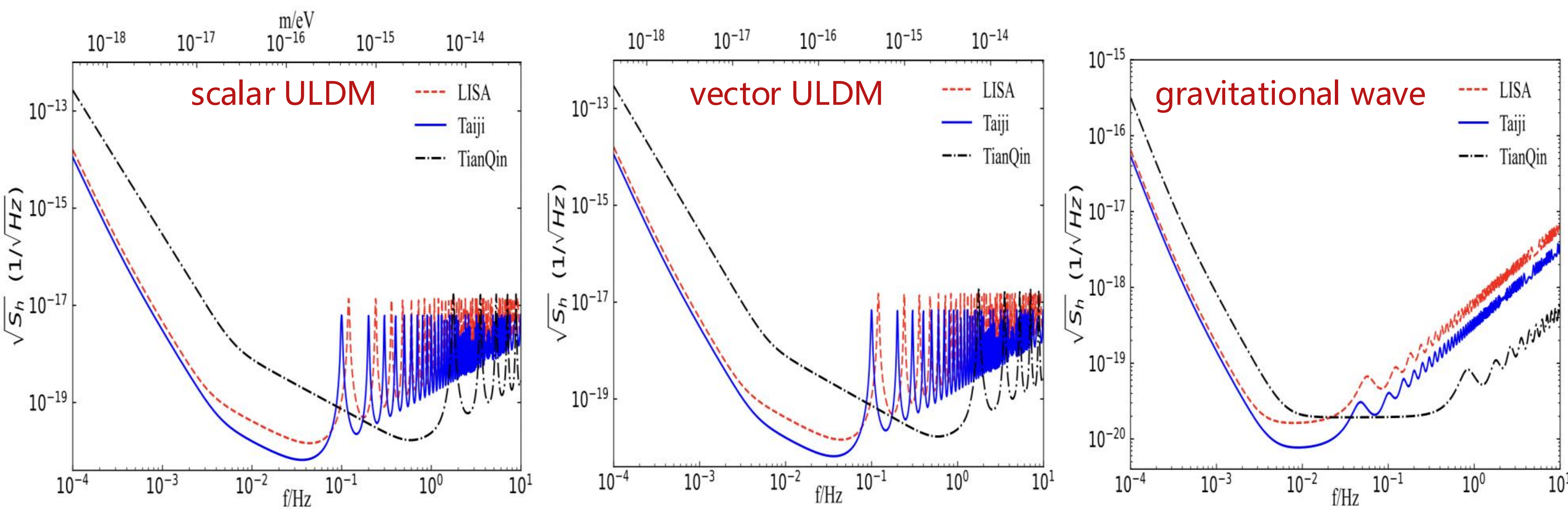
$$S_{oms}(f) = \left(s_{oms}\frac{2\pi f}{c}\right)^2\left[1+\left(\frac{2\times10^{-3}\ \mathrm{Hz}}{f}\right)^4\right]\frac{1}{\mathrm{Hz}},$$

$$S_{acc}(f) = \left(\frac{s_{acc}}{2\pi f c}\right)^2\left[1+\left(\frac{0.4\times10^{-3}\ \mathrm{Hz}}{f}\right)^2\right]\left[1+\left(\frac{f}{8\times10^{-3}\ \mathrm{Hz}}\right)^4\right]\frac{1}{\mathrm{Hz}},$$

$$\text{LISA}:\quad s_{oms} = 15\times10^{-12}\ \mathrm{m},\ s_{acc} = 3\times10^{-15}\ \mathrm{m/s^2},$$

$$\text{Taiji}:\quad s_{oms} = 8\times10^{-12}\ \mathrm{m},\ s_{acc} = 3\times10^{-15}\ \mathrm{m/s^2},$$

$$\text{TianQin}:\quad s_{oms} = 1\times10^{-12}\ \mathrm{m},\ s_{acc} = 1\times10^{-15}\ \mathrm{m/s^2}.$$

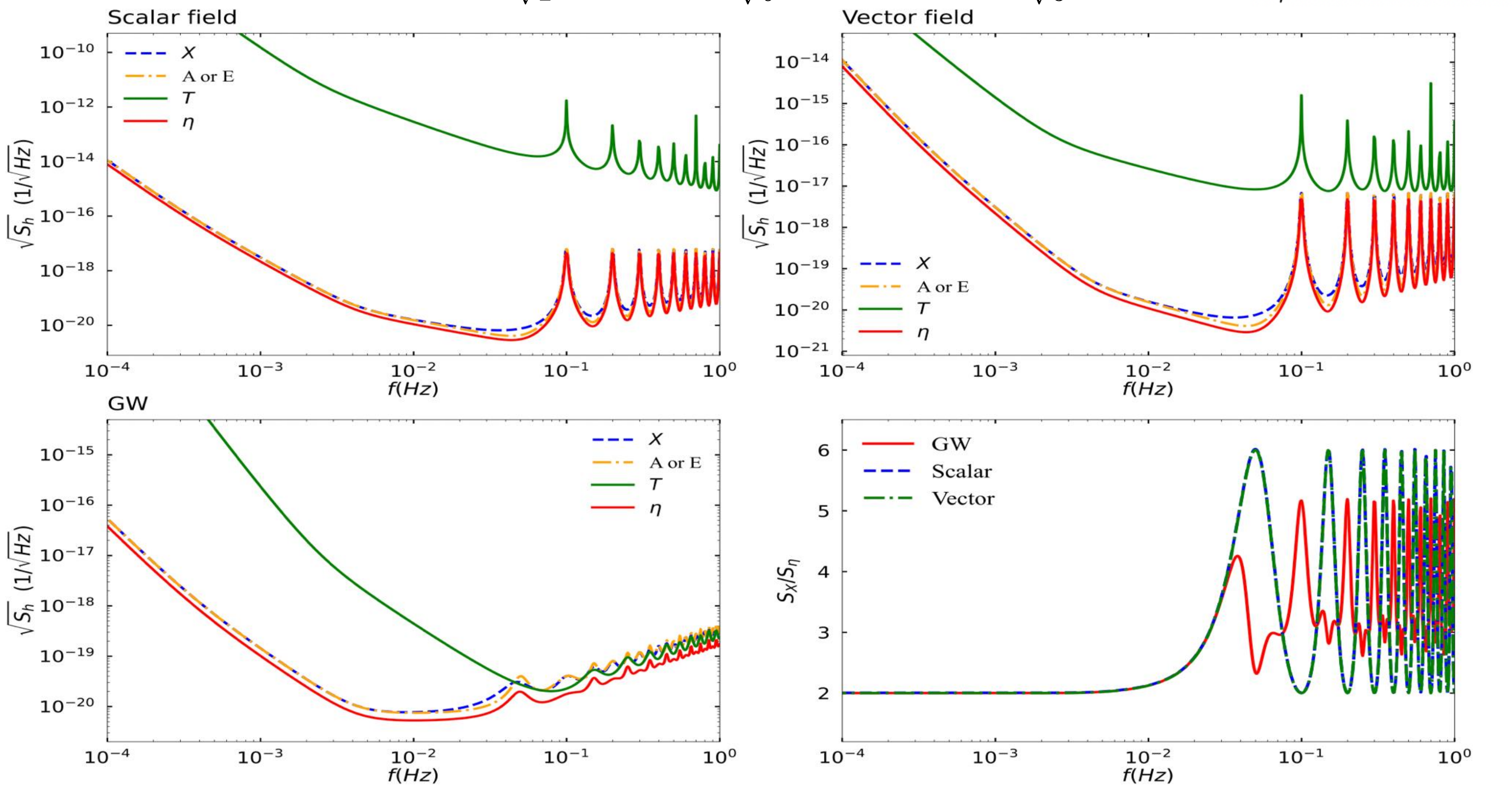


Yu, 姚跃辉, Tang, Wu, 2307.09197

Sensitivity – Other Interferometers

Prince, Tinto, Larson & Armstrong

- Optimal channels $A = \frac{1}{\sqrt{2}}\left[Z - X\right], E = \frac{1}{\sqrt{6}}\left[X - 2Y + Z\right], T = \frac{1}{\sqrt{3}}\left[X + Y + Z\right].$ $\frac{1}{S_\eta} = \frac{1}{S_A} + \frac{1}{S_E} + \frac{1}{S_T}$



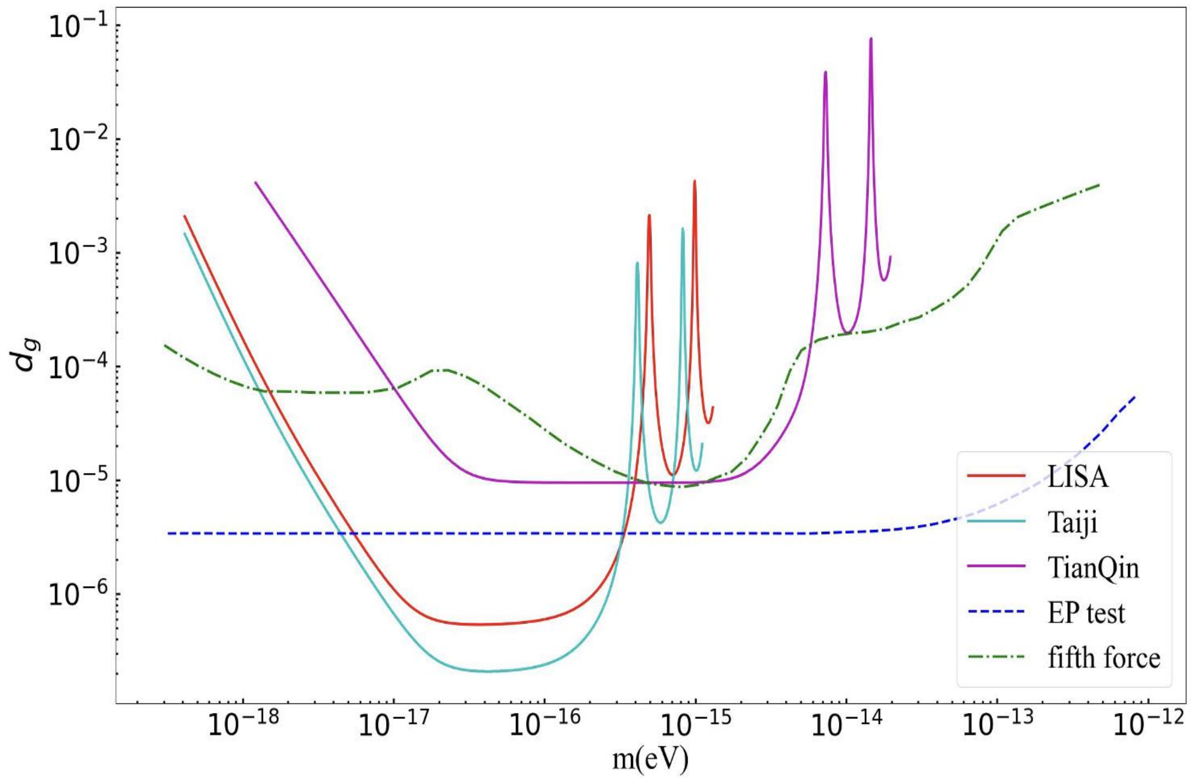
Sensitivity on scalar DM

- Strong sector $$\delta\mathcal{L} = \frac{\phi}{M_P}\left[-\frac{d_g\beta_3}{2g_3}F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=u,d}\left(d_{m_i} + \gamma_{m_i}d_g\right)m_i\bar{\psi}_i\psi_i\right]$$

Damour & Donoghue

- At nucleus level, coupling is not ∝ mass
- Equivalence principle is violated.
- MICROSCOPE
- Pulsars

