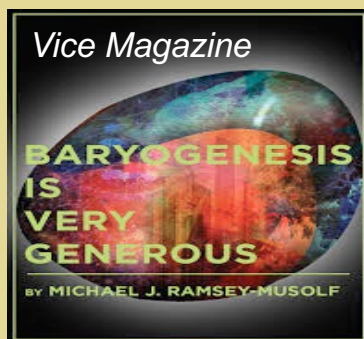


Primordial Gravitational Waves: The Collider-Theory Interface

M.J. Ramsey-Musolf

- *T.D. Lee Institute/Shanghai Jiao Tong Univ.*
- *UMass Amherst (on leave)*
- *Caltech*

About MJRM:



Science



Family



Friends

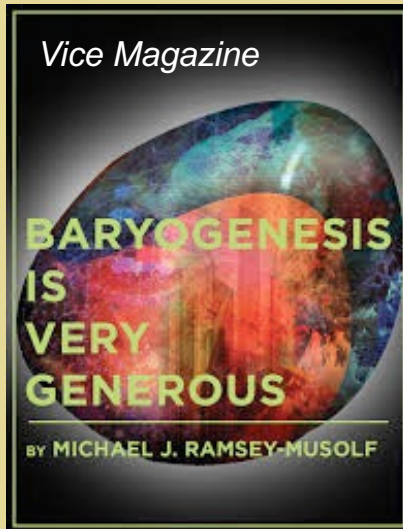
*My pronouns: he/him/his
MeToo*

Gravitational Wave Physics Conference
Hangzhou, October 18, 2025

Michael Ramsey-Musolf

T.D. Lee Chair Professor, TDLI & Chair Professor, SJTU

Theoretical Physics



- *Why does the Universe contain more matter than antimatter ?*
- *What are the laws of nature beyond those of the Standard Model & General Relativity ?*
- *How do quantum field theories work ? How do they apply to processes in the early Universe ?*
- *How can experiments test our theoretical ideas?*

- *Ph.D. Princeton*
- *Post-doc MIT*
- *美国 → 中国 2019*

- [*mjrm@sjtu.edu.cn*](mailto:mjrm@sjtu.edu.cn)
- *微信 : mjrm-china*

Goals for This Talk

- Convince you that the possibility of *primordial gravitational waves* generated from various particle physics dynamics is an *exciting area of exploration*
- Illustrate how the possibility of an *electroweak phase transition*, catalyzed by the presence of physics beyond the Standard Model, is a rich “*meeting ground*” between particle physics, gravitational wave physics, collider phenomenology, and quantum field theory
- Illustrate the *quantum field theory challenges* associated with relevant to particle physics based gravitational wave sources
- Show some *highlights of recent progress in meeting these challenges*

Key Themes for This Talk

- *The possibility of primordial gravitational waves generated from various particle physics dynamics has become an exciting area of exploration*
- *There exist many creative ideas for novel phenomena and dynamics that could have generated GW*
- *Realizing which, if any, of these ideas was realized in nature requires input from additional observables and performing the most rigorous theoretical calculations*
- *The electroweak phase transition provides a unique “laboratory” for testing our theoretical methods and ideas, with LHC and next generation collider measurements providing key input*

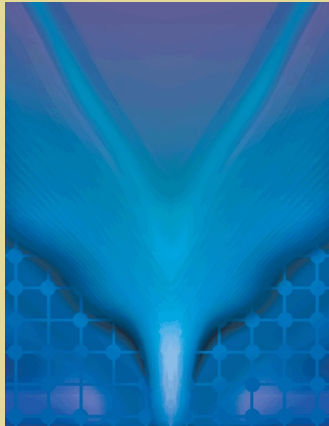
Outline

- I. Primordial GW: Particle Physics & Cosmology Perspective*
- II. Electroweak Phase Transition*
- III. Gravitational Wave-Collider Interplay*
- IV. Quantum Field Theory Challenges*
- V. Model Illustrations & Results*

I. Primordial GW: Particle Physics & Cosmology Perspective

Particle Physics & GW: Spontaneous Symmetry Breaking

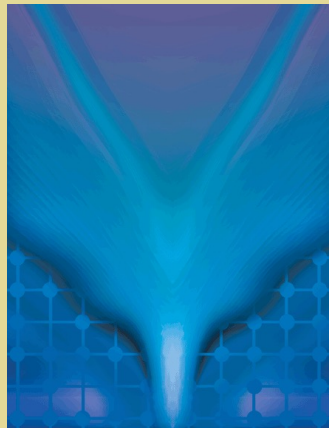
Spontaneous Symmetry Breaking



*Cooling
cosmos:
more order,
less symmetry*

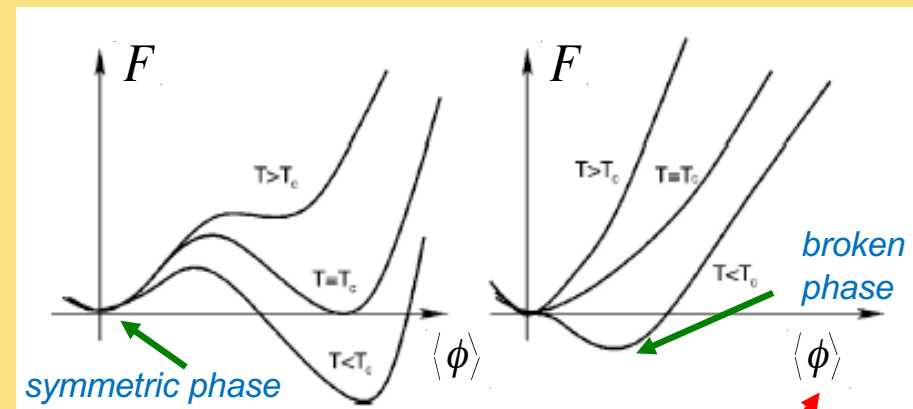
Particle Physics & GW: Spontaneous Symmetry Breaking

Spontaneous Symmetry Breaking



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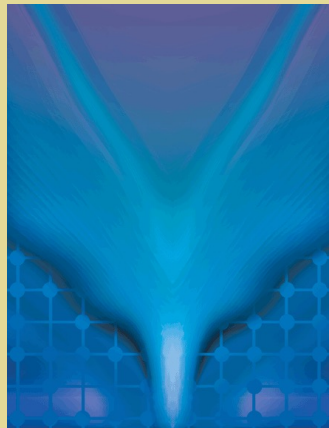
Effective potential



“order parameter” : value of
classical field $\rightarrow$ vacuum expect
value (VEV) of quantum field (v)

Particle Physics & GW: Spontaneous Symmetry Breaking

Spontaneous Symmetry Breaking



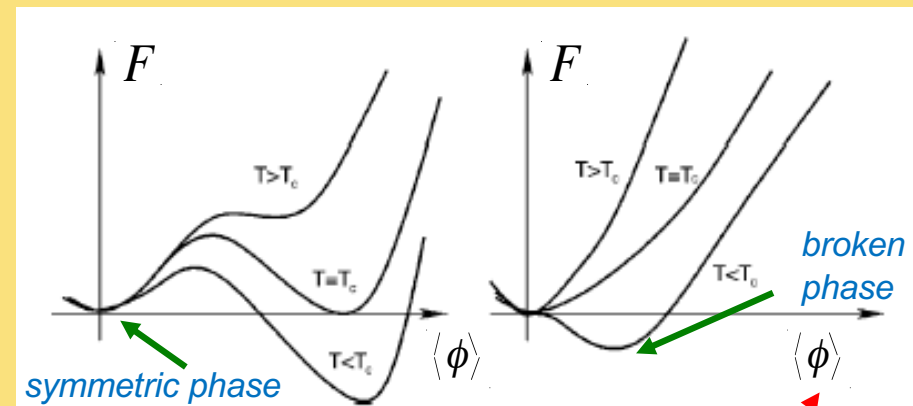
Cooling
cosmos:
more order,
less symmetry

Symmetry encoded in group
theoretic structure of
underlying particle physics
model

Standard Model:

$SU(3)_C \times SU(2)_L \times U(1)_Y$

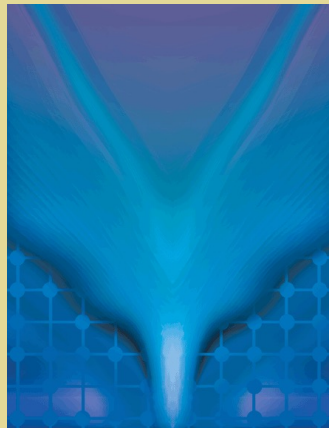
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Particle Physics & GW: Spontaneous Symmetry Breaking

Spontaneous Symmetry Breaking

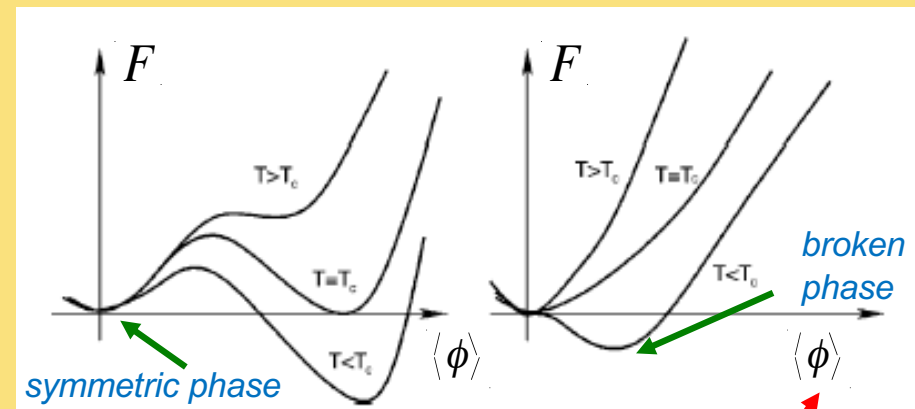


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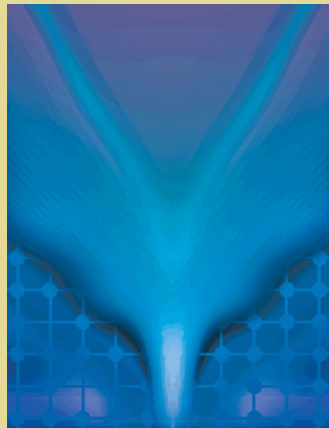
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Particle Physics & GW: Spontaneous Symmetry Breaking

Spontaneous Symmetry Breaking

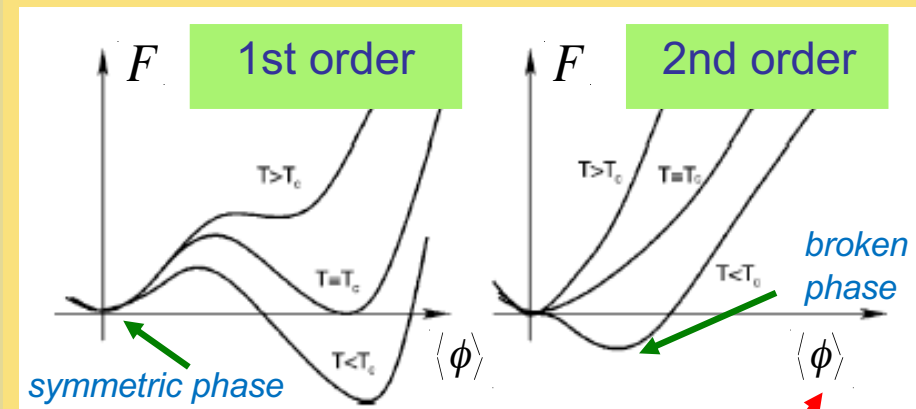


Cooling
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Symmetry encoded in group
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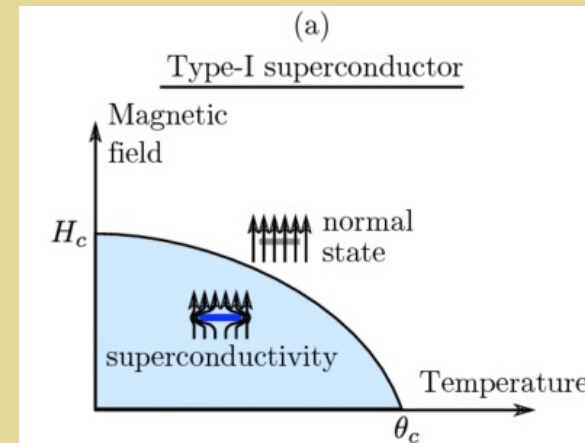
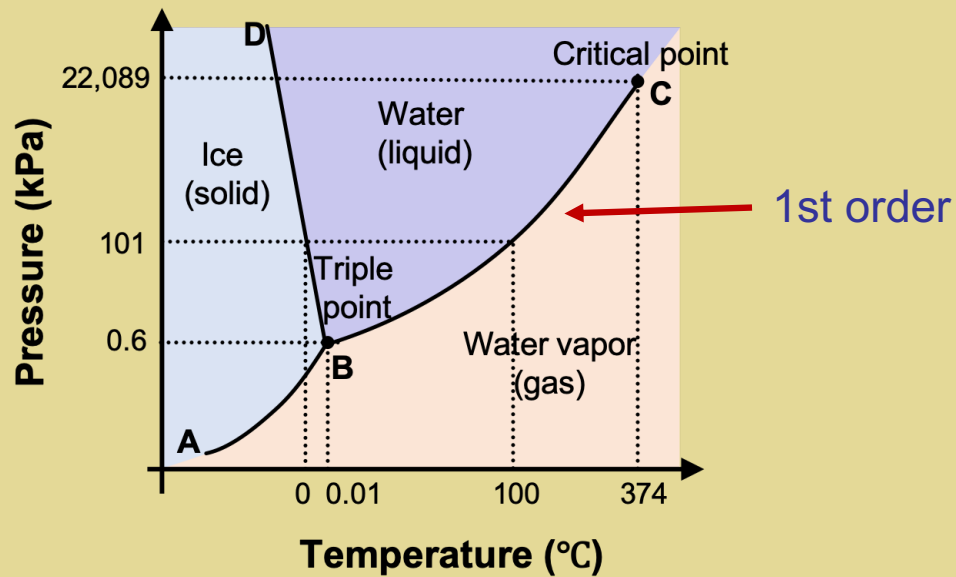
Standard Model:
 $SU(3)_C \times SU(2)_L \times U(1)_Y$ $\rightarrow$ $U(1)_{EM}$

Effective potential $\rightarrow$ Phase transitions

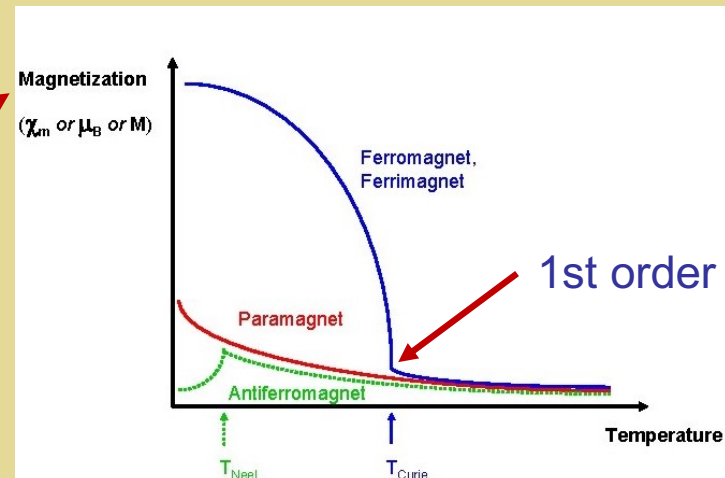


"order parameter" : value of
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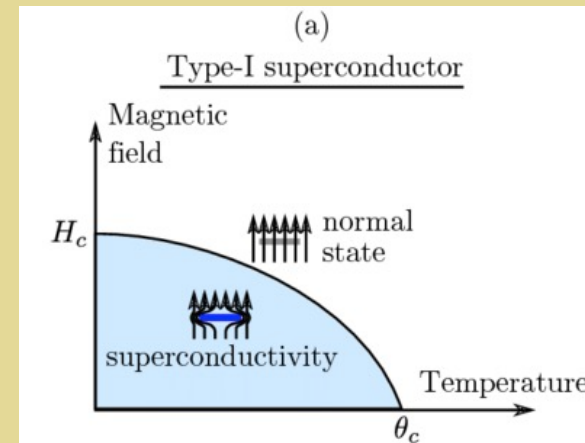
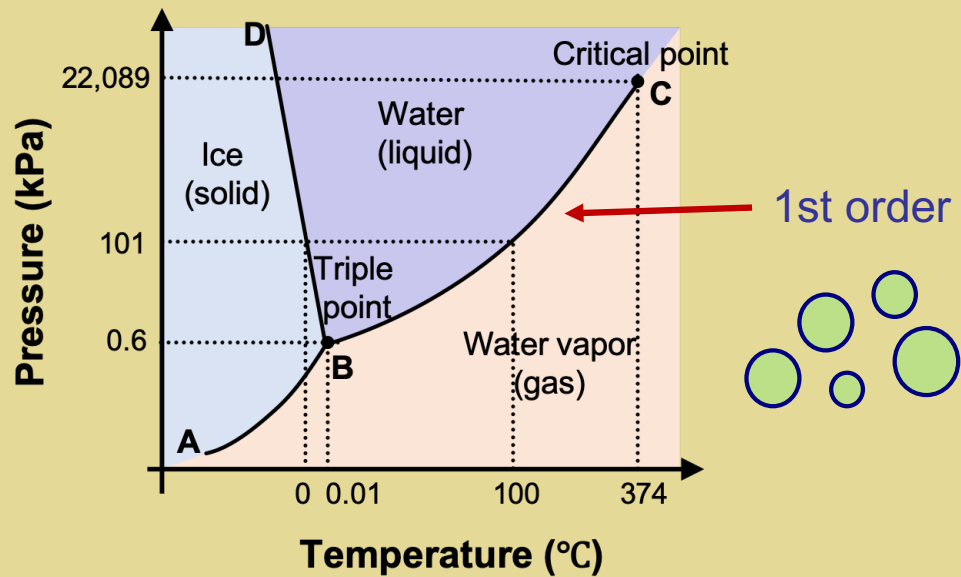
Phase Transitions



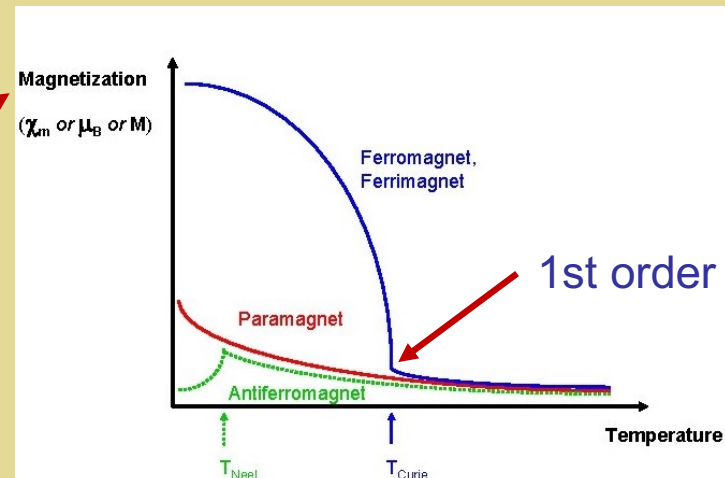
Order parameter



Phase Transitions



Order parameter



Phase Transitions

Ehrenfest classification:

1st order PT: discontinuous first derivative of free energy

$$\left(\frac{\partial F}{\partial T}\right)_V = -S \qquad \left(\frac{\partial G}{\partial T}\right)_P = -S$$

Phase Transitions

Ehrenfest classification:

1st order PT: discontinuous first derivative of free energy

$$\left(\frac{\partial F}{\partial T}\right)_V = -S \qquad \left(\frac{\partial G}{\partial T}\right)_P = -S$$

- *nth order PT: discontinuous nth derivative of free energy*
- *smooth “crossover” transition: no discontinuities in derivatives of the free energy*

Phase Transitions

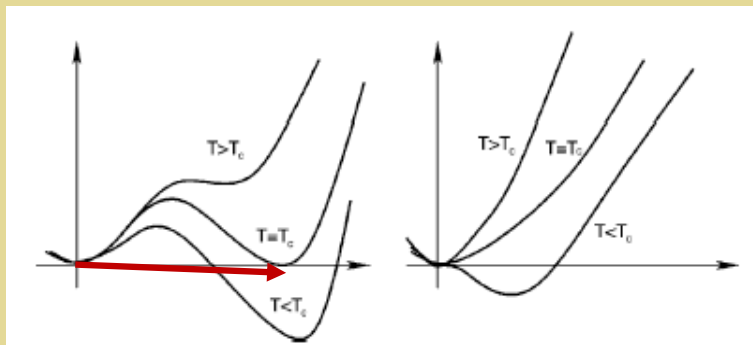
Ehrenfest classification:

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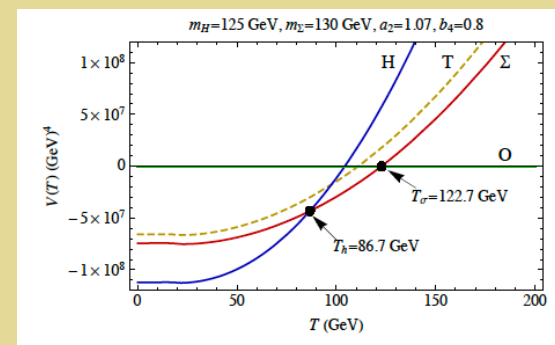
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$$\left(\frac{\partial G}{\partial T}\right)_P = -S$$

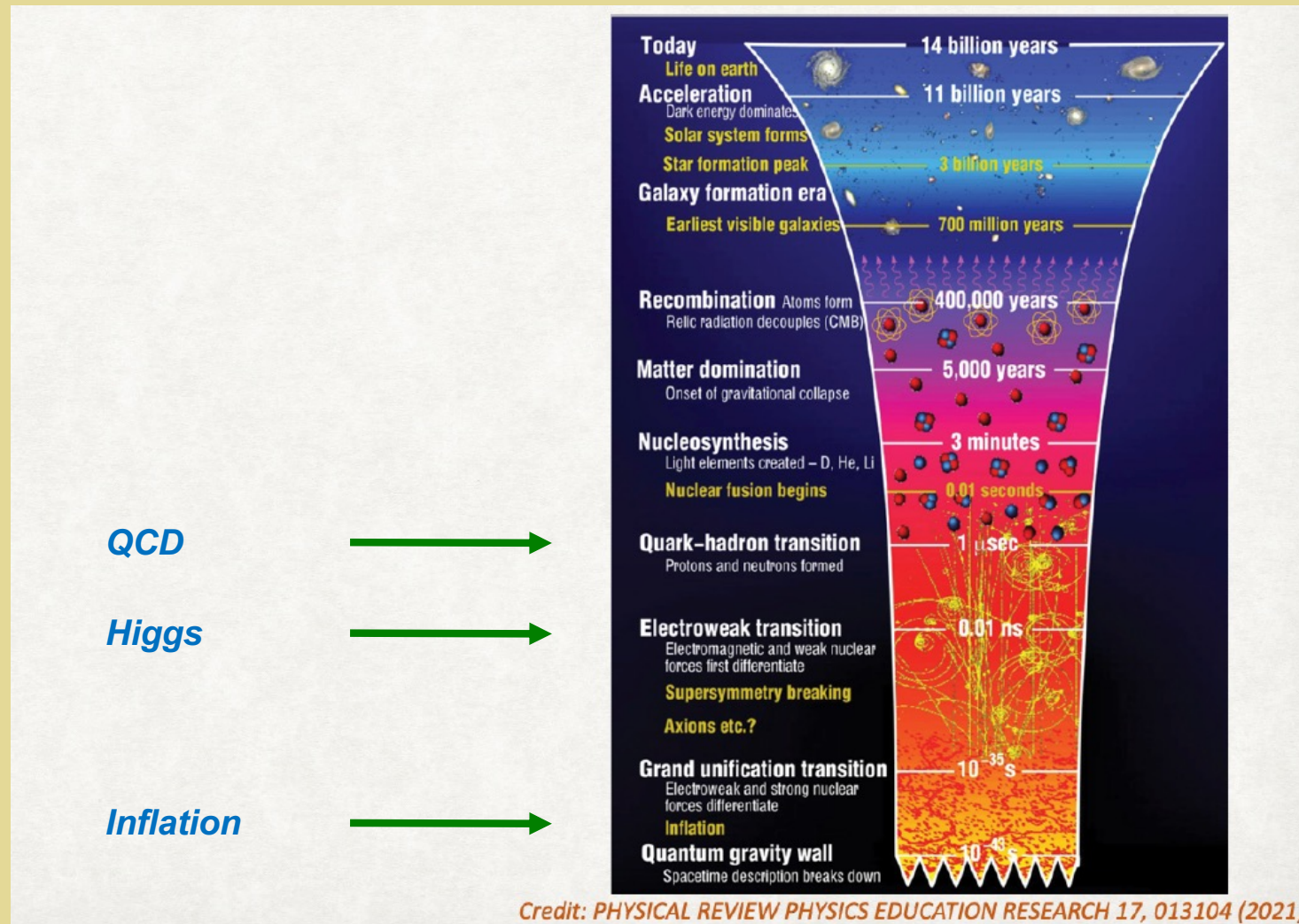
Jump in entropy (i.e., entropy injection)



Real triplet model

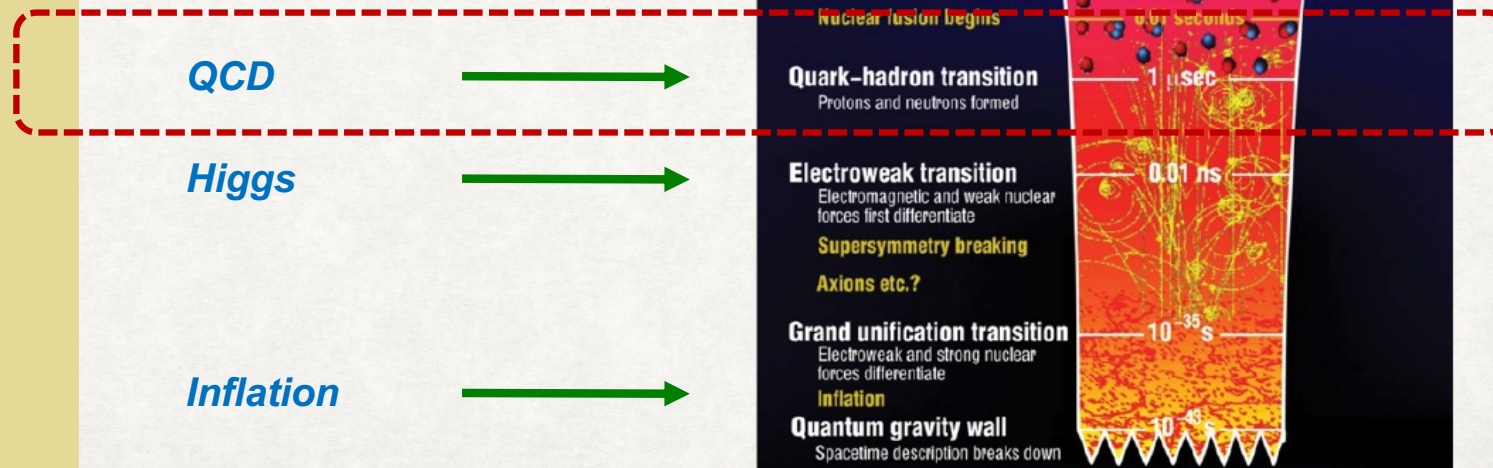


Thermal History of Symmetry Breaking



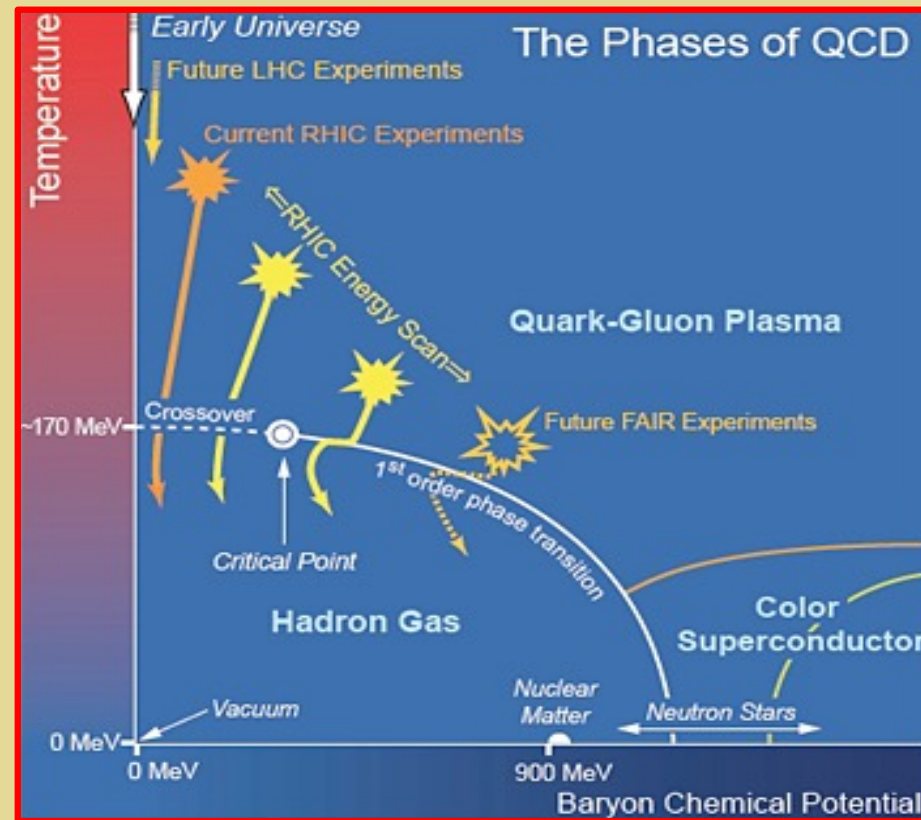
Thermal History of Symmetry Breaking

- **Free quarks & gluons $\rightarrow$ hadrons**
- **Chiral sym breaking $\rightarrow$ pions...**



Credit: PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH 17, 013104 (2021)

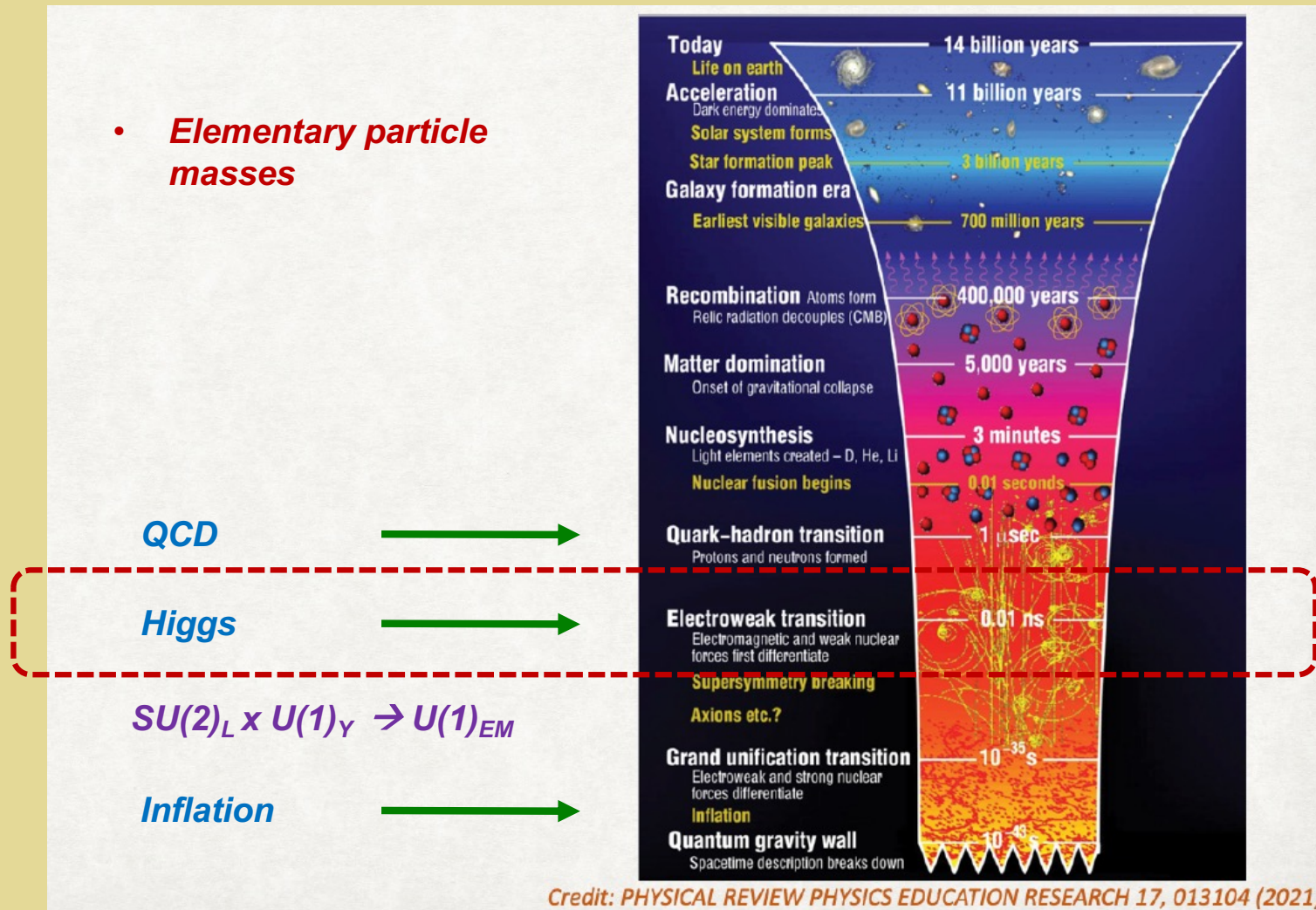
Thermal History of Symmetry Breaking



*QCD Phase Diagram → EW Theory Analog ?
“Dark Sector” ? Cosmological consequences ?*

