Phase Transition Gravitational Waves as a Unique Discriminant for Warm Inflation

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Outline

Why Inflation?

- Solves Big Bang cosmology puzzles: horizon, flatness, monopole problems
- Generates primordial perturbations for large-scale structure formation

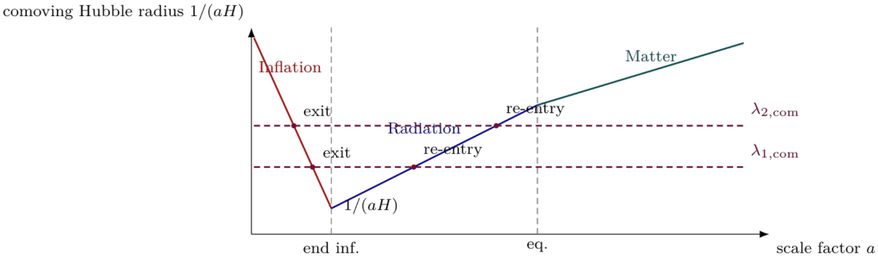


Fig. 1: Cosmic scale factor evolution

Inflationary

Fundamental Assumptions

1. *Isolated Inflaton*: No interactions between inflaton ϕ and other fields during inflation
2. *Potential Dominance*: Energy density dominated by inflaton potential: $\rho_\phi \approx V(\phi) \gg \rho_r$ (radiation negligible)
3. *Post-Inflation Reheating*: Particle production (reheating) occurs *only after* inflation ends

CI Inflaton Evolution Equation

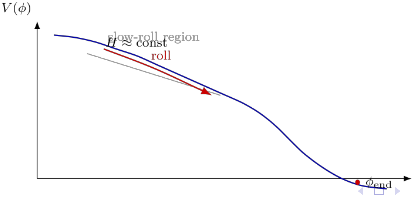
Inflaton Dynamics From Einstein equations + inflaton action, inflaton follows:

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0 \tag{1}$$

Where:

- $\ddot{\phi}$: Inflaton acceleration
- $3H\dot{\phi}$: Hubble damping
- $V_{,\phi}$: Potential force

To sustain inflation, $\ddot{\phi} \ll 3H\dot{\phi}, V_{,\phi} \rightarrow$ requires ultra-flat potential ($V_{,\phi} \ll 3H\dot{\phi}$).



Inflationary

1. **η-Problem (Mass Crisis)**
   - Inflaton mass $m_\phi \sim \sqrt{V_{,\phi\phi}}$; quantum corrections easily push $m_\phi > H$
   - Spoils slow-roll: $3H\dot{\phi}$ cannot balance $V_{,\phi}$, terminating inflation prematurely
   - Example: Quadratic potential $V = \frac{1}{2}m_\phi^2\phi^2$ needs $m_\phi \ll H$, but corrections give $m_\phi \sim H$ (contradiction)

2. **Super-Planckian Field Excursion**
   - Monomial potentials ($V \propto \phi^n$) require $\Delta\phi > M_{Pl}$ for $N \sim 60$ e-folds
   - Violates EFT constraints (EFT breaks down above M_{Pl}) and swampland distance conjecture

3. **Reheating Uncertainty**
   - CI needs inflaton decay to radiation *post-inflation*, but no robust microphysical model
   - Reheating efficiency, temperature, and timescale are highly uncertain

Warm Inflation (WI): Motivation

Core Insight

Introduce *dissipative interactions* between inflaton and a thermal bath during inflation:

- Enables concurrent particle production $\rightarrow$ no separate reheating phase
- Thermal bath provides extra damping

Key Distinction from cold inflation (CI)

CI

- $T \ll H$ (negligible thermal bath)
- Only quantum fluctuations
- Reheating required post-inflation

WI

- $T > H$ (sustained thermal bath)
- Thermal + quantum fluctuations
- Smooth radiation transition

1. Inflaton Evolution

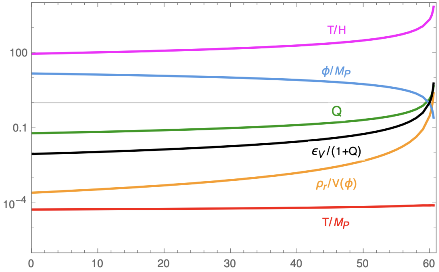
$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_{,\phi} = 0 \tag{2}$$

The $\Upsilon\dot{\phi}$ term describes energy loss from inflaton to thermal bath.

2. Radiation Energy Conservation

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2 \tag{3}$$

Where ρ_r is radiation energy density during WI .