Shao, Wex, Kramer 2019



Yu, 姚跃辉, Tang, Wu, 2307.09197

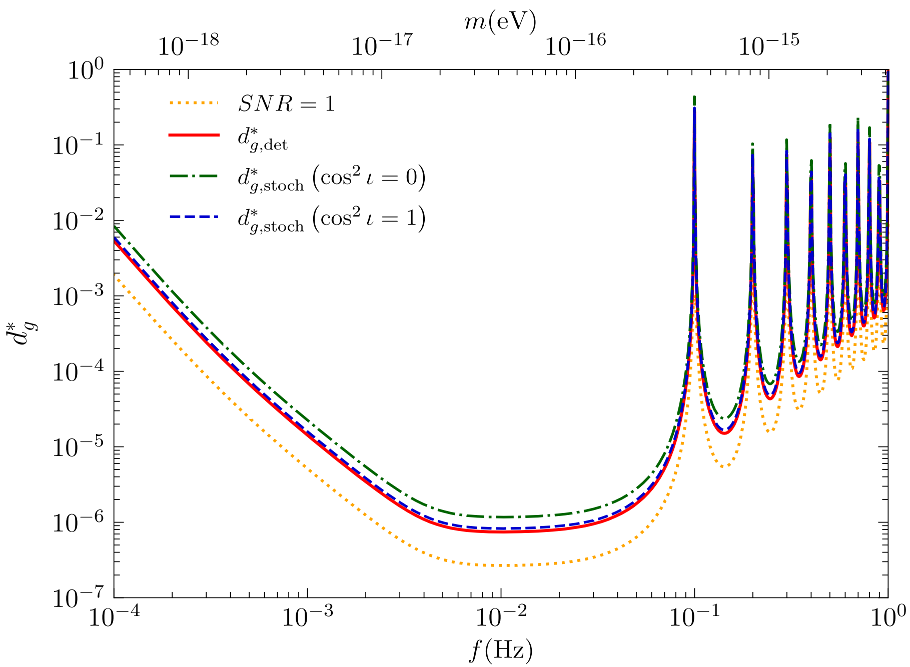
Statistical Effects

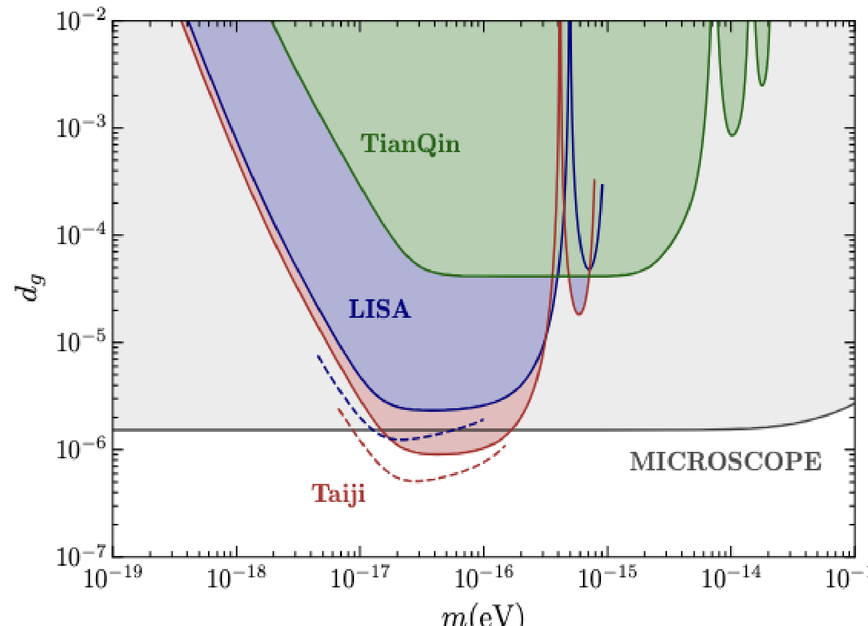
➢ Velocity distributions, likelihood analysis

$$S_O(\lambda_{\min}) = \left(\frac{\ln\alpha}{\ln\gamma} - 1\right) N_O, \quad \lambda_{\min}^2 = \frac{N_O}{\Gamma_O S_\Phi}\left(\frac{\ln\alpha}{\ln\gamma} - 1\right).$$

$$\gamma = \int_{P_*(\alpha)}^{\infty} dP\, \mathcal{L}_{\rm stoch}\left(\tilde{d}\middle| S_O(\lambda_{\min}), N_O\right),$$ detection probability

$$\alpha = \int_{P_*}^{\infty} dP\, \mathcal{L}_{\rm stoch}\left(P\middle|\lambda = 0, N_O\right).$$ false alarm rate

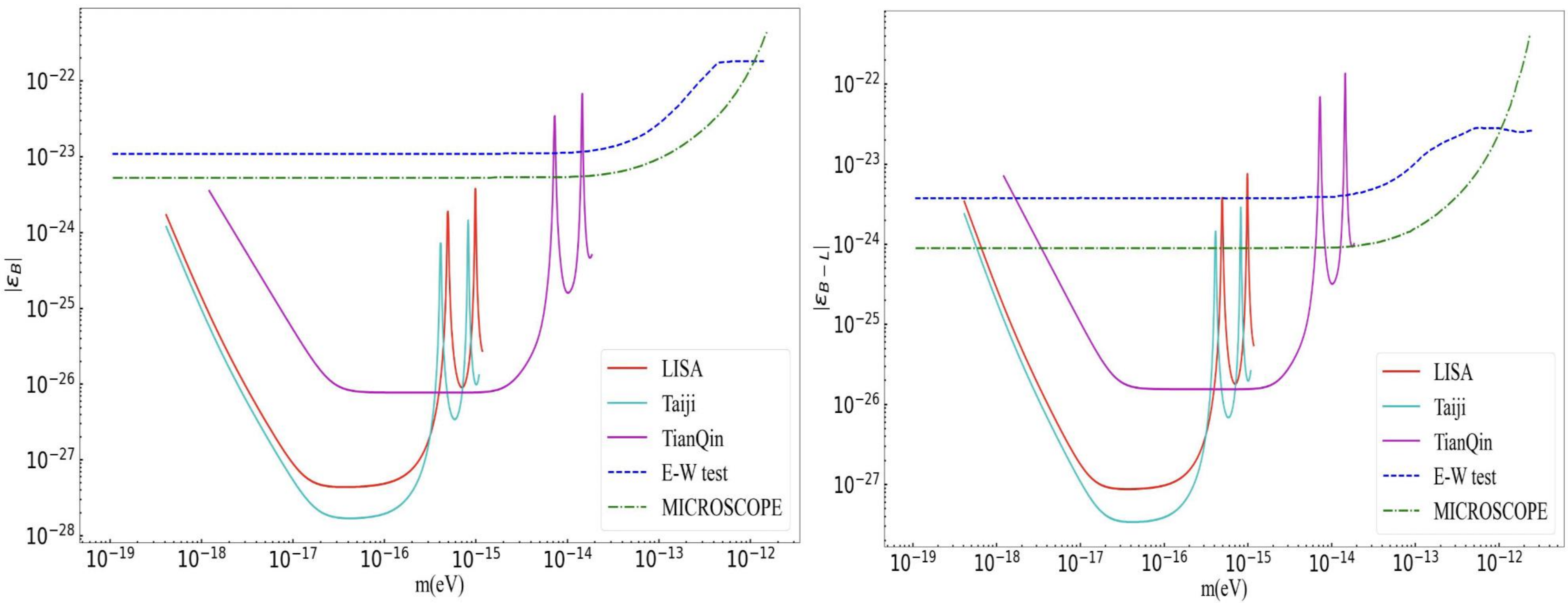




姚跃辉, Tang, 2404.01494(PRD)

Sensitivity on vector DM

- For example, vector fields couple to baryon number *B*, or *B-L*, effectively neutron number. Sensitivity on ratio $\epsilon_D = e_D/e$



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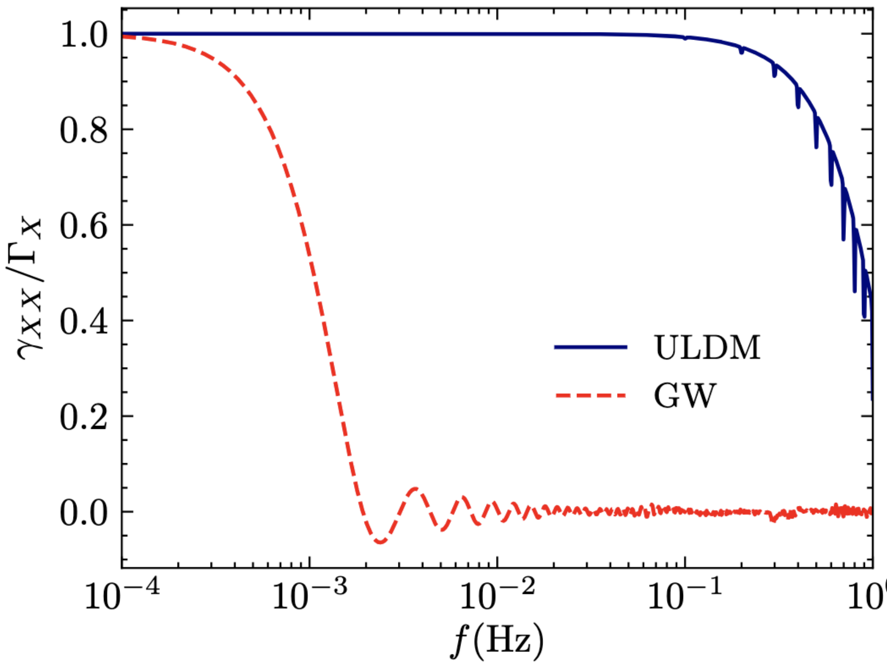
Correlation

- Network of detectors

$$\gamma_{ab}(f, v) := \int d\hat{k}\; \chi_{ab}(f, \hat{k}, v), \quad \lambda^2_{\min} = \frac{N}{2\Gamma_X S_\Phi}\left(\frac{\ln\alpha}{\ln\gamma} - 1\right).$$

- Correlation length is larger
- Sensitivity is enhanced by $\sqrt{n}$
- Different from SGWB searches which has $\sqrt{T}$ enhancement.
- How to distinguish them.
- Anisotropy and TDI



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Sensitivity on Axion-Photon Coupling

- Axion-photon coupling

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m^2 a^2 - \frac{g_{a\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu},$$

- Modifies Maxwell's eqs

$$\nabla \cdot \boldsymbol{E} = -g_{a\gamma}\nabla a \cdot \boldsymbol{B},$$

$$\frac{\partial \boldsymbol{E}}{\partial t} - \nabla \times \boldsymbol{B} = -g_{a\gamma}\left(\frac{\partial a}{\partial t} \cdot \boldsymbol{B} + \nabla a \times \boldsymbol{E}\right), \quad \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\boldsymbol{B} = g_{a\gamma}\dot{a}\nabla \times \boldsymbol{B}.$$

- Dispersion relation for two polarizations, $\omega^2 - k^2 = \pm g_{a\gamma}\dot{a}k$ Birefringence
- Axion dark matter background affects the phase speed of laser with different polarization, $v_\pm = \frac{\omega}{k} \simeq 1 \pm \frac{g_{a\gamma}\dot{a}}{2\omega}.$
- Equivalent phase changes
- Might be detected by LISA and Taiji detectors, etc
- Need modifications

Melissinos 2009,
DeRocco & Hook 2018
Nagano+2019
DarkGEO 2024
LIDA 2024, ···

Sensitivity on Axion-Photon Coupling

- Signal response $L = \int_{t-\Delta T_\pm}^{t} dt\, v_\pm = \Delta T_\pm \pm \frac{g_{a\gamma}}{2\omega}\left[a(t) - a(t-\Delta T_\pm)\right],$
- Equivalent phase changes

$$\Delta T_\pm \simeq L \mp \frac{g_{a\gamma}}{2\omega}\left[a(t) - a(t-L)\right].$$

$$\eta_{rs}(t) = -\frac{d\left(\Delta T_\pm\right)}{dt} = \pm\frac{im g_{a\gamma}}{2\omega}\left[a(t) - a(t-L)\right],$$

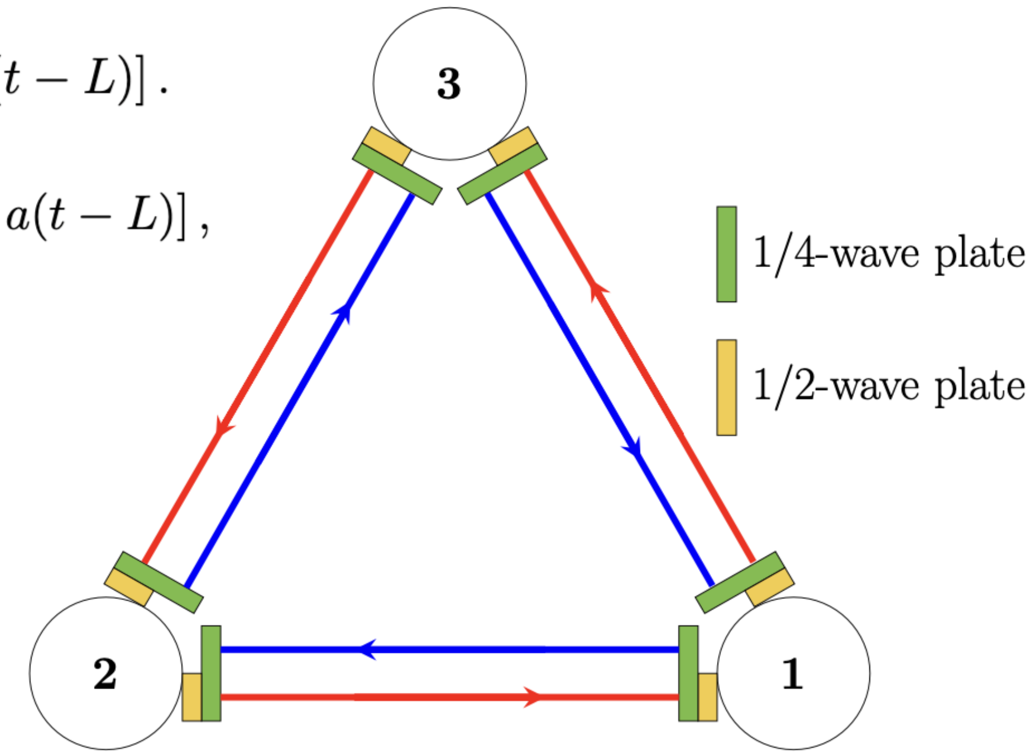
- Linearly polarized – pol angle

$$\boldsymbol{E}_r(t) = \begin{bmatrix}1\\ i\end{bmatrix}\frac{e^{i\omega(t-\Delta T_+)}}{2} + \begin{bmatrix}1\\ -i\end{bmatrix}\frac{e^{i\omega(t-\Delta T_-)}}{2}.$$