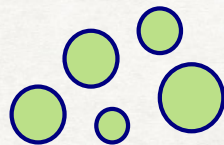
Thermal History of Symmetry Breaking

- **Elementary particle masses**



Thermal History of Symmetry Breaking

- **Elementary particle masses**
- **Matter-antimatter asym ?**



QCD

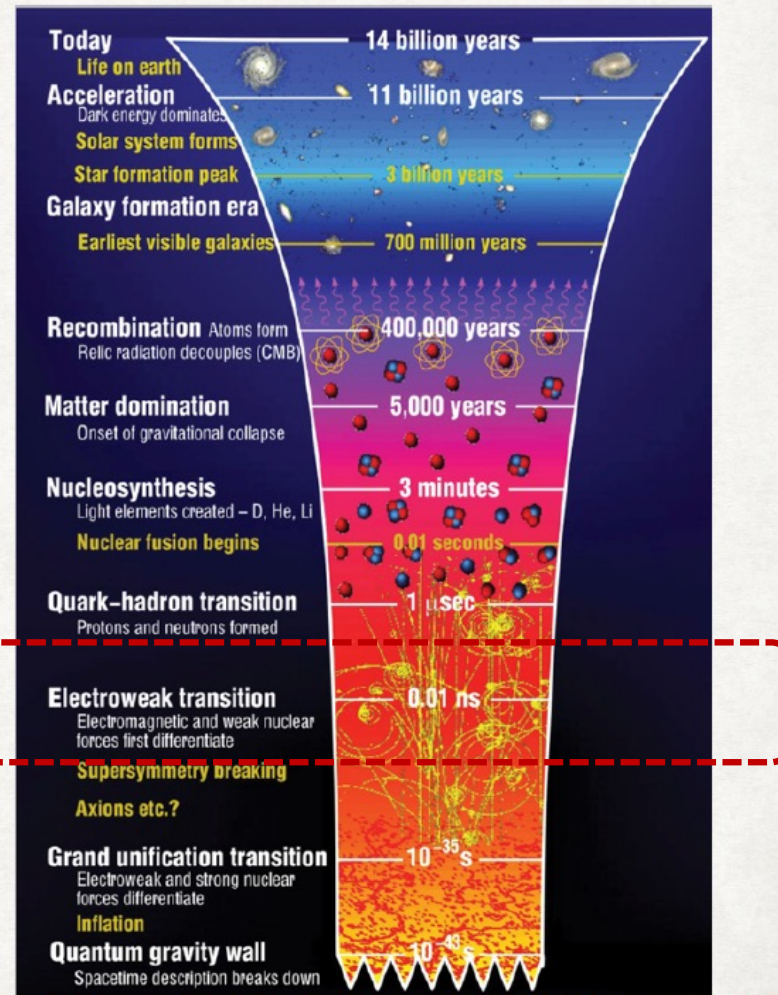


Higgs



$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

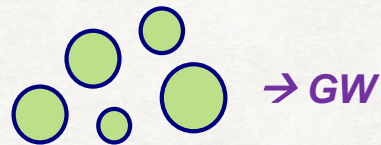
Inflation



Credit: PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH 17, 013104 (2021)

Thermal History of Symmetry Breaking

- **Elementary particle masses**
- **Matter-antimatter asym ?**



QCD

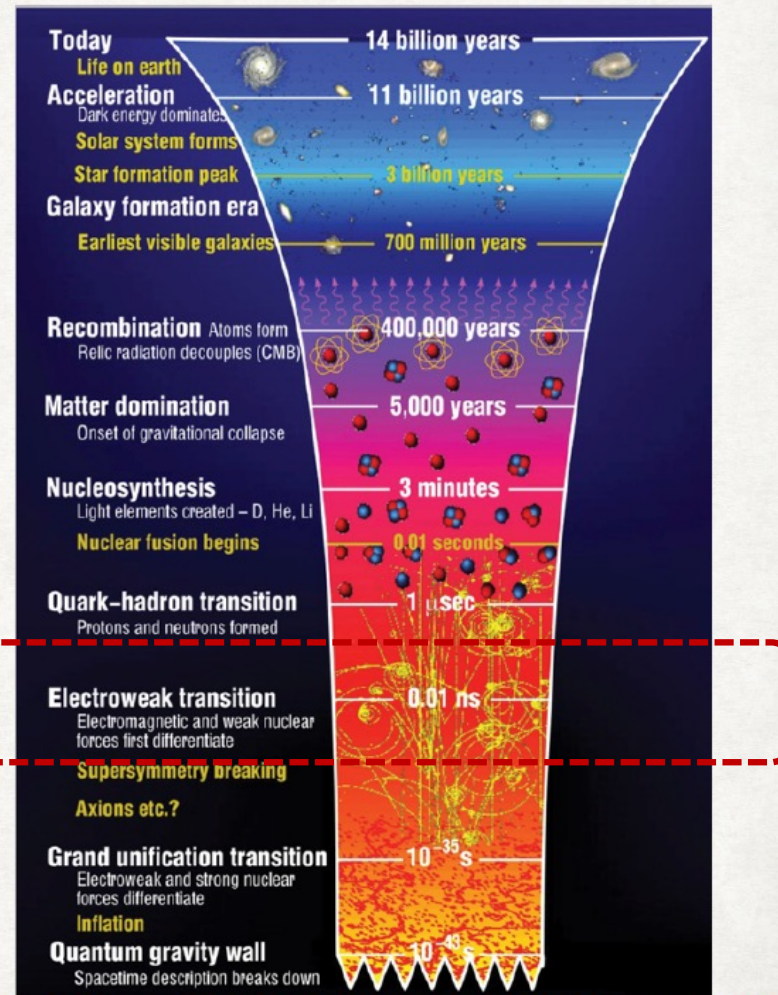


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Thermal History of Symmetry Breaking

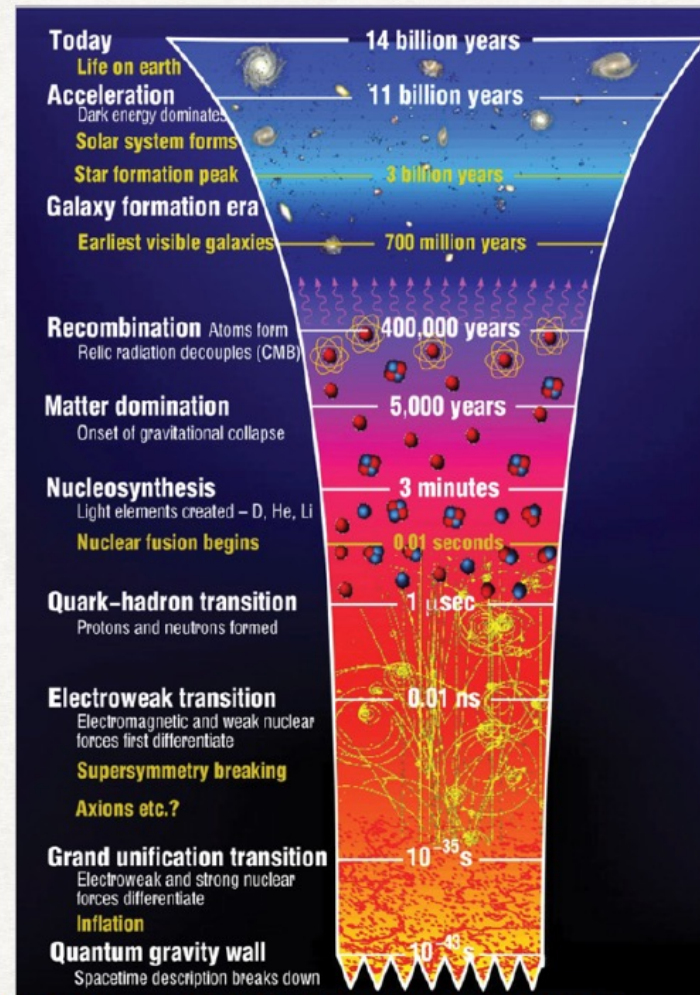
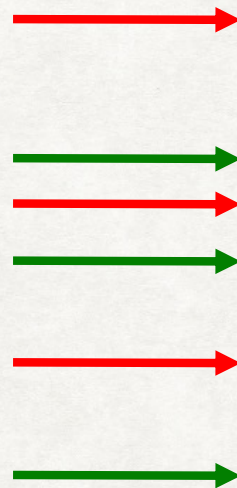
- **Neutrino masses ?**
- **Cosmic strings ?**
- **Dark matter ?**
- **Primordial black holes ?**
- **Hubble tension ?**
- ...

Other ?

QCD

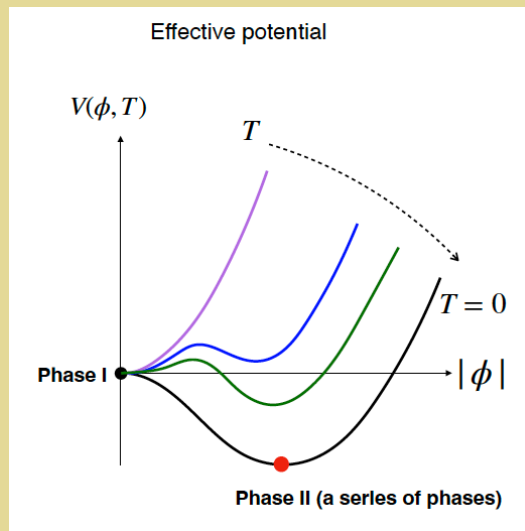
Higgs

Inflation



Credit: PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH 17, 013104 (2021)

Particle Physics & GW: Spontaneous Symmetry Breaking

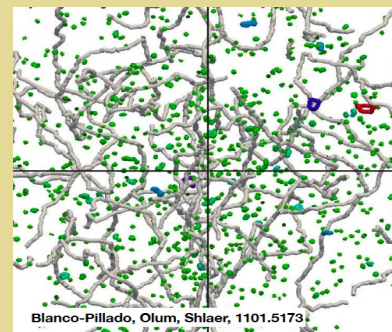
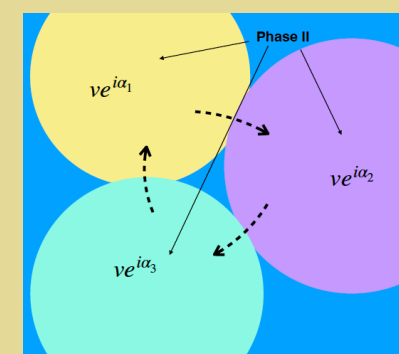
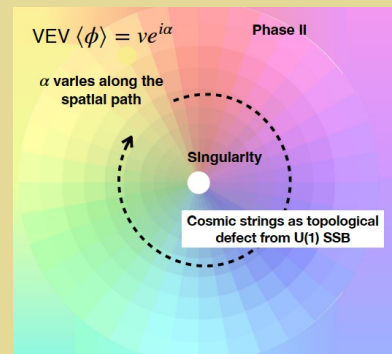
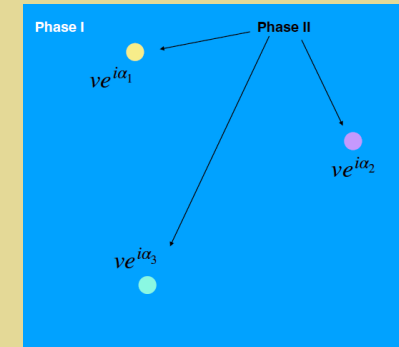
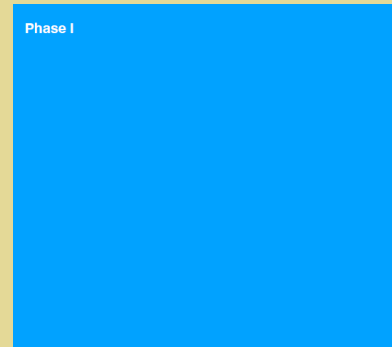


Thanks: Ye-Ling Zhou CPCS '24

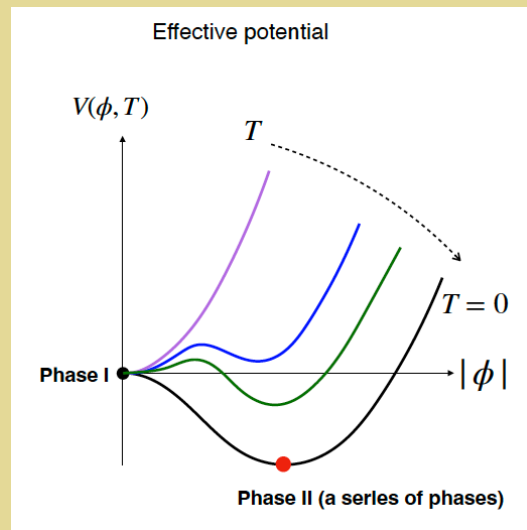
Spontaneous breaking of global $U(1)_X$

→ Cosmic strings:

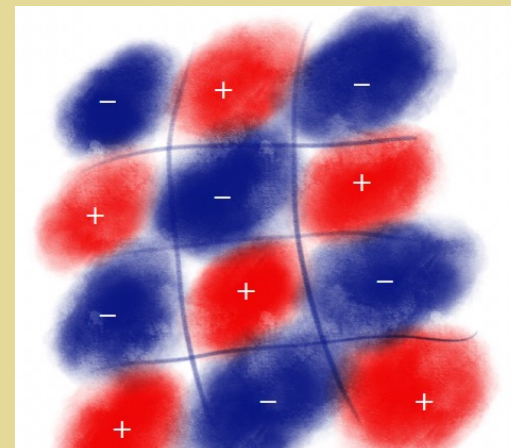
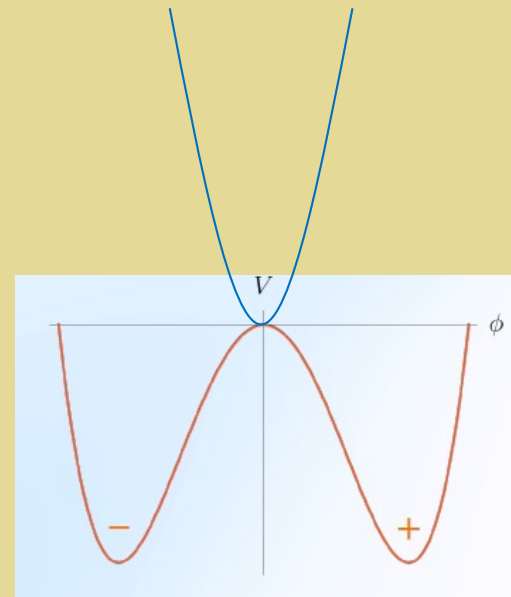
Dynamics can generate GW sources



Particle Physics & GW: Spontaneous Symmetry Breaking



Thanks: Ye-Ling Zhou CPCS '24



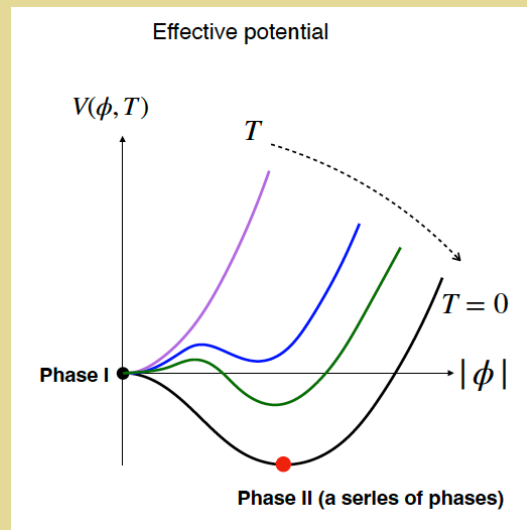
Thanks: Bowen Fu CPCS '24

Spontaneous breaking of discrete sym: e.g., $\mathbb{Z}_2$

→ Domain walls:

Dynamics can generate GW sources

Particle Physics & GW: Spontaneous Symmetry Breaking

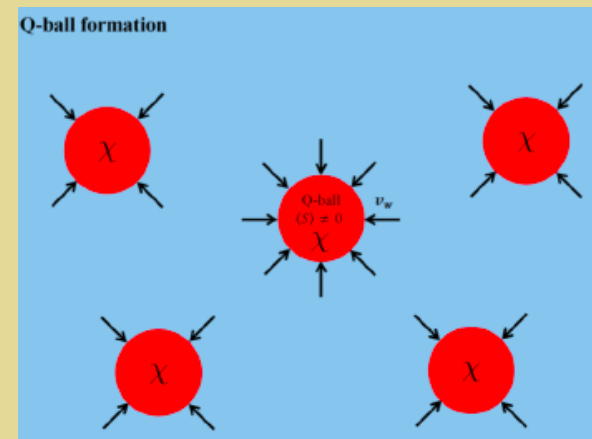
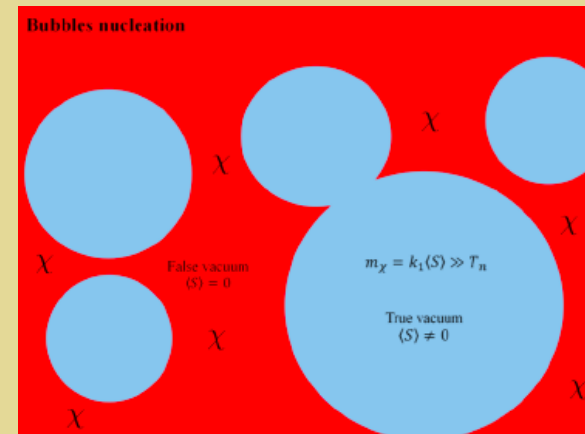


Thanks: Ye-Ling Zhou CPCS '24

Trapping of heavy particles in bubbles

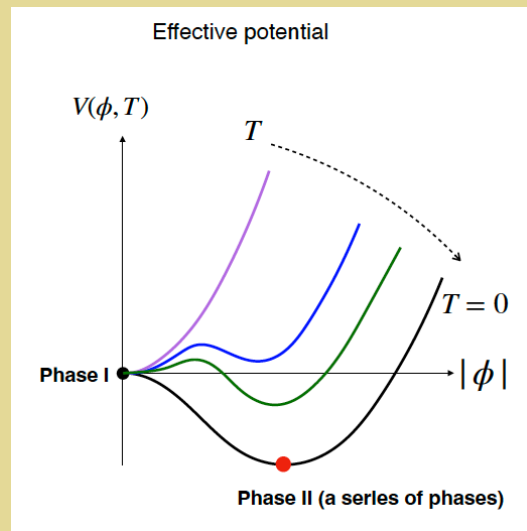
→ **Q-Ball dark matter**

GW from bubble collisions



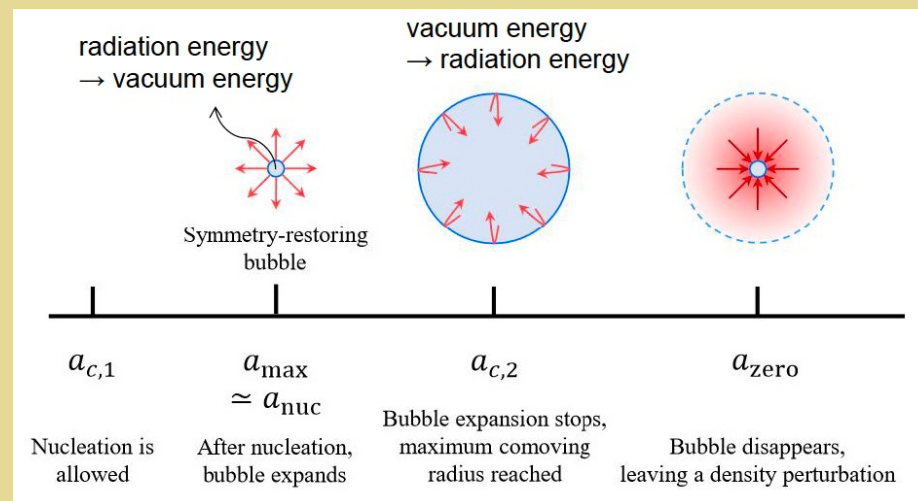
Thanks: Fa Peng Huang CPCS '24

Particle Physics & GW: Spontaneous Symmetry Breaking



Thanks: Ye-Ling Zhou CPCS '24

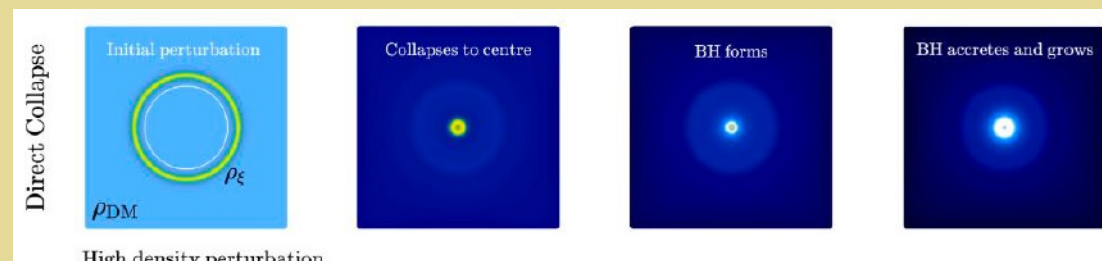
Symmetry restored during reheating



Aborted 1st order PT

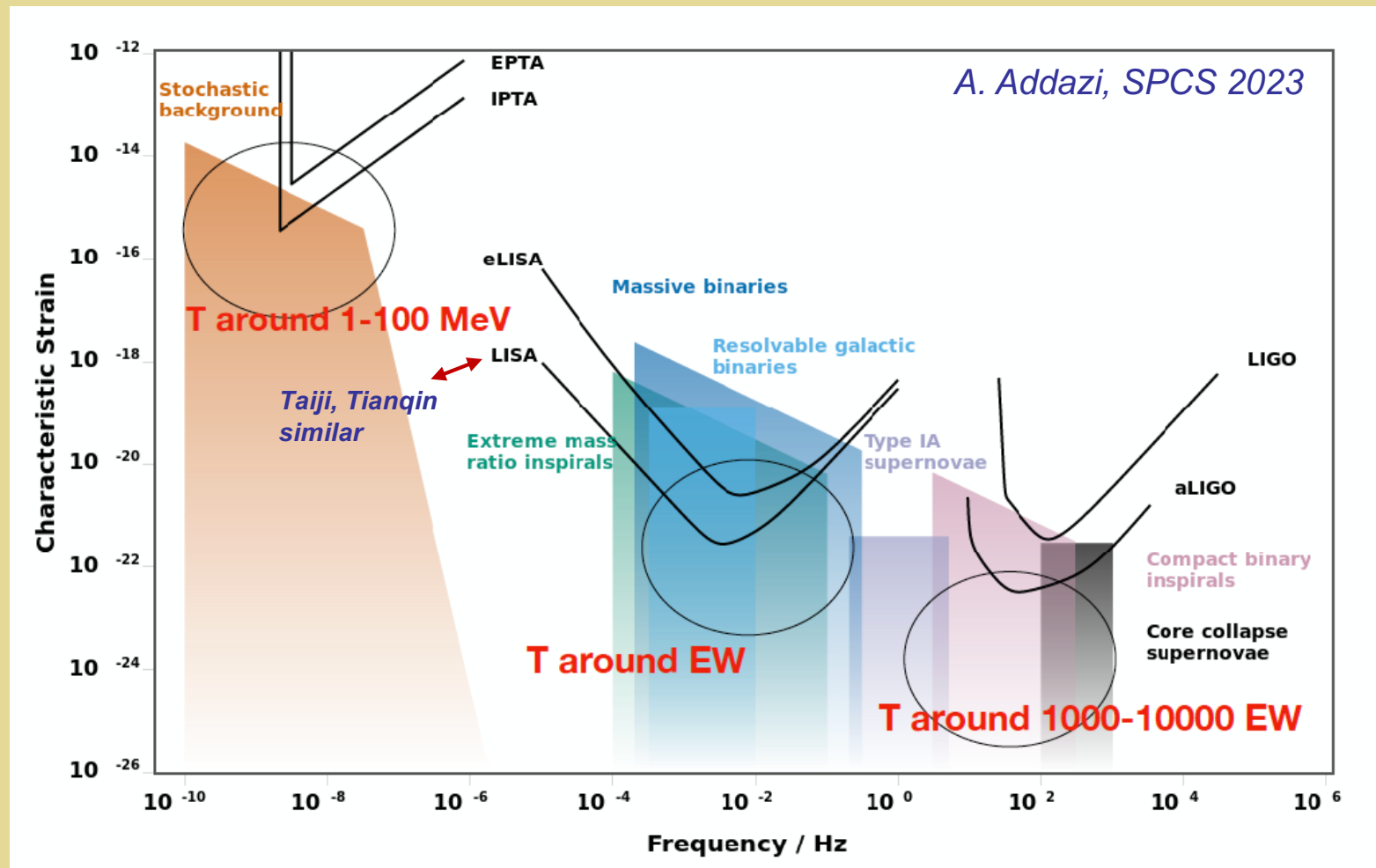
$\rightarrow$ Collapsed bubbles $\rightarrow \rho$ perturb $\rightarrow$ Black holes

No bubble collisions $\rightarrow$ no GW

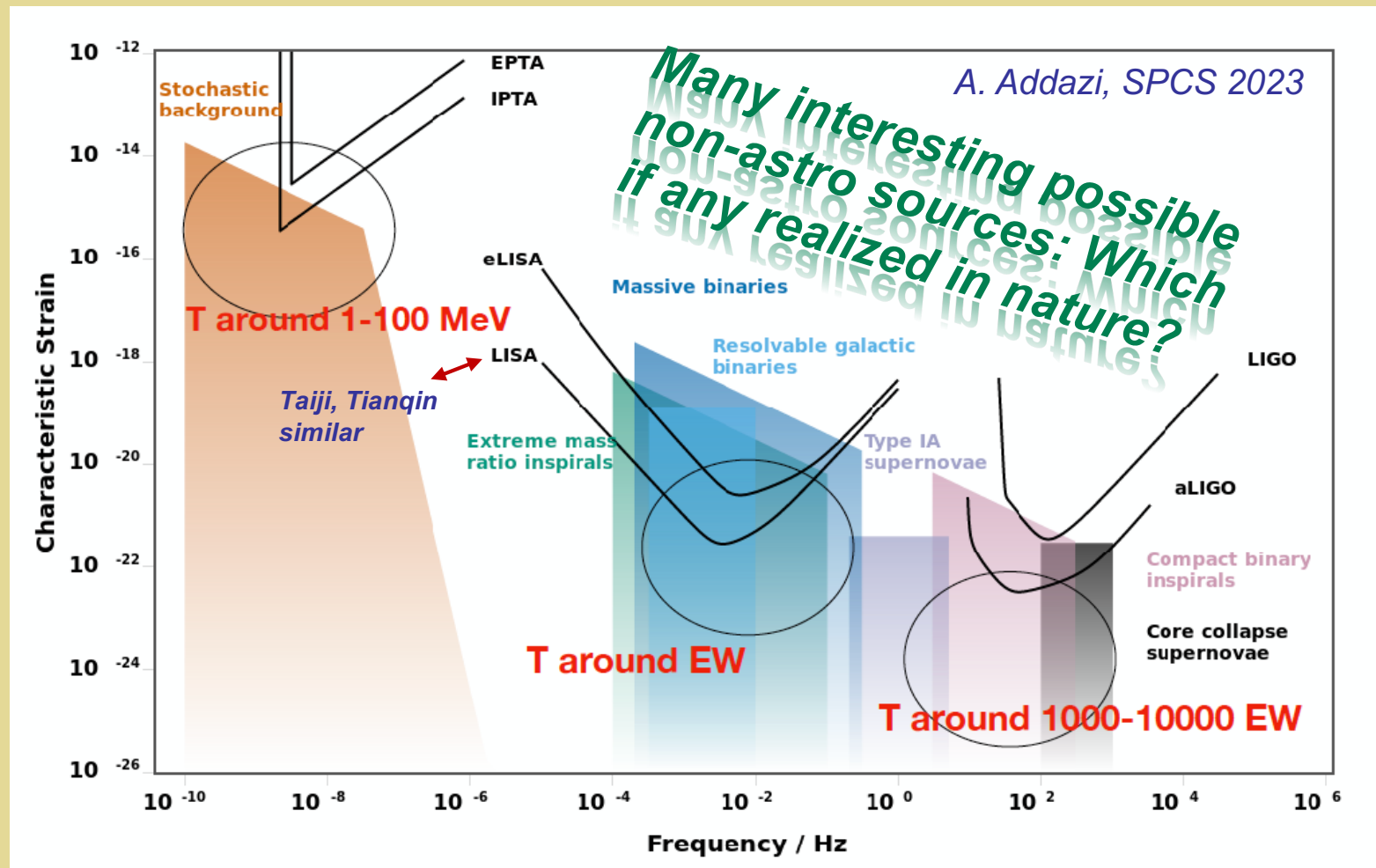


Thanks: Wenyuan Ai CPCS '24

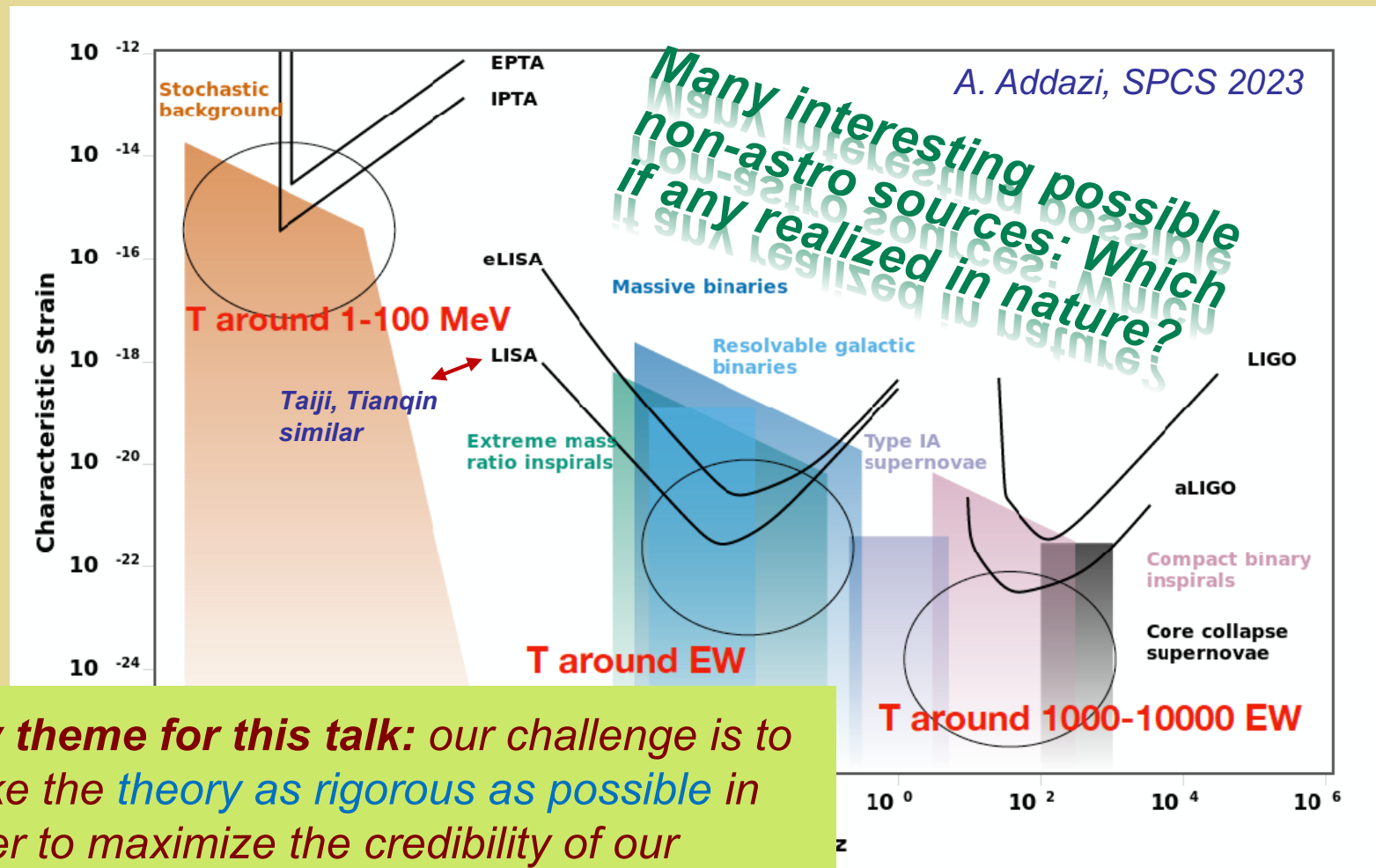
Gravitational Waves



Gravitational Waves

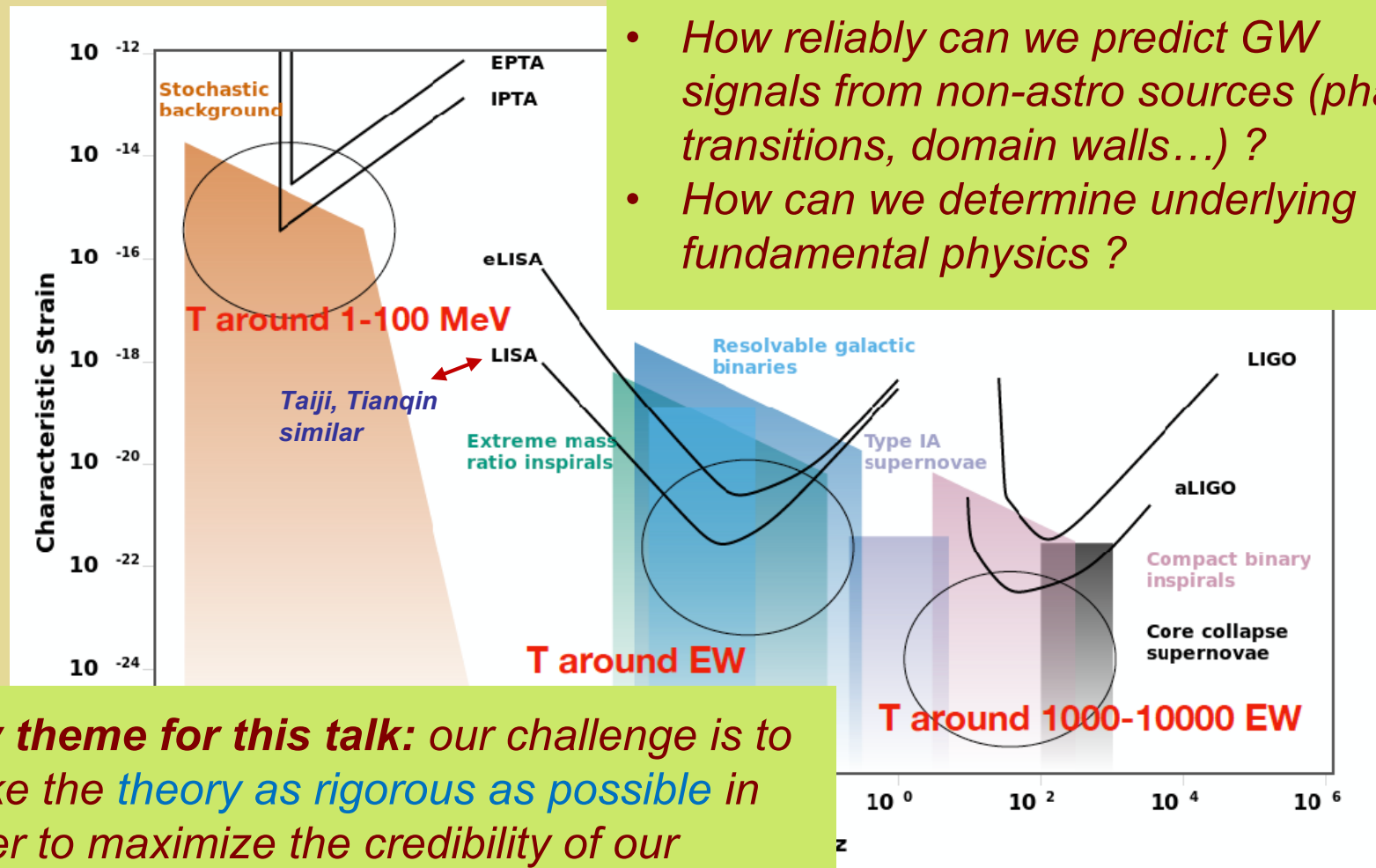


Gravitational Waves



Key theme for this talk: our challenge is to make the *theory as rigorous as possible* in order to maximize the credibility of our creative ideas and enable a robust confrontation with experiment to address this question

