Testing GUT phase transition via inflated gravitational waves

Ye-Ling Zhou

2025-10-19

國科大杭州高等研究院
Hangzhou Institute for Advanced Study,UCAS

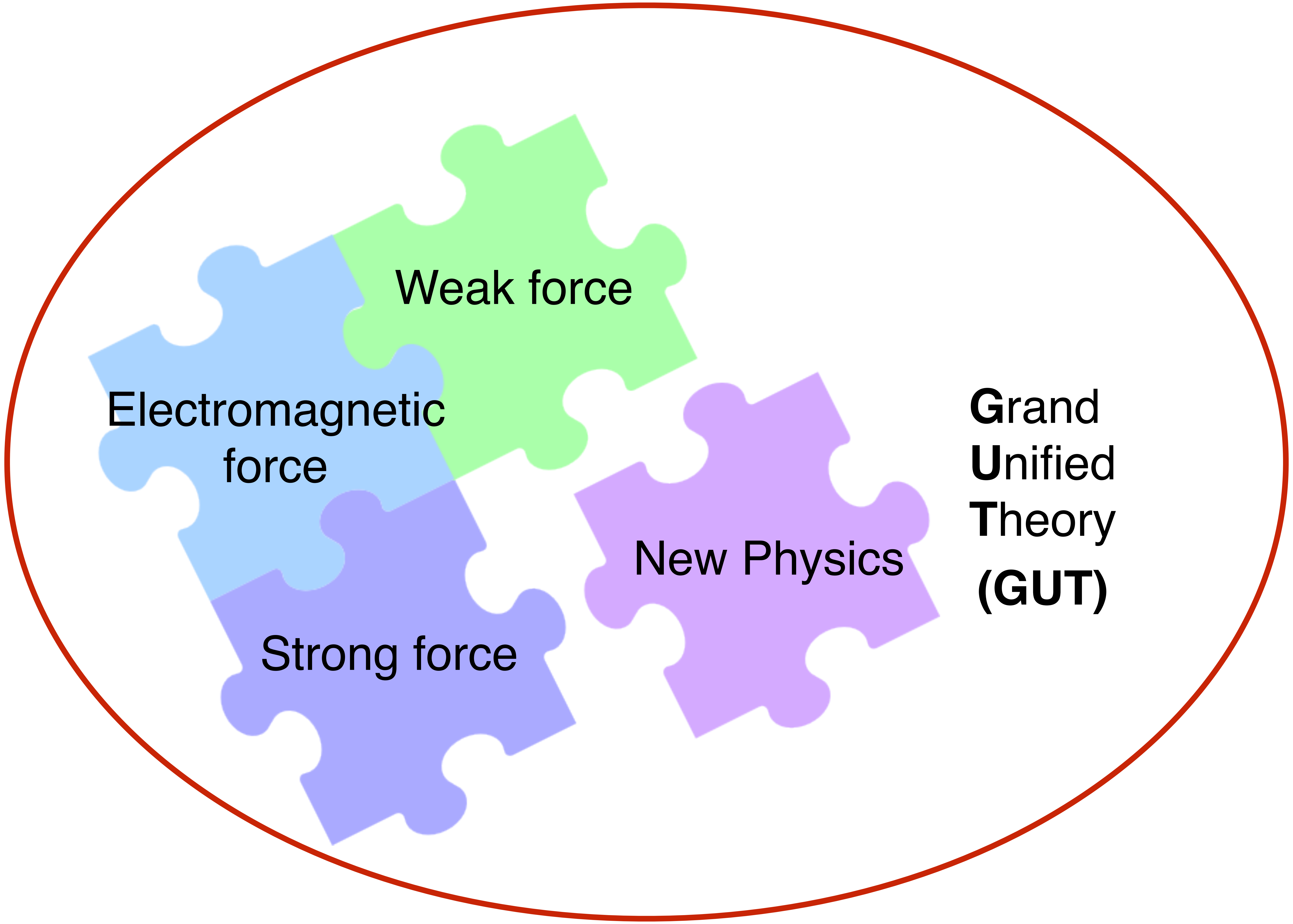
基础物理与数学科学学院
School of Fundamental Physics and Mathematical Sciences

Contents

- Difficulty of direct testing GUT phase transition in cosmology
- Way out: mechanism of inflated GWs via FOPT during inflation
- Application of the mechanism to GUT phase transition

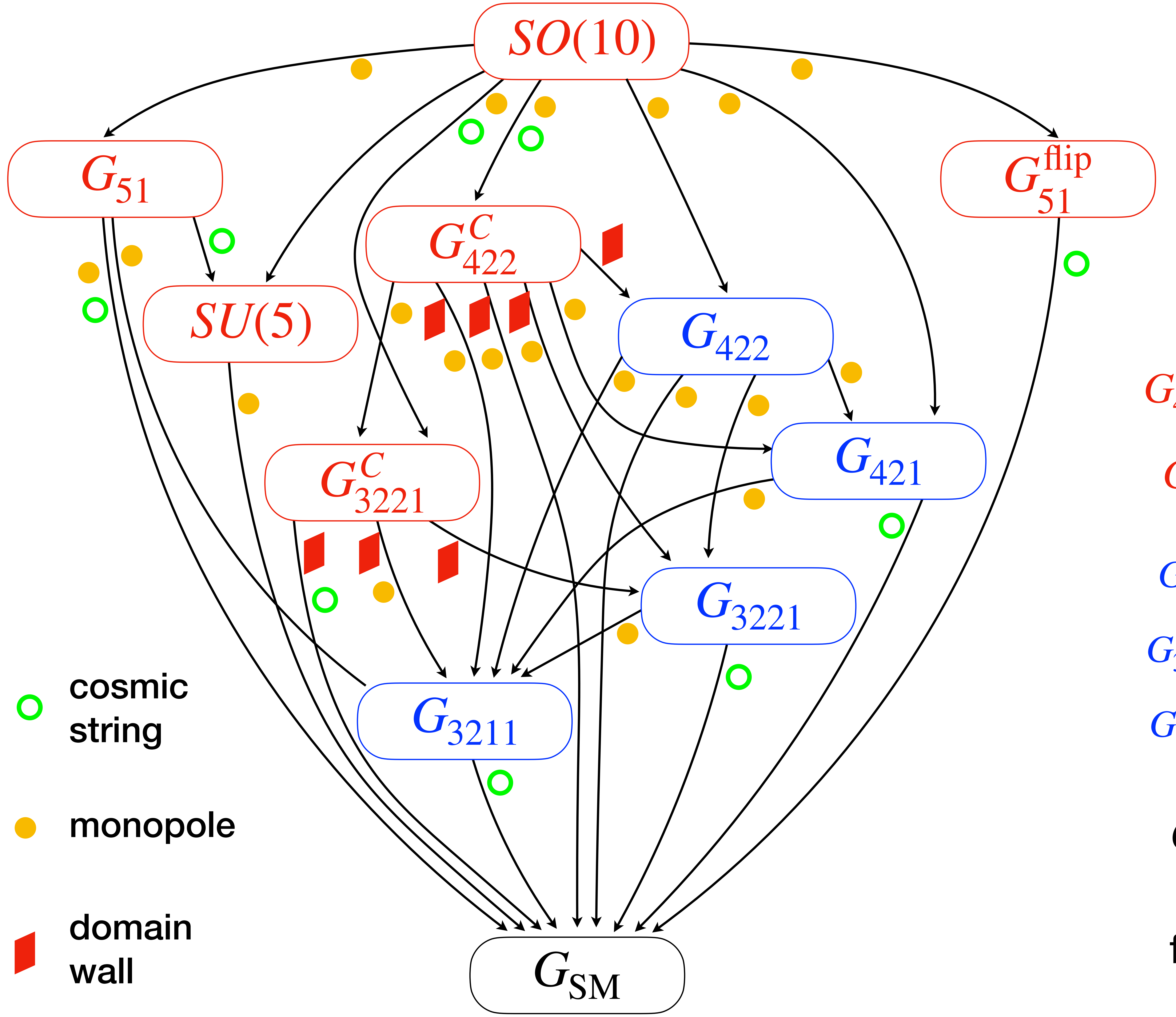
2501.01491, collaboration with Xi-He Hu

Introduction



Gravity… not included

Monopoles in grand unified theories



$G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$

$G_{51} = SU(5) \times U(1)$

$G_{421} = SU(4)_c \times SU(2)_L \times U(1)_R$

$G_{3221} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$G_{3211} = SU(3)_c \times SU(2)_L \times SU(1)_Y \times U(1)_{B-L}$

C: parity $\psi_L \leftrightarrow \psi_R^C$

flip: $u \leftrightarrow d, \nu \leftrightarrow e$

GUT monopole problem

- GUT monopoles are produced after the breaking of GUTs, with masses naturally around the GUT scale $M_{\rm mono} \simeq \Lambda_{\rm GUT}/\alpha_{\rm GUT}$ and number density $n_\star = H_\star^3$.

