彭桓武理论物理创新研究中心“2025
引力波物理研讨会”

$f(T)$ thick brane and QNMs

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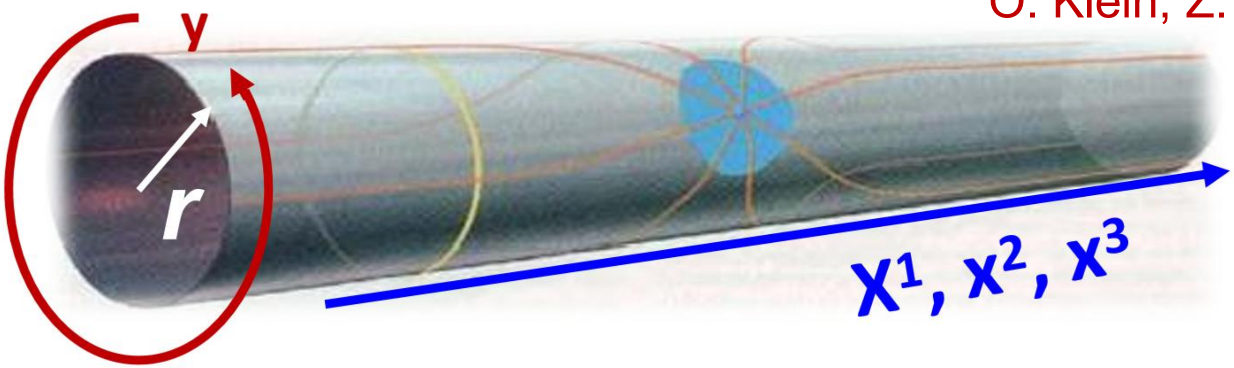
Contents

Kaluza-Klein theory Unify GR and EM

T. Kaluza, Math. Phys. 1921
O. Klein, Z. Phys. 1926



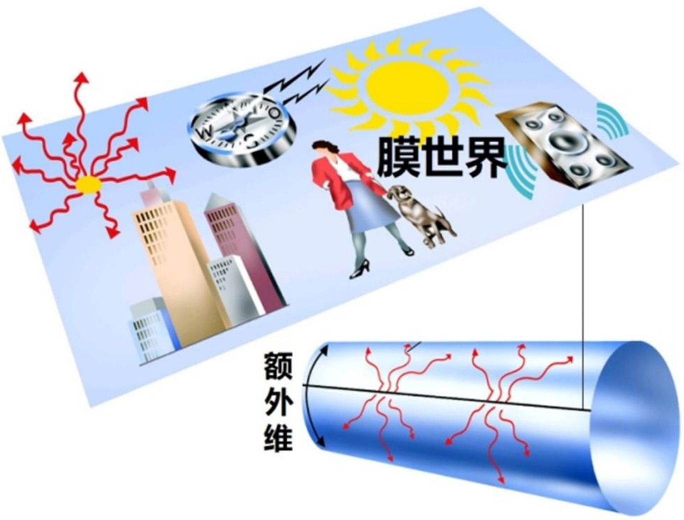
$$S = \frac{1}{16\pi G_5}\int d^4x dy \sqrt{-g} R \qquad g_{MN} = \phi^{-1/3}\begin{pmatrix} \hat{g}_{\alpha\beta} + \phi A_\alpha A_\beta & \phi A_\alpha \\ \phi A_\beta & \phi \end{pmatrix}$$

$$= \frac{1}{16\pi \hat{G}}\int d^4x \sqrt{-\hat{g}}\left(R_4 - \frac{1}{4}\phi F_{\alpha\beta}F^{\alpha\beta} - \frac{1}{\phi^2}\right)$$