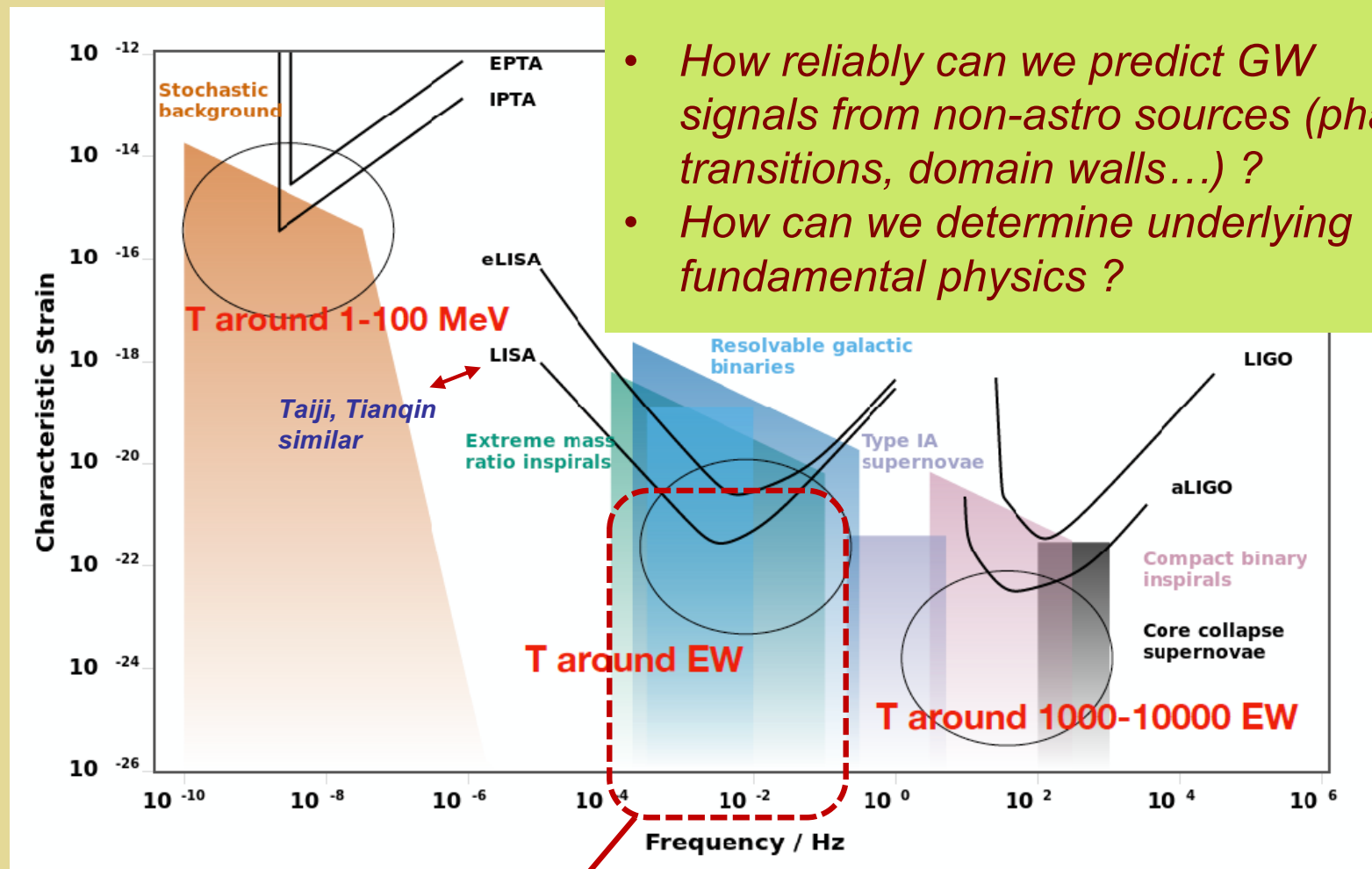
Gravitational Waves



- How reliably can we predict GW signals from non-astro sources (phase transitions, domain walls...)?
- How can we determine underlying fundamental physics?

Key theme for this talk: our challenge is to make the *theory as rigorous as possible* in order to maximize the credibility of our creative ideas and enable a robust confrontation with experiment to address this question

GW: Electroweak Phase Transition



- How reliably can we predict GW signals from non-astro sources (phase transitions, domain walls...)?
- How can we determine underlying fundamental physics?

EWPT laboratory for GW micro-physics: colliders can probe particle physics responsible for non-astro GW sources → test our framework for GW microphysics at other scales

II. The Electroweak Phase Transition

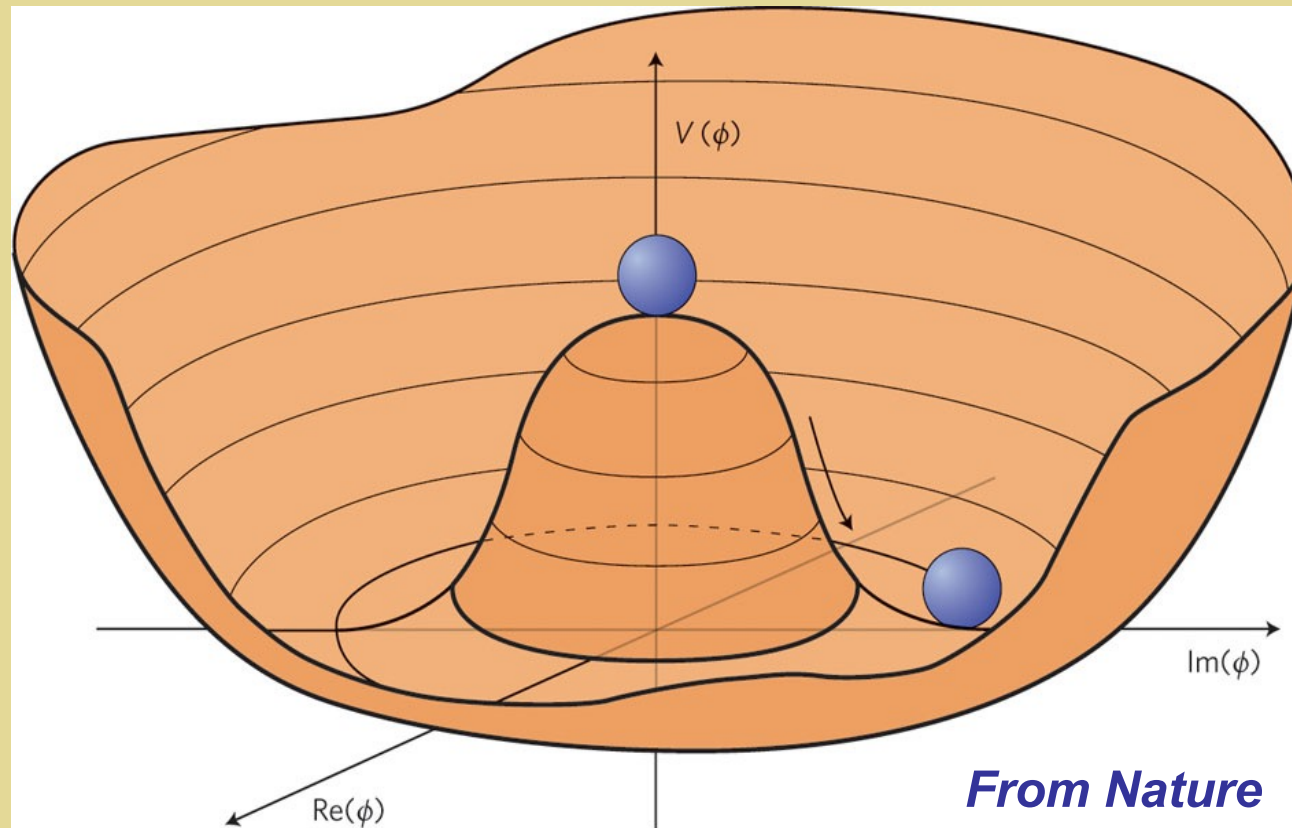
Was There and Electroweak Phase Transition

- *Higgs discovery → What was the thermal history of EWSB ?*
- *Baryogenesis → Was the matter-antimatter asymmetry generated in conjunction with EWSB (EW baryogenesis) ?*
- *Gravitational waves → If a signal observed in in next generation probes, could a cosmological phase transition be responsible ?*
- *Laboratory for testing phase transition physics → How reliable is the theory ?*

Was There and Electroweak Phase Transition

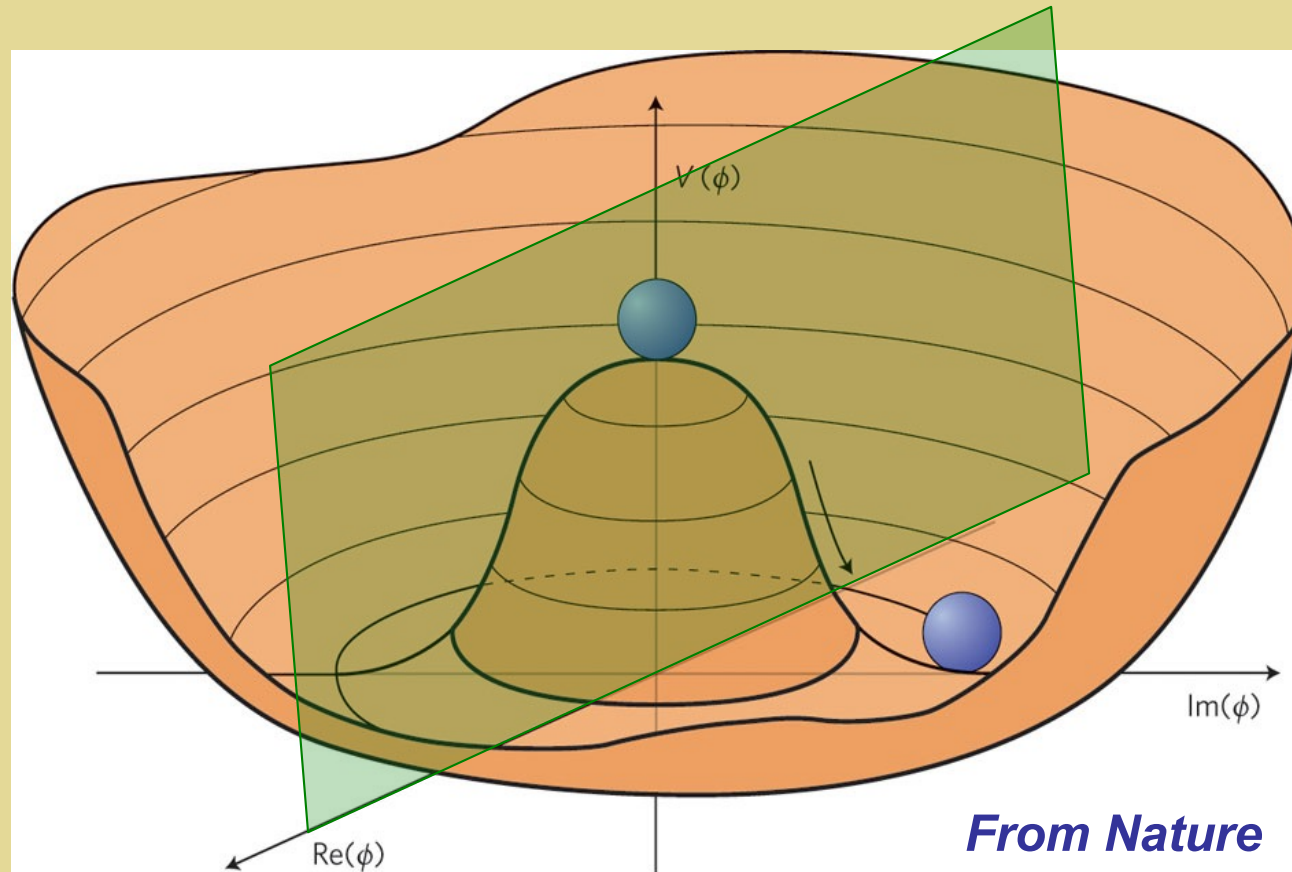
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EWSB: The Scalar Potential



What was the thermal history of EWSB ?

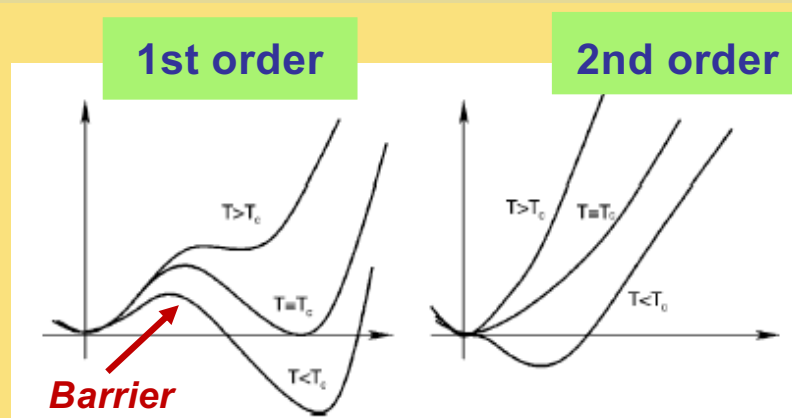
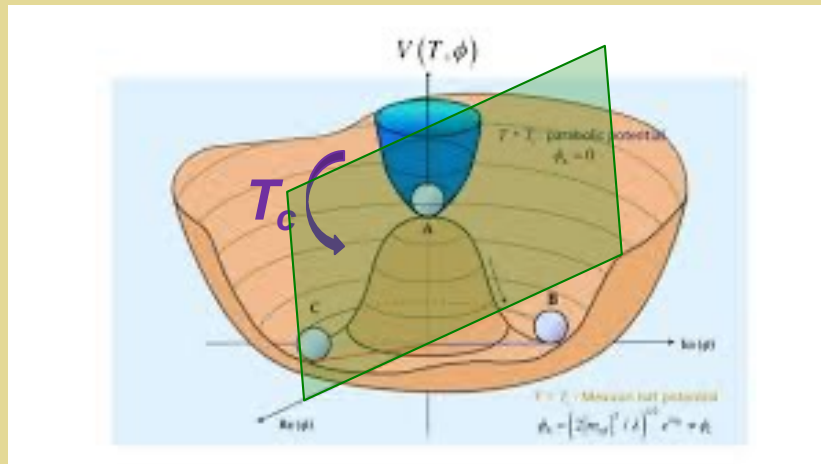
EWSB: The Scalar Potential



What was the thermal history of EWSB ?

Spontaneous Symmetry Breaking

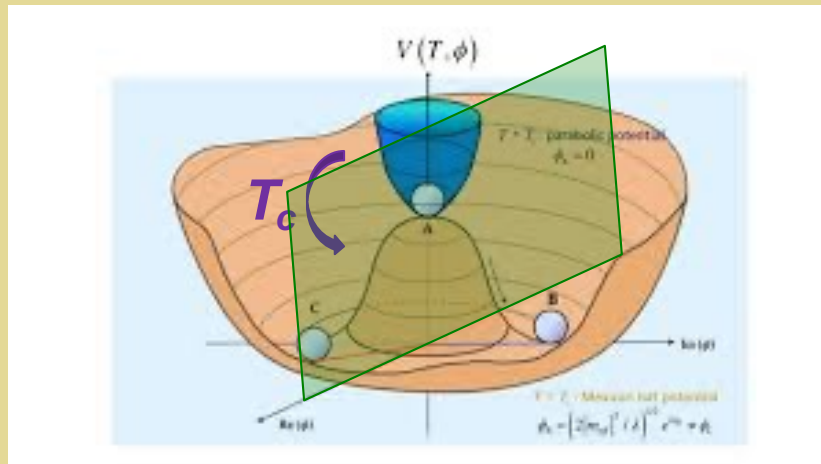
Higgs Boson potential energy



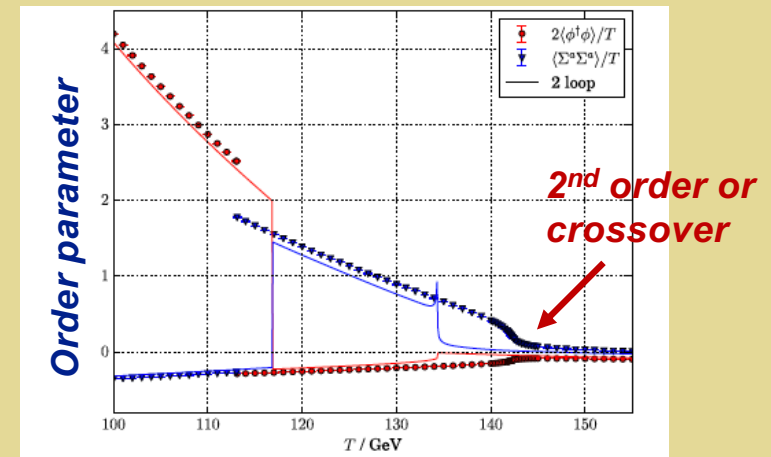
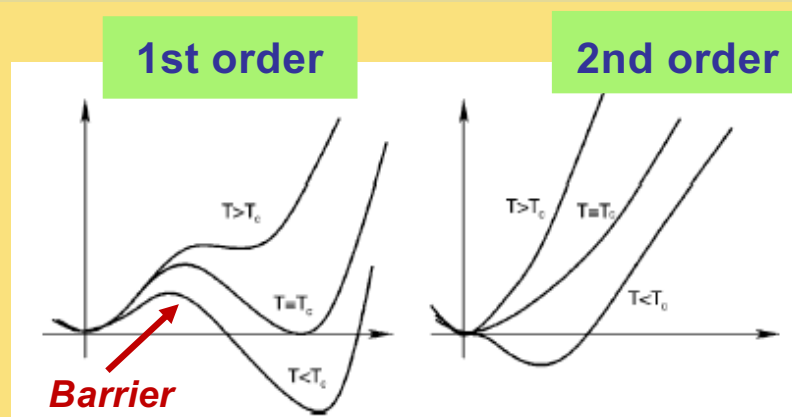
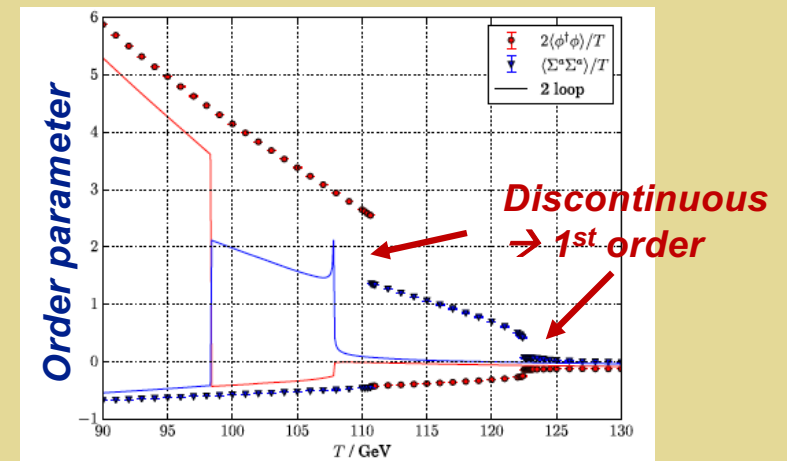
Increasing m_h $\longrightarrow$

Spontaneous Symmetry Breaking

Higgs Boson potential energy

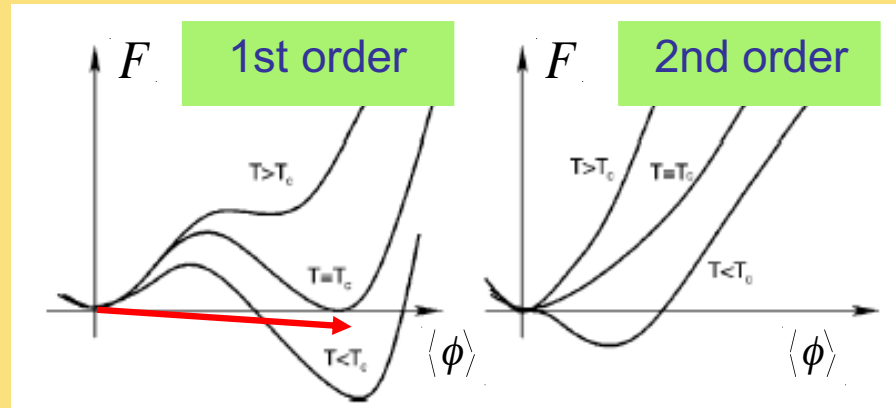


Nature of the transition



Increasing m_h $\longrightarrow$

EWSB Transition: St'd Model



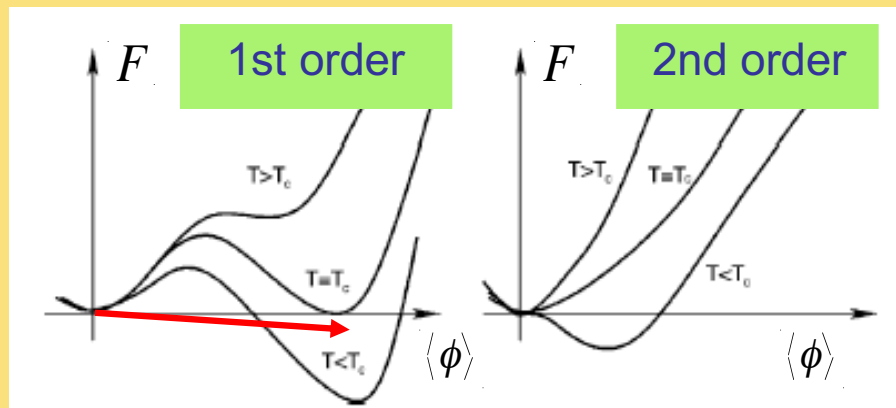
Increasing m_h $\longrightarrow$

Higgs potential: $T=0$

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

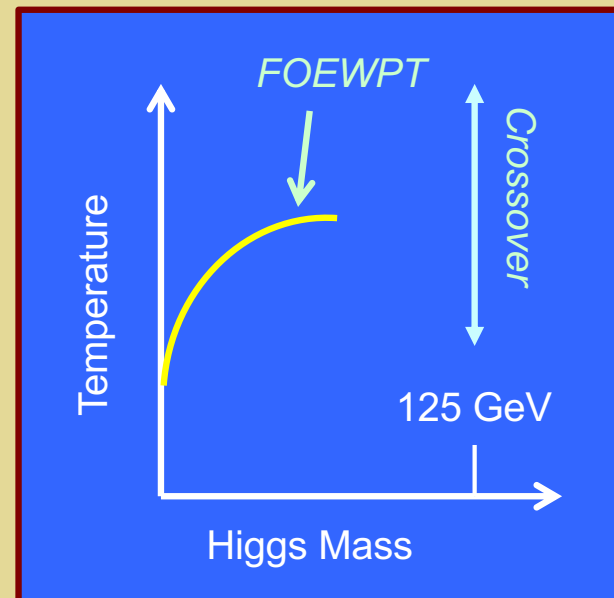
$$m_h^2 = 2\lambda v^2$$

EWSB Transition: St'd Model



Increasing m_h $\longrightarrow$

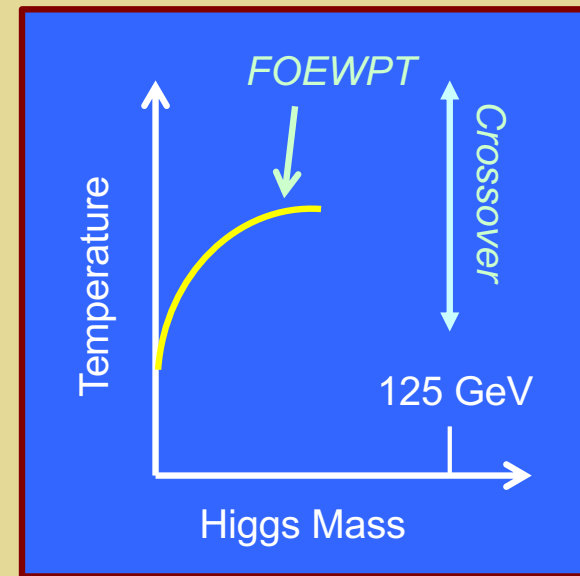
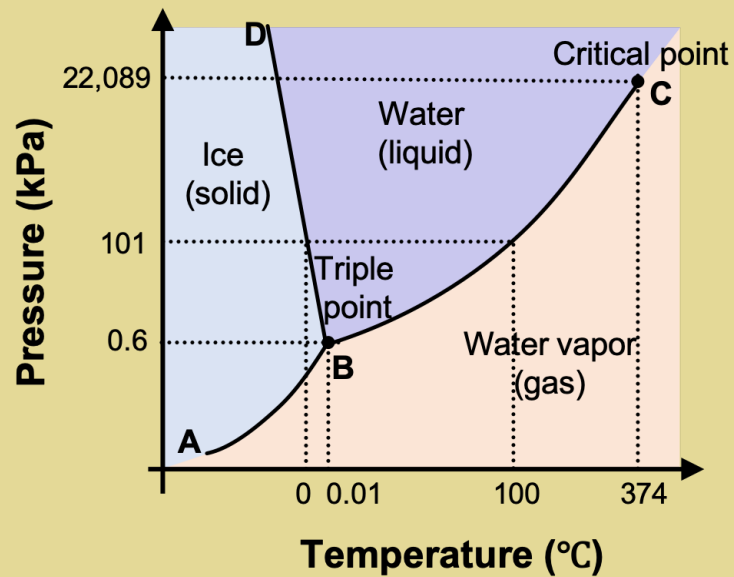
Lattice	Authors	M_h^C (GeV)
4D Isotropic	[76]	80 ± 7
4D Anisotropic	[74]	72.4 ± 1.7
3D Isotropic	[72]	72.3 ± 0.7
3D Isotropic	[70]	72.4 ± 0.9



EW Phase Diagram

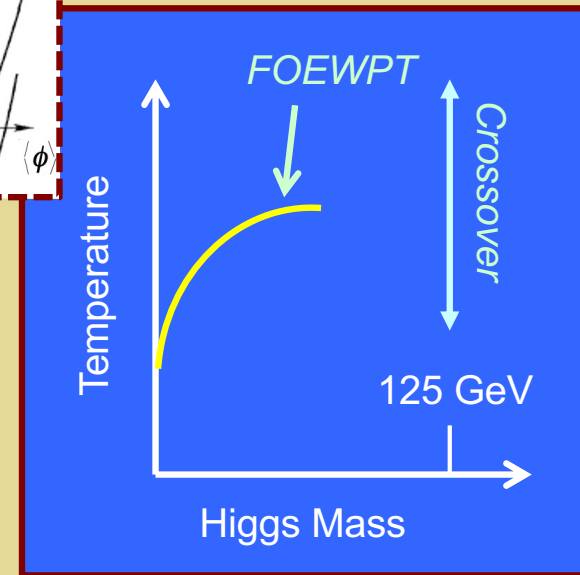
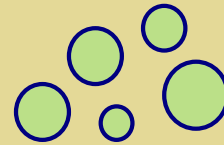
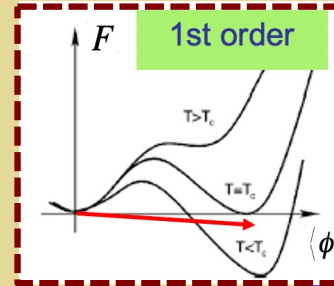
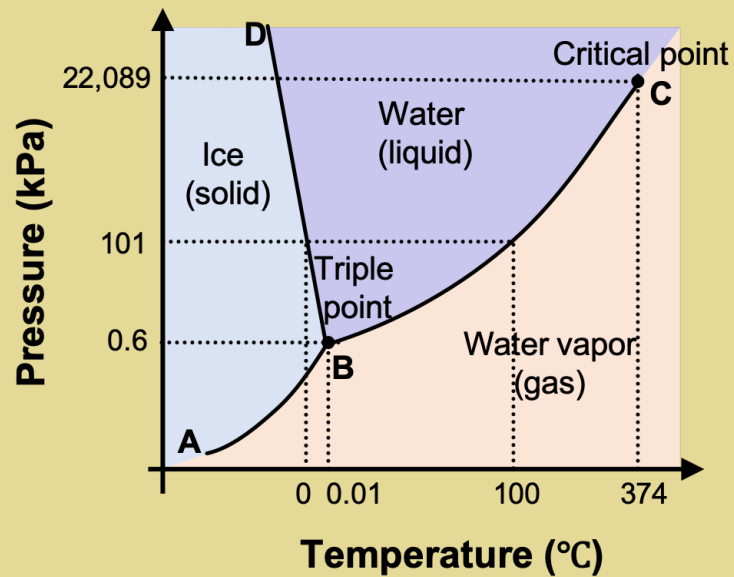
SM EW: Cross over transition

Was There an Electroweak Phase Transition ?



