



Effective field theory for general baryon-number-violating nucleon decays

South China Normal University

Xiao-Dong Ma

Wei-Qi Fan, Yi Liao, **XDM**, Hao-Lin Wang, arXiv: 2412.20774 (PLB 862 (2025) 139335)

Yi Liao, **XDM**, Hao-Lin Wang, arXiv: 2504.14855 (Accepted by PRL)

Yi Liao, **XDM**, Hao-Lin Wang, arXiv: 2506.05052 (PRD 112, L031704 (2025))

Wei-Qi Fan, Yi Liao, **XDM**, Hao-Lin Wang, arXiv:2507.11844

XDM, Michael Schmidt, Wei-Hang Zhang, arXiv:2510.xxxxx

+ ongoing works

The 2025 Beijing Particle Physics and Cosmology Symposium (BPCS 2025)

2025.9.27 Beijing

Outline

- **Introduction**
- **General BNV nucleon decay interactions in the LEFT**
- **Chiral realizations**
- **Applications**
- **Summary**

Baryon number violation (**BNV**) is related to many BIG questions

- Baryogenesis

Sakharov's conditions: BNV; C, CP violation; Out of thermal equilibrium

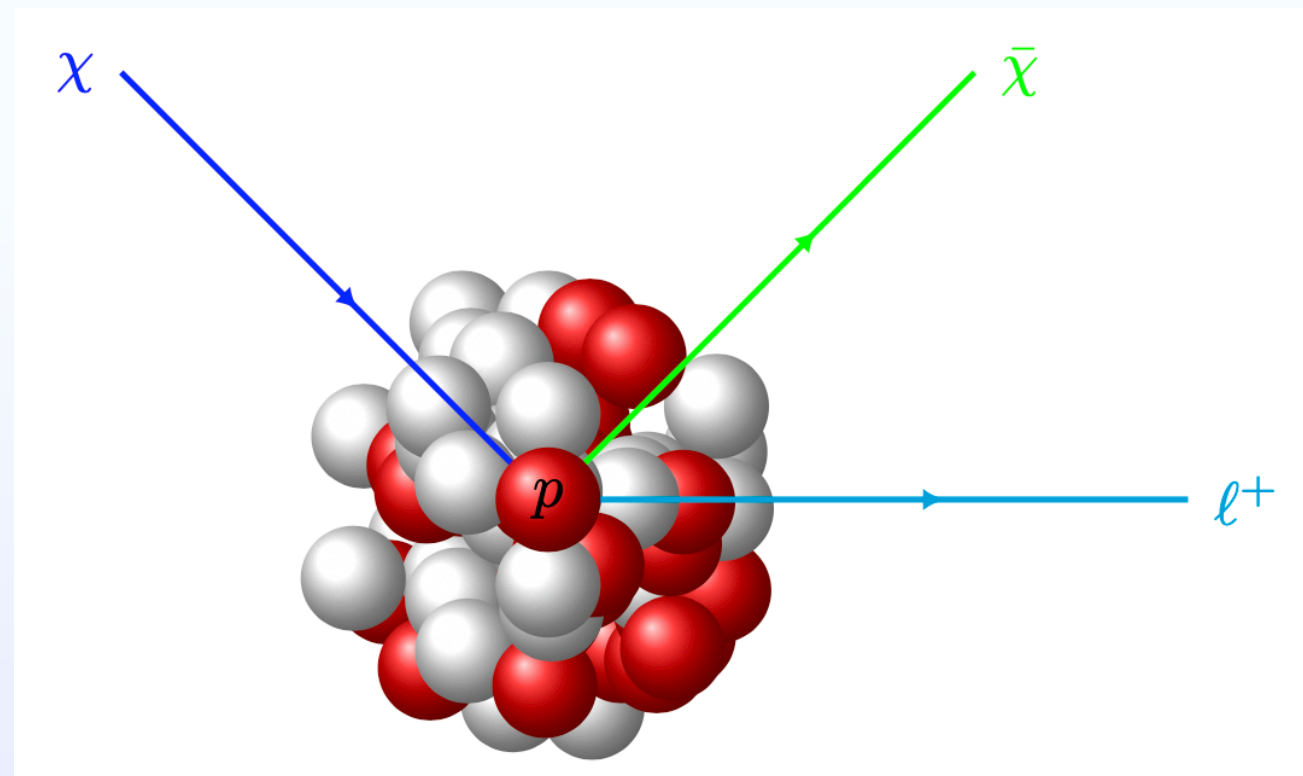
$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 6 \times 10^{-10}$$

Sakharov, 1967

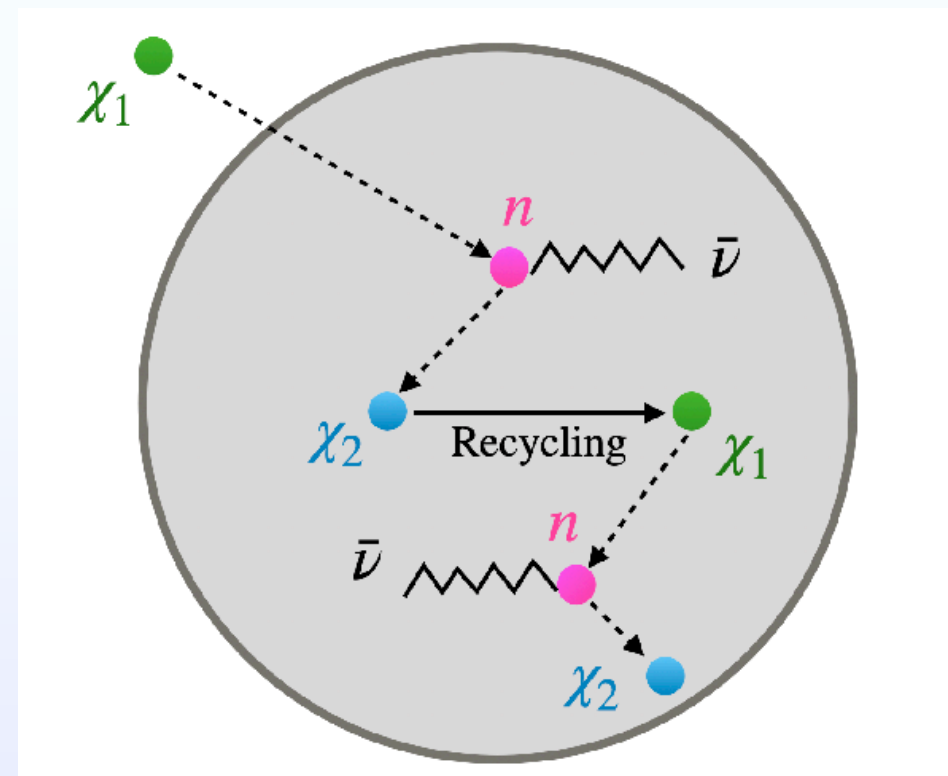
- Dark matter

Nucleon consumption induced by DM

DM-catalyzed baryon destruction inside a NS

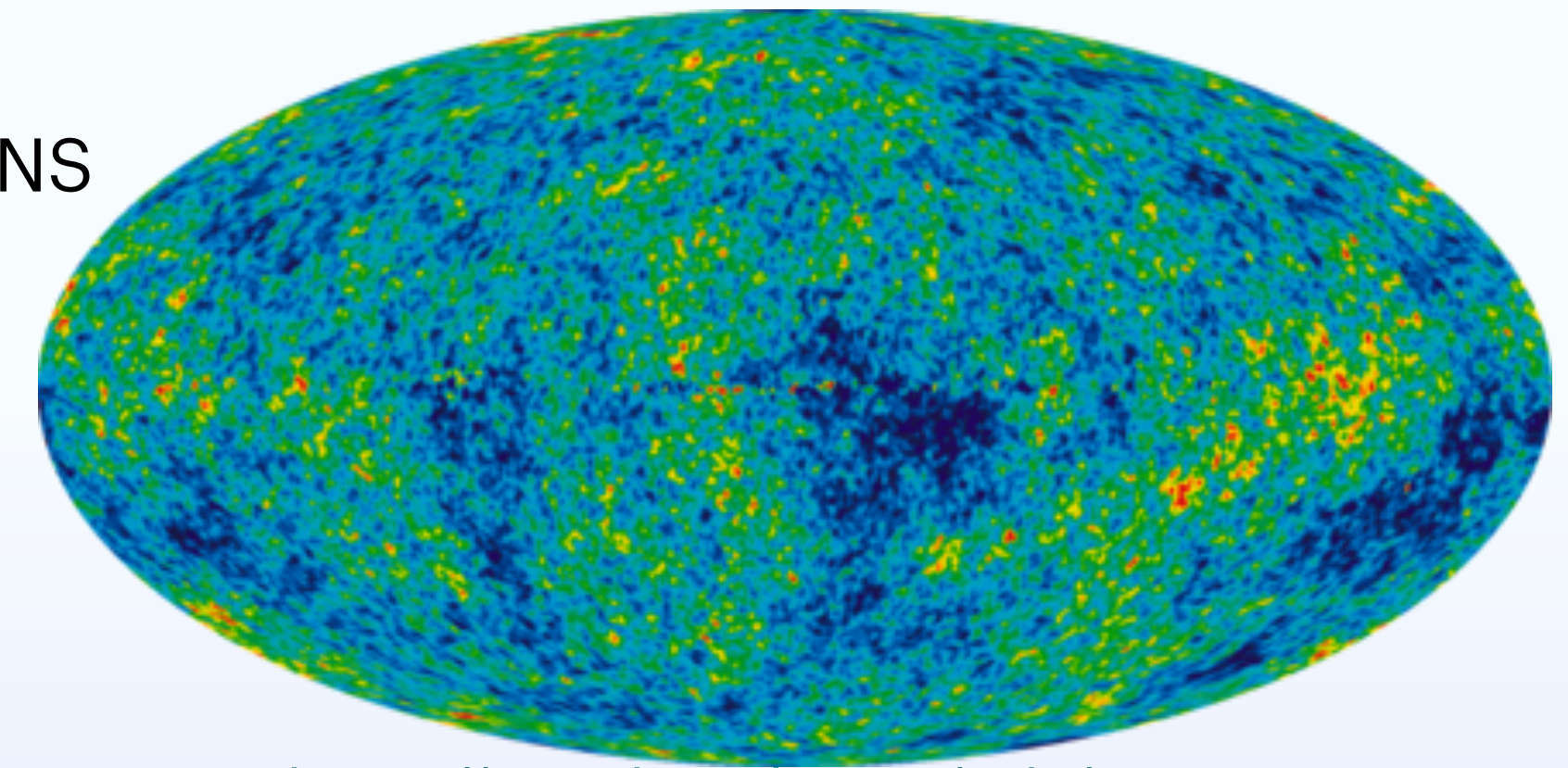


S.-F. Ge and XDM: 2406.00445



Y. Ema, R. McGehee, M. Pospelov, and A. Ray, 2405.18472

+ many many works along this direction

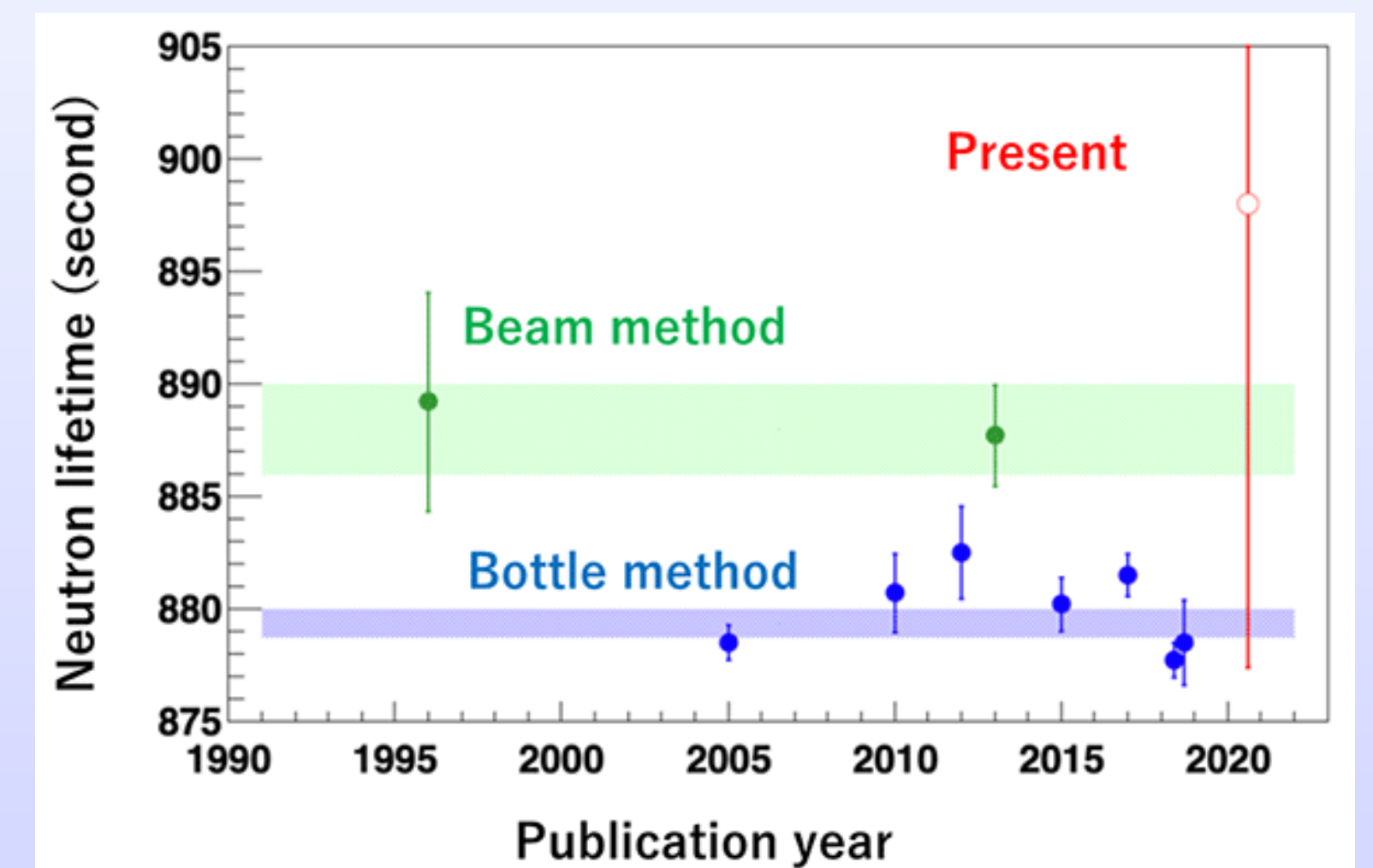


<https://en.wikipedia.org/wiki/Baryogenesis>

- Grand unified theories: SU(5), SO(10), ...

