Beyond Symmetry-Reduced Models:

Bouncing Cosmology, Regular Black Holes and Gravitational Waves from a 4d Theory Perspective

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Based on

- Ferrero, Han, Hongguang Liu, One-loop effective action from the coherent state path integral of loop quantum gravity, Phys.Rev.D 112 (2025) 2, 024033, arXiv: 2502.07696

- K. Giesel, Hongguang Liu, P. Singh and S. Weigl, Regular black holes and their relationship to polymerized models and mimetic gravity, Phys.Rev.D 111 (2025) 6, 064064, arXiv: 2405.03554

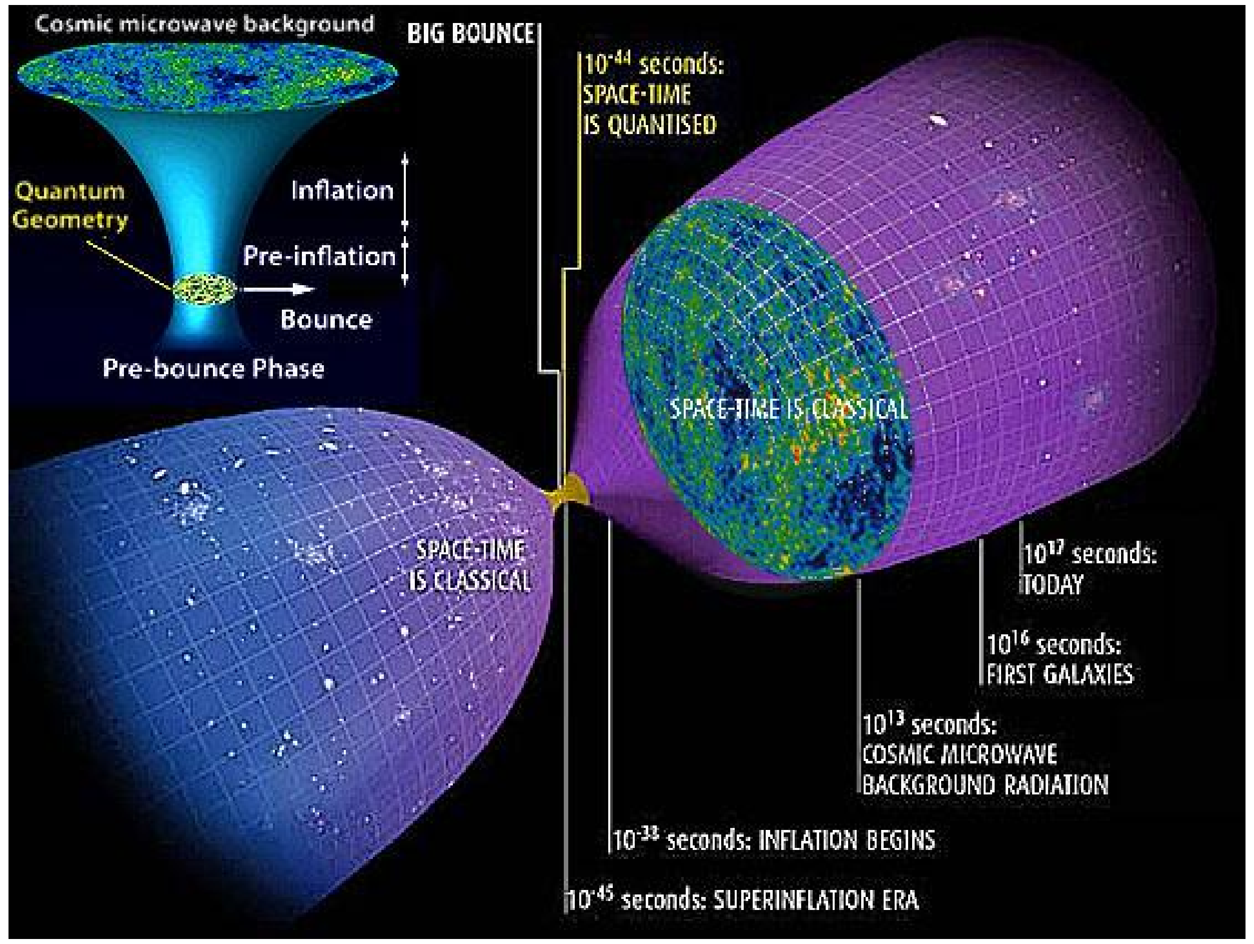
- K. Giesel, Hongguang Liu, E. Rullit, P. Singh and S. Weigl, Embedding generalized Lemaître-Tolman-Bondi models in polymerized spherically symmetric spacetimes, Phys.Rev.D 110 (2024) 10, 104017, arXiv: 2308.10949

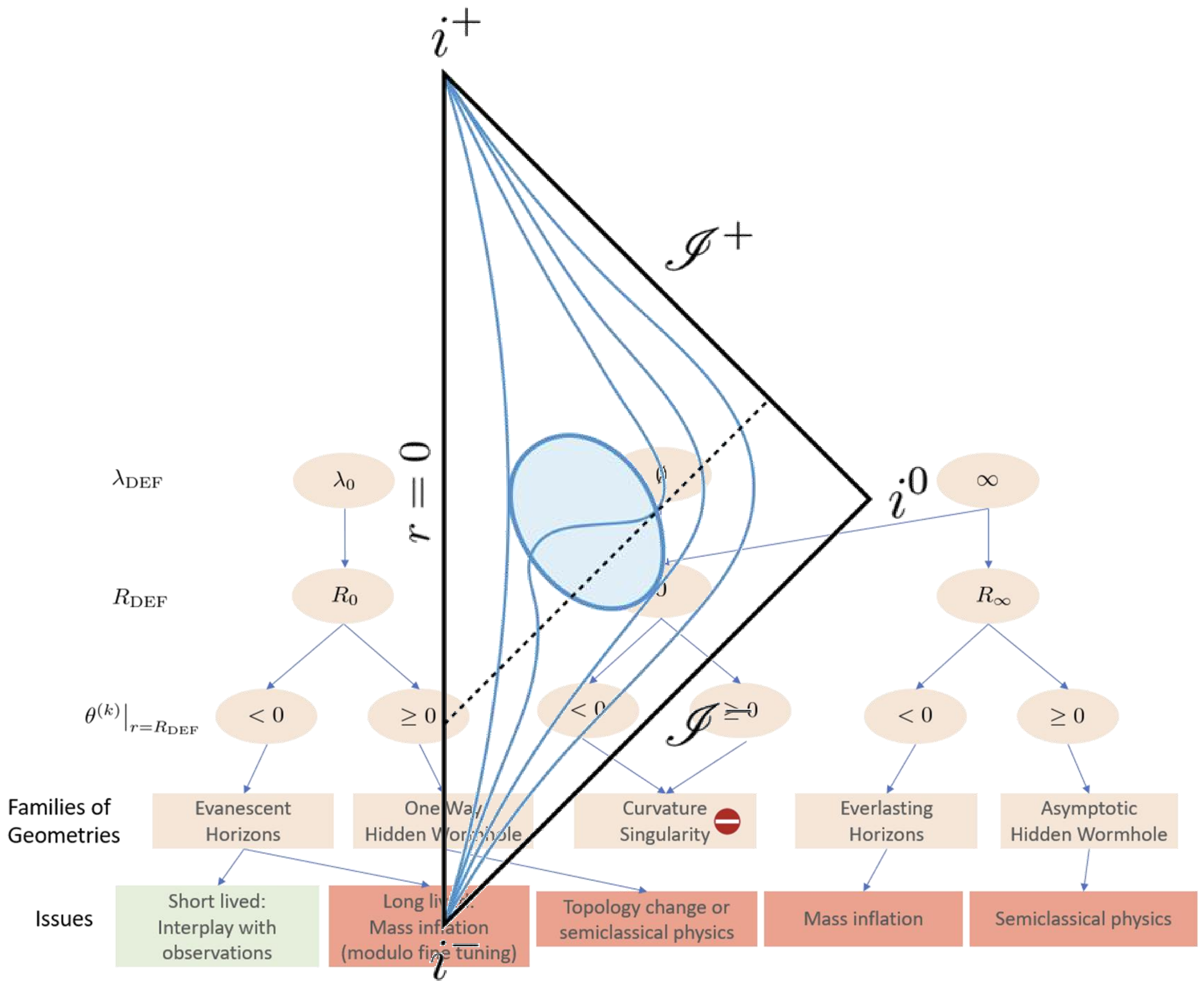
- M. Han, Hongguang Liu, Covariant μ-scheme effective dynamics, mimetic gravity, and nonsingular black holes: Applications to spherically symmetric quantum gravity, Phys.Rev.D 109 (2024) 8, arXiv: 2212.04605