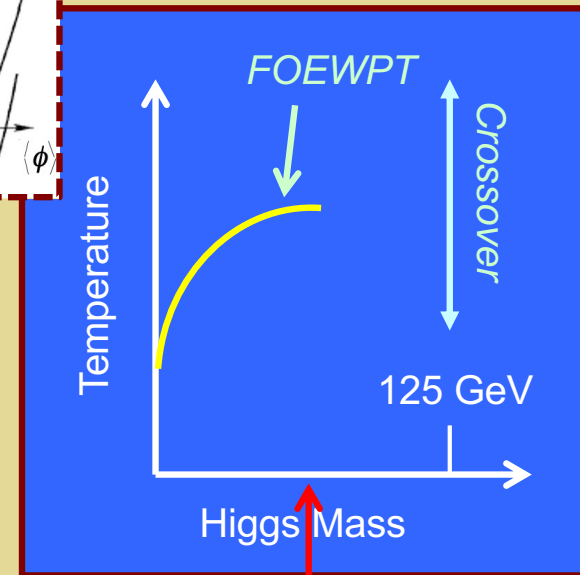
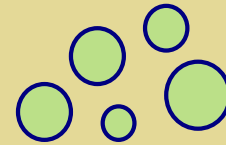
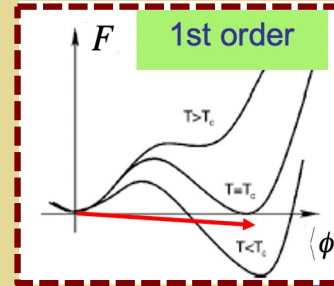
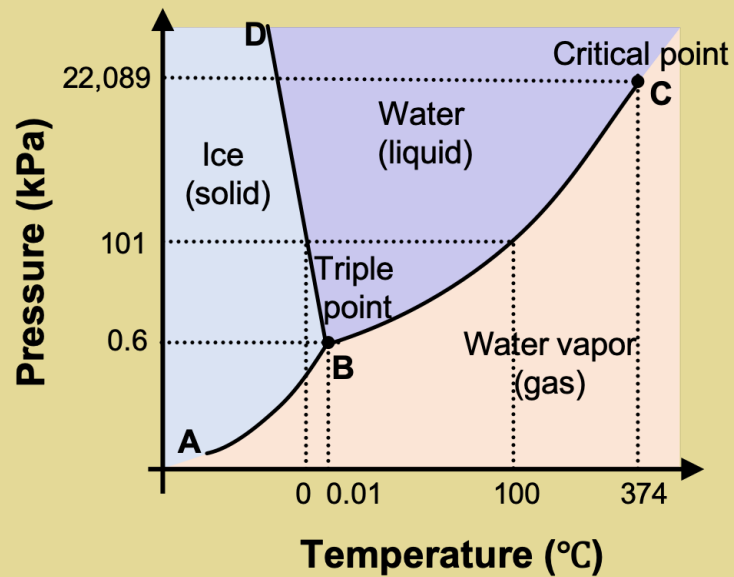
EW Phase Diagram

Was There an Electroweak Phase Transition ?



EW Phase Diagram

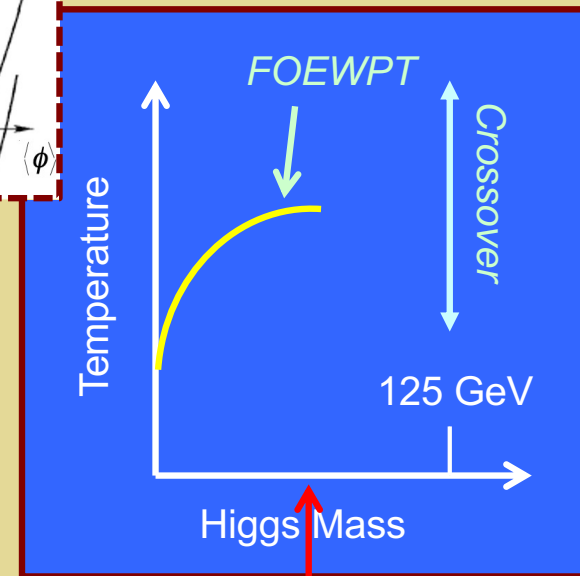
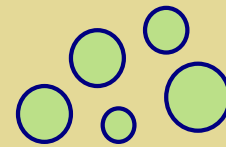
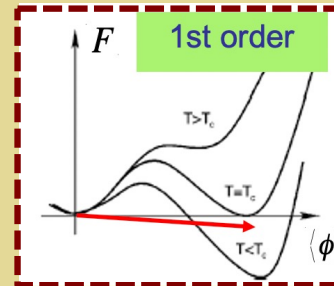
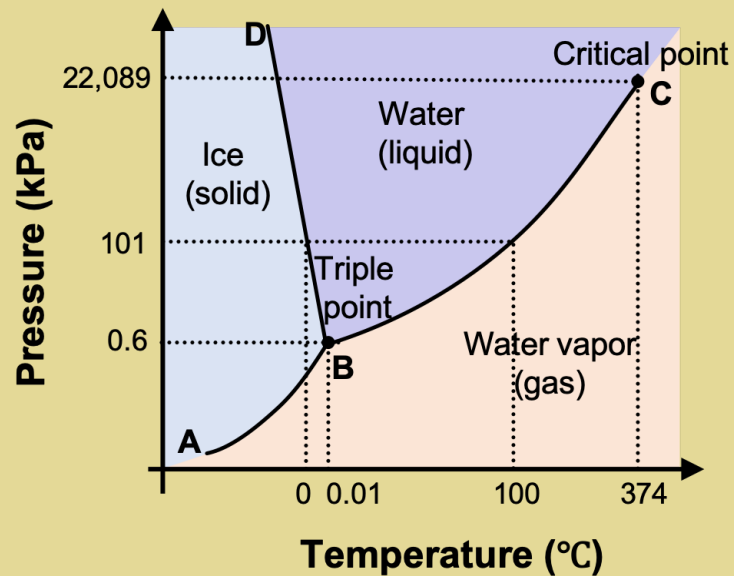
Was There an Electroweak Phase Transition ?



EW Phase Diagram

Lattice	Authors	M_h^C (GeV)
4D Isotropic	[76]	80 ± 7
4D Anisotropic	[74]	72.4 ± 1.7
3D Isotropic	[72]	72.3 ± 0.7
3D Isotropic	[70]	72.4 ± 0.9

Was There an Electroweak Phase Transition ?

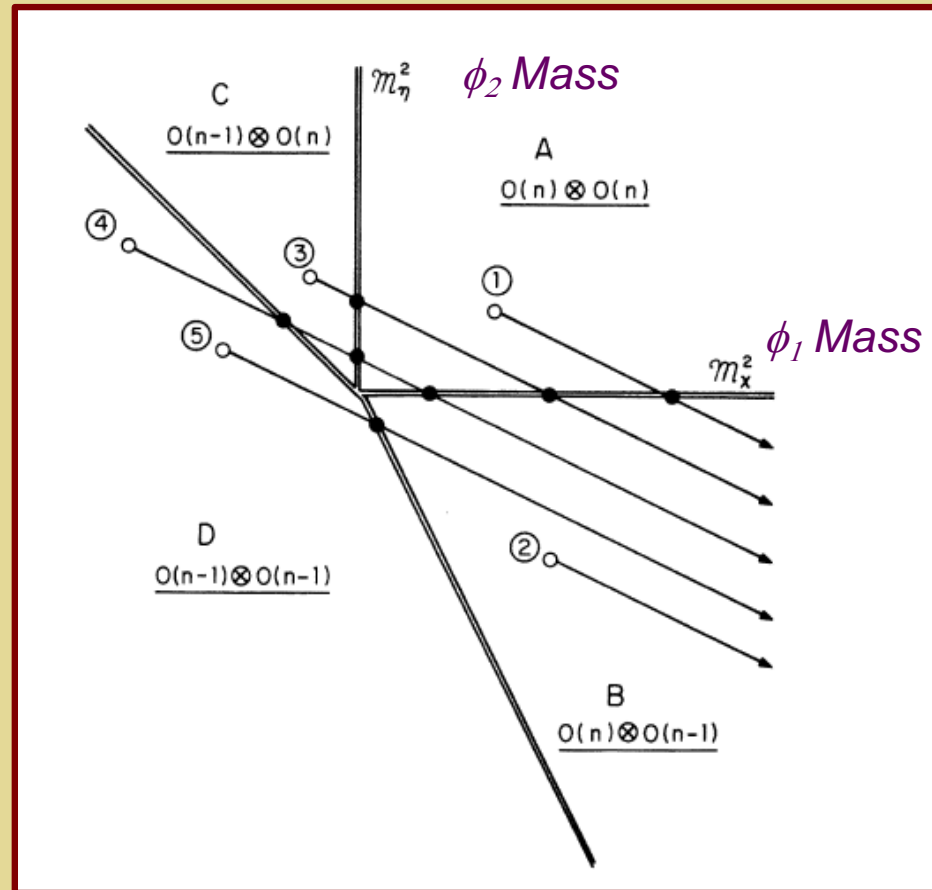


EW Phase Diagram

How does this picture change in presence of new TeV scale physics ? What is the phase diagram ? SFOEWPT ?

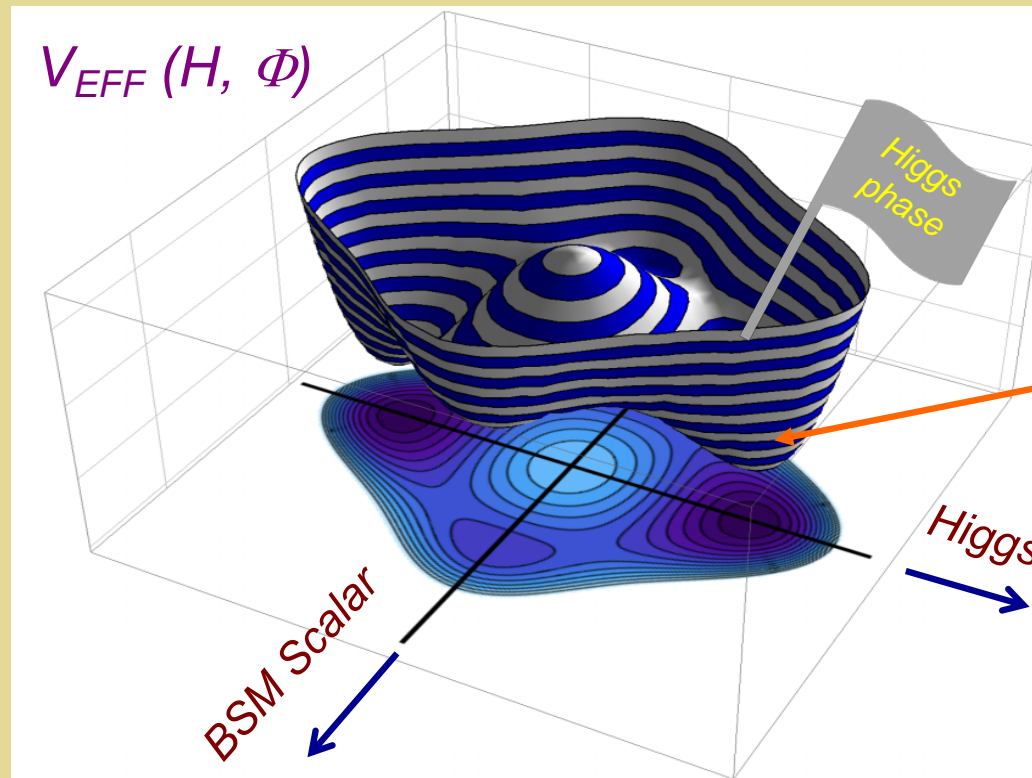
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Patterns of Symmetry Breaking



S. Weinberg, PRD 9 (1974) 3357

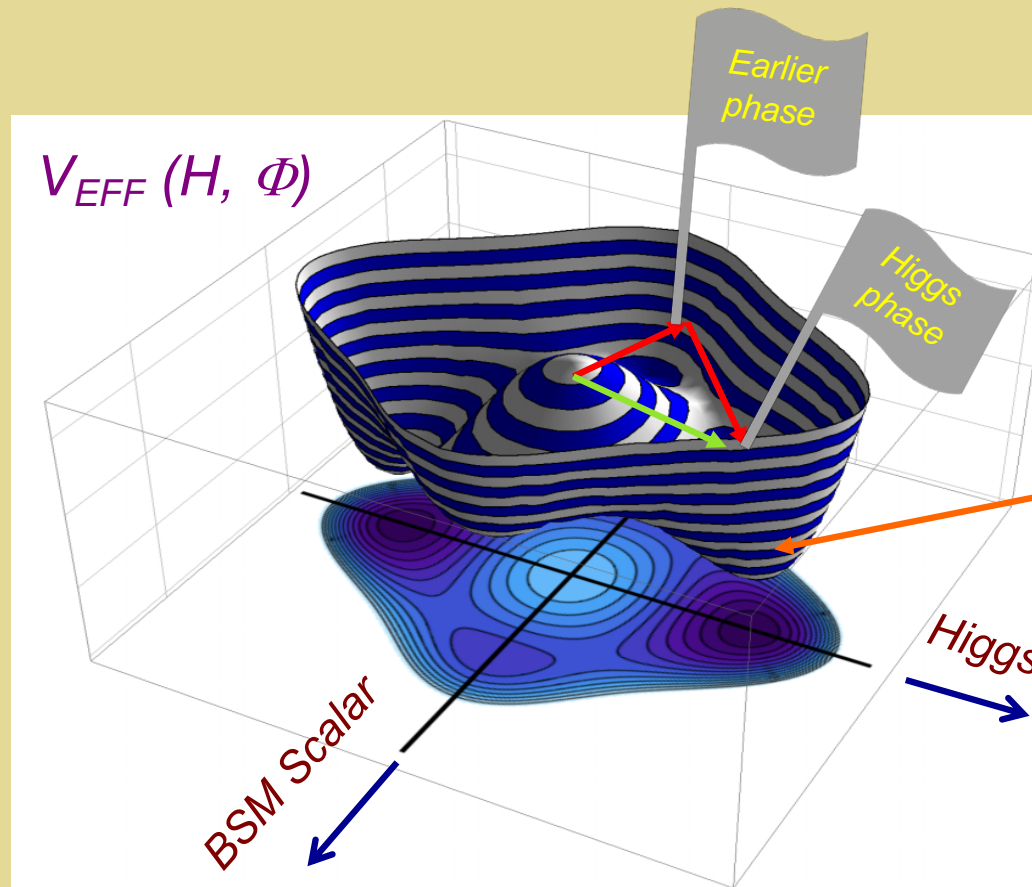
Patterns of Symmetry Breaking



*How did we
end up here ?*

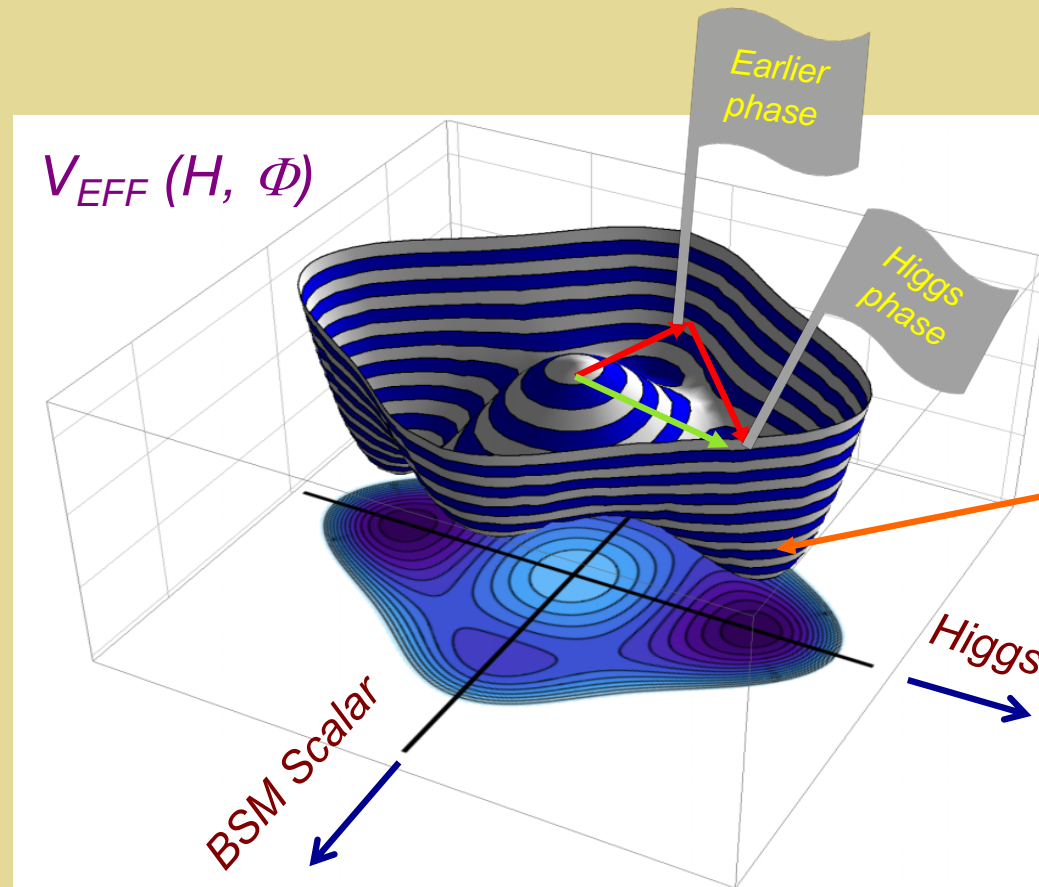
***Extrema can evolve differently as T evolves →
rich possibilities for symmetry breaking***

Patterns of Symmetry Breaking



Extrema can evolve differently as T evolves → rich possibilities for symmetry breaking

What Was the EWSB Thermal History ?



*How did we
end up here ?*

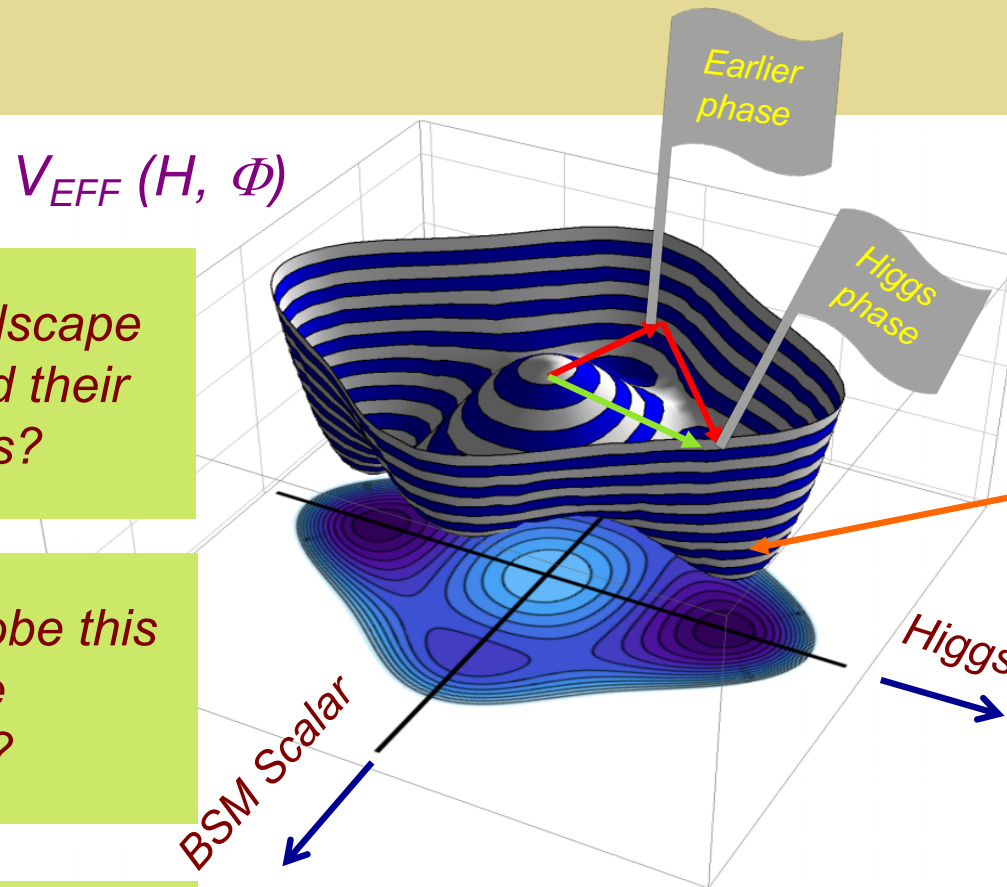
***Extrema can evolve differently as T evolves →
rich possibilities for symmetry breaking***

What Was the *EWSB* Thermal History ?

- What is the landscape of potentials and their thermal histories?

- How can we probe this $T > 0$ landscape experimentally ?

- How reliably can we compute the thermodynamics ?



How did we end up here ?

n evolve differently as T evolves $\rightarrow$
ilities for symmetry breaking

$T_{EW} \rightarrow$ Scale for Colliders & GW probes

High-T SM Effective Potential

$$V(h, T)_{\text{SM}} = D(T^2 - T_0^2) h^2 + \lambda h^4 + \dots$$

$$T_0 \sim 140 \text{ GeV}$$

$$\equiv T_{EW}$$

$T_{EW} \rightarrow$ Scale for Colliders & GW probes

High- T SM Effective Potential

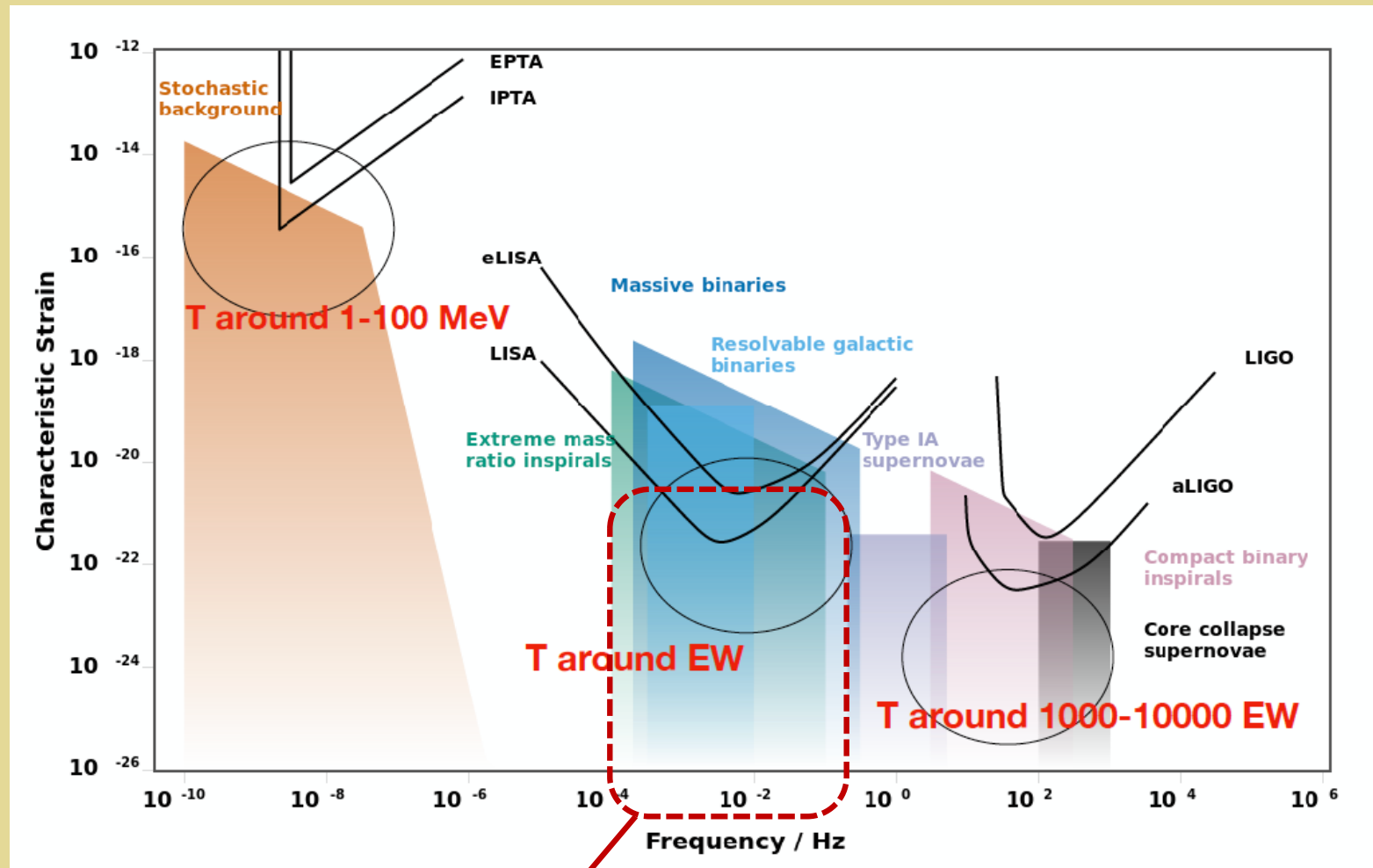
$$V(h, T)_{\text{SM}} = D(T^2 - T_0^2) h^2 + \lambda h^4 + \dots$$

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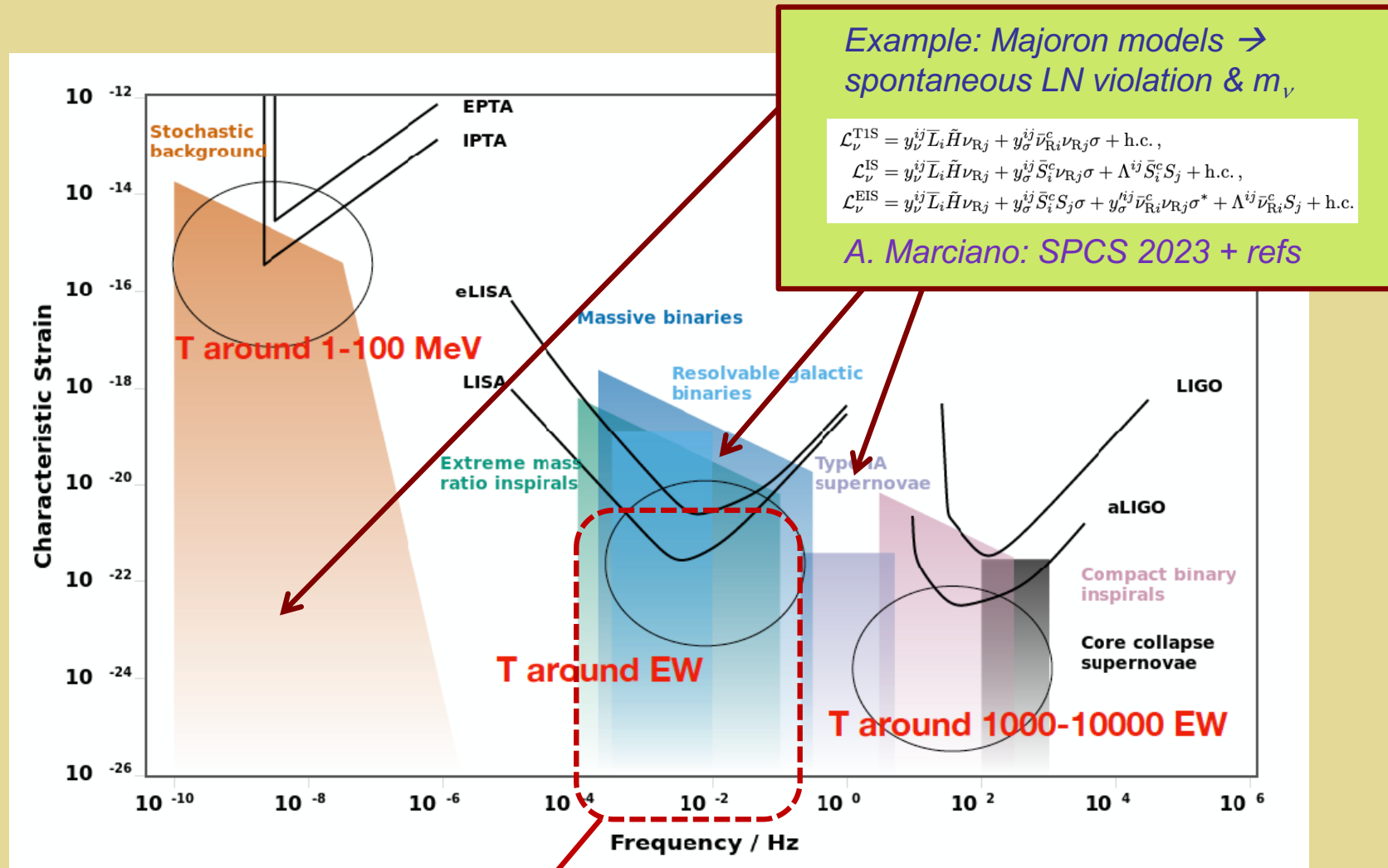
Collider target

GW: Electroweak Phase Transition



EWPT laboratory for GW micro-physics: colliders can probe particle physics responsible for non-astro GW sources → test our framework for GW microphysics at other scales

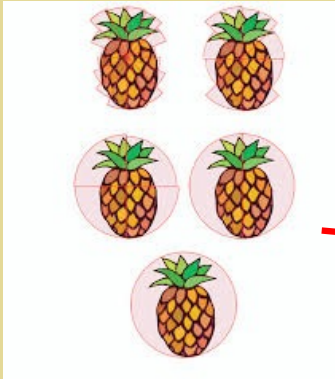
GW: Electroweak Phase Transition



EWPT laboratory for GW micro-physics: colliders can probe particle physics responsible for non-astro GW sources $\rightarrow$ test our framework for GW microphysics at other scales

Was There an EW Phase Transition?

Bubble Collisions

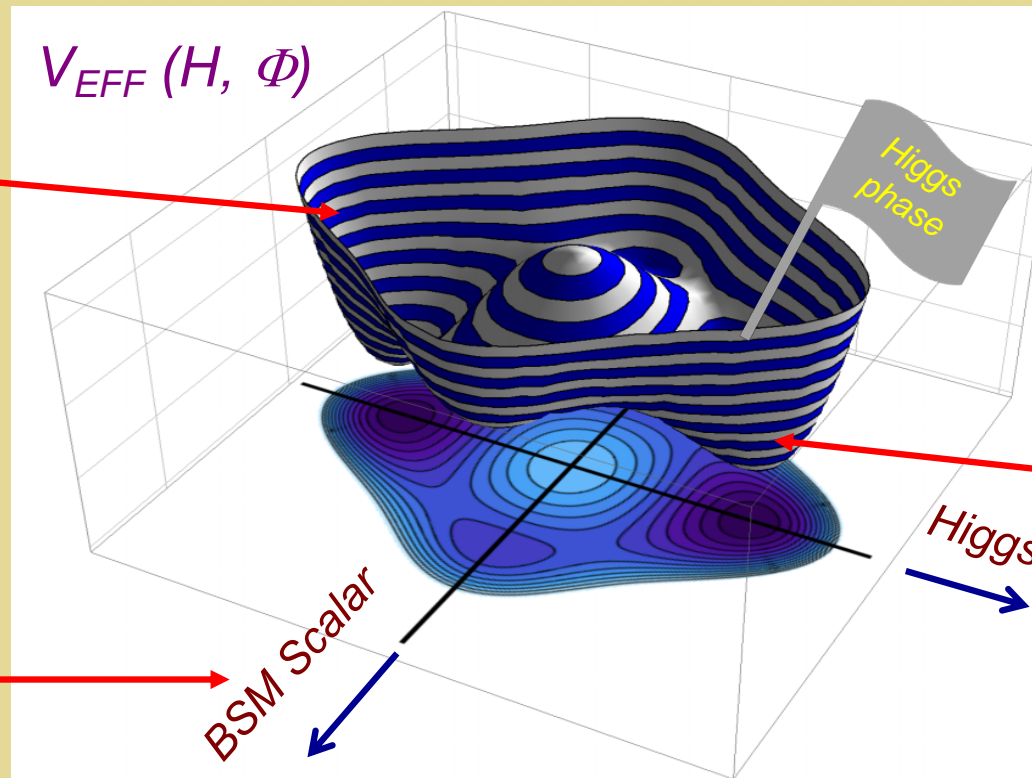


Grav Radiation

Direct Production



BSM Higgs



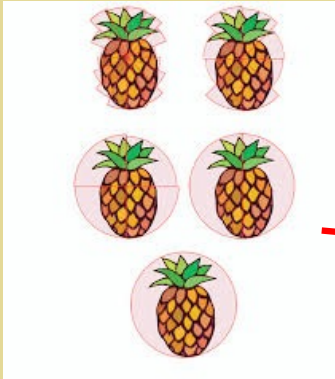
Higgs precision tests



**Extrema can evolve differently as T evolves →
rich possibilities for symmetry breaking**

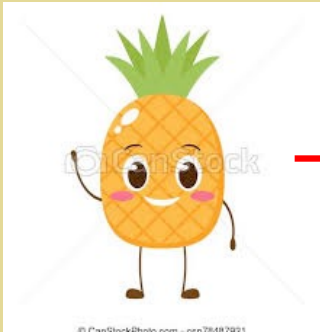
Was There an EW Phase Transition?

