



Strong backreaction of gauge quanta produced during inflation and the sourced GWs

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Based on:

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1

An Overview

2

Model 1: Linear Potential

3

Model 2: Cosine Potential

4

Conclusion



An Overview



Given that the energy scale of inflation far exceeds that of the standard model of particle physics, it is natural to anticipate the presence of many beyond-standard model (BSM) fields during inflation.

On the other hand, there are many axion in BSMs:

- axion-like particles
- QCD axion
- axion from string theory

So it's natural to study axion during inflation



An Overview

some early literatures (axion as inflaton) see: [1011.1500, N. Barnaby, M. Peloso] [1101.1525, L. Sorbo] [1102.4333, N. Barnaby et al.] [1110.3327, N. Barnaby et al.]



Lagrangian with an inflaton and a spectator axion:

$$\mathcal{L} = \frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - U(\chi) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4f} \chi \tilde{F}^{\mu\nu} F_{\mu\nu}$$

Equation of Motion (background)

$$H^2 = \frac{1}{3M_{\text{pl}}^2} \rho,$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

$$\frac{\ddot{a}}{a} + \frac{1}{2} \left(\frac{\dot{a}}{a} \right)^2 = -\frac{1}{2M_{\text{pl}}^2} P,$$

$$\ddot{\chi} + 3H\dot{\chi} + U_{,\chi} = \frac{1}{f} \langle \mathbf{E} \cdot \mathbf{B} \rangle$$

strong backreaction condition

$$f^{-1} \langle \vec{E} \cdot \vec{B} \rangle \sim U_{,\chi}$$

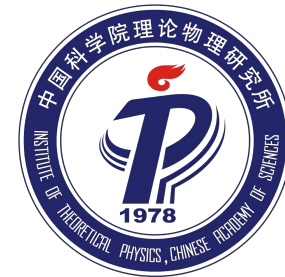
E_i and B_i are defined by

$$E_i(t) = -\dot{A}_i/a,$$

$$B_i(t) = \epsilon_{ijk} \partial_j A_k / a^2.$$



An Overview



E^2 and B^2 also appear in energy density and pressure

$$\begin{aligned}\rho &= \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + V(\phi) + U(\chi) + \frac{1}{2} \langle \mathbf{E}^2 + \mathbf{B}^2 \rangle, \\ P &= \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 - V(\phi) - U(\chi) + \frac{1}{6} \langle \mathbf{E}^2 + \mathbf{B}^2 \rangle.\end{aligned}$$

The ensemble averages are computed by integral over the gauge field

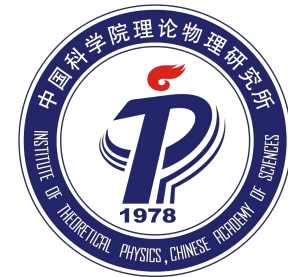
$$\langle \mathbf{E} \cdot \mathbf{B} \rangle = -\frac{1}{4\pi^2 a^4} \sum_{\lambda=\pm} \lambda \int_0^\infty dk k^3 \frac{d}{d\tau} |A^\lambda(k)|^2,$$

$$\langle E^2 \rangle = \frac{1}{2\pi^2 a^4} \sum_{\lambda=\pm} \int_0^\infty dk k^2 \left| \frac{dA^\lambda(k)}{d\tau} \right|^2,$$

$$\langle B^2 \rangle = \frac{1}{2\pi^2 a^4} \sum_{\lambda=\pm} \int_0^\infty dk k^4 |A^\lambda(k)|^2.$$



An Overview



Decomposition of the gauge field

$$\hat{A}_i(t, \boldsymbol{x}) = \int \frac{d^3 \boldsymbol{k}}{(2\pi)^{3/2}} \sum_{\lambda=\pm} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \epsilon_i^\lambda(\hat{\boldsymbol{k}}) A^\lambda(t, \boldsymbol{k}) \hat{a}^\lambda(\boldsymbol{k}) + \text{h.c.}$$

where the basis of the left/right hand circular polarization is defined by

$$\begin{aligned} k_i \epsilon_i^\pm(\hat{\boldsymbol{k}}) &= 0, \quad \varepsilon_{ijk} k_j \epsilon_k^\pm(\hat{\boldsymbol{k}}) = \mp i k \epsilon_i^\pm(\hat{\boldsymbol{k}}), \\ \epsilon_i^\pm(\hat{\boldsymbol{k}}) \epsilon_i^\pm(\hat{\boldsymbol{k}}) &= 0, \quad \epsilon_i^\pm(\hat{\boldsymbol{k}}) \epsilon_i^\mp(\hat{\boldsymbol{k}}) = 1. \end{aligned}$$

Note the non-vanishing cross product $\vec{k} \times \vec{\epsilon}$ implies the basis is "rotating".



Equation of motion of the gauge field

Tachyonic instability happen when the effective mass becomes negative;
Outside the horizon, oscillation frequency exponentially decrease and **finally freeze**

$$\ddot{A}^{\pm}(\mathbf{k}) + H\dot{A}^{\pm}(\mathbf{k}) + \left(\frac{k^2}{a^2} \mp \frac{k}{a} \frac{1}{f} \dot{\chi} \right) A^{\pm}(\mathbf{k}) = 0$$

$$A^{\pm''}(\mathbf{k}) + (k^2 \pm 2\xi a H k) A^{\pm}(\mathbf{k}) = 0$$

Definition of the dimensionless parameter ξ

$$\xi \equiv -\frac{\dot{\chi}}{2fH}$$

In our study, $\dot{\chi} < 0$, thus the minus mode A^{-} is the enhanced mode. The initial condition is the Bunch-Davis vacuum

$$A_{\text{BD}}^{\pm}(\mathbf{k}) \equiv A(\tau, \mathbf{k})|_{k \gg aH} = \frac{1}{\sqrt{2k}} e^{-ik\tau}$$

Equations

$$\frac{\ddot{a}}{a} + \frac{1}{2} \left(\frac{\dot{a}}{a} \right)^2 = -\frac{1}{2M_{\text{pl}}^2} P$$

$$P = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\chi}^2 - V(\phi) - U(\chi) + \frac{1}{6} \langle \mathbf{E}^2 + \mathbf{B}^2 \rangle$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

$$\ddot{\chi} + 3H\dot{\chi} + U_{,\chi} = \frac{1}{f} \langle \mathbf{E} \cdot \mathbf{B} \rangle$$

$$\ddot{A}^{\pm}(\mathbf{k}) + H\dot{A}^{\pm}(\mathbf{k}) + \left(\frac{k^2}{a^2} \mp \frac{k}{a} \frac{1}{f} \dot{\chi} \right) A^{\pm}(\mathbf{k}) = 0$$

