Entanglement features from intermediate heavy particle in scattering

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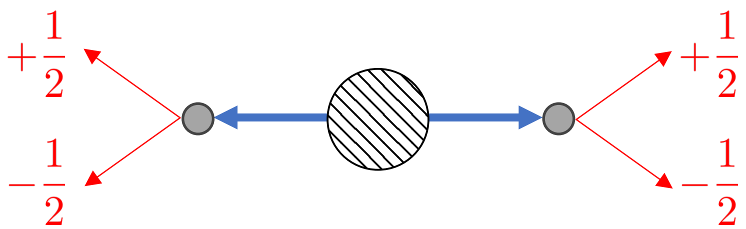
Based on: arXiv 2507.03555 with **Yi Wang & Xingkai Zhang (HKUST)**

Quantum information at collider

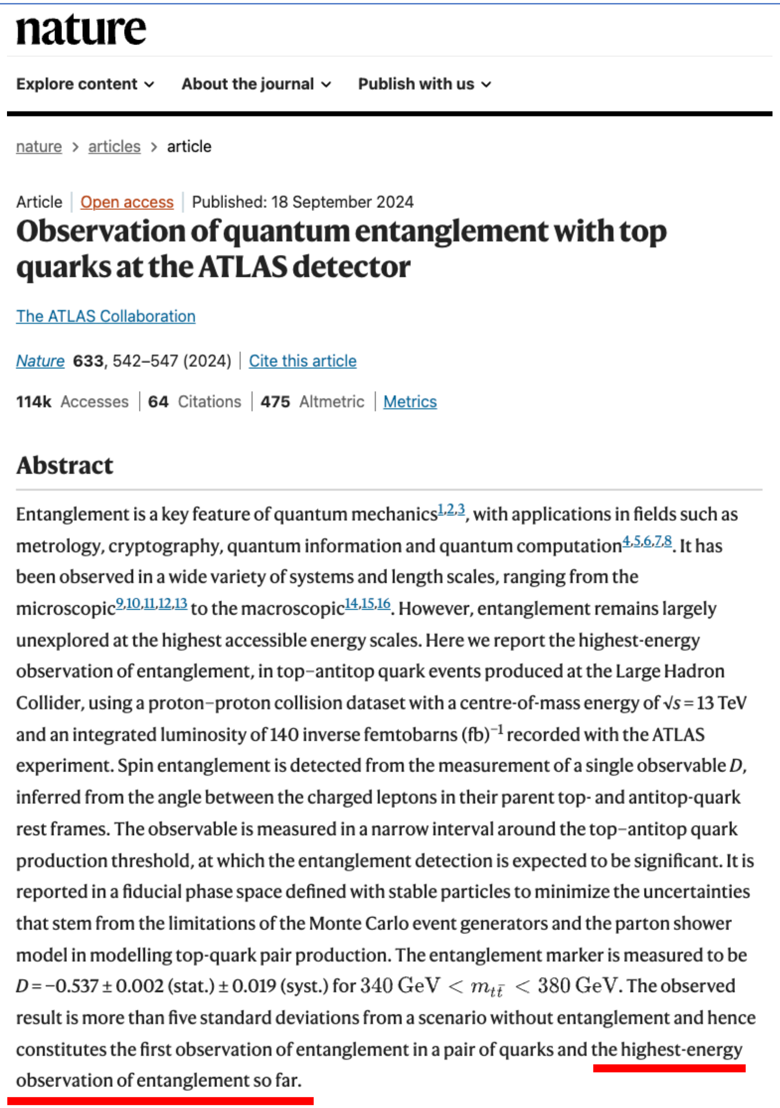
Quantum entanglement can now be probed at the **highest scale accessible by humans**

- Violation of Bell inequality measured with top-quark spins (Afik & de Nova, 2003.02280) (ATLAS, Nature 633 (2024) 8030, 542-547)



Growing interest in probing QI quantities in more processes

- Higgs→W (Barr, 2106.01377), weak decays (Ashby-Pickering, Barr & Wierzchucka, 2209.13990), leptons (Fabbrichesi, Floreanini & Gabrielli, 2208.11723), light quarks (Cheng & Yan, 2501.03321) …



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Observation of quantum entanglement with top quarks at the ATLAS detector

The ATLAS Collaboration

Nature **633**, 542–547 (2024) | Cite this article

114k Accesses | **64** Citations | **475** Altmetric | Metrics

Abstract

Entanglement is a key feature of quantum mechanics[1,2,3], with applications in fields such as metrology, cryptography, quantum information and quantum computation[4,5,6,7,8]. It has been observed in a wide variety of systems and length scales, ranging from the microscopic[9,10,11,12,13] to the macroscopic[14,15,16]. However, entanglement remains largely unexplored at the highest accessible energy scales. Here we report the highest-energy observation of entanglement, in top–antitop quark events produced at the Large Hadron Collider, using a proton–proton collision dataset with a centre-of-mass energy of $\sqrt{s}$ = 13 TeV and an integrated luminosity of 140 inverse femtobarns (fb)$^{-1}$ recorded with the ATLAS experiment. Spin entanglement is detected from the measurement of a single observable D, inferred from the angle between the charged leptons in their parent top- and antitop-quark rest frames. The observable is measured in a narrow interval around the top–antitop quark production threshold, at which the entanglement detection is expected to be significant. It is reported in a fiducial phase space defined with stable particles to minimize the uncertainties that stem from the limitations of the Monte Carlo event generators and the parton shower model in modelling top-quark pair production. The entanglement marker is measured to be $D = -0.537 \pm 0.002$ (stat.) $\pm$ 0.019 (syst.) for $340\ \mathrm{GeV} < m_{t\bar{t}} < 380\ \mathrm{GeV}$. The observed result is more than five standard deviations from a scenario without entanglement and hence constitutes the first observation of entanglement in a pair of quarks and the highest-energy observation of entanglement so far.

Implications of quantum information in scattering

Growth of scattering entanglement entropy
(Cheung, He & Sivaramakrishnan, 2304.13052)

“Area law” in 2⟶2 scattering
Entanglement entropy ∝ cross section
(Low & Yin, 2405.08056 & 2410.22414)

Positivity of EFT from 2⟶2 scattering
Positivity = entanglement entropy
(Aoude, Elor, Remmen & Sumensari, 2402.16956)

Entanglement suppression ➪ emergent spin-flavor symmetry
(Beane, Kaplan, Klco & Savage, 1812.03138)

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Our motivation: relation closer to experiment
e.g. pole structure
How does the breakdown of EFT manifest as features of QI quantities in scattering?

⬇

Universal entanglement pattern induced by intermediate heavy particle

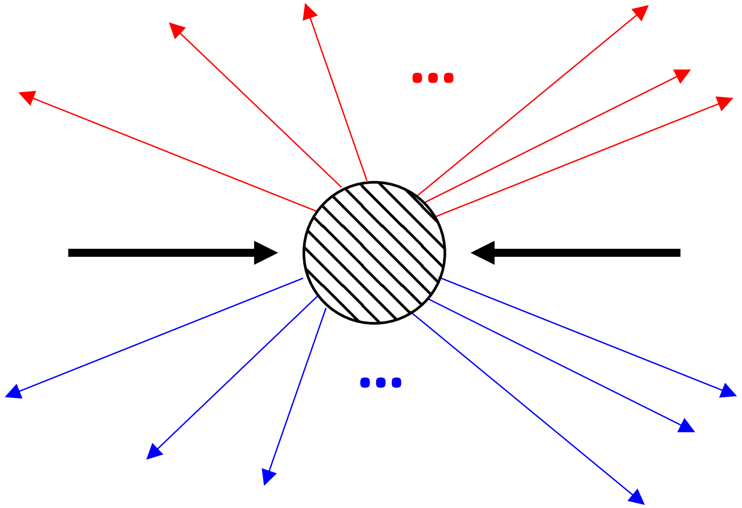
Entanglement pattern quantified by entanglement entropy

