Bouncing behaviour in Non-Commutative Space-Time

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Outline

- Why Quantum Gravity?
- QG Approaches & NC space-time
- Entropic gravity in κ-Minkowski space-time
- κ-Friedmann equation & bounce behaviour
- Summary

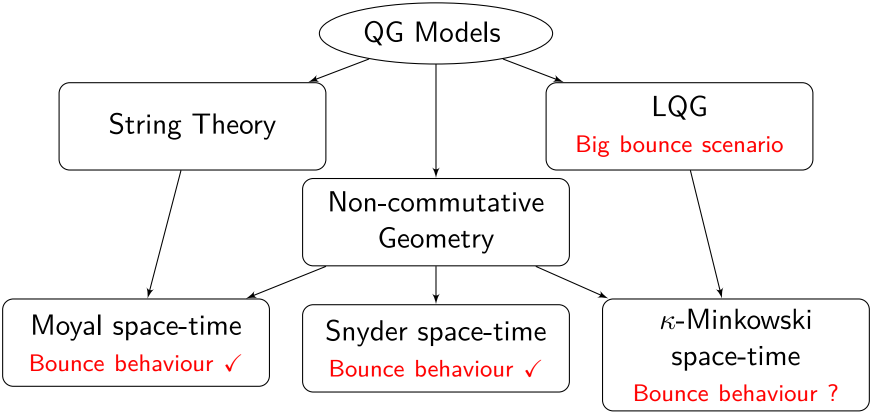
Why Quantum Gravity?

- General Relativity predicts two kinds of singularities:
  - Big Bang singularity occured at begining of universe
  - Black Hole singularities existing inside the black hole
- Break down of physics at singularities
- Quantum fluctuations become significant at Planck's length.
- Classical GR is incomplete

General Relativity + Quantum Mechanics $\rightarrow$ Quantum Gravity

- Singularity resolution through QG !

QG approaches



NC space-times carry the imprints of QG signals

Features of NC space-times

- $[\hat{x}_\mu, \hat{x}_\nu] = \Theta_{\mu\nu}(\hat{x})$
- Existence of minimal length scale and space-time uncertainties
- Modified dispersion relation
- Deformed Poincare / deformed Diffeomorphism symmetry
- Non-local field theory

Affects the propagation of GW

GW propagating through NC space-times could reveal QG signatures!

κ-Minkowski space-time

- Lie-algebraic type NC relation

$$[\hat{x}_i, \hat{x}_0] = i\kappa^{-1}\hat{x}_i, \quad [\hat{x}_i, \hat{x}_j] = 0$$

- Symmetry algebra can be realised as
  - κ-Poincare algebra: Both algebra and co-algebra sectors are deformed
  - undeformed κ-Poincare algebra: Only co-algebra sector is deformed
- Appears in the context of Doubly Special Relativity (DSR)

Entropic Force Approach in κ-Minkowski space-time

- Newton's force is the emergent entropic force

$$F = T\frac{\Delta S}{\Delta x}$$

κ-Newton's gravity

- Consider holographic screen at κ-corrected Unruh temperature:

$$T = \frac{\mathcal{A}}{2\pi}\left(1 + \frac{a\mathcal{A}}{4\pi}\right)$$

$$F = -\frac{GMm}{r^2}\left(1 - \frac{\lambda GM}{4\pi r^2}\right)$$

Logarithmic entropy

- Modified entropy relation

$$S = \frac{A}{4G} + s(A)$$

- Logarithmic correction as κ-deformed correction

$$S = \frac{A}{4G} - \frac{\lambda M}{4}\ln\left(\frac{A}{4\pi}\right)$$

κ-Friedmann Equations

- From κ-deformed Newton's gravitational force we obtain κ-Friedmann Equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(1+3\omega)\rho\left(1 - \frac{\lambda G}{3}\sqrt{\frac{3}{8\pi G}}(1+3\omega)\sqrt{\rho}\right)$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho\left(1 - \frac{\lambda G}{9}\sqrt{\frac{3}{8\pi G}}\frac{(1+3\omega)^2}{(1+2\omega)}\sqrt{\rho}\right)$$