Pseudo Code

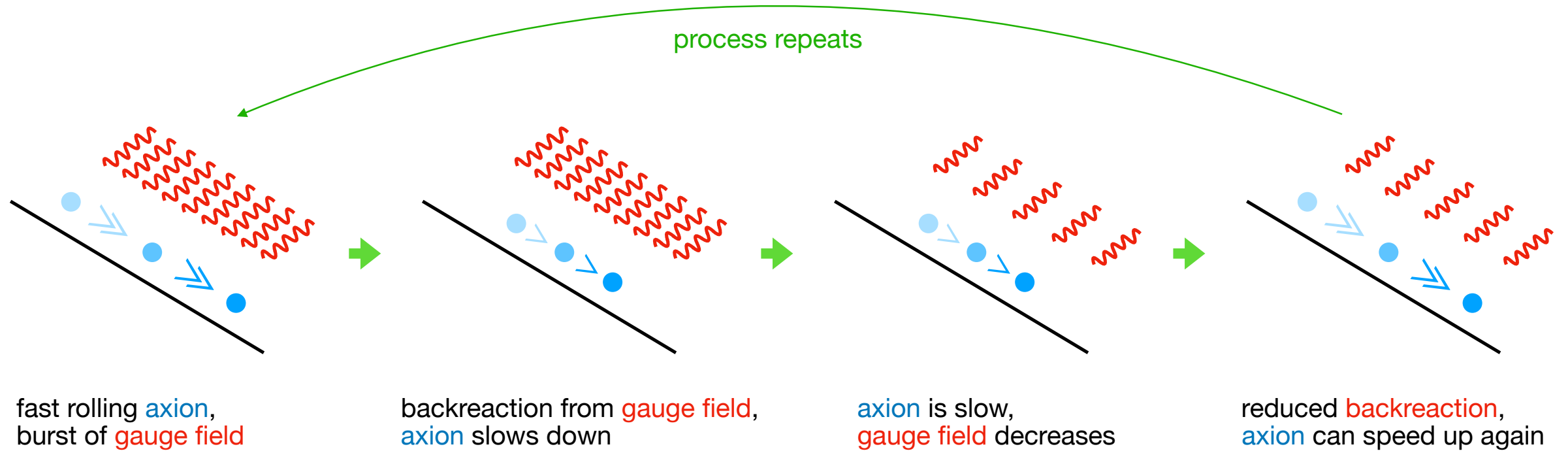
```

for it < itmax:
    // background evolution
    rk4_iterate(phi, dphi, chi, dchi,
                a, da, EEplusBB_ave, EB_ave)

for iq < iqmax:
    // gauge field evolution
    rk2_iterate(Aq)

    // check if tachyonic instability
    // happen, then sum to compute
    // backreaction
    if mass_eff < 0:
        EB_ave += EB_integrand(Aq)
        EEplusBB_ave += EEplusBB_integrand(Aq)

```



From Fig. 1 of [2407.13405, A. Caravano, M. Peloso, Unveiling the nonlinear dynamics of a rolling axion during inflation]



An Overview



Tensor perturbations of FRW background

$$ds^2 = a^2(\tau) [-d\tau^2 + (\delta_{ij} + h_{ij}(\tau, \mathbf{x}))dx_i dx_j]$$

Evolution of the sourced tensor perturbation

$$h''_{ij} + 2 \frac{a'}{a} h'_{ij} - \Delta h_{ij} = \frac{2}{M_P^2} \Pi_{ij}{}^{lm} T_{lm}^{EM}$$

Stress-energy tensor and the traceless-transverse projection tensor are

$$T_{ij}^{EM} = -a^2 (E_i E_j + B_i B_j) + \delta_{ij} (\dots)$$

$$\Pi_{lm}^{\pm}(\mathbf{k}) \equiv \epsilon_l^{\mp}(\hat{k}) \epsilon_m^{\mp}(\hat{k})$$

Decomposition of the tensor mode

$$\hat{h}_{ij}(\tau, \mathbf{x}) = \int \frac{d^3 \mathbf{k}}{(2\pi)^{3/2}} \sum_{\lambda=+,-} (\epsilon_i^\lambda(\hat{\mathbf{k}}) \epsilon_j^\lambda(\hat{\mathbf{k}}) \hat{a}^\lambda(\mathbf{k}) h^\lambda(\tau, \mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}} + \text{h.c.})$$

Solution of the polarization $\pm$

$$\begin{aligned} \hat{h}^\pm(\tau, \mathbf{k}) = & -\frac{2H^2}{M_{\text{pl}}^2} \int_{-\infty}^{\tau} d\tau' G_k(\tau, \tau') \tau'^2 \int \frac{d^3 \mathbf{q}}{(2\pi)^{3/2}} \Pi_{lm}^\pm(\mathbf{k}) \left[\tilde{A}'_l(\mathbf{q}, \tau') \hat{A}'_m(\mathbf{k} - \mathbf{q}, \tau') \right. \\ & \left. - \varepsilon_{lab} q_a \hat{A}_b(\mathbf{q}, \tau') \varepsilon_{mcd} (k_c - q_c) \hat{A}_d(\mathbf{k} - \mathbf{q}, \tau') \right] \end{aligned}$$

Green's function of the de-Sitter background is

$$G_k(\tau, \tau') = \frac{1}{k^3 \tau'^2} \left[(1 + k^2 \tau \tau') \sin(k(\tau - \tau')) - k(\tau - \tau') \cos(k(\tau - \tau')) \right] \Theta(\tau - \tau')$$

Power spectrum

$$\langle h^\pm(k, \tau) h^\pm(k', \tau) \rangle \equiv \frac{2\pi^2}{k^3} \mathcal{P}_h^\pm(k, \tau) \delta(\mathbf{k} + \mathbf{k}')$$

$$\begin{aligned} \mathcal{P}_h^\lambda(k, \tau_{\text{end}}) = & \frac{H^4 k^3}{\pi^4 M_{\text{pl}}^4} \int_0^\infty q^2 dq \int_{-1}^1 du \left[\left| \epsilon_i^\lambda(\mathbf{k}) \epsilon_i^-(-\mathbf{q}) \right|^2 \left| \epsilon_j^\lambda(\mathbf{k}) \epsilon_j^-(\mathbf{q} - \mathbf{k}) \right|^2 \right. \\ & \times \left. \left| \int_{-\infty}^0 d\tau' \tau'^2 G_k(\tau_{\text{end}}, \tau') \left(A'^-(\tau', \mathbf{q}) A'^-(\tau', \mathbf{k} - \mathbf{q}) + q|\mathbf{k} - \mathbf{q}| A^-(\tau', \mathbf{q}) A^-(\tau', \mathbf{k} - \mathbf{q}) \right) \right|^2 \right] \end{aligned}$$

Energy spectrum, where $\Omega_{r,0}$ is the the current fraction of the radiation energy density

$$\Omega_{\text{GW},0}(k) h^2 = \frac{\Omega_{r,0} h^2}{24} (\mathcal{P}_h^+(k, \tau_{\text{end}}) + \mathcal{P}_h^-(k, \tau_{\text{end}}))$$

