Universal gravitational self-force for a point mass orbiting around a compact star

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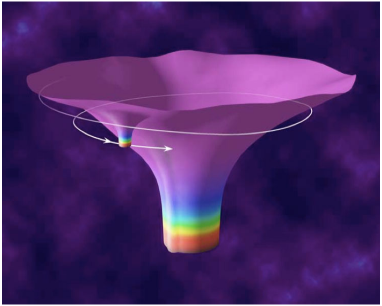
SIMIS

Extreme mass-ratio inspirals

Extreme mass-ratio inspirals (EMRIs) are compact binaries with a mass ratio $\mu/M \simeq 10^{-4} - 10^{-7}$.

- Small environmental forces.
- Multipole moments of the background spacetime.
- Black hole nature of the central supermassive compact object.
- ......



Motivation

- Charactering the tidal effect in binary neutron star systems with the black hole perturbation approach (mass ratio expansion).
- Characterizing the tidal effect of black hole mimickers for fundamental physics tests using EMRIs.

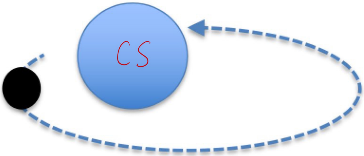


Figure 1: Compact Star

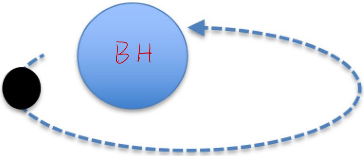
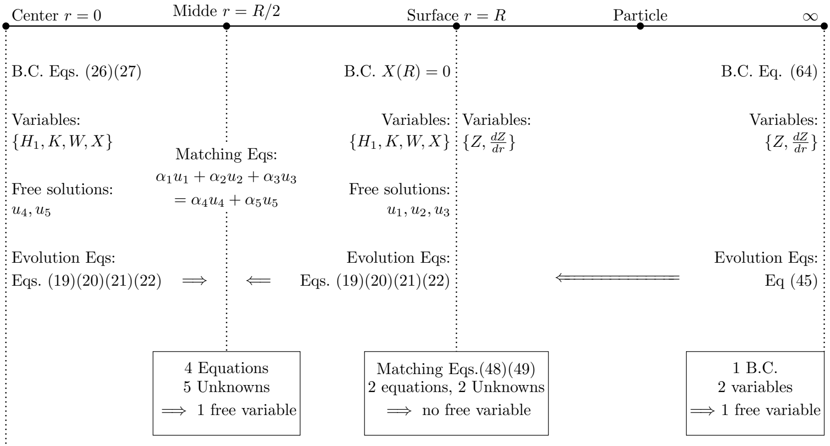


Figure 2: Black Hole

1. Motivation
2. Gravitational flux and self-force
   - Tide-induced Gravitational flux
   - Gravitational Self-force
   - Higher ℓ modes contributions
3. Tide-induced phase shift
4. Summary

Numerical method



Tide-induced gravitational wave flux

- The gravitational wave flux at infinity can be computed. We evaluate the difference in flux between a BH interior and a star interior (with same mass and the same orbital frequency):

$$P^{\rm tide} = P^{\rm star} - P^{\rm BH} \,.$$

- The flux can be computed for various stars and different orbital frequencies of the point mass. How to summarize the results? An ansatz:

$$P^{\rm tide} = P^{\rm tide}(\lambda_2^{\rm dyn}, \, P_0^{\rm tide}(\omega)) \,, \quad \lambda_2^{\rm dyn} = \lambda_2 \frac{\omega_f^2}{\omega_f^2 - \omega^2} \,.$$

Tide-induced gravitational wave flux

The flux P^{tide} actually falls onto a single curve when plotted against the dynamical tidal deformability λ_2^{dyn}.

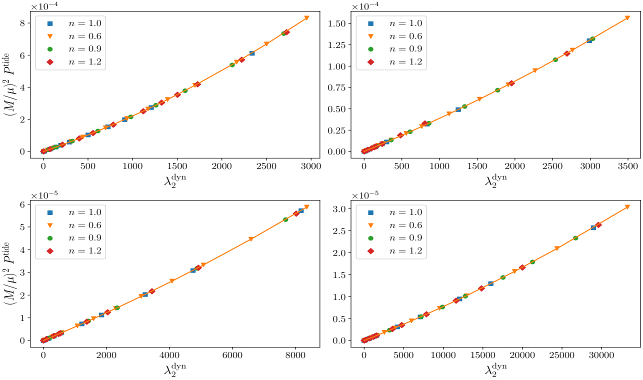
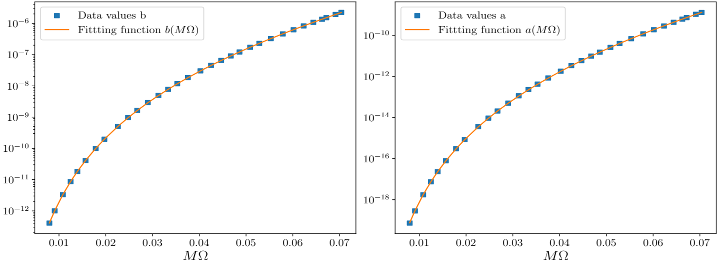


Figure 3: The values of $M\Omega$ are 0.0516, 0.0413, 0.032 and 0.0237.