- J. B. Achour, F. Lamy, Hongguang Liu, K. Noui, Non-singular black holes and the Limiting Curvature Mechanism: A Hamiltonian perspective, JCAP 05 (2018) 072, arXiv: 1712.03876

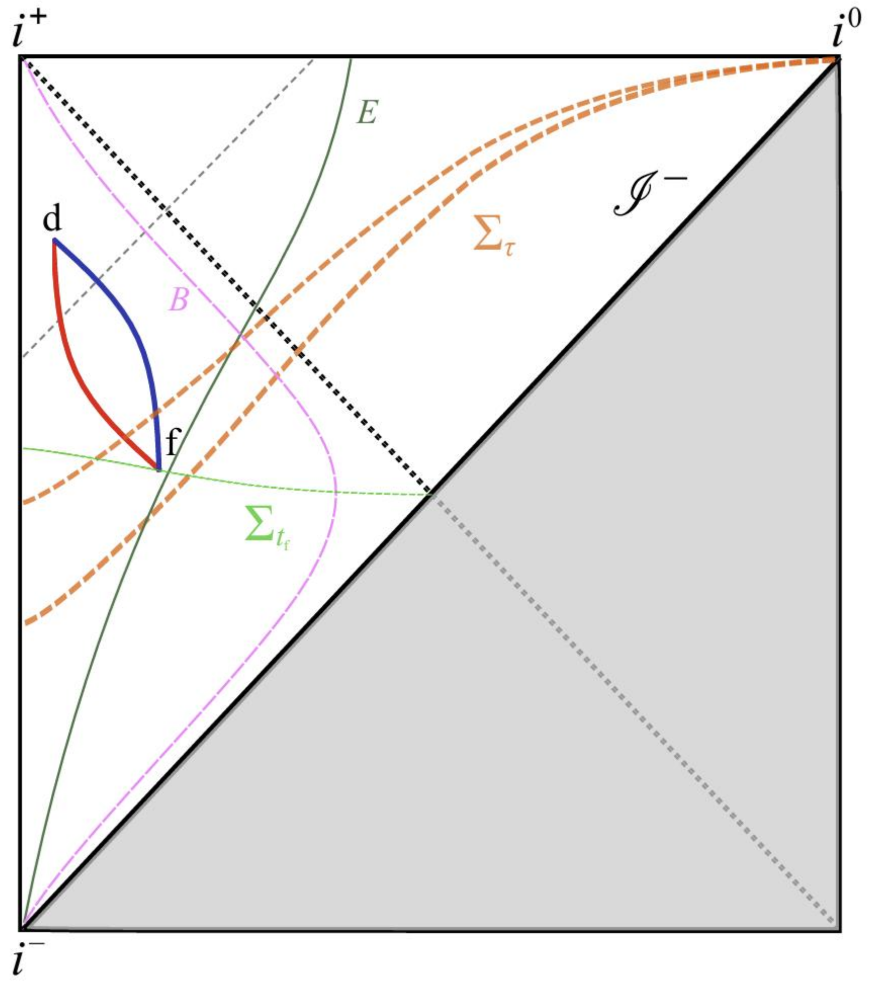
- D. Langlois, Hongguang Liu, K. Noui, E. Wilson-Ewing, Effective loop quantum cosmology as a higher-derivative scalar-tensor theory, Class.Quant.Grav. 34 (2017) 13, 135008, arXiv: 1702.06793

Bouncing cosmology & Physical regular BHs





[R. Carballo-Rubio， F。 Di Filippo, S. Liberati, M. Visser 20'



I Soranidis, D. R. Terno 25'

Bouncing cosmology

Physical BHs: geodesically compelete and formation of trapped region in finite time (distant observer)

These are all symmetry reduced models in 1D or 2D, where they do not have gravitational waves

Physical regular BHs

Can we recontruct a covariant theory (Lagrangian) given a BH metric?

Yes, and it quite easy in 2D (spherically symmetric case, symmetry reduced models)

$$\mathrm{d}s^2 = A(r)\mathrm{d}t^2 + \frac{1}{A(r)}\mathrm{d}r^2,$$

Vacuum spherical solution

$$I_b = -\frac{1}{2}\int \mathrm{d}^2x\sqrt{g}\,(\phi R + W(\phi)) \qquad A' = W(r)$$

Generalized 2D Dilaton gravity

[Witten 20']

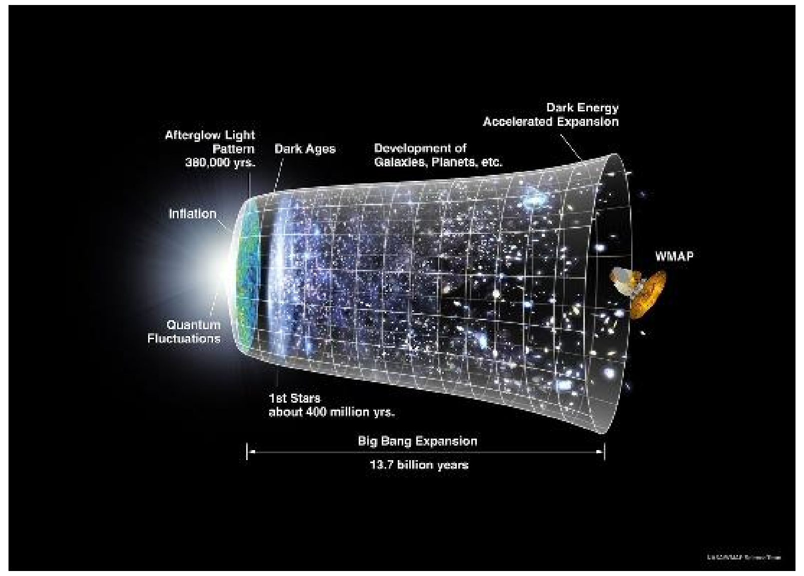
Topological theory: All solutiones are characterised by a constant (BH mass): Birkhoff theorem (Uniqueness)