- Monopoles, once they are produced, evolve as matter during Hubble expansion. The number density today is given by

$$n_{\rm mono}(t_0) = \left(\frac{a(t_\star)}{a(t_0)}\right)^3 n_\star$$

- Their energy density fraction $\Omega_{\rm mono} = M_{\rm mono} n(t_0)/\rho_c$ is given by

$$\Omega_{\rm mono} = \frac{8\pi G M_{\rm mono} H_\star^3}{3H_0^2(1+z_{\rm Rh})^3} \sim 10^{14}\left(\frac{\Lambda_{\rm GUT}}{10^{15}\ {\rm GeV}}\right)^4 \gg 1$$

Zeldovieh and Khlopov, PLB 1978;
Preskill, PRL 1979

- Solving the monopole problem is one of the initial motivations of inflation

Guth, PRD, 1981

GWs via cosmic strings predicted in GUTs

Dror, Hiramatsu, Kohri, Murayama, White,1908.03227;
Buchmuller, Domcke, Murayama, Schmitz, 1912.03695;
King, Pascoli, Turner, YLZ, 2005.13549, 2106.15634

GUT — Broken at M_{GUT} ← Proton decay

Inflation

Gauge unification

$G_1 \supset G_{\rm SM} \times U(1)_{B-L}$ — Broken at M_{B-L} ← Gravitational waves

Standard Model

Correlation of fermion masses and mixing
Predicting right neutrino mass spectrum
Explaining matter-antimatter asymmetry

Fu, King, Marsili, Pascoli, Turner, YLZ,
2209.00021; 2308.05799;
King, Leontaris, YLZ, 2407.02701

Also see e.g.,
Babu *et al*, 2409.03840; 2508.14969;
Saad *et al*, 2506.11806

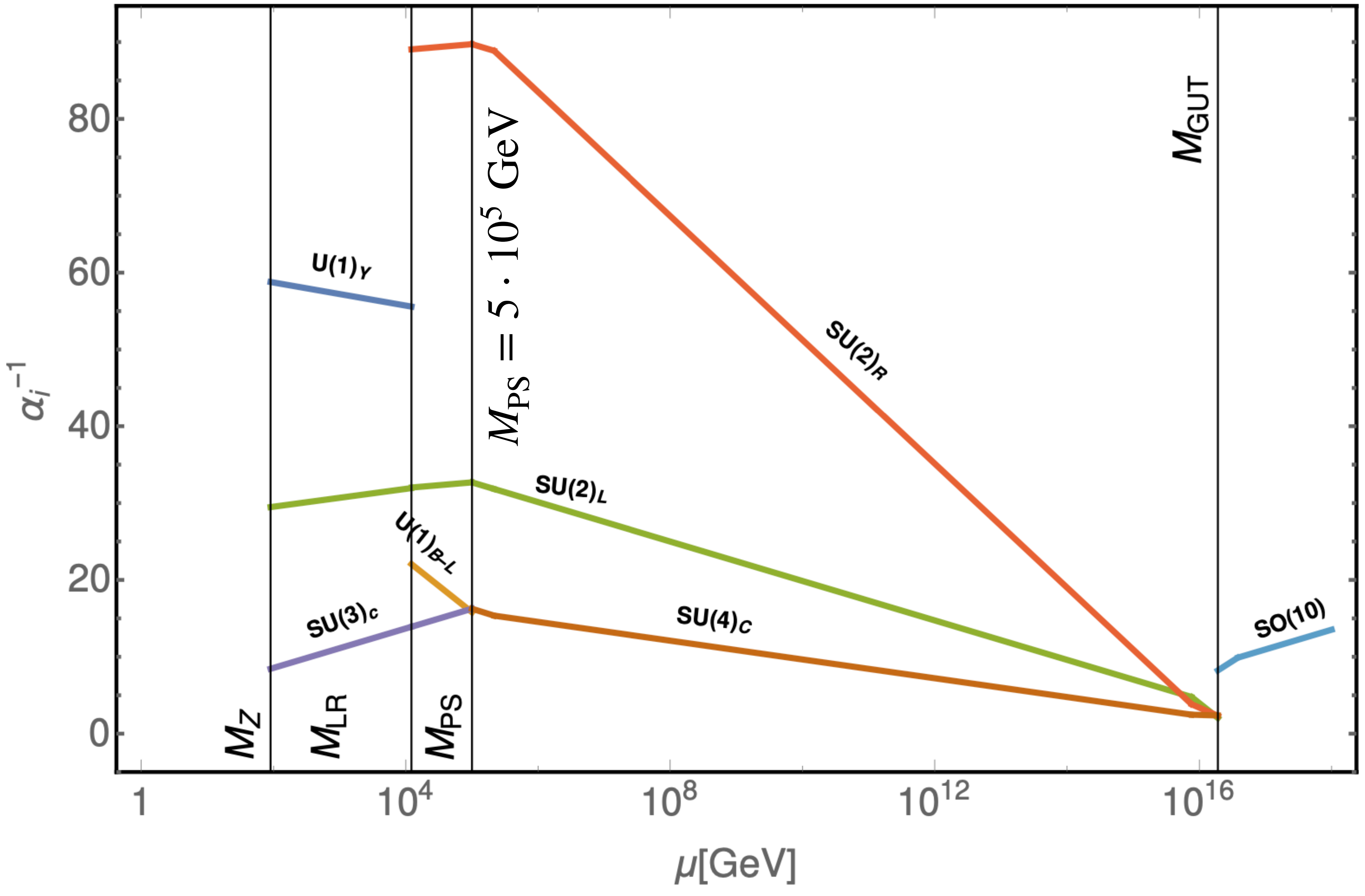
(SU(5) does not include $U(1)_{B-L}$)

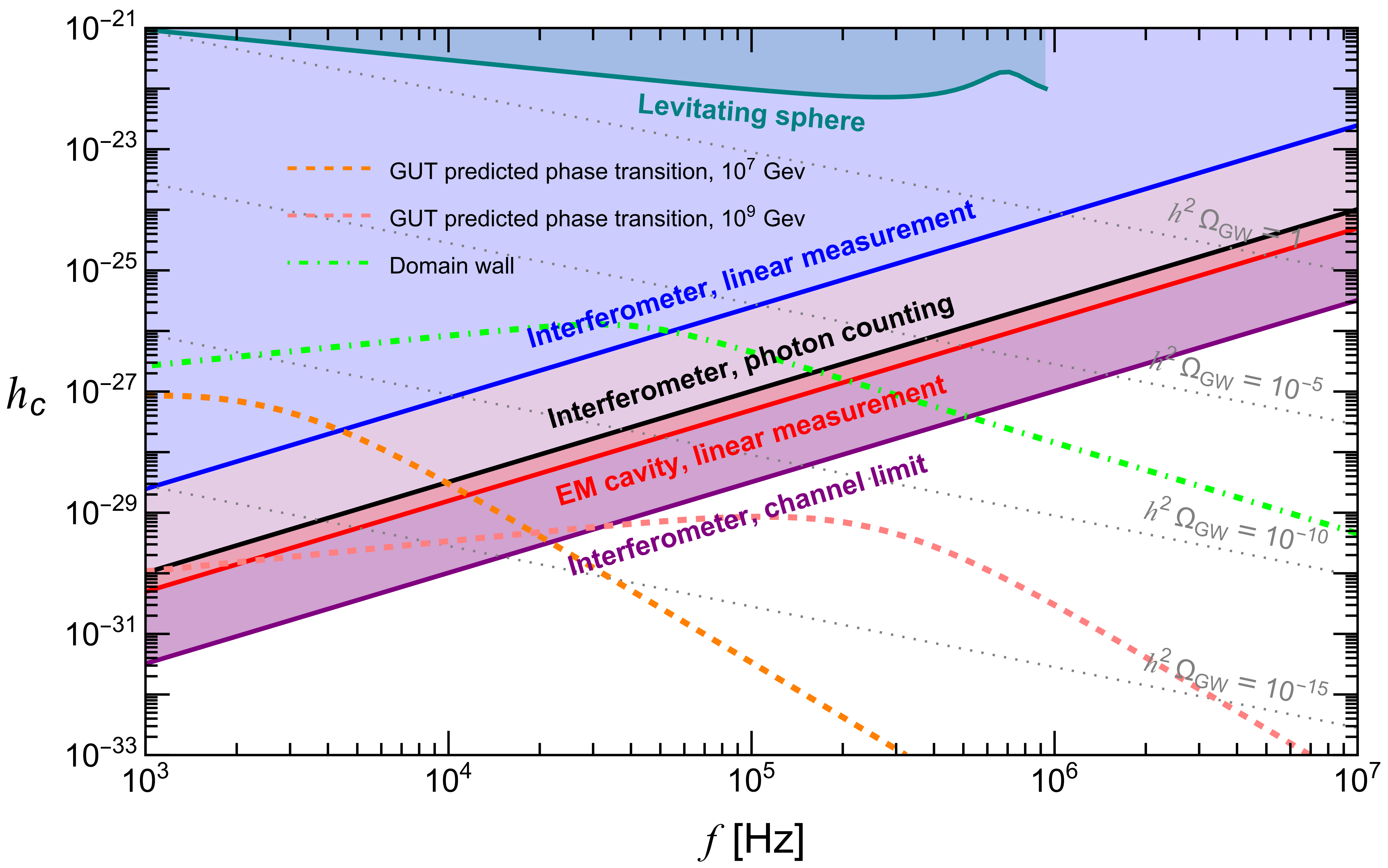
GWs via phase transition motivated by GUTs