1 Overview of Spectator Axion



2 Model 1: Linear Potential

3 Model 2: Cosine Potential

4 Conclusion



2.1 Model

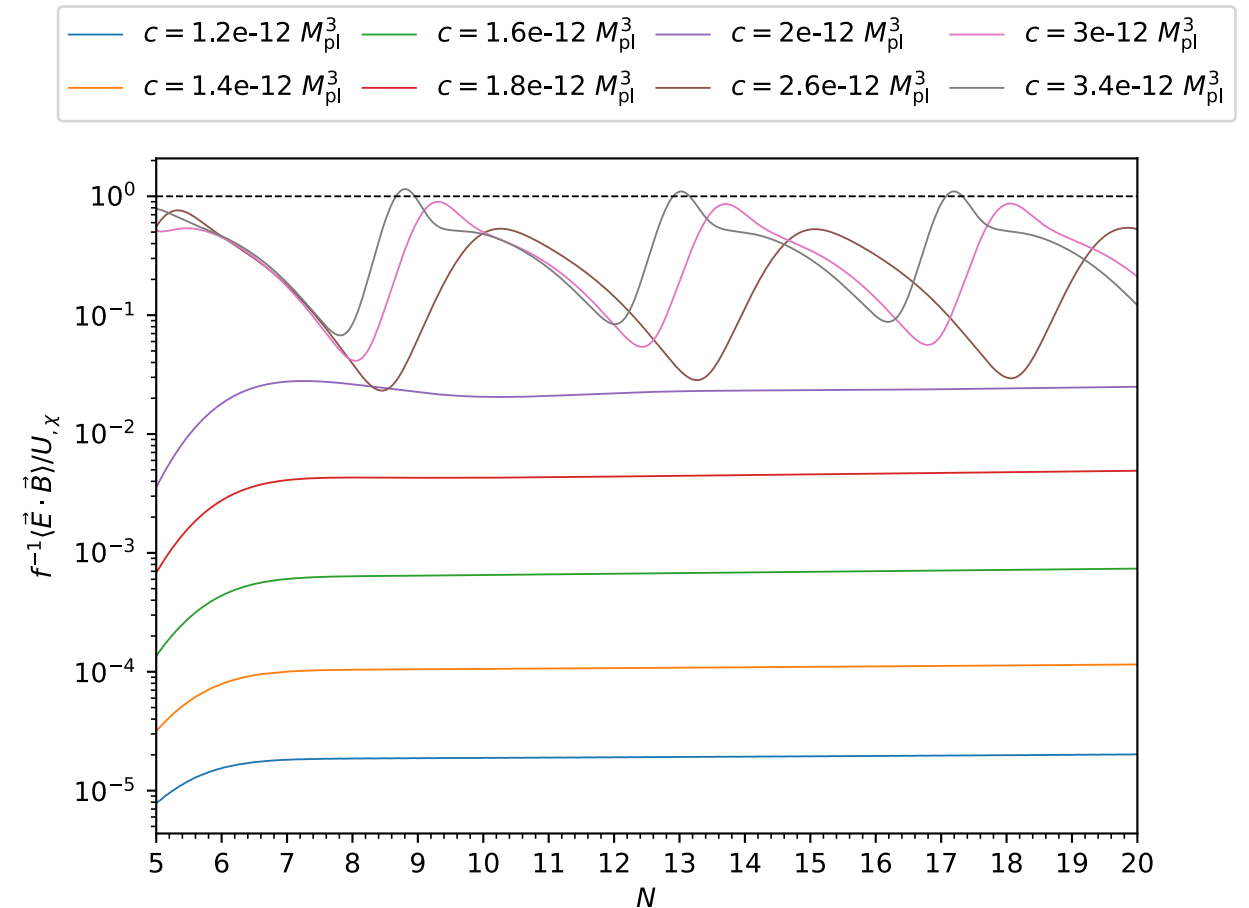
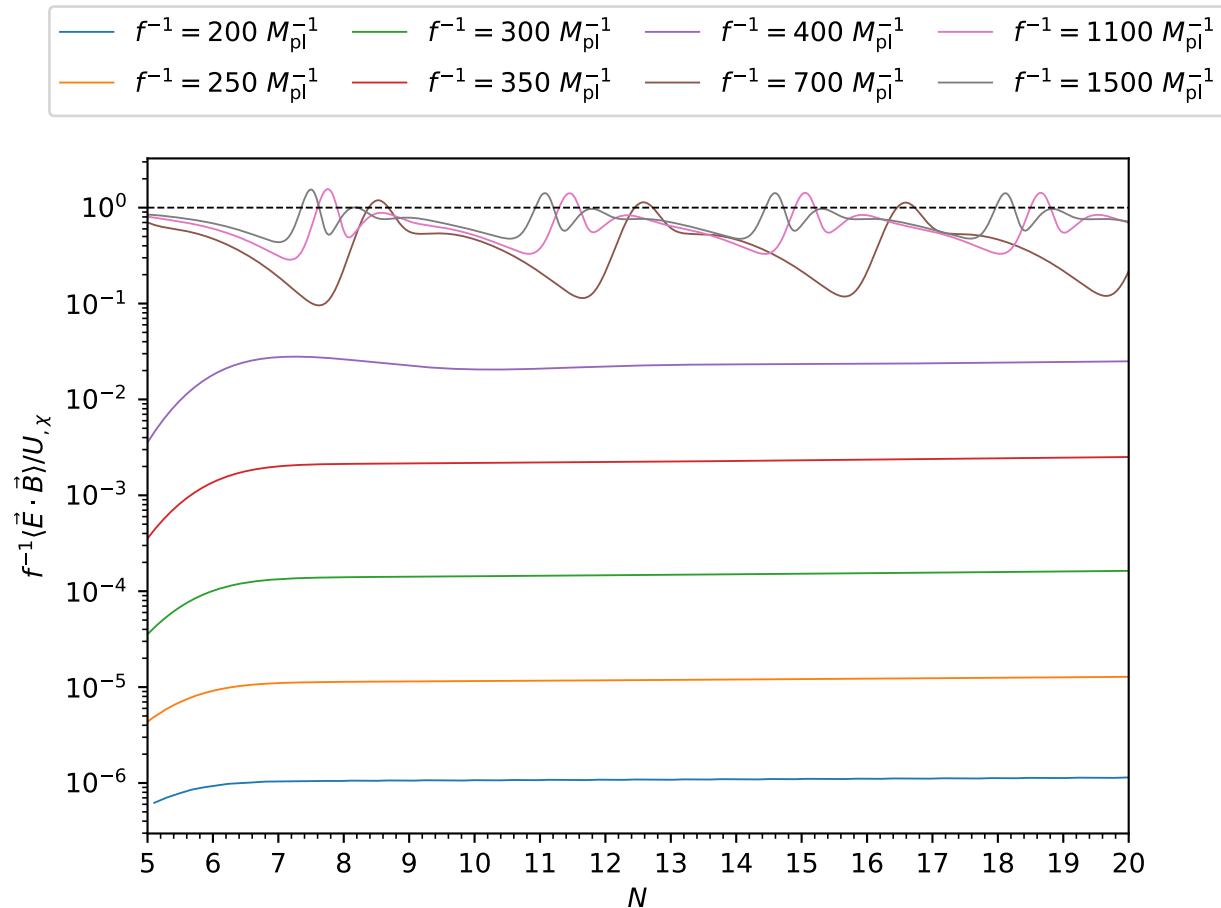
$$\mathcal{L} = \frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - U(\chi) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4f} \chi \tilde{F}^{\mu\nu} F_{\mu\nu}$$

Since any potential can be locally treated as linear, we choose a linear potential to investigate the influence of the slope:

$$V(\phi) = V_0 \left[1 - \exp \left(-\sqrt{\frac{2}{3}} \frac{\phi}{M_{\text{pl}}} \right) \right]^2 \quad \text{Starobinsky potential}$$