Dissipation & Regimes

1. Dissipation Ratio Q

Q $= \Upsilon_{\overline{3H}}$, where Υ = dissipative coefficient (energy transfer rate from ϕ to thermal bath).

2. Two Dissipative Regimes

Weak Dissipation ($Q < 1$)

- Hubble damping ($3H\dot{\phi}$) dominates
- Background dynamics similar to CI
- Thermal fluctuations affect perturbations

Strong Dissipation ($Q > 1$)

- Dissipative damping ($\Upsilon\dot{\phi}$) dominates
- Allows $m_\phi > H$ (solves η-problem)
- Sub-Planckian inflaton excursion ($\phi < M_{Pl}$)

Overview of Warm Inflation Models

1. Two-Stage Dissipation Model Inflaton doesn't directly feed the thermal bath: first, it passes energy to heavy intermediate fields. These heavy fields then decay into light particles.

2. Warm Little Inflaton (WLI) Model Inflaton is a "pseudo-Nambu-Goldstone boson" (from broken symmetry). It couples to fermions with bounded masses (masses never get too big). These fermions thermalize easily, keeping the bath warm.

3. Axion-Like Inflaton Model Inflaton acts like an axion (has a "shift symmetry" that stops unwanted mass growth). It twists gauge fields (e.g., photon-like fields) via a special interaction.

4. Scalar-Driven Thermal Bath Model Inflaton directly couples to light scalar fields. As the inflaton rolls, it excites these light scalars. .

Dissipation Coefficient Υ

1. **Distributed Mass Model (DMM)**

$$\Upsilon^F(T) = \sum_{j=1}^{N_{th}} C_T^F T,$$

2. **Two-Stage Mechanism Model**

$$\Upsilon = C_\phi \frac{T^3}{\phi^2},$$

3. **Warm Little Inflaton (WLI) Model**

$$\Upsilon = C_T T,$$

4. **Axion-Like Inflaton Model**

$$\Upsilon = C_\Upsilon \frac{T^3}{f^2},$$

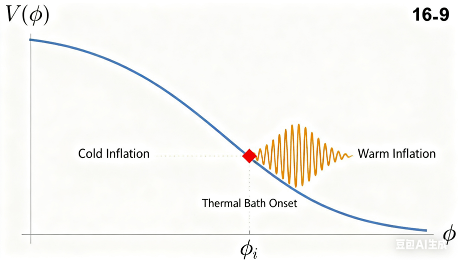
Warm Inflation: Core Evolution Stages

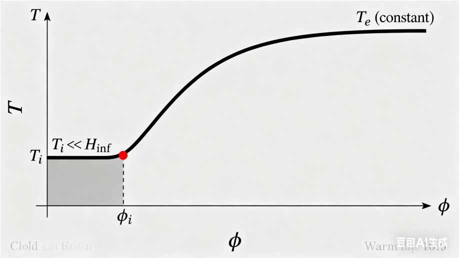
1. Cold Inflation Initial Stage - Inflaton field: $\phi \gg \phi_i$ - Thermal bath: $T = T_i \ll H_{inf}$ - No significant interactions; universe expands in cold regime

2. Transition at $\phi = \phi_i$ - Interactions between inflaton and other fields become prominent - Thermal bath starts to form and heat up

3. Warm Inflation Final Stage - Thermal bath: Stabilizes at constant $T = T_e$ - Warm inflion

$$\Upsilon(\phi, T) = C_\Upsilon T^n \phi^j M^{1-p-c}$$





Equations

Original Core Equations

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_{,\phi} = 0$$

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2$$

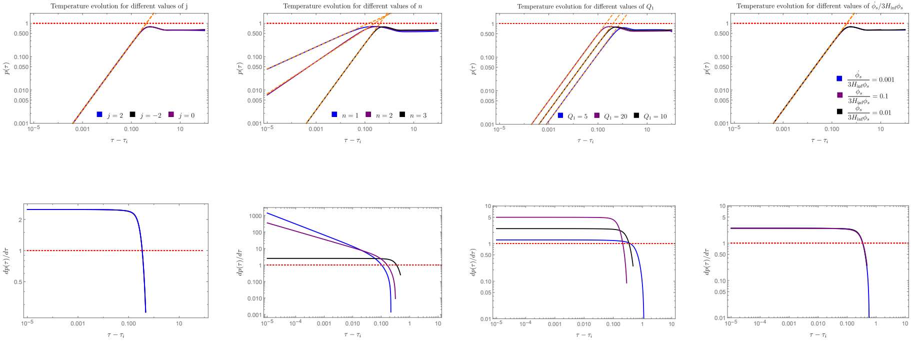
Reparametrization Definitions: $x = \frac{\dot{\phi}}{\dot{\phi}_s}$, $p = \frac{\overline{T}}{\sqrt{\dot{\phi}_s}}$, $y = \frac{\phi}{\phi_i}$, and $\tau = 3H_{\inf} t$, $\quad (\epsilon_V = \frac{M_{Pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2)$