For example, GWs via phase transition in Pati-Salam model

$\Omega_{\rm GW} \propto f^2 h_c^2$

Croon, Gonzalo, White, 1812.02747;
Huang, Sannino, Wang, 2004.02332;
Athron, Balazs, Gonzalo, Pearce, 2307.02544



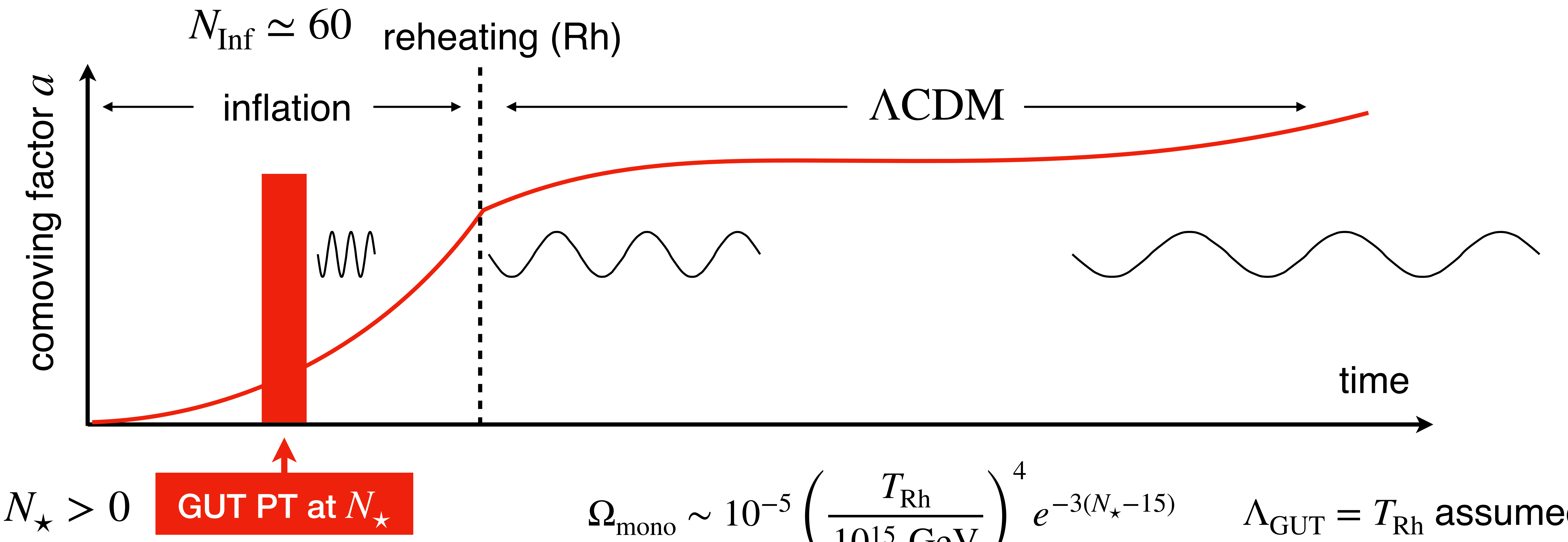


The testability of GWs via ultra-high scale PT conflict with quantum limits

Guo, Miao, Wang, Yang, YLZ, 2501.18146

GUT breaking during inflation

- Locating the GUT PT during inflation can also solve the monopole problem.
  And if the PT is first order …



$N_{\rm Inf} > N_\star > 0$

$$\Omega_{\rm mono} \sim 10^{-5} \left(\frac{T_{\rm Rh}}{10^{15}\ {\rm GeV}} \right)^4 e^{-3(N_\star - 15)}$$

$\Lambda_{\rm GUT} = T_{\rm Rh}$ assumed

- Explicit model constructions — Hu, Ouyang, YLZ, in progress

  SU(5) inflation, Vilenki, Shafi, PRL, 1984

  Smooth hybrid inflation, Lazarides, Panagiotakopoulos, hep-ph/9506325

  Shifted hybrid inflation in PS model, Jeannerot, Khalil, Lazarides, Shafi, hep-ph/0002151, …

GWs from phase transition during inflation: formalism

- The mechanism was originally proposed in [An, Lyu, Wang, Zhou, 2009.12381, 2201.05171]
- We repeat and generalise the formalism to PT happens at any e-folds to the end of inflation (=reheating), and interpret the formalism in a more transparent way (specifying IR, UV, FUV band)
- Conformal frame $ds^2 = a^2(\tau)\left[\mathrm{d}\tau^2 - \mathrm{d}\mathbf{x}^2\right],\ \mathrm{d}t = a\mathrm{d}\tau$
- GWs as perturbation of the metric $ds^2 = a^2(\tau)\left[\mathrm{d}\tau^2 - (\delta_{ij} + h_{ij}(\tau,\mathbf{x}))\mathrm{d}x^i\mathrm{d}x^j\right]$
- EOM of GWs $h''_{ij}(\tau,\mathbf{x}) + 2a(\tau)H\,h'_{ij}(\tau,\mathbf{x}) - \nabla^2 h_{ij}(\tau,\mathbf{x}) = 16\pi G_{\mathrm{N}} a^2(\tau)\sigma_{ij}(\tau,\mathbf{x})$
- Fourier Trans. $\tilde{h}''_{ij}(\tau,\mathbf{k}) + 2a(\tau)H\,\tilde{h}'_{ij}(\tau,\mathbf{k}) + k^2\tilde{h}_{ij}(\tau,\mathbf{k}) = 16\pi G_{\mathrm{N}} a^2(\tau)\tilde{\sigma}_{ij}(\tau,\mathbf{k})$
- GW metric proportional from τ' to τ ($\tau' < \tau$) is given by

$$h_1^{\mathrm{Inf}}(\tau,\mathbf{k}) = \cos k\tau + k\tau\sin k\tau$$

$$h_2^{\mathrm{Inf}}(\tau,\mathbf{k}) = \sin k\tau - k\tau\cos k\tau$$

$$\tilde{h}_{ij}(\tau,\mathbf{k}) = 16\pi G_{\mathrm{N}}\int \mathrm{d}\tau'\,\theta(\tau-\tau')\,a^2(\tau')\,\tilde{\sigma}_{ij}(\tau',\mathbf{k}) \times \mathcal{G}(\tau,\tau';\mathbf{k})$$

Green function: $$\mathcal{G}(\tau,\tau';\mathbf{k}) = \left[\frac{\partial h_1}{\partial\tau} - \frac{h_1}{h_2}\frac{\partial h_2}{\partial\tau}\right]^{-1}_{\tau=\tau'} h_1(\tau,\mathbf{k}) + \left[\frac{\partial h_2}{\partial\tau} - \frac{h_2}{h_1}\frac{\partial h_1}{\partial\tau}\right]^{-1}_{\tau=\tau'} h_2(\tau,\mathbf{k})$$

GWs from phase transition during inflation: formalism

- GW propagation in Inflation (Inf) era

$$\tilde{h}_{\mu\nu}(\tau^{\rm Inf}, \mathbf{k}) = C^{\rm Inf}_{\mu\nu,1}\, h^{\rm Inf}_1(\tau^{\rm Inf}, \mathbf{k}) + C^{\rm Inf}_{\mu\nu,2}\, h^{\rm Inf}_2(\tau^{\rm Inf}, \mathbf{k})$$

$$h^{\rm Inf}_1(\tau, \mathbf{k}) = \cos k\tau + k\tau \sin k\tau$$
$$h^{\rm Inf}_2(\tau, \mathbf{k}) = \sin k\tau - k\tau \cos k\tau$$

- GW propagation in Radiation Domination (RD) era

$$\tilde{h}_{\mu\nu}(\tau^{\rm RD}, \mathbf{k}) = C^{\rm RD}_{\mu\nu,1}\, h^{\rm RD}_1(\tau^{\rm RD}, \mathbf{k}) + C^{\rm RD}_{\mu\nu,2}\, h^{\rm RD}_2(\tau^{\rm RD}, \mathbf{k})$$

$$h^{\rm RD}_1(\tau, \mathbf{k}) = \frac{\cos k\tau}{k\tau} \qquad h^{\rm RD}_2(\tau, \mathbf{k}) = \frac{\sin k\tau}{k\tau}$$

- Matching between the end of Inflation and the beginning of RD (ignoring the influence of reheating)

Dirichlet condition $$\tilde{h}^{\rm Inf}_{ij}(\tau^{\rm Inf}, \mathbf{k})\Big|_{\rm Rh} = \tilde{h}^{\rm RD}_{ij}(\tau^{\rm RD}, \mathbf{k})\Big|_{\rm Rh}$$

Neumann condition $$\partial_t \tilde{h}^{\rm Inf}_{ij}(\tau^{\rm Inf}, \mathbf{k})\Big|_{\rm Rh} = \partial_t \tilde{h}^{\rm RD}_{ij}(\tau^{\rm RD}, \mathbf{k})\Big|_{\rm Rh}$$

- GW energy density

$$\rho_{\rm GW} = \frac{1}{32\pi G_{\rm N} a^2(t)} \int_{T_\tau} \frac{{\rm d}\tau}{T_\tau} \int \frac{{\rm d}^3\mathbf{k}}{(2\pi)^3 V} \left| h'_{ij}(\tau, \mathbf{k}) \right|^2$$

And the ratio to the critical energy density

$$h^2\Omega_{\rm GW}(f) = \frac{h^2}{\rho_{\rm c}} \frac{{\rm d}\rho_{\rm GW}}{{\rm d}\log k}\Bigg|_{t = t_0,\, k = 2\pi a_0 f}$$

- PS. Matching from RD to Matter Domination (MD) and further to Vacuum Domination (VD) eras are also considered. These periods induce no new effect, thus ignored in the discussion.