Entanglement entropy between two groups of final particles

$$\sum_i \sqrt{p_i}\,|i\rangle_A \otimes |i\rangle_B \implies S_{EE} = -\sum_i p_i \log p_i$$

Scattering can cause **entanglement** between final particles

- Characterized by **the uncertainty of quantum state**, by only knowing part of the particles (e.g. subsystem *A*) and neglecting the remaining part (e.g. *B*)
- Quantified by the **entanglement entropy**

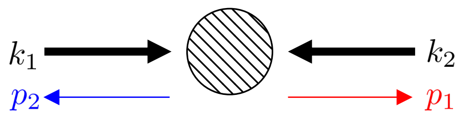


Entanglement entropy of 2⟶2 scattering in the literature

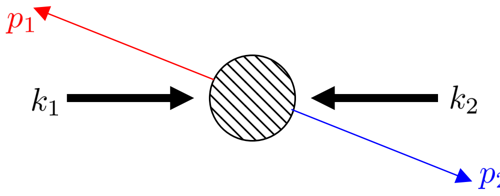
The "area law" of 2⟶2 elastic scattering (entropy ∝ cross section)

- (Seki, Park & Sin, 1412.7894) (Fan, Deng & Huang, 1703.07911) (Low & Yin, 2405.08056 & 2410.22414)

$$S_{EE}^{(2\to 2)} = \underbrace{-\frac{P_F}{P^{(2\to 2)}}\log\frac{P_F}{P^{(2\to 2)}} - \frac{P_{NF}}{P^{(2\to 2)}}\log\frac{P_{NF}}{P^{(2\to 2)}}}_{\text{Shannon entropy of forward/non-forward scattering (mixture part: depends on probability)}} + \frac{P_{NF}}{P^{(2\to 2)}}\log\left(\underbrace{\frac{1}{T}\frac{|\vec{p}_{CM}|E_1E_2}{\pi\sqrt{s}}V}_{\text{Size of 2-body phase space (state's part: depends on microscopic details)}}\right)$$



Forward scattering



Non-forward scattering

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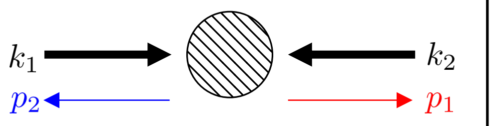
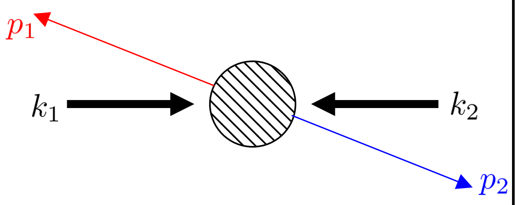
$$\approx \underbrace{-\frac{P_{NF}}{P^{(2\to2)}}\log\frac{P_{NF}}{P^{(2\to2)}}}_{\text{Non-forward probability scales with cross section}} + \mathcal{O}(\Delta)$$



Forward scattering



Non-forward scattering

- Assume isotropic scattering for simplicity
- The discretization factor of the 2-body phase space $\Delta = \dfrac{T}{4E_{k_1}E_{k_2}V}$ ⟵ Interaction time (T), ⟵ Volume (V)
- For Rényi or Tsallis entropies, they are **directly proportional to cross section** (without $\log\frac{P_{NF}}{P^{(2\to2)}}$)

Entanglement entropy of decay in the literature

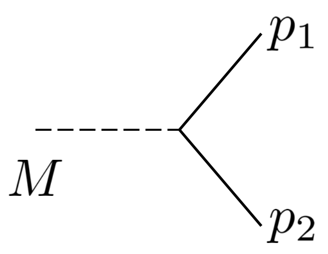
Entanglement from 1⟶2 decay of heavy particle

- The Wigner-Weisskopf method approximating **time-dependent evolution** (Lello, Boyanovsky & Holman, 1304.6110)

Decay rate

$$S_{EE}^{(1\to 2)} = \log\left(\Gamma \underbrace{\frac{|\vec{p}_{CM}| E_1 E_2}{\pi M}}\, V\right)$$

Size of 2-body phase space around $M \sim E_1 + E_2$
(**state's part: depends on microscopic details**)



- Contrast to the 2⟶2 elastic scattering (**only relies on mixture part**), the entanglement structure **is only related to the microscopic details**
- Practically, it is included in **a larger scattering process**, where the initial and final asymptotic states are well-defined

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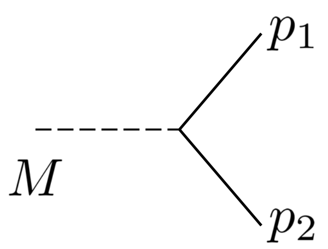

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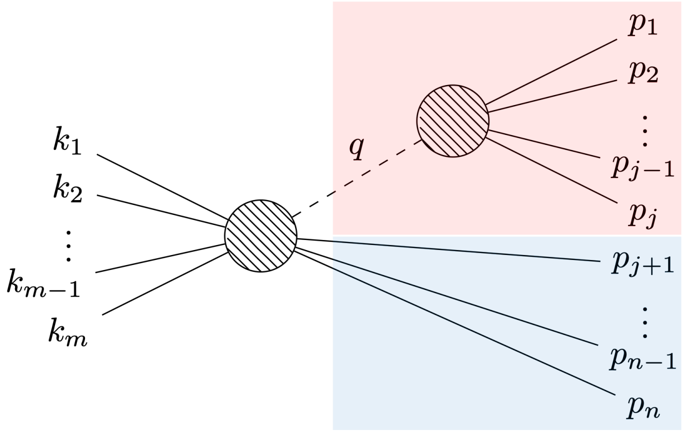
Motivation: investigate entanglement structure in more general scatterings

Outline of our results

1. The novel entanglement structure from <u>**heavy particle**</u> in *m*⟶*n* (**n≥3**) inelastic scattering
    - **Bipartition** *n*-body phase space into decay products and other particles
    - **Universal** entropy suppression from **on-shell heavy particle**



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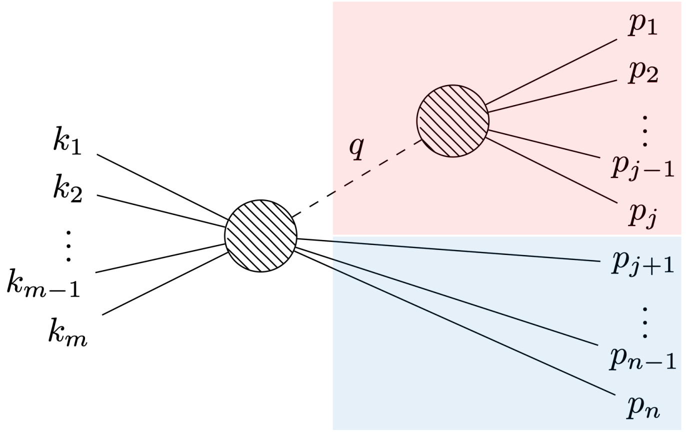
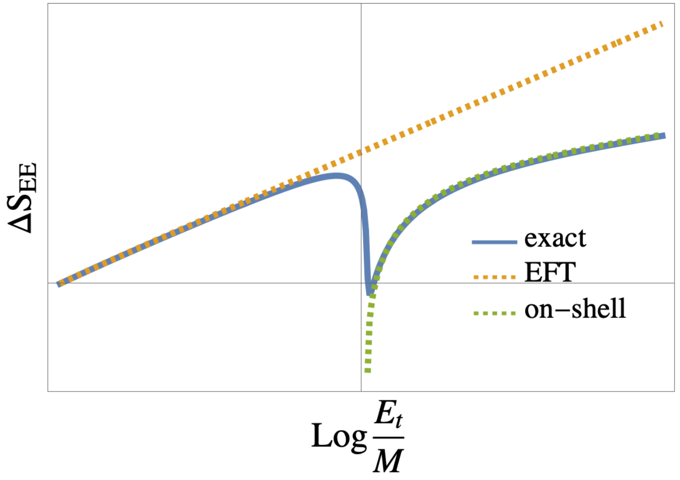