all zero modes that describe the observed particles are neutral

1. Background

ADD model   hierarchy problem   Arkani-Hamed et al, PLB 1998



$$M_{\mathrm{Pl}}^2 = M^{n+2}(2\pi r)^n$$

Matters are localized on the brane

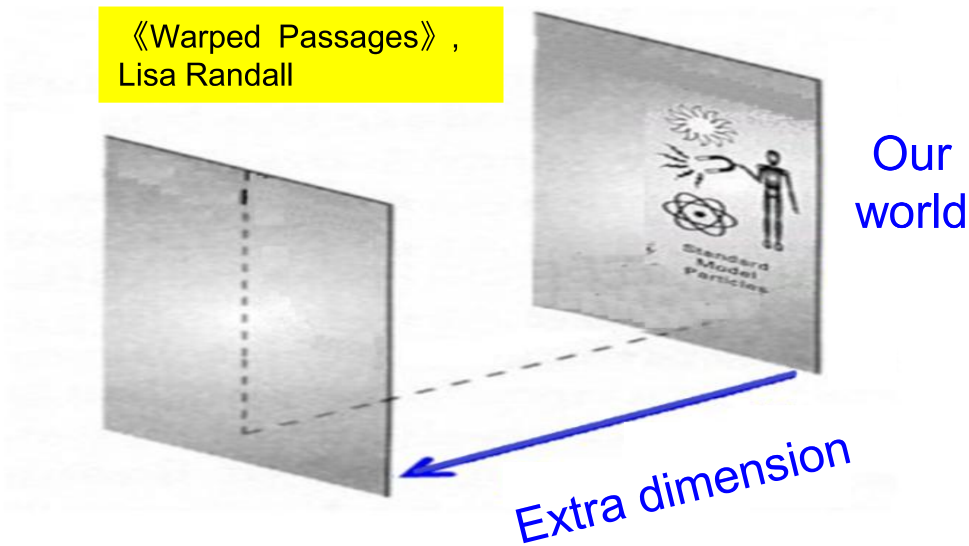
The fundamental energy scale is Tev, M~1Tev, M_{EW} ~ 1 TeV

M_{Pl} is an effective energy scale, because of the huge volume of extra dimensions

New hierarchy: $r \sim 10^{32/n}\frac{1}{M} \gg \frac{1}{M} \sim 10^{-19}\mathrm{m}$

1. Background

Randall-Sundrum model
brane tension



Randall and Sundrum, PRL 1999

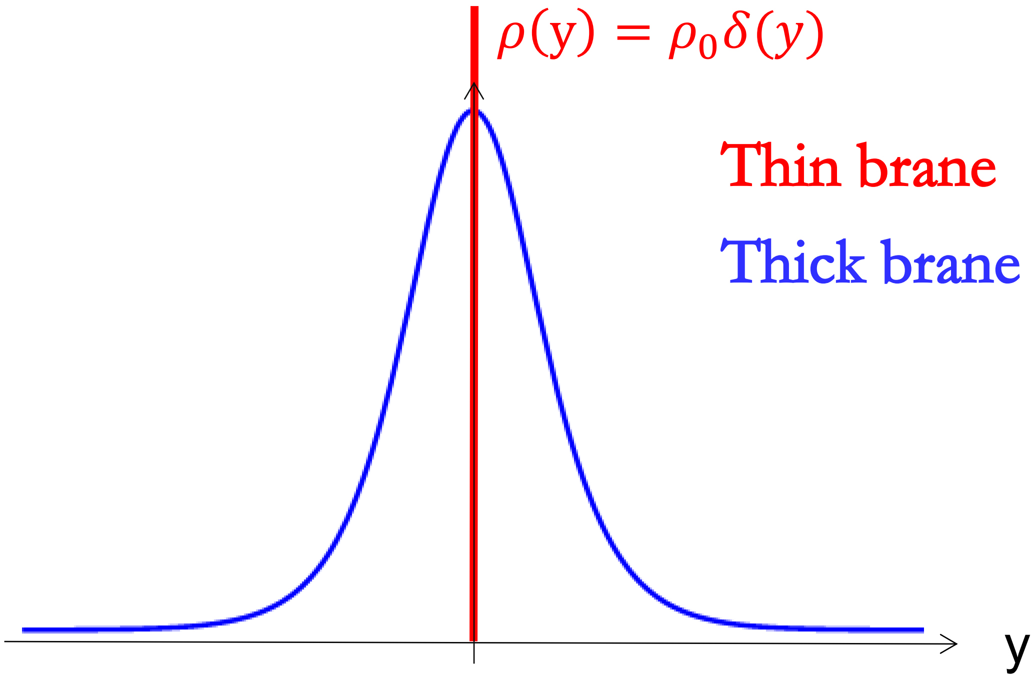
$$S = \int d^5x \sqrt{g} \left(\frac{1}{\kappa^2} R + \Lambda \right) + S_b$$