GW spectrum: inflated vs uninflated

Inflated GW   Uninflated GW

$$h^2\Omega_{\mathrm{GW}}(f) = h^2\widetilde{\Omega}_{\mathrm{GW}}(fe^{N_\star}) \times S(f)$$

redshift   deformation

$$h^2\widetilde{\Omega}_{\mathrm{GW}}(\tilde{f}) \equiv \frac{h^2}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{GW}}^{\mathrm{flat}}}{\mathrm{d}\log k} \times \frac{a_{\mathrm{Rh}}^4}{a_0^4}$$

Instant-source approximation (瞬时源近似) $S(f) = S_0(f) + S_1(f)$

$$S_0(f) = \left\{\frac{\cos[y(1-\epsilon)]}{y^2} - \frac{\sin[y(1-\epsilon)]}{y^3}\right\}^2 \qquad y = \frac{2\pi a_0 f}{a_\star H_\star}$$

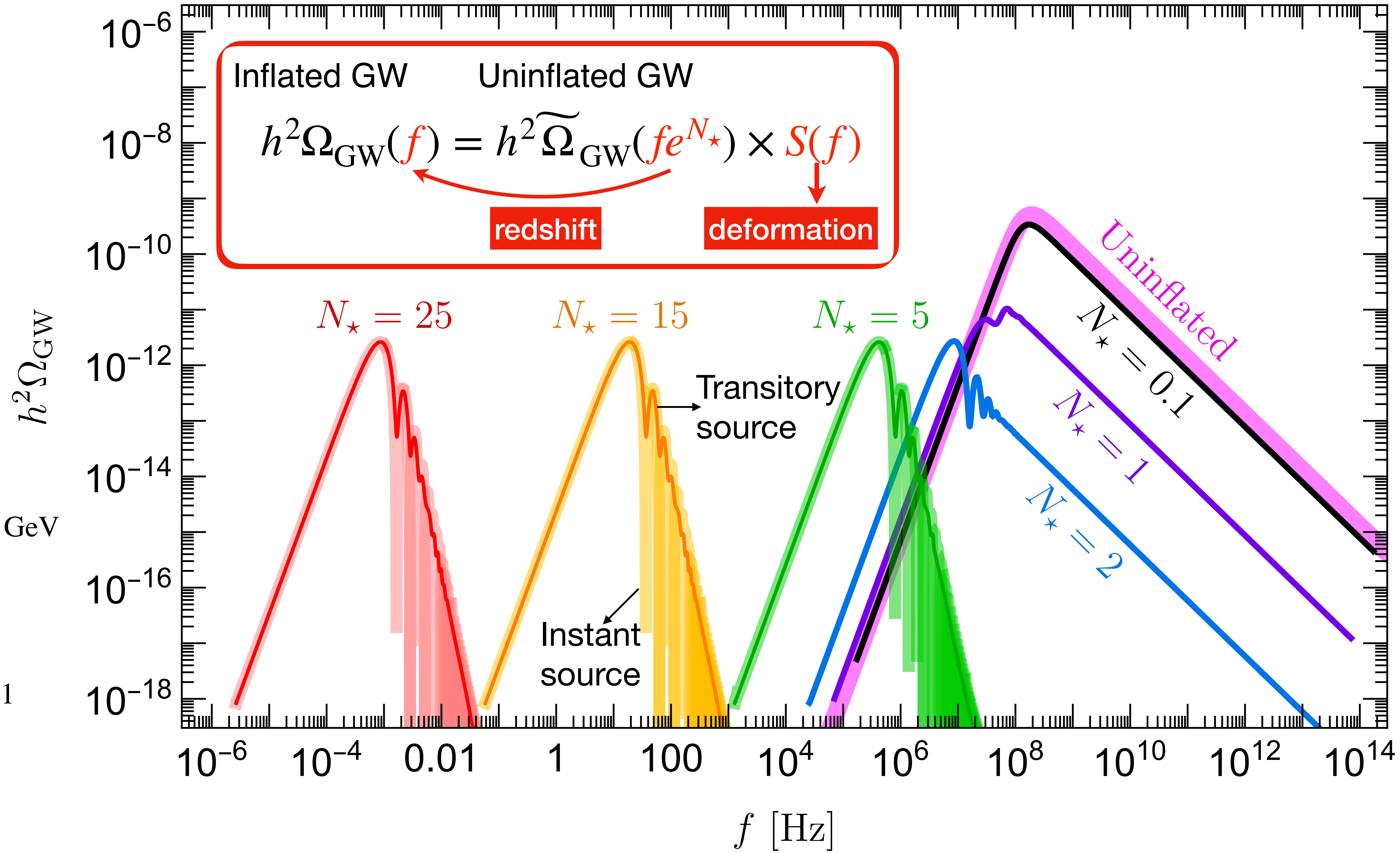
Derived in [An et al, 2009.12381]

$$S_1(f) = y\epsilon \times \left\{\left[\frac{1}{y^2} + 2\epsilon - 1\right]\frac{\sin[2y(1-\epsilon)]}{y^4} - \left[\frac{2-\epsilon}{y^2} + \epsilon\right]\frac{\cos[2y(1-\epsilon)]}{y^3} + \frac{\epsilon^3}{y}\left(\frac{1}{y^2}+1\right)\right\}$$

Our correction

Transitory-source approximation (短时源近似) $S(f) \to \bar{S}(f) = \frac{1}{\Delta_y}\int_{\bar{y}-\Delta_y/2}^{\bar{y}+\Delta_y/2} \mathrm{d}y\, S(f)\Big|_{\bar{y}=\frac{2\pi a_0 f}{a_\star H_\star}}$ $\Delta_y = \frac{a_\star \Delta_\tau}{1/H_\star}\bar{y}$

GW spectrum: inflated vs uninflated



$T_{\rm Rh} = 10^{15}$ GeV

$\frac{\beta}{H_\star} = 5$

$\frac{\rho_{\rm PT}}{\rho_{\rm tot}} = 0.1$

Three bands of frequencies

	Infrared (IR)	Ultraviolet (UV)	Far Ultraviolet (FUV)
Wave length	$\lambda_\star = \frac{2\pi a_\star}{k} \gg H_\star^{-1}$	$\lambda_\star \ll H_\star^{-1} \ll \lambda_\star \frac{a_{\rm Rh}}{a_\star}$	$\lambda_\star \frac{a_{\rm Rh}}{a_\star} \ll H_\star^{-1}$

A quantity with subscript ★ means its value during phase transition, which happens during the inflation

The Hubble rate is the same as that during inflation $H_\star = H_{\rm Inf}$

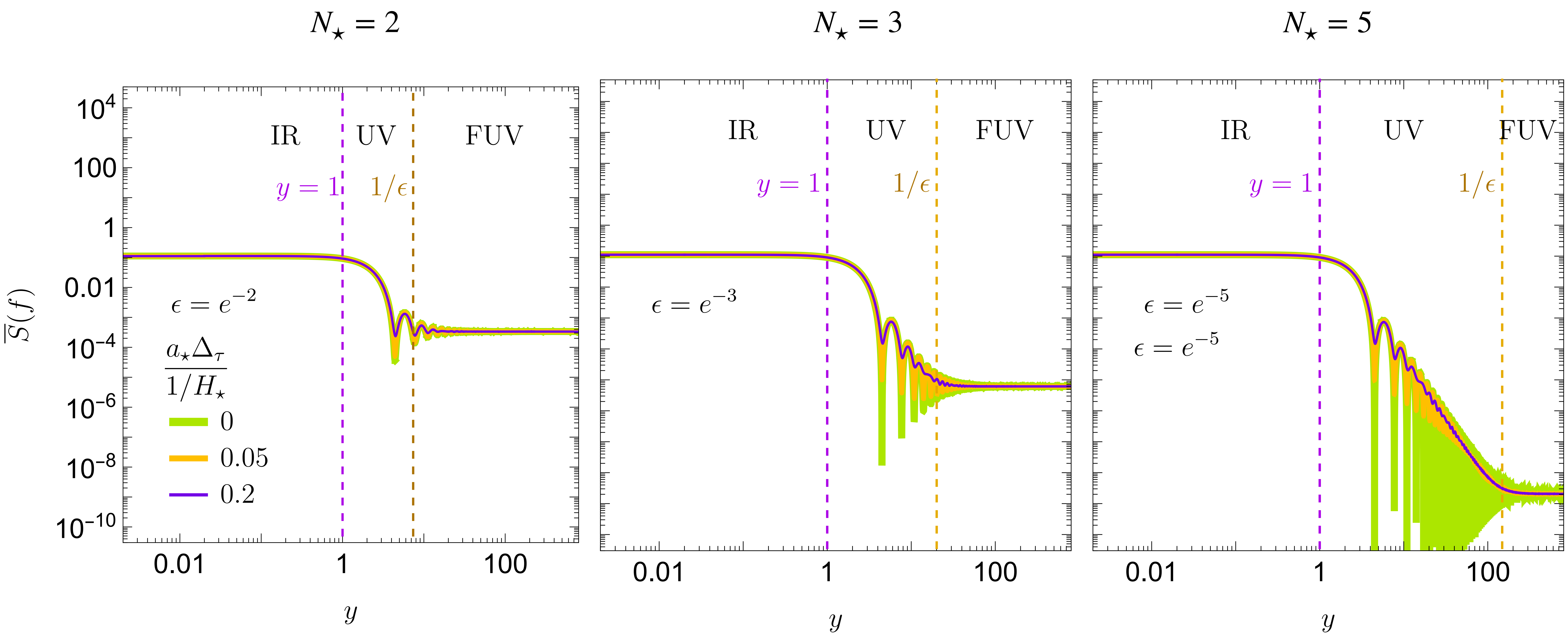
The coming factor is much smaller than that at reheating $a_\star = a_{\rm Rh} e^{-N_\star}$