2. **Universal entanglement feature** from low-energy EFT to high energy theory
   - **"Dip" feature of entanglement suppression** when total energy reaches the heavy mass
   - Verify in simple 2⟶3 and 2⟶4 models



Outline of our results

1. The novel entanglement structure from **heavy particle** in $m \longrightarrow n$ (**n≥3**) inelastic scattering
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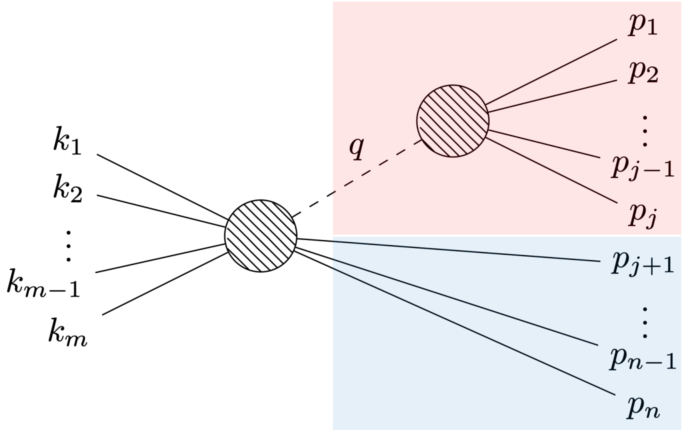

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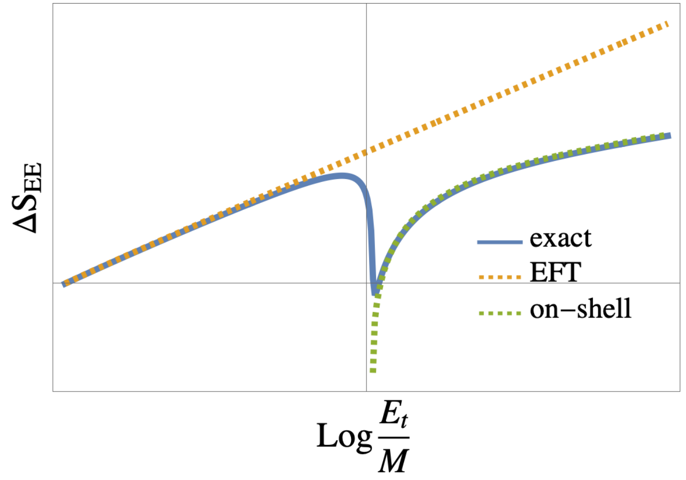


Key demonstration

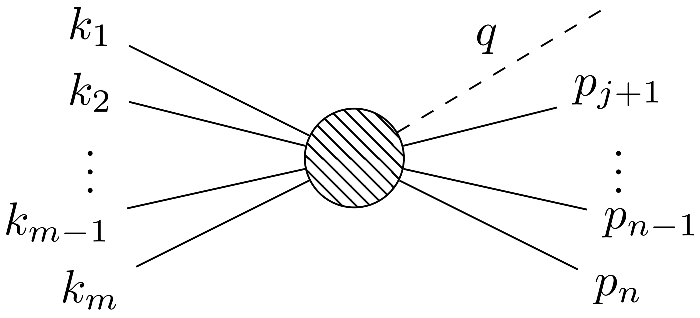
Pole structure in amplitude ➪ Entanglement feature

$m \longrightarrow n$ inelastic scattering with intermediate heavy particle

Setup of $m \longrightarrow n$ ($n \geq 3$) inelastic scattering with intermediate heavy particle

Initial unentangled *m* particles with momenta
$$\{k_1, \ldots, k_m\}$$

Intermediate heavy particle with momentum
$$q$$



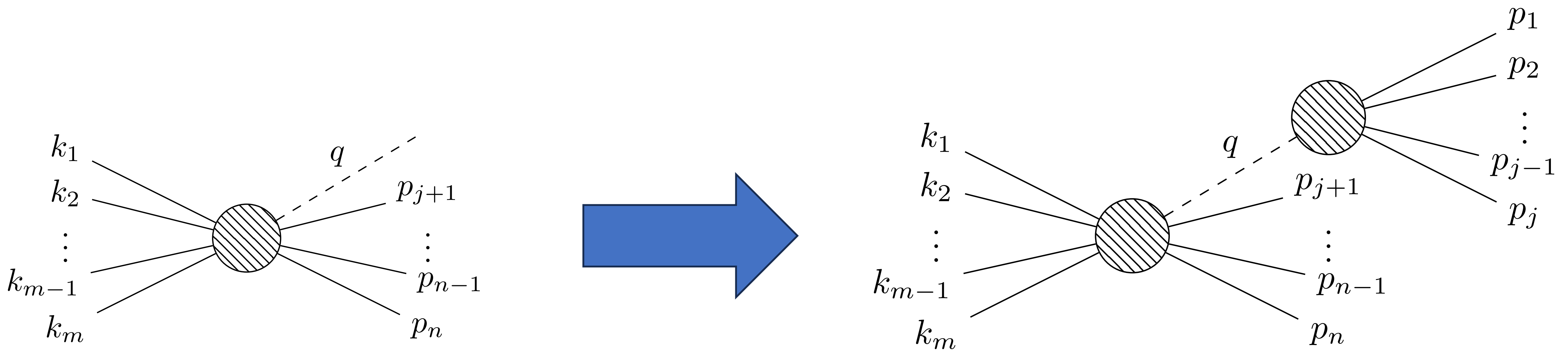
- All particles are distinguishable for simplicity
- Focus on **momentum space for simplicity**, ignoring internal degrees of freedom (e.g. spin, polarization …)

Setup of *m*⟶*n* (*n*≥3) inelastic scattering with intermediate heavy particle

Initial unentangled *m* particles with momenta $\{k_1, \ldots, k_m\}$

Intermediate heavy particle with momentum q

Decay products with momenta $\{p_1, \ldots, p_j\}$ and other final particles $\{p_{j+1}, \ldots, p_n\}$



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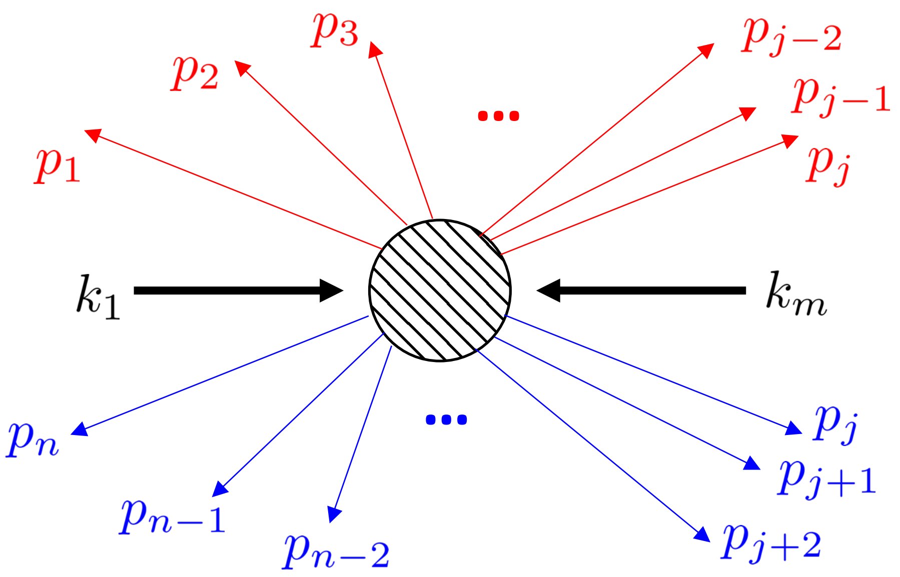
Entanglement in the n-body final state