$$E_x = +\cos\left[g_{a\gamma}\frac{a(t)-a(t-L)}{2}\right]\cos[\omega(t-L)]$$

$$E_y = -\sin\left[g_{a\gamma}\frac{a(t)-a(t-L)}{2}\right]\cos[\omega(t-L)]$$

- Need modifications



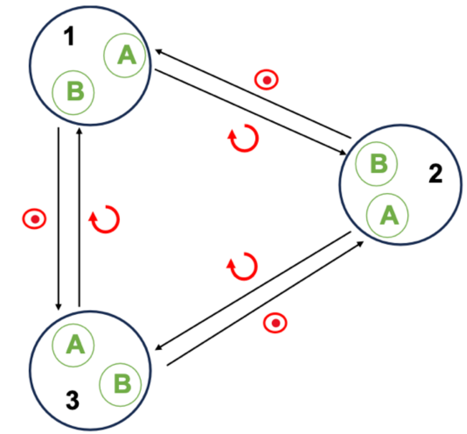
姚跃辉, Jiang, YT, 2410.22072

Sensitivity on Axion-Photon Coupling

- Signal response
- Equivalent phase changes

$$\Delta T_{\pm} \simeq L \mp \frac{g_{a\gamma}}{2\omega}\left[a(t) - a(t-L)\right].$$

$$\eta_{rs}(t) = -\frac{d\left(\Delta T_{\pm}\right)}{dt} = \pm\frac{i m g_{a\gamma}}{2\omega}\left[a(t) - a(t-L)\right],$$

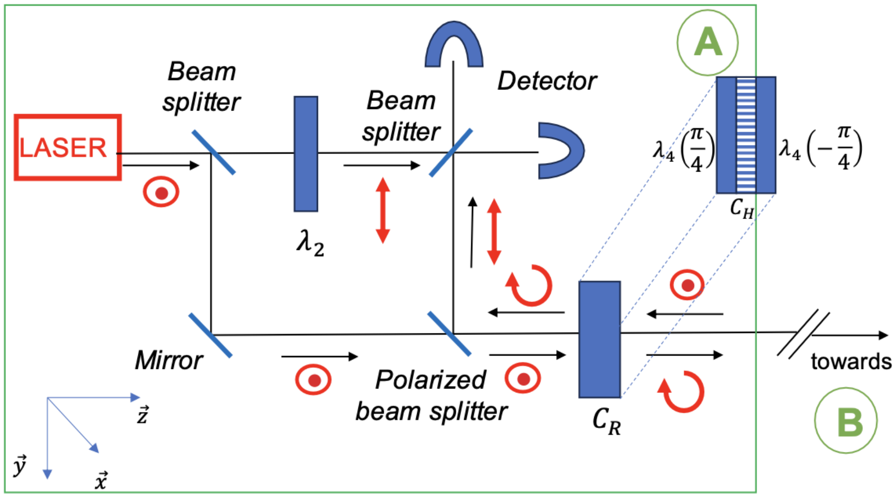


- Linearly polarized – pol angle

$$\boldsymbol{E}_r(t) = \begin{bmatrix} 1 \\ i \end{bmatrix} \frac{e^{i\omega(t-\Delta T_+)}}{2} + \begin{bmatrix} 1 \\ -i \end{bmatrix} \frac{e^{i\omega(t-\Delta T_-)}}{2}.$$

$$E_x = +\cos\left[g_{a\gamma}\frac{a(t)-a(t-L)}{2}\right]\cos[\omega(t-L)]$$

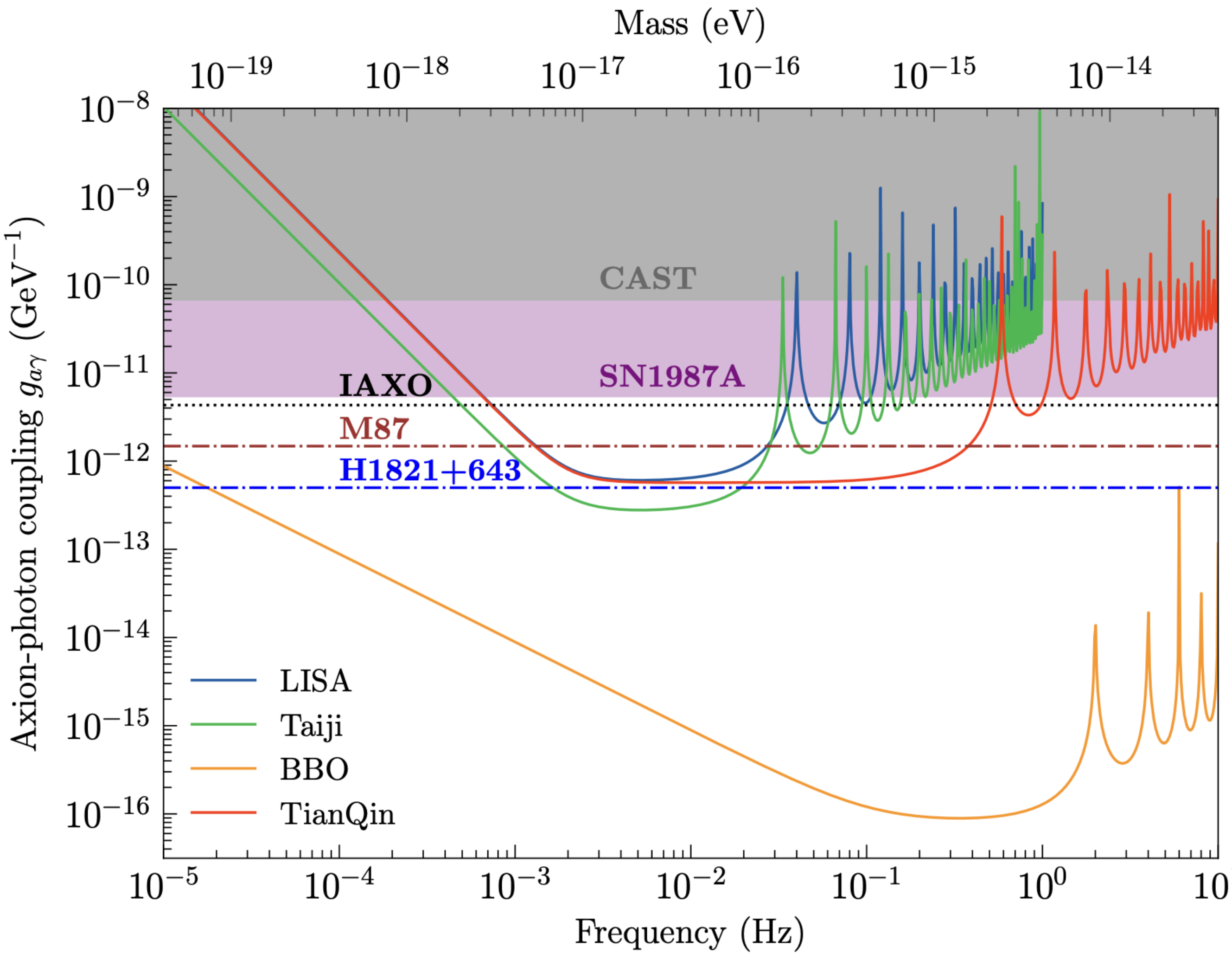
$$E_y = -\sin\left[g_{a\gamma}\frac{a(t)-a(t-L)}{2}\right]\cos[\omega(t-L)]$$



- Need modifications

Gue, Hees, Wolf, 2410.17763

Sensitivity on Axion-Photon Coupling



Yao, Jiang, YT,

2410.22072, Oct 29, 2024

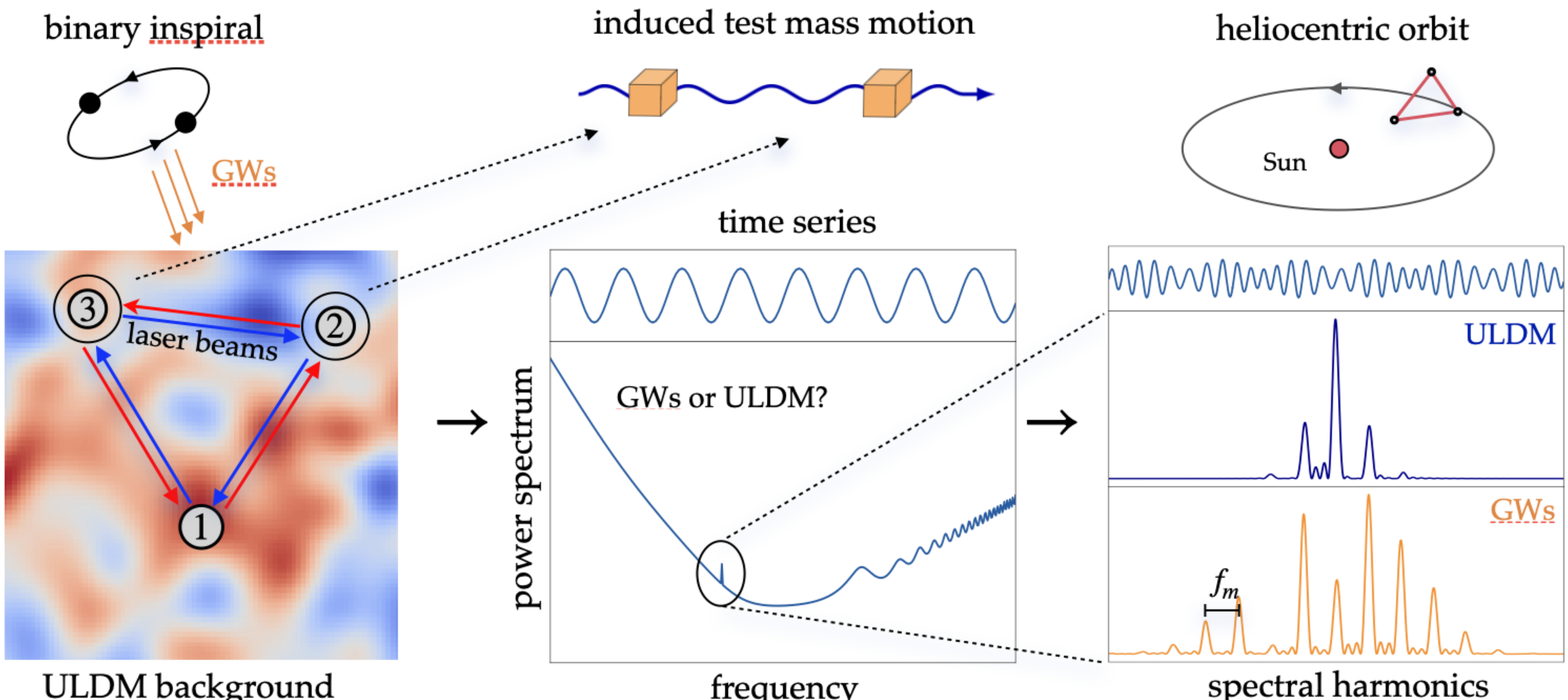
Gue, Hees, Wolf,

2410.17763, Oct 23, 2024

Yao, Bi, Yin, Huang, 2504.10083

Orbital Modulation

- Spectral split $y_{rs}(t) \supset \hat{n}_{rs} : \frac{\boldsymbol{h}(t_s - \hat{k}\cdot \mathbf{x}_s) - \boldsymbol{h}(t - \hat{k}\cdot \mathbf{x}_r)}{2(1-\hat{k}\cdot \hat{n}_{rs})} : \hat{n}_{rs},$ $y_{rs}(t) \supset g\hat{n}_{rs}\cdot[\mathbf{A}(t,\mathbf{x}_r) - \mathbf{A}(t_s,\mathbf{x}_s)],$



姚跃辉 *et al*, 2508.14655

Orbital Modulation

- Spectral split $y_{rs}(t) \supset \hat{n}_{rs} : \frac{\boldsymbol{h}(t_s - \hat{k}\cdot\mathbf{x}_s) - \boldsymbol{h}(t - \hat{k}\cdot\mathbf{x}_r)}{2(1-\hat{k}\cdot\hat{n}_{rs})} : \hat{n}_{rs},$ $y_{rs}(t) \supset g\hat{n}_{rs}\cdot[\mathbf{A}(t,\mathbf{x}_r) - \mathbf{A}(t_s,\mathbf{x}_s)],$

$$X(t) = (y_{13} + y_{31,2} + y_{12,22} + y_{21,322}) - (y_{12} + y_{21,3} + y_{13,33} + y_{31,233}),$$