$$\boxed{M_{Pl}^2 = \frac{M_*^3}{k}(1 - e^{-2kb})}$$

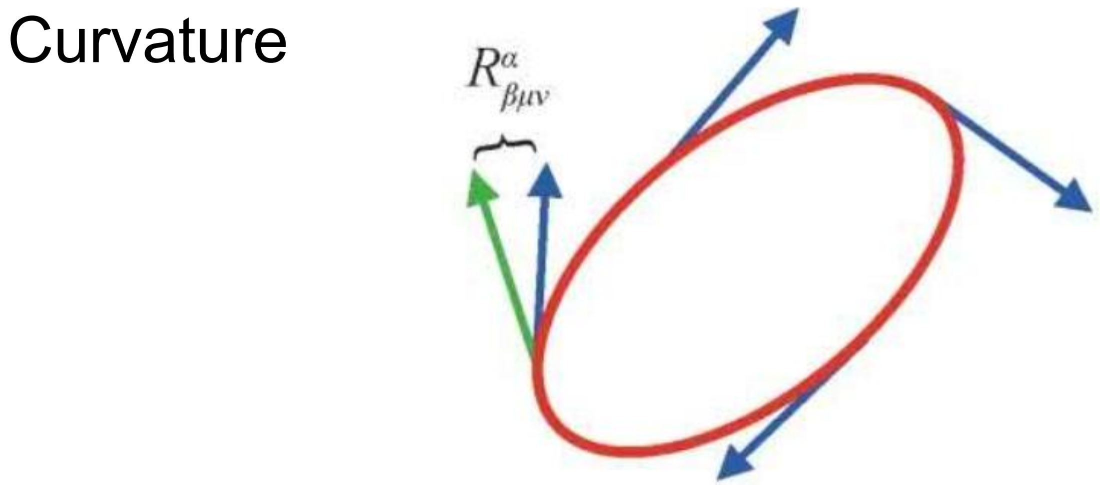
$$\mathrm{d}s^2 = e^{2A(y)} \eta_{\mu\nu} \mathrm{d}x^{\mu}\, x^{\nu} + \mathrm{d}y^2$$