Fitting for the flux

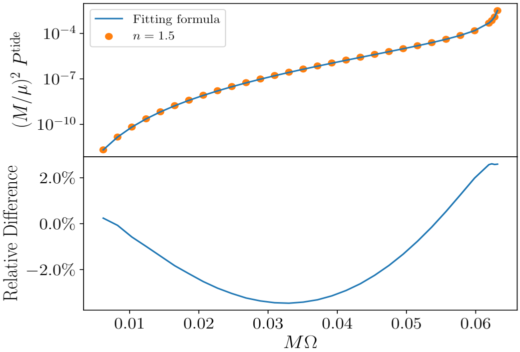
Nonlinearities at large λ: possibly from higher-order modes. We take one type of star, say $n = 0.6$, vary the central density, and compute $P^{\rm tide}$.

$$
\begin{aligned}
P^{\rm tide} &= (\lambda_2^{\rm dyn})^2 a(M\Omega) + \lambda_2^{\rm dyn} b(M\Omega)\,, \\
a(M\Omega) &= \frac{32}{5}(M\Omega)^{30/3}\left(3.648\, e^{40.48(M\Omega)} + 6.737\right)\,, \\
b(M\Omega) &= \frac{32}{5}(M\Omega)^{20/3}\left(1.405\, e^{30.63(M\Omega)} + 4.614\right)\,.
\end{aligned}
$$



Check the fitting formula

Check the fitting formula with EOS not used, say $n = 1.5$ for fitting.
Error everywhere less than 3%.



Gravitational Self-force

- Small body perturbs a spacetime:

$$g_{\mu\nu} = g^0_{\mu\nu} + \epsilon\, h^{(1)}_{\mu\nu} + \epsilon^2\, h^{(2)}_{\mu\nu} + \ldots$$

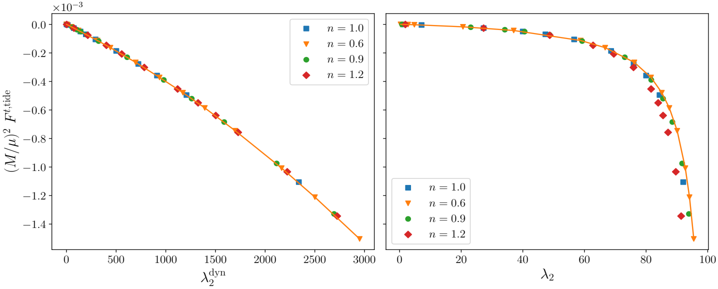
  where $\epsilon \propto \mu/M$.

- This deformation of the geometry affects μ's motion (gravitational self-force)

$$\frac{D^2 z^\mu}{d\tau^2} = \epsilon\, F^\mu_{(1)} + \epsilon^2\, F^\mu_{(2)} + \ldots$$

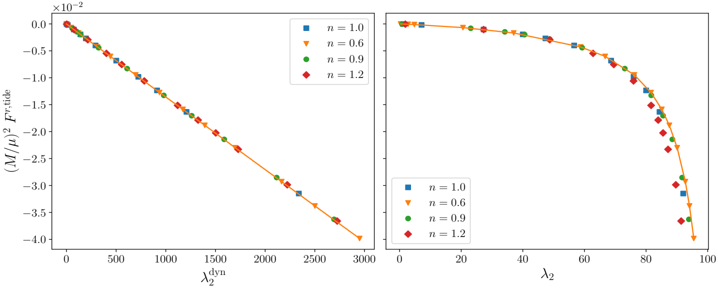
t-component self force $F^{\mathrm{t,tide}}$

- t-component of gravitational SF $F^{\mathrm{t,tide}}$ agrees with the flux.
- If we replace λ_2^{dyn} with the equilibrium tidal deformability λ_2, this nice universal relation is broken.



r-component self force $F^{r,\text{tide}}$

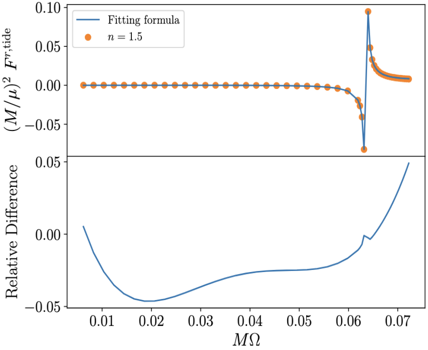
The r-component self-force $F^{r,\text{tide}}$ satisfies a rather linear relation with the dynamical tidal deformability.