Bubble Collisions

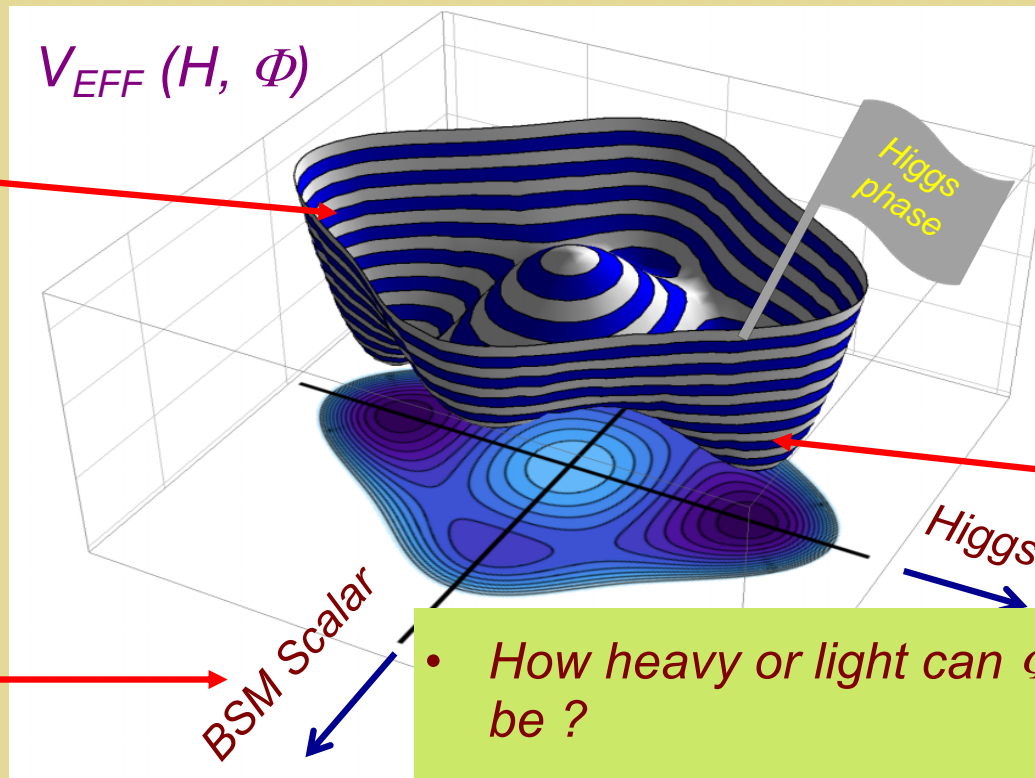


Grav Radiation

Direct Production



BSM Higgs



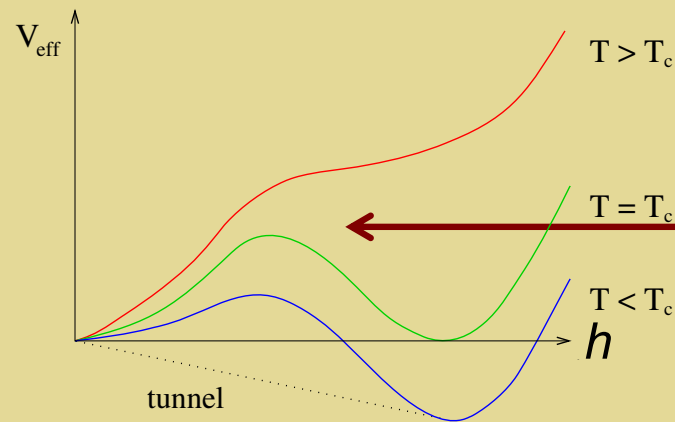
Higgs precision tests



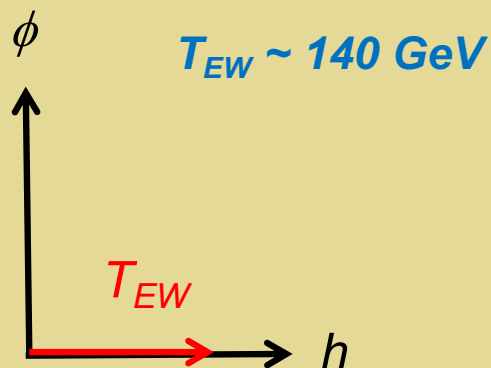
- How heavy or light can Φ be ?
- How coupled to H ?
- Can it be discovered with colliders & GW probes ?

Extrema can evolve
rich possibilities for

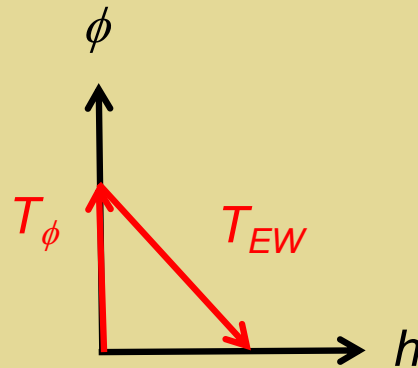
First Order EWPT from BSM Physics



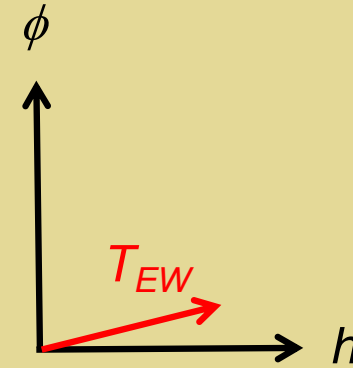
Representative thermal histories $\rightarrow$ barrier for SFOEWPT



$a_2 H^2 \phi^2 : T > 0$
loop effect

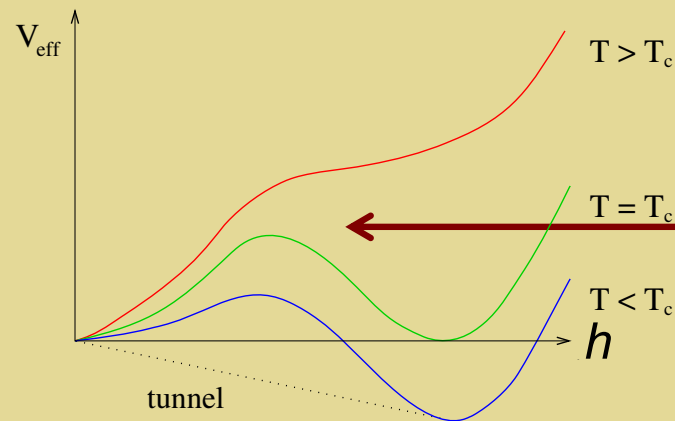


$a_2 H^2 \phi^2 : T = 0$
tree-level effect

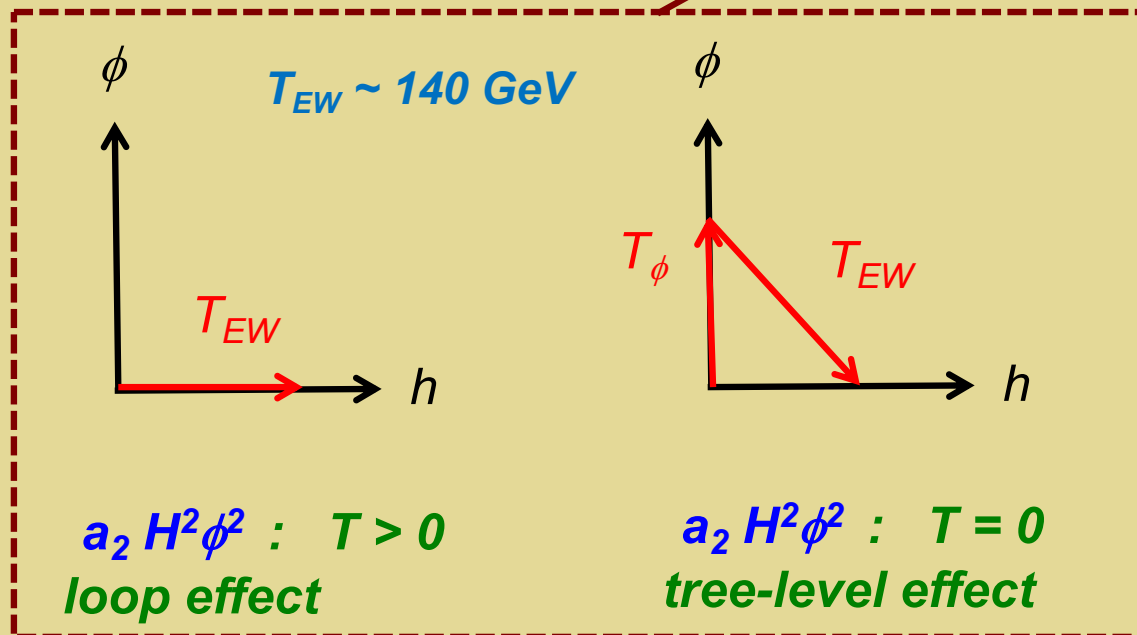


$a_1 H^2 \phi : T = 0$
tree-level effect

First Order EWPT from BSM Physics



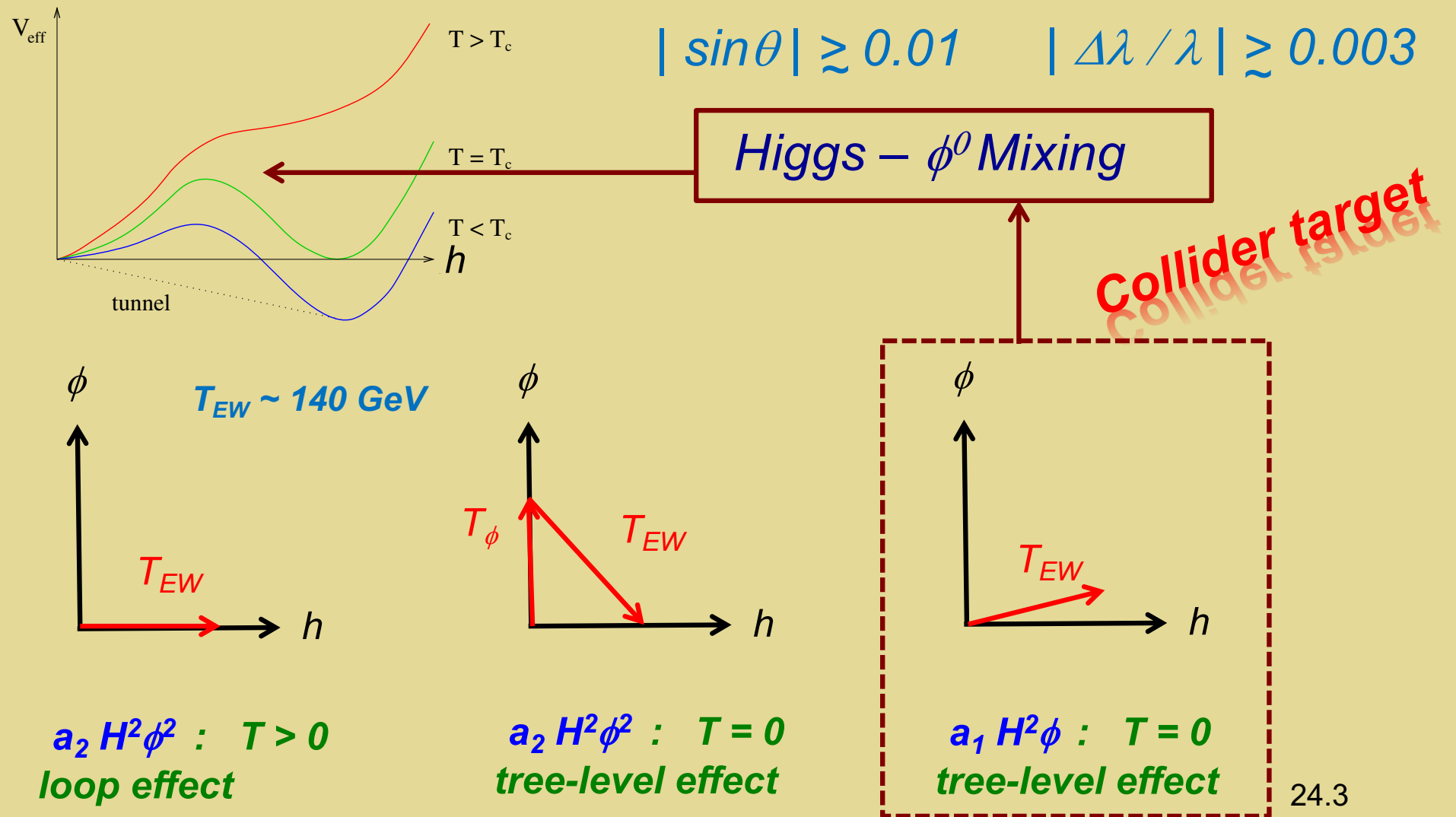
Simple arguments: $T_{EW} +$
first order EWPT $\rightarrow$
 $M_\phi \lesssim 700 \text{ GeV}$



Collider target

$a_1 H^2 \phi : T = 0$
tree-level effect

First Order EWPT from BSM Physics



Experimental Probes: Energy Frontier



ATLAS

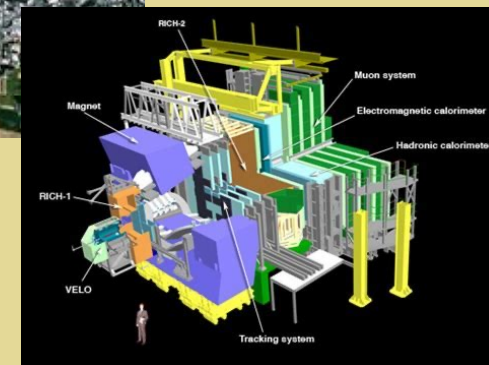
*Large Hadron
Collider (LHC)*



CMS



ALICE



LHCb

Experimental Probes: Energy Frontier

LHC

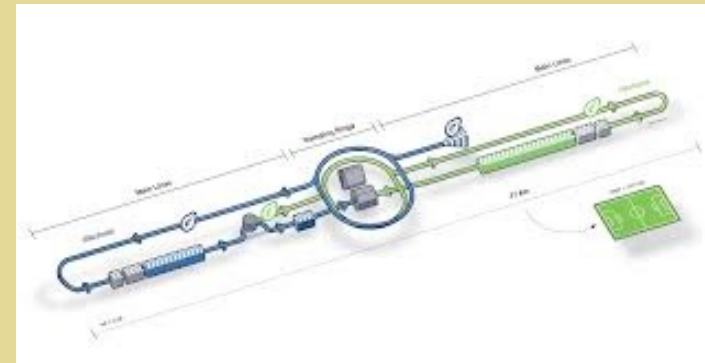


ATLAS

CMS

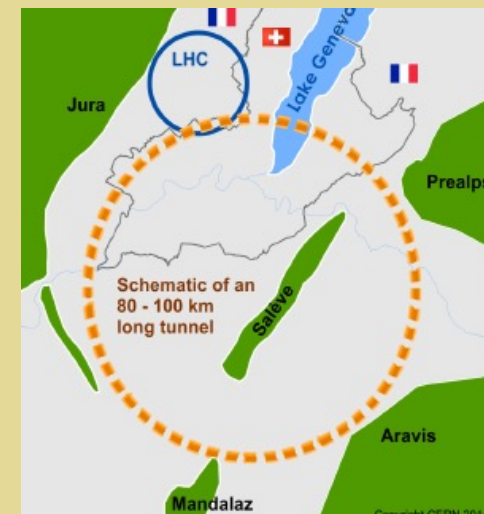
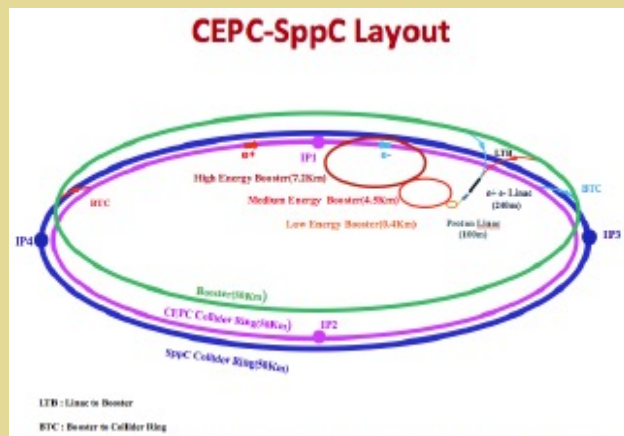


International Linear Collider



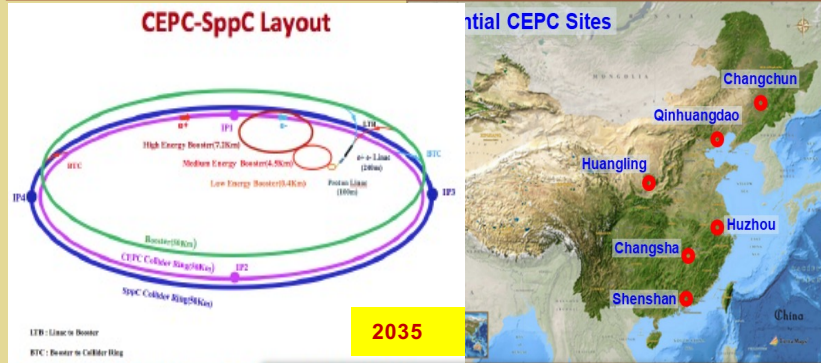
Future Circular e^+e^- & pp

Future Circular e^+e^- & pp



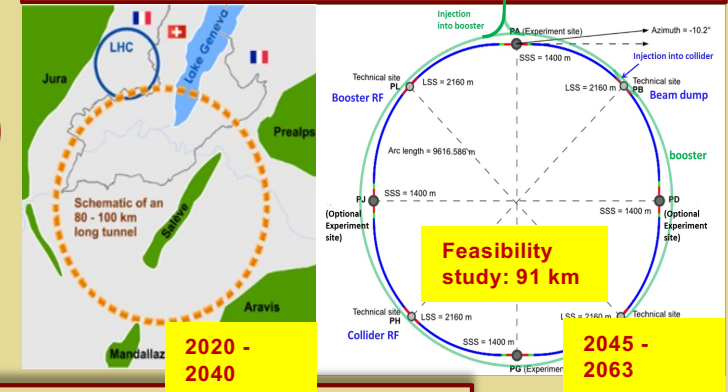
Energy Frontier: e^+e^- “Higgs & Z Factory”

Circular Electron-Positron Collider CEPC (China)

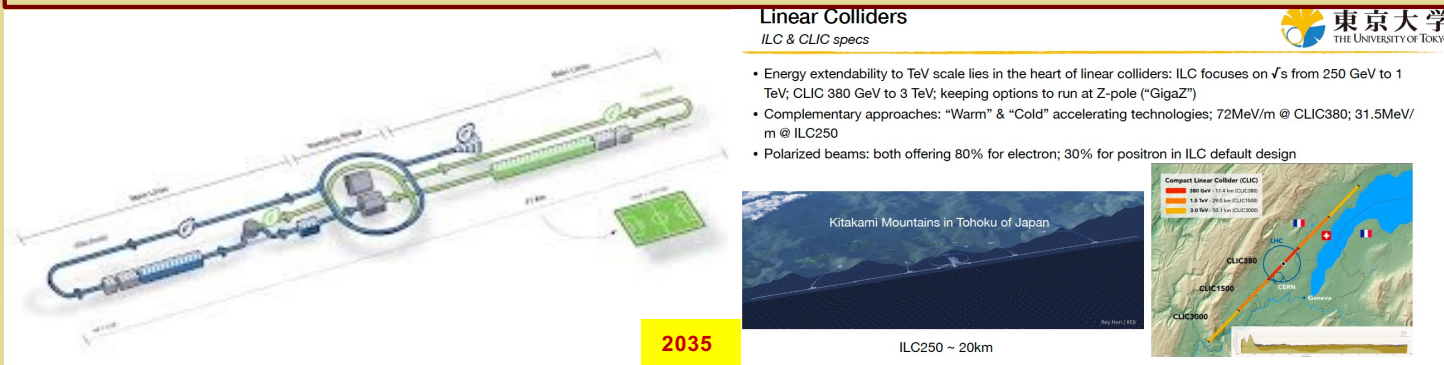


**High Energy +
High Intensity**

Future Circular Collider FCC-ee (Europe)



International Linear Collider & Compact Linear Collider ILC (Japan) & CLIC (Europe)



+ Muon Collider (USA)

First Order EWPT: Collider Probes

Higgs property

hadron

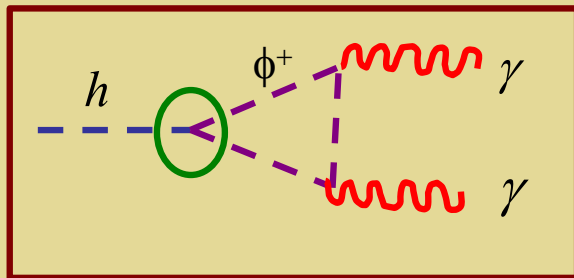
lepton

- $\Gamma(h \rightarrow \gamma\gamma)$
- *Exotic Higgs Decays*
- *Higgs signal strengths*
- *Higgs self-coupling*
- *Resonant di-Higgs production*
- *Heavy Higgs $\rightarrow VV$*
- *Associated production (Zh)*

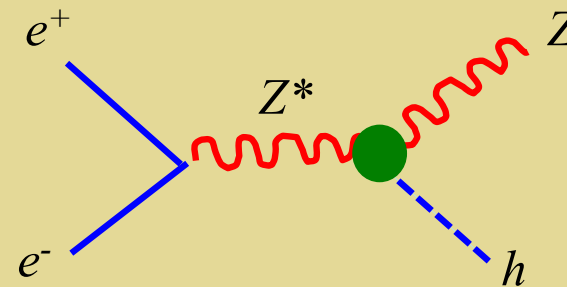


First Order EWPT: Collider Probes

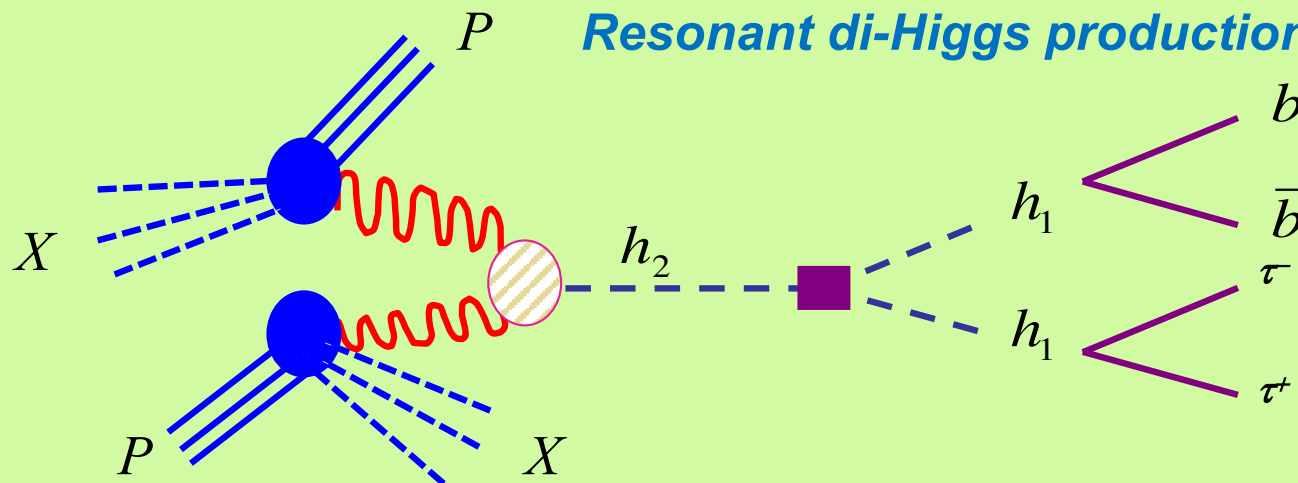
Higgs di-photon decay



Associated production



Resonant di-Higgs production



III. Gravitational Wave-Collider Interplay

BSM EWPT: Inter-frontier Connections

*Robust theory:
EFT + lattice*



**Collider
Signatures**

*Observables:
model specific*

Mapping

Combination

**GW
Signals**

*Bubble
dynamics:
 $\alpha, \beta / H_*, v_w$*

Nucleation @ $T > 0$: Gravitational Waves

Amplitude & frequency: latent heat & intrinsic time scale

Normalized latent heat

$$\Delta Q = \Delta F + T \Delta S$$

$$S = -\partial F / \partial T$$

$$F \approx V$$

$$\Delta Q \approx \Delta V - T \partial \Delta V / \partial T$$

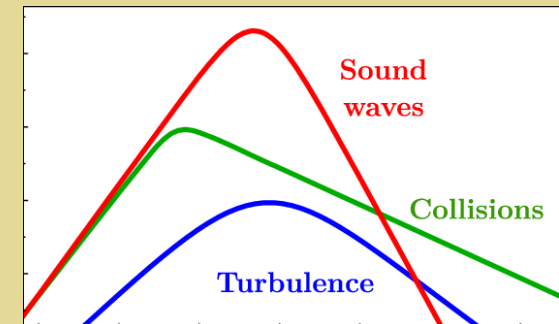
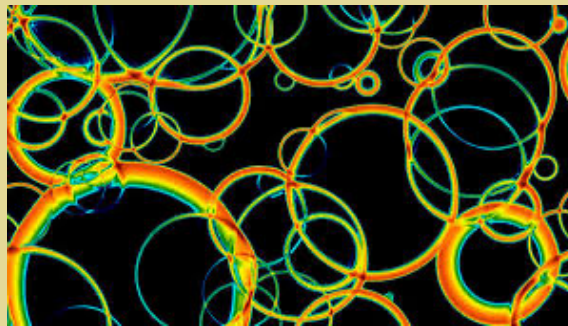
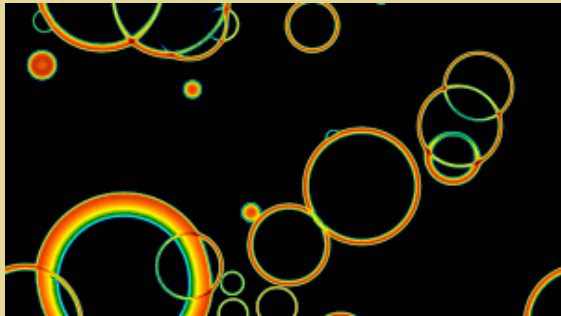
$$\alpha = \frac{30 \Delta q}{\pi^2 g_* T^4}$$

Time scale

$$\frac{\beta}{H_*} = T \frac{d}{dT} \frac{S_3}{T}$$

Gravitational Radiation

$$\Omega_{\text{GW}} = \Omega_{\text{coll}} + \Omega_{\text{sw}} + \Omega_{\text{turb}}$$



$\alpha, \beta / H_*, v_w, \dots \rightarrow$ inputs for GW sources: **collisions, sound waves, hydrodynamic turbulence**

Gravitational Radiation

$$\frac{d \log \Omega_{\text{GW}}}{d \log k} = K^2 (k L_f)^3 \int \frac{dt_1}{t_1} \int \frac{dt_2}{t_2} \cos [k(t_1 - t_2)] \tilde{\Pi}(k, t_1, t_2)$$

$$K = \frac{\kappa \alpha_n}{1 + \alpha_n}$$

Latent heat $\rightarrow$ KE fraction

κ : “efficiency factor”

Mean bubble separation

$$L_f \sim R_*$$

$$R_* = \frac{(8\pi)^{1/3}}{\beta} \text{Max}(v_w, c_s)$$

Wall velocity

Sound speed

Gravitational Radiation

Spectrum

$$h^2 \Omega_{\text{env}}(f) = 1.67 \times 10^{-5} \left(\frac{H_*}{\beta} \right)^2 \left(\frac{\kappa \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} \left(\frac{0.11 v_w^3}{0.42 + v_w^2} \right) S_{\text{env}}(f)$$

Fluid

$$h^2 \Omega_{\text{sw}}(f) = 2.65 \times 10^{-6} \left(\frac{H_*}{\beta} \right) \left(\frac{\kappa_v \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} v_w S_{\text{sw}}(f)$$

Sound

$$h^2 \Omega_{\text{turb}}(f) = 3.35 \times 10^{-4} \left(\frac{H_*}{\beta} \right) \left(\frac{\kappa_{\text{turb}} \alpha}{1 + \alpha} \right)^{\frac{3}{2}} \left(\frac{100}{g_*} \right)^{1/3} v_w S_{\text{turb}}(f)$$

Turbulence

Peak frequency

$$f_{\text{env}} = 16.5 \times 10^{-3} \text{ mHz} \left(\frac{f_*}{\beta} \right) \left(\frac{\beta}{H_*} \right) \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}}$$

$$f_{\text{sw}} = 1.9 \times 10^{-2} \text{ mHz} \frac{1}{v_w} \left(\frac{\beta}{H_*} \right) \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}}$$

$$f_{\text{turb}} = 2.7 \times 10^{-2} \text{ mHz} \frac{1}{v_w} \left(\frac{\beta}{H_*} \right) \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}}$$

Caprini et al, 1512.06239 [astro-ph]

Gravitational Radiation

Spectrum

Thermal QFT

$$h^2 \Omega_{\text{env}}(f) = 1.67 \times 10^{-5} \left(\frac{H_*}{\beta} \right)^2 \left(\frac{\kappa \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} \left(\frac{0.11 v_w^3}{0.42 + v_w^2} \right) S_{\text{env}}(f)$$

Fluid

$$h^2 \Omega_{\text{sw}}(f) = 2.65 \times 10^{-6} \left(\frac{H_*}{\beta} \right) \left(\frac{\kappa_v \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} v_w S_{\text{sw}}(f)$$

Sound

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Caprini et al, 1512.06239 [astro-ph]

Gravitational Radiation

Spectrum

Thermal QFT

Transport Theory

$$h^2 \Omega_{\text{env}}(f) = 1.67 \times 10^{-5} \left(\frac{H_*}{\beta} \right)^2 \left(\frac{\kappa \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} \left(\frac{0.1 v_w^3}{0.42 + v_w^2} \right) S_{\text{env}}(f)$$

Fluid

$$h^2 \Omega_{\text{sw}}(f) = 2.65 \times 10^{-6} \left(\frac{H_*}{\beta} \right) \left(\frac{\kappa_v \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} v_w S_{\text{sw}}(f)$$

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Gravitational Radiation

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Fluid

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Sound

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Turbulence

Peak frequency

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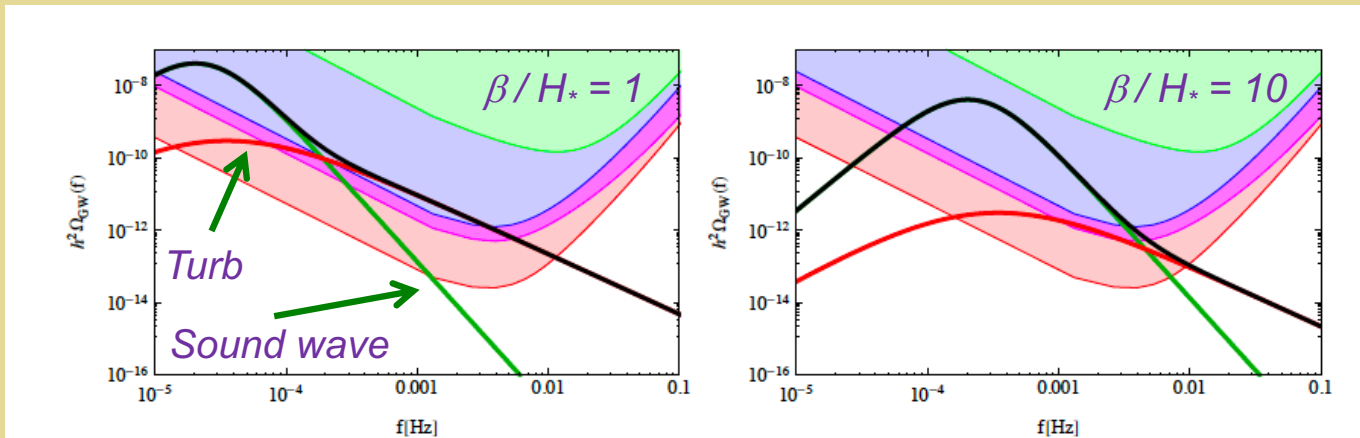
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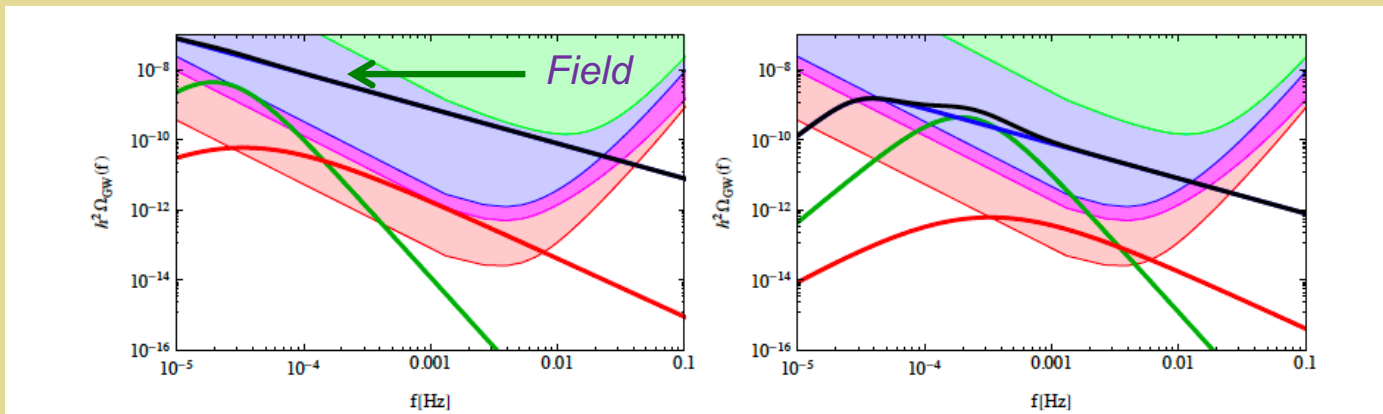
Caprini et al, 1512.06239 [astro-ph]