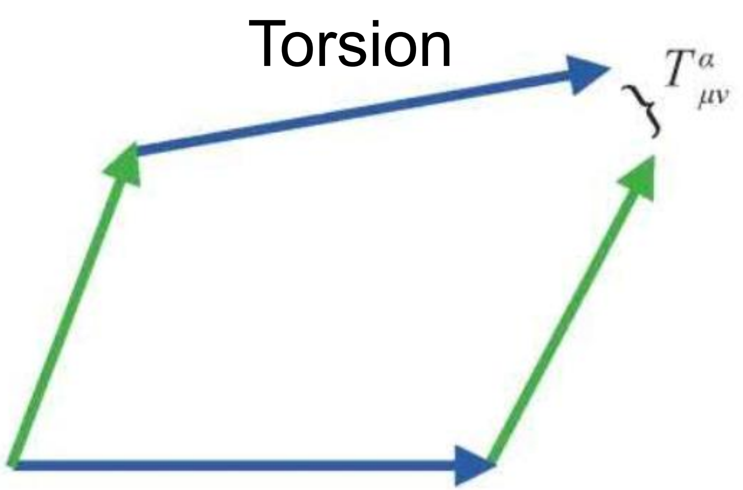
Thick brane

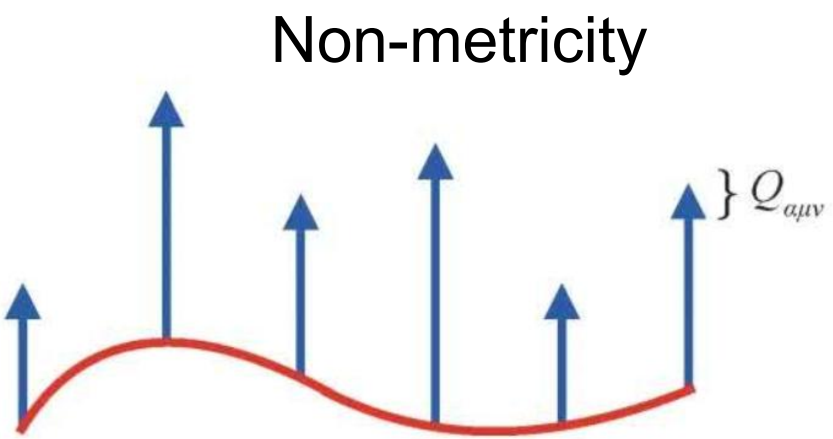
Dewolfe etc. hep-th/9909134



1. Background

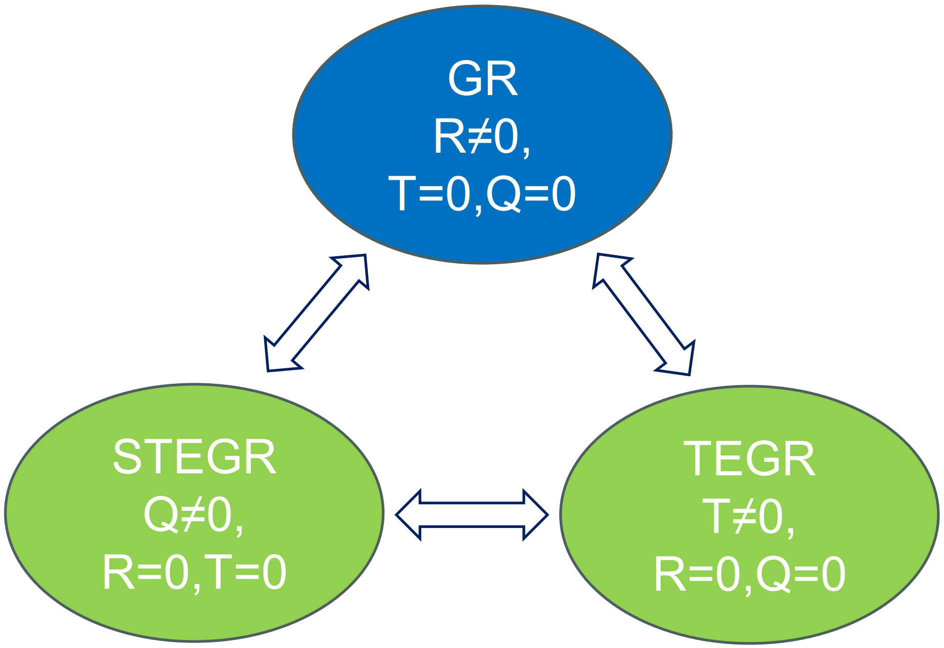






王海天，徐立昕，超越爱因斯坦的旅程：修正引力理论与宇宙学模型

Geometrical Trinity



王海天，徐立昕，超越爱因斯坦的旅程：修正引力理论与宇宙学模型

1. Background

$f(T)$ theory?

Curvature
$R(g_{\mu\nu})$

Torsion
$T(e_\mu^a)$

$$R = -T - 2\nabla^\mu (T^\nu{}_{\mu\nu})$$

TEGR

$f(R)$[a]

$f(T)$[b]

$f(R)$ theory and $f(T)$ theory are not equivalent

Higher order field equations

Second order field equations

$f(R)$ thick brane[c], $f(T)$ thick brane?

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[a] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. 82 (2010)
[b] G. Bengochea and R. Ferraro, Phys. Rev. D 79, (2008)
[c] Y. Zhong, Y.-X. Liu, and K. Yang, Phys. Lett. B 699 (2011)

1. Background

What we want to do

- Find the analytical solution of $f(T)$ thick brane
- Analyze the stability and the localization of graviton zero mode
- Find the QNMs of $f(T)$ thick brane

$f(T)$ thick brane

Function of T

action $$S = -\frac{M_*^3}{4}\int d^5x\, e\, f(T) + \int d^5x\, \mathcal{L}_M$$

Fundamental energy scale, we set to 1

Determinant of tetrad

Lagrangian of matter

metric $$ds^2 = \boxed{e^{2A(y)}}\,\boxed{\eta_{\mu\nu}}\,dx^\mu dx^\nu + dy^2$$

Warp factor

Flat brane

2 Thick brane in f(T) gravity

Canonical scalar field $\mathcal{L}_M = e(-\frac{1}{2}\partial^M\phi\partial_M\phi - V(\phi))$

Field equations

$$6A'^2 f_T + \frac{1}{4}f = -V + \frac{1}{2}\phi'^2$$

$$\frac{1}{4}f + \left(\frac{3}{2}A'' + 6A'^2\right)f_T - (36A'^2A'')f_{TT} = -V - \frac{1}{2}\phi'^2$$

$$\phi'' + 4A'\phi' = \frac{dV}{d\phi}$$

Second order derviatives

We need to solve $A(y)$, $\phi(y)$, and $V(\phi)$. Since $\nabla^\mu(\mathcal{T}_{\mu\nu}) = 0$, only two equations are independent, so we can choose the warp factor $A(y)$.