- Neutron lifetime anomaly: **Neutron dark decay**

B. Fornal and B. Grinstein: 1801.01124; 1810.00862



Low energy probes of BNV signals

$\Delta B = 1$ ✓

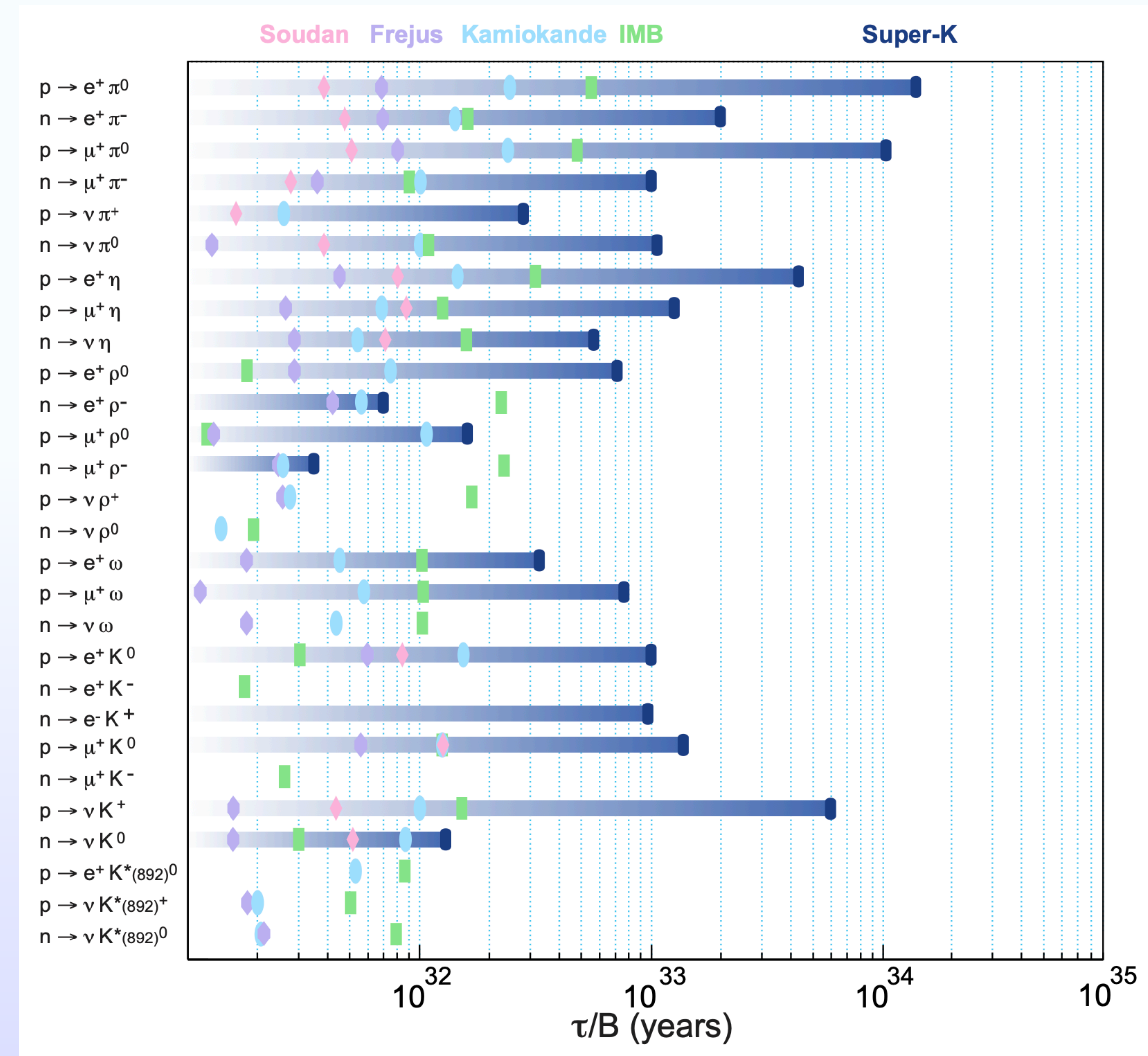
- The most sensitive probe of BNV is through **nucleon decay**
- A lot of experimental efforts in the past: **IMB, SNO+, KamLAND, Super-Kamiokande, ...**

→ Null result but stringent bound

$\Delta B = 2$

- $n - \bar{n}$ oscillation D.G. Phillips et al, 1410.1100
- $H - \bar{H}$ oscillation Feinberg, Goldhaber and Steigman, 1978

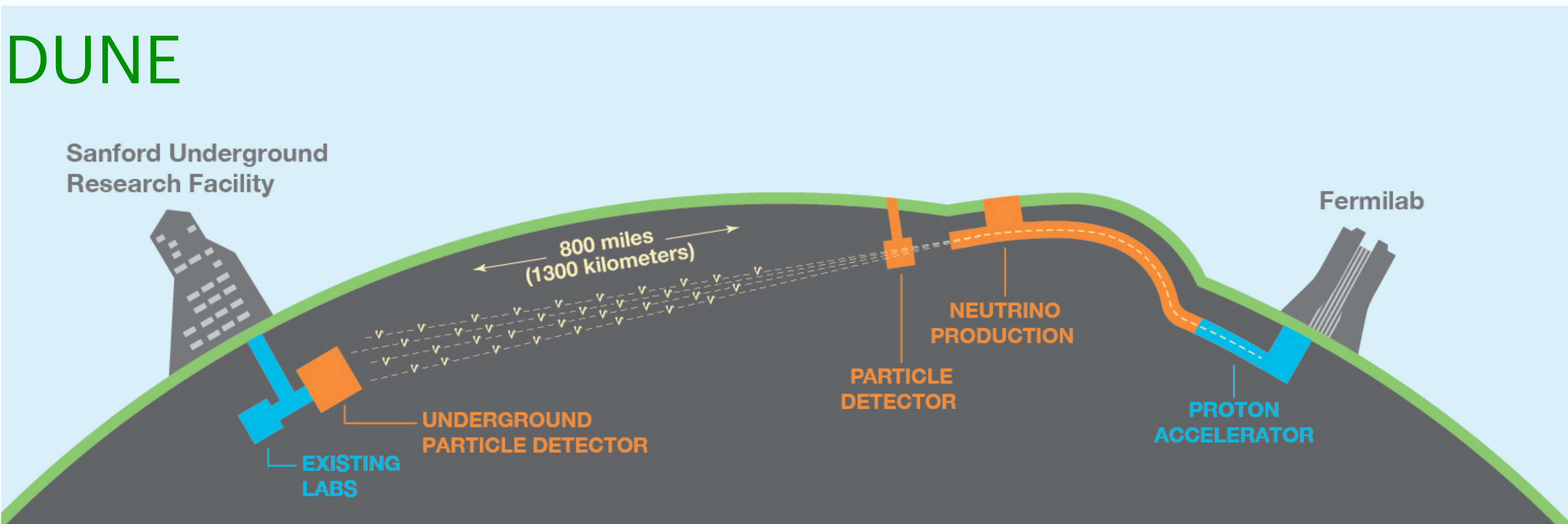
- Dinucleon decays: $NN' \rightarrow MM', \ell\ell', \ell\nu', \nu\nu'$ Xiao-Gang He and XDM, 2102.02562