	Infrared (IR)	Ultraviolet (UV)	Far Ultraviolet (FUV)
Co-moving momentum	$k \ll a_\star H_\star$	$a_\star H_\star \ll k \ll a_{\rm Rh} H_\star$	$k \gg a_{\rm Rh} H_\star$
Frequency today	$f \ll \frac{a_\star H_\star}{2\pi a_0}$	$\frac{a_\star H_\star}{2\pi a_0} \ll f \ll \frac{a_{\rm Rh} H_\star}{2\pi a_0}$	$f \gg \frac{a_{\rm Rh} H_\star}{2\pi a_0}$

GW propagation to the radiation domination (RD)

	Infrared (IR)	Ultraviolet (UV)	Far Ultraviolet (FUV)
	$h_{\rm RD}^{\rm IR} \simeq \frac{1}{k}\frac{a_{\rm Rh}^2}{a(t)} \times \frac{1}{3} \sin\left(k\tau - \frac{k}{a_{\rm Rh} H_\star}\right)$	$h_{\rm RD}^{\rm UV} \simeq \frac{-1}{k}\frac{a_{\rm Rh}^2}{a(\tau)} \frac{\cos[y(1-\epsilon)]}{y^2} \sin(k\tau - y\epsilon)$	$h_{\rm RD}^{\rm FUV} \simeq \frac{1}{k}\frac{a_\star^2}{a(t)} \sin[k\tau + y(1-2\epsilon)]$
Deformation function	$S(f)^{\rm IR} \simeq \frac{1}{9}$	$S(f)^{\rm UV} \simeq \frac{\cos[2y(1-\epsilon)]+1}{2y^4}$	$S(f)^{\rm FUV} \simeq \frac{a_\star^4}{a_{\rm Rh}^4}$
	Frozen	Oscillation and partial dilution	Dilution as radiation

Deformation function in three regimes



Source of GWs during phase transition

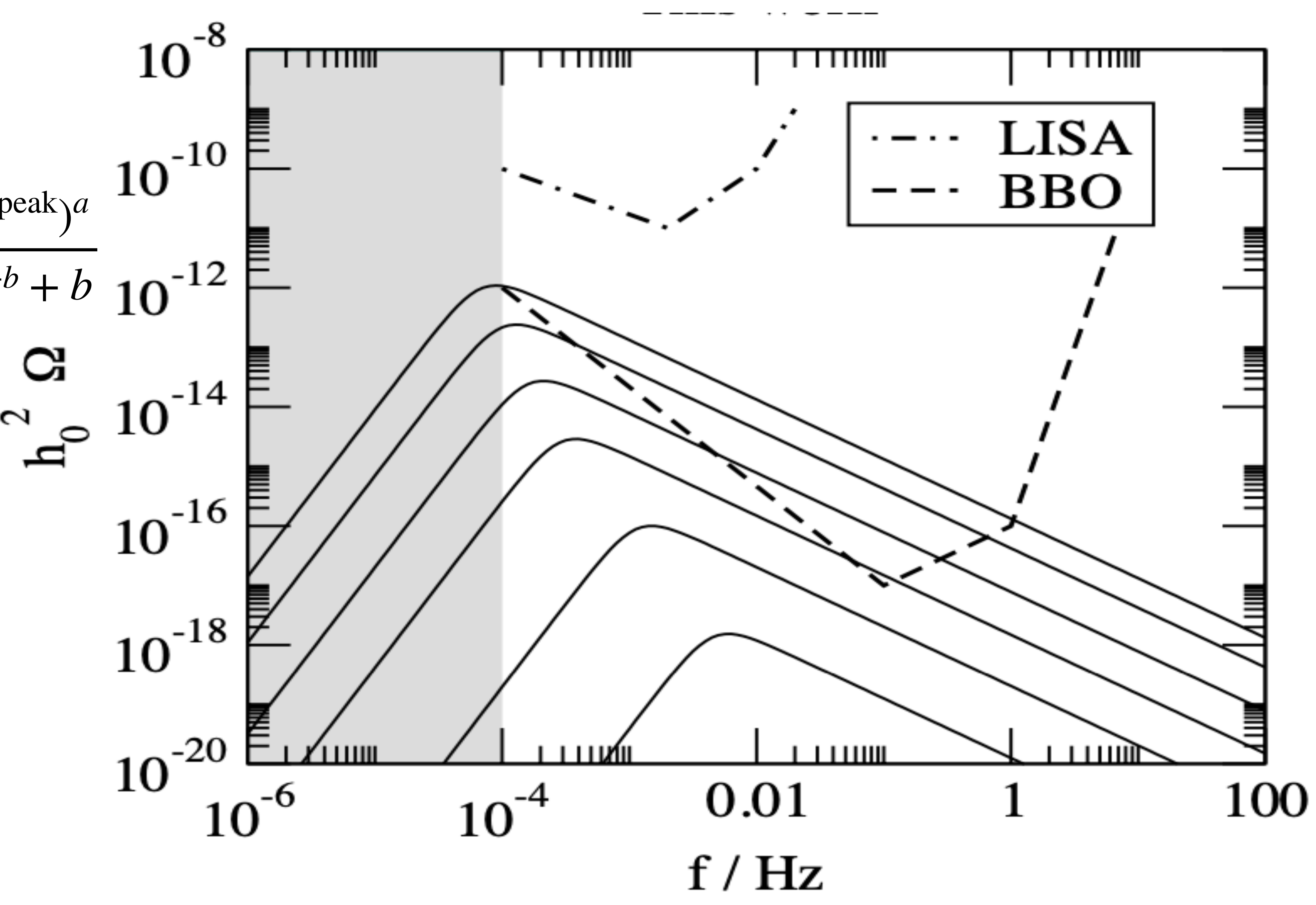
- GWs via bubble collisions in FOPT

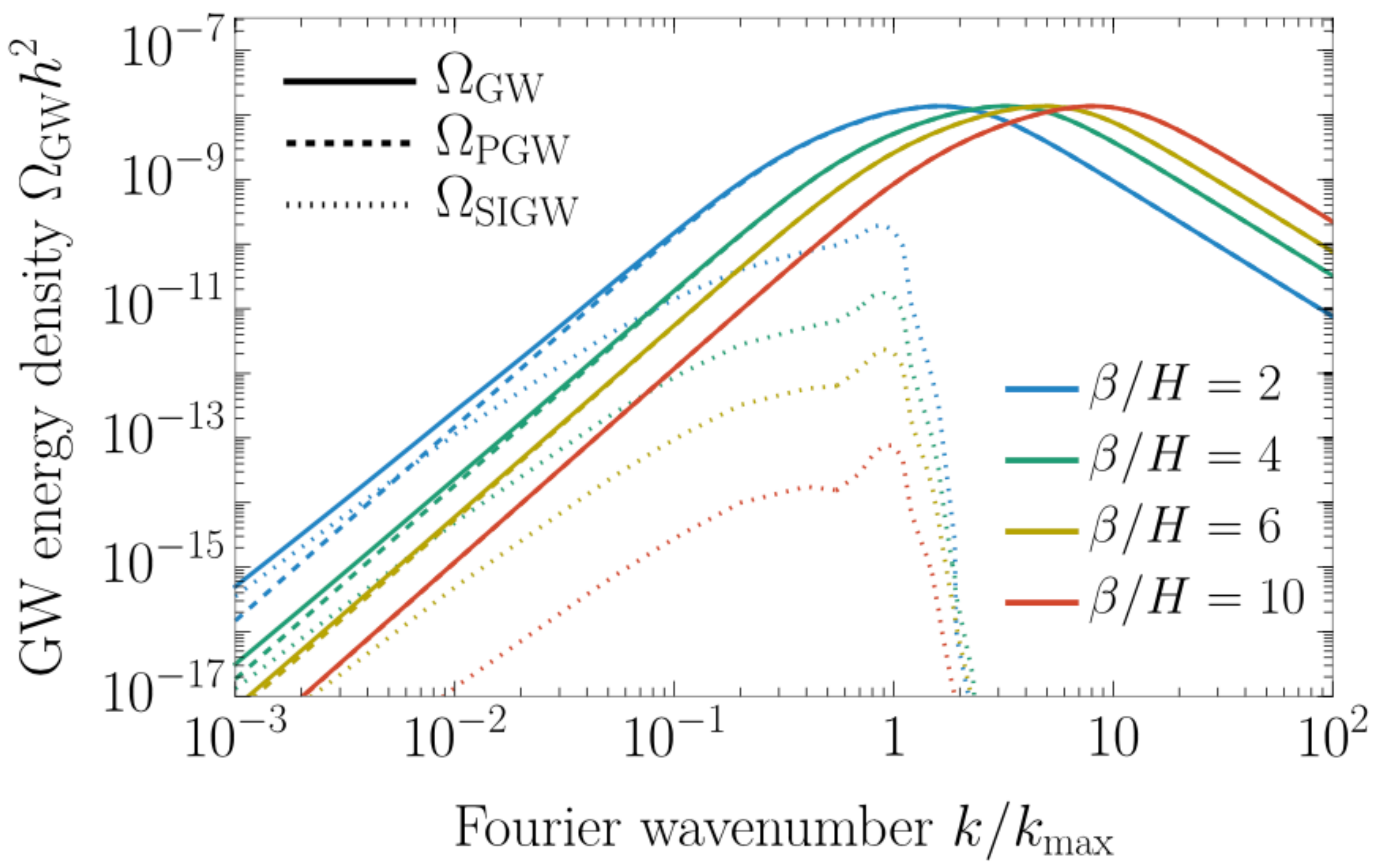
$$h^2\widetilde{\Omega}_{\rm GW}(\tilde f) = 1.27\times 10^{-6}\times\left(\frac{H_\star}{\beta}\right)^2\left(\frac{\rho_{\rm PT}}{\rho_{\rm tot}}\right)^2\left(\frac{100}{g_\star}\right)^{1/3}\times\frac{(a+b)(\tilde f/\tilde f^{\rm peak})^a}{a(\tilde f/\tilde f^{\rm peak})^{a+b}+b}$$