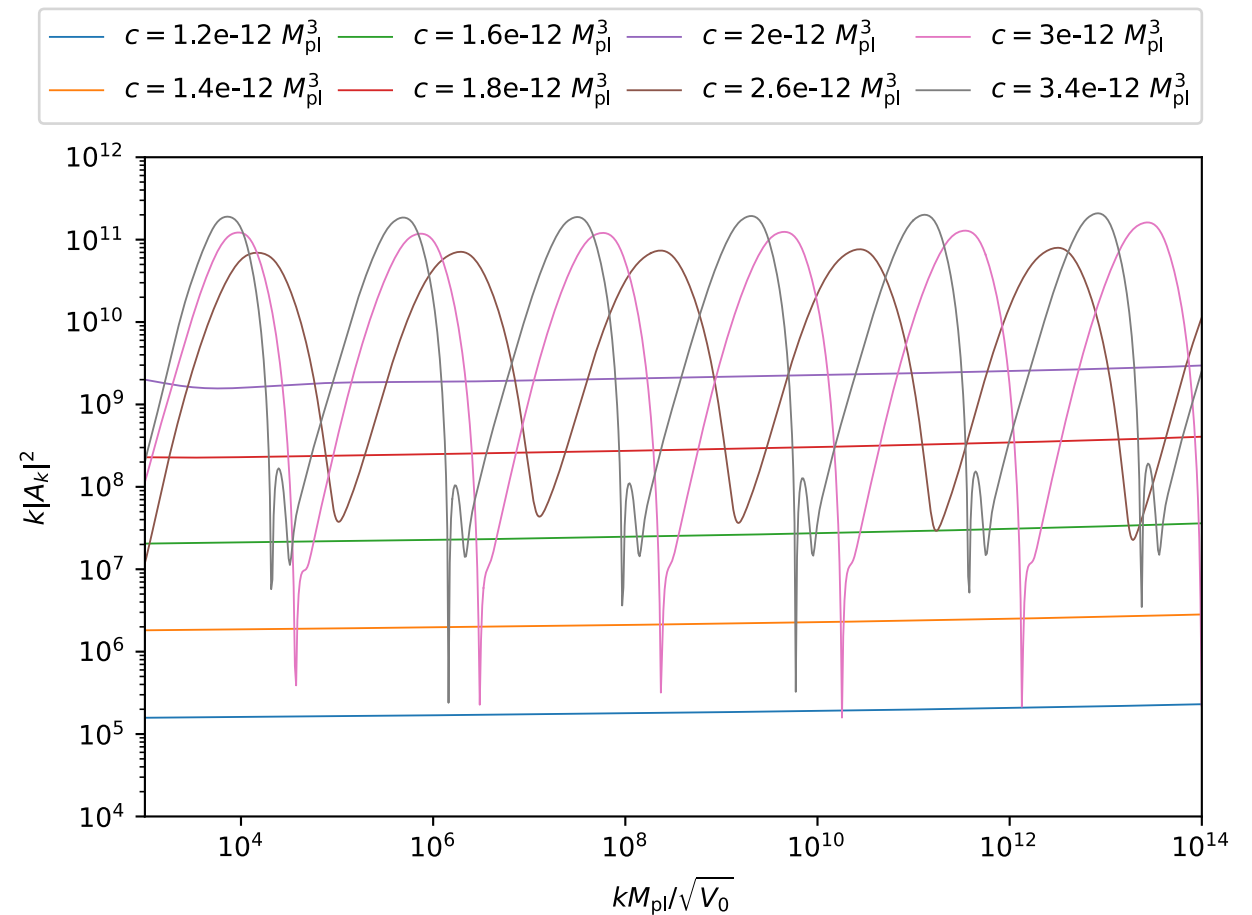
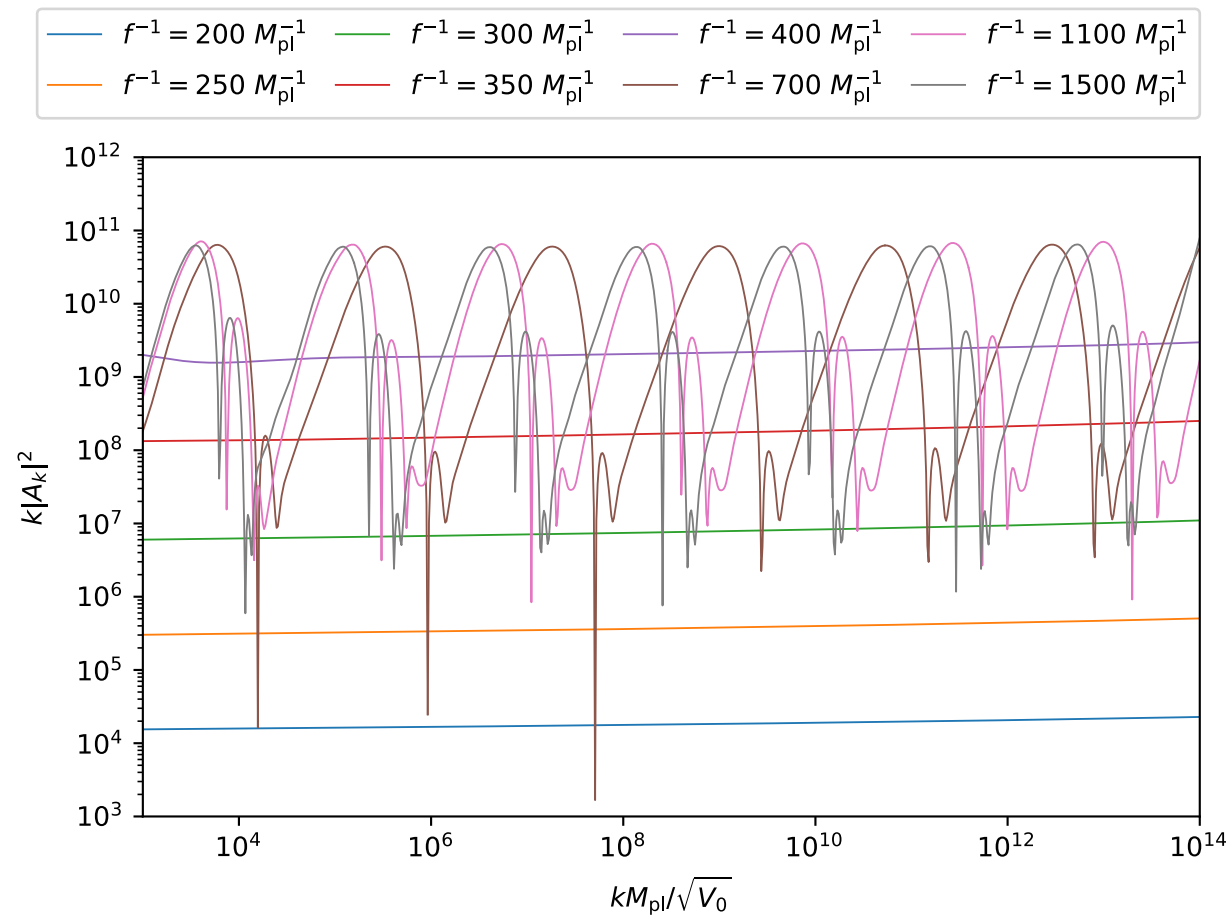
$$U(\chi) = c\chi \quad \text{linear potential}$$