The n-body final state is a superposition according to the amplitude

➪ **momentum-space entanglement**

$$|f^{(m\to n)}\rangle = \frac{1}{\sqrt{\mathcal{N}^{(m\to n)}}}\int d\Pi_n(K;p_1,\ldots,p_n)i\mathcal{M}(m\to n)|p_1\rangle\otimes|p_2\rangle\otimes\cdots\otimes|p_n\rangle$$



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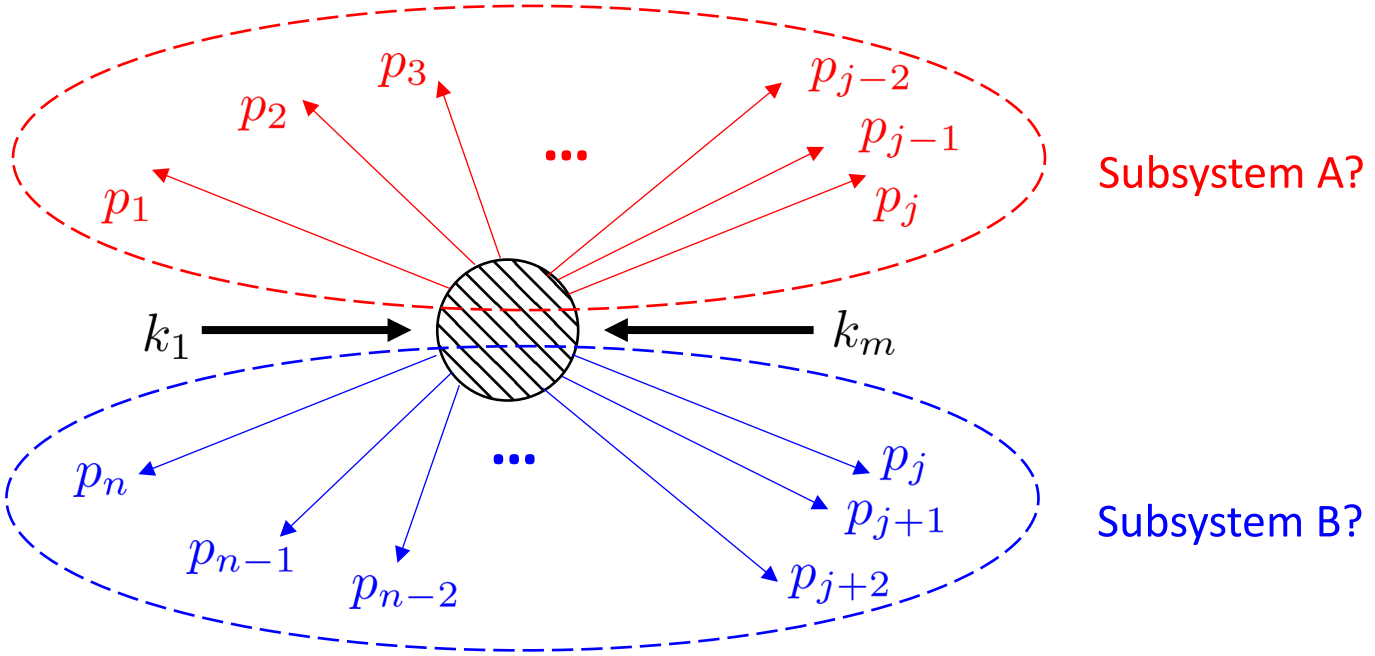

For (simplest) bipartite entanglement, which bipartition is non-trivial and utilizes features with heavy pole?

Key points for bipartite entanglement entropy in scattering

Entanglement entropy from the Schmidt decomposition

$$\sum_i \sqrt{p_i}\,|i\rangle_A \otimes |i\rangle_B \implies S_{EE} = -\sum_i p_i \log p_i$$

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Counting the states

n-body phase space

$$\int d\Pi_n(K; \overbrace{p_1, \ldots, p_n}^{\text{Momenta of } n \text{ final particles}})$$

$$= \int \left(\prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}\right)(2\pi)^4 \delta^4\left(K - \sum_{i=1}^{n} p_i\right)$$

Total initial momentum

Probability distribution

Determined by the amplitude with pole

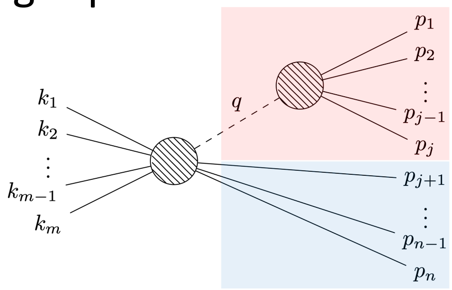
$$i\mathcal{M}(m \to n)$$

$$\approx -i\mathcal{M}(m \to n-j+1)\frac{1}{q^2 - M^2 + i\Gamma M}\mathcal{M}(1 \to j)$$

Breit-Wigner formula

Bipartition the full system

Bipartition by decay products and other light particles



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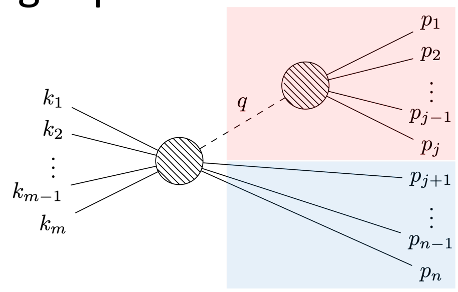


Bipartition of *n*-body phase space

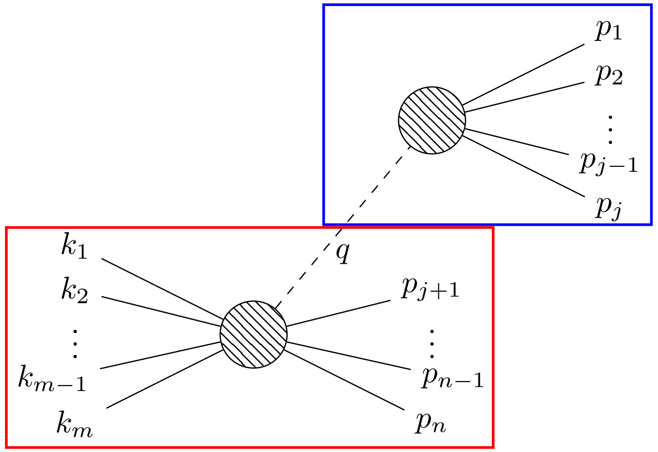
Bipartition of n-body phase space and entanglement structure

Decomposition of the n-body phase space

Recursive relation of n-body phase space

$$\int d\Pi_n(K; p_1, \ldots, p_n) = \int \frac{dq^2}{2\pi} d\Pi_{n-j+1}(K; q, p_{j+1}, \ldots, p_n) d\Pi_j(q; p_1, \ldots, p_j)$$

- q is the intermediate four momentum with integrating out its square ("invariant mass")



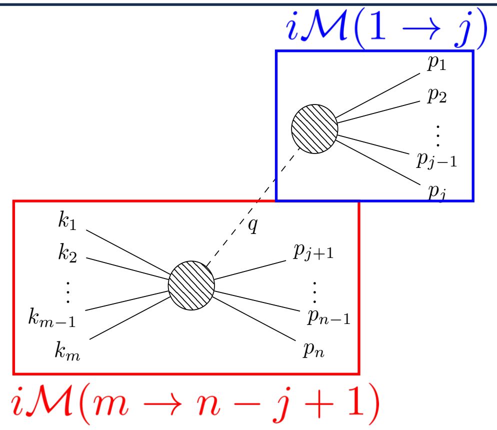
Decompose the phase space with (q^2, q)

The reduced density matrix of decay products

The entanglement between the decay products and other particles is characterized by the entropy of **the reduced density matrix**

$$\rho_{1-j}^{(m\to n)} = \mathrm{Tr}_{p_{j+1},\ldots,p_n}\left(|f^{(m\to n)}\rangle\langle f^{(m\to n)}|\right)$$

What is the basis of this matrix?