- Evolution equation of Hubble parameter in κ-Minkowski space-time

$$\dot{H} = -H^2 - \frac{1}{2}(1+3\omega)\left(H^2+\frac{k}{a^2}\right)\left(1-\frac{\lambda(1+3\omega)(2+3\omega)}{24\pi(1+2\omega)}\sqrt{H^2+\frac{k}{a^2}}\right)$$

Evolution of scale factor in κ-deformed space-time

$$a(\tau) = \tau^{2/3(1+\omega)}\left(1 - \frac{4\lambda(1+3\omega)^2(2+3\omega)}{81\pi(1+2\omega)}\frac{(1+\ln\tau)}{\tau}\right),$$

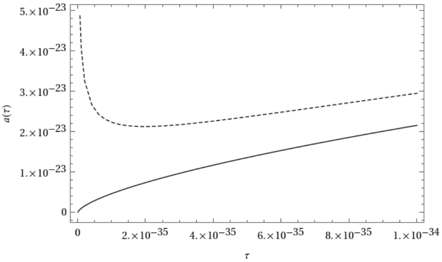


图 1: $a(\tau) = \tau^{2/3} - \frac{8\alpha}{81\pi}\frac{1}{\tau^{1/3}} - \frac{8\alpha}{81\pi}\frac{\ln\tau}{\tau^{1/3}}$.

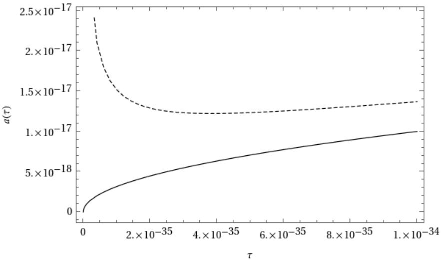


图 2: $a(\tau) = \tau^{1/2} - \frac{3\alpha}{20\pi}\frac{1}{\tau^{1/2}} - \frac{3\alpha}{20\pi}\frac{\ln\tau}{\tau^{1/2}}$

Bouncing Universe

Bouncing behaviour Mechanism

- **Contraction $\rightarrow$ Bounce $\rightarrow$ Expansion**
- Minimum scale factor a_b
- For bounce

$$\dot{a}_b = 0,\ \ddot{a}_b > 0$$

- Finite density at bounce point

$$\rho_b = \frac{3k}{8\pi G a_b^2}$$

- For $\ddot{a}_b > 0$, we need

$$\rho + 3p < 0, \quad \rho > 0$$

Bounce in κ-Minkowski space-time

- Finite density at bounce

$$\rho_b = \frac{\rho_\lambda}{4}\left(2 - \frac{\lambda\sqrt{k}}{12\pi a_b}\frac{(1+3\omega)^2}{(1+2\omega)} \pm 2\sqrt{1 - \frac{\lambda\sqrt{k}}{12\pi a_b}\frac{(1+3\omega)^2}{(1+2\omega)}}\right)$$

where $\rho_\lambda = \frac{864\pi}{\lambda^2 G}\frac{(1+2\omega)}{(1+3\omega)^2}$

- For bounce in $k = 0$, we need $\omega > -\frac{5}{9}$
- For bounce in $k = 1$, we need $\omega > -\frac{1}{3}$ and $\frac{\lambda}{24\pi}\frac{(1+3\omega)^2(8+15\omega)}{(1+2\omega)(5+9\omega)} < a_b < \frac{\lambda(1+3\omega)}{8\pi}$
- No bounce in $k = -1$

Summary

- Generalized entropic force description to κ-Minkowski space-time
- First order correction to κ-deformed Newton's potential scales as $1/r^3$
- κ-deformation introduces logarithmic corrections to Bekenstein-Hawking entropy
- Minimal length scale of κ-Minkowski space-time avoids initial singularity through bouncing behavior

Thanks!