Gravitational Radiation

Non-run away bubbles

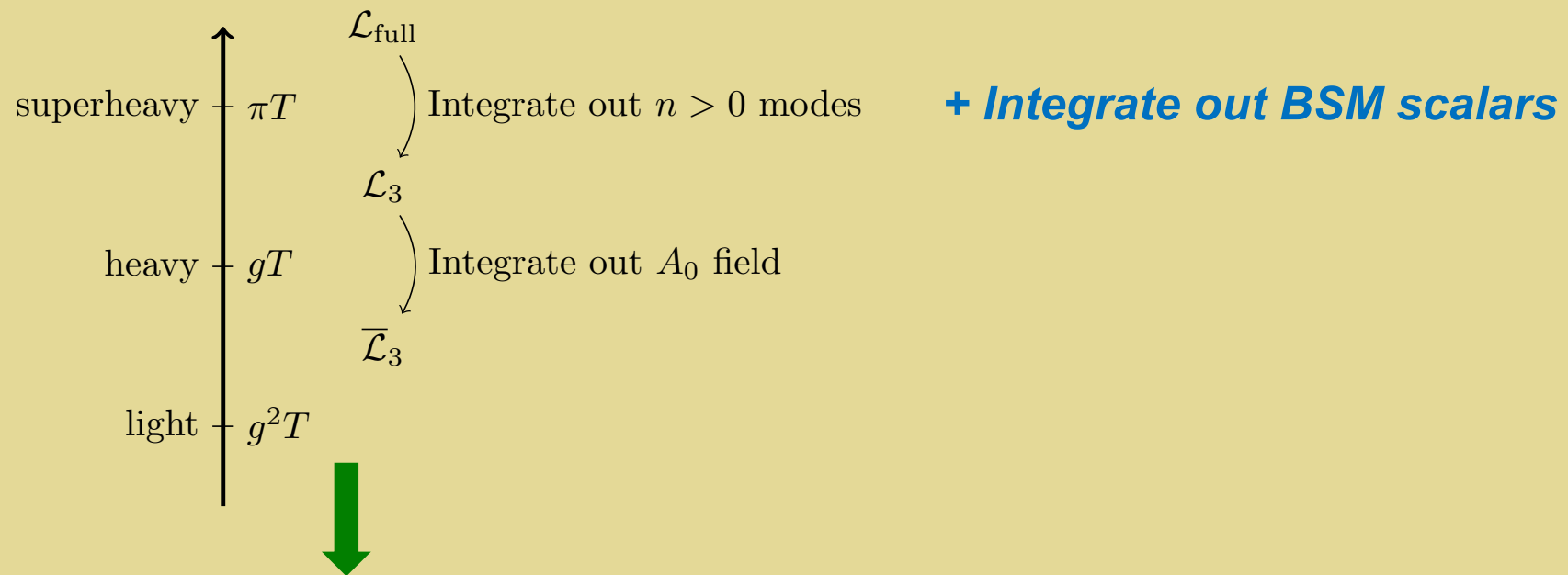


Run away bubbles



Particle Physics Ingredients @ $T > 0$

Framework: 3-D high- T effective theory



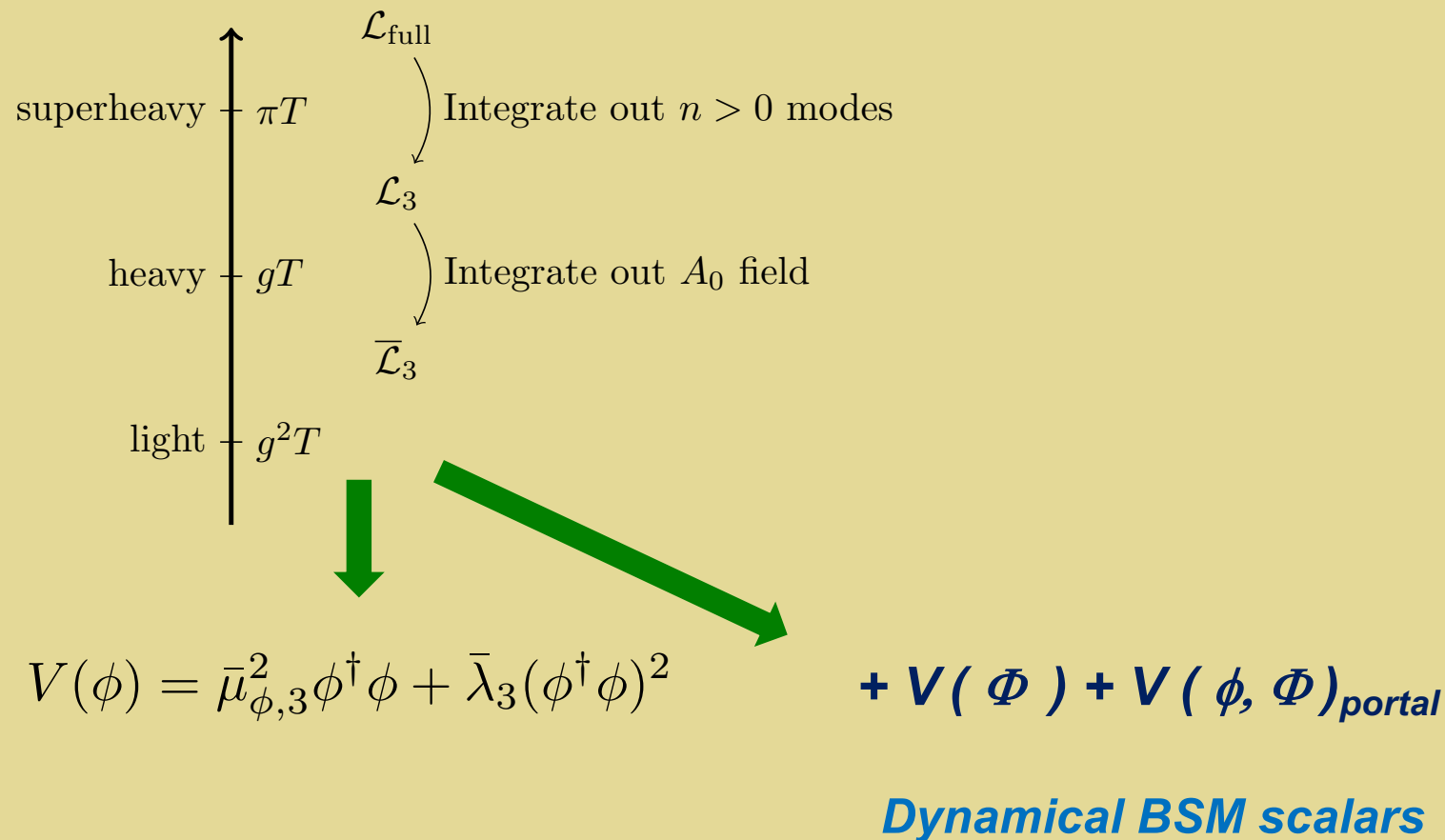
$$V(\phi) = \bar{\mu}_{\phi,3}^2 \phi^\dagger \phi + \bar{\lambda}_3 (\phi^\dagger \phi)^2$$

Non-dynamical BSM scalars

"Standard Model-like"

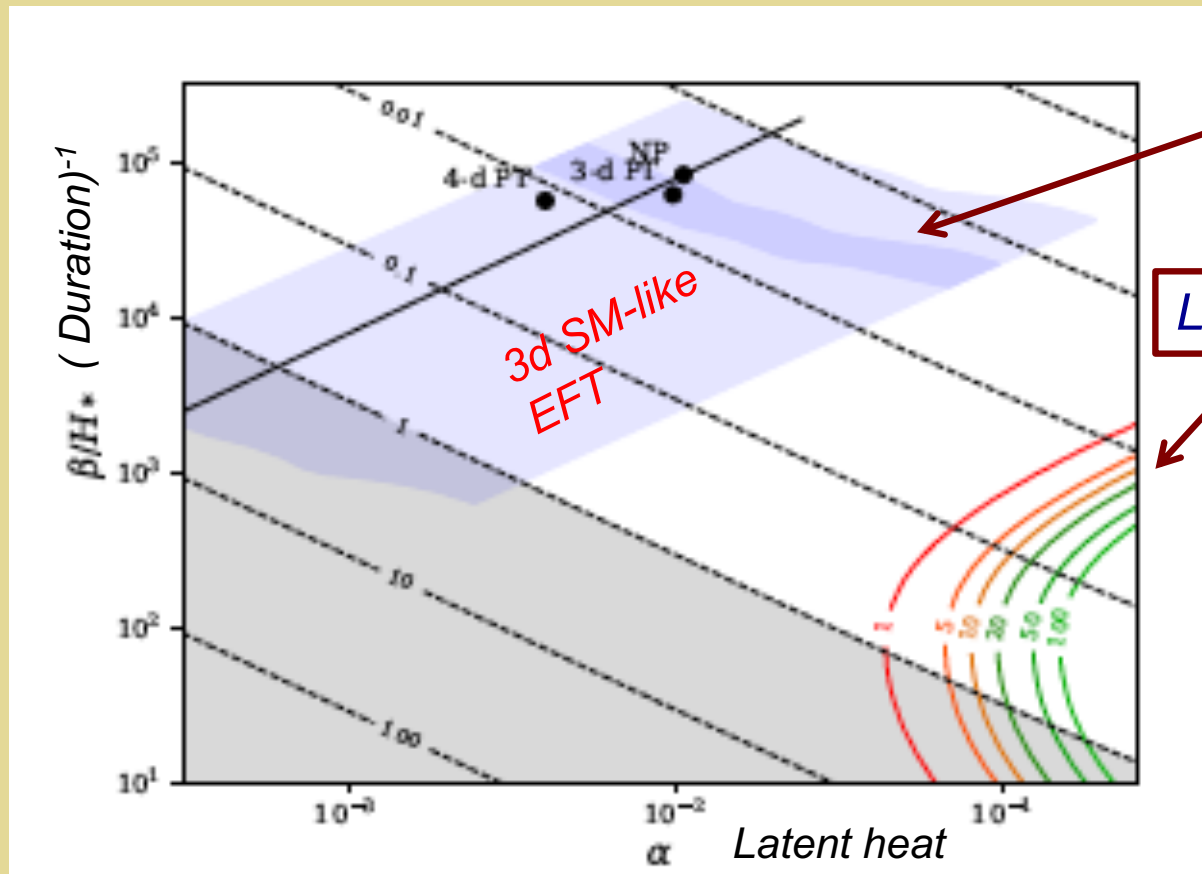
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BSM Scalar: EWPT & GW

High- T dimensional reduction: DR 3d EFT

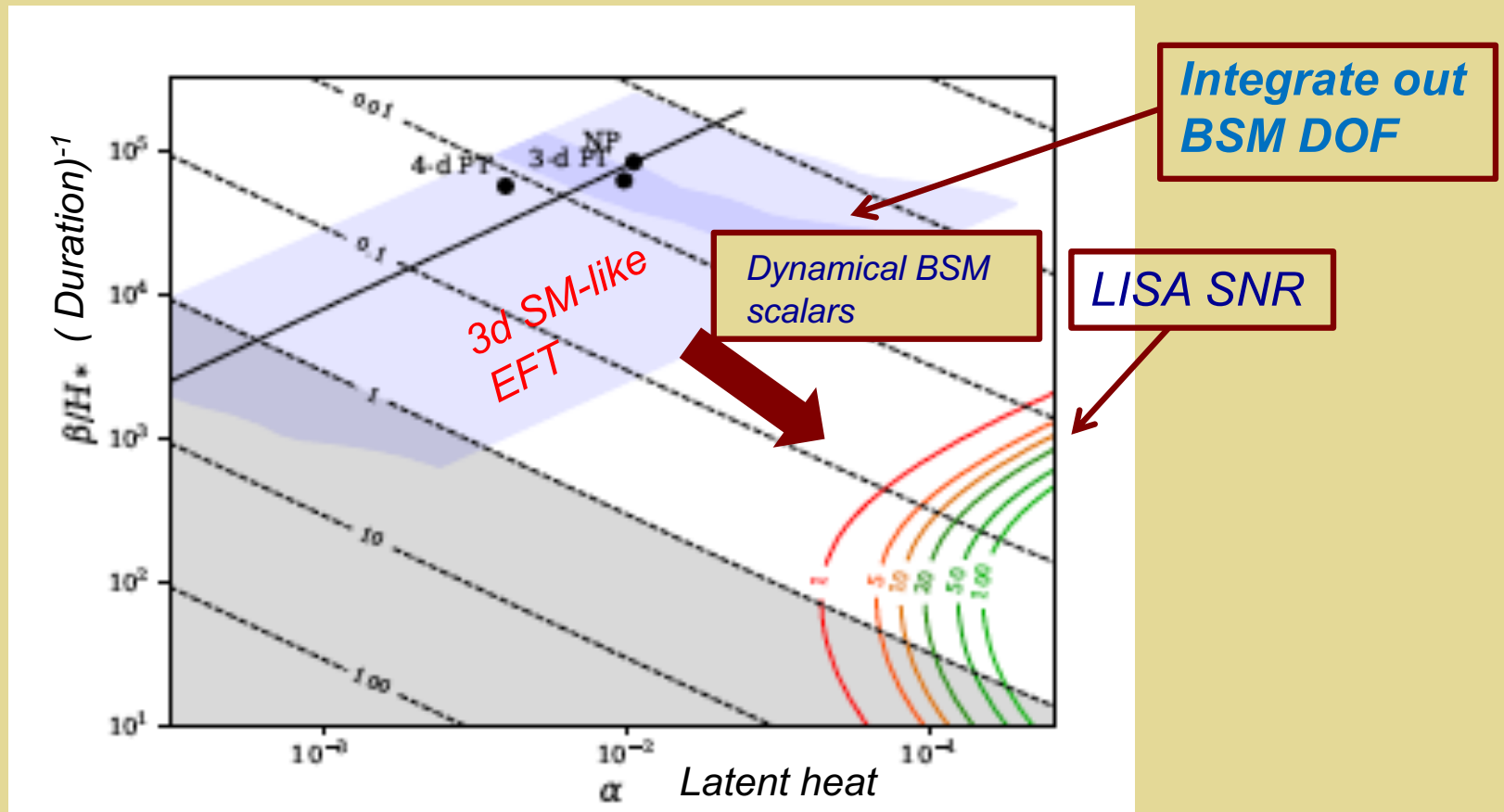


Integrate out
BSM DOF

LISA SNR

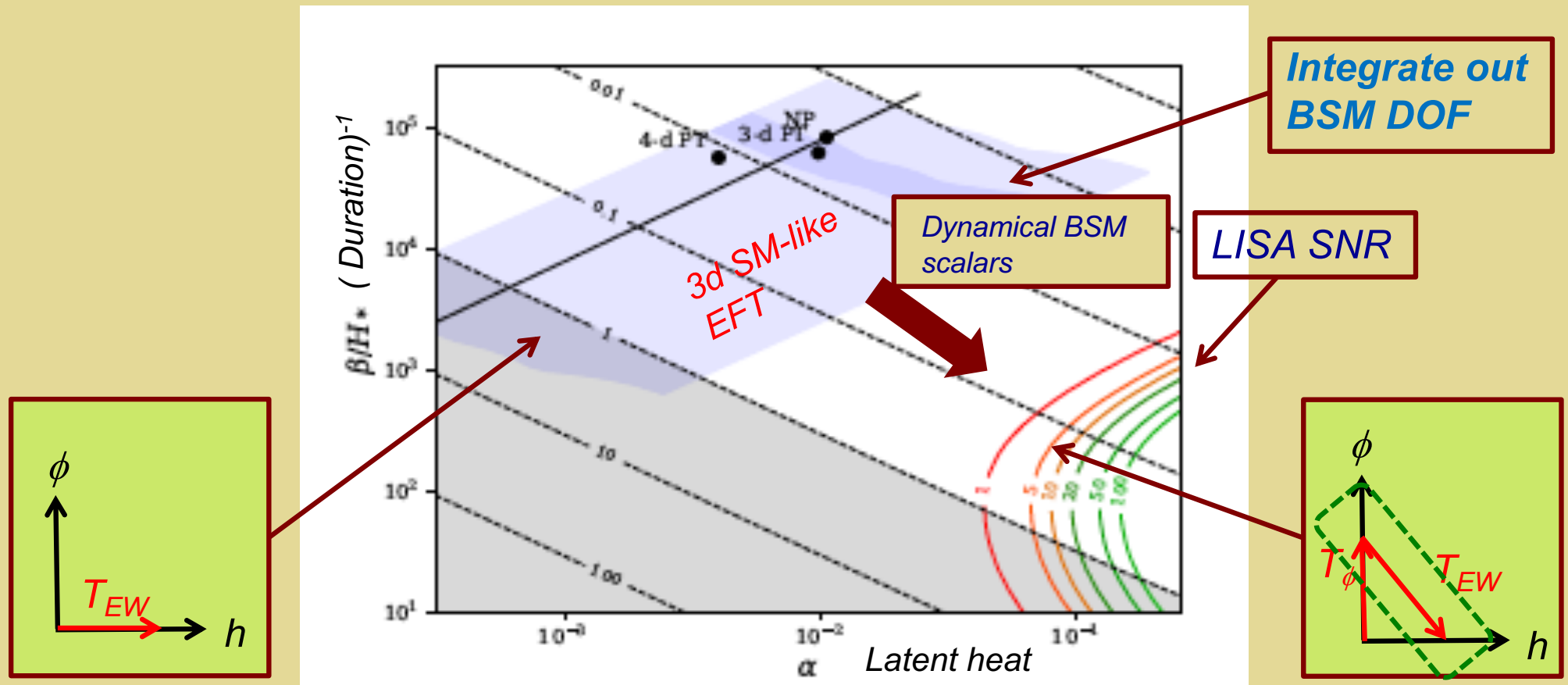
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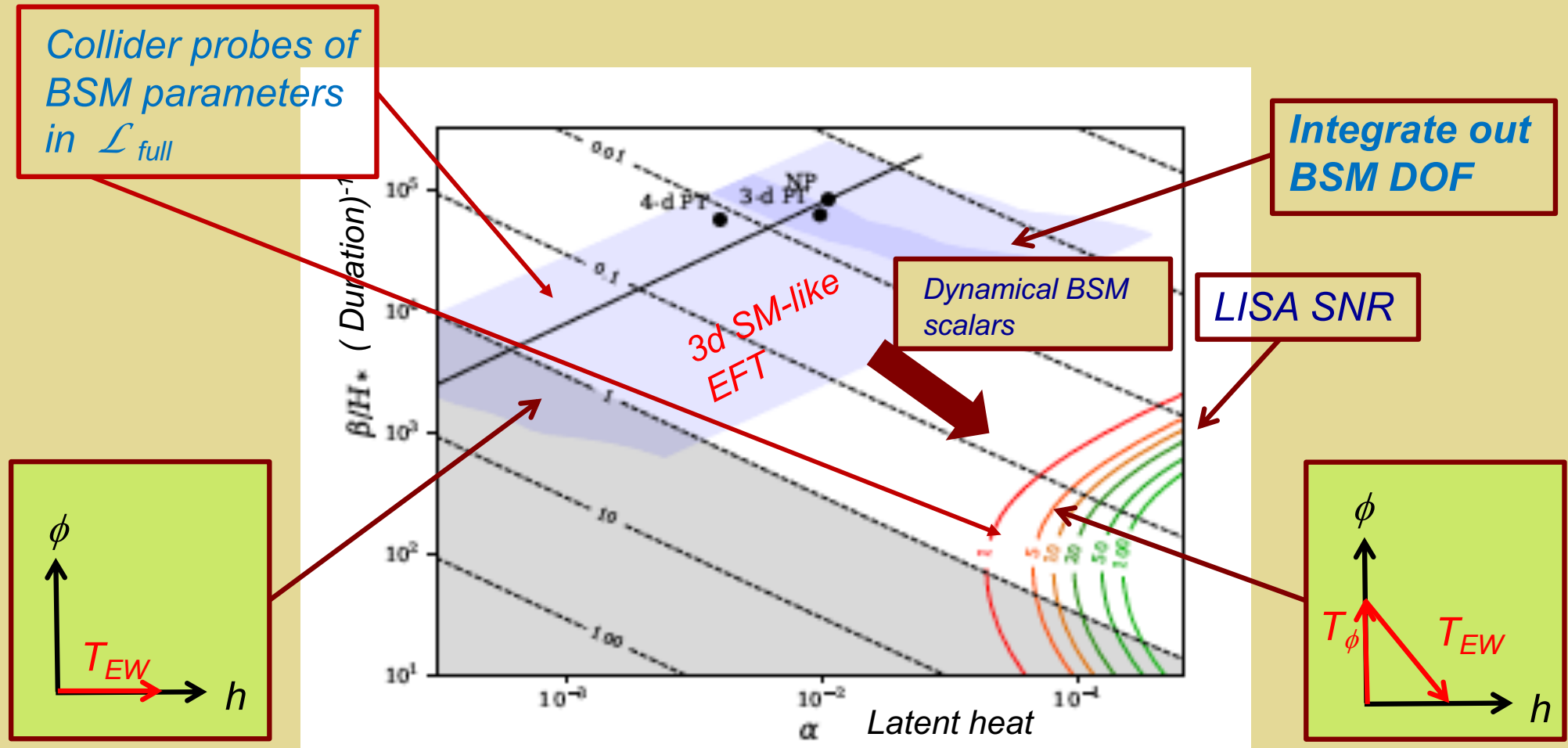


BSM Scalar: EWPT & GW

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BSM Scalar: EWPT & GW



IV. Quantum Field Theory Challenges

Was There and Electroweak Phase Transition

- *Higgs discovery → What was the thermal history of EWSB ?*
 - *Baryogenesis → Was the matter-antimatter asymmetry generated in conjunction with EWSB (EW baryogenesis) ?*
 - *Gravitational waves → If a signal observed in next generation probes, could a cosmological phase transition be responsible ?*
- *Laboratory for testing phase transition physics
→ How reliable is the theory ?*

Tunneling @ $T > 0$: Gravitational Waves

Amplitude & frequency: latent heat & intrinsic time scale

Normalized latent heat

$$\Delta Q = \Delta F + T \Delta S$$

$$S = -\partial F / \partial T$$

$$F \approx V$$

Time scale

$$\frac{\beta}{H_*} = T \frac{d}{dT} \frac{S_3}{T}$$

$$\Delta Q \approx \Delta V - T \partial \Delta V / \partial T$$

$$\alpha = \frac{30 \Delta q}{\pi^2 g_* T^4}$$

How Reliable?
How Believable?

Theoretical Robustness

- ***IR Problem***
- ***Nucleation @ finite T : gauge invariance***
- ***Wall velocity***

Theoretical Robustness

- *IR Problem*
- *Nucleation @ finite T : gauge invariance*
- *Wall velocity*

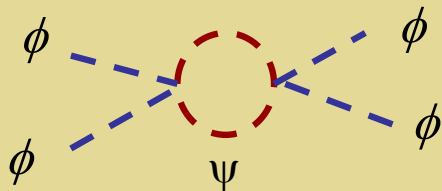
EWPT & Perturbation Theory: IR Problem

Bosonic loop at $T>0$

$$I(T) = g^2 \int \frac{d^3p}{(2\pi)^3} f_B(E, T) \frac{1}{(p^2 + m^2)^n}$$

Bose dist fn





EWPT & Perturbation Theory: IR Problem

Bosonic loop at $T>0$

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Small p regime

$$f_B(E, T) \longrightarrow \frac{T}{m}$$

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Small p regime

$$f_B(E, T) \longrightarrow \frac{T}{m}$$

Effective expansion parameter

Field-dependent thermal mass

$$m^2(\varphi, T) \sim C_1 g^2 \varphi^2 + C_2 g^2 T^2 \equiv m_T^2(\varphi)$$

- Near phase transition: $\varphi \sim 0$
- $m_T(\varphi) < g T$

EWPT & Perturbation Theory

Expansion parameter

$$g_{\text{eff}} \equiv \frac{g^2 T}{\pi m_T(\varphi)}$$



*Infrared sensitive
near phase trans*

SM lattice studies: $g_{\text{eff}} \sim 0.8$ in vicinity of EWPT for $m_H \sim 70 \text{ GeV}$ *

** Kajantie et al, NPB 466 (1996) 189; hep/lat 9510020 [see sec 10.1]*

Challenges for Theory

Perturbation theory

- *I.R. problem: poor convergence*
- *Thermal resummations*
- *Gauge Invariance
(radiative barriers)*
- *RG invariance at $T>0$*

Non-perturbative (I.R.)

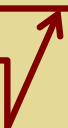
- *Computationally and labor intensive*

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BSM proposals



Non-perturbative (I.R.)

- *Computationally and labor intensive*

Simplest Model

SM Higgs + BSM Scalar

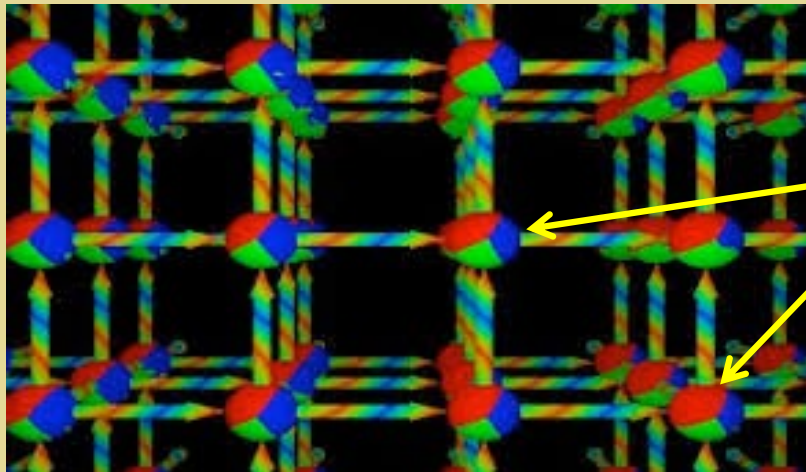


H



ϕ

“Non-perturbative” (lattice) methods



- ***Discretize space***
- ***H, ϕ take on different values at each site***
- ***Vary field values using Monte Carlo methods***
- ***Parameters depend on T***

Theory Meets Phenomenology

A. Non-perturbative

- *Most reliable determination of character of EWPT & dependence on parameters*
- *Broad survey of scenarios & parameter space not viable*

B. Perturbative

- *Most feasible approach to survey broad ranges of models, analyze parameter space, & predict experimental signatures*
- *Quantitative reliability needs to be verified*

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V. Model Illustrations & Results

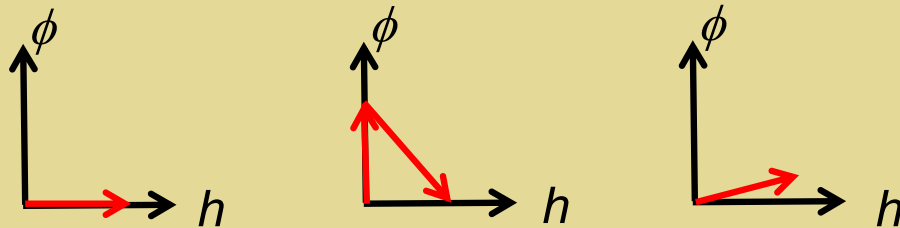
Theory-Pheno Interface



Simple Higgs portal models:

- *Real gauge singlet (SM + 1)*
- *Real EW triplet (SM + 3)*

$$V \subset a_1 H^2 \phi + a_2 H^2 \phi^2$$



Theory-Pheno Interface

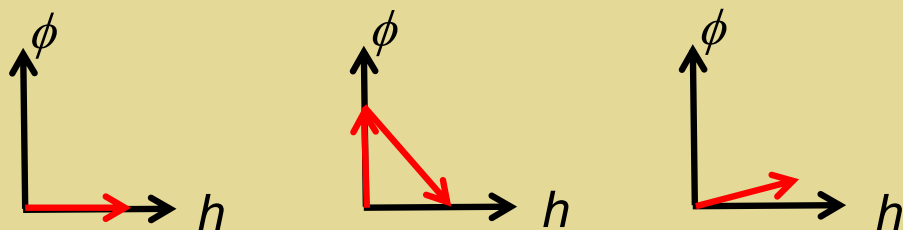


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$$\phi = \frac{1}{2} \begin{pmatrix} \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^0 \end{pmatrix}$$



Theory-Pheno Interface



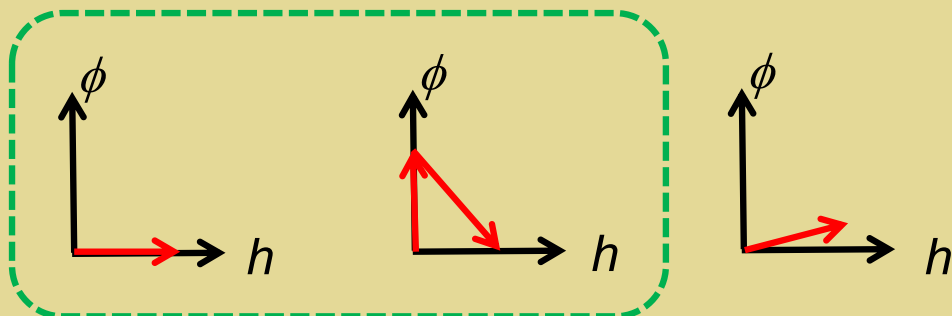
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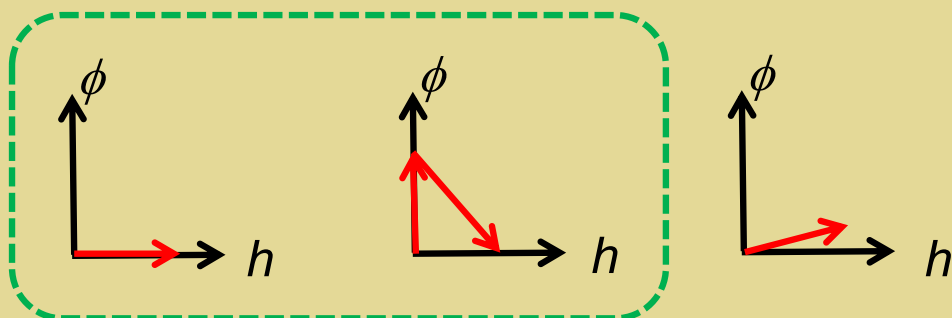


