Acoustic gravitational waves from primordial curvature perturbations

Zhuan Ning (宁专), UCAS

Based on: *Sound waves from primordial black hole formations*, arXiv: 2504.12243
Relic gravitational waves from primordial gravitational collapses, arXiv: 2504.11275
Numerical simulations of acoustic gravitational waves from primordial curvature perturbations, arXiv: 2510.xxxxx

Co-authors: Xiang-Xi Zeng, Zi-Yan Yuwen, Shao-Jiang Wang, Heling Deng, Rong-Gen Cai

2025 Gravitational Wave Physics Conference

2025.10.19 @ Chun'an

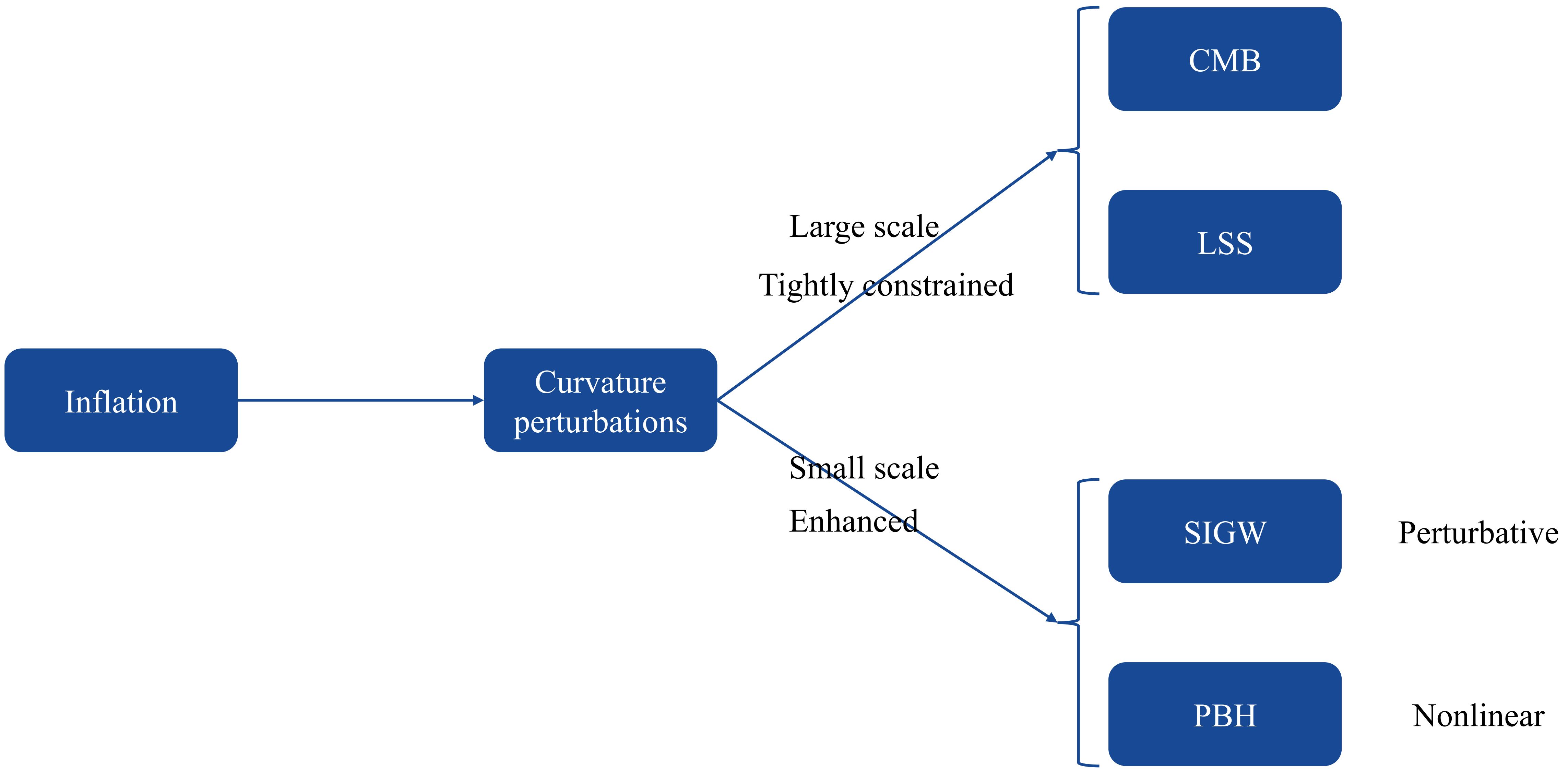
Outline

- Introduction
- Sound Shell Model
- Simulation Setup
- Numerical Results
- Conclusions and Discussion