2.2 Evolution of the Backreaction



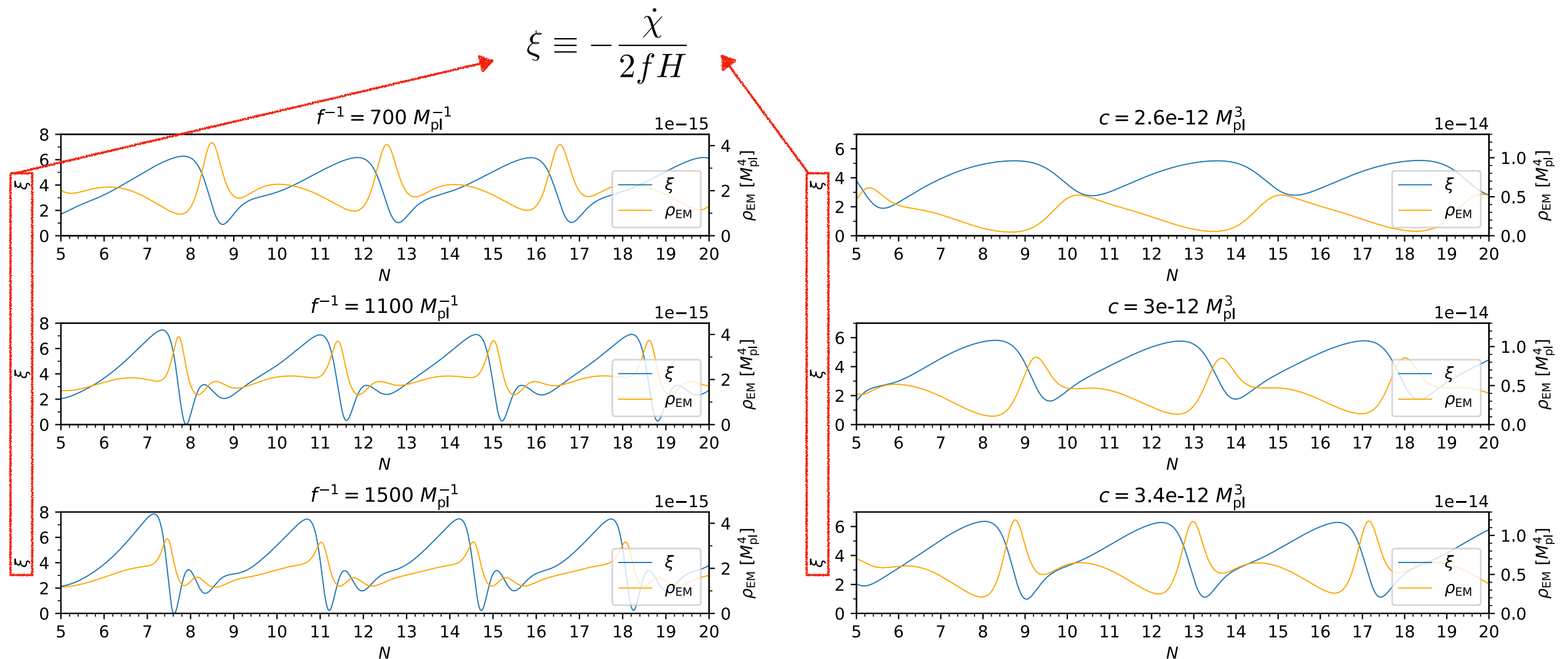
Black dashed line represents $f^{-1} \langle \vec{E} \cdot \vec{B} \rangle / U_{\chi} = 0$. They show oscillation only happen when $f^{-1} \langle \vec{E} \cdot \vec{B} \rangle \sim U_{\chi}$, which verifies our argument about strong backreaction condition.

2.3 Spectrum of the Gauge Field



In the strong backreaction region, a larger f^{-1} don't significantly influence $k|A_k|^2$,
but a larger $U_{,\chi}$ can improve it.

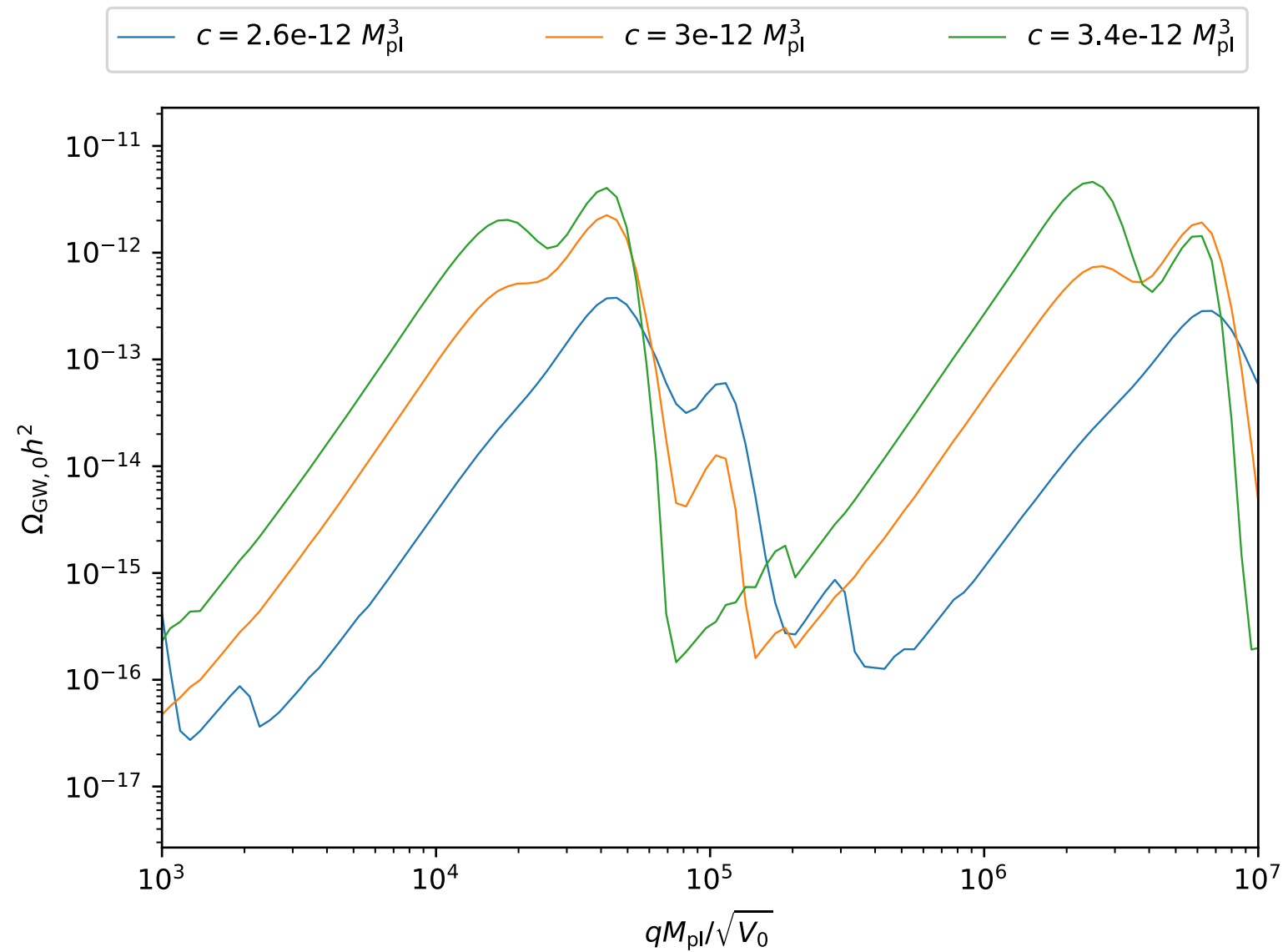
2.4 Time Delay between Axion Speed and Gauge Quanta Production