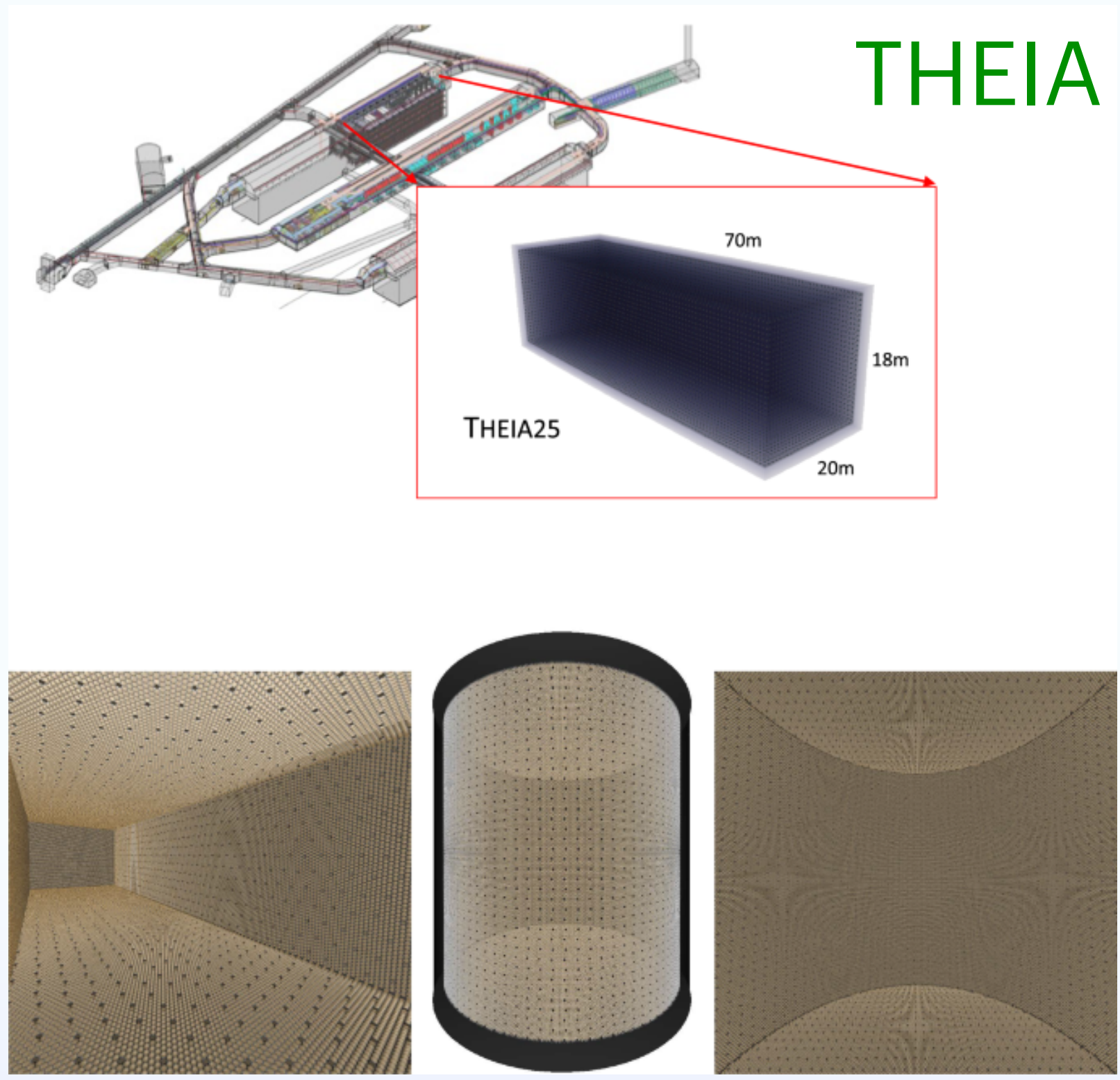
Snowmass 2013, arXiv: 1311.5285

Nucleon decay search as experimental frontiers

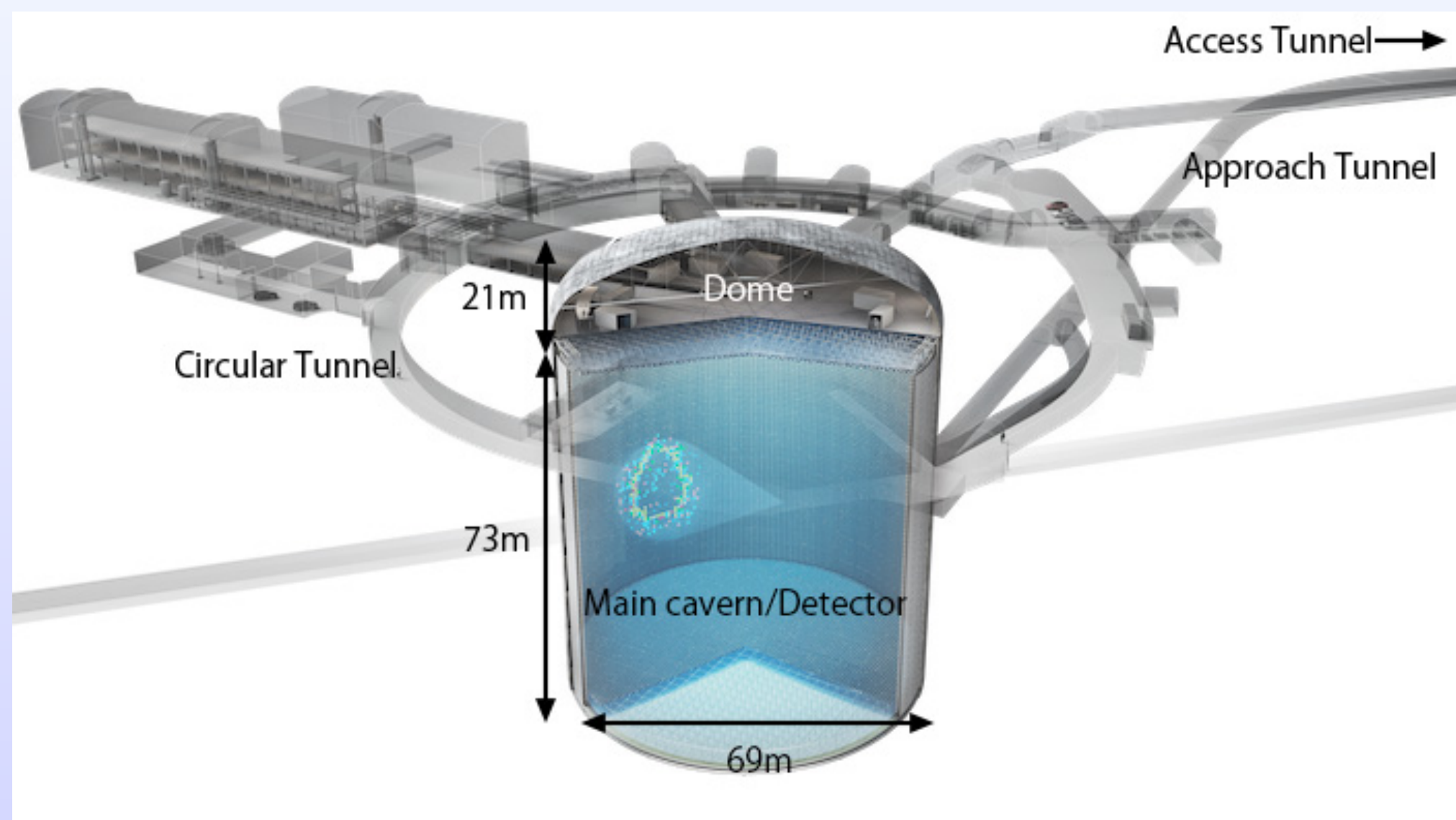
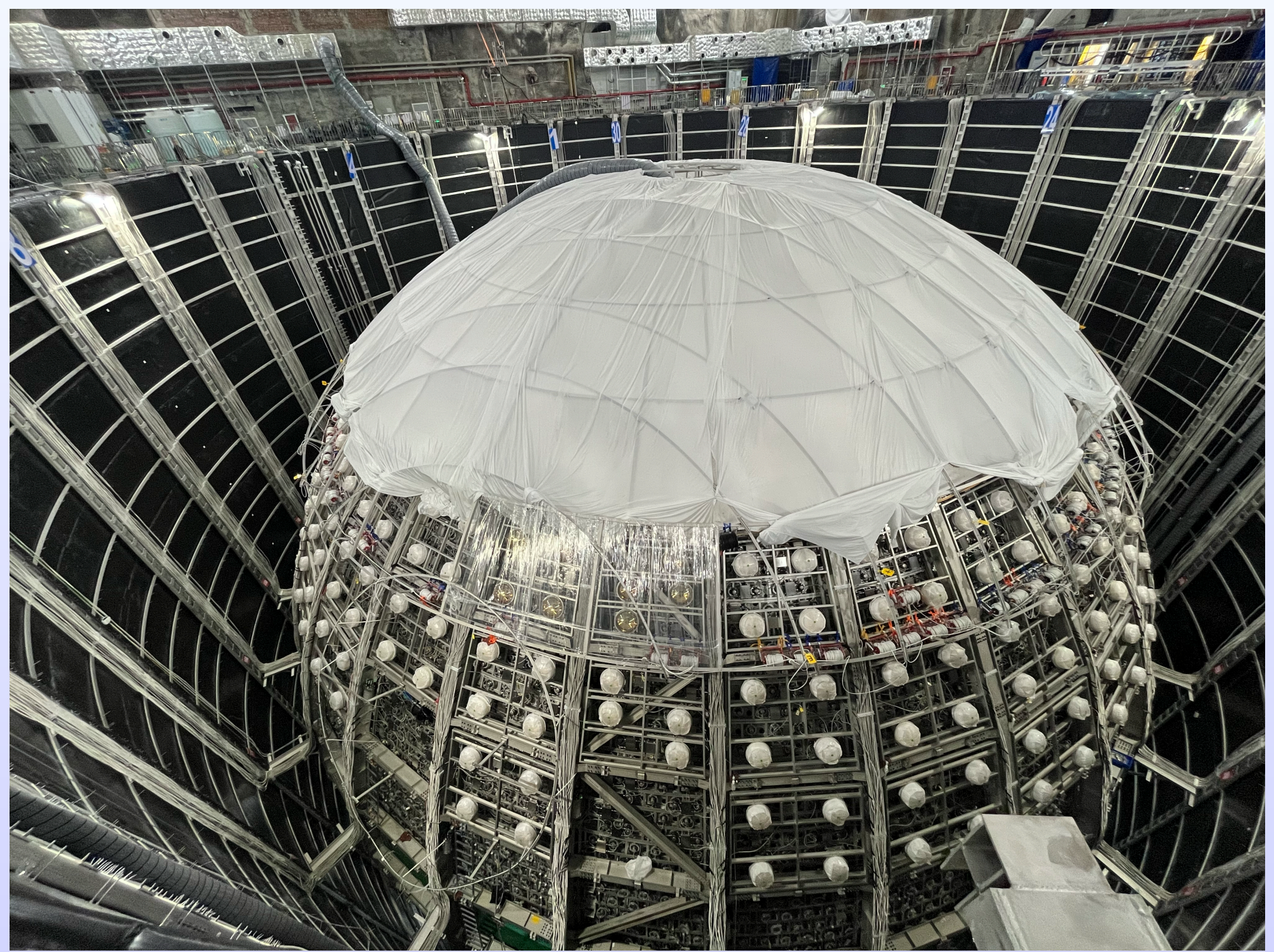
DUNE



<https://www.dunescience.org/>



JUNO

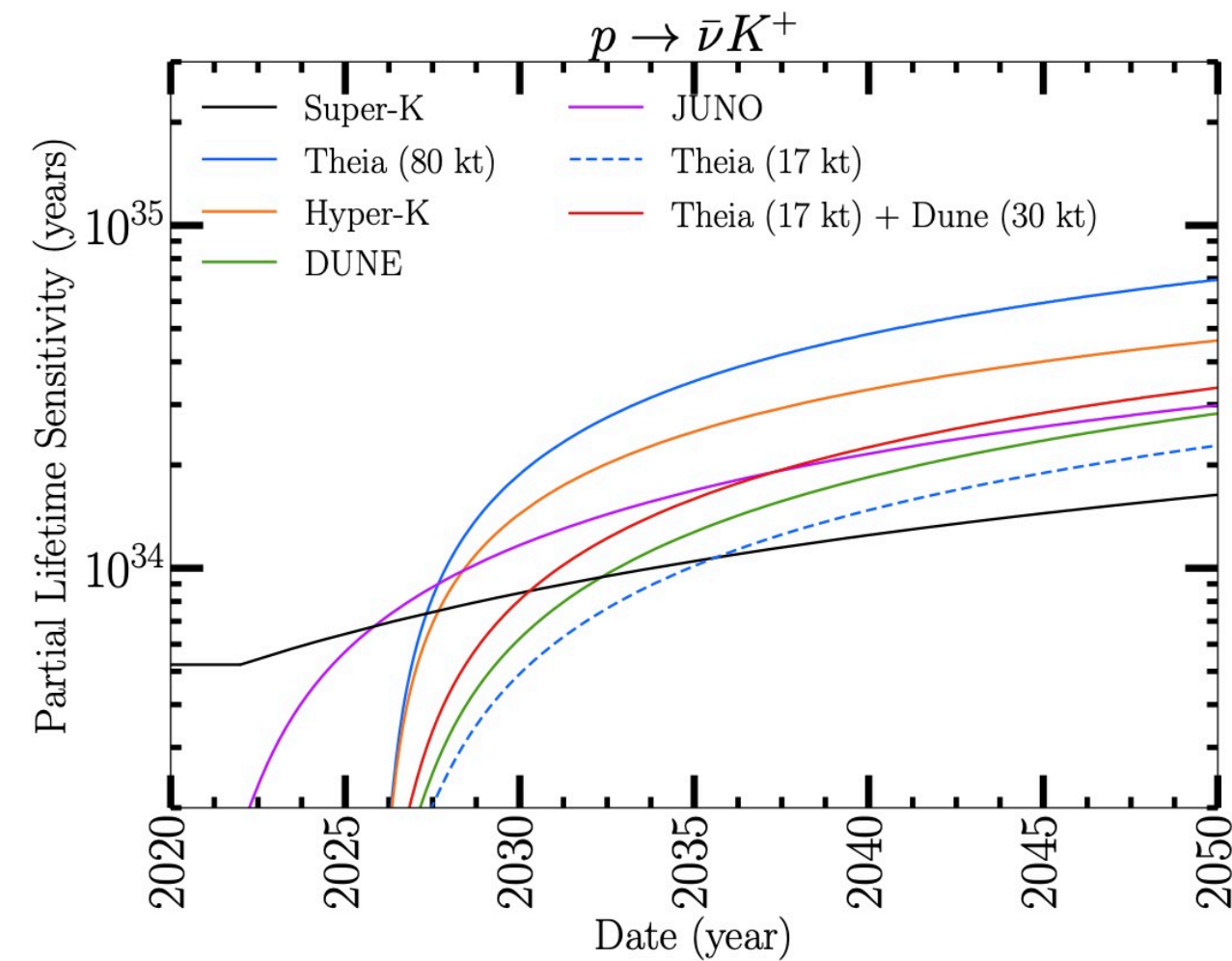


<https://www-sk.icrr.u-tokyo.ac.jp/en/news/detail/684>

Hyper-K

Experiments meet theory

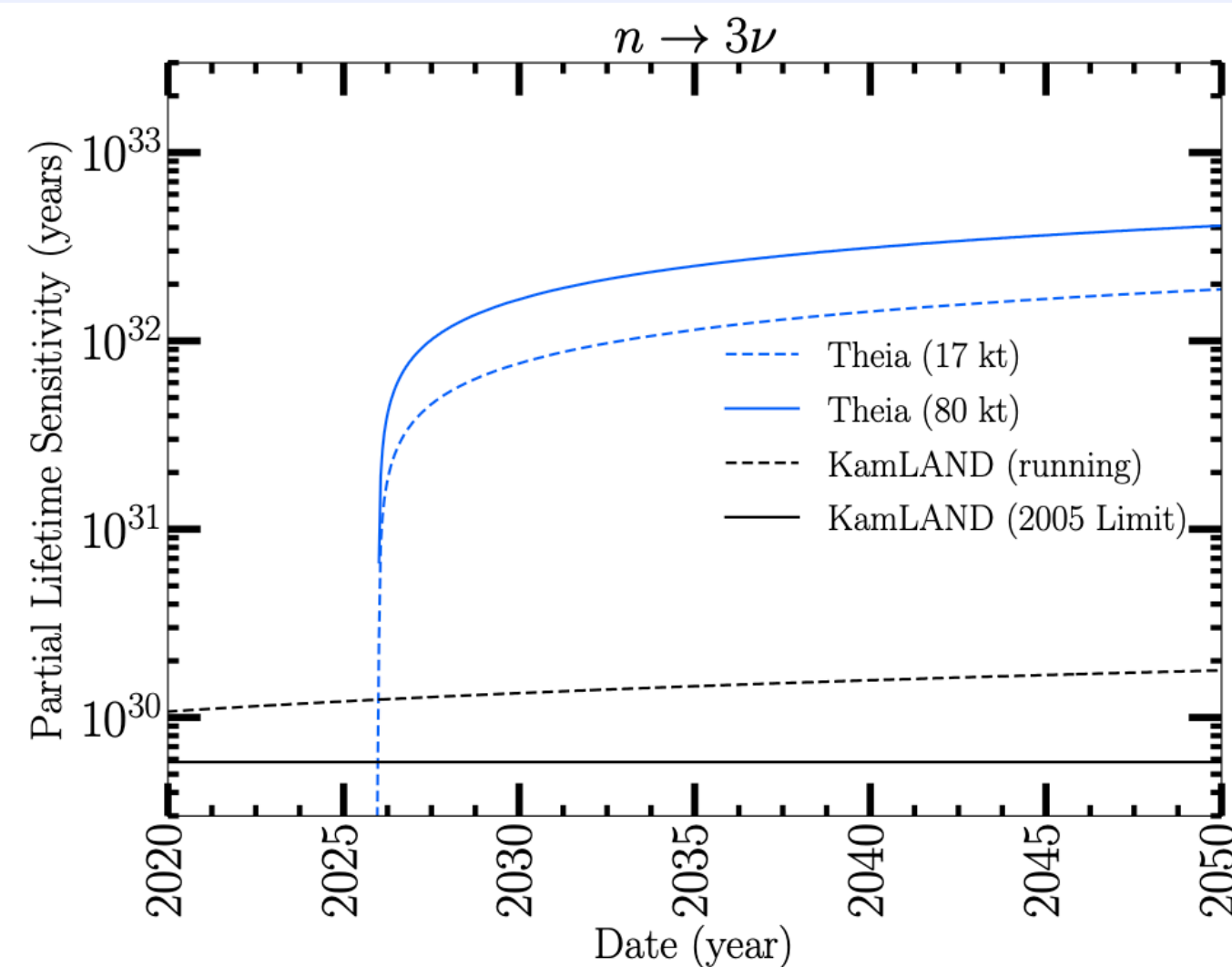
- Improving sensitivity to conventional modes



All possible decay channels should be attempted

conventional and exotic

- Searching for exotic modes



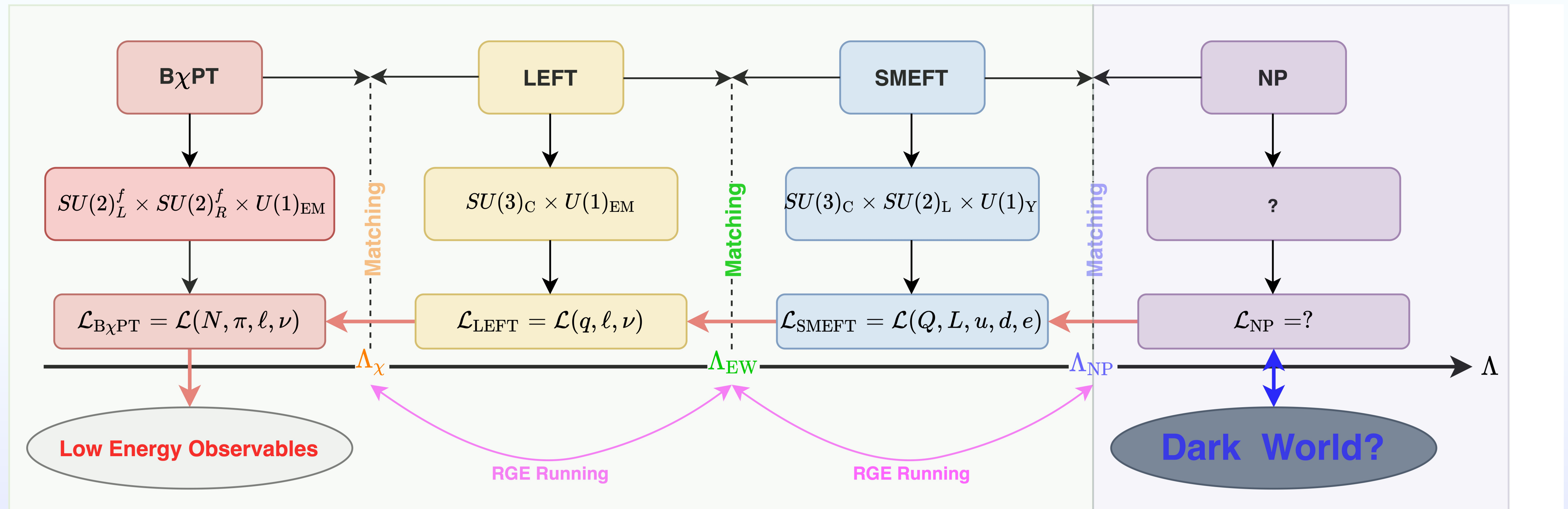
- 2-body vs 3-body modes
- Pseudoscalar vs vector meson modes
- SM particles vs new light particles
-