$$\tilde f^{\rm peak} = 37.8\ {\rm MHz}\times\left(\frac{\beta}{H_\star}\right)\left(\frac{T_\star}{10^{15}\ {\rm GeV}}\right)\left(\frac{g_\star}{100}\right)^{1/6}$$

$$a = 2.8,\ b = 1$$

Kosowsky, Turner, Watkins, PRD, 92;
Huber, Konstandin, 0806.1828





- Scalar-induced GWs

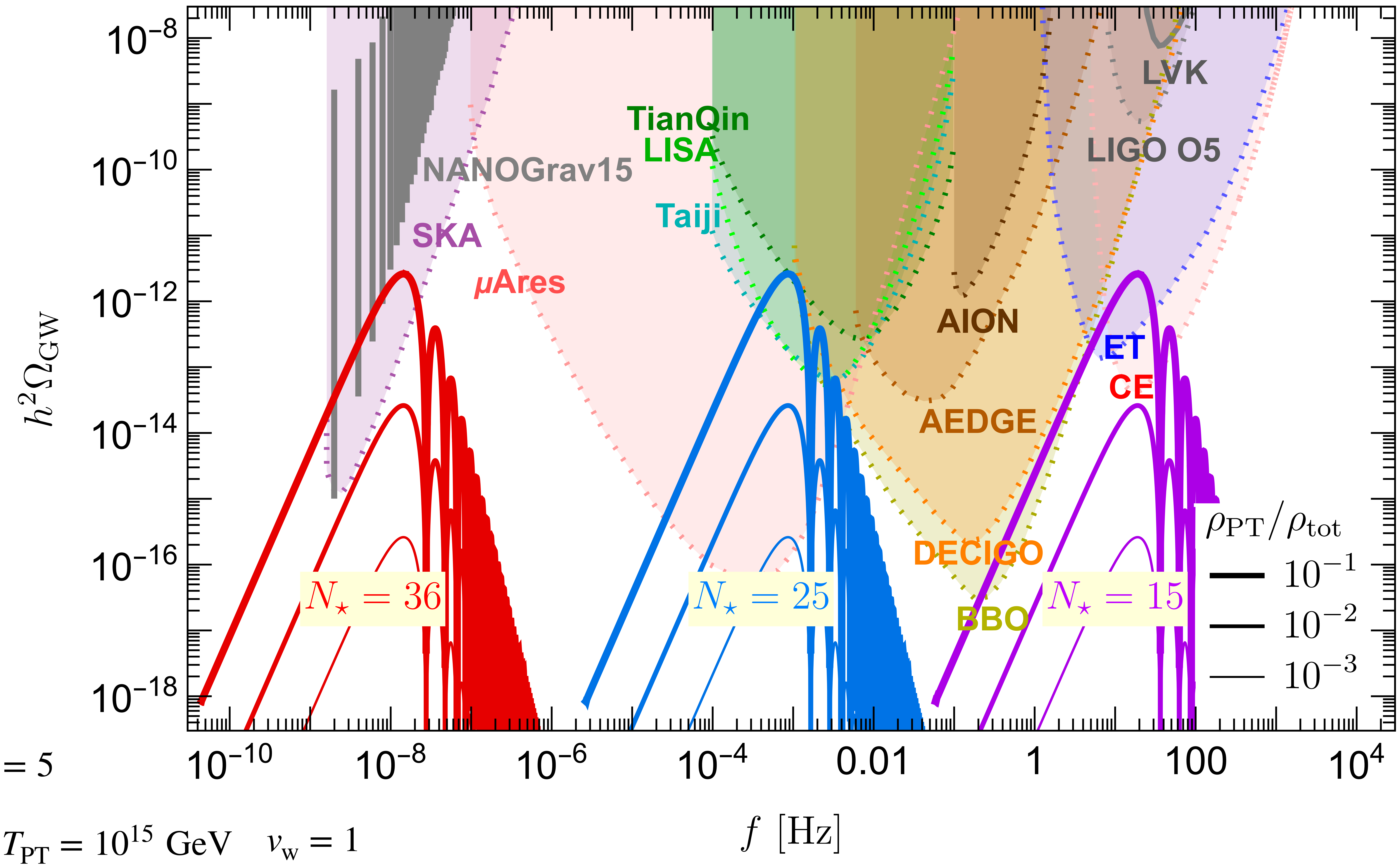
Gauge dependence should be properly accounted