2 Thick brane in f(T) gravity

Warp factor $e^{2A(y)} = \cosh^{-2b}(ky)$

Strength of torsion

for $f(T) = T + \alpha T^2$

Analytical solution

$$\begin{aligned}
\phi(y) &= \sqrt{\frac{3b}{2}}\Bigg(i\Big(\mathrm{E}(iky;u) - \mathrm{F}(iky;u)\Big) \\
&\quad + \sqrt{\left(1 + u\sinh^2(ky)\right)}\tanh(ky)\Bigg), \\
V(\phi(y)) &= \frac{3}{8}bk^2\Bigg(288\alpha b^3k^2\tanh^4(ky) - 8b\tanh^2(ky) \\
&\quad + \mathrm{sech}^4(ky)\left(u\cosh(2ky) - u + 2\right)\Bigg),
\end{aligned}$$

Stability under perturbation

Linear perturbation

$$e^A{}_M = \begin{pmatrix} e^{A(y)}(\delta^a{}_\mu + h^a{}_\mu) & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Downarrow$$

$$g_{MN} = e^A{}_M \, e^B{}_N \eta_{AB}$$

$$\Downarrow$$

$$g_{MN} = \begin{pmatrix} e^{2A(y)}(\eta_{\mu\nu} + \gamma_{\mu\nu}) & 0 \\ 0 & 1 \end{pmatrix}$$

$$\gamma_{\mu\nu} = (\delta^a{}_\mu h^b{}_\nu + \delta^b{}_\nu h^a{}_\mu)\eta_{ab}$$

2 Thick brane in f(T) gravity

Transvers-Traceless gauge

$$\partial_\mu \gamma^{\mu\nu} = 0 = \eta^{\mu\nu}\gamma_{\mu\nu}$$

$$\partial_\mu(\delta_a{}^\mu h_b{}^\nu + \delta_b{}^\nu h_a{}^\mu)\eta^{ab} = 0$$

$$\delta_a{}^\mu h^a{}_\mu = 0$$

Perturbation equation

$$\left(e^{-2A}\Box^{(4)}\gamma_{\mu\nu} + \gamma''_{\mu\nu} + 4A'\gamma'_{\mu\nu}\right) f_T$$
$$-24A'A''\gamma'_{\mu\nu} f_{TT} = 0$$

KK decomposition

$$\gamma_{\mu\nu}(x^\rho, y) = \epsilon_{\mu\nu}(x^\rho)\Psi(y)$$

2 Thick brane in f(T) gravity

Coordinate transformation

$$dz = e^{-A} dy$$

Field transformation

$$\Psi(z) = \exp\left(-\int H(z)dz\right)\psi(z)$$

$$H = \frac{3}{2}\partial_z A + 12e^{-2A}\left((\partial_z A)^3 - \partial_z^2 A \partial_z A\right)\frac{f_{TT}}{f_T}$$

Klein-Gorden equation

Mass of KK graviton

Schrodinger-like equation

$$\left(\Box^{(4)} + m^2\right)\epsilon_{\mu\nu}(x^\rho) = 0$$

$$\left(-\partial_z^2 + U(z)\right)\psi = m^2\psi$$

$$U(z) = H^2 + \partial_z H$$

$$\left(-\partial_z + H\right)\left(\partial_z + H\right)\psi = m^2\psi \quad \Longrightarrow \quad m^2 \geq 0$$