Needing a complete theoretical framework

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- **General BNV nucleon decay interactions in the LEFT**
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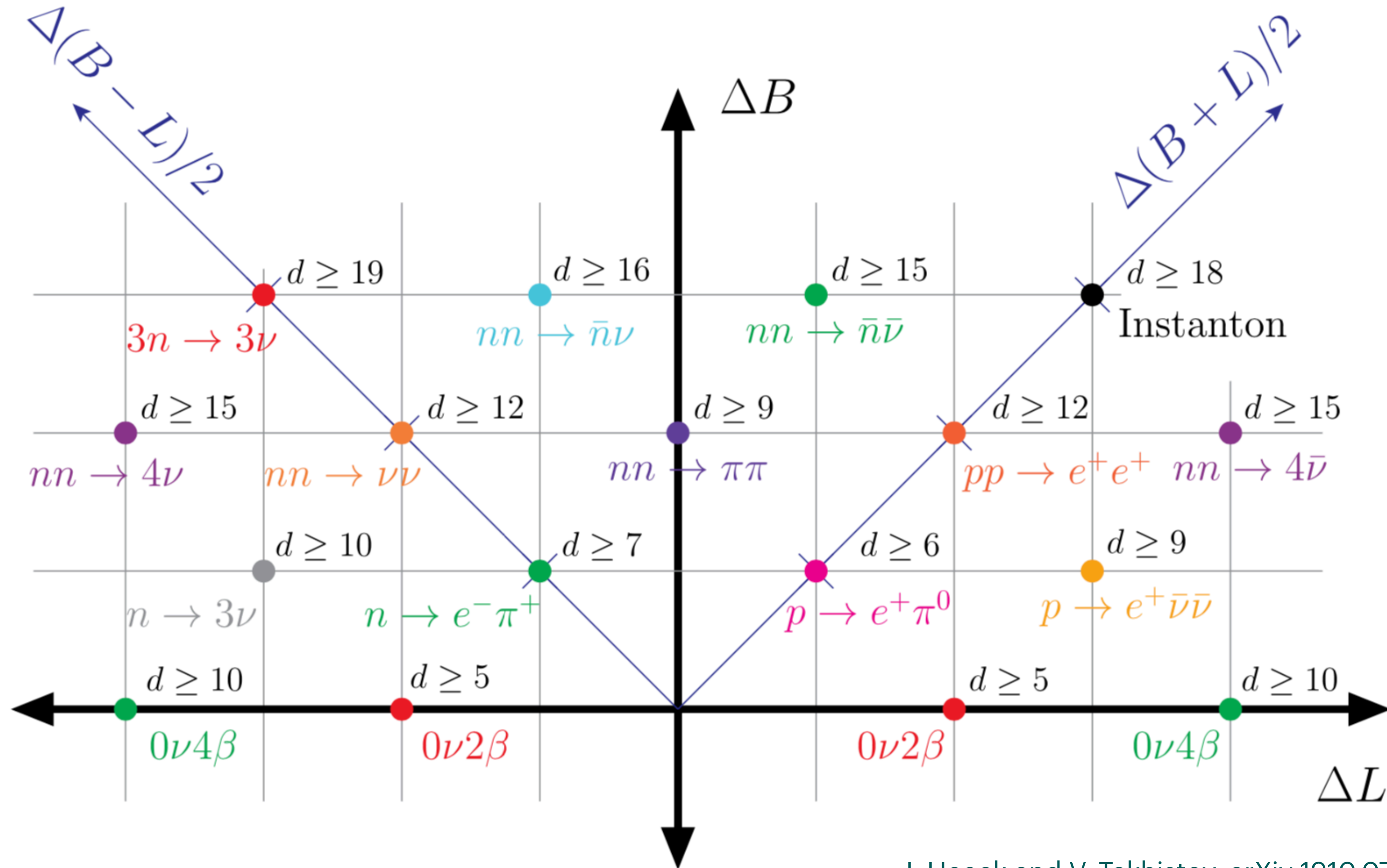
Nucleon decay in the effective field theory (EFT) landscape



Xiao-Gang He and XDM, 2102.02562

Hadron level: ChPT $\rightarrow$ quark level 1: LEFT $\rightarrow$ quark level 2: SMEFT

BNV in the SMEFT framework



J. Heeck and V. Takhistov, arXiv:1910.07647

- Dim-6 case: $\Delta(B - L) = 0$
 $p \rightarrow e^+\pi^0, p \rightarrow \bar{\nu}K^+, \dots$
- Dim-7 case: $\Delta(B + L) = 0$
 $n \rightarrow e^-\pi^+, p \rightarrow \nu K^+, \dots$
- Exotic nucleon decay modes beyond dim 6 & 7
 $n \rightarrow 3\nu, p \rightarrow \ell_1^+\ell_2^+\ell_3^-, \dots$

Working framework: low energy effective field theory (LEFT)

- Fields: $u, d, s, \cancel{c, b}; e, \mu, \tau; \nu_e, \nu_\mu, \nu_\tau$
- Symmetry: $SU(3)_c \times U(1)_{em}$
- Power counting: canonical dimension d
- Range: $\ll \Lambda_{EW}$

$\Delta(B - L) = 0$		$\Delta(B + L) = 0$	
$\mathcal{O}_{\nu dud}^{LL}$	$(\bar{\nu}_L^c d_L^\alpha)(\bar{u}_L^{\beta c} d_L^\gamma) \epsilon_{\alpha\beta\gamma}$	$\mathcal{O}_{\bar{\ell} ddd}^{LL}$	$(\bar{\ell}_R d_L^\alpha)(\bar{d}_L^{\beta c} d_L^\gamma) \epsilon_{\alpha\beta\gamma}$
$\mathcal{O}_{\ell udu}^{LL}$	$(\bar{\ell}_L^c u_L^\alpha)(\bar{d}_L^{\beta c} u_L^\gamma) \epsilon_{\alpha\beta\gamma}$	$\mathcal{O}_{\bar{\nu} dud}^{RL}$	$(\bar{\nu}_L d_R^\alpha)(\bar{u}_L^{\beta c} d_L^\gamma) \epsilon_{\alpha\beta\gamma}$
$\mathcal{O}_{\ell duu}^{RL}$	$(\bar{\ell}_R^c d_R^\alpha)(\bar{u}_L^{\beta c} u_L^\gamma) \epsilon_{\alpha\beta\gamma}$	$\mathcal{O}_{\bar{\nu} udd}^{RL}$	$(\bar{\nu}_L u_R^\alpha)(\bar{d}_L^{\beta c} d_L^\gamma) \epsilon_{\alpha\beta\gamma}$
$\mathcal{O}_{\ell udu}^{RL}$	$(\bar{\ell}_R^c u_R^\alpha)(\bar{d}_L^{\beta c} u_L^\gamma) \epsilon_{\alpha\beta\gamma}$	$\mathcal{O}_{\bar{\ell} ddd}^{RL}$	$(\bar{\ell}_L d_R^\alpha)(\bar{d}_L^{\beta c} d_L^\gamma) \epsilon_{\alpha\beta\gamma}$
$\mathcal{O}_{\ell duu}^{LR}$	$(\bar{\ell}_L^c d_L^\alpha)(\bar{u}_R^{\beta c} u_R^\gamma) \epsilon_{\alpha\beta\gamma}$	$\mathcal{O}_{\bar{\ell} ddd}^{LR}$	$(\bar{\ell}_R d_L^\alpha)(\bar{d}_R^{\beta c} d_R^\gamma) \epsilon_{\alpha\beta\gamma}$
$\mathcal{O}_{\ell udu}^{LR}$	$(\bar{\ell}_L^c u_L^\alpha)(\bar{d}_R^{\beta c} u_R^\gamma) \epsilon_{\alpha\beta\gamma}$	$\mathcal{O}_{\bar{\nu} dud}^{RR}$	$(\bar{\nu}_L d_R^\alpha)(\bar{u}_R^{\beta c} d_R^\gamma) \epsilon_{\alpha\beta\gamma}$
$\mathcal{O}_{\nu ddu}^{LR}$	$(\bar{\nu}_L^c d_L^\alpha)(\bar{d}_R^{\beta c} u_R^\gamma) \epsilon_{\alpha\beta\gamma}$	$\mathcal{O}_{\bar{\ell} ddd}^{RR}$	$(\bar{\ell}_L d_R^\alpha)(\bar{d}_R^{\beta c} d_R^\gamma) \epsilon_{\alpha\beta\gamma}$
$\mathcal{O}_{\nu udd}^{LR}$	$(\bar{\nu}_L^c u_L^\alpha)(\bar{d}_R^{\beta c} d_R^\gamma) \epsilon_{\alpha\beta\gamma}$	Jenkins, Manohar, Stoffer, 2018	
$\mathcal{O}_{\ell udu}^{RR}$	$(\bar{\ell}_R^c u_R^\alpha)(\bar{d}_R^{\beta c} u_R^\gamma) \epsilon_{\alpha\beta\gamma}$		

Jenkins, Manohar, Stoffer, 2018

Li, Ren, Xiao, Yu, Zheng, 2020

$$\mathcal{L}_{LEFT} = \mathcal{L}_{\dim \leq 4} + \sum_{\dim 5, i} \frac{\hat{C}_{5,i}}{\Lambda} Q_{\dim-5}^i + \sum_{\dim 6, i} \frac{\hat{C}_{6,i}}{\Lambda^2} Q_{\dim-6}^i + \sum_{\dim 7, i} \frac{\hat{C}_{7,i}}{\Lambda^3} Q_{\dim-7}^i + \sum_{\dim 8, i} \frac{\hat{C}_{8,i}}{\Lambda^4} Q_{\dim-8}^i + \sum_{\dim 9, i} \frac{C_{9,i}}{\Lambda^5} Q_{\dim-9}^i + \dots$$

Yi Liao, XDM, Quan-Yu Wang, 2020

Murphy, 2020

Yi Liao, XDM, Hao-Lin Wang, 2019

F. Wilczek and A. Zee, PRL 43 (1979)

J. R. Ellis, M. k. Gaillard, and D. V. Nanopoulos, PLB 88 (1979)

S. Weinberg, PRL 43 (1979) & PRD 22 (1980)

L. F. Abbott and M. B. Wise, PRD 22 (1980)

Easily to bridge with the SMEFT framework

Exotic nucleon decay involving new light particles

LEFT + new light particles $\Rightarrow$ LEFT-like framework $\Rightarrow$ SMEFT-like framework

- LEFT + sterile neutrino (N): $p \rightarrow N\pi^+$, $n \rightarrow N\pi^0$, $\dots$

$$(\overline{N_R} d_L^\alpha)(\overline{u_L^{\beta C}} d_L^\gamma) \epsilon_{\alpha\beta\gamma}, \dots$$

- LEFT + ALP (a): $p \rightarrow e^+ a$, $n \rightarrow e^+ \pi^- a$, $\dots$

$$(\partial_\mu a)(\overline{e_L^C} u_L^\alpha)(\overline{u_L^{\beta C}} \gamma^\mu d_L^\gamma) \epsilon_{\alpha\beta\gamma}, \dots$$

- LEFT + dark photon (X): $p \rightarrow e^+ X$, $n \rightarrow e^+ \pi^- X$, $\dots$

$$X_{\mu\nu}(\overline{\ell_R} d_L^\alpha)(\overline{d_L^{\beta C}} \sigma^{\mu\nu} d_L^\gamma) \epsilon_{\alpha\beta\gamma}, \dots$$

- LEFT + scalar (φ): $p \rightarrow e^+ \varphi$, $n \rightarrow e^+ \pi^- \varphi$, $\dots$

$$\varphi(\overline{\nu_L^C} d_L^\alpha)(\overline{u_L^{\beta C}} d_L^\gamma) \epsilon_{\alpha\beta\gamma}, \dots$$



General $\Delta B = 1$ nucleon decay operator structures

- Must involve an odd number of light quarks: $qqq, qqqG, qqqq\bar{q}, \dots$
- Leading-order interactions: involve only **three light quarks**
- Only **four** general triple-quark (without derivative) structures

$$\mathcal{O}_a^{yzw} = (\overline{\Psi}_a q_{L,y}^\alpha) (\overline{q_{L,z}^{\beta C}} q_{L,w}^\gamma) \epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_b^{yzw} = (\overline{\Psi}_b q_{R,y}^\alpha) (\overline{q_{L,z}^{\beta C}} q_{L,w}^\gamma) \epsilon_{\alpha\beta\gamma}$$

+ their **chiral partners** with $L \leftrightarrow R$

$$\mathcal{O}_c^{yzw} = (\overline{\Psi}_{c,\mu} q_{L,\{y}^\alpha) (\overline{q_{L,z}^{\beta C}} \gamma^\mu q_{R,w}^\gamma) \epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_d^{yzw} = (\overline{\Psi}_{d,\mu\nu} q_{L,\{y}^\alpha) (\overline{q_{L,z}^{\beta C}} \sigma^{\mu\nu} q_{L,w}^\gamma) \epsilon_{\alpha\beta\gamma}$$

❖ $\overline{\Psi}_a, \overline{\Psi}_b, \overline{\Psi}_{c,\mu}, \overline{\Psi}_{d,\mu\nu}$: combinations of **non-QCD** fields

❖ $y, z, w = 1, 2, 3$: quark flavor indices with $q_{1,2,3} = u, d, s$

❖ $\{y, z\}$ and $\{y, z, w\}$: total symmetrization of flavor indices

Newly identified structures

- Form a basis for any triple-quark operators

Yi Liao, XDM, Hao-Lin Wang, arXiv: 2504.14855

QCD running effect

- 1-loop RGE due to QCD correction

$$\frac{dC_{a,b}^{yzw}}{d \ln \mu} = -2 \frac{\alpha_s}{2\pi} C_{a,b}^{yzw}, \quad \frac{dC_c^{yzw}}{d \ln \mu} = +\frac{2}{3} \frac{\alpha_s}{2\pi} C_c^{yzw}, \quad \frac{dC_d^{yzw}}{d \ln \mu} = +2 \frac{\alpha_s}{2\pi} C_d^{yzw}$$