Franciolini, Gouttenoire, Jinno, 2503.01962

$\Longrightarrow$ Jinno's talk in BPCS 2025

Inflated GWs via GUT phase transition

Instant source approximation

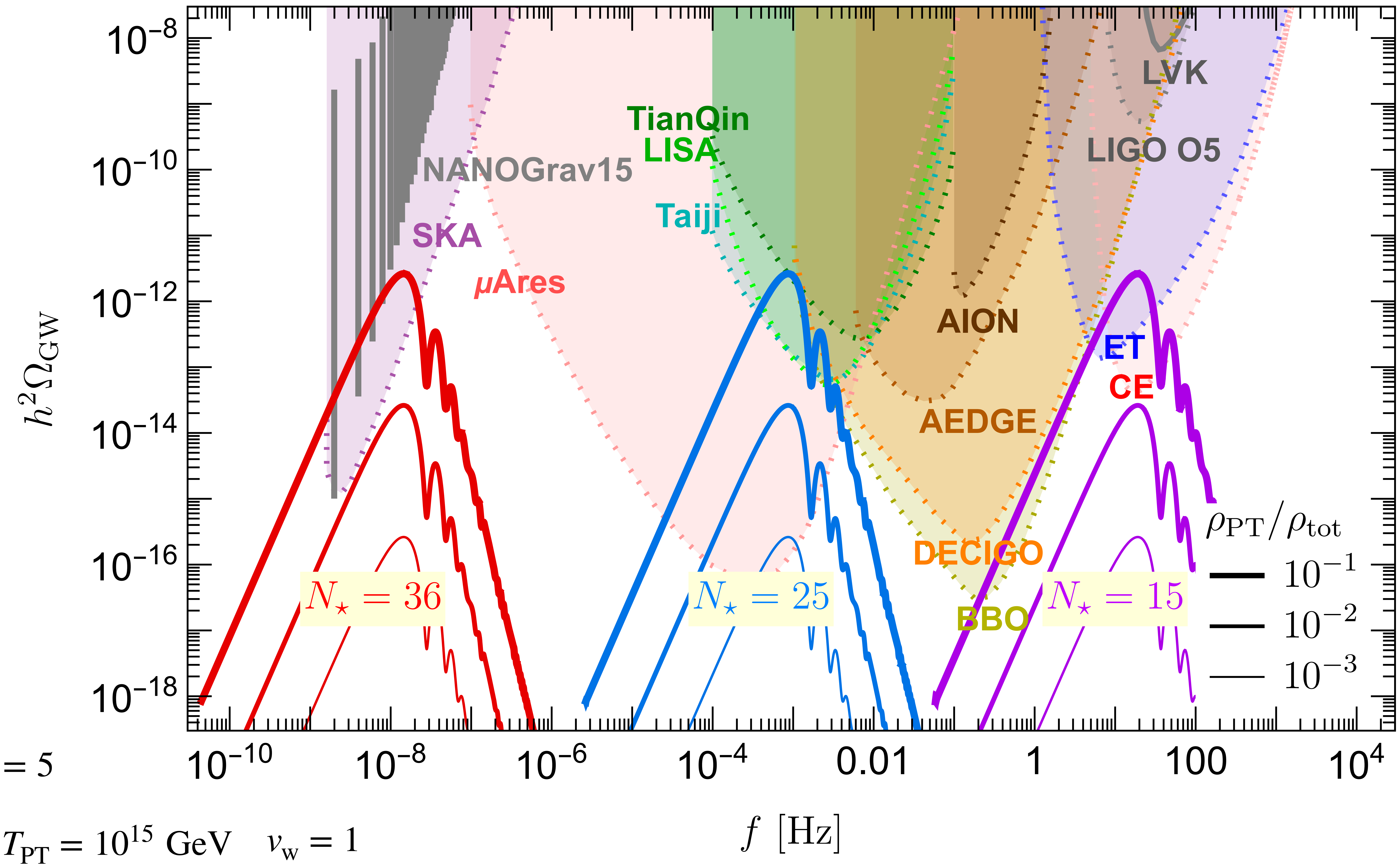


$\beta/H_\star = 5$

$T_\star = T_{PT} = 10^{15}$ GeV $\quad v_w = 1$

Inflated GWs via GUT phase transition

Transitory source approximation

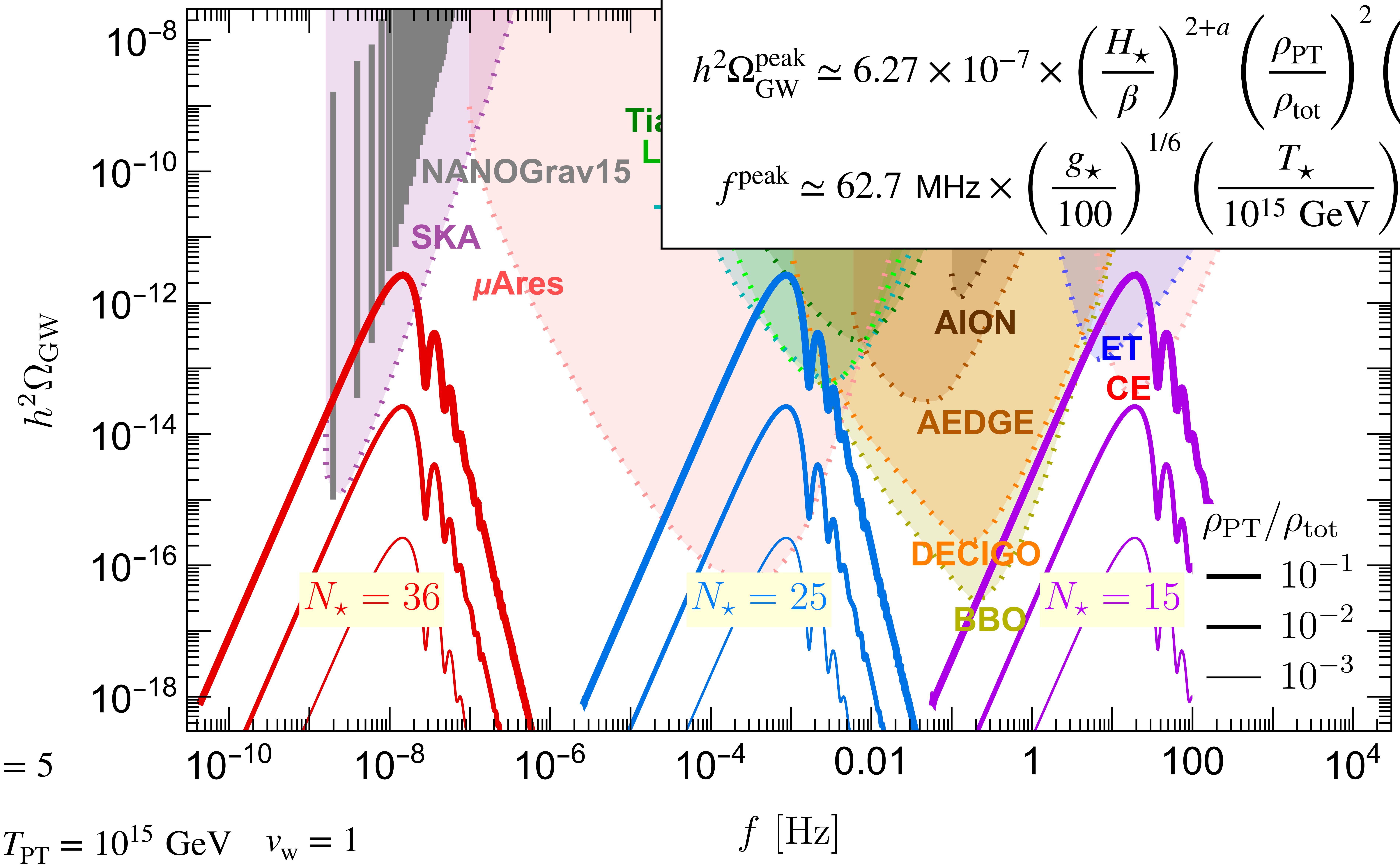


$\beta/H_\star = 5$

$T_\star = T_{\rm PT} = 10^{15}$ GeV $\quad v_{\rm w} = 1$

Inflated GWs via GUT phase transition

Transitory source approximation



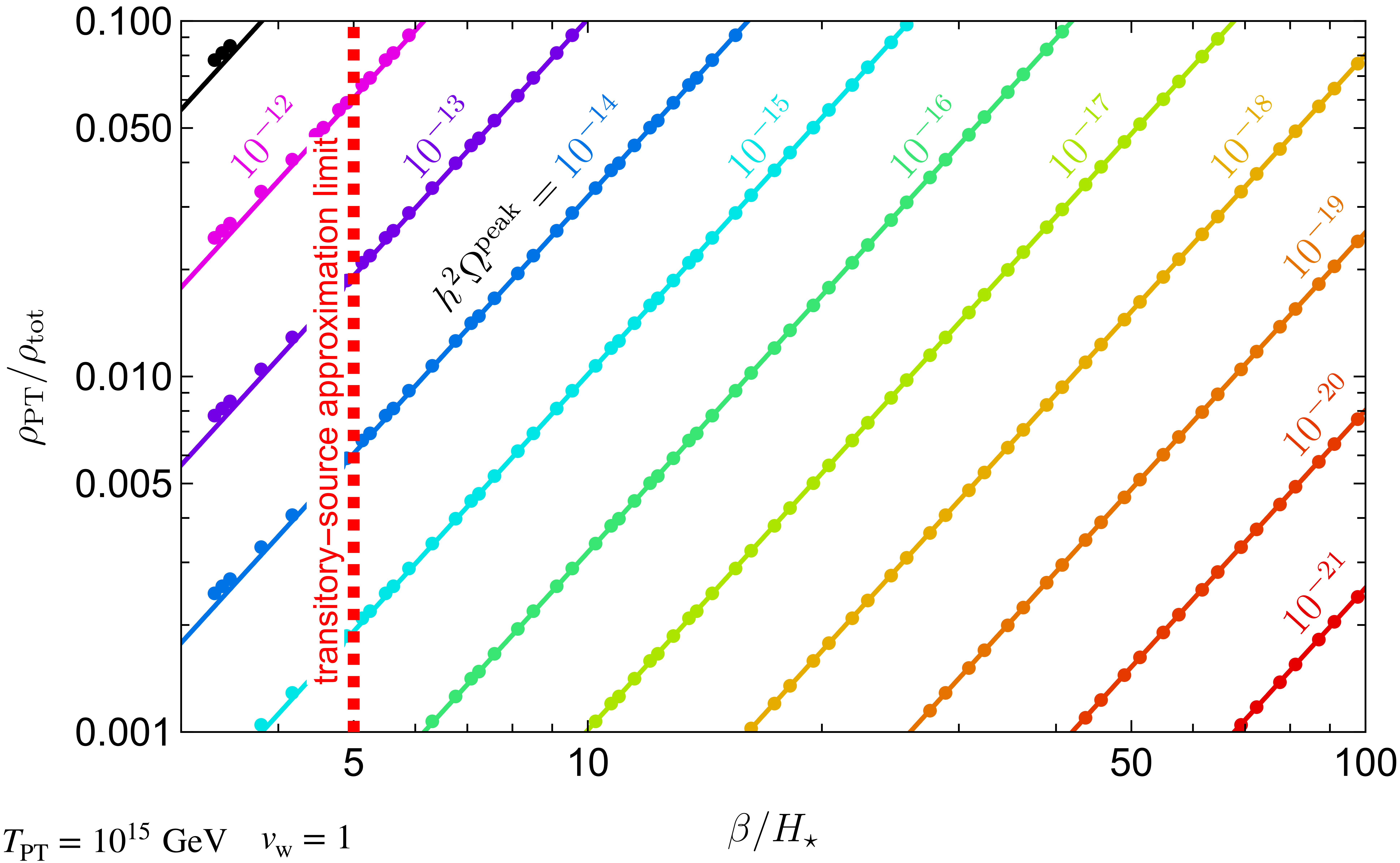
$$h^2\Omega_{GW}^{peak} \simeq 6.27\times10^{-7}\times\left(\frac{H_\star}{\beta}\right)^{2+a}\left(\frac{\rho_{PT}}{\rho_{tot}}\right)^2\left(\frac{100}{g_\star}\right)^{1/3}$$

$$f^{peak} \simeq 62.7\ \text{MHz}\times\left(\frac{g_\star}{100}\right)^{1/6}\left(\frac{T_\star}{10^{15}\ \text{GeV}}\right)\times e^{-N_\star}$$