- We adopt the Keplerian orbit

$$\begin{aligned}
x_i(t) &= r_\odot \cos\alpha + \frac{1}{2}er_\odot[\cos(2\alpha-\beta_i) - 3\cos\beta_i] \\
&+ \frac{1}{8}e^2 r_\odot[3\cos(3\alpha-2\beta_i) - 10\cos\alpha - 5\cos(\alpha-2\beta_i)], \\
y_i(t) &= r_\odot \sin\alpha + \frac{1}{2}er_\odot[\sin(2\alpha-\beta_i) - 3\sin\beta_i] \\
&+ \frac{1}{8}e^2 r_\odot[3\sin(3\alpha-2\beta_i) - 10\sin\alpha + 5\sin(\alpha-2\beta_i)], \\
z_i(t) &= -\sqrt{3}er_\odot\cos(\alpha-\beta_i) \\
&+ \sqrt{3}e^2 r_\odot\left[\cos^2(\alpha-\beta_i) + 2\sin^2(\alpha-\beta_i)\right], \qquad \text{(A1)}
\end{aligned}$$

e is the orbital eccentricity, $\alpha = 2\pi f_m t + \kappa$

$f_m = 1/\text{yr}$, $\beta_i = 2\pi(i-1)/3 + \lambda$

姚跃辉 *et al*, 2508.14655

Orbital Modulation

- Spectral split $y_{rs}(t) \supset \hat{n}_{rs} : \frac{\boldsymbol{h}(t_s - \hat{k}\cdot\mathbf{x}_s) - \boldsymbol{h}(t - \hat{k}\cdot\mathbf{x}_r)}{2(1-\hat{k}\cdot\hat{n}_{rs})} : \hat{n}_{rs}$, $y_{rs}(t) \supset g\hat{n}_{rs}\cdot[\mathbf{A}(t,\mathbf{x}_r) - \mathbf{A}(t_s,\mathbf{x}_s)]$,

$$X(t) = (y_{13} + y_{31,2} + y_{12,22} + y_{21,322}) - (y_{12} + y_{21,3} + y_{13,33} + y_{31,233}),$$

- For GWs, it can be rewritten as

$$X(t) \simeq 16\pi^2 f^2 L^2 \sum_{n=-\infty}^{+\infty} e^{i2\pi(f+nf_m)t} \times \sum_{l=-2j}^{2j} \mathcal{D}^{(n-l)}(f,\hat{k}) \sum_{p=+,\times} G_p^{(l)}(\hat{k}) H_p,$$

- For ULDM, it is

$$X(t) \simeq -2im^3L^3 \sum_{n=-j}^{j} e^{i2\pi(f_c+nf_m)t} \sum_p \mathcal{G}_p^{(n)} a_p(\mathbf{x}_1),$$

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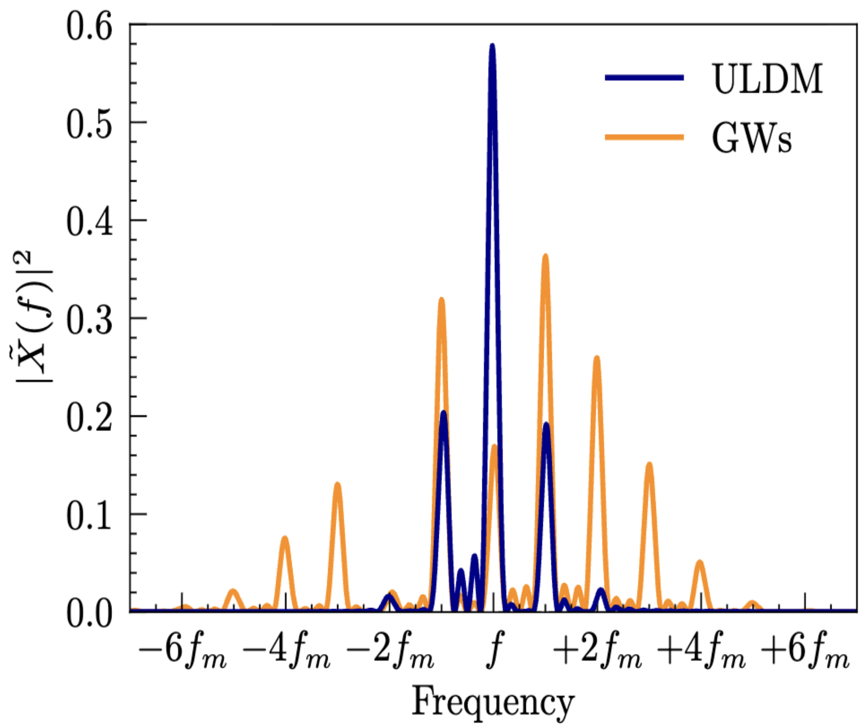
Orbital Modulation

- Spectral split $y_{rs}(t) \supset \hat{n}_{rs} : \frac{\boldsymbol{h}(t_s - \hat{k}\cdot\mathbf{x}_s) - \boldsymbol{h}(t - \hat{k}\cdot\mathbf{x}_r)}{2(1-\hat{k}\cdot\hat{n}_{rs})} : \hat{n}_{rs},$ $y_{rs}(t) \supset g\hat{n}_{rs}\cdot[\mathbf{A}(t,\mathbf{x}_r) - \mathbf{A}(t_s,\mathbf{x}_s)]\,,$

$$X(t) = (y_{13} + y_{31,2} + y_{12,22} + y_{21,322}) - (y_{12} + y_{21,3} + y_{13,33} + y_{31,233})\,,$$

n	$G_+^{(n)}$	$G_\times^{(n)}$
0	$-\frac{9\sqrt{3}}{128}[3+\cos(2\theta)]\sin(2(\phi-\lambda))$	$\frac{9\sqrt{3}}{32}\cos\theta\cos(2(\phi-\lambda))$
1	$-\frac{9i}{64}\sin(2\theta)e^{i(\kappa+\phi-2\lambda)}$	$-\frac{9}{32}\sin\theta e^{i(\kappa+\phi-2\lambda)}$
2	$\frac{9\sqrt{3}i}{128}[1-\cos(2\theta)]e^{i2(\kappa-\lambda)}$	0
3	$\frac{3i}{64}\sin(2\theta)e^{i(3\kappa-2\lambda-\phi)}$	$-\frac{3}{32}\sin\theta e^{i(3\kappa-2\lambda-\phi)}$
4	$\frac{\sqrt{3}i}{256}[3+\cos(2\theta)]e^{2i(2\kappa-\lambda-\phi)}$	$-\frac{\sqrt{3}}{64}\cos\theta e^{2i(2\kappa-\lambda-\phi)}$

n	$\mathcal{G}_x^{(n)}$	$\mathcal{G}_y^{(n)}$	$\mathcal{G}_z^{(n)}$
0	$-\frac{3}{4}\sin\lambda$	$\frac{3}{4}\cos\lambda$	0
1	0	0	$-\frac{\sqrt{3}i}{4}e^{i(\kappa-\lambda)}$
2	$\frac{i}{8}e^{i(2\kappa-\lambda)}$	$\frac{1}{8}e^{i(2\kappa-\lambda)}$	0



姚跃辉 *et al*, 2508.14655

Null-Response Channel

- For a source from some direction, we construct a null-response channel that is insensitive to ULDM or GWs,

$$\tilde{\eta} = a_1\tilde{\alpha} + a_2\tilde{\beta} + a_3\tilde{\gamma}.$$

- The Signal-to-Noise Ratio (SNR)

$$\mathrm{SNR}_\eta^2 = \int df \, \frac{\left|a_1\tilde{\alpha}_s + a_2\tilde{\beta}_s + a_3\tilde{\gamma}_s\right|^2}{\mathbf{a}^\dagger \mathbf{S}_\alpha(f)\mathbf{a}},$$

- SNR = 0 gives

$$a_1\tilde{\alpha}_s + a_2\tilde{\beta}_s + a_3\tilde{\gamma}_s = 0.$$

徐恒弢, 姚跃辉, Tang, Wu, 2506.09744

Null-Response Channel

- In GR, two polarizations of GW, the response can be written as

$$\tilde{\alpha}_s = \alpha_+^{\rm gw}(f,\hat{k})\tilde{h}_+(f) + \alpha_\times^{\rm gw}(f,\hat{k})\tilde{h}_\times(f),$$

- The condition $a_1\tilde{\alpha}_s + a_2\tilde{\beta}_s + a_3\tilde{\gamma}_s = 0$ can be satisfied by

$$\left(\mathbf{a}\cdot\mathbf{x}_+^{\rm gw}(\hat{k})\right)\tilde{h}_+ + \left(\mathbf{a}\cdot\mathbf{x}_\times^{\rm gw}(\hat{k})\right)\tilde{h}_\times = 0,$$

- Where

$$\mathbf{x}_p^{\rm gw}(\hat{k}) = [\alpha_p^{\rm gw}(\hat{k}), \beta_p^{\rm gw}(\hat{k}), \gamma_p^{\rm gw}(\hat{k})]^T$$

- This can be solved by

$$\mathbf{a}_c^{\rm gw}(\hat{k}) = \mathbf{x}_+^{\rm gw}(\hat{k}) \times \mathbf{x}_\times^{\rm gw}(\hat{k}).$$

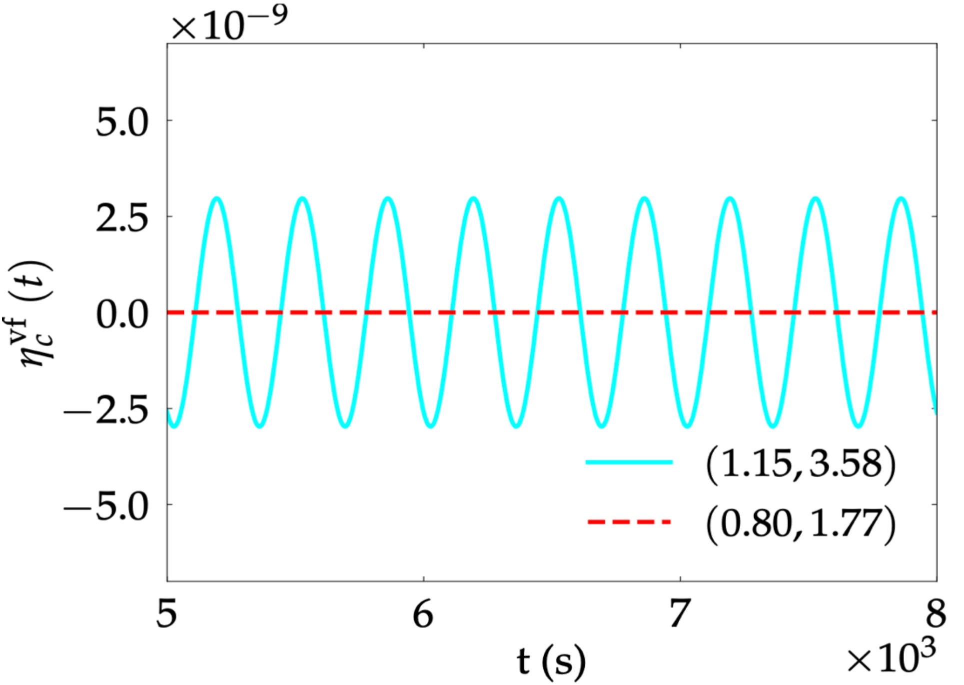
- So NRC is given by $\tilde{\eta}_c^{\rm gw}(f,\hat{k};\hat{k}_c) = \mathbf{a}_c^{\rm gw}(\hat{k}_c)\cdot[\tilde{\alpha},\tilde{\beta},\tilde{\gamma}]$