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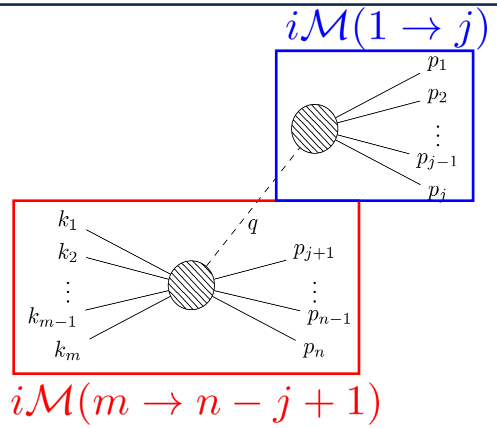

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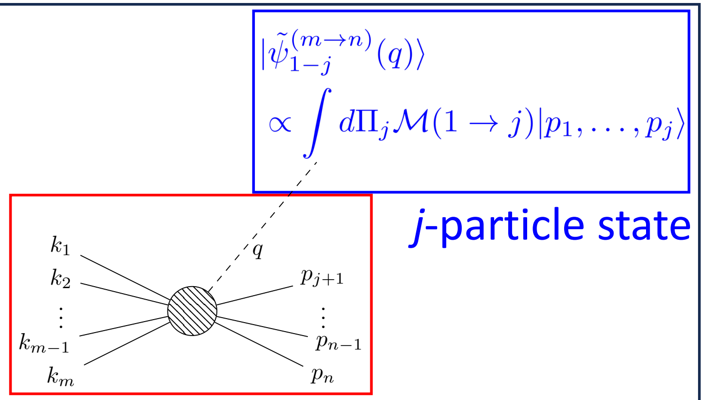
Decomposing phase space with (q^2, q)

For fixed “invariant mass” q^2, **the basis is the scattered *j*-particle state**

$$\sigma_{1-j}^{(m\to n)}(q^2) \propto \int d\Pi_{n-j+1}|\mathcal{M}(m\to n-j+1)|^2|\tilde{\psi}_{1-j}^{(m\to n)}(q)\rangle\langle\tilde{\psi}_{1-j}^{(m\to n)}(q)|$$



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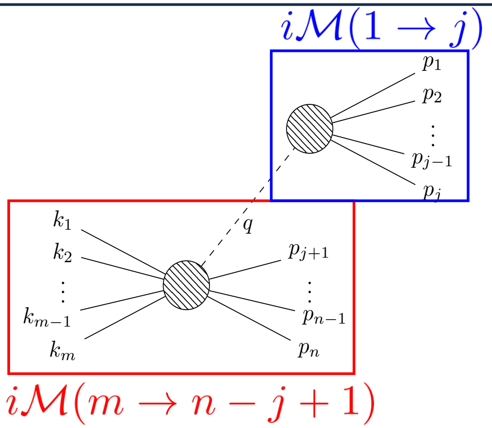

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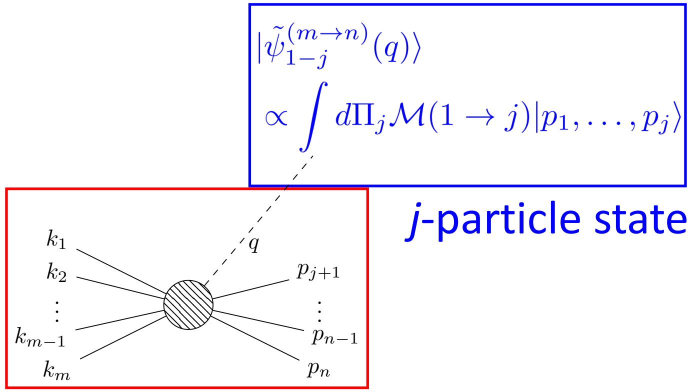

Finally, **probabilistic mixture** (linear combination) of the basis of q^2

$$\rho_{1-j}^{(m\to n)} \propto \int dq^2\, \mathcal{P}_{\Gamma,M}(q^2)\mathcal{I}^{(m\to n-j+1)}(q^2)\mathcal{I}^{(1\to j)}(q^2)\sigma_{1-j}^{(m\to n)}(q^2)$$

- Cauchy distribution by heavy propagator $\dfrac{\Gamma M}{\pi}\dfrac{1}{(q^2-M^2)^2+\Gamma^2 M^2}$
- Phase-space integral $\mathcal{I}^{(a\to b)} = \int d\Pi_b|\mathcal{M}(a\to b)|^2$

<u>Universal entanglement structure</u> from the mixture of quantum states

Theorem (Nielsen & Chuang, 2010): for a mixture of quantum states (linear combination of density matrices on **orthogonal subspaces**)

$$\rho = \sum_i P_i \sigma_i$$

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The entropy is decomposed into: (1) Classical Shannon entropy (**mixture part, probability**) and (2) averaged quantum non-Neumann entropy (**state's part, microscopic details**)

$$S(\rho) = H(P_i) + \sum_i P_i S(\sigma_i)$$

- Recall that for 2⟶2 scattering, it is a **mixture of forward and non-forward scatterings**, leading to the "area law"

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The entanglement entropy includes a mixture part related to the **heavy pole**

(**Sou**, Wang & Zhang, 2507.03555)

$$\rho_{1-j}^{(m\to n)} = \int dq^2\, P_{1-j}^{(m\to n)}(q^2)\sigma_{1-j}^{(m\to n)}(q^2)$$

$$\Longrightarrow S_{EE}\left(\rho_{1-j}^{(m\to n)}\right) = h\left(P_{1-j}^{(m\to n)}\right) + \int dq^2 P_{1-j}^{(m\to n)}(q^2) s\left(\sigma_{1-j}^{(m\to n)}(q^2)\right) + \log\left(\frac{VT}{2\pi}\right)$$

Shannon entropy of distribution with pole (∝ Cauchy distribution) $P_{1-j}^{(m\to n)} \propto \mathcal{P}_{\Gamma,M}(q^2)$