Outline

- Introduction
- Sound Shell Model
- Simulation Setup
- Numerical Results
- Conclusions and Discussion

Introduction



Introduction

- SIGWs omit some nonlinear effects!
  - Early matter-dominated era [0901.0989, 1002.3278, 2212.00425, 2312.12499]
    - Collapse of halos
    - Tidal interactions
    - Evaporation of halos into radiation
    - Dissipation after reheating
  - Non-spherical collapse of peaks of curvature perturbations [1905.13459]
  - Sound waves

Sound waves from primordial black hole formations, arXiv: 2504.12243

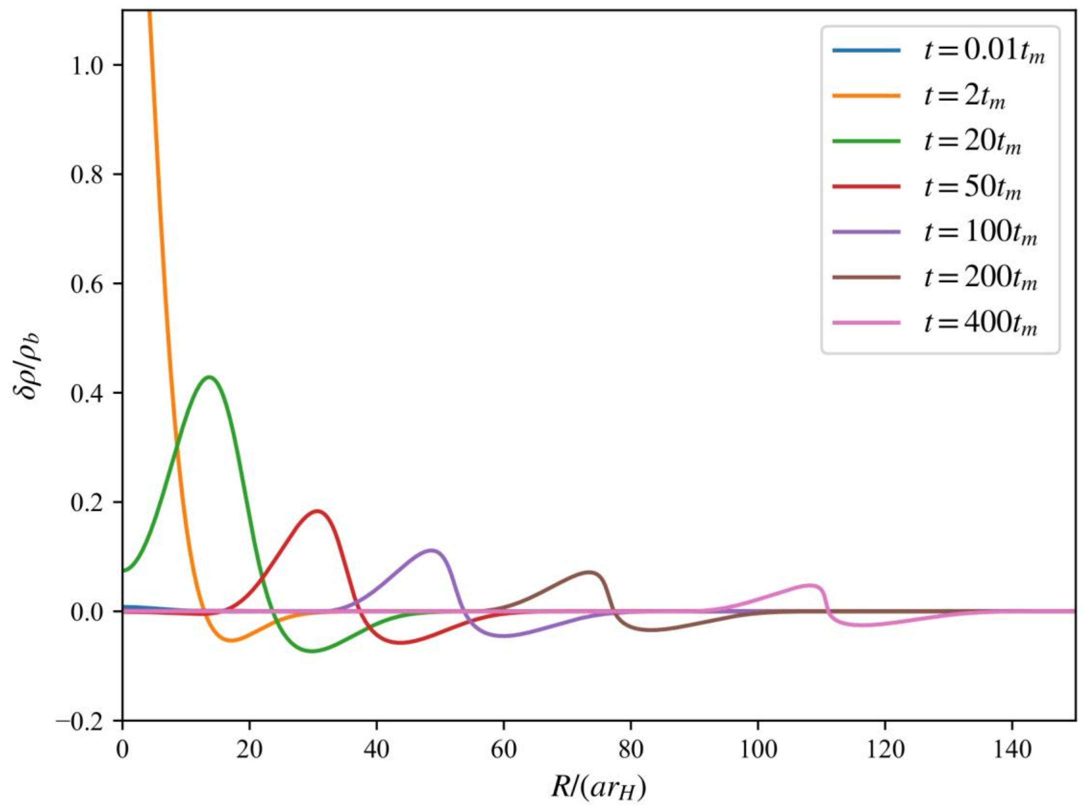
Relic gravitational waves from primordial gravitational collapses, arXiv: 2504.11275