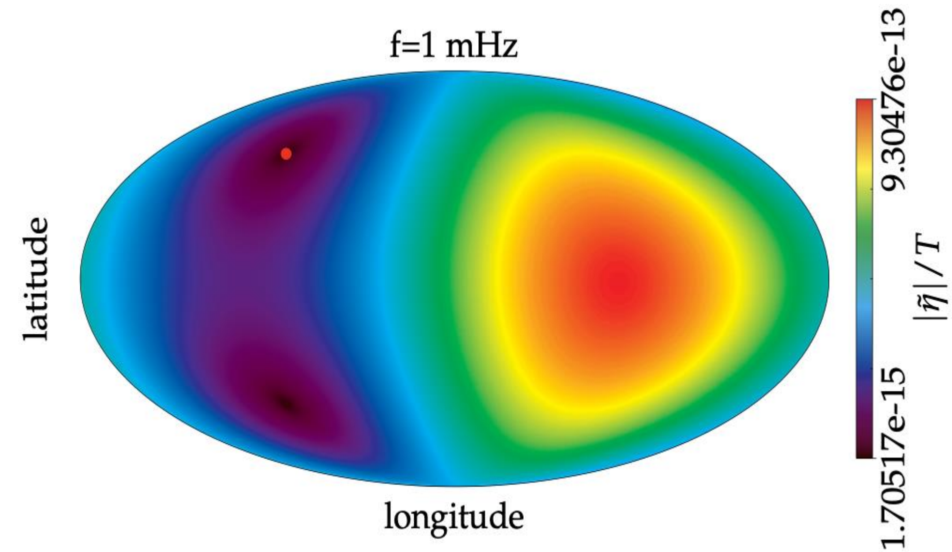
徐恒弢, 姚跃辉, Tang, Wu, 2506.09744

Null-Response Channel

- For a source from some direction, we construct a null-response channel that is insensitive to ULDM or GWs,

$$\tilde{\eta} = a_1\tilde{\alpha} + a_2\tilde{\beta} + a_3\tilde{\gamma}.$$





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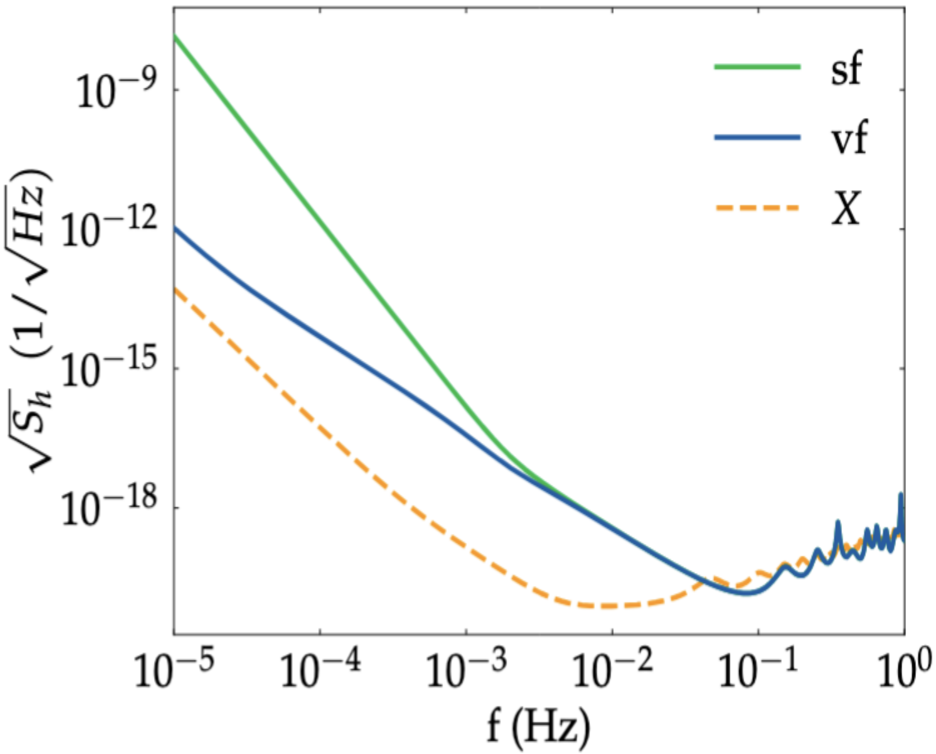
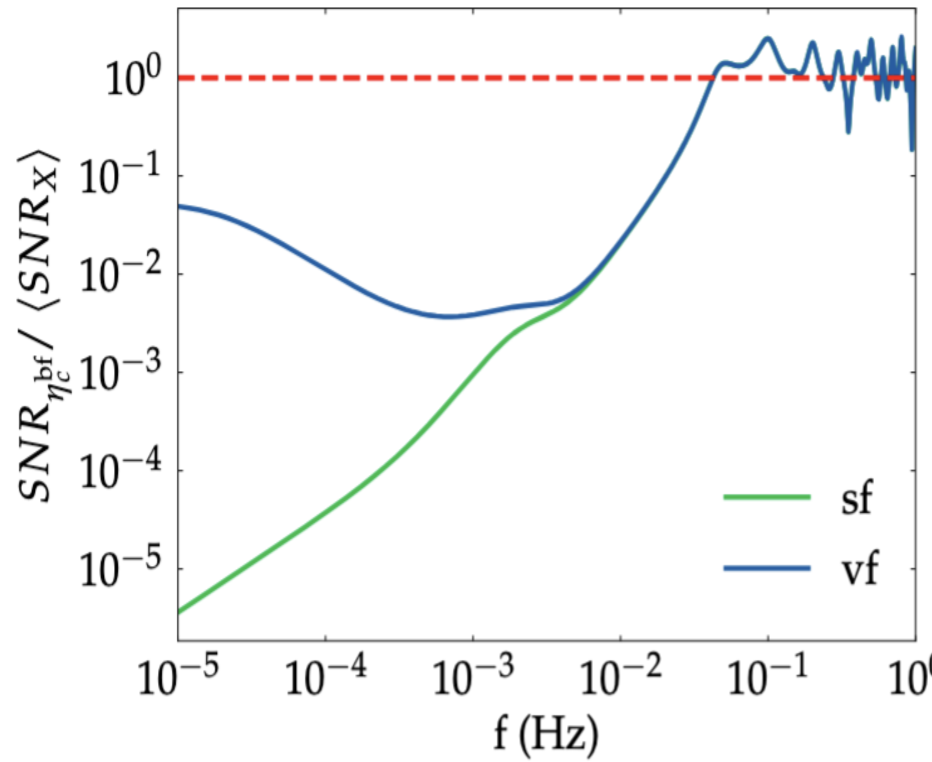
Null-Response Channel

➢ For a source from some direction, we construct a null-response channel that is insensitive to ULDM or GWs





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Detecting Ultralight Dark Matter Gravitationally

- Can we probe DM with gravitational interaction in solar system
- What if dark matter has only gravitational interaction?
- Motivations from theories of gravity, particle physics, ···
- Viable scenarios in cosmology
  - Axion or axion-like particle, misalignment
  - Dark photon, vacuum fluctuation

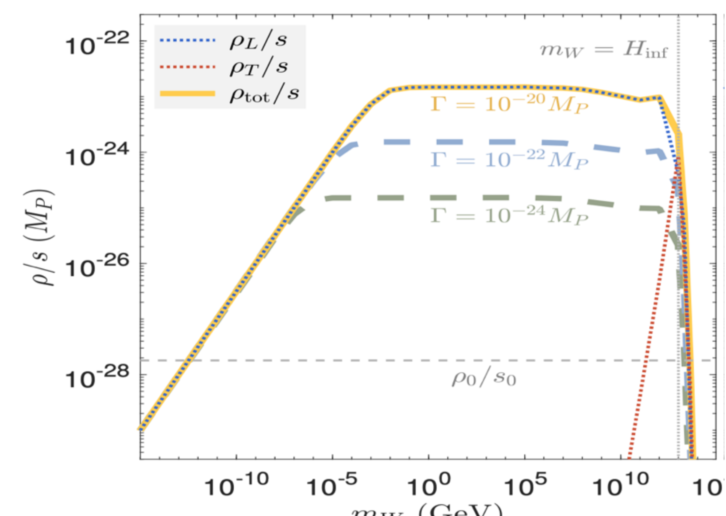
Graham, Mardon, Rajendran,1504.02102(PRD)

Ema, Nakayama, Tang, 1903.10973(JHEP),

Ahmed, Grzadkowski, Socha, 2005.01766(JHEP)

Kolb, Long, 2009.03828(JHEP), ···

王清扬, Tang, Wu, 2203.15452(PRD)



Detecting Ultralight Dark Matter Gravitationally

- Metric perturbation in solar system

$$ds^2 = -(1+2\Psi)dt^2 + (1-2\Phi)\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j$$

PTA, e.g. Khmelnitsky & Rubakov 2014,⋯

- Einstein equations

$$\partial_i \partial^i \Phi = 4\pi G T_{00},$$

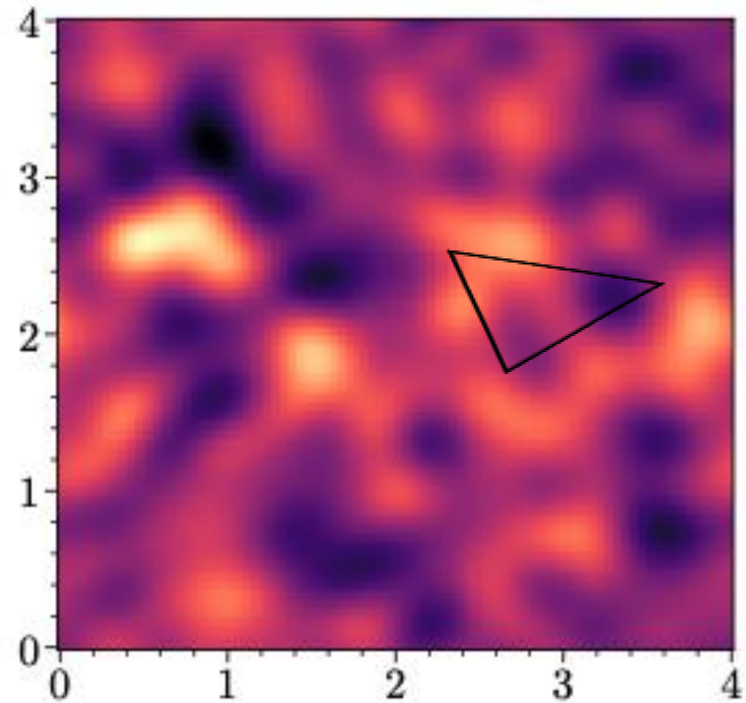
$$3\ddot{\Phi} + \partial_i \partial^i (\Psi - \Phi) = 4\pi G T^k_k,$$

$$\ddot{h}_{ij} = 16\pi G \left(T_{ij} - \frac{1}{3}\delta_{ij} T^k_k \right),$$

$$\Psi^j \simeq \Phi^j \simeq \pi G \frac{\rho}{m^2} = \frac{7\times 10^{-26}\rho}{0.4\,\mathrm{GeV/cm^3}} \left(\frac{10^{-18}\mathrm{eV}}{m}\right)^2,$$

$$h^v_{ij} \propto h_0 \simeq \frac{8}{3}\pi G \frac{\rho}{m^2} = \frac{2\times 10^{-25}\rho}{0.4\,\mathrm{GeV/cm^3}} \left(\frac{10^{-18}\mathrm{eV}}{m}\right)^2,$$

$$h^s_{ij} \simeq h_0 v^2/2$$



- Perturbation for scalar is suppressed.

H. Kim 2023

- Not for h_{ij} from vector

Nomura, Ito & Soda 2020

Detecting Ultralight Dark Matter Gravitationally

➢ Tensor perturbation

$$S_\Phi^s \simeq \frac{64}{15}\kappa^4\rho^2 v^2 L^4 T\left[v^2\sin^2\gamma + 5m^2L^2\sin^2\frac{\gamma}{2}\right],$$

scalar

$$S_h^v(\epsilon_{ij}) \simeq \frac{256}{9}\kappa^4\rho^2 L^4 T[(\hat{n}_{12}^i\hat{n}_{12}^j - \hat{n}_{13}^i\hat{n}_{13}^j)\epsilon_{ij}]^2$$

vector

TABLE I. Arm lengths and instrumental noises of several planned laser interferometers in space. In the last row we give the sensitivities on vector ULDM with mass 5.0×10^{-19} eV. Here L is the nominal arm length of triangle constellation, while $s_{\rm acc}$ and $s_{\rm oms}$ are the acceleration noise of test mass and noise from optical metrology system, respectively. Note that LISA/LISAmax/Taiji/Tianqin adopt frequency-dependent noise power spectra [101], $s_{\rm acc}^2 \propto [1 + (0.4 \times 10^{-3}\ {\rm Hz}/f)^2][1 + (f/8 \times 10^{-3}\ {\rm Hz})^4]$ and $s_{\rm oms}^2 \propto 1 + (2 \times 10^{-3}\ {\rm Hz}/f)^4$.