Differential (continuous) entropy

From discretizing phase space

Entanglement suppression at the on-shell limit

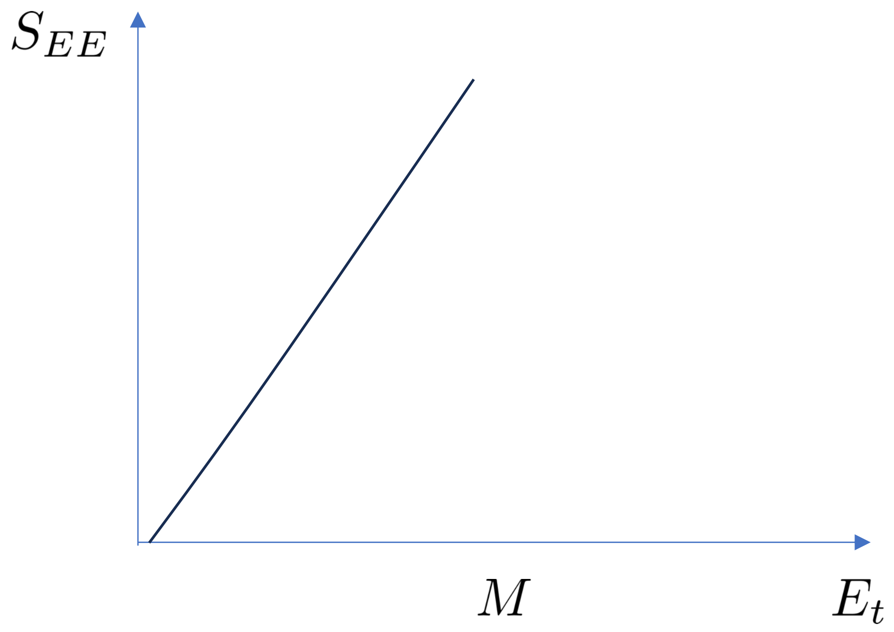
Intuition for the entanglement entropy with heavy-field propagator

Since the entanglement entropy is determined by a probability distribution with pole

$$P_{1-j}^{(m\to n)}(q^2) \propto \frac{1}{(q^2-M^2)^2+\Gamma^2 M^2}$$

we expect the following variations

- For total energy $E_t \ll M$, all kinematically allowed final states **contribute evenly**, with multiplicity grows with energy, so **increasing entropy**



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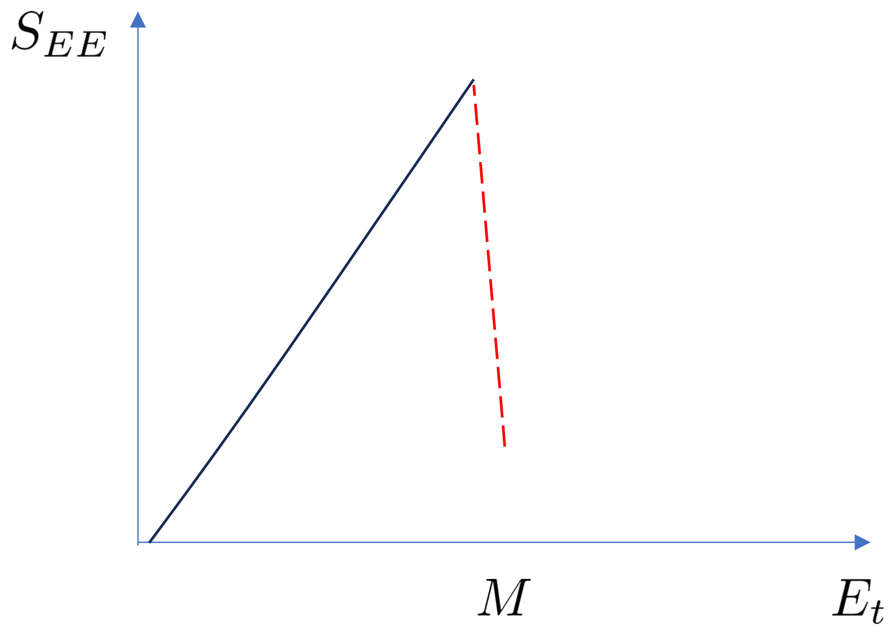

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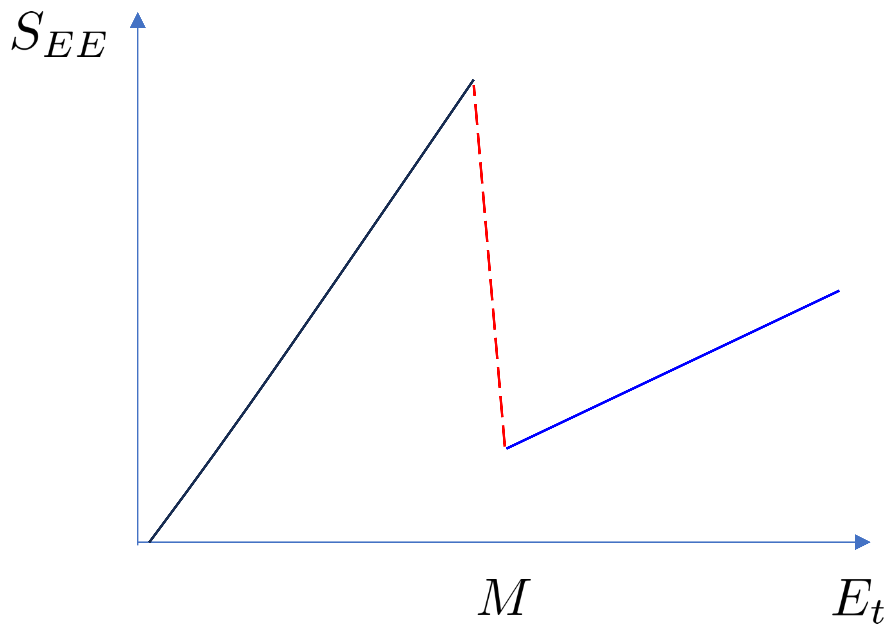

On-shell approximation with the Cauchy distribution

The **entanglement suppression** by the on-shell heavy particle can be proved with complex analysis

- Entropy for distribution with poles

$$-\int_{-\infty}^{+\infty} dq^2\, \mathcal{P}_{\Gamma,M}(q^2) f(q^2) \log(\mathcal{P}_{\Gamma,M}(q^2) f(q^2))$$

Factorized into $\dfrac{1}{q^2 - M^2 + i\Gamma M} \times \dfrac{1}{q^2 - M^2 - i\Gamma M}$

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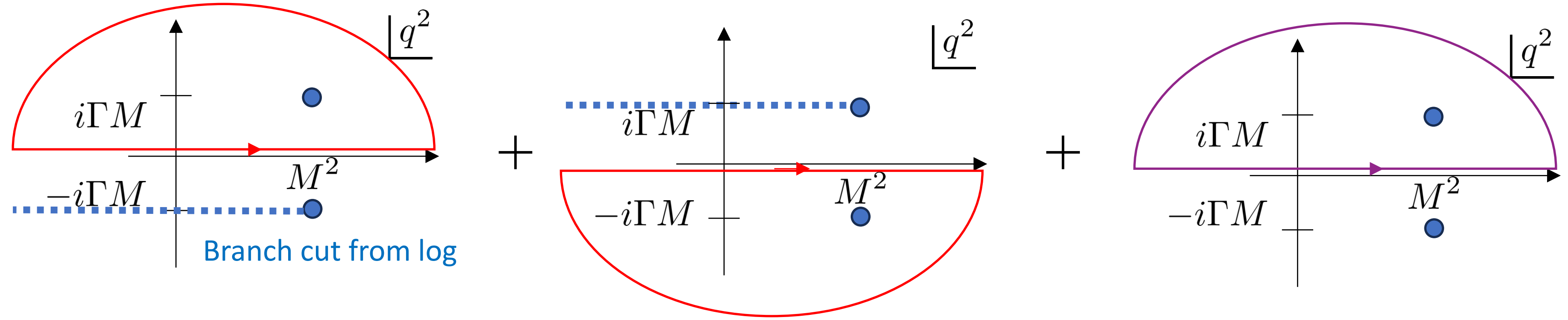