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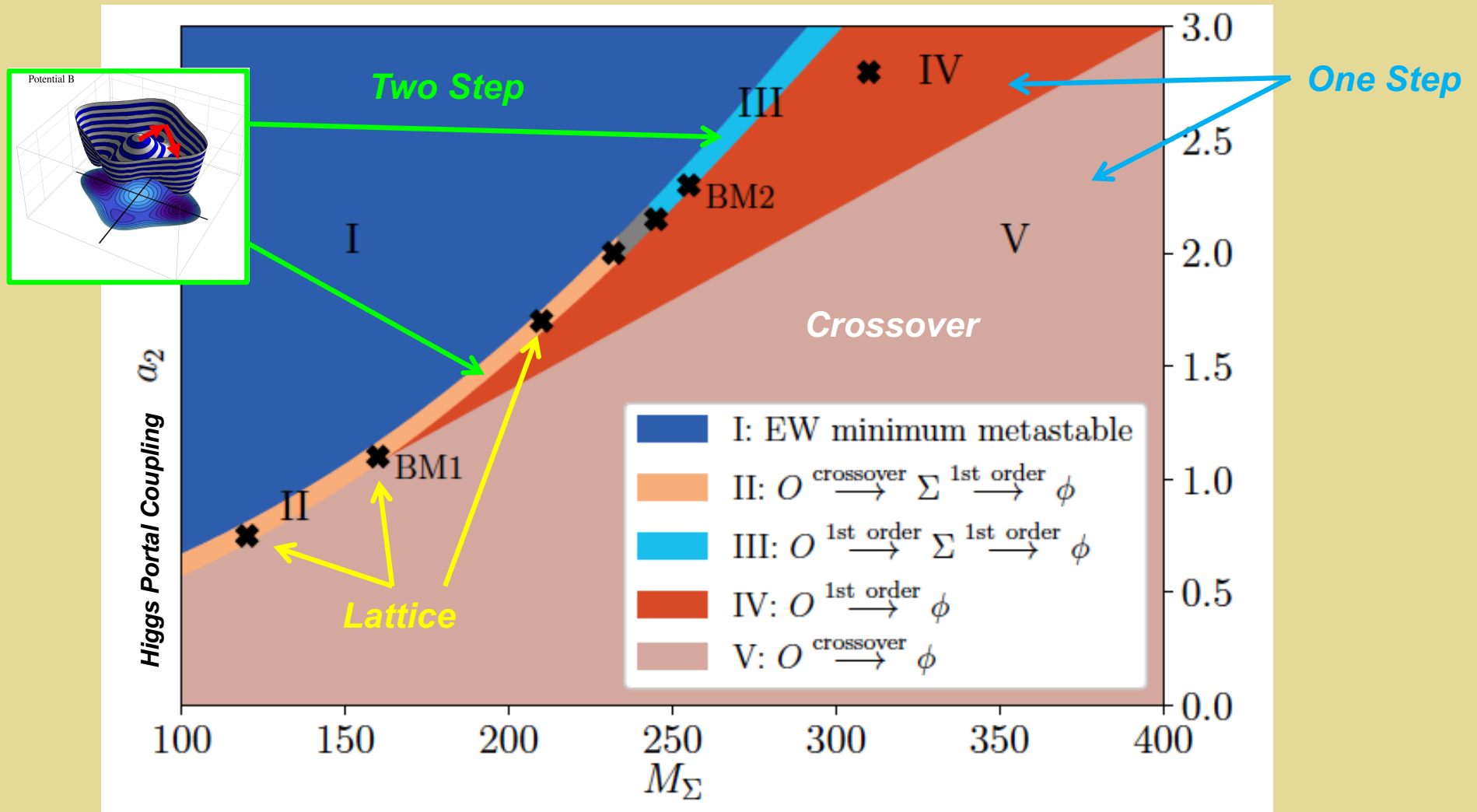
$$V \subset a_1 H^2 \phi + a_2 H^2 \phi^2$$



Phenomenology

- Gravitational waves
- Collider: $h \rightarrow \gamma\gamma$, dis charged track, NLO $e^+e^- \rightarrow Zh...$

Real Triplet & EWPT: Novel EWSB



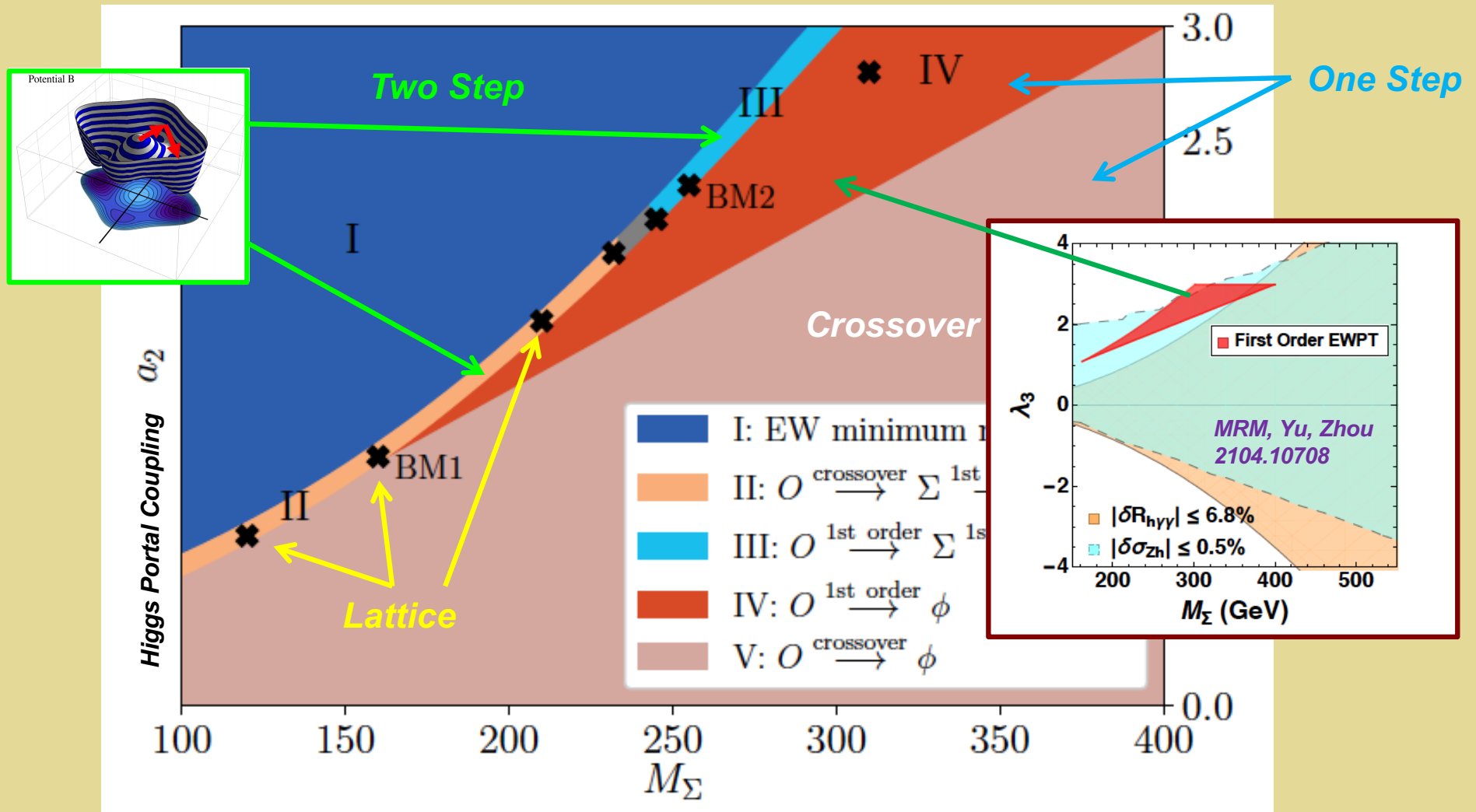
The figure is a phase diagram showing the Higgs portal coupling α_2 on the y-axis (ranging from 0.0 to 3.0) versus the mass M_Σ on the x-axis (ranging from 100 to 400). The diagram is divided into five regions (I, II, III, IV, V) and a 'Crossover' region. Region I is blue, II is orange, III is light blue, IV is red, and V is brown. The 'Crossover' region is indicated by a dashed green line. The diagram is labeled 'Two Step' (green) and 'One Step' (blue). A green box highlights the 'Distinguishing crossover – PT boundary: non perturbative (lattice)'. A yellow box highlights the 'Lattice' region. A legend in the bottom right corner describes the regions:

- I: EW minimum metastable
- II: $O \xrightarrow{\text{crossover}} \Sigma \xrightarrow{1st\ order} \phi$
- III: $O \xrightarrow{1st\ order} \Sigma \xrightarrow{1st\ order} \phi$
- IV: $O \xrightarrow{1st\ order} \phi$
- V: $O \xrightarrow{\text{crossover}} \phi$

Key features include the 'BM1' and 'BM2' points marked with black crosses, and the 'Crossover' region highlighted by a dashed green line. An inset in the top left shows a 3D plot of the potential landscape.

- Niemi, R-M, Tenkanen, Weir 2005.11332**

Real Triplet & EWPT: Novel EWSB

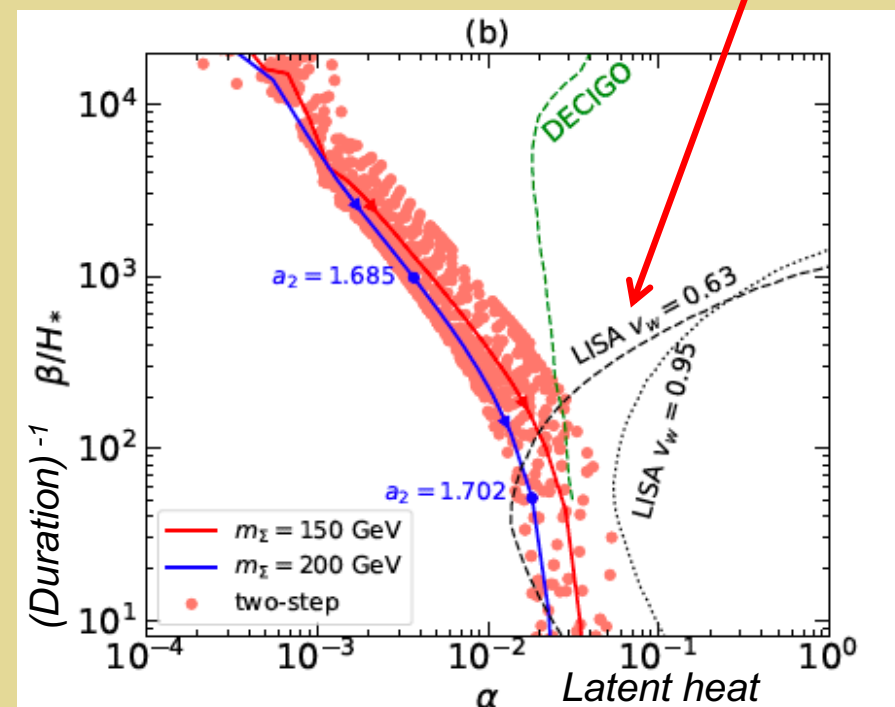
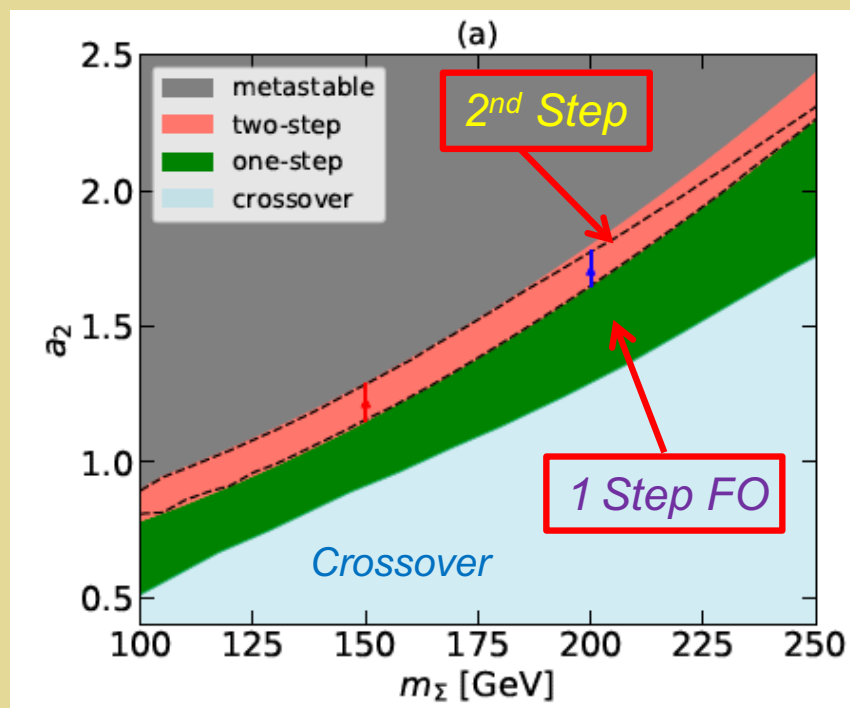


Niemi, R-M, Tenkanen, Weir 2005.11332

- 1 or 2 step
- Non-perturbative

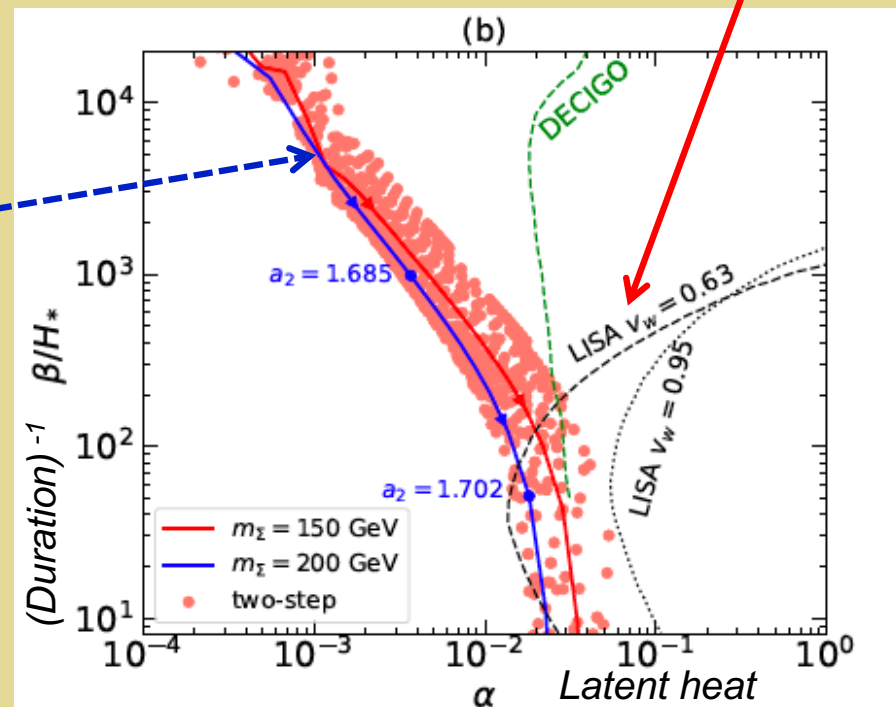
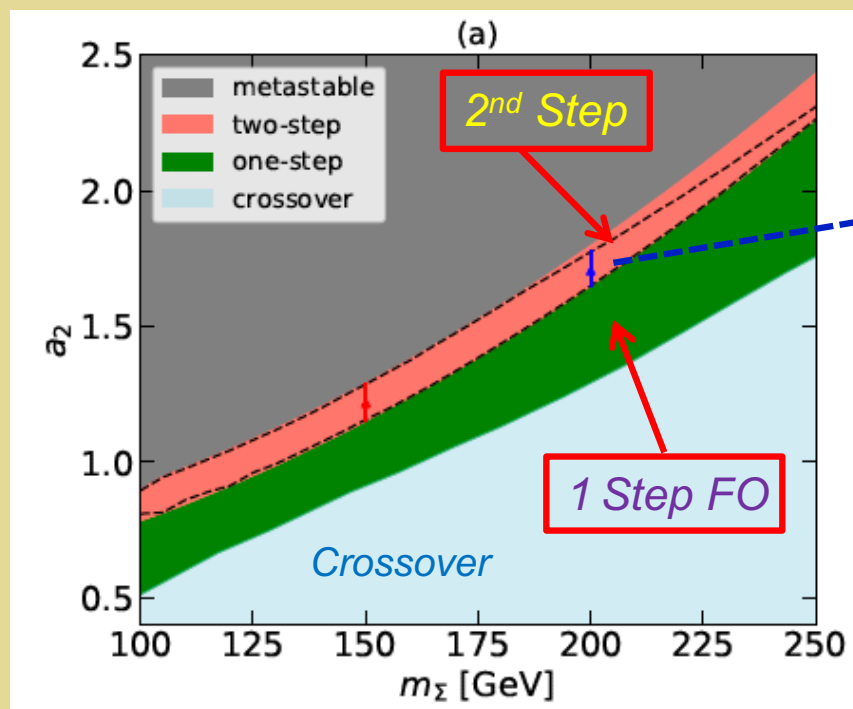
GW & EWPT Phase Diagram

Real triplet extension



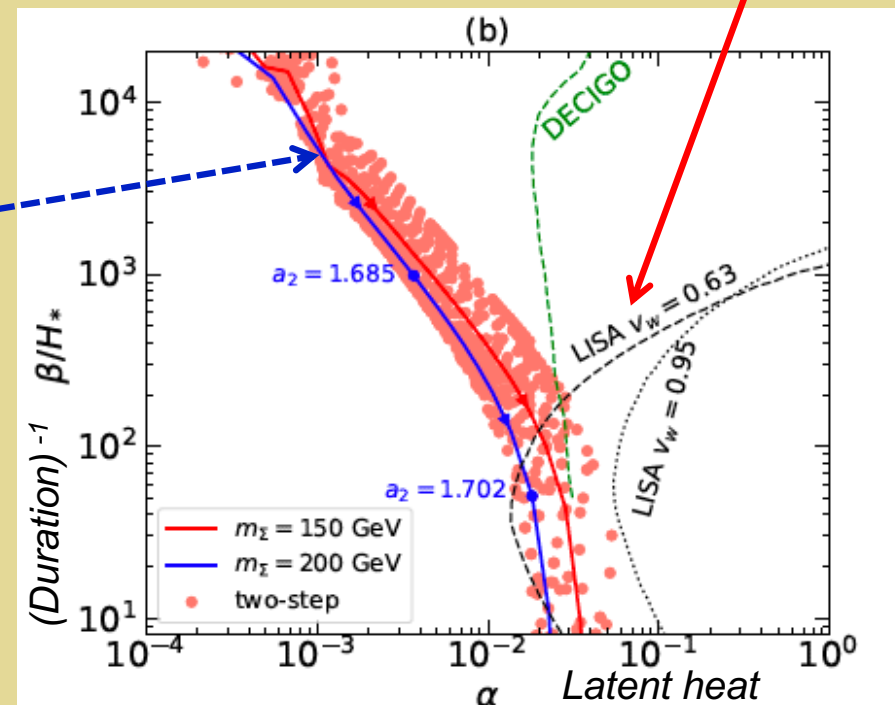
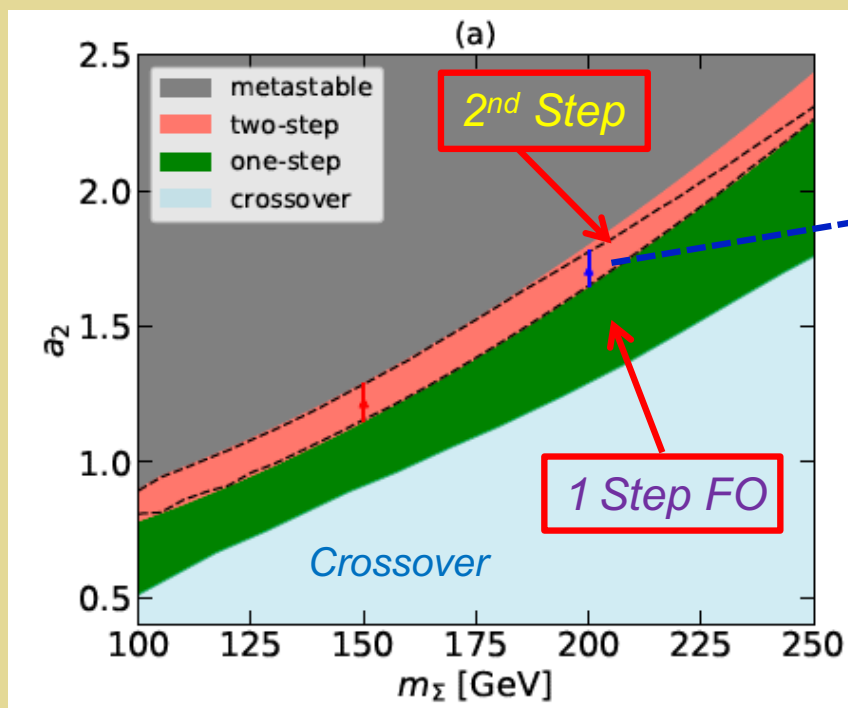
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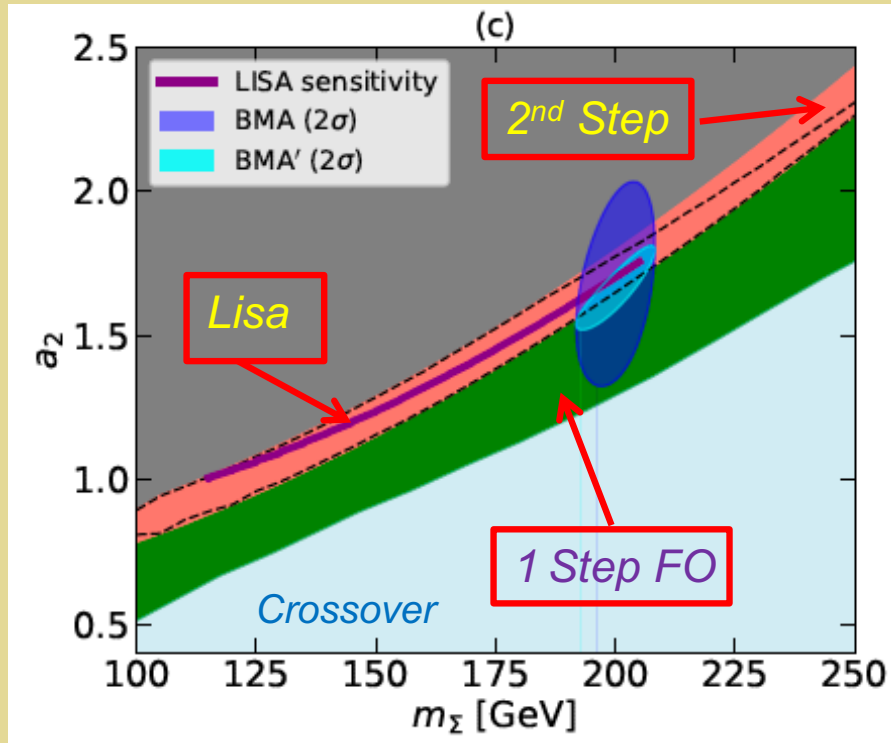
Real triplet extension



- *Single step transition: GW well outside LISA sensitivity*
- *Second step of 2-step transition can be observable*
- *Significant GW sensitivity to portal coupling*

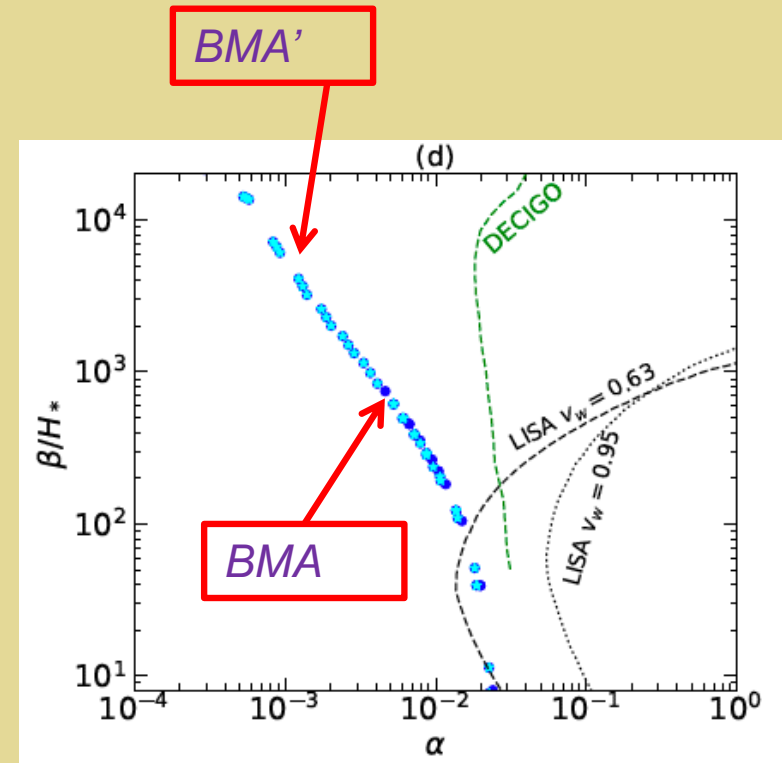
GW & Collider : EWPT Phase Diagram

Real triplet extension



BMA: $m_\Sigma + h \rightarrow \gamma\gamma$

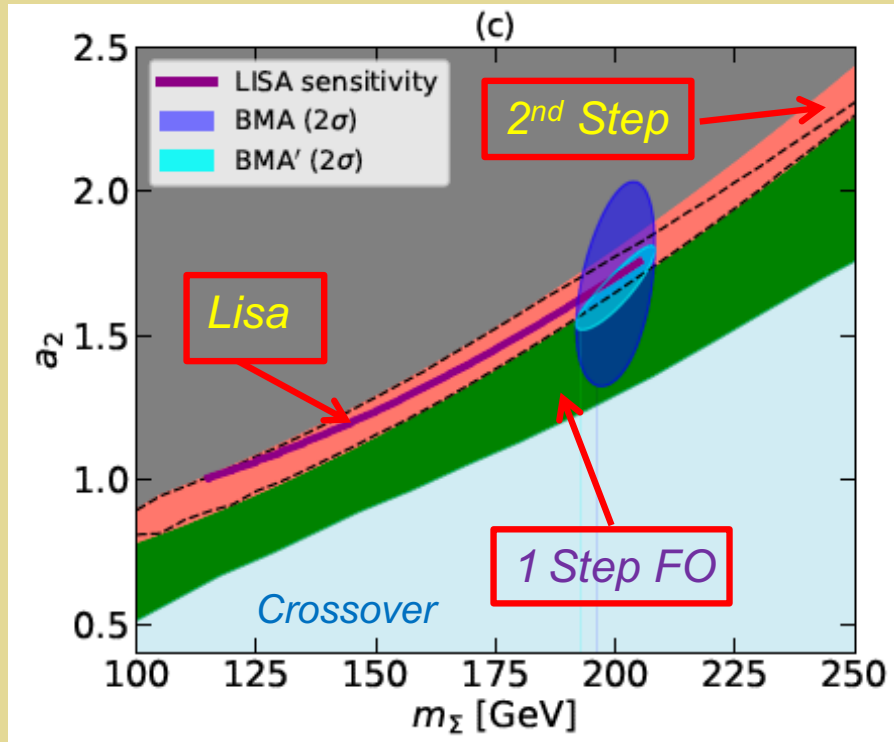
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- Two-step
- EFT+ Non-perturbative

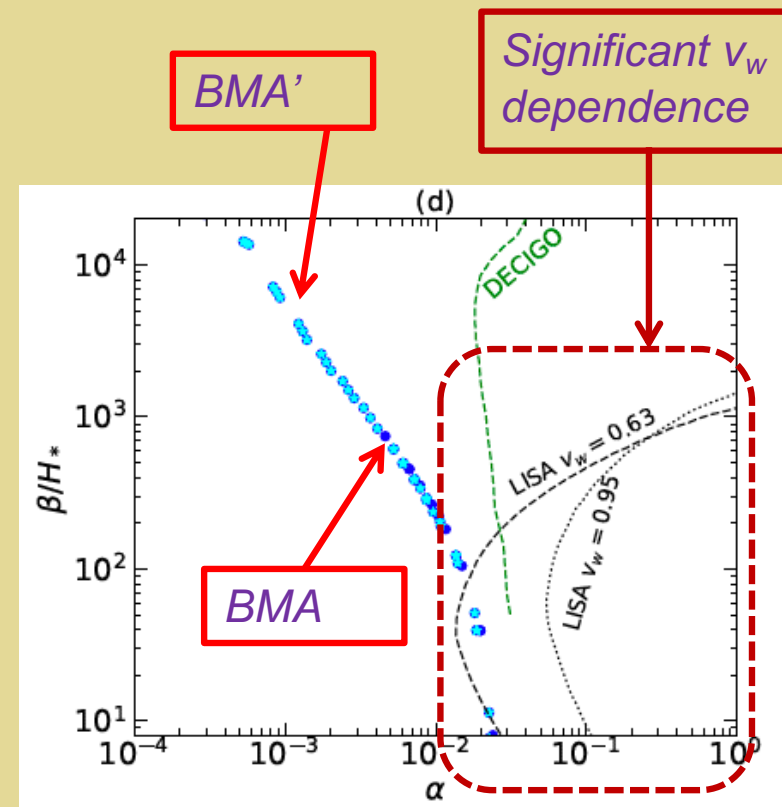
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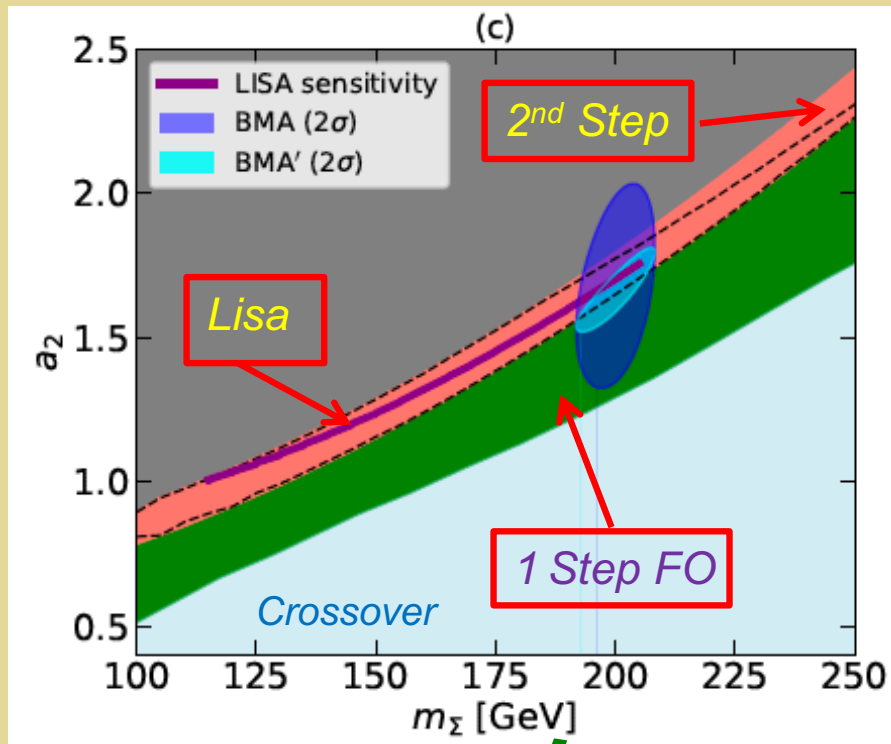
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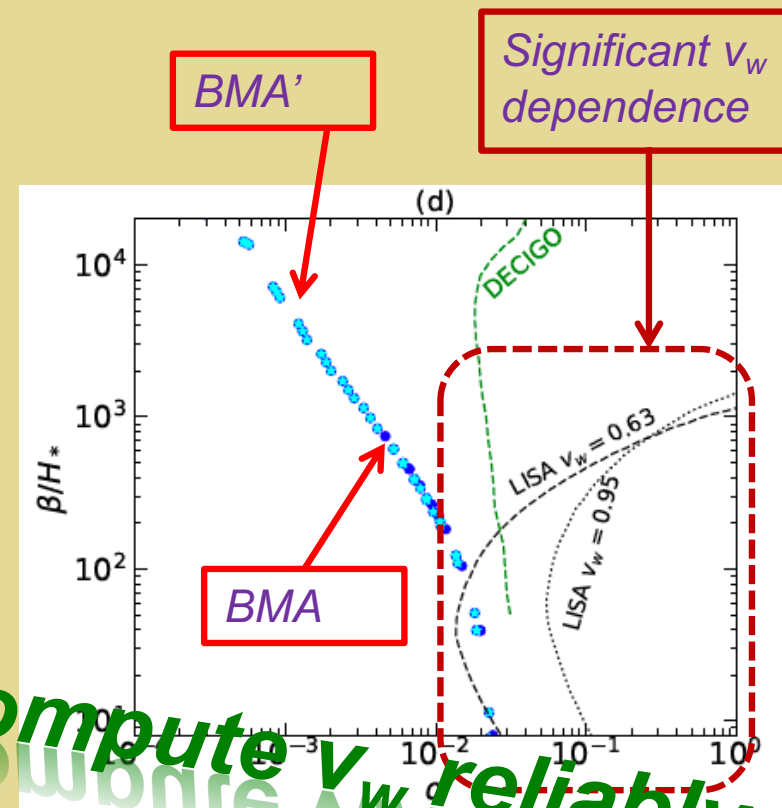
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How to compute v_w reliably?

- Two-step
- EFT+ Non-perturbative

IV. Outlook

- *The possibility of primordial gravitational waves generated from various particle physics dynamics has become an exciting area of exploration*
- *There exist many creative ideas for novel phenomena and dynamics that could have generated GW*
- *Realizing which, if any, of these ideas was realized in nature requires input from additional observables and performing the most rigorous theoretical calculations*
- *The electroweak phase transition provides a unique “laboratory” for testing our theoretical methods and ideas, with LHC and next generation collider measurements providing key input*

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谢谢!