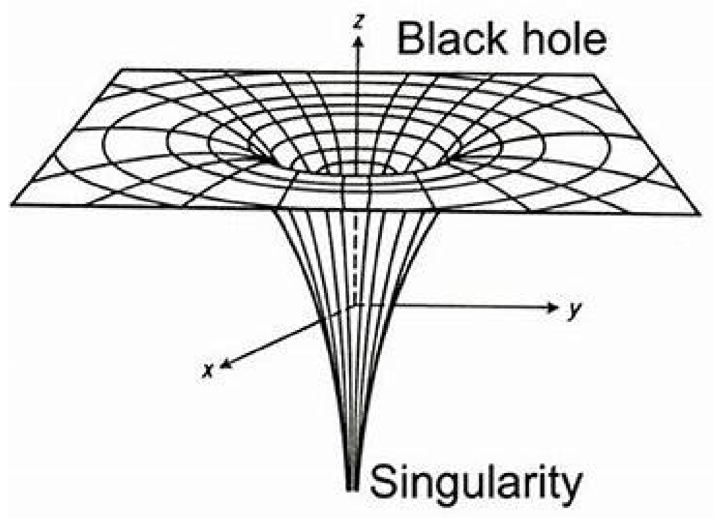
However, this can not be lifted directly to 4D: there is no gravitational wave!!

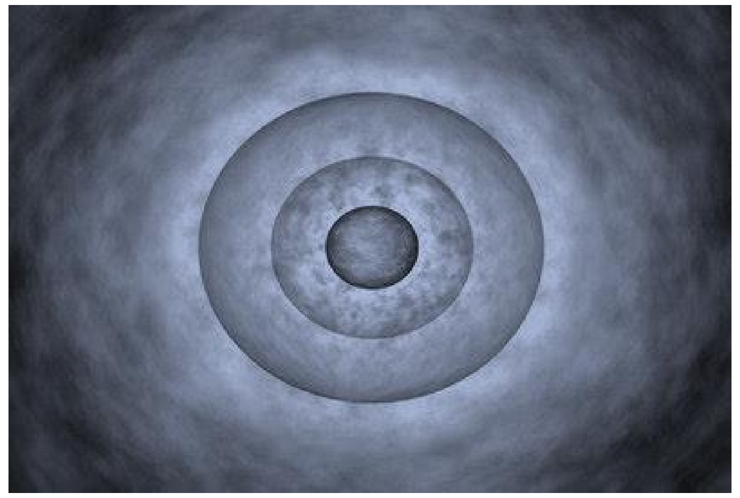
In 2D, LoveLock theorem is not that restrictive, one do have not trivial LoveLock gravity, which is **not** ture in 4D

Warm up: what happens in GR

FLRW cosmology = Spherically symmetric stationary BH = Spherically symmetric star (dust) collapse = Kerr BH







e.g. Neuman-Janis algorithm

Dynamics of scale factor a(t)
No field d.o.f
(homogeneity)

No field d.o.f
because of symmetry (time translation)

1+1d field theory
field d.o.f !

$$ds^2 = -dt^2 + \frac{(R')^2(t,x)}{1+\mathcal{E}(x)}dx^2 + R^2(t,x)d\Omega^2$$

Lemaître–Tolman-Bondi (LTB) spacetime

[Lemaître ‘33; Tolman ‘34; Bondi ‘47

One-loop divergenges in perturbative QFT

Can we have a 4d covariant theory given certain (physical) BH metrics

[Torre 93’, Brown and Kuchaˇr 94’
S. Giddings, 25

From QG point view:

- extra matter fields as “clock” (relational framework, no observable in pure GR)
- Infinie higher order couplings encoded in a general (non-polynomial)function (perturbatively non-renormalizablity <==> non-perturbatively renormalzablility)

[Goroff and Sagnotti

Consistency requirement:

- Unique lift from flat cosmology to BHs
- A definition of “modified” vacuum
- A kind of Birkhoff theorem (Uniqueness of “modified” vacuum solution)
- Allows (dust) collapse and BH formation (Physical BHs)

This can be achived in a special class of extended Mimetic gravity

Extended Mimetic Gravity

$\phi_\mu = \nabla_\mu \phi, \phi_{\mu\nu} = \nabla_\mu \nabla_\nu \phi$

$\chi_1 = \phi^\mu_\mu$

$\chi_n = [\phi^\nu_\mu]^n$

$$S[\tilde{g}_{\mu\nu}, \phi] = \frac{1}{8\pi G} \int_{\mathcal{M}_4} \mathrm{d}^4x \sqrt{-g} \left[\tfrac{1}{2}\mathcal{R}^{(4)} + L_\phi(\phi, \chi_1, \cdots, \chi_n)\right]$$

Why mimetic? $g_{\mu\nu} = -\widetilde{X}\widetilde{g}_{\mu\nu}, \quad \widetilde{X} = \widetilde{g}^{\mu\nu}\phi_\mu\phi_\nu$ Conformally invaraint for $S[\widetilde{g}_{\mu\nu}, \phi]$

Extra conformal gauge symmetry removes extra d.o.f from higher derivatives

Scalar tenosr theory (only propagates 2 (gravity) +1 (scalar) d.o.f., beyond Hordonski, subclass of DHOST)

$$S[g_{\mu\nu}, \phi, \lambda] = \frac{1}{8\pi G} \int_{\mathscr{M}_4} \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2}\mathcal{R}^{(4)} + L_\phi(\phi, \chi_1, \cdots, \chi_n) + \frac{1}{2}\lambda(\phi_\mu\phi^\mu + 1)\right]$$

higher derivative coupling

mimetic condition

Non rotational dust

contraction of higher order derivatives

Only **2** independent terms in spherically symmetric spacetime

dust density

$\lambda:$ lagrangian multiplier

[Chamseddine, Mukhanov 13,16
[Langlois, HL, Noui, Wilson-Ewing, 17
[Langlois, Mancarella, Noui, Vernizzi, 18
[Han, HL, 22
[Giesel, HL, Singh, Weigl, 24-25
....