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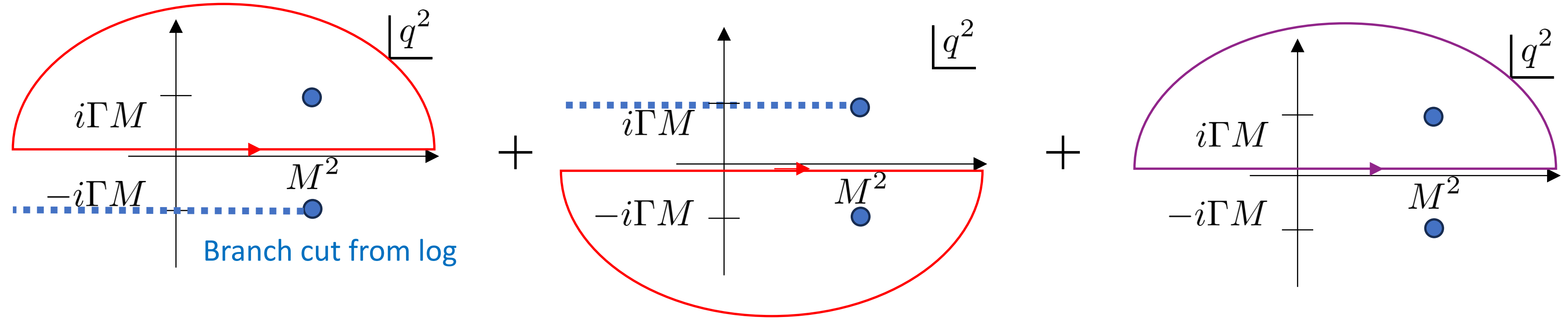

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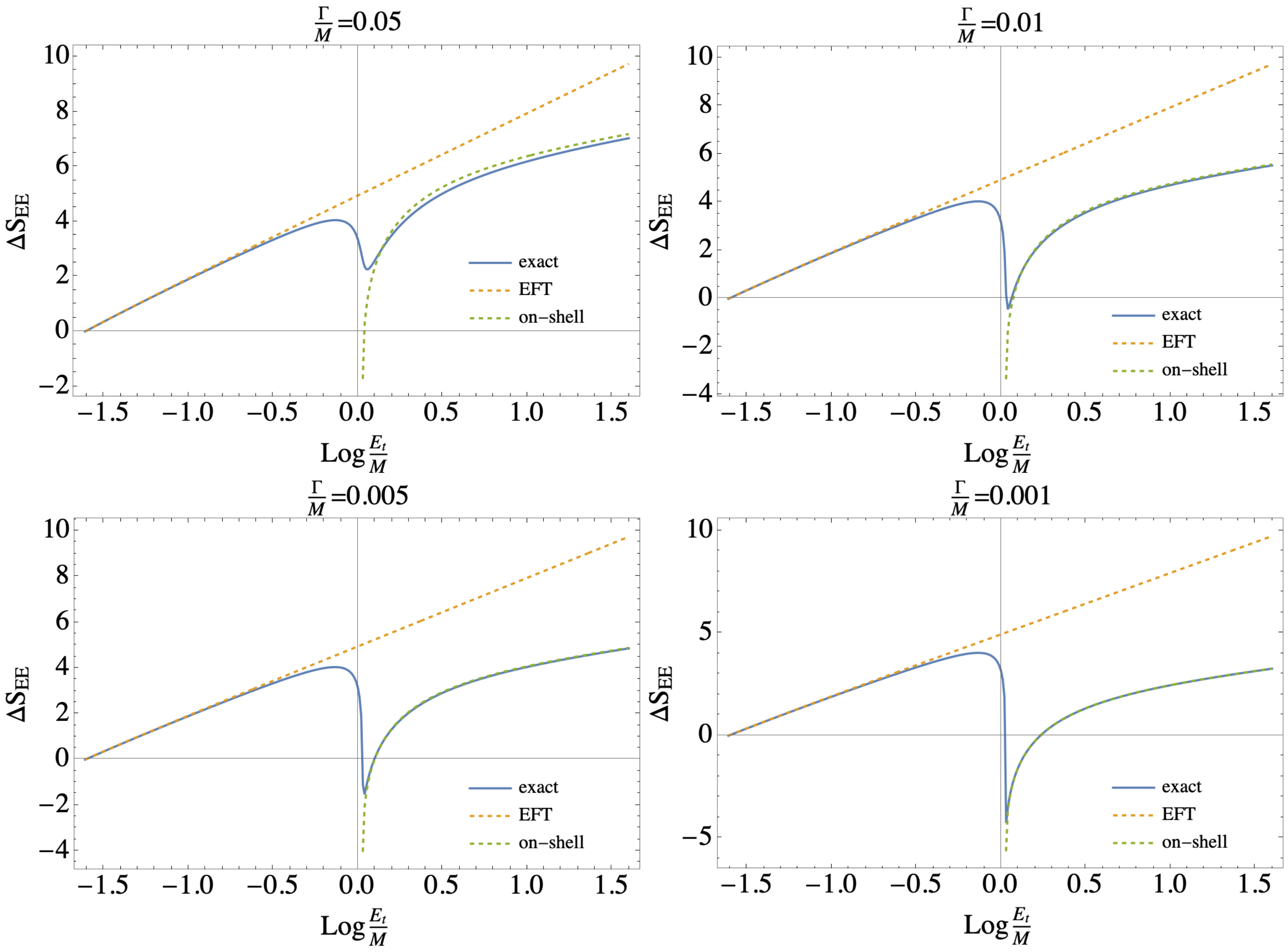
The entanglement entropy from on-shell heavy particle is **suppressed by decay rate**

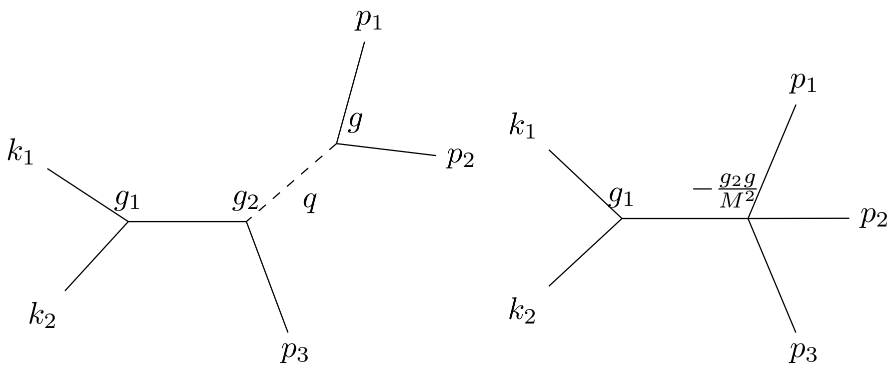
(**Sou**, Wang & Zhang, 2507.03555)

$$S_{EE}\left(\rho_{1-j}^{(m\to n)}\right) \approx \log(4\pi\Gamma M) + s\left(\sigma_{1-j}^{(m\to n)}(q^2)\right)\Big|_{q^2=M^2} + \log\left(\frac{VT}{2\pi}\right) + \mathcal{O}\left(\frac{\Gamma}{M}\right)$$

Examples: entanglement features in 2⟶3 and 2⟶4 scatterings

Concrete model of 2⟶3 scattering





Low-energy EFT

$$\mathcal{L}_{\text{int}} = g_1\phi_A\phi_B\chi + g_2\chi\sigma\phi_3 + g\sigma\phi_1\phi_2$$