	LISA	Taiji	Tianqin	BBO	DECIGO	μAres	LISAmax	ASTROD-GW
L (10^9 m)	2.5	3	0.17	0.05	1×10^{-3}	395	260	260
$s_{\rm acc}$ ($10^{-15}\ \frac{\rm m/s^2}{\sqrt{\rm Hz}}$)	3	3	1	3×10^{-2}	4×10^{-4}	1	3	3
$s_{\rm oms}$ ($10^{-12}\ \frac{\rm m}{\sqrt{\rm Hz}}$)	15	8	1	1.4×10^{-5}	2×10^{-6}	50	15	100
$\frac{\rho}{\rho_0}$ (5.0×10^{-19} eV)	7.95×10^2	6.53×10^2	3.82×10^3	2.00×10^2	1.34×10^2	0.44	7.67	4.32

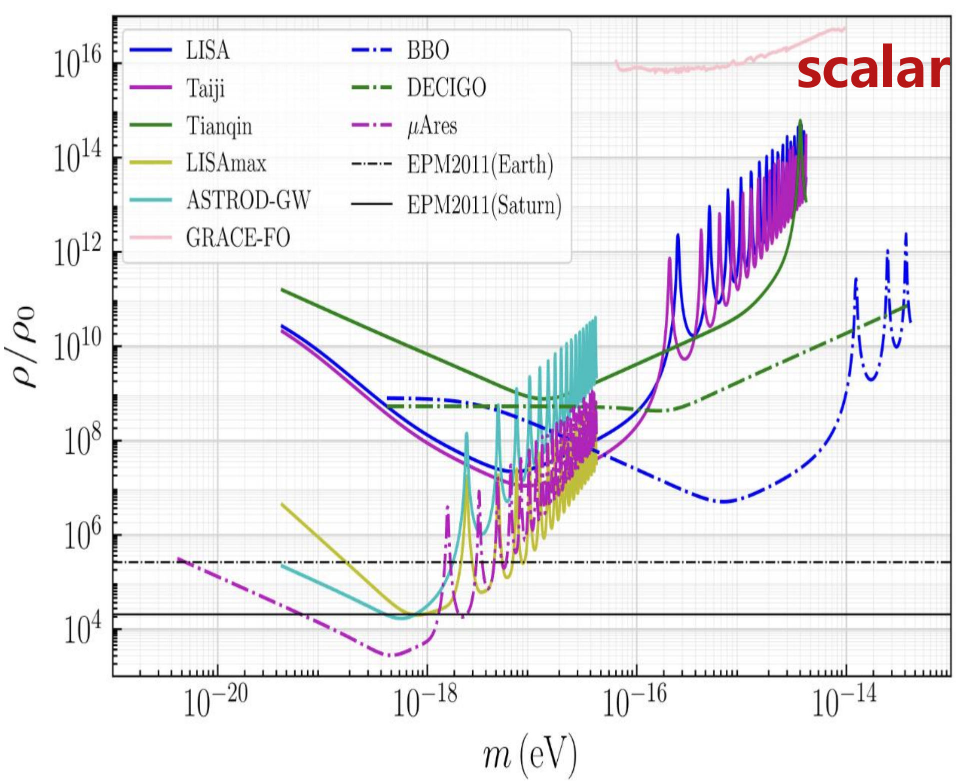
Yu, Cao, Tang, Wu, 2404.04333

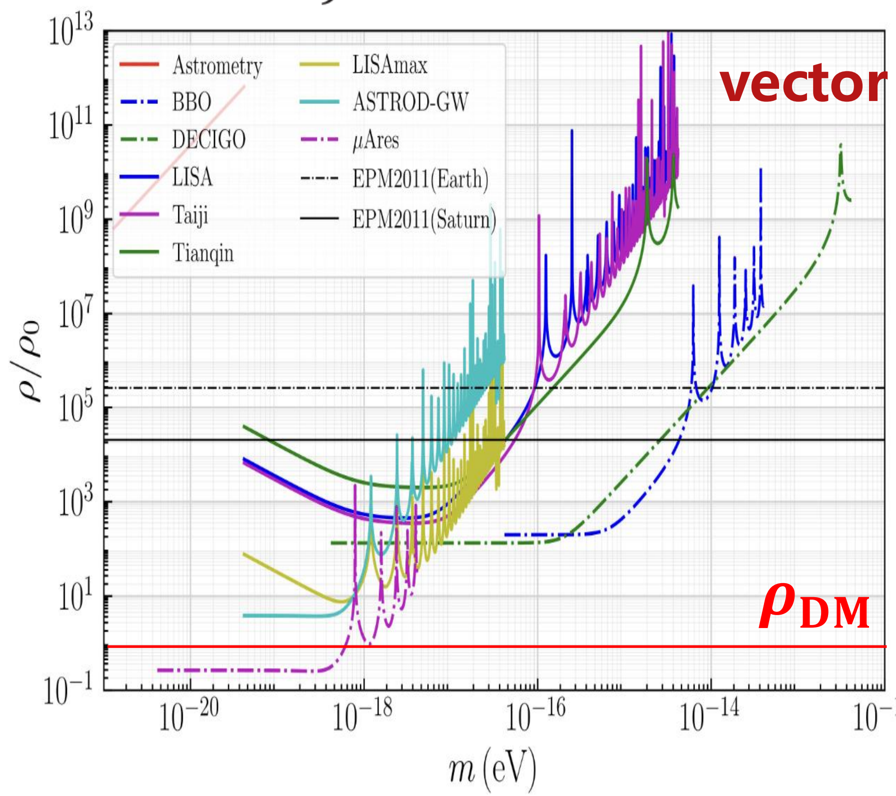
Detecting Ultralight Dark Matter Gravitationally

- Tensor perturbation

$$S^s_\Phi \simeq \frac{64}{15}\kappa^4\rho^2 v^2 L^4 T\left[v^2\sin^2\gamma + 5m^2L^2\sin^2\frac{\gamma}{2}\right],$$

$$S^v_h(\epsilon_{ij}) \simeq \frac{256}{9}\kappa^4\rho^2 L^4 T[(\hat{n}^i_{12}\hat{n}^j_{12} - \hat{n}^i_{13}\hat{n}^j_{13})\epsilon_{ij}]^2$$





Yu, Cao, Tang, Wu, 2404.04333

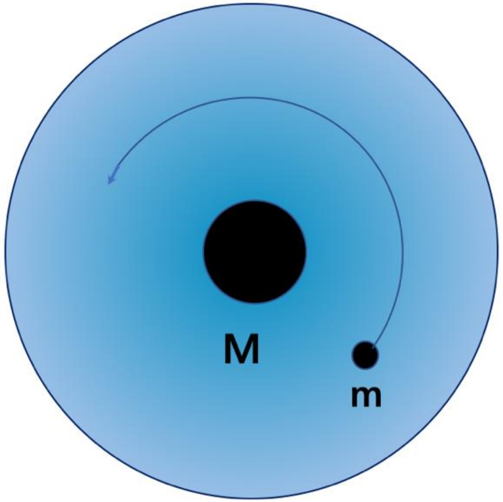
WIMP DM Spikes around Black Holes

- WIMP DM particles accretion around BH → DM spike
- NFW profile → spiky density profile **Gondolo & Silk (1999)**

$$\rho(r) \propto r^{-\gamma}, 0 \leq \gamma \leq 2 \quad \Rightarrow \quad \rho_{\rm spike}(r) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\alpha}, \quad \alpha = \frac{9-2\gamma}{4-\gamma}$$

- Dynamical friction → Gravitational wave

Extreme-mass-ratio Inspiral (EMRI)

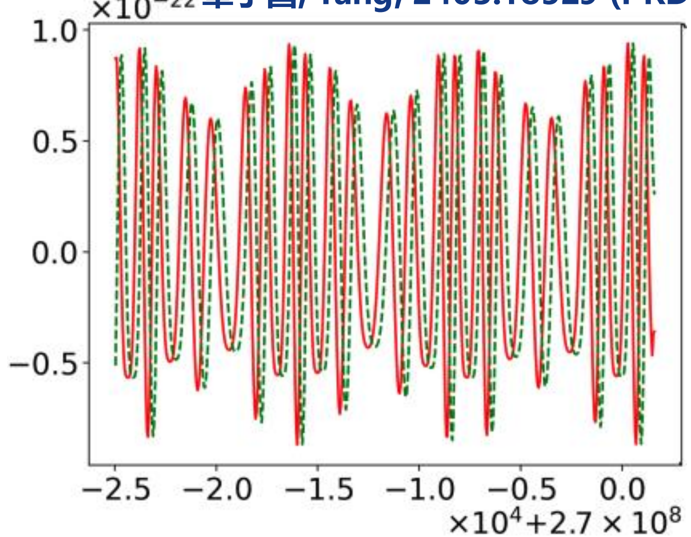


$$m\frac{d^2\mathbf{x}}{dt^2} = \mathbf{F}_{\rm G} + \mathbf{F}_{\rm DF}$$

$$h_{ij} \sim \frac{G}{d}\frac{d^2 Q_{ij}}{dt^2},$$

$$Q_{ij} \sim m\left(x_i x_j - \frac{1}{3}x^2\delta_{ij}\right)$$

章子昌, Tang, 2403.18529 (PRD)



Eda, Itoh, Kuroyanagi & Silk, Yue & Cao, Yue, Han & Chen , Gong, Bertone et al ... ...

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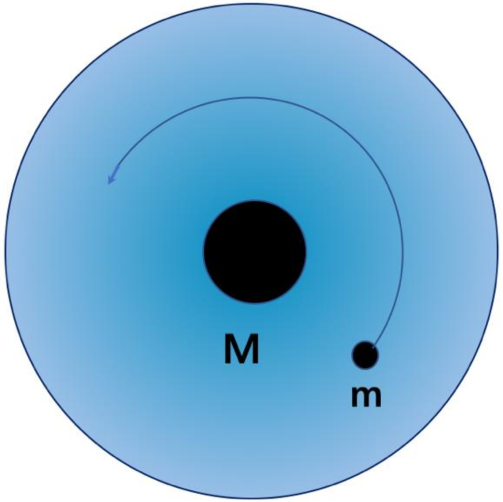

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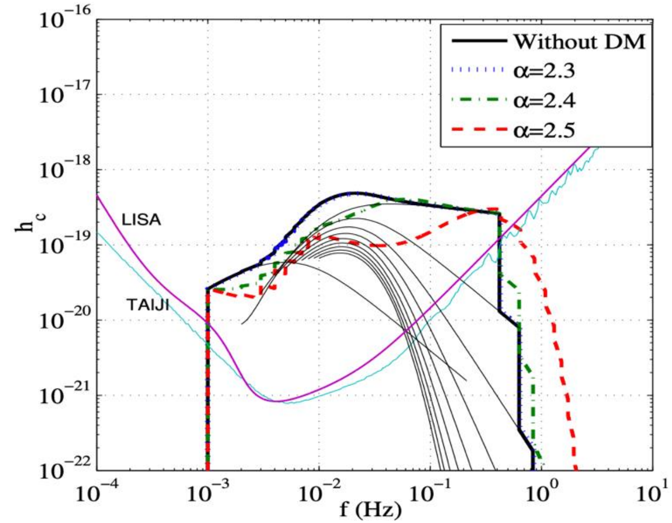
Li, Tang, Wu, 2112.14041



Eda, Itoh, Kuroyanagi & Silk, Yue & Cao, Yue, Han & Chen , Gong, Bertone et al ... ...

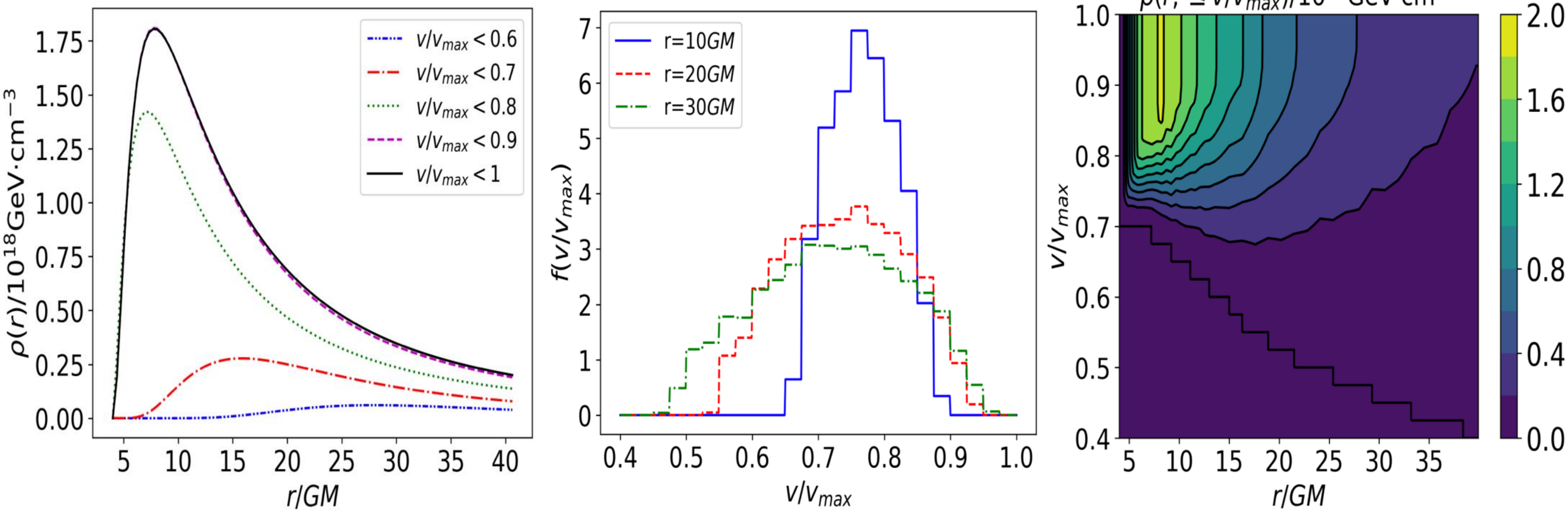
Density and Velocity Distribution

- Relativistic treatment (Will et al)

$$\rho(r, \alpha = 1) = \frac{\kappa}{(r/GM)^{\omega}} \left(1 - \frac{4GM}{r}\right)^{\eta}.$$