Known from dim-6 LEFT counterparts

E. E. Jenkins, A. V. Manohar, and P. Stoffer, arXiv:1711.05270v3

- No mixing: QCD interactions preserve chiral symmetry
- Running effects

$$C_{a,b}^{yzw}(\Lambda_\chi) \approx 1.32 C_{a,b}^{yzw}(\Lambda_{EW}), \quad C_c^{yzw}(\Lambda_\chi) \approx 0.91 C_c^{yzw}(\Lambda_{EW}), \quad C_d^{yzw}(\Lambda_\chi) \approx 0.76 C_d^{yzw}(\Lambda_{EW})$$

Enhancement by 30%

Suppression

Chiral structures

- 3-flavor (u, d, s) QCD has a global chiral symmetry: $SU(3)_L \otimes SU(3)_R$
in the massless limit
- Restrict to triple-quark sector

$$\mathcal{N}_{yzw}^{LL} \equiv q_{L,y}^\alpha (\overline{q_{L,z}^{\beta C}} q_{L,w}^\gamma) \epsilon_{\alpha\beta\gamma} \in \mathbf{8}_L \otimes \mathbf{1}_R$$

$$\mathcal{N}_{yzw}^{RL} \equiv q_{R,y}^\alpha (\overline{q_{L,z}^{\beta C}} q_{L,w}^\gamma) \epsilon_{\alpha\beta\gamma} \in \bar{\mathbf{3}}_L \otimes \mathbf{3}_R$$

$$+L \leftrightarrow R$$

$$\mathcal{N}_{yzw}^{LR,\mu} \equiv q_{L,\{y}^\alpha (\overline{q_{L,z}^{\beta C}} \gamma^\mu q_{R,w}^\gamma) \epsilon_{\alpha\beta\gamma} \in \mathbf{6}_L \otimes \mathbf{3}_R$$

$$\mathcal{N}_{yzw}^{LL,\mu\nu} \equiv q_{L,\{y}^\alpha (\overline{q_{L,z}^{\beta C}} \sigma^{\mu\nu} q_{L,w}^\gamma) \epsilon_{\alpha\beta\gamma} \in \mathbf{10}_L \otimes \mathbf{1}_R$$

Newly identified chiral structures

- Different isospin property

$$\mathcal{N}_{yzw}^{LL} \text{ and } \mathcal{N}_{yzw}^{RL} \Rightarrow \Delta I = 0, 1/2, 1$$

$$\mathcal{N}_{yzw}^{LR,\mu} \text{ and } \mathcal{N}_{yzw}^{LL,\mu\nu} \Rightarrow \Delta I = 0, 1/2, 1, \textcolor{red}{3/2} \Rightarrow \textcolor{red}{n \rightarrow e^- \pi^+, n \rightarrow \mu^- \pi^+}$$

Non-trivial Lorentz structures

- Usual structures—spin-1/2 objects:

$$\mathcal{N}_{\mathbf{8}_L \otimes \mathbf{1}_R}, \mathcal{N}_{\mathbf{3}_L \otimes \bar{\mathbf{3}}_R} \in (1/2, 0), \quad \mathcal{N}_{\mathbf{1}_L \otimes \mathbf{8}_R}, \mathcal{N}_{\bar{\mathbf{3}}_L \otimes \mathbf{3}_R} \in (0, 1/2)$$

- New structures—spin-3/2 objects:

Vector-spinor object: $\mathcal{N}_{yzw}^{\text{LR},\mu} \in (1, 1/2), \quad \mathcal{N}_{yzw}^{\text{RL},\mu} \in (1/2, 1) \quad \gamma_\mu \mathcal{N}_{yzw}^{\text{LR},\mu} = \gamma_\mu \mathcal{N}_{yzw}^{\text{RL},\mu} = 0$

Tensor-spinor object: $\mathcal{N}_{yzw}^{\text{LL},\mu\nu} \in (3/2, 0), \quad \mathcal{N}_{yzw}^{\text{RR},\mu\nu} \in (0, 3/2) \quad \gamma_\mu \mathcal{N}_{yzw}^{\text{LL},\mu\nu} = \gamma_\mu \mathcal{N}_{yzw}^{\text{RR},\mu\nu} = 0$



Complicating the chiral matching $\longrightarrow$ Needing proper Lorentz projectors

$$\Gamma_{\mu\nu}^{\text{L,R}} \equiv \left(g_{\mu\nu} - \frac{1}{4} \gamma_\mu \gamma_\nu \right) P_{\text{L,R}}$$

$$\Gamma_{\mu\rho}^{\text{L,R}} \Gamma_{\nu}^{\text{L,R}}{}^\rho = \Gamma_{\mu\nu}^{\text{L,R}} \quad \gamma^\mu \Gamma_{\mu\nu}^{\text{L,R}} = 0$$

$$\hat{\Gamma}_{\mu\nu\alpha\beta}^{\text{L,R}} \equiv \frac{1}{24} \left(2\{\sigma_{\mu\nu}, \sigma_{\alpha\beta}\} - [\sigma_{\mu\nu}, \sigma_{\alpha\beta}] \right) P_{\text{L,R}}$$

$$\hat{\Gamma}_{\mu\nu\rho\sigma}^{\text{L,R}} \hat{\Gamma}_{\alpha\beta}^{\text{L,R}}{}^{\rho\sigma} = \hat{\Gamma}_{\mu\nu\alpha\beta}^{\text{L,R}} \quad \gamma^\mu \hat{\Gamma}_{\mu\nu\alpha\beta}^{\text{L,R}} = 0$$

The general LEFT Lagrangian involving triple quarks

Non-quark factor as spurion fields

$$\mathcal{L}_{q^3}^{\mathcal{B}} = \sum_i C_i^{yzw} \mathcal{O}_i^{yzw} = \sum_i \boxed{C_i^{yzw} \bar{\psi}} \Gamma_1 q (\bar{q}^c \Gamma_2 q) \quad \text{Quark factor}$$

$$\equiv \mathcal{P}_{yzw}^i \quad \equiv \mathcal{N}_{yzw}^i$$

$$\begin{aligned} \mathcal{L}_{q^3}^{\mathcal{B}} = & \text{Tr}[\mathcal{P}_{\mathbf{8}_L \otimes \mathbf{1}_R} \mathcal{N}_{\mathbf{8}_L \otimes \mathbf{1}_R} + \mathcal{P}_{\mathbf{1}_L \otimes \mathbf{8}_R} \mathcal{N}_{\mathbf{1}_L \otimes \mathbf{8}_R}] \\ & + \text{Tr}[\mathcal{P}_{\mathbf{3}_L \otimes \bar{\mathbf{3}}_R} \mathcal{N}_{\bar{\mathbf{3}}_L \otimes \mathbf{3}_R} + \mathcal{P}_{\bar{\mathbf{3}}_L \otimes \mathbf{3}_R} \mathcal{N}_{\mathbf{3}_L \otimes \bar{\mathbf{3}}_R}] \\ & + [\mathcal{P}_{\bar{\mathbf{6}}_L \otimes \bar{\mathbf{3}}_R}^{\{yz\}w, \mu} \mathcal{N}_{\mathbf{6}_L \otimes \mathbf{3}_R, \mu}^{\{yz\}w} + \mathcal{P}_{\bar{\mathbf{3}}_L \otimes \bar{\mathbf{6}}_R}^{\{yz\}w, \mu} \mathcal{N}_{\mathbf{3}_L \otimes \mathbf{6}_R, \mu}^{\{yz\}w}] \\ & + [\mathcal{P}_{\mathbf{10}_L \otimes \mathbf{1}_R}^{\{yzw\}, \mu\nu} \mathcal{N}_{\mathbf{10}_L \otimes \mathbf{1}_R, \mu\nu}^{\{yzw\}} + \mathcal{P}_{\mathbf{1}_L \otimes \mathbf{10}_R}^{\{yzw\}, \mu\nu} \mathcal{N}_{\mathbf{1}_L \otimes \mathbf{10}_R, \mu\nu}^{\{yzw\}}] \\ & + \text{h.c.} \end{aligned}$$

Matrix form for easy chiral realization

$$\mathcal{N}_{\mathbf{8}_L \otimes \mathbf{1}_R} = \begin{pmatrix} \mathcal{N}_{uds}^{\text{LL}} \mathcal{N}_{usu}^{\text{LL}} \mathcal{N}_{uud}^{\text{LL}} \\ \mathcal{N}_{dds}^{\text{LL}} \mathcal{N}_{dsu}^{\text{LL}} \mathcal{N}_{dud}^{\text{LL}} \\ \mathcal{N}_{sds}^{\text{LL}} \mathcal{N}_{ssu}^{\text{LL}} \mathcal{N}_{sud}^{\text{LL}} \end{pmatrix} \quad \mathcal{N}_{\bar{\mathbf{3}}_L \otimes \mathbf{3}_R} = \begin{pmatrix} \mathcal{N}_{uds}^{\text{RL}} \mathcal{N}_{usu}^{\text{RL}} \mathcal{N}_{uud}^{\text{RL}} \\ \mathcal{N}_{dds}^{\text{RL}} \mathcal{N}_{dsu}^{\text{RL}} \mathcal{N}_{dud}^{\text{RL}} \\ \mathcal{N}_{sds}^{\text{RL}} \mathcal{N}_{ssu}^{\text{RL}} \mathcal{N}_{sud}^{\text{RL}} \end{pmatrix}$$

Wei-Qi Fan, Yi Liao, XDM, Hao-Lin Wang, 2412.20774

Chiral building blocks

$$\mathcal{P}_{\mathbf{8}_L \otimes \mathbf{1}_R} = \begin{pmatrix} 0 & \mathcal{P}_{dds}^{\text{LL}} & \mathcal{P}_{sds}^{\text{LL}} \\ \mathcal{P}_{usu}^{\text{LL}} & \mathcal{P}_{dsu}^{\text{LL}} & \mathcal{P}_{ssu}^{\text{LL}} \\ \mathcal{P}_{uud}^{\text{LL}} & \mathcal{P}_{dud}^{\text{LL}} & \mathcal{P}_{sud}^{\text{LL}} \end{pmatrix} \quad \mathcal{P}_{\mathbf{3}_L \otimes \bar{\mathbf{3}}_R} = \begin{pmatrix} \mathcal{P}_{uds}^{\text{RL}} \mathcal{P}_{dds}^{\text{RL}} \mathcal{P}_{sds}^{\text{RL}} \\ \mathcal{P}_{usu}^{\text{RL}} \mathcal{P}_{dsu}^{\text{RL}} \mathcal{P}_{ssu}^{\text{RL}} \\ \mathcal{P}_{uud}^{\text{RL}} \mathcal{P}_{dud}^{\text{RL}} \mathcal{P}_{sud}^{\text{RL}} \end{pmatrix} \quad \mathcal{P}_{yzw}^{\text{LR}, \mu} \quad \mathcal{P}_{yzw}^{\text{LL}, \mu\nu}$$

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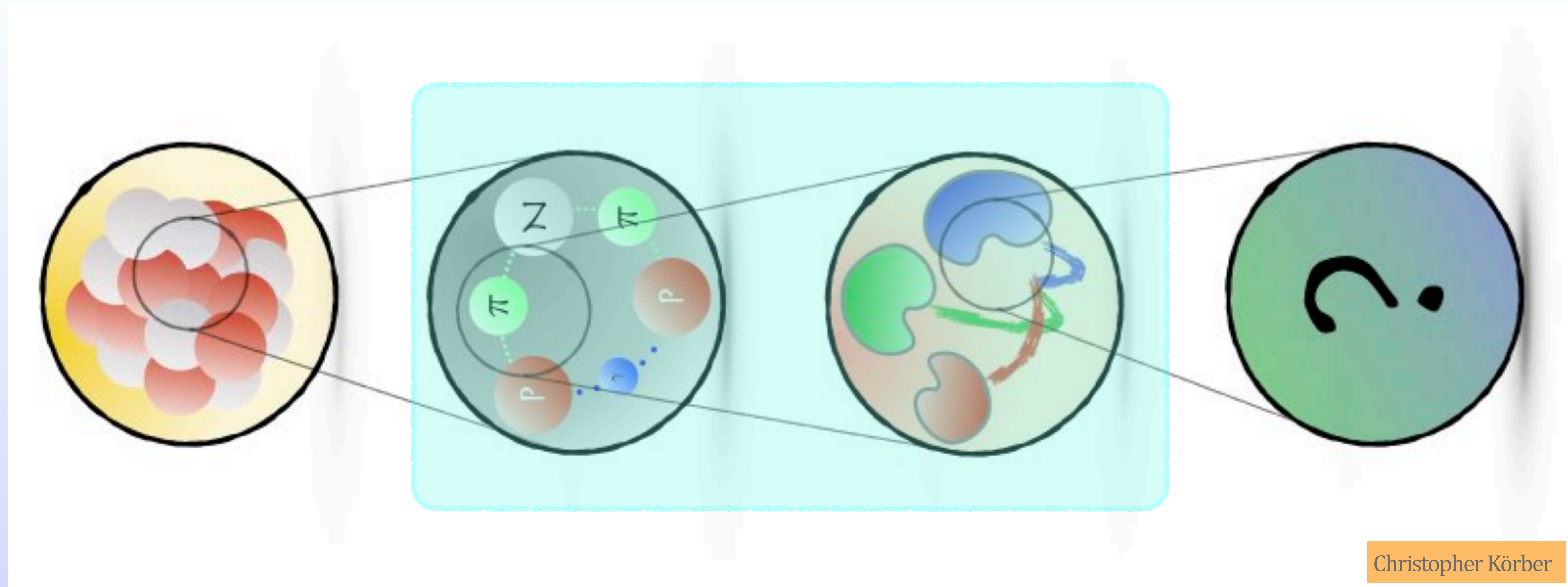
Chiral Perturbation Theory

$$SU(3)_L \otimes SU(3)_R$$

u, d, s

Nucleon level operators

Quark level operators



Christopher Körber

Chiral matching procedures

- Building blocks in ChPT: **Octet baryons, pseudoscalars, vector mesons + spurions**

$$\Sigma(x) = \xi^2(x) = \exp\left(\frac{i\sqrt{2}\Pi(x)}{F_0}\right) \quad \Pi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- - \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & & K^0 \\ K^- & \bar{K}^0 - \sqrt{\frac{2}{3}}\eta & \end{pmatrix}, \quad B(x) = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- - \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & & n \\ \Xi^- & \Xi^0 - \sqrt{\frac{2}{3}}\Lambda^0 & \end{pmatrix}, \quad V_\mu(x) = \begin{pmatrix} \frac{\rho_\mu^0}{\sqrt{2}} + \frac{\phi_\mu^{(8)}}{\sqrt{6}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & -\frac{\rho_\mu^0}{\sqrt{2}} + \frac{\phi_\mu^{(8)}}{\sqrt{6}} & K_\mu^{*0} \\ K_\mu^{*-} & \bar{K}_\mu^{*0} & -\sqrt{\frac{2}{3}}\phi_\mu^{(8)} \end{pmatrix}$$