Heavy particle

- **Dip feature** of sharp entanglement suppression
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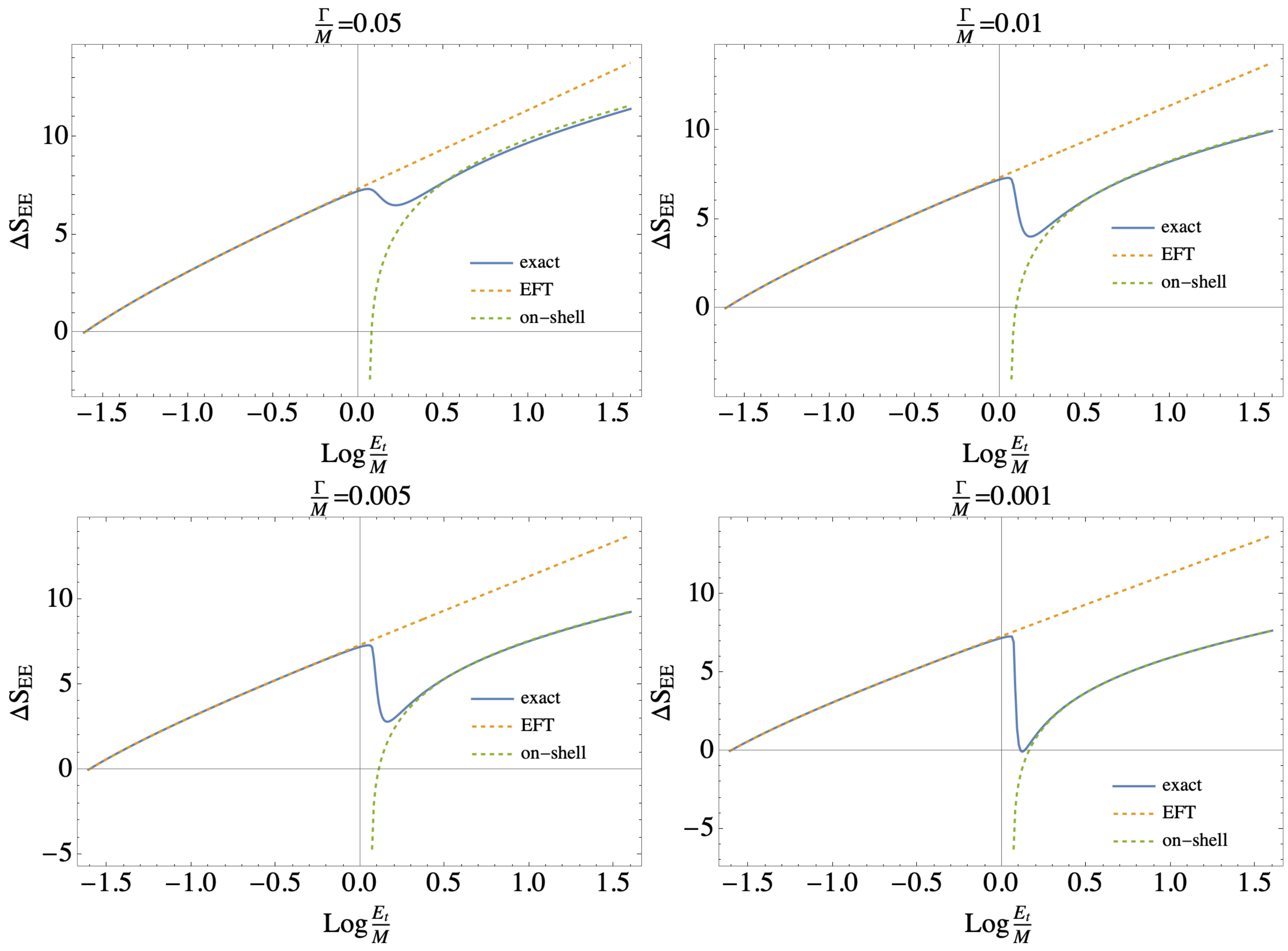

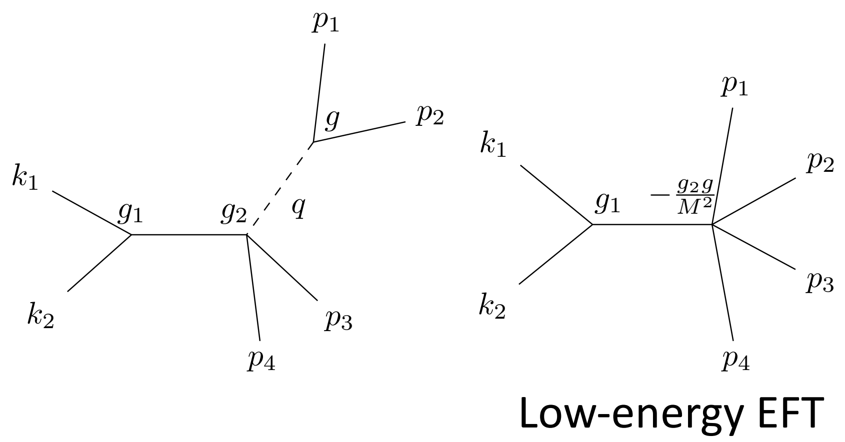

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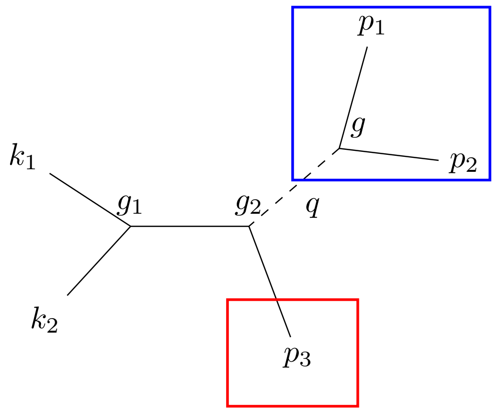
Comments on measuring the entanglement features

For the simplest 2⟶3 scattering, the entanglement entropy can be complementarily obtained by tracing out the decay products (1 and 2)

- Reduced density matrix of particle 3



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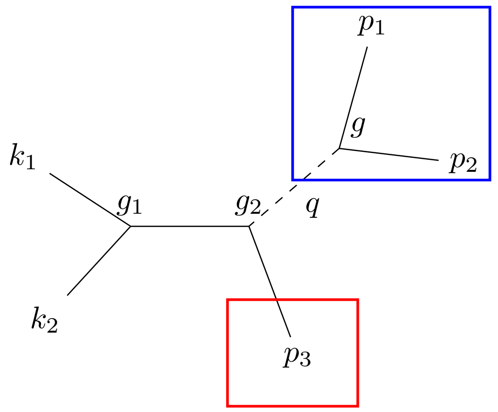

Marginalizing p_1 and p_2 ➪ **Coefficients** of the matrix for p_3
➪ **Entanglement entropy** $S_{EE}\left(\rho_3^{(2\to3)}\right) = S_{EE}\left(\rho_{12}^{(2\to3)}\right)$

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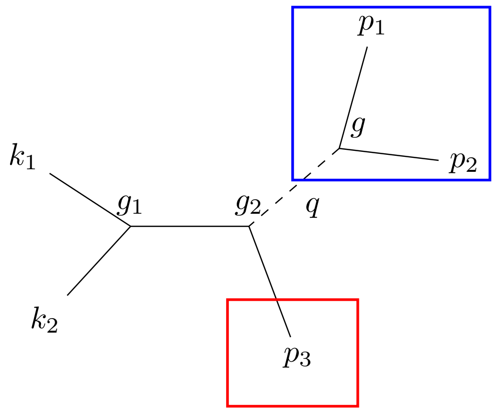

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The entanglement feature may be measured by **suitably marginalizing the phase-space distribution** of final particles

Conclusion

1. **Universal entanglement features** mediated by heavy particle in inelastic scatterings with $n \geq 3$ particles
    - **Dip feature** (sharp reduction) of entanglement entropy when total energy reaches the mass scale
    - The entanglement suppression comes from the **on-shell heavy particle**, analytically suppressed by **decay rate**
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3. In practice, the entanglement features may be probed by **suitably marginalizing the phase-space distribution** of final particles
   - Potential guide for quantum-information observables at collider