Numerical simulations of acoustic gravitational waves from primordial curvature perturbations, arXiv: 2510.xxxxx

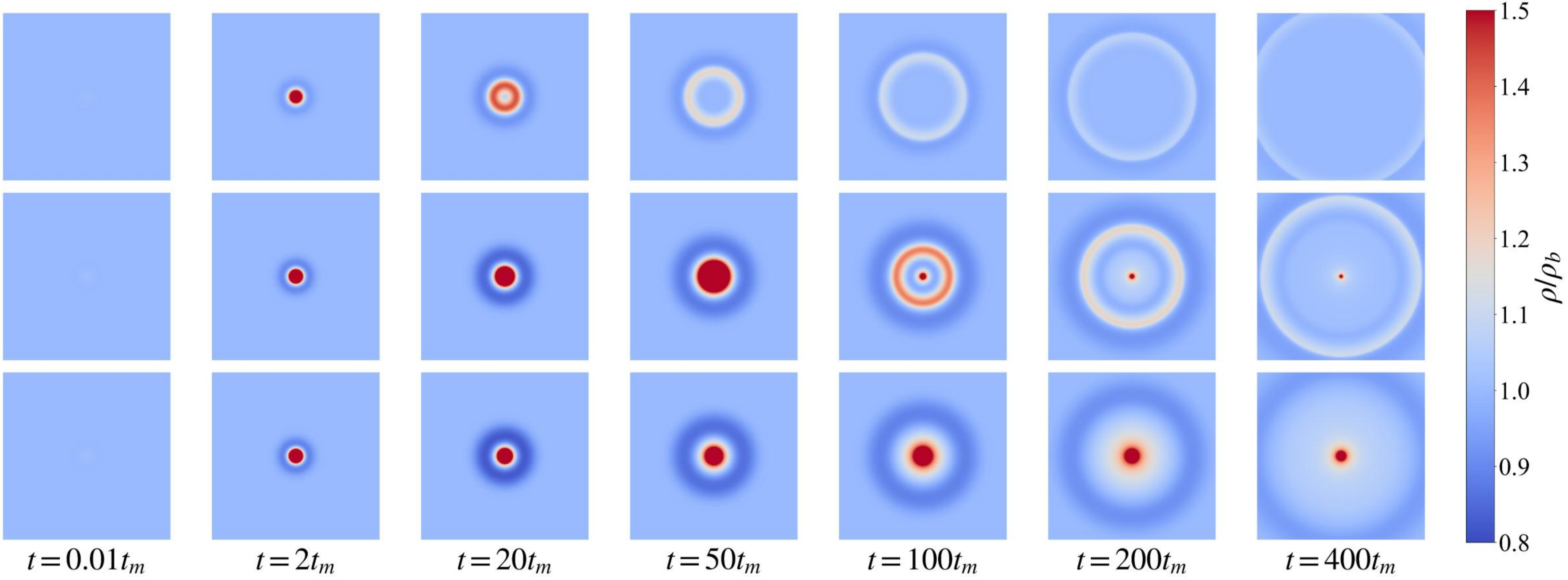
Introduction

- Misner-Sharp formalism
  - Metric ansatz: $ds^2 = -A(t,r)^2\mathrm{d}t^2 + B(t,r)^2\mathrm{d}r^2 + R(t,r)^2\mathrm{d}\Omega^2$
  - Energy-momentum tensor: $T_{\mu\nu} = (\rho+p)u_\mu u_\nu + p g_{\mu\nu}$
  - Equation of state: $p = \omega\rho$

$$
\begin{aligned}
D_r A &= \frac{-A}{\rho+p} D_r p, \\
D_r M &= 4\pi\Gamma\rho R^2, \\
\Gamma &= \sqrt{1+U^2-\frac{2M}{R}}, \\
D_t R &= U, \\
D_t U &= -\left[\frac{\Gamma}{(\rho+p)} D_r p + \frac{M}{R^2} + 4\pi R p\right], \\
D_t M &= -4\pi R^2 U p, \\
D_t \rho &= -\frac{(\rho+p)}{\Gamma R^2} D_r (U R^2).
\end{aligned}
$$