$\beta/H_\star = 5$

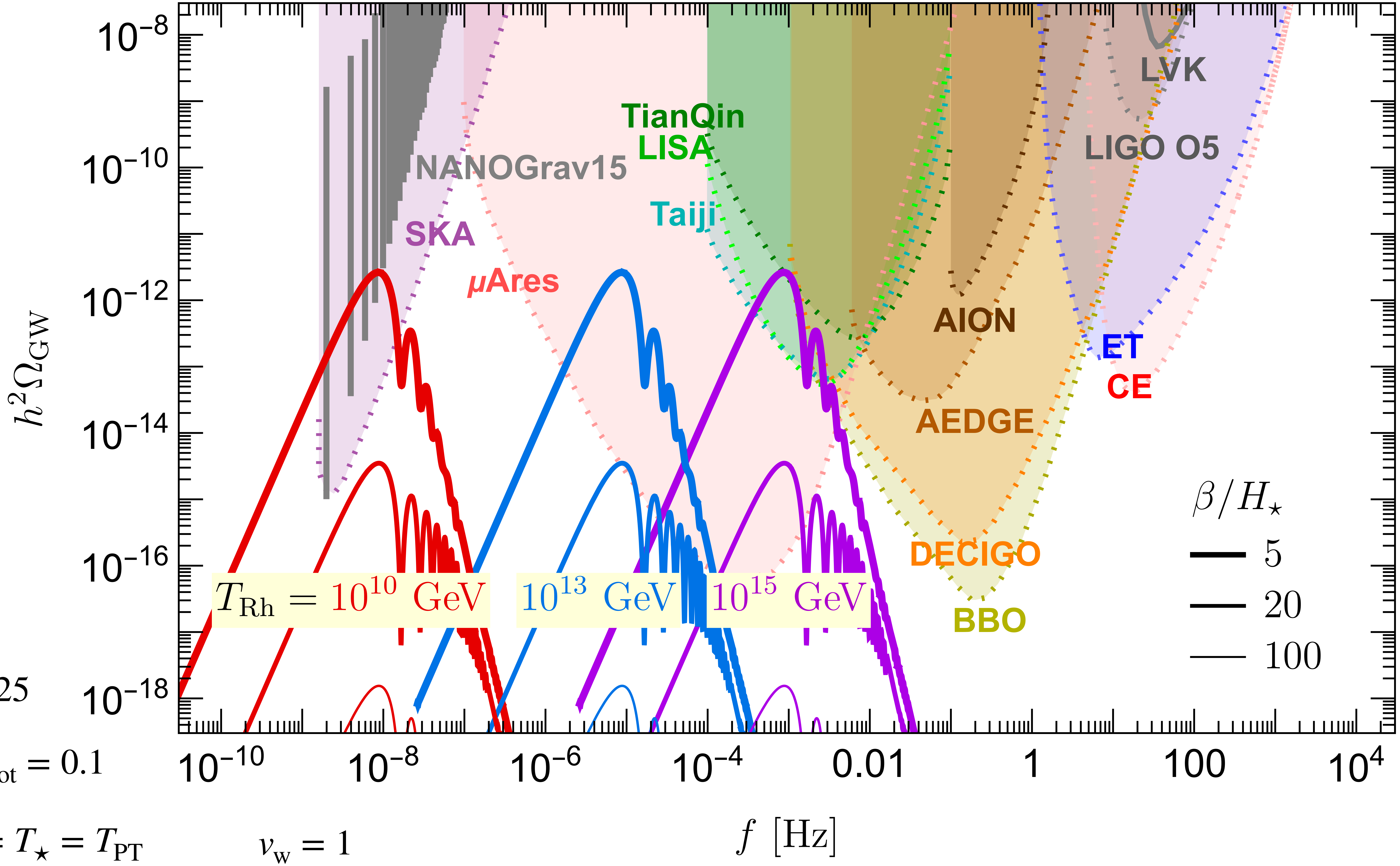
$T_\star = T_{PT} = 10^{15}$ GeV $v_w = 1$

Testable region respect to the peak of GW spectrum



$T_\star = T_{\rm PT} = 10^{15}$ GeV $\quad v_{\rm w} = 1$

Application to other ultra-high energy physics not at GUT scale

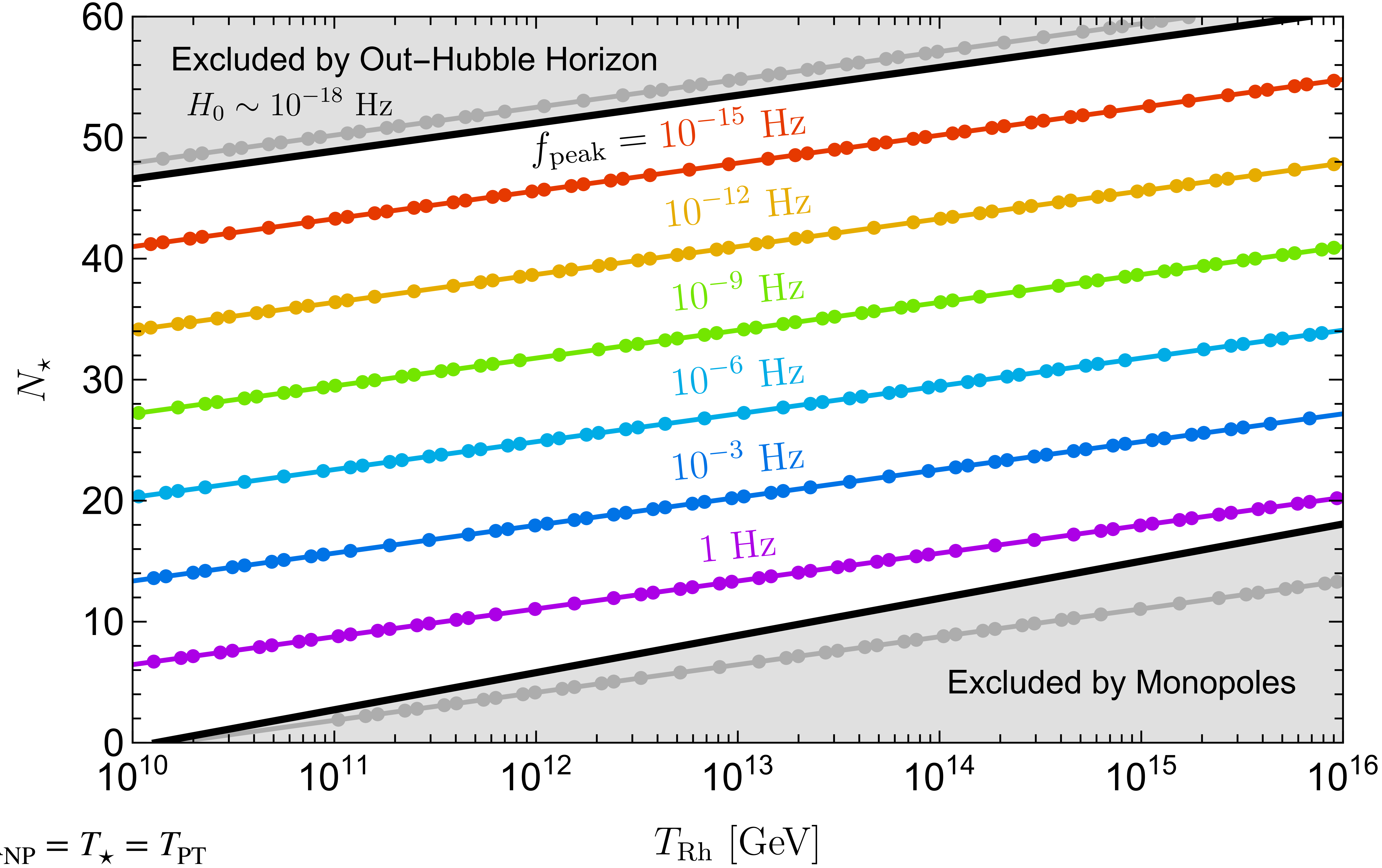


$N_\star = 25$

$\rho_{\rm PT}/\rho_{\rm tot} = 0.1$

$\Lambda_{\rm NP} = T_\star = T_{\rm PT}$ $\quad$ $v_{\rm w} = 1$

Application to other ultra-high energy physics not at GUT scale



$\Lambda_{\rm NP} = T_\star = T_{\rm PT}$

Conclusion

- We present a scenario that GUT phase transition can be directly tested in GW observation

  It happens during inflation
  The phase transition is strong first order

- We develop the formalism of GWs via phase transition during inflation

  Both instant- and transitory-source approximations are considered
  Consistent with [An et al], but generalised to PT at any e-folds before before the end of inflation

- We specify three frequency bands: IR, UV and FUV.

  IR: The GW metric is frozen
  UV: GW spectrum oscillates and is partially diluted
  FUV: GW is high diluted, similar to radiation

- The mechanism is applied to GUT phase transition

  GW source: bubble collision
  Testable regimes: e-folds 15 → 36, 100 Hz → nHz; $\rho_{PT}/\rho_{tot} > 10^{-3}$, $5 < \beta/H_\star < 100$

- Outlook: explicit model building, application to another ultra-high-energy physics ...

@千岛湖 Qiandao Lake

Thanks and questions!