Extended Mimetic Gravity

ϕ oplays the role of "time" and gives physical observables in QG: reduced phase space quantization

$$S[h_{ij}] = \frac{1}{8\pi G}\int_{\mathcal{M}_4} \mathrm{d}t\mathrm{d}^3x\sqrt{h}\left[\frac{1}{2}\left(K_{ij}K^{ij} - K^2 + \mathcal{R}^{(3)}\right) + L_\phi\left(K, K_{ij}K^{ij}, \cdots, [K]^n\right)\right]$$

Similar to LQC, but as 4d covariant

Spatially covariant theory which leads to a **physical** Hamiltonian **H**

allow to define the Euclidean path integral $e^{-\beta H}$

H is the one for Lorentzian gravity instead of Euclidean ones (they are different!!)

dust density

Modified Einstein Eq: $G^{(\alpha)}_{\mu\nu} := G_{\mu\nu} - T^{\phi}_{\mu\nu} = -\lambda\partial_\mu\phi\partial_\nu\phi$ Non rotational dust energy-momentum

Effective Einstein tensor

higher derivative coupling (possible QG effect)

"modified" vacuum: $\lambda = 0$

What about the **reconstruction**?

⇩

Conservation of dust energy-mome ntum $\nabla_\mu T^{\mu\nu} = \nabla_\mu(\lambda\, \phi^\mu \phi^\nu) = 0$

⇩

$k = 0$ cosmological dynamic = Sationary (vacuum) solution of BHs $\mathrm{d}s^2 = -(1-\mathcal{G}_{(i)}(r)^2)\mathrm{d}\tau^2 + \frac{1}{(1-\mathcal{G}_{(i)}(r)^2)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$ = $k = 0$ dust collapse

2D-dilaton-mimetic gravity dynamics (not unique)

⇩

Uniqueness of "modified" vacuum

⇩

Fix ambiguities in LQC

unique $k \neq 0$ cosmological dynamic = $k \neq 0$ dust collapse

Unique 2D-dlaton-mimetic gravity dynamics(as 1+1 field theory)

⇩

4d extended mimetic gravity

Background field method and effective action

$$S[\tilde{g}_{\mu\nu},\phi] = \frac{1}{8\pi G}\int_{\mathcal{M}_4} \mathrm{d}^4x\sqrt{-g}\left[\tfrac{1}{2}\mathcal{R}^{(4)} + L_\phi(\phi,\chi_1,\cdots,\chi_n)\right]$$

$$\tilde{g}_{\mu\nu} = e^{2\omega}\tilde{\tilde{g}}_{\mu\nu}$$

Conformal gauge

+

$$\tilde{\tilde{g}}_{\mu\nu} = \bar{\tilde{g}}_{\mu\nu} + h_{\mu\nu} \qquad \phi = \overline{\phi} + \delta\phi$$

$$h_{\mu\nu} = h^{\mathbb{T}}_{\mu\nu} + \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu - \frac{1}{2}g_{\mu\nu}\nabla_\mu\xi^\mu + \frac{1}{4}g_{\mu\nu}h$$

Background field method + York decomposition

Hessian matrix

$$\left[\Gamma^{(2)}\right]^{ij} = \frac{\delta^2(S+S_{\text{measure}})}{2\delta\varphi\delta\varphi} - M_{\text{ghost}}$$

$$\left[\Gamma^{(2)}_k\right]^{ij} = \underbrace{\mathbb{K}_i(\Delta)\,\delta^{ij}\,\mathbf{1}_i}_{\text{kinetic terms}} + \underbrace{\mathbb{D}^{ij}(D_\mu)}_{\text{uncontracted derivatives}} + \underbrace{\mathbb{M}^{ij}(R,D_\mu)}_{\text{background curvature}},$$

STrace can be calculated by off-diagonal heat-kernal method, e.g. [Groh, Saueressig, Zanusso 11

Beyond Effective description:
Gravity + matter in reduced phase space

Matter couplings as reference frame:

$$\mathcal{L}_{\text{GD}} = \frac{1}{2}\sqrt{|g|}\left(\rho\left(g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}+1\right)+W_j g^{\mu\nu}\phi_{,\mu}X^j_{,\nu}\right)$$

Physical Hamiltonian

$$\mathbf{H}_0 = \int_{\mathcal{S}} \mathrm{d}^3\sigma\, \mathcal{C}(\sigma,\tau) \qquad \mathcal{C} = \mathcal{C}^{GR}+\mathcal{C}^{YM}+\mathcal{C}^{F}+\mathcal{C}^{S}$$

In order to better corporate with YM and fermions

Densitized Triad formulation of GR

β a parameter which has nothing to do classically (topological coupling)

$$\mathcal{C}^{GR} = \frac{1}{\kappa}\left[F^a_{jk} - \left(\beta^2+1\right)\varepsilon_{ade}K^d_j K^e_k\right]\varepsilon^{abc}\frac{E^j_b E^k_c}{\sqrt{\det(q)}} + \frac{2\Lambda}{\kappa}\sqrt{\det(q)}$$

$F(A)$, $A = \beta\, K + \Gamma$ gravity SU(2) connection, $E = \sqrt{q}e$ electric field (densitized triad)

$$\mathcal{D}_j\xi = \left[\partial_j + A^a_j\frac{\tau^a}{2} + \underline{A}^I_j T^I_{R_f}\right]\xi$$

Fermion field ξ

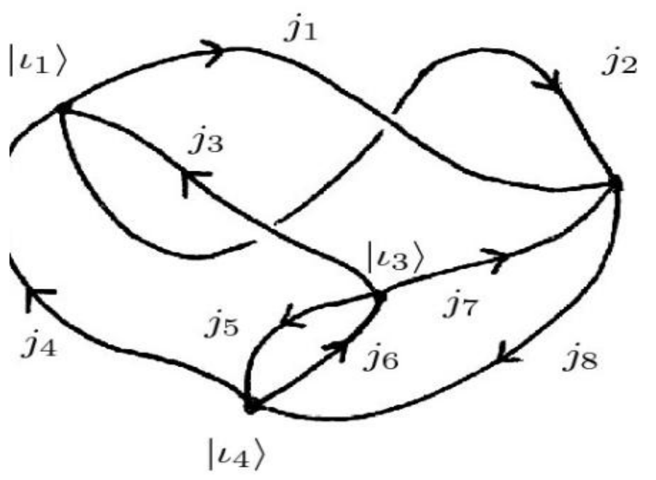
The theory has a spatial diffeomorphism symmetry which corresponds to infinitely many charges