Introduction

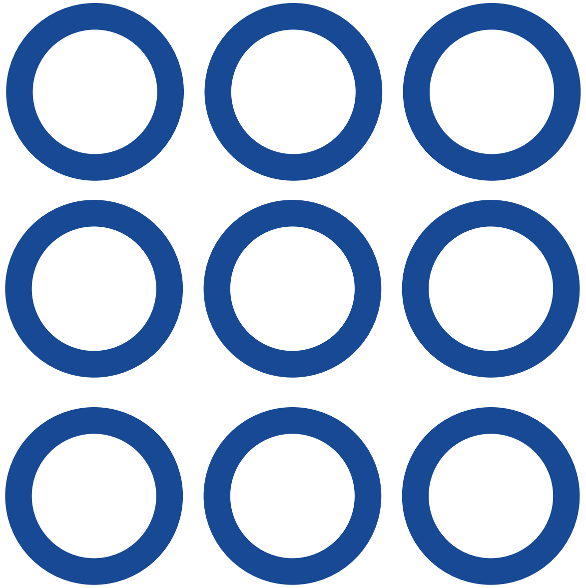


Sub-critical, near-critical, super-critical

Outline

- Introduction
- Sound Shell Model
- Simulation Setup
- Numerical Results
- Conclusions and Discussion

Sound Shell Model



- A semi-analytical method adapted from FOPTs with approximations:
  - Linear superposition of sound shells
  - Uniform distribution of peaks
  - Same amplitude of peaks

$$\mathcal{P}_{\rm GW}(y, kr_H) = 3\Gamma^2 (H_s a_s R_{*c}) \left(\frac{r_H}{R_{*c}}\right)^7 \frac{(kr_H)^3}{2\pi^2} \times \tilde{P}_{\rm GW}(kr_H)\Upsilon(y)$$