The peak of ξ is always earlier than the peak of ρ_{EM} ; large f^{-1} can linearly suppress ρ_{EM} , which can be explained by strong backreaction condition $f^{-1}\langle \vec{E} \cdot \vec{B} \rangle \sim U_{\chi}$

early research about the delay: [2002.02952,
Domcke etc.] [2209.08131, M. Peloso, L. Sorbo]

2.5 Energy Spectrum of the GWs



similar oscillating
spectrum:
[2303.13425, J.
Garcia-Bellido et al.]

Since $\Omega_{\text{GW},0}(k)$ is mainly determined by A_k , the oscillation of A_k leads to the oscillation of $\Omega_{\text{GW},0}(k)$

1 Overview of Spectator Axion

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3 Model 2: Cosine Potential

4 Conclusion

3.1 Model

notation of the coefficient now
is α/f_a instead of $1/f$

$$\mathcal{L} = \frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - U(\chi) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \boxed{\frac{\alpha}{4f_a}} \chi \tilde{F}^{\mu\nu} F_{\mu\nu}$$

where $U(\chi)$ is the most commonly used cosine potential

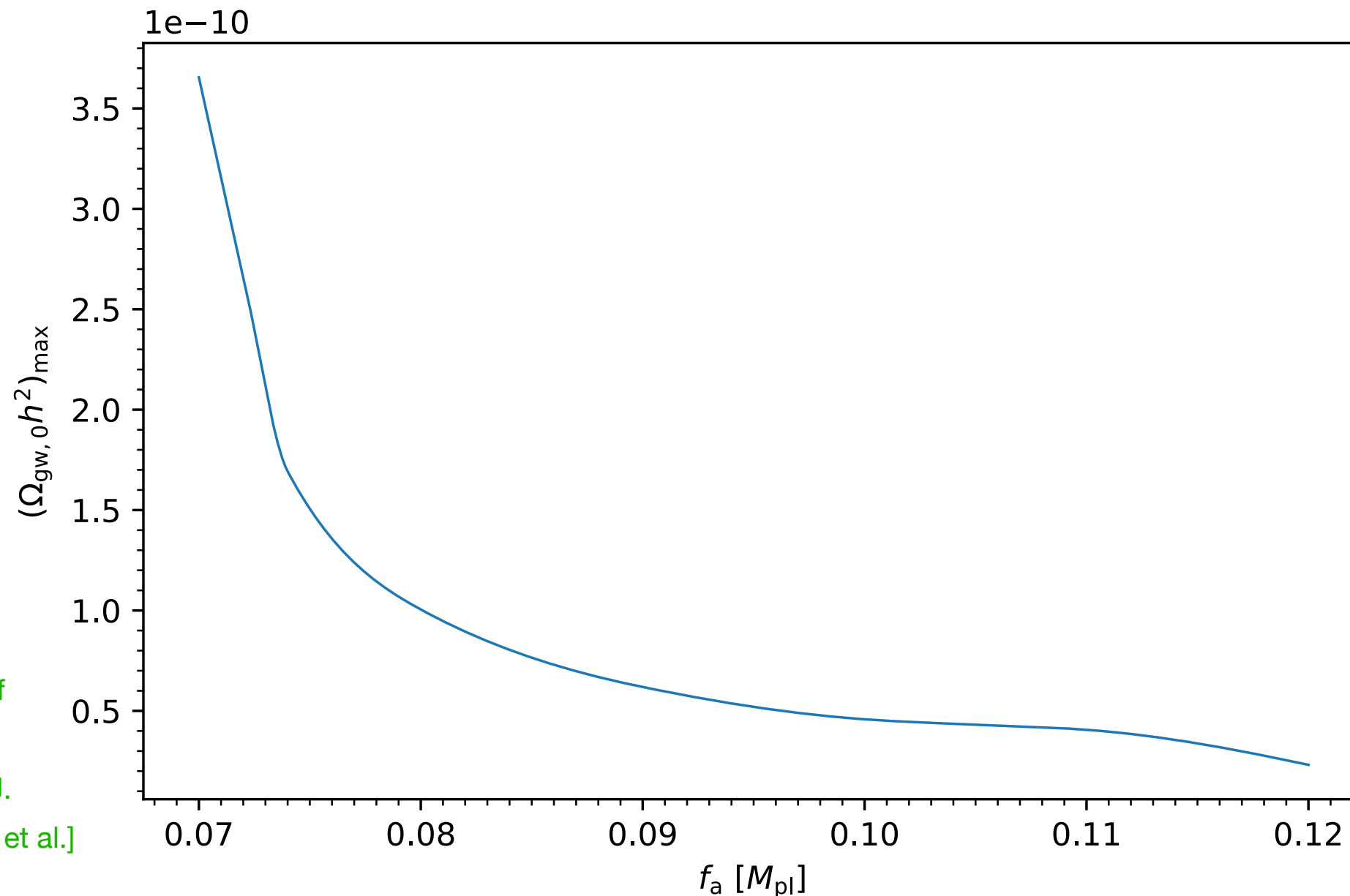
$$U(\chi) = \frac{\Lambda^4}{2} \left[1 + \cos \left(\frac{\chi}{f_a} \right) \right]$$

The slope is given by

$$U_{,\chi}(\chi) = -\frac{\Lambda^4}{2f_a} \sin \left(\frac{\chi}{f_a} \right)$$

To obtain a stronger particle production, one need a larger $U_{,\chi}$. Since Λ^4 should satisfy $\Lambda^4 \ll V(\phi)$, the only free parameter left is f_a .