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_{,\phi} = 0 \implies \frac{dx}{d\tau} + (1 + Q)x + \frac{v_\phi}{3H_{\inf\dot{\phi}_s}}$$

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2 \implies \frac{dp}{d\tau} = \frac{1}{4}\frac{Qx^2}{p^3} - \frac{1}{3}p$$

$$\frac{dy}{d\tau} = x\frac{\dot{\phi}_s}{3H_{\inf}\phi_s}, \quad Q(\tau, p) = Q_1 y^j p^n$$

Thermal Evolution Stages: $p(\tau)$ (Top) & $dp(\tau)/d\tau$ (Bottom)



Note: Black solid line: $j = -2$, $n = 3$, $Q_1 = 10$, $\dot{\phi}_s/(3H_{inf}\phi_s) = 0.01$;
Blue/purple lines: Other parameter sets.

Summary: Temperature Evolution Key Parameter Dependence

1. Temperature Evolution Pattern

- Heating phase: Temperature rises with *increasing rate*; - Near asymptotic maximum: Rise rate slows rapidly, then stabilizes;
- Most stages: Follows approximate *power-law behavior*.

2. Dominant Influential Parameters

Q_1 (Energy Transfer Efficiency)
- Larger $Q_1 \rightarrow$ Faster temperature rise rate (higher energy transfer efficiency);
- From $T_e^4 \propto \frac{Q}{(1+Q)^2}$ ($p \approx 1, y \approx 1$ post-heating): $Q_1 \approx 1 \rightarrow$ Max T_e; $Q_1 > 1 \rightarrow$ Larger $Q_1 \rightarrow$ Lower T_e.

n (Model-Specific Exponent) - Typical range: $0 < n \leq 3$ (most WI models);
- Smaller $n \rightarrow$ Faster temperature rise rate;
- Special case: $n = 3 \rightarrow$ Constant temperature change rate (most heating stages).

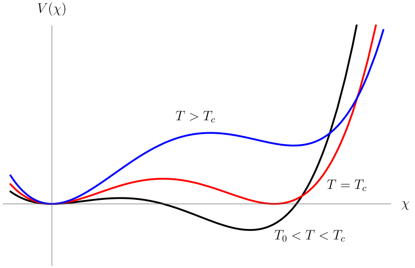
Finite-Temperature Potential for PTs

Potential Form

$$V(\chi, T) = \frac{\mu^2}{2}(T^2 - T_0^2)\chi^2 - \frac{A}{3}T\chi^3 + \frac{\lambda}{4!}\chi^4 \quad (4)$$

Where:

- μ, A, λ are model parameters; T_c : critical temperature



- *Heating Phase Transition (hPT)*: During warm inflation heating, temperature rises above T_c (from $T_s < T_c$ to $T_e > T_c$).
- *Cooling Phase Transition (cPT)*: During radiation-dominated era, temperature cools below T_c.

GW Generation from First-Order PTs

Three Primary Sources of GWs

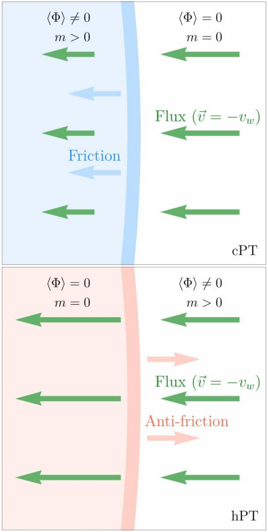
Bubble Collisions

- Energy derived from expanding bubble walls
- Dominant contribution for hPT (due to runaway bubble expansion)

Sound Waves

- Plasma pressure waves triggered by bubble motion
- Dominant contribution for cPT

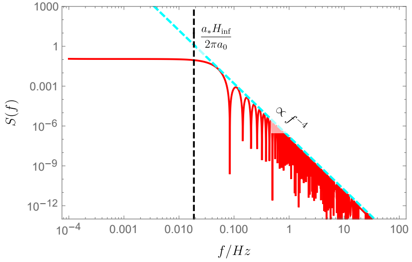
hPT

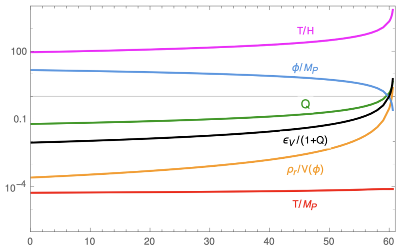


$$h^2\Omega_{GW,hPT}(f) = h^2\hat{\Omega}_{GW}(f\exp(N_*))\frac{H_{inf}^2}{\pi^2 g_*/90 \cdot T_e^4/M_{Pl}^2}S(f) \qquad (5)$$