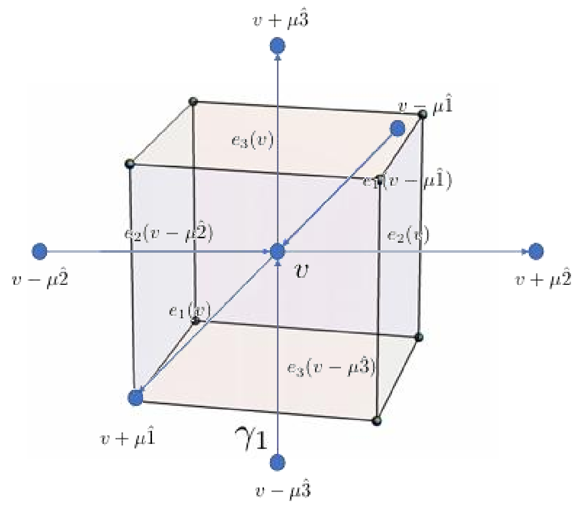
Reduced phase space quantization

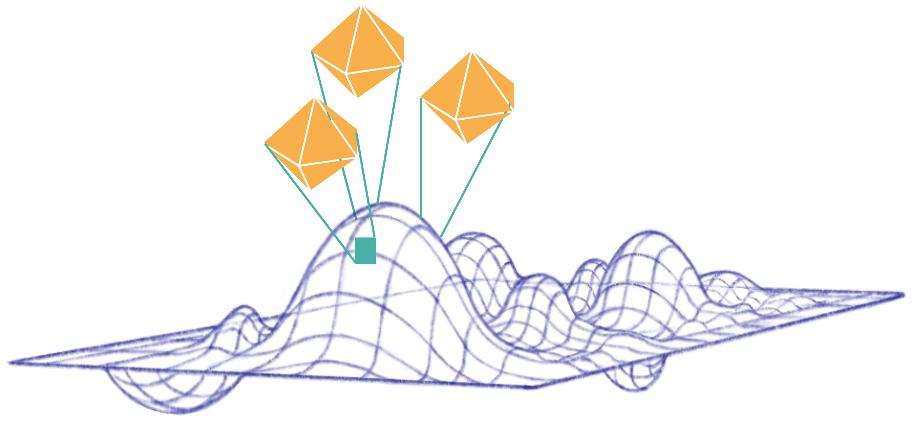
The quantization can follow the standard constructive QFT methods

Separable Hilbert space

1. IR cut-off: compact spatial manifold e.g. 3-torus with size R to avoid infra-red (IR) singularities
2. mode (UR) cut-off, e.g. a lattice γ_M to avoid possible UV singularities (diffeo symmetry is breaking)







- A unique cyclic representation for the Weyl algebra generated from canonical commutation relations and $*$ −relations by von Neumann theorem

$$A(c) := \mathcal{P}\exp\left(\int_c A\right),\quad E_f(S) = \int_S \mathrm{Tr}(f * E) \qquad \mathcal{H} = \otimes_e L_2(SU(2))/\otimes_v SU(2)$$

Discrete spectrum for Geometric operators (area, volume, …)

[Thiemann 20’, 22’ for review

- A representation of H as symmetric operators, up to operator ordering ambiguities since H is a highly non-linear operator

1. ordering problems are due to $\sqrt{q}$: renormalisation is typically required
2. Very difficult to compute ground state: extra difficulties in defining measure (path integral) from Feynman-Kac

Coherent state path integral for cLQG

Using coherent states, Path integral expression for transition amplitude generated by physical Hamiltonian

Configuration space on γ_M: $h_e = e^{i\theta^a \sigma_a} \in SU(2)$

Complexifier coherent states （gauge field） $\Psi^t_{[g]}$

→ Semi-classical parameter

→ coherent state label $g = e^{p^a \sigma_a} h = \quad \in$ SL(2,C)

Transition amplitude
$$A_{[g],[g']} = \langle \Psi^t_{[g]} | \exp[-\frac{i}{\hbar}\hat{\mathrm{H}}T] | \Psi^t_{[g']} \rangle = \int \mathrm{d}h \prod_{i=1}^{N+1} \boxed{\mathrm{d}g_i \, \nu[g]} \, e^{S[g,h]/t}$$

measure

Lorentzian action in the time continuum limit
$$S = i \int_0^T dt \left(\sum_{e \in E(\gamma)} G_{ab} p^a \frac{d\theta^b}{dt} - \kappa(H(p,\theta) + \mathcal{O}(\hbar)) \right)$$

All operator ordering ambiguities are in $\mathcal{O}(\hbar)$

We can define the Euclidean path integral $e^{-\beta H}$

H is the one for Lorentzian gravity instead of Euclidean ones

Hall 98' 99', Thiemann, 02,03'

Coherent state path integral for cLQG

This opens the possibility to use standard QFT techniques with partition function

$$Z = \int dg \mu(g) e^{i^s S[g]}$$

s=0 Thermal (Euclidean)
s=1 Real time

1. Perturbative calculation around (complex) saddle points
2. Resurgence
3. MCMC (on Lefschetz thimble)
4. Effective action
5. Functional renormalization group and effective average action.

In the following, we will work with the flat background to see some properties of the theory

1L EA from coherent state path integral in cLQG

Propagator on the long wavelength approx.

$\vec{k} = (k_x, 0, 0)$

We compute this on a flat saddle

$$\left\{ -\frac{a^2\sqrt{P_0}}{\left(\frac{2\pi}{T}k_0\right)^2}, -\frac{a^2\sqrt{P_0}}{\left(\frac{2\pi}{T}k_0\right)^2}, -\frac{a^2\sqrt{P_0}}{\left(\frac{2\pi}{T}k_0\right)^2} \right\}$$

Gauss modes $\mathcal{Y}^a(e_I) - \mathcal{Y}^I(e_a), a \neq I$

$$\left\{ \frac{a^2\sqrt{P_0}}{\left(\frac{2\pi}{T}k_0\right)^2}, \frac{a^2\sqrt{P_0}}{\left(\frac{2\pi}{T}k_0\right)^2} \right\}$$

vector modes $\mathcal{Y}^1(e_2) + \mathcal{Y}^2(e_1)$ and $\mathcal{Y}^1(e_3) + \mathcal{Y}^3(e_1)$

$$\left\{ \frac{a^2\sqrt{P_0}}{\left(\frac{2\pi}{T}k_0\right)^2 - \frac{\vec{k}^2}{P_0}}, \frac{a^2\sqrt{P_0}}{\left(\frac{2\pi}{T}k_0\right)^2 - \frac{\vec{k}^2}{P_0}} \right\}$$

tensor modes $\mathcal{Y}^2(e_3) + \mathcal{Y}^3(e_2)$ and $\mathcal{Y}^2(e_2) - \mathcal{Y}^3(e_3)$

$$\left\{ \frac{a^2\sqrt{P_0}}{\left(\frac{2\pi}{T}k_0\right)^2 + \frac{\vec{k}^2}{P_0} + \sqrt{8\left(\frac{2\pi}{T}k_0\right)^4 + \left(\left(\frac{2\pi}{T}k_0\right)^2 + \frac{\vec{k}^2}{P_0}\right)^2}}, \frac{a^2\sqrt{P_0}}{\left(\frac{2\pi}{T}k_0\right)^2 + \frac{\vec{k}^2}{P_0} - \sqrt{8\left(\frac{2\pi}{T}k_0\right)^4 + \left(\left(\frac{2\pi}{T}k_0\right)^2 + \frac{\vec{k}^2}{P_0}\right)^2}} \right\}$$

scalar modes $\mathcal{Y}^1(e_1)$ and $\mathcal{Y}^2(e_2) + \mathcal{Y}^3(e_3)$

Only tensor modes have non-zero poles: tensor modes are the only dynamical d.o.f (graviton), the other 4 are dust co-moving with the reference frame with poles at zero

1L EA from coherent state path integral in LQG

Effective action on the long wavelength approx.