$f(T)$ thick brane is stable under linear tensor perturbation

2 Thick brane in f(T) gravity

Localization of gravitational zero mode

Zero mode solution $\psi_0 = \exp\left(\int H\, dz\right)$

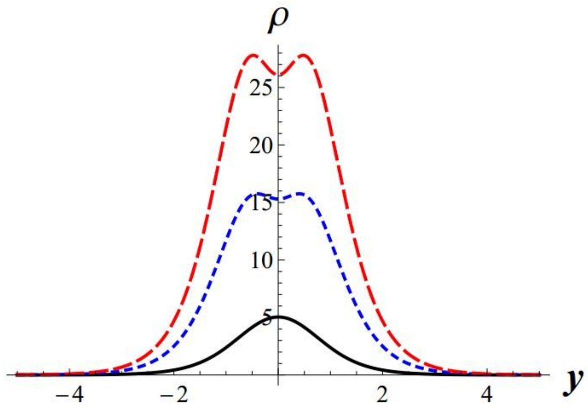
It satisfy the following normalization condition

$$\int_{-\infty}^{\infty} dz\, \psi_0^2(z) = 1$$

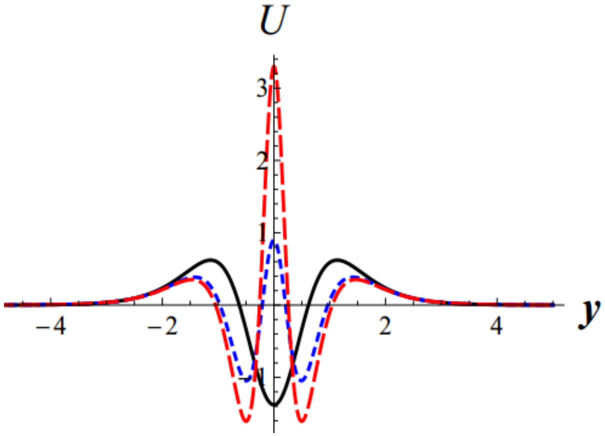
Gravitational zero mode can be localized near the brane， four-dimensional gravity can be restored.

2 Thick brane in f(T) gravity

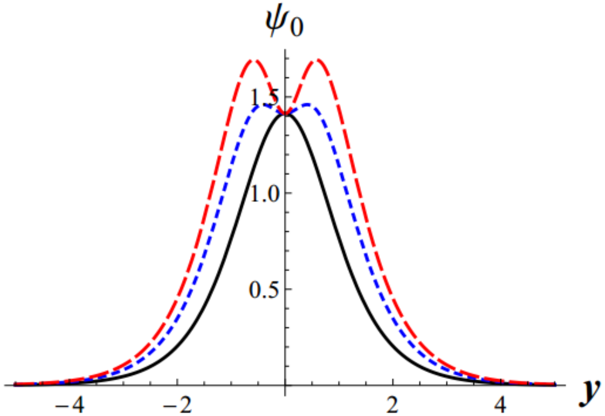
$$f(T) = T + \alpha T^2$$



Energy density



Effective potential



Zero mode

$\alpha=-0.005$ (black), $\alpha=-0.1$ (blue), $\alpha=-0.2$ (red)

Gravitational zero mode also split and it corresponds to the energy density.

3 QNMs in $f(T)$ thick brane

QNMs: characteristic modes of dissipative system $\begin{cases} \text{Black hole} \\ \text{Thick brane} \end{cases}$

QNMs of thick brane?

There is a set of discrete QNMs in thin brane (RS II)
Phys. Rev. D 72, 066002 (2005)