3.2 Maximum GWs due to Backreaction



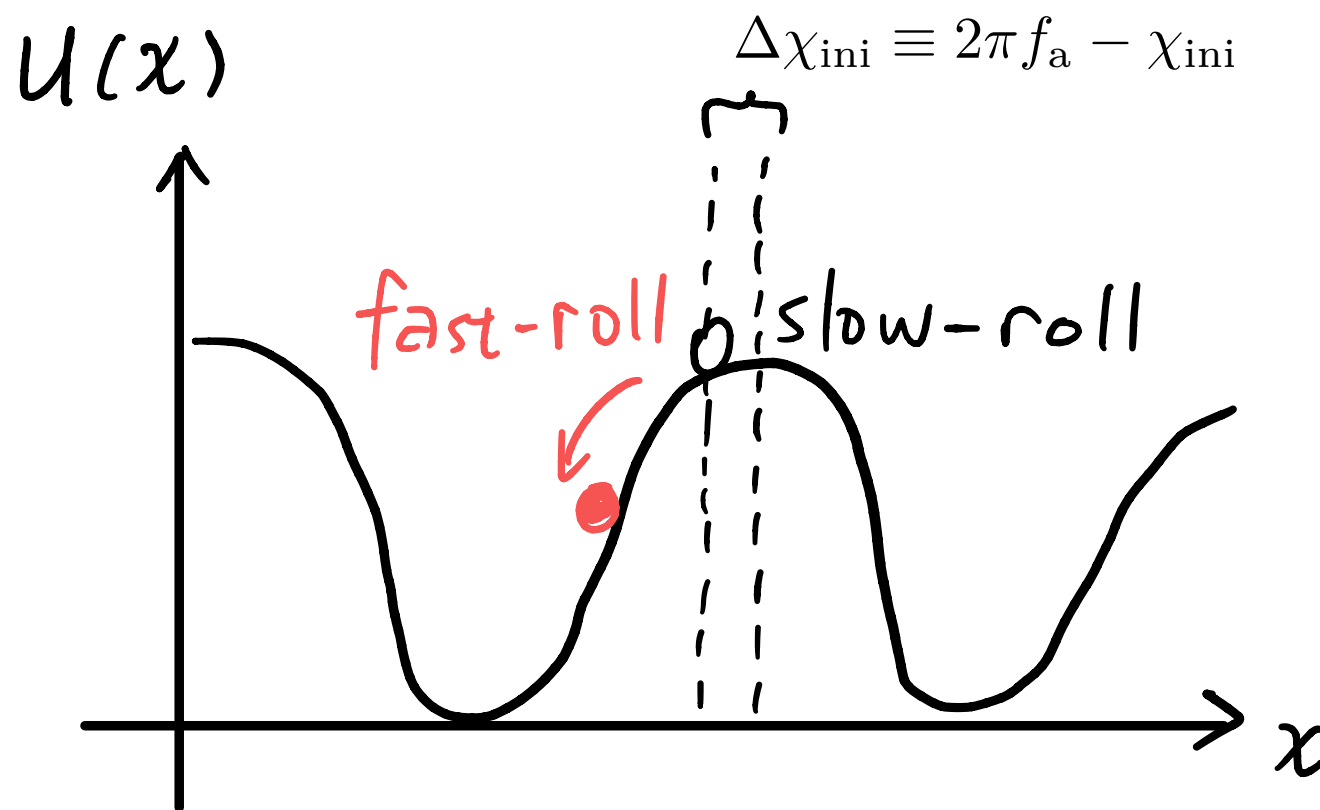
similar order of
magnitude:
[2303.13425, J.
Garcia-Bellido et al.]

Maximum value of $\Omega_{GW,0}(k)h^2$ as a function of f_a . For each value of f_a , we scan the parameter space of α to determine the maximum spectrum.



3.3 Constraint of f_a

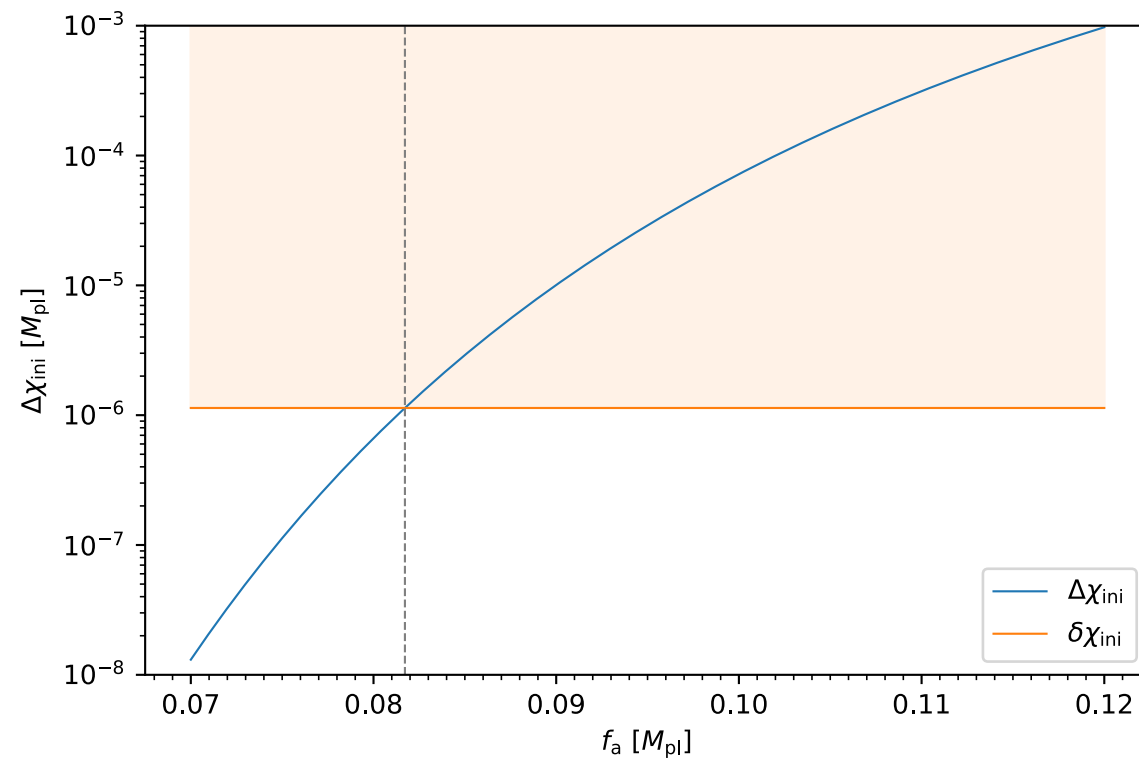
However, f_a cannot be arbitrary small. If f_a is too small, the potential will be too sharp, and the quantum fluctuation will prevent one to put the axion near the peak of $U(\chi)$, which means one cannot obtain a long-enough slow-roll stage.