Comparison of $F^{\mathrm{r,tide}}$

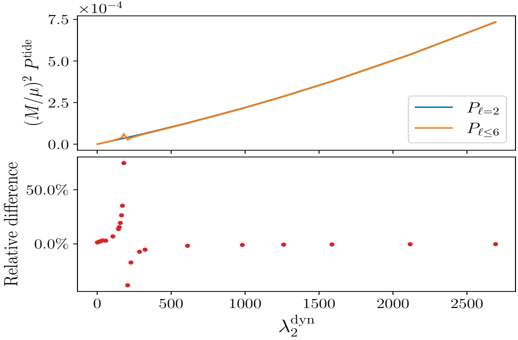
Fitting formula for $F^{\mathrm{r,tide}}$

$$F^{\mathrm{r,tide}} = \lambda_2^{\mathrm{dyn}} c(M\Omega)\,,$$
$$c(M\Omega) = (M\Omega)^{14/3}\big(-7.6023 - 3.6672^3 (M\Omega)^2 + 1.1071^6 (M\Omega)^4 - 1.1531 \times 10^7 (M\Omega)^5 \big)\,.$$



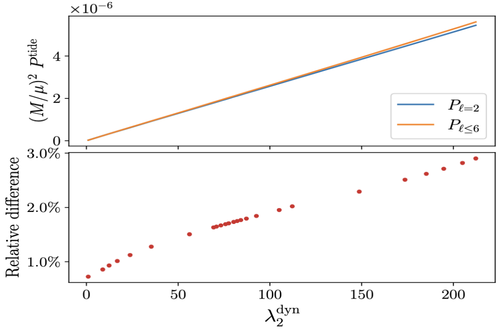
Higher ℓ modes for $P^{\rm tide}$ I

Higher ℓ modes contribute ($< 3\%$ away from resonance) to the flux at $M\Omega = 0.0516$. Some noticeable deviations due to resonance with the $\ell = 3$ f-mode are shown.



Higher ℓ modes for $P^{\rm tide}$ II

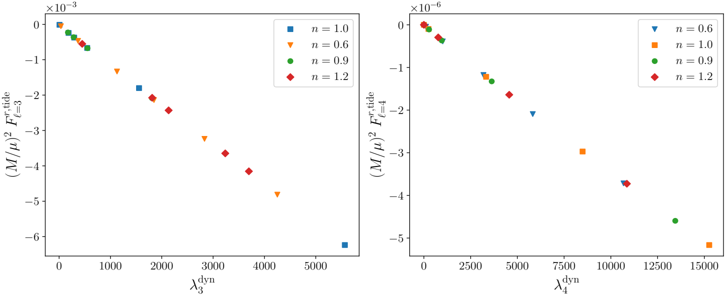
Higher ℓ modes contribute ($< 3\%$ away from resonance) to the flux at $M\Omega = 0.0392$. None of the $\ell = 2, 3, 4$ f-modes are resonantly excited.



Higher ℓ modes $F^{\mathrm{r,tide}}$

Higher-order tidal deformations (higher-ℓ contributions may not be negligible):

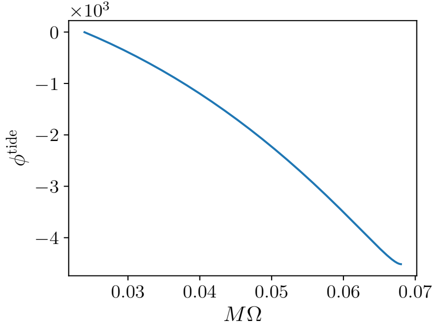
$$F^{\mathrm{r,tide}} = \sum_{\ell\geq 2} \lambda_\ell^{\mathrm{dyn}} c_\ell(\Omega)\,, \quad \lambda_\ell^{\mathrm{dyn}} = \lambda_\ell \frac{\omega_{\ell,f}^2}{\omega_{\ell,f}^2 - \omega^2}\,.$$



Tide-induced phase shift

Tidal phase correction to the waveform representing a four-year inspiral of an EMRI system which has masses $(M, \mu) = (10^6, 10)M_\odot$. Consider a star-like BH mimicker with polytropic EOS

$$\rho_c = 1.4 \times 10^6\,\mathrm{g/cm^3}, \quad \kappa = 1.608 \times 10^9\,\mathrm{km^2}, \quad n = 1.5.$$



Summary

- Tide-induced self-force can be well characterized by the dynamical tidal deformability of the central object.
- Tide-induced phase shift can be significant for BH mimickers.
- Maybe be useful to check for more EOS, other BH mimickers and more general orbits.

Thank you for listening!