+ Spurion fields

$$\mathcal{P}_{\mathbf{8}_L \otimes \mathbf{1}_R} = \begin{pmatrix} 0 & \mathcal{P}_{dds}^{\text{LL}} & \mathcal{P}_{sds}^{\text{LL}} \\ \mathcal{P}_{usu}^{\text{LL}} & \mathcal{P}_{dsu}^{\text{LL}} & \mathcal{P}_{ssu}^{\text{LL}} \\ \mathcal{P}_{uud}^{\text{LL}} & \mathcal{P}_{dud}^{\text{LL}} & \mathcal{P}_{sud}^{\text{LL}} \end{pmatrix} \quad \mathcal{P}_{\mathbf{3}_L \otimes \bar{\mathbf{3}}_R} = \begin{pmatrix} \mathcal{P}_{uds}^{\text{RL}} & \mathcal{P}_{dds}^{\text{RL}} & \mathcal{P}_{sds}^{\text{RL}} \\ \mathcal{P}_{usu}^{\text{RL}} & \mathcal{P}_{dsu}^{\text{RL}} & \mathcal{P}_{ssu}^{\text{RL}} \\ \mathcal{P}_{uud}^{\text{RL}} & \mathcal{P}_{dud}^{\text{RL}} & \mathcal{P}_{sud}^{\text{RL}} \end{pmatrix} \quad \mathcal{P}_{yzw}^{\text{LR},\mu} \quad \mathcal{P}_{yzw}^{\text{LL},\mu\nu}$$

- Chiral symmetry: $SU(3)_L \otimes SU(3)_R$
- Chiral power counting: momentum p :

$$\{\Sigma, \xi, B, D_\mu B, V, D_\mu V\} \sim \mathcal{O}(p^0) \quad D_\mu \Sigma \sim \mathcal{O}(p^1)$$

- Low energy constant (LEC): associate an unknown LEC for each indep. operator

Leading-order chiral Lagrangian for nucleon decay

Yi Liao, XDM, Hao-Lin Wang, arXiv: 2504.14855

$$\begin{aligned}
 \mathcal{L}_B^{\mathcal{B},0} = & c_1 \text{Tr} \left[\mathcal{P}_{\bar{3}_L \otimes 3_R} \xi B_L \xi - \mathcal{P}_{3_L \otimes \bar{3}_R} \xi^\dagger B_R \xi^\dagger \right] \\
 & + c_2 \text{Tr} \left[\mathcal{P}_{8_L \otimes 1_R} \xi B_L \xi^\dagger - \mathcal{P}_{1_L \otimes 8_R} \xi^\dagger B_R \xi \right] \\
 & + \frac{c_3}{\Lambda_\chi} \left[\mathcal{P}_{yzi}^{\text{LR},\mu} \Gamma_{\mu\nu}^L (\xi i D^\nu B_L \xi)_{yj} \Sigma_{zk} \epsilon_{ijk} - \mathcal{P}_{yzi}^{\text{RL},\mu} \Gamma_{\mu\nu}^R (\xi^\dagger i D^\nu B_R \xi^\dagger)_{yj} \Sigma_{kz}^* \epsilon_{ijk} \right] \\
 & + \text{h.c.}, \\
 \mathcal{L}_B^{\mathcal{B},1} = & \frac{c_4}{\Lambda_\chi^2} \left[\mathcal{P}_{yzw}^{\text{LL},\mu\nu} \hat{\Gamma}_{\mu\nu\alpha\beta}^L (\xi D^\alpha B_L \xi)_{yi} \Sigma_{zj} (D^\beta \Sigma)_{wk} \epsilon_{ijk} - \mathcal{P}_{yzw}^{\text{RR},\mu\nu} \hat{\Gamma}_{\mu\nu\alpha\beta}^R (\xi^\dagger D^\alpha B_R \xi^\dagger)_{iy} \Sigma_{jz}^* (D^\beta \Sigma)_{kw}^* \epsilon_{ijk} \right] \\
 & + \text{h.c.}.
 \end{aligned}$$

Related to new structures

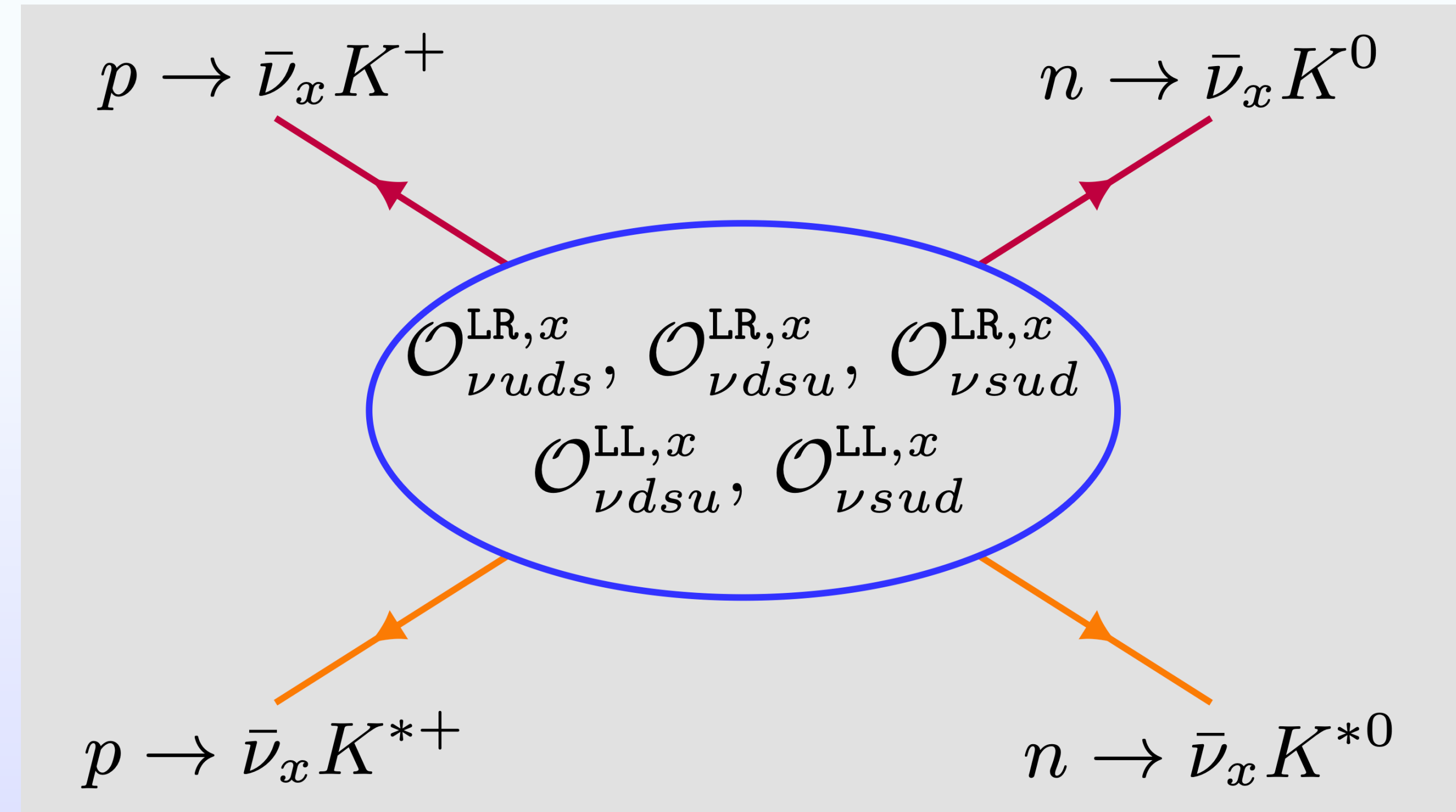
$$\Gamma_{\mu\nu}^{\text{L,R}} \equiv \left(g_{\mu\nu} - \frac{1}{4} \gamma_\mu \gamma_\nu \right) P_{\text{L,R}} \quad \hat{\Gamma}_{\mu\nu\alpha\beta}^{\text{L,R}} \equiv \frac{1}{24} \left(2 \{ \sigma_{\mu\nu}, \sigma_{\alpha\beta} \} - [\sigma_{\mu\nu}, \sigma_{\alpha\beta}] \right) P_{\text{L,R}}$$

Expanding these Lagrangian terms lead to relevant interacting vertices, then the relevant amplitude can be computed as simple as in the perturbative QFT

Chiral Lagrangian involving octet vector mesons

$$\begin{aligned}
 \mathcal{L}_{BV}^{\#} = & d_1 \text{Tr} [\mathcal{P}_{\bar{\mathbf{3}}_L \otimes \mathbf{3}_R} \xi \gamma_\mu V^\mu B_R \xi - \mathcal{P}_{\mathbf{3}_L \otimes \bar{\mathbf{3}}_R} \xi^\dagger \gamma_\mu V^\mu B_L \xi^\dagger] \\
 & + d'_1 \text{Tr} [\mathcal{P}_{\bar{\mathbf{3}}_L \otimes \mathbf{3}_R} \xi \gamma_\mu B_R V^\mu \xi - \mathcal{P}_{\mathbf{3}_L \otimes \bar{\mathbf{3}}_R} \xi^\dagger \gamma_\mu B_L V^\mu \xi^\dagger] \\
 & + d_2 \text{Tr} [\mathcal{P}_{\mathbf{8}_L \otimes \mathbf{1}_R} \xi \gamma_\mu V^\mu B_R \xi^\dagger - \mathcal{P}_{\mathbf{1}_L \otimes \mathbf{8}_R} \xi^\dagger \gamma_\mu V^\mu B_L \xi] \\
 & + d'_2 \text{Tr} [\mathcal{P}_{\mathbf{8}_L \otimes \mathbf{1}_R} \xi \gamma_\mu B_R V^\mu \xi^\dagger - \mathcal{P}_{\mathbf{1}_L \otimes \mathbf{8}_R} \xi^\dagger \gamma_\mu B_L V^\mu \xi] \\
 & + d_3 [\mathcal{P}_{yzi}^{\text{LR},\mu} \Gamma_{\mu\nu}^L (\xi B_L \xi)_{yj} (\xi V^\nu \xi)_{zk} \epsilon_{ijk} \\
 & - \mathcal{P}_{yzi}^{\text{RL},\mu} \Gamma_{\mu\nu}^R (\xi^\dagger B_R \xi^\dagger)_{yj} (\xi^\dagger V^\nu \xi^\dagger)_{kz} \epsilon_{ijk}] \\
 & + d'_3 [\mathcal{P}_{yzi}^{\text{LR},\mu} \Gamma_{\mu\nu}^L (\xi V^\nu B_L \xi)_{yj} \Sigma_{zk} \epsilon_{ijk} \\
 & - \mathcal{P}_{yzi}^{\text{RL},\mu} \Gamma_{\mu\nu}^R (\xi^\dagger V^\nu B_R \xi^\dagger)_{yj} \Sigma_{kz}^* \epsilon_{ijk}] \\
 & + d''_3 [\mathcal{P}_{yzi}^{\text{LR},\mu} \Gamma_{\mu\nu}^L (\xi B_L V^\nu \xi)_{yj} \Sigma_{zk} \epsilon_{ijk} \\
 & - \mathcal{P}_{yzi}^{\text{RL},\mu} \Gamma_{\mu\nu}^R (\xi^\dagger B_R V^\nu \xi^\dagger)_{yj} \Sigma_{kz}^* \epsilon_{ijk}] \\
 & + \frac{d_4}{\Lambda_\chi} [\mathcal{P}_{yzw}^{\text{LL},\mu\nu} \hat{\Gamma}_{\mu\nu\alpha\beta}^L (\xi D^\alpha B_L \xi)_{yi} \Sigma_{zj} (\xi V^\beta \xi)_{wk} \epsilon_{ijk} \\
 & - \mathcal{P}_{yzw}^{\text{RR},\mu\nu} \hat{\Gamma}_{\mu\nu\alpha\beta}^R (\xi^\dagger D^\alpha B_R \xi^\dagger)_{yi} \Sigma_{jz}^* (\xi V^\beta \xi)_{kw}^* \epsilon_{ijk}] \\
 & + \text{h.c.},
 \end{aligned}$$

Complementarity between
pseudoscalar and vector meson



Comparison to known results: ~40 years ago

Octet pseudoscalar case

CHIRAL LAGRANGIAN FOR DEEP MINE PHYSICS*

Mark CLAUDSON and Mark B. WISE¹

Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

Lawrence J. HALL

Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, USA

Received 25 August 1981

The chiral lagrangian for baryon number violating nucleon decay is derived and applied to nucleon decays into strange and non-strange final states. The uncertainties in our predictions are discussed.