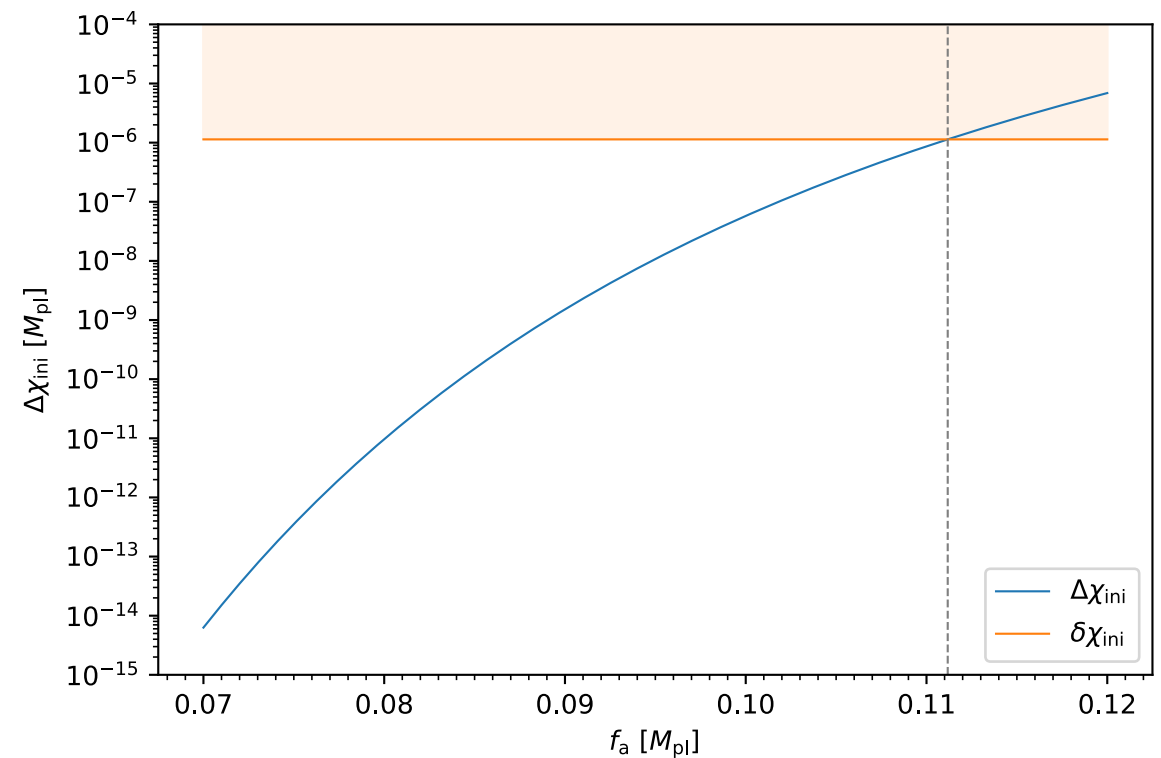
$$\Delta\chi_{\text{ini}} > \delta\chi_{\text{ini}} \sim \sqrt{\mathcal{P}_{\delta\chi}} \sim \frac{H}{2\pi}$$

$$\Rightarrow 2f_a e^{-\frac{\Lambda^4}{6H^2 f_a^2} (N_{\text{ini}} - \boxed{N_*})} > \frac{H}{2\pi}$$

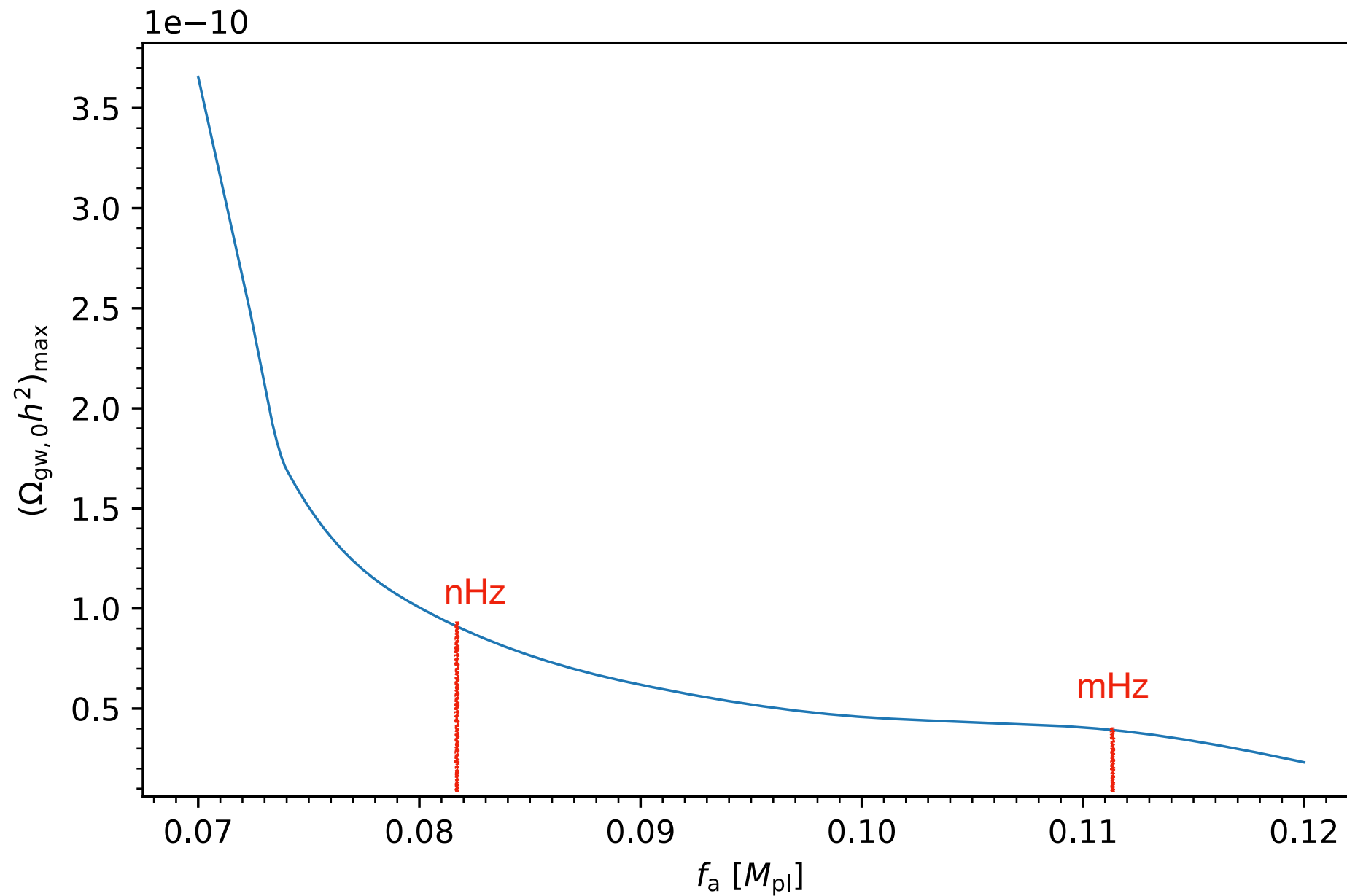
the time where the fast-roll happened

3.3 Constraint of f_a 

Constraint on the parameter f_a for GW spectrum peaking at nHz region



Constraint on the parameter f_a for GW spectrum peaking at mHz region

3.3 Constraint of f_a 

For both nHz and mHz cases, the maximum $\Omega_{gw,0} h^2$ is both $\sim 10^{-10}$

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2 Model 1: Linear Potential

3 Model 2: Cosine Potential



4 Conclusion



- peak of $\dot{\chi}$ is always prior than ρ_{EM} , and their time-delay leads to the oscillation behavior
- system **enter strong backreaction** when $f^{-1}\langle \vec{E} \cdot \vec{B} \rangle \sim U_{,\chi}$
- in **strong backreaction** region peak value of $k |A_k|^2$ nearly linearly relay on $U_{,\chi}$, but is almost not influenced by f^{-1}
- $\Omega_{\text{GW},0}(k)$ mainly relay on $k |A_k|^2$, thus oscillation of $k |A_k|^2$ leads to oscillation of $\Omega_{\text{GW},0}(k)$
- For cosine potential, the maximum $\Omega_{\text{GW},0}$ are roughly 10^{-10} for both nHz and mHz

Thanks for Listening!