Halo parameters (M_{halo}, r_s)	Index (γ)	Fitting parameters $(\kappa,\ \eta\ ,\ \omega)$
$(10^{12} M_\odot, 20\ \text{kpc})$	-	$(5.33 \times 10^{20}\ \text{GeV/cm}^3,\ 1.99,\ 2.07)$
$(4.5 \times 10^{8} M_\odot, 1.85\ \text{kpc})$	-	$(6.15 \times 10^{24}\ \text{GeV/cm}^3,\ 2.03,\ 2.11)$
$(7.3 \times 10^{8} M_\odot, 1.85\ \text{kpc})$	7/4	$(5.83 \times 10^{26}\ \text{GeV/cm}^3,\ 2.04,\ 2.16)$

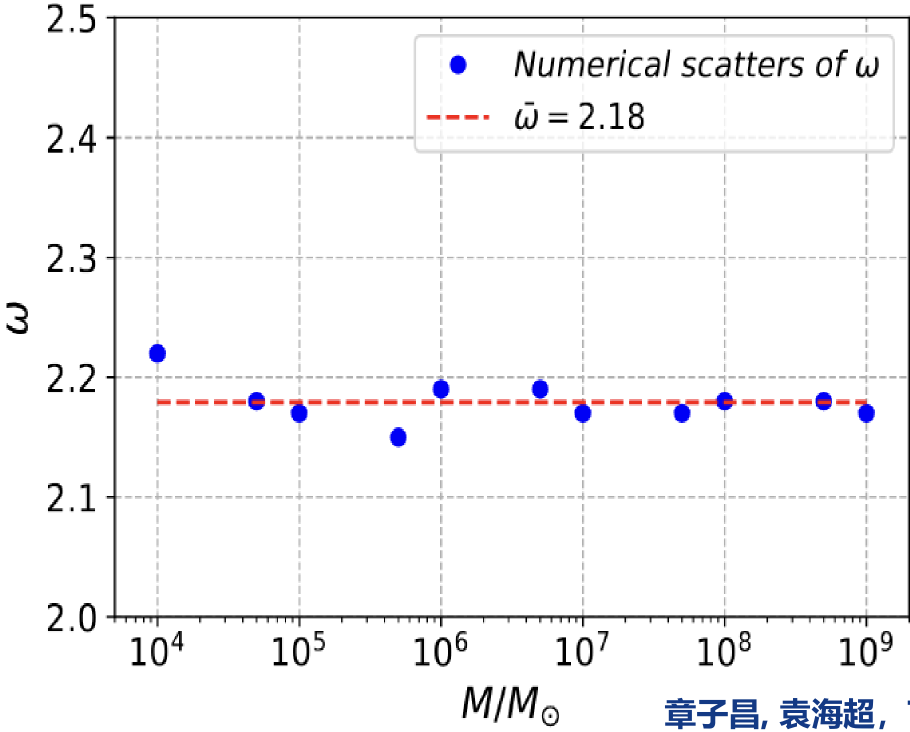
章子昌, Tang, 2403.18529 (PRD)

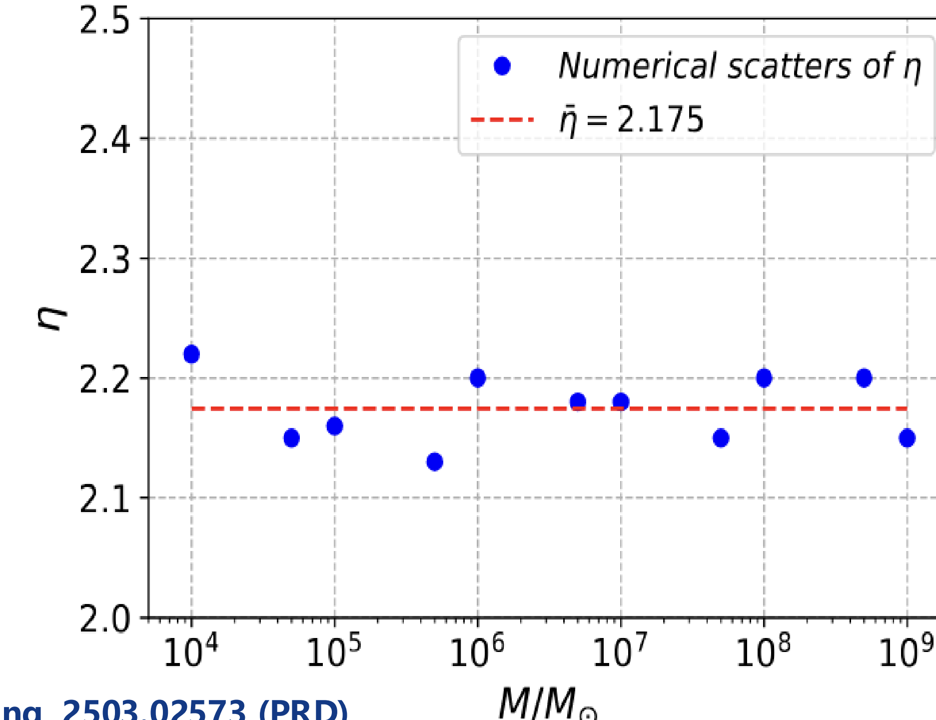


Dark Matter Spike

➢ Universal profile $\rho(r) = \frac{\kappa(\mathrm{GeV}\cdot\mathrm{cm}^{-3})}{(r/GM)^{\omega}}\left(1-\frac{4GM}{r}\right)^{\eta}$

$$\log\left(\kappa/(\mathrm{GeV}\cdot\mathrm{cm}^{-3})\right) = a\cdot\log(M/M_{\odot}) + b \qquad (a,b) = (-1.62, 31.5).$$





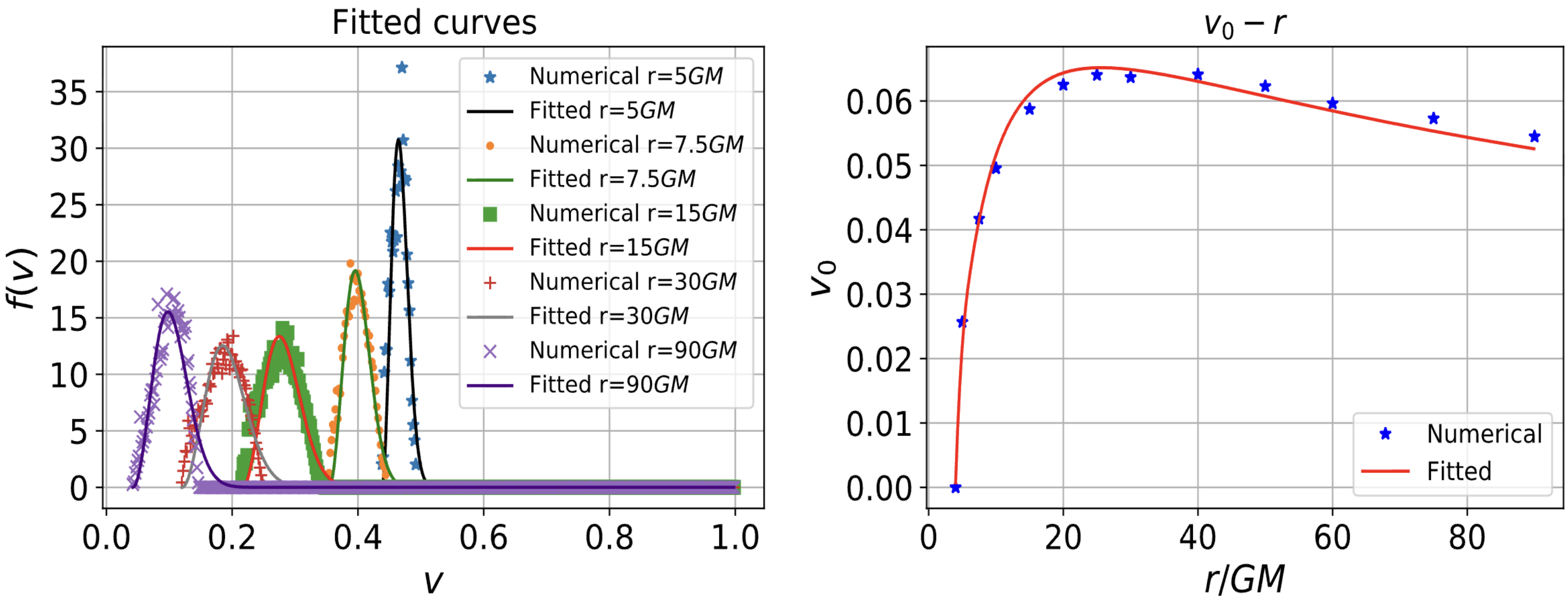
章子昌, 袁海超，Tang, 2503.02573 (PRD)

Velocity Distribution

- Fitted with Maxwell-Boltzmann distribution

$$f_r(v) = \frac{4\pi\left[v - v_{\min}(r)\right]^2}{\left[\pi v_0^2(r)\right]^{\frac{3}{2}}} e^{-[v-v_{\min}(r)]^2/v_0^2(r)},$$

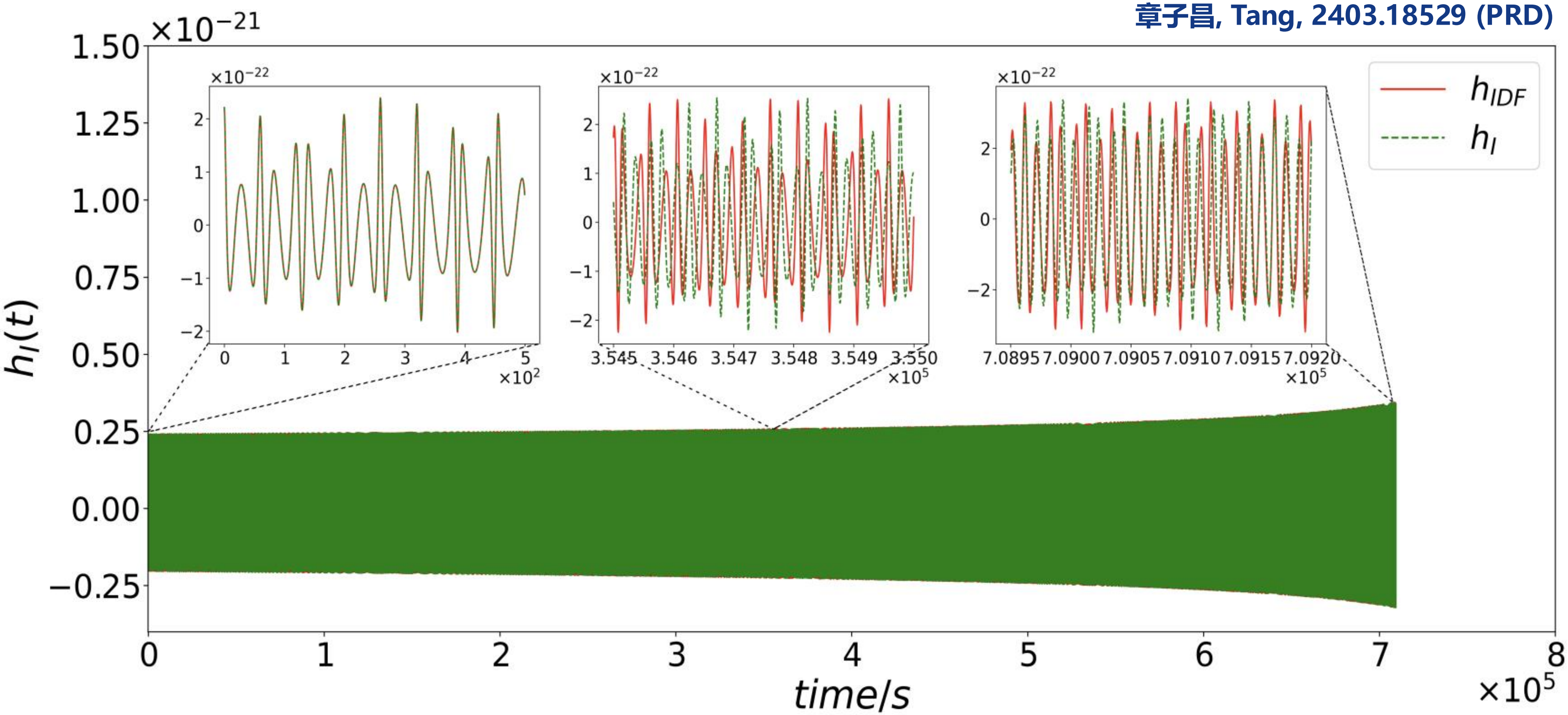
章子昌, 袁海超, Tang, 2503.02573(PRD)