Key Features

- Redshifted by inflation: $f \to f\exp(-N_*)$ ($N_* =$ e-folds since hPT)
- Oscillatory structure: From $S(f)$
- Low-frequency band





cPT GW Power Spectrum

Key Features

- No inflationary redshift (post-inflationary)
- Smooth peak: Dominated by sound waves
- High-frequency band

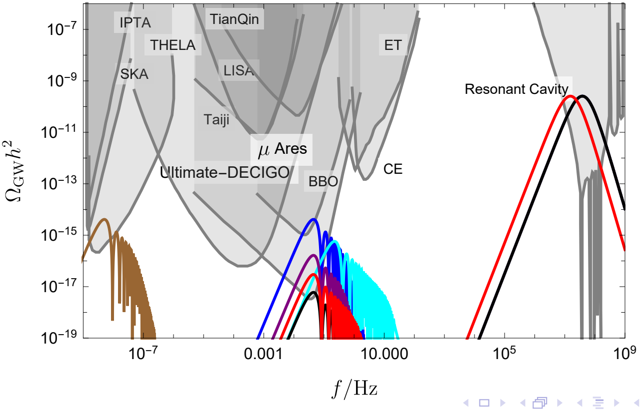
Spectrum Formula

$$h^2\Omega_{GW,cPT}(f) = 2.56 \times 10^{-6} \left(\frac{H_*}{\beta_{cPT}}\right) \left(\frac{\kappa_{cPT}\alpha_{cPT}}{1+\alpha_{cPT}}\right)^2 \times \left(\frac{100}{g_*}\right)^{1/3} v_{cPT} \left(\frac{f}{f_{cPT}}\right)^3 \left(\frac{7}{4+3(f/f_{cPT})^2}\right)^{7/2} \quad (6)$$

GW Detectability & WI-CI Discrimination

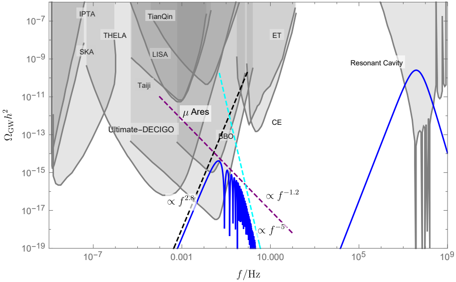
Double-Peak SGWB: Unique WI Signature

- hPT peak: Low frequency $\rightarrow T_e$, N_* and H_{inf}
- cPT peak: High frequency $\rightarrow T_c$



GW Generation from First-Order PTs

- for $f < \frac{a_* H_{\text{inf}}}{2\pi a_0}$, $\Omega_{\text{GW,hPT}} \propto f^{2.8}$;
- for $\frac{a_* H_{\text{inf}}}{2\pi a_0} < f < f_{\text{hPT}} \exp(-N_*)$, $\Omega_{\text{GW,hPT}} \propto f^{-1.2}$
- for $f > f_{\text{hPT}} \exp(-N_*)$, $\Omega_{\text{GW,hPT}} \propto f^{-5}$.



two constraints

- $\frac{a_* H_{\text{inf}}}{2\pi a_0} \propto \exp(-N_*) \frac{T_0}{T_e} H_{\text{inf}}$ and $f_{\text{hPT}} \propto \frac{\beta_{\text{hPT}}}{H_{\text{inf}}} T_e$
- the peak of the GW signal from the hPT depends on $\frac{H_{\text{inf}}}{\beta_{\text{hPT}}} \frac{\alpha^2}{\mathcal{R}^2} \frac{H_{\text{inf}}^2 M_{\text{pl}}^2}{T_e^4}$

- these constraints exactly help us determine the value of $\frac{\beta_{\text{hPT}}}{H_{\text{inf}}}$.
- distinguish whether inflation is WI but also gain deeper insights into the generation of the thermal bath.

Summary: WI Dual PTs GW Signatures

- 1. Phase transition gravitational waves can be used to determine inflation models, WI or CI.
- 2. Phase transition gravitational waves can be used to determine the warm inflation parameter $\frac{\beta_{\mathrm{hPT}}}{H_{\mathrm{inf}}}$.
- 3. The constraints on the not strong dissipation region, $Q \sim 1$, can be supplemented.

Thank You!