Evaluate det:

$$\det(H) = \prod_{k_0 \neq 0} \prod_{\vec{k}} \det M(k_0, \vec{k}) = \prod_{k_0, \vec{k}} \frac{\left(\frac{2\pi}{T} k_0\right)^{18} \mu^{36}}{a^{36} \beta^{18}} \prod_{k_0, \vec{k}} \left(1 - \left(\frac{T}{2\pi\sqrt{P_0}}\right)^2 \frac{|\vec{k}|^2}{k_0^2}\right)^2$$

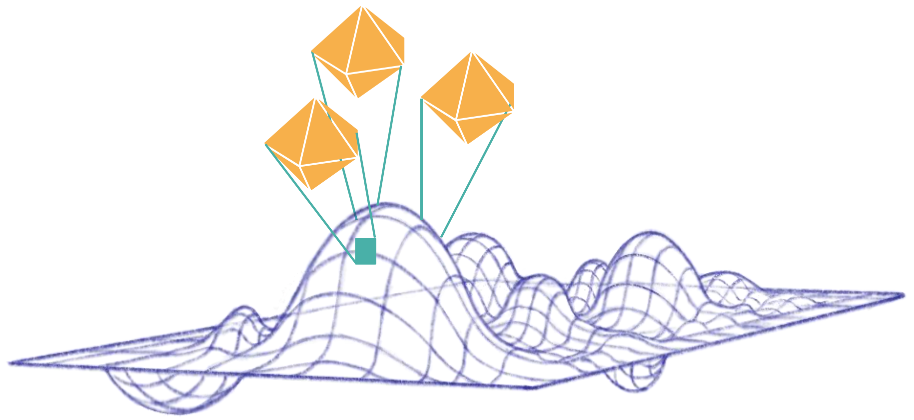
$$\left[\prod_{k_0=1}^{\infty} \left(1 - \left(\frac{T}{2\pi\sqrt{P_0}}\right)^2 \frac{|\vec{k}|^2}{k_0^2}\right)^2\right]^2 = \left[\frac{\sin(\xi|\vec{m}|)}{\xi|\vec{m}|}\right]^4 , \quad \xi = \frac{\pi T}{L\sqrt{P_0}}$$

Log-det formula

$$S_{1L}(\xi) = -\log\det(H) = -4 \sum_{\vec{m}} \log\left[\frac{\sin(\xi|\vec{m}|)}{\xi|\vec{m}|}\right] + \text{const.}$$

↓ $i\varepsilon$-regularize

$$S_{1L}(j_0) = i\mathcal{N}^3 \frac{T}{\sqrt{j_0}\ell_P} \frac{4\sqrt{2}\pi c}{\sqrt{\beta}} - 2\mathcal{N}^3 \log(j_0)$$



1L EA from coherent state path integral in LQG

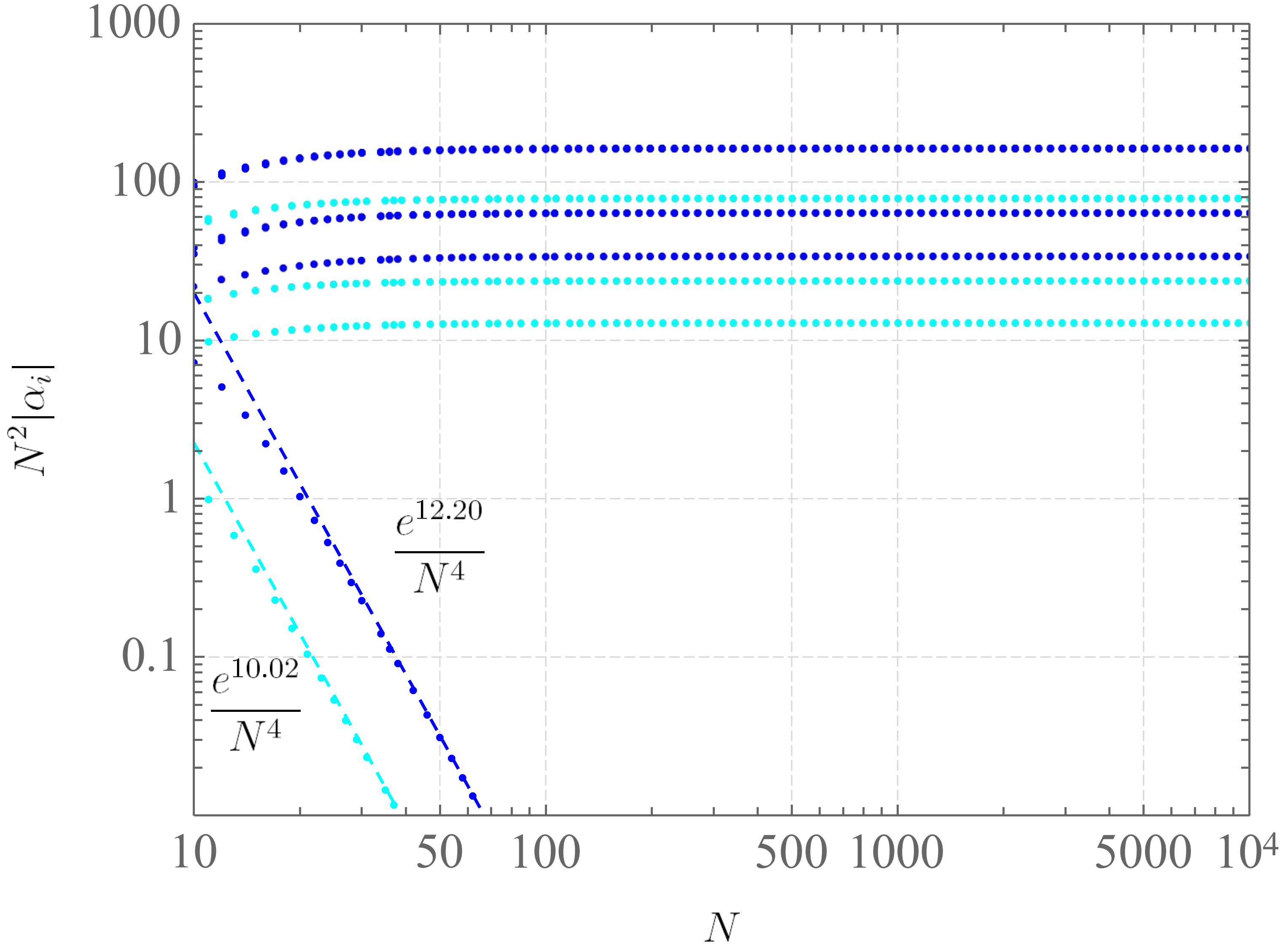
Beyond the long wavelength approx.