Back Up Slides

Gravitational Radiation

$$\Omega_{\text{GW}} = \Omega_{\text{coll}} + \Omega_{\text{sw}} + \Omega_{\text{turb}}$$

$$\frac{d \log \Omega_{\text{GW}}}{d \log k} \sim K^{\sigma} (\beta / H_{*})^{\delta}$$

$$\sigma = 2$$

$$\delta = -2$$

Collision (φ)

$$\sigma = 2$$

$$\delta = -2$$

Sound waves, large k

$$\sigma = 3/2$$

$$\delta = -19/3$$

Turbulence, large k

Gravitational Radiation

$$\frac{d \log \Omega_{\text{GW}}}{d \log k} = K^2 (k L_f)^3 \int \frac{dt_1}{t_1} \int \frac{dt_2}{t_2} \cos [k(t_1 - t_2)] \tilde{\Pi}(k, t_1, t_2)$$

Greens function

Source

$$\tilde{\Pi}_{ij}(\vec{k}, t) = \Lambda_{ijkl} \tilde{T}_{kl}(\vec{k}, t)$$

TT projector

$$\langle \tilde{\Pi}_{ij}(\vec{k}, t_1) \tilde{\Pi}_{ij}(\vec{q}, t_2) \rangle = (2\pi)^3 \delta(\vec{k} - \vec{q}) \tilde{\Pi}(k, t_1, t_2)$$

Inputs from Thermal QFT

Thermodynamics

- *Phase diagram: first order EWPT?*
- *Latent heat: GW*

Dynamics

- *Nucleation rate: transition occurs? T_N ? Transition duration (GW) ?*
- *EW sphaleron rate: baryon number preserved?*

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How reliable is the theory ?

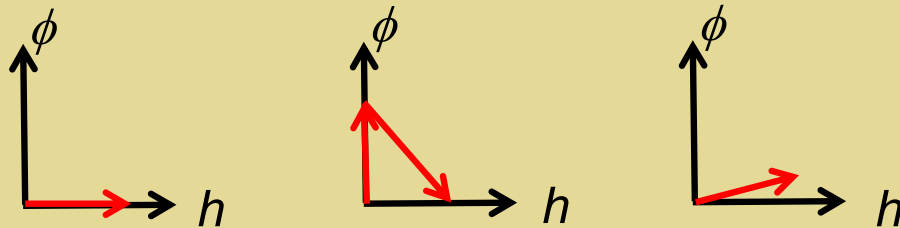
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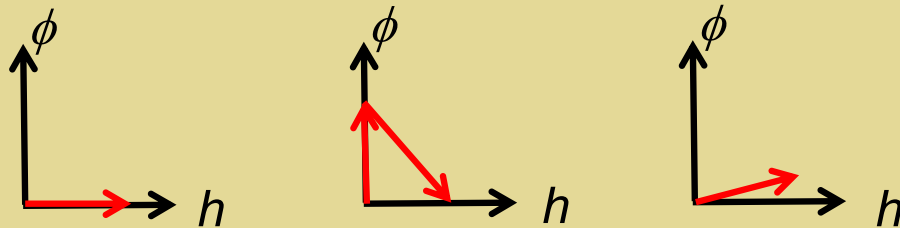


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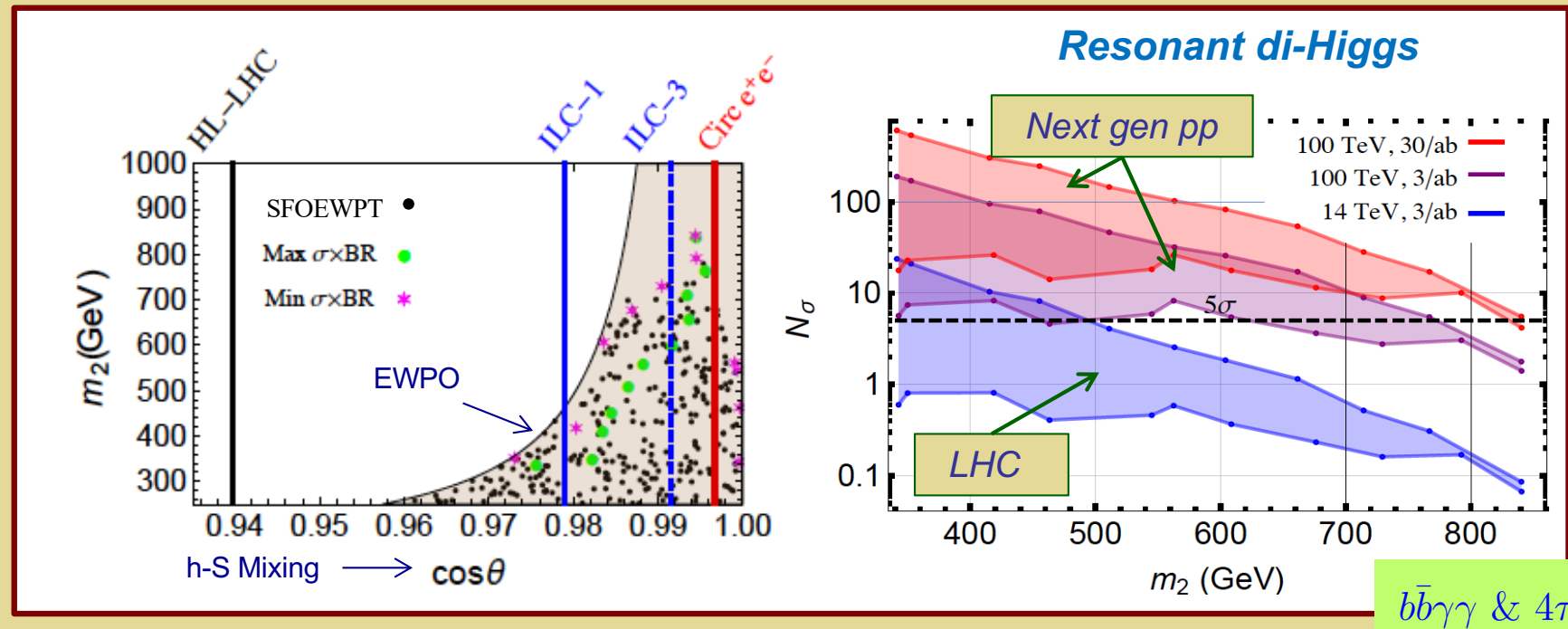
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H- ϕ mixing



Singlets: Precision & Res Di-Higgs Prod

SFOEWPT Benchmarks^{**}: Resonant di-Higgs & precision Higgs studies



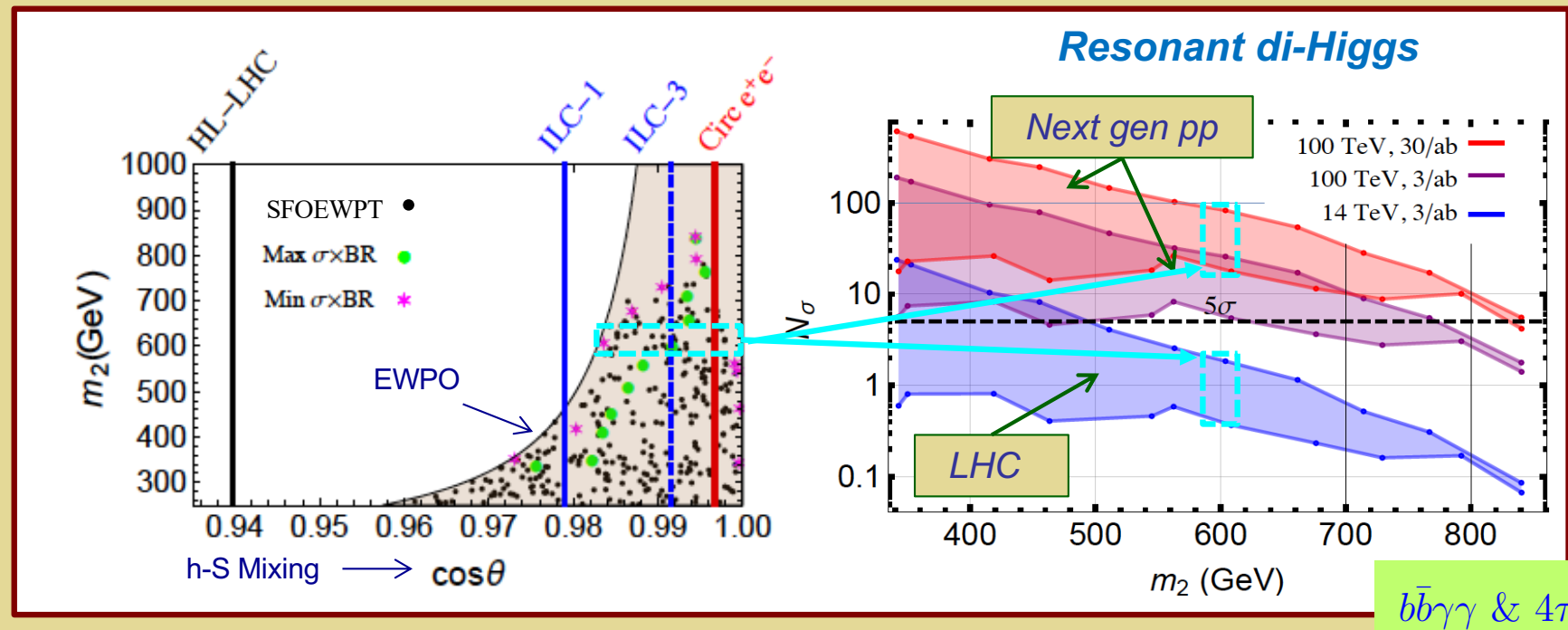
Kotwal, No, R-M, Winslow 1605.06123

^{**} Perturbative thermal QFT

See also: Huang et al, 1701.04442;
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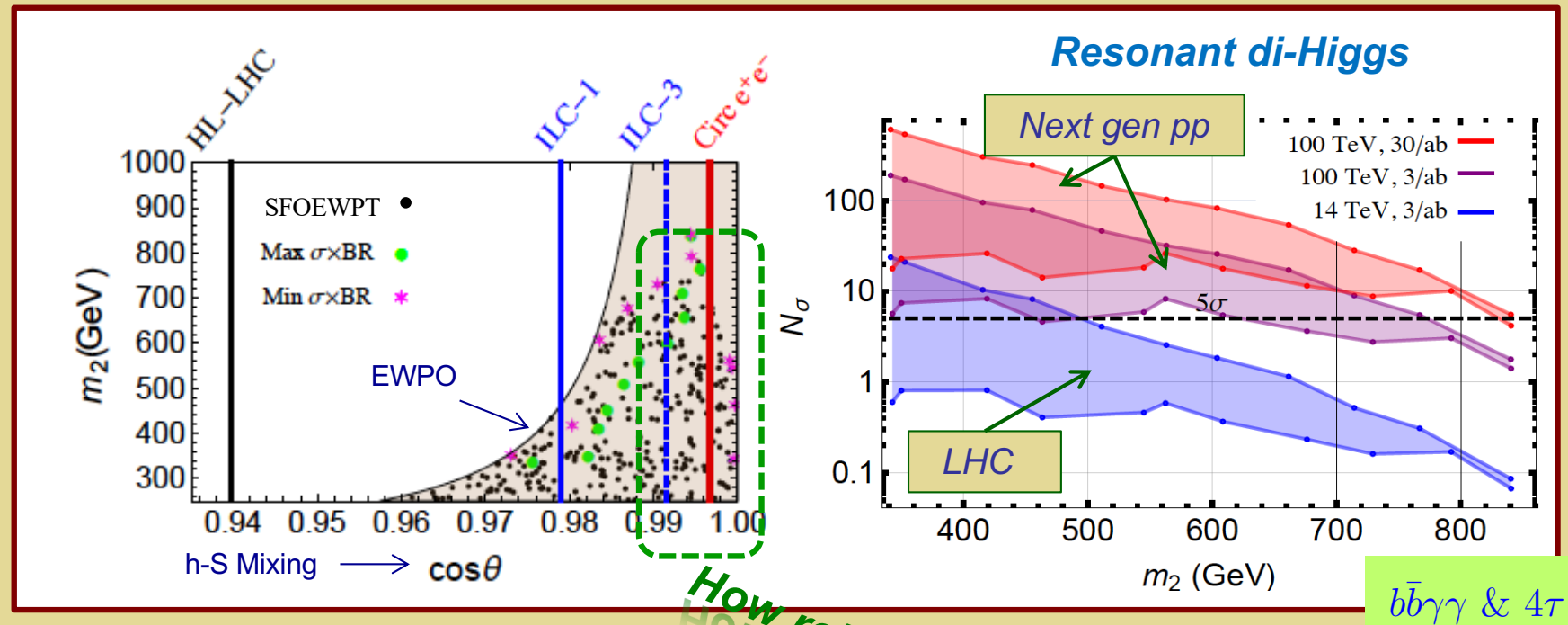
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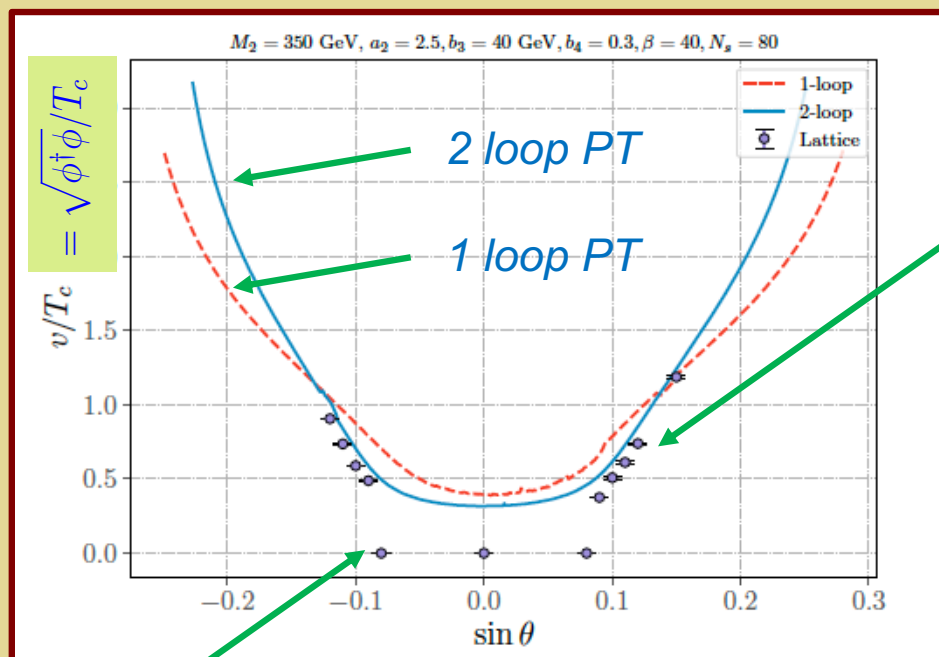


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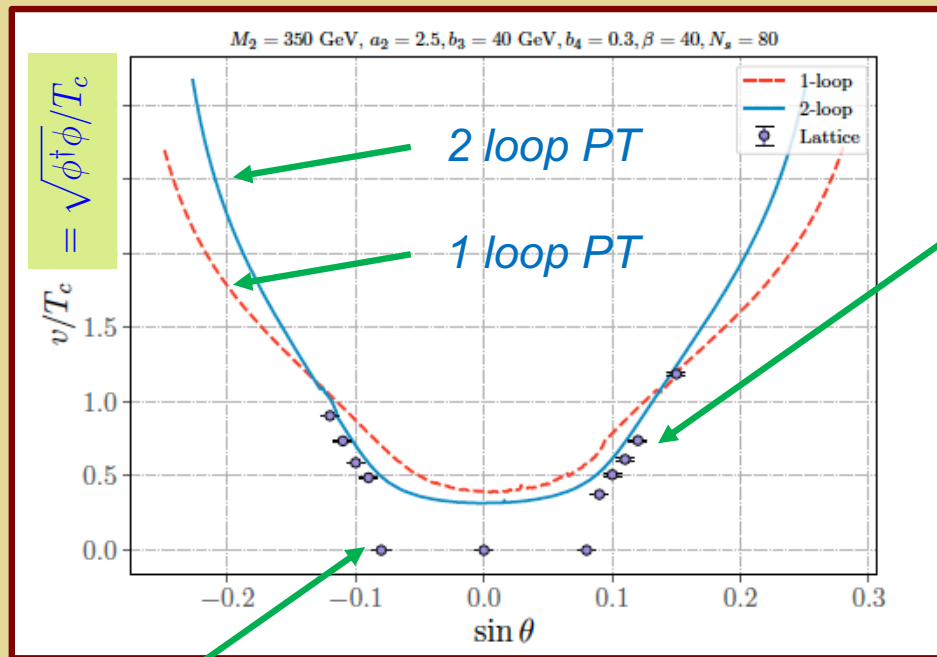
Singlets: Lattice vs. Pert Theory



Lattice:
Crossover

Lattice:
FOEWPT

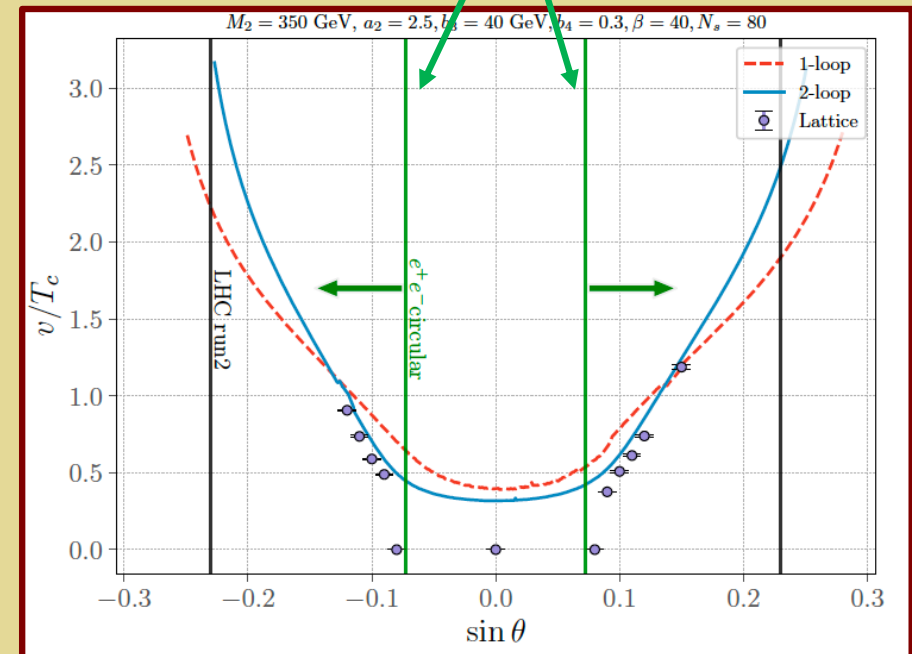
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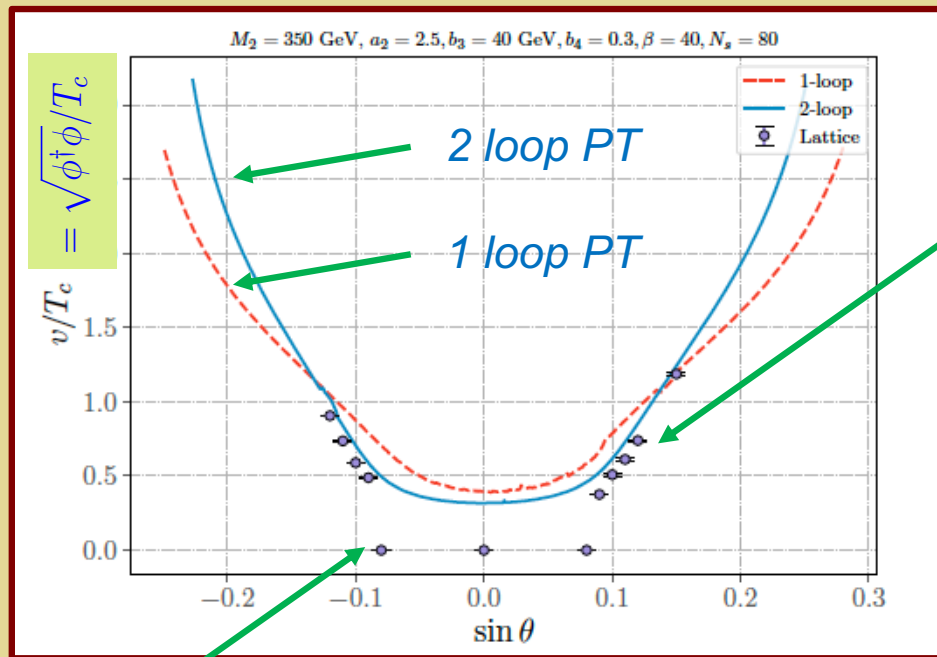
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Future e^+e^-



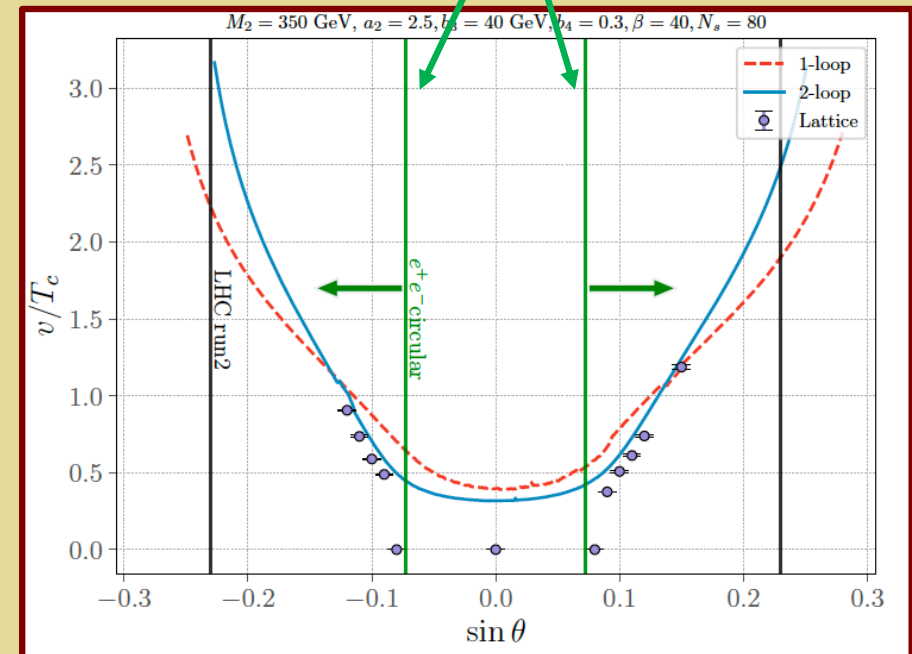
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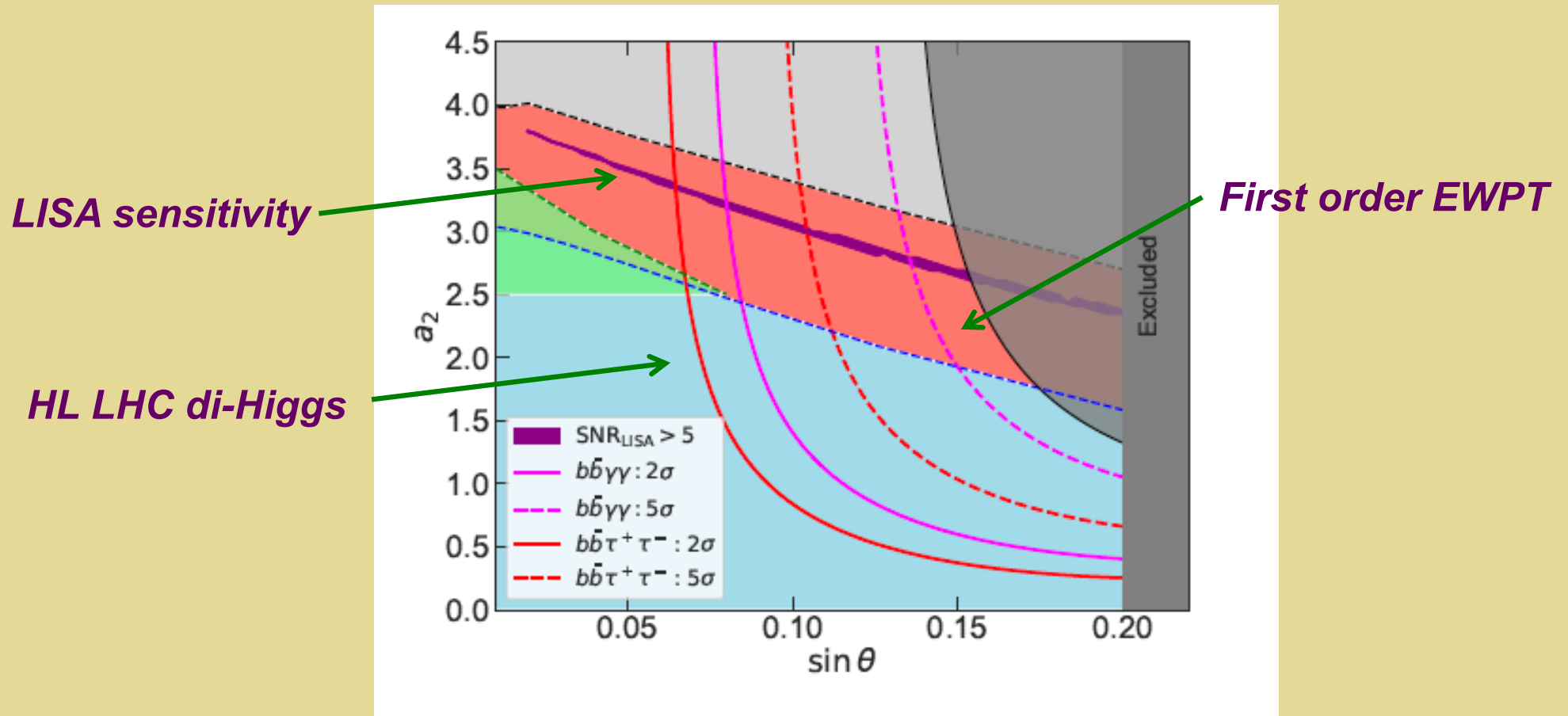
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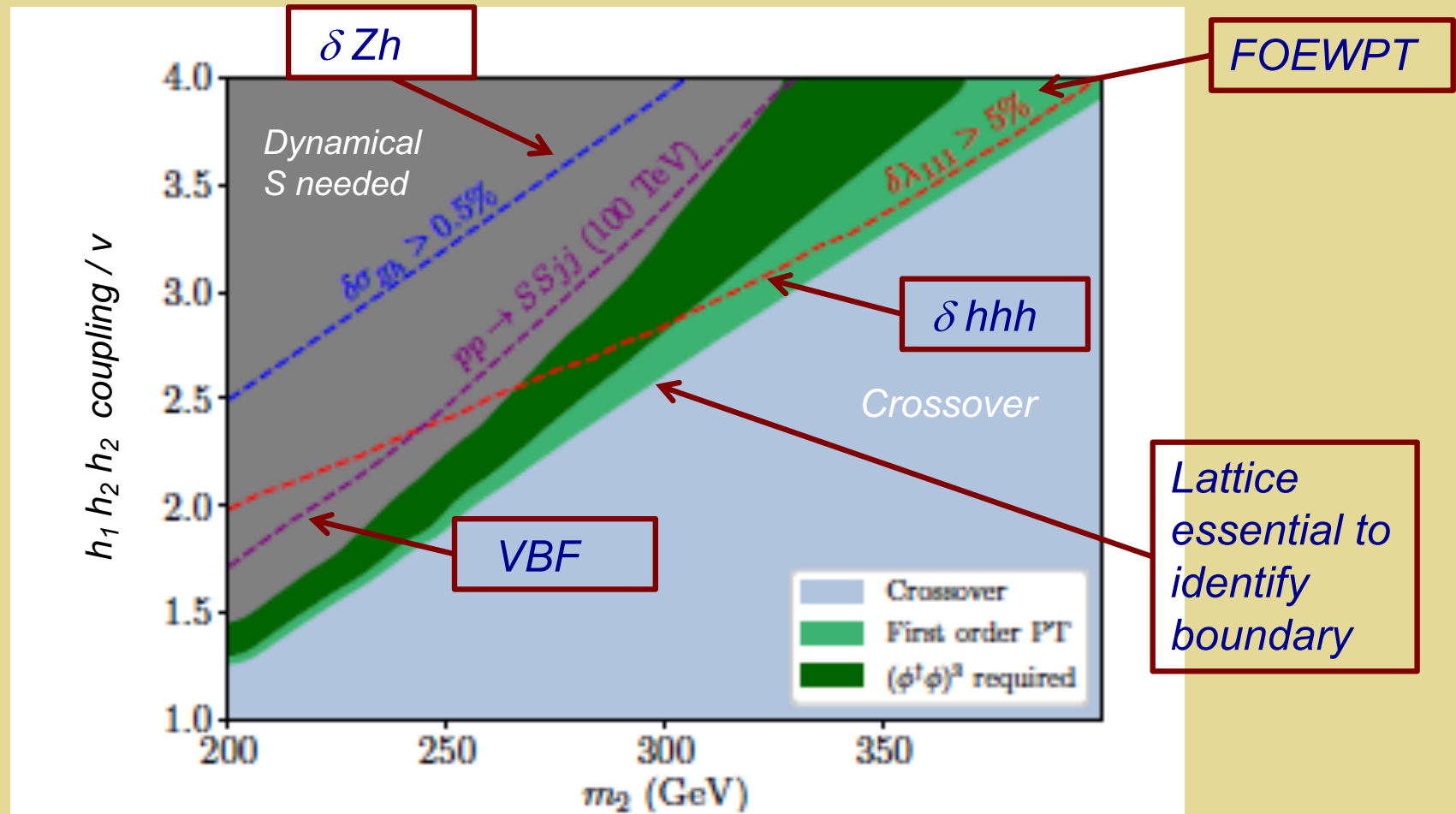


- Lattice: crossover-FOEWPT boundary
- FOEWPT region: PT-lattice agreement
- Pheno: precision Higgs studies may be sensitive to a greater portion of FOEWPT-viable param space than earlier realized

Gravitational Radiation & Colliders



Non-Dynamical Real Singlet & EWPT: Probes



III. Theoretical Robustness

- *IR Problem*
- *Nucleation @ finite T: gauge invariance*
- *Wall velocity*

$$\boxed{\frac{\beta}{H_*} = T \frac{d}{dT} \frac{S_3}{T}} \quad + T_N$$

BSM Scalar: EWPT & GW

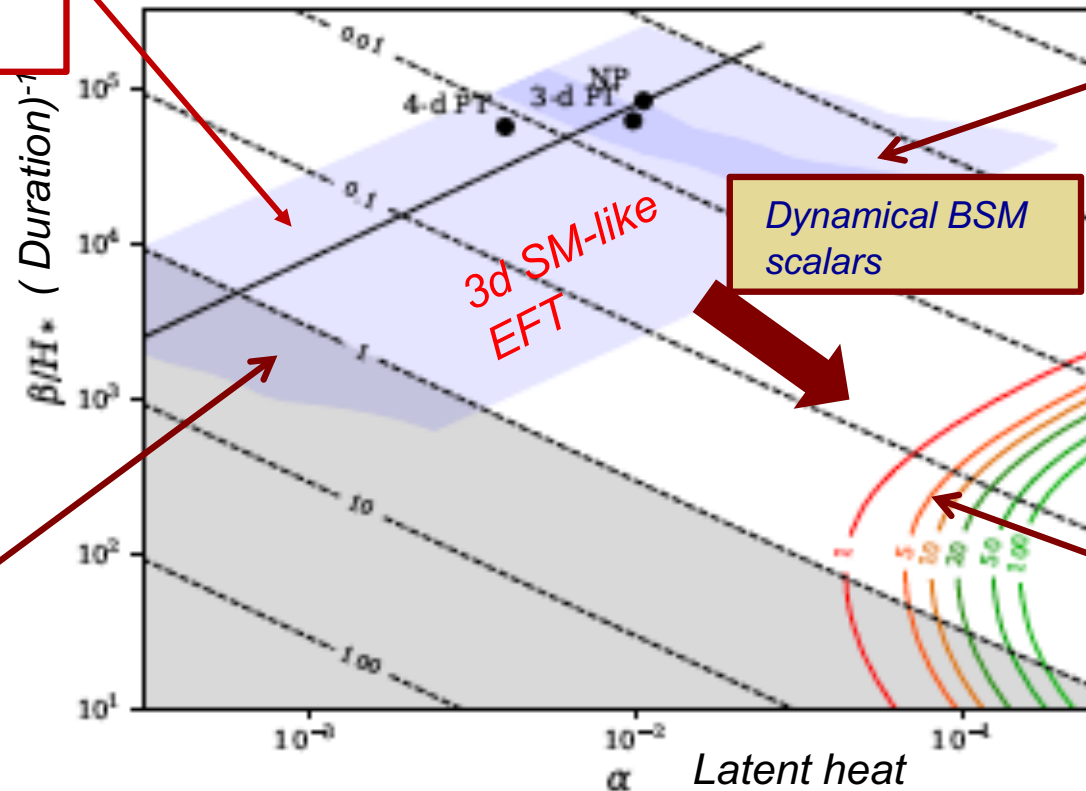
Collider probes of BSM parameters in $\mathcal{L}_{full}$

Integrate out BSM DOF

LISA SNR

Dynamical BSM scalars

3d SM-like
EFT



Radiative barrier from gauge sector

Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

B8.2