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M. Claudson et al. / Chiral lagrangian

with $SU(3) \times SU(2) \times U(1)$ symmetry are [5, 6]

$$O_{abcd}^{(1)} = (d_{\alpha\alpha R} u_{\beta\beta R})(q_{i\gamma c L} l_{j d L}) \epsilon_{\alpha\beta\gamma} \epsilon_{ij}, \quad (1a)$$

$$O_{abcd}^{(2)} = (q_{i\alpha a L} q_{j\beta b L})(u_{\gamma c R} l_{d R}) \epsilon_{\alpha\beta\gamma} \epsilon_{ij}, \quad (1b)$$

$$O_{abcd}^{(3)} = (q_{i\alpha a L} q_{j\beta b L})(q_{k\gamma c L} l_{d L}) \epsilon_{\alpha\beta\gamma} \epsilon_{ik} \epsilon_{jk}, \quad (1c)$$

$$O_{abcd}^{(4)} = (d_{\alpha\alpha R} u_{\beta\beta R})(u_{\gamma c R} l_{d R}) \epsilon_{\alpha\beta\gamma}. \quad (1d)$$

$$\begin{aligned} \mathcal{L}^{|\Delta B|=1} = & \alpha \sum_{d=1}^2 \{ C_d^{(1)} [e_{dL} \text{Tr} O \xi B_L \xi - \nu_{dL} \text{Tr} O' \xi B_L \xi] \\ & + C_d^{(2)} e_{dR} \text{Tr} O \xi^+ B_R \xi^+ + \tilde{C}_d^{(1)} [e_{dL} \text{Tr} \tilde{O} \xi B_L \xi - \nu_{dL} \text{Tr} \tilde{O}' \xi B_L \xi] \\ & + \tilde{C}_d^{(2)} e_{dR} \text{Tr} \tilde{O} \xi^+ B_R \xi^+ + \tilde{C}_d^{(5)} \nu_{dL} \text{Tr} \tilde{O}'' \xi B_L \xi \} \\ & + \beta \sum_{d=1}^2 \{ C_d^{(3)} [e_{dL} \text{Tr} O \xi B_L \xi^+ - \nu_{dL} \text{Tr} O' \xi B_L \xi^+] \\ & + C_d^{(4)} e_{dR} \text{Tr} O \xi^+ B_R \xi + \tilde{C}_d^{(3)} [e_{dL} \text{Tr} \tilde{O} \xi B_L \xi^+ - \nu_{dL} \text{Tr} \tilde{O}' \xi B_L \xi^+] \\ & + \tilde{C}_d^{(4)} e_{dR} \text{Tr} \tilde{O} \xi^+ B_R \xi + \tilde{C}_d^{(6)} \nu_{dL} \text{Tr} \tilde{O}'' \xi B_L \xi^+] + \text{h.c.}, \end{aligned} \quad (16)$$

Octet vector meson case

PHYSICAL REVIEW D

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Chiral Lagrangian for proton decay

Ömer Kaymakçalan, Lo Chong-Huah, and Kameshwar C. Wali

Physics Department, Syracuse University, Syracuse, New York 13210

(Received 6 June 1983)

We extend the recent chiral model of Claudson, Wise, and Hall to include vector and axial-vector mesons as gauge bosons of an $SU(3)_L \times SU(3)_R$ chiral symmetry. The resulting baryon-number-violating interaction Lagrangian contains an additional free parameter and modifies significantly the two-body branching ratios of protons. Without some experimental input, it is not possible to predict

$$\begin{aligned} \mathcal{L}_{(2)}^{\Delta B=1} = & \frac{\gamma}{M_p} \sum_d \{ \overline{e_{dR}^+} i \gamma_\mu \text{Tr} [C_d^{(1)} \underline{\xi} (D^\mu \underline{B}) \underline{\xi}] - \overline{e_{dL}^+} i \gamma_\mu \text{Tr} [C_d^{(2)} \underline{\xi}^\dagger (D^\mu \underline{B}) \underline{\xi}^\dagger] - \overline{\nu_{dR}^c} i \gamma_\mu \text{Tr} [F_d^{(1)} \underline{\xi} (D^\mu \underline{B}) \underline{\xi}] \} + \text{H.c.} \\ & + \frac{\delta}{M_p} \sum_p \{ \overline{e_{dR}^+} i \gamma_\mu \text{Tr} [C_d^{(3)} \underline{\xi} (D^\mu \underline{B}) \underline{\xi}^\dagger] e_{dL}^+ i \gamma_\mu \text{Tr} [C_d^{(4)} \underline{\xi}^\dagger (D^\mu \underline{B}) \underline{\xi}] - \overline{\nu_{dR}^c} i \gamma_\mu \text{Tr} [F_d^{(3)} \underline{\xi} (D^\mu \underline{B}) \underline{\xi}^\dagger] \} + \text{H.c.} \end{aligned} \quad (3.7)$$

Not complete, double counting issues, ...

Our results provide a complete and consistent framework for more precise calculations of a variety of nucleon decay amplitudes.

Only for dim-6 $\Delta(B - L) = 0$ case

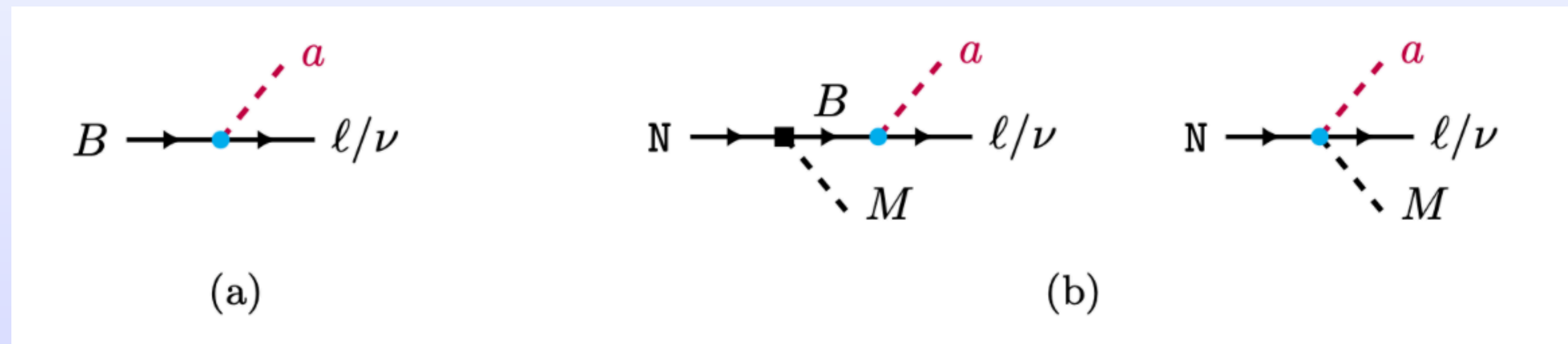
- ~~Introduction~~
- ~~General BNV nucleon decay interactions in the LEFT~~
- ~~Chiral realizations~~
- **Applications**
- Summary

Calculation of relevant transition matrix element and decay rate

Standard ChPT interactions

$$\mathcal{L}_{\text{ChPT}}^{B,0} \supset \frac{D}{2} \text{Tr}(\bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\}) + \frac{F}{2} \text{Tr}(\bar{B} \gamma^\mu \gamma_5 [u_\mu, B]) \\ + G_D \text{Tr}(\bar{B} \gamma_\mu \{V^\mu, B\}) + G_F \text{Tr}(\bar{B} \gamma_\mu [V^\mu, B]), \quad (11)$$

Expanding the chiral Lagrangian $\longrightarrow \mathcal{L} \supset M^n(\mathcal{P}B) \longrightarrow$ Draw diagram and calculate



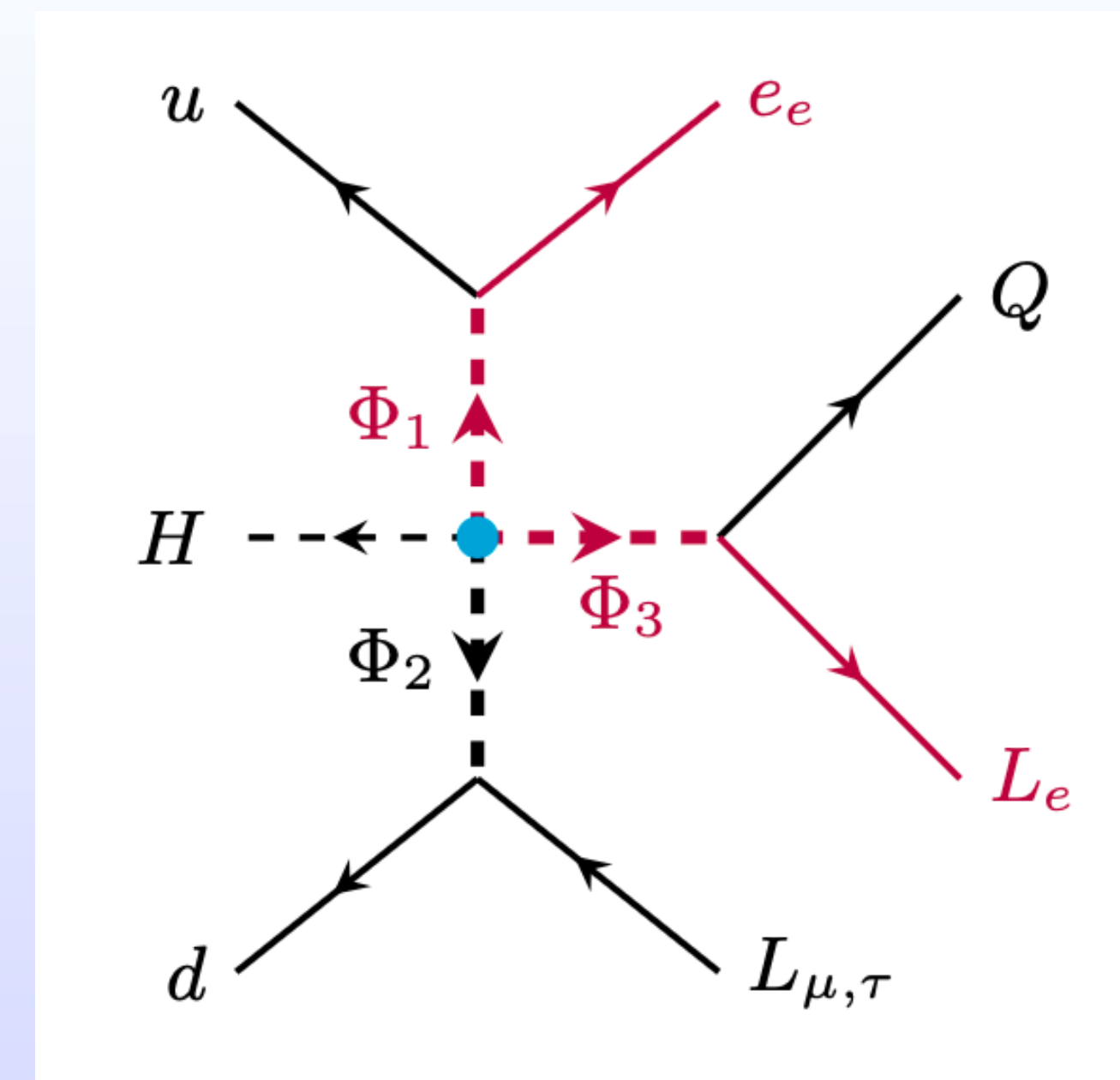
Triple-lepton example: $p \rightarrow e^+ e^+ \mu^-, \mu^+ \mu^+ e^-$

- Among the experimentally most constrained proton decay processes
- They can only be mediated by **dim-9 and higher** LEFT operators
- Associated with irreps $\mathbf{6}_{L(R)} \otimes \mathbf{3}_{R(L)}$ ← dim-10 or higher in SMEFT ↑

$$\mathcal{O}_{\ell\ell'} = (\bar{\ell}_L^C \gamma_\mu \ell_R) (\bar{\ell}_R' u_L^\alpha) (u_L^{\beta C} \gamma^\mu d_R^\gamma) \epsilon_{\alpha\beta\gamma}$$

$$\mathcal{P}_{uud}^{\text{LR},\mu} = \Lambda_{\ell\ell'}^{-5} (\bar{\ell}_L^C \gamma^\mu \ell_R) \bar{\ell}_R'$$

$$\Gamma(p \rightarrow e^+ e^+ \mu^- / \mu^+ \mu^+ e^-) \sim \frac{1}{10^{34} \text{ yr}} \left(\frac{4 \times 10^5 \text{ GeV}}{\Lambda_{\mu e}} \right)^{10}$$



Leptoquark model

Nucleon decay with a light scalar: $N \rightarrow l\varphi$

$$\mathcal{L} \supset \left[R_1^\dagger (y_{Lpr} \overline{Q}_p^{iC} \epsilon_{ij} L_r^j + y_{Rpr} \overline{u}_p^C e_r) + S_1^\alpha (z_{Lpr} \overline{Q}_p^{i\beta C} \epsilon_{ij} Q_r^{j\gamma} + z_{Rpr} \overline{u}_p^{\beta C} d_r^\gamma) \epsilon_{\alpha\beta\gamma} - \kappa R_1^\dagger S_1 \varphi + \text{h.c.} \right], \quad \text{Leptoquark model}$$