3 QNMs in $f(T)$ thick brane

$$A(z) = \frac{-1}{2}\ln\left(1+\left(\frac{kz}{\delta}\right)^{2s}\right)$$

$$\left(-\partial_z^2 + U(z)\right)\psi = m^2\psi$$

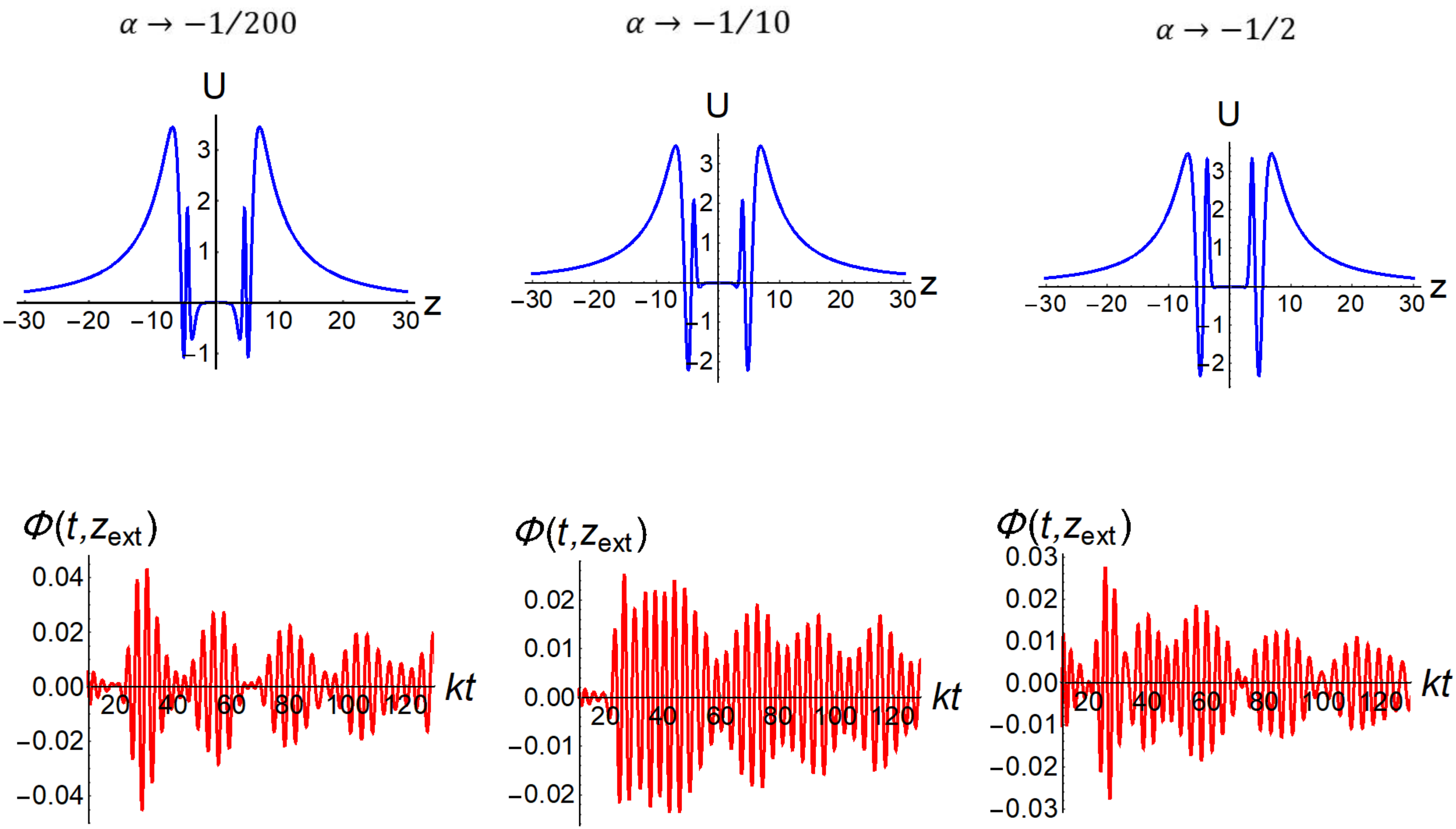
Boundary conditions

$$\psi(z) \propto \begin{cases} e^{imz}, & z \to \infty. \\ e^{-imz}, & z \to -\infty. \end{cases}$$

3 QNMs in $f(T)$ thick brane

		Asymptotic iteration method		Bernstein spectral method	
	n	Re(m/k)	Im(m/k)	Re(m/k)	Im(m/k)
α =-1/200	1	0.95920	-0.50492	0.95920	-0.50492
	2	0.57397	-1.78665	0.57206	-1.78504
α =-1/20	1	0.72109	-0.33141	0.72109	-0.33141
	2	0.65695	-1.56934	0.65506	-1.57100
α =-1/10	1	0.50881	-0.19034	0.50881	-0.19034
	2	0.72040	-1.52845	0.72071	-1.52861

3 QNMs in $f(T)$ thick brane



4 Conclusion and outlook

We find the analytical solution of $f(T)$ thick brane

$f(T)$ thick brane is stable under linear tensor perturbation and Gravitational zero mode can be localized near the brane

There is gravitational echo in $f(T)$ thick brane, it need further investigation

Thanks for your attention!