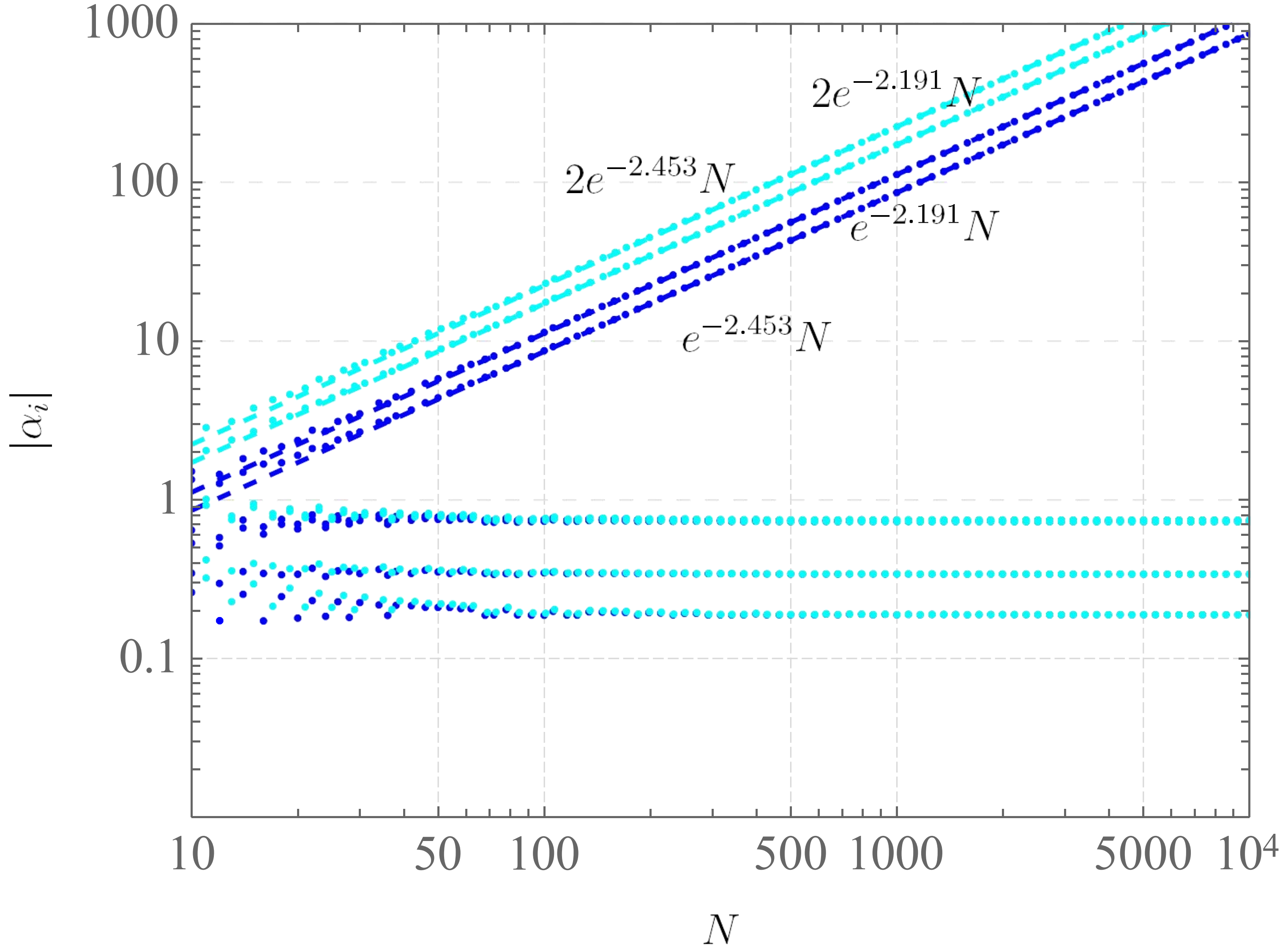
$$P(k_0,\vec{m}) = \frac{1}{\det M_{VV}(k_0,\vec{m})} = \frac{a^{36}\beta^{18}}{\mu^{36}} \frac{1}{\left(\frac{2\pi}{T}k_0\right)^6 \prod_{i=1}^{6}\left(\left(\frac{2\pi}{T}k_0\right) - \frac{\alpha_i}{\sqrt{P_0}}\right)\left(\left(\frac{2\pi}{T}k_0\right) - \frac{\overline{\alpha_i}}{\sqrt{P_0}}\right)}, \quad \alpha_i \in \mathbb{C}$$

3 unphysical (Gauss constraint) + **6** physical dofs （2 gravity + 4 matter)

Convergence to semiclassical limit: $|k| \leq k_{\max,\mathcal{N}}$
k fixed, $\mathcal{N} \to \infty$

Scaling with $\mathcal{N}$: $|k| \leq k_{\max,\mathcal{N}} \equiv \frac{\pi}{\mu} = \frac{\pi\mathcal{N}}{L}$