Γ: $(\bar{\rho} + \bar{p})/\bar{\rho} \approx 4/3$

$(H_s a_s R_{*c})$: const

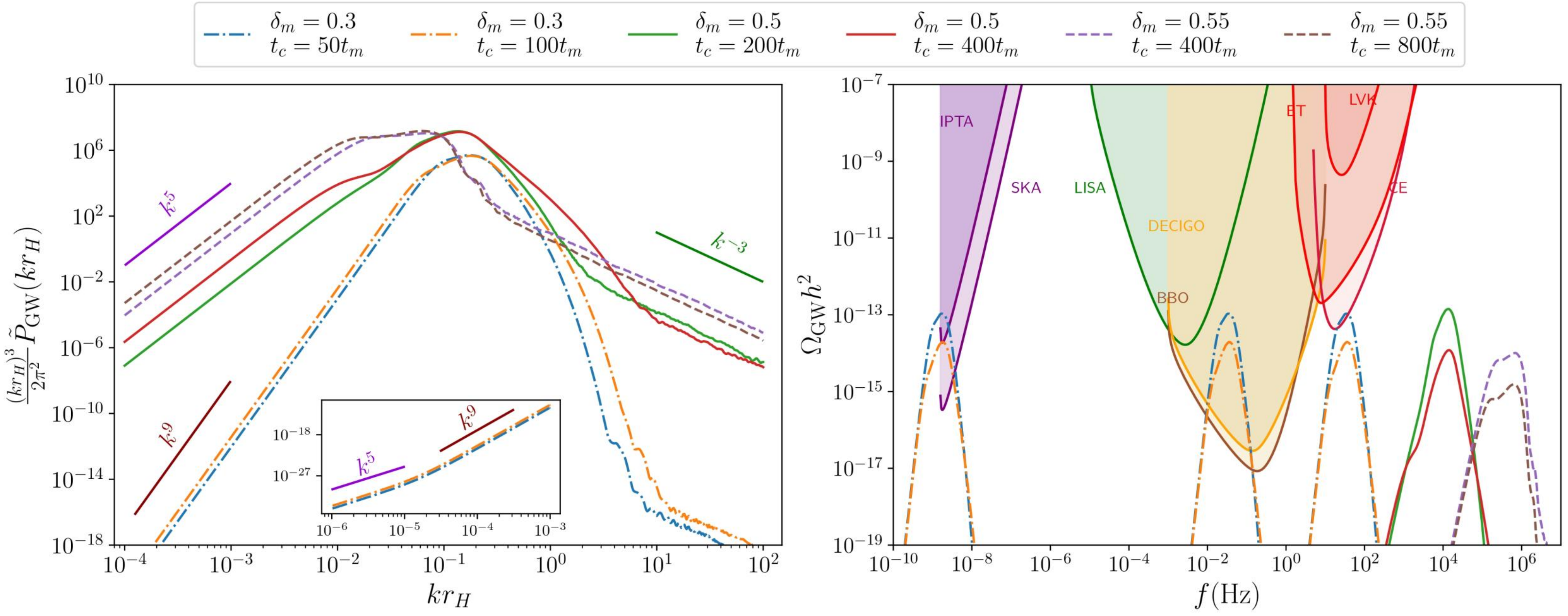
$\tilde{P}_{\rm GW}(kr_H)$: convolution of the velocity power spectrum

$\Upsilon(y)$: 1

R_{*c}: mean comoving separation between sound shells (peaks)