Effects on EMRI

- Phase shift of the waveform of GW

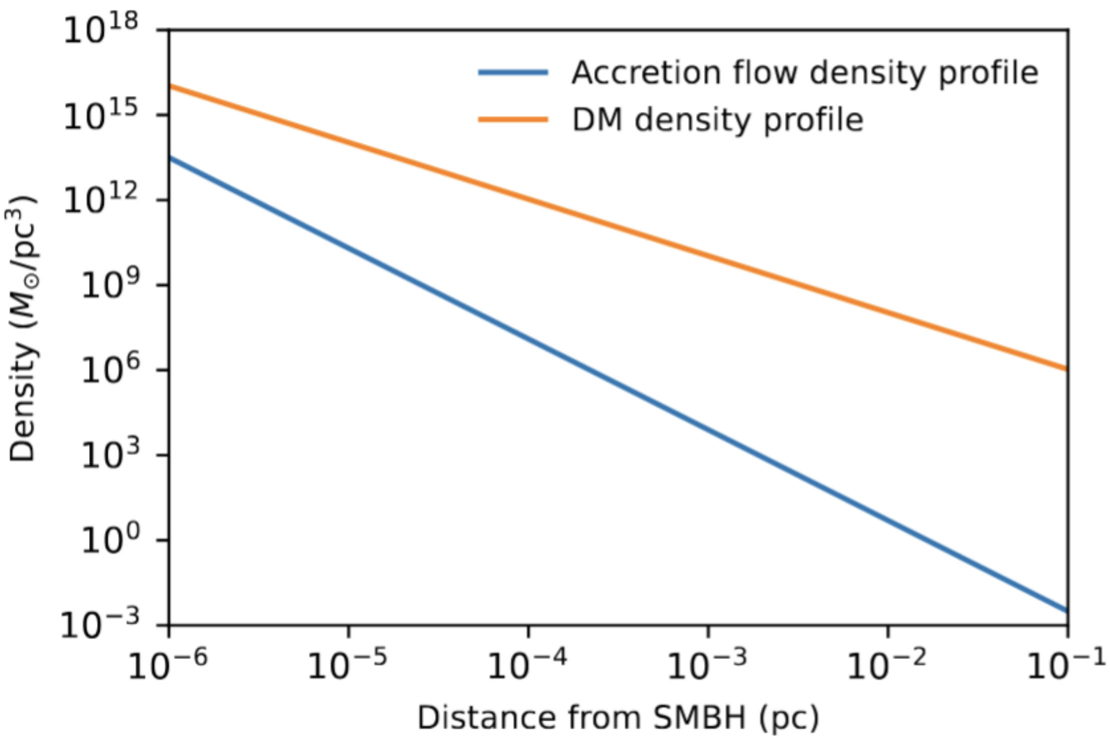
章子昌, Tang, 2403.18529 (PRD)



Effects on Early EMRIs

- In Milky Way center, SMBH with mass $4 \times 10^6 M_{\odot}$, possible many EMRIs at early stages, GW background

Feng, Tang, Wu, 2506.02937



$$\rho(r) = \rho_{\rm c} \left(\frac{r_{\rm c}}{r} \right)^{\gamma}, \ \gamma = \frac{9-2\beta}{4-\beta},$$

$$\left\langle \frac{da}{dt} \right\rangle = \left\langle \frac{da}{dt} \right\rangle_{\rm df} + \left\langle \frac{da}{dt} \right\rangle_{\rm gw},$$
$$\left\langle \frac{de}{dt} \right\rangle = \left\langle \frac{de}{dt} \right\rangle_{\rm df} + \left\langle \frac{de}{dt} \right\rangle_{\rm gw}.$$

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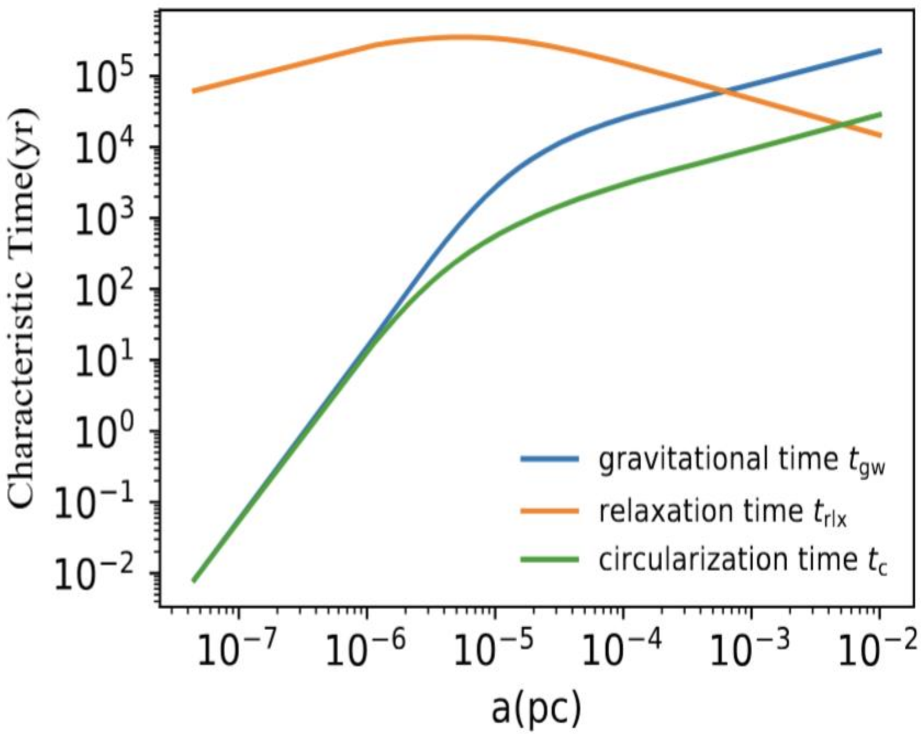

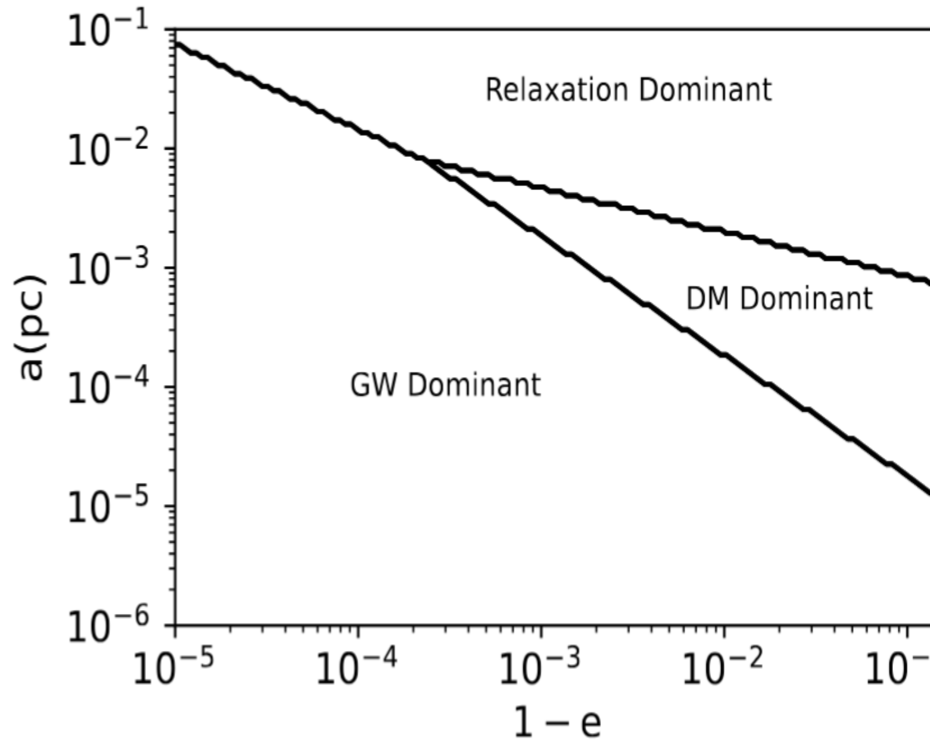

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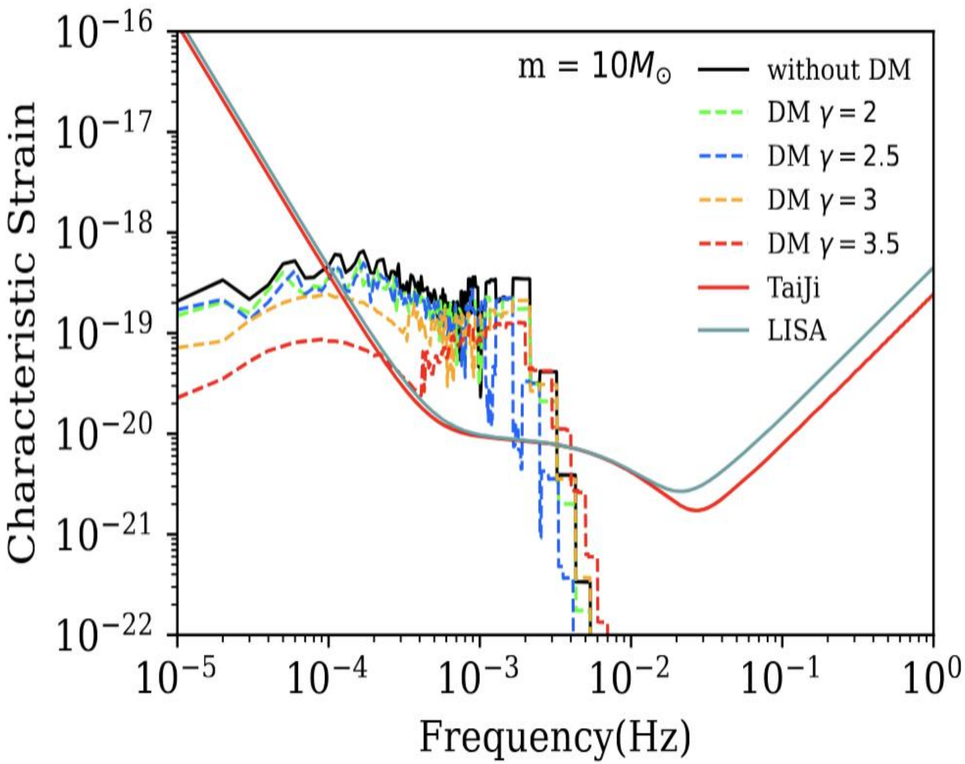

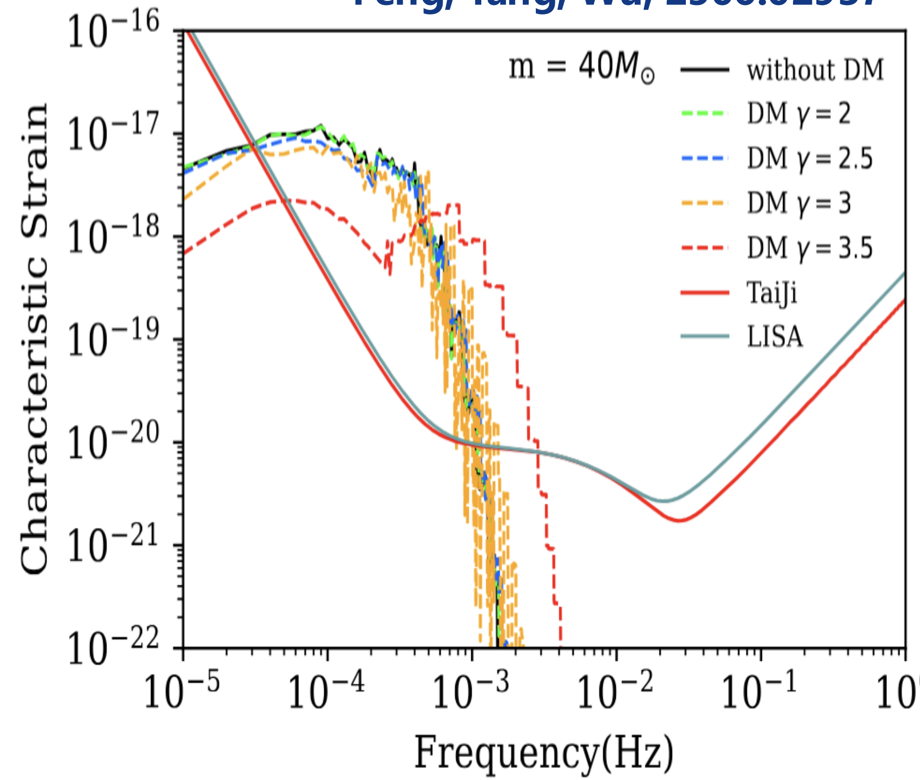

Summary

- We discuss how GW laser interferometers in space may help to understand the nature of dark matter.
- Wave/Ultralight Dark Matter
  - induce the oscillation of test masses, leading to signals in detectors. Also sensitive to axion-photon coupling
  - Metric perturbation by vector ULDM may be detectable in next-generation interferometers.
- Weakly-interacting massive particles
  - can form spikes around black holes and affect the orbiting compact objects, imprinting on the waveform of GW.

Thanks!

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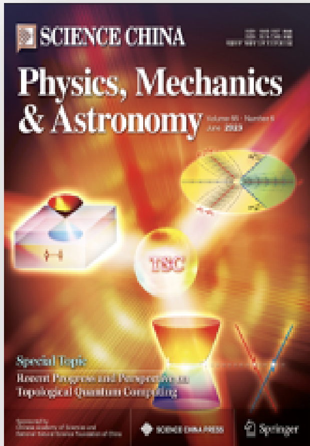

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