Dim-7 in φ SMEFT

$$C_{LQdu\varphi}^{prst} = -\frac{\kappa^* [y_L]_{rp} [z_R]_{ts}}{m_S^2 m_R^2},$$

$$C_{LQQQ\varphi}^{prst} = \frac{2\kappa^* [y_L]_{rp} [z_L]_{st}}{m_S^2 m_R^2},$$

$$C_{euQQ\varphi}^{prst} = -\frac{\kappa^* [y_R]_{rp} [z_L]_{st}}{m_S^2 m_R^2},$$

$$C_{eudu\varphi}^{prst} = \frac{\kappa^* [y_R]_{rp} [z_R]_{ts}}{m_S^2 m_R^2}.$$

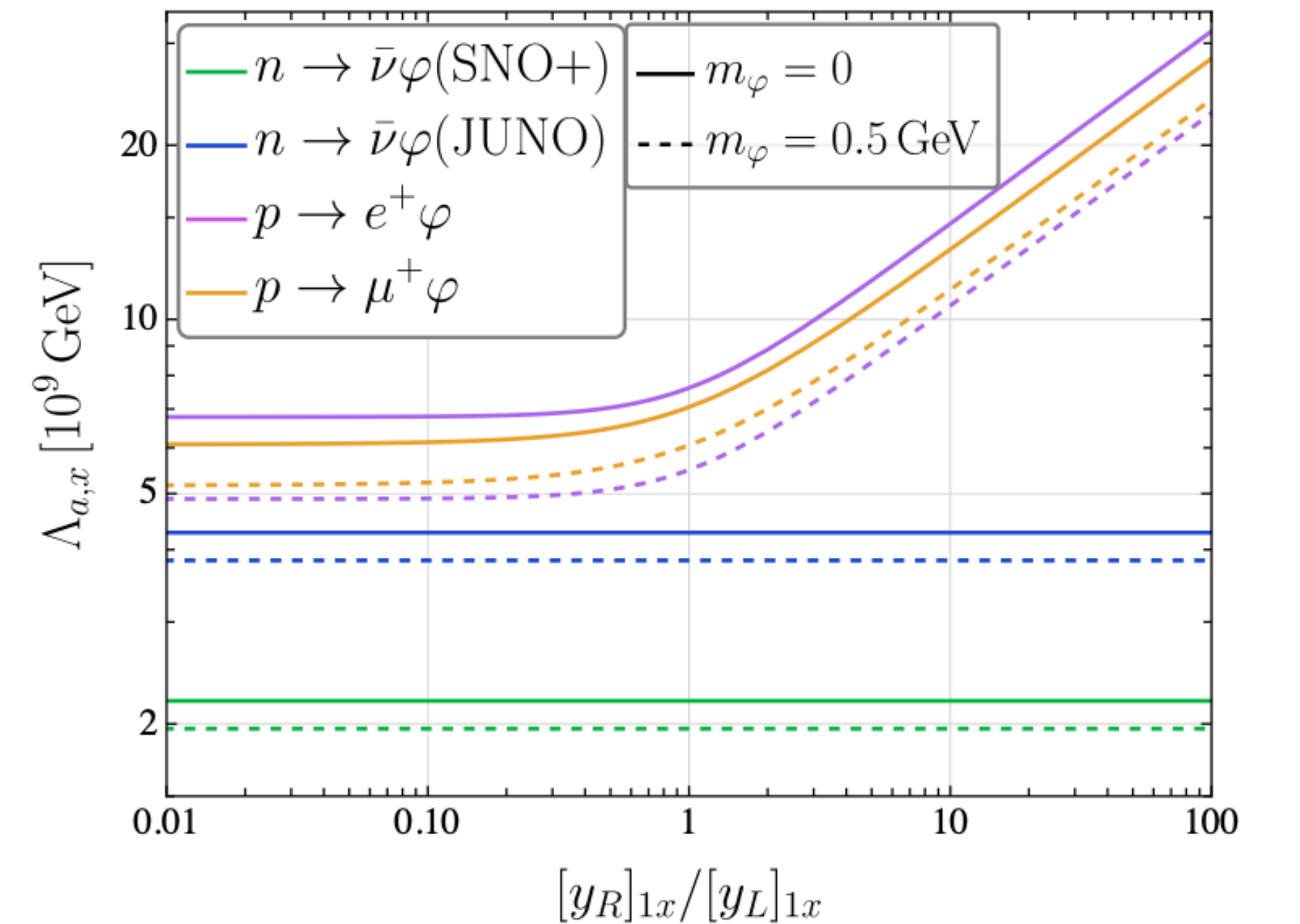
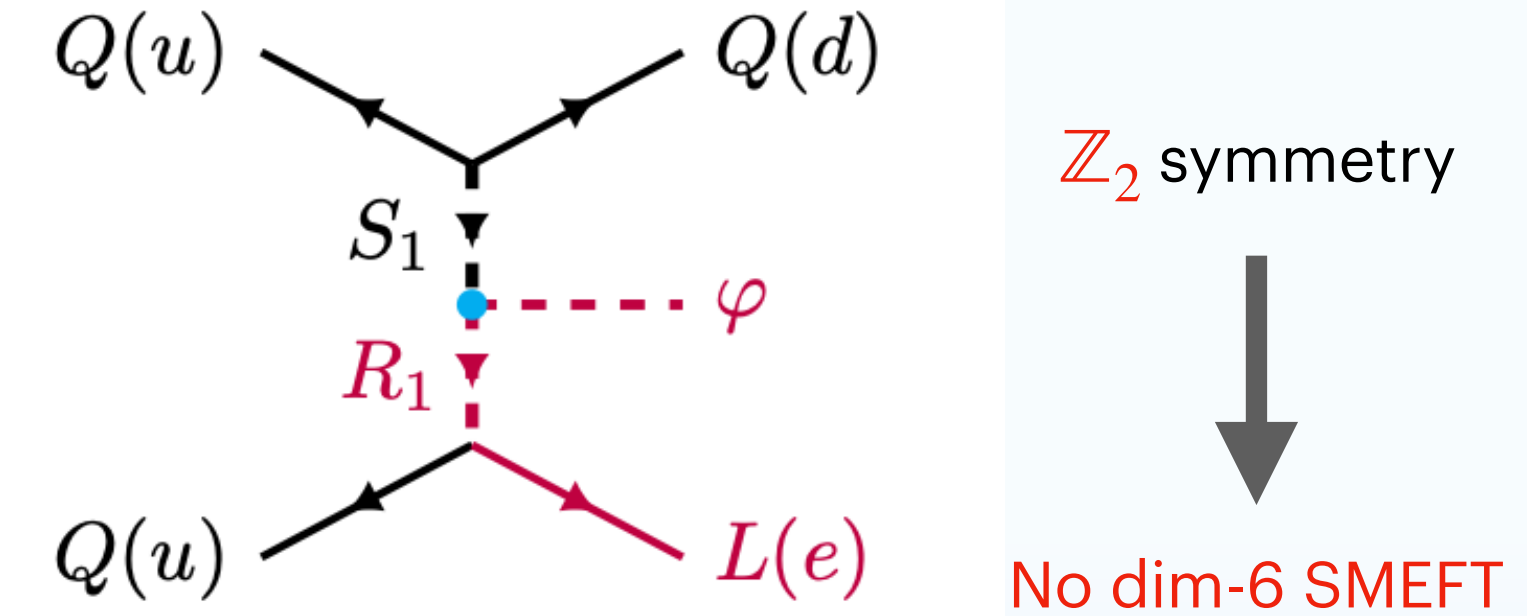
Dim-7 in φ LEFT

$$C_{\varphi\nu dud}^{\text{LR},x} = 1.32 \frac{\kappa^* [y_L]_{1x} [z_R]_{11}}{m_S^2 m_R^2}, \quad C_{\varphi luud}^{\text{LR},x} = -C_{\varphi\nu dud}^{\text{LR},x}, \quad C_{\varphi luud}^{\text{RL},x} = -1.32 \frac{2\kappa^* [y_R]_{1x} [z_L]_{11}}{m_S^2 m_R^2},$$

$$C_{\varphi\nu dud}^{\text{LL},x} = 1.32 \frac{2\kappa^* [y_L]_{1x} [z_L]_{11}}{m_S^2 m_R^2}, \quad C_{\varphi luud}^{\text{LL},x} = -C_{\varphi\nu dud}^{\text{LL},x}, \quad C_{\varphi luud}^{\text{RR},x} = -1.32 \frac{\kappa^* [y_R]_{1x} [z_R]_{11}}{m_S^2 m_R^2},$$

Spurion fields

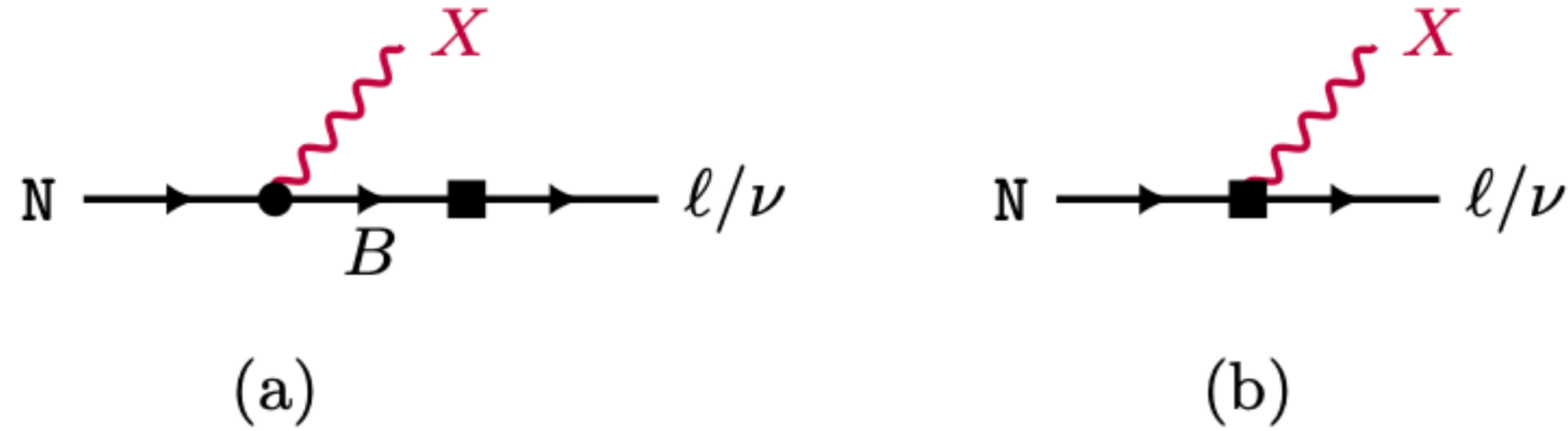
$$\mathcal{N}_{yzw}^{\text{LL}} \text{ and } \mathcal{N}_{yzw}^{\text{RL}} + L \longleftrightarrow R$$



$$\Lambda_{a,x} \equiv (m_S^2 m_R^2 / |\kappa^* [y_L]_{1x} ([z_R]_{11} - 2[z_L]_{11})|)^{1/3}$$

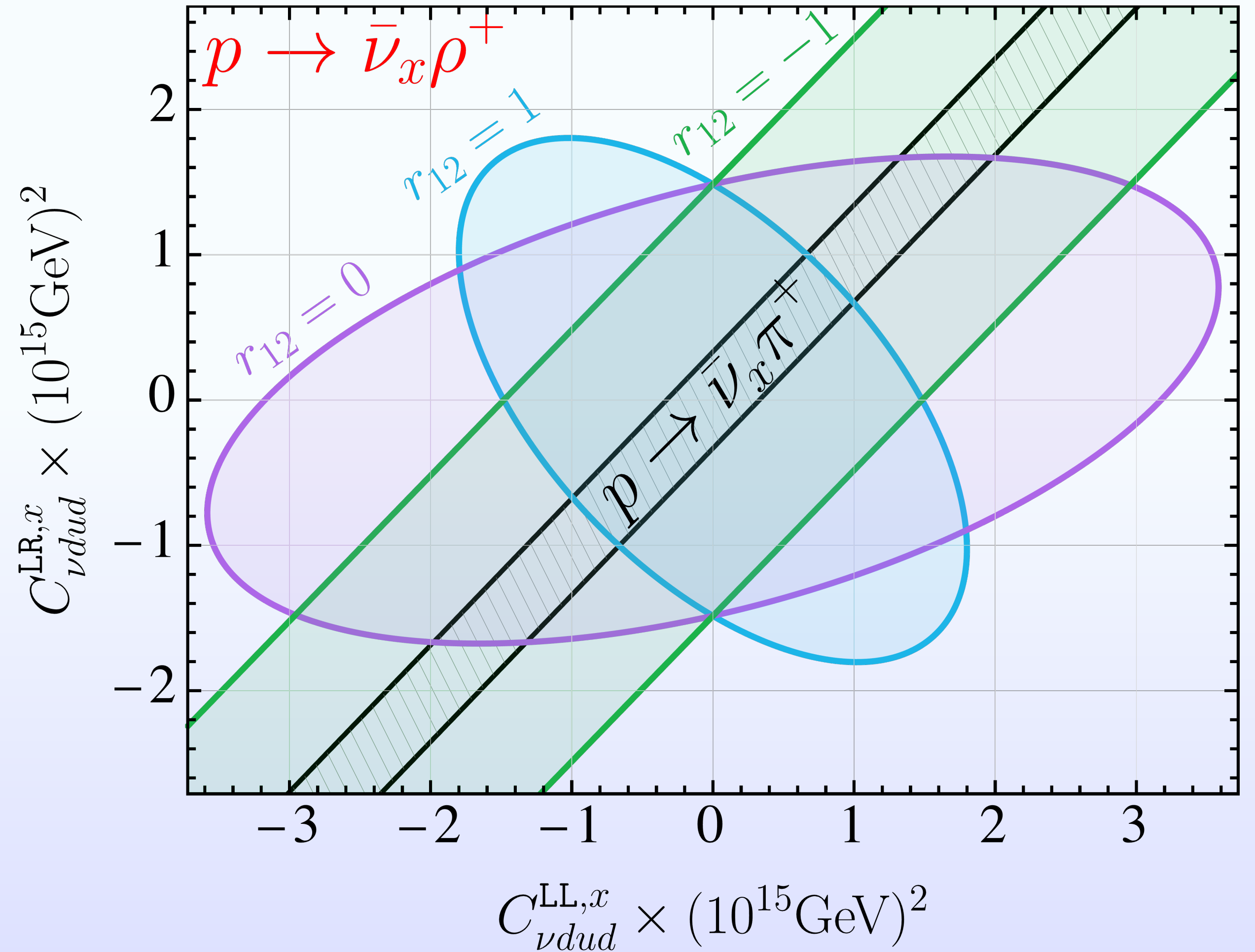
Vector meson modes help breaking degeneracy

Yi Liao, XDM, Hao-Lin Wang, 2506.05052



$$\Gamma_{p \rightarrow \bar{\nu}_x \rho^+} = \frac{m_p(1 - 3x_\rho^2 + 2x_\rho^3)}{32\pi x_\rho} \left(|d_1 C_{\nu d u d}^{\text{LR},x} + d_2 C_{\nu d u d}^{\text{LL},x}|^2 + \frac{G_F^2}{m_n^2} |c_1 C_{\nu d u d}^{\text{LR},x} + c_2 C_{\nu d u d}^{\text{LL},x}|^2 \right), \quad (15a)$$

$$\Gamma_{p \rightarrow \bar{\nu}_x \pi^+} = \frac{m_p(1 - x_\pi)^2 [1 + (D + F)m_p m_n^{-1}]^2}{64\pi F_0^2} \times |c_1 C_{\nu d u d}^{\text{LR},x} + c_2 C_{\nu d u d}^{\text{LL},x}|^2. \quad (15b)$$



Complementarity between the pseudoscalar and vector modes

Summary

- General triple-quark interactions without a ∂ related to nucleon decay are identified: $\bar{\mathbf{3}}_{L(R)} \otimes \mathbf{3}_{R(L)}$, $\mathbf{8}_{L(R)} \otimes \mathbf{1}_{R(L)}$, $\mathbf{6}_{L(R)} \otimes \mathbf{3}_{R(L)}$ and $\mathbf{10}_{L(R)} \otimes \mathbf{1}_{R(L)}$;
- Their LO chiral matching are realized, and the new LECs are estimated by the NDA;
- For the nucleon decay involving a vector meson, this is the first consistent chiral framework;
- Our results provide a reliable framework for further experimental and theoretical search.

Thanks for your attention!