Sound Shell Model

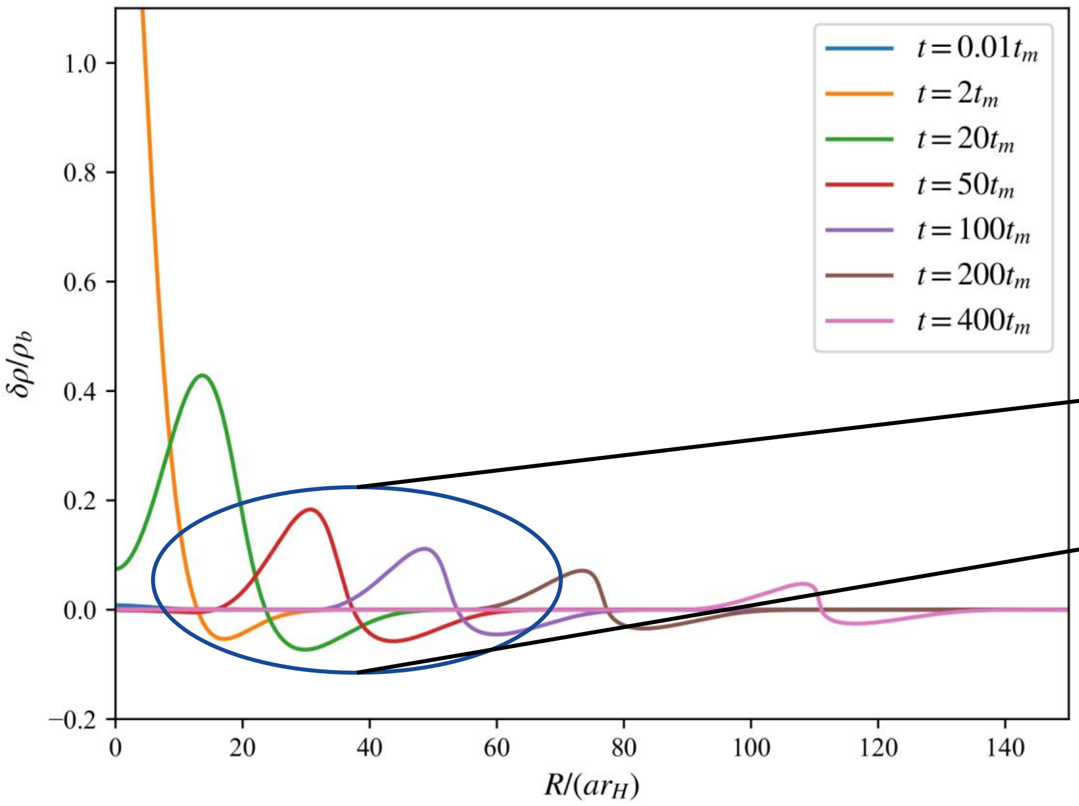
- The dimensionless GW power spectrum



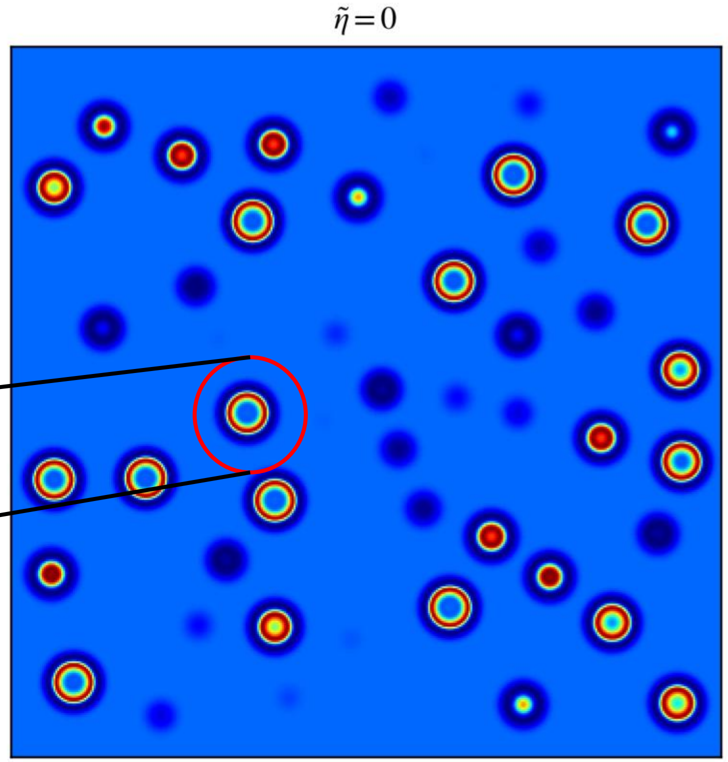
Outline

Hybrid Simulations

- First stage: gravitational collapse of peaks to generate sound waves
- Second stage: collisions between sound shells to produce GWs



Fully GR, spherically symmetric



Neglect gravitational back-reaction, 3D

Hybrid Simulations

- Length of the curvature perturbations: $r_m = 10 r_h$
- Delta-function power spectrum
- Sound shells are formed at the same time

$$\Omega_{\mathrm{GW}} = \frac{1}{\rho_{\mathrm{c}}} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\log k}$$

$$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\log k} = \frac{m_{\mathrm{p}}^2 k^3}{8\pi^2} P_{\dot{h}}(k,t)$$

Equations of Motion

$$\mathrm{d}s^2 = -a(\eta)^{2\alpha}\mathrm{d}\eta^2 + a(\eta)^2(\delta_{ij} + h_{ij})\mathrm{d}x^i\mathrm{d}x^j$$

η: α-time (CosmoLattice). $\alpha = 0$: cosmic time, $\alpha = 1$: conformal time

- Relativistic fluid:

$$T^{\mu\nu} = (\rho + p)U^\mu U^\nu + pg^{\mu\nu}$$

$$U^0 = \gamma/a^\alpha, \quad U^i = \gamma u^i/a$$

$$\gamma = \frac{1}{\sqrt{1-u^2}} = \frac{1}{\sqrt{1-\delta_{ij}u^i u^j}}$