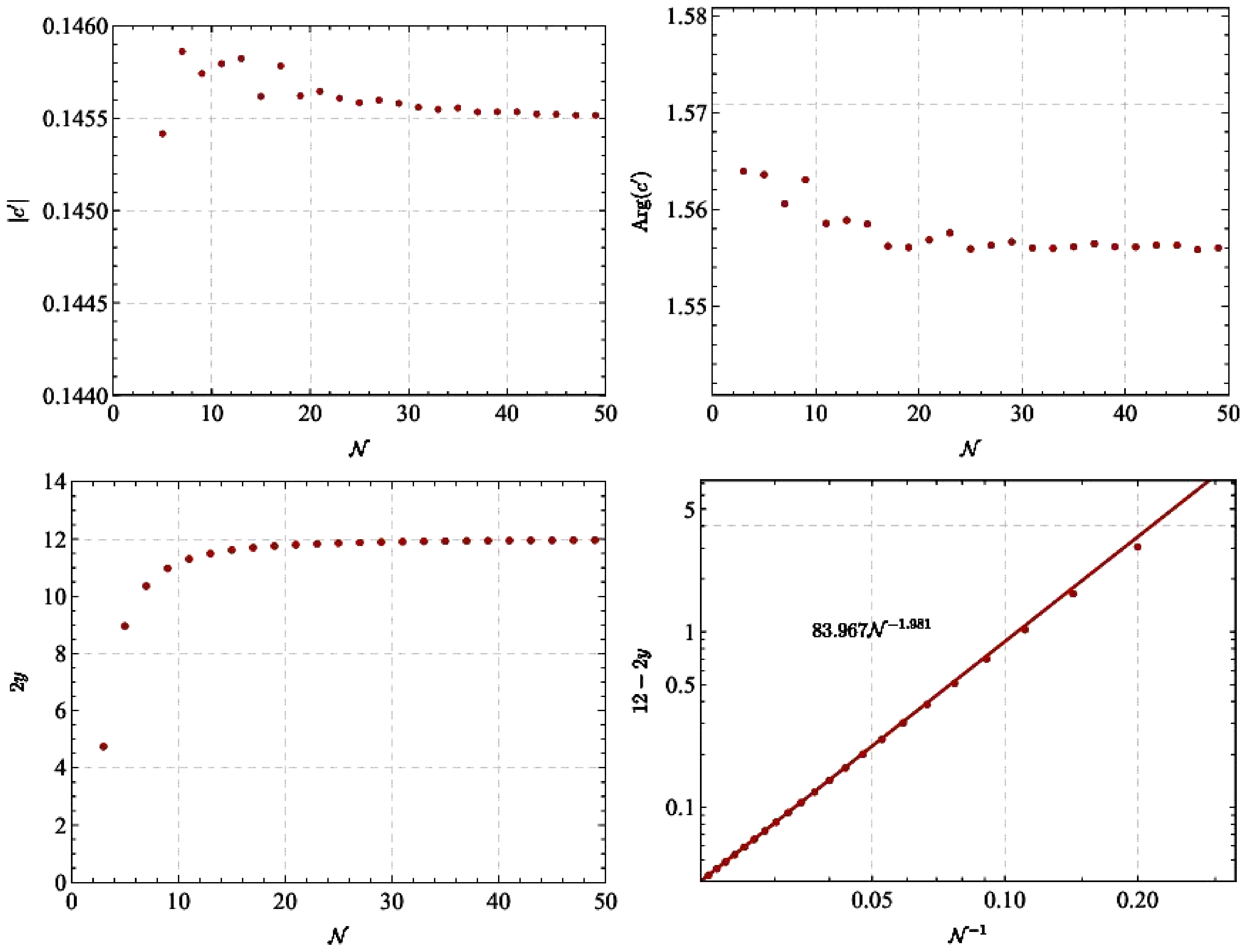


1L EA from coherent state path integral in LQG

Numerics & convergence

Effective action
$$S_{1L}(j_0) = i\mathcal{N}^3 \frac{T}{\sqrt{j_0}\,\ell_P}\,\frac{4\sqrt{2}\pi c}{\sqrt{\beta}} - y\mathcal{N}^3 \log(j_0)$$

Summing over different configurations (numerically) without long wave length approximation



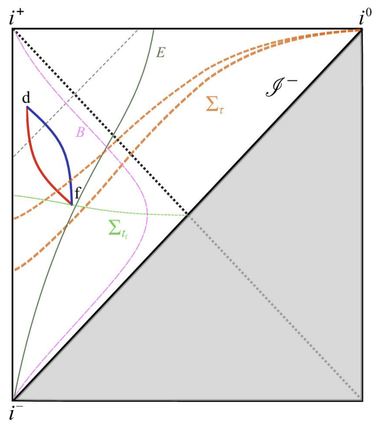
Quick convergence but with complex values:

Complex critical points for quantum EoM

Can we check this in spinfoam and effective spinfoams

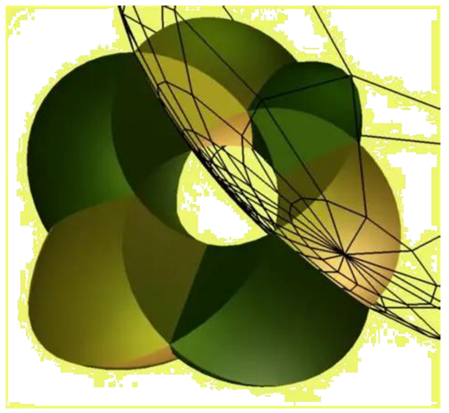
Conclusion

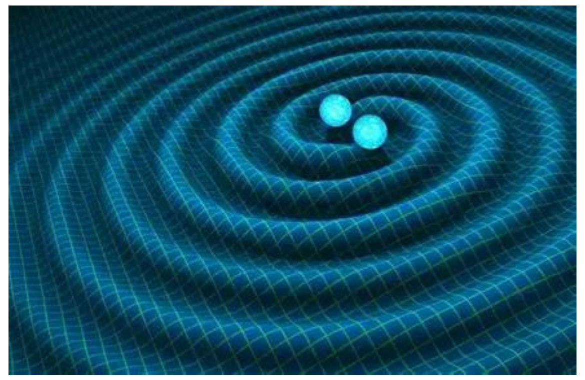
Physical BHs



Unique (re)construction of 2d mimetic dilaton theory

Lift to 4d mimetic theory





What we aim to achieve

- QNMs or 1-loop effective action calculation in full 4d mimetic
- Asymptotical safe analysis and full 1PI effective averaged action
- Perturbations from a full QG

Thank you

Why Gravity are special

Completely constrained system:

$$\begin{aligned}\{H[\vec{N}], H[\vec{M}]\} &= -H[\mathcal{L}_{\vec{M}}\vec{N}] \\ \{H[N]\,,\, H[\vec{M}]\} &= -H[\mathcal{L}_{\vec{M}}N] \\ \{H[N]\,,\, H[M]\} &= +H[\vec{V}] \qquad \text{with} \quad V^a = q^{ab}\left(M\partial_b N - N\partial_b M\right)\end{aligned}$$

Lie algeboid instead of Lie algebra
[Wald, 80'

Diffeomorphism invariance: no local observables in gravity theories

$$\delta_\xi O(x) = -\xi^\mu \partial_\mu O(x)$$

Problem of time

proved by Torre 93'

"nothing seems to happen in quantum gravity"

Relational framework:

Matter dressed	Gravitationally dressed
Can be defined on compact manifold	Need boundary symmetry

Brown and Kuchaˇr 94'

[S. Giddings, 25

Relational Dirac Observables

Gravity $\{P_a,\ Q^a\}$ + Matter $\{p_{\phi_i},\ \phi_i\}$

Linearized Constraints

$$C_i \to \hat{C}_i = p_{\phi_i} + h_i(P, Q; \phi)$$

Deparametrized: when h does not depend on ϕ

Schrodinger picture: $\tilde{C}_I \Psi = 0 \to i\partial_{\phi_I}\Psi = H_I \Psi$

ϕ plays the role of time

Relational framework:

Dirac Observable $O_F(t) := \left[e^{s^i V_{\hat{C}_i}} \cdot F \right]_{s=-G}$

defined from gauge fixing functions

$$G_i(t) = \phi_i - X_i(t)$$

Gauge invariant reduced phase space on (P, Q) $\left\{ O_{F_1}(t),\ O_{F_2}(t) \right\} = O_{\{F_1, F_2\}}(t)$

Keeps Poisson structure

Physical Hamiltonian $H(t) := \dot{k}^i(t) h_i\big(O_P(0), O_Q(0), k(t)\big)$

$$\frac{d}{dt} O_F(t) := \{H(t),\ O_F(t)\}$$

Dittrich 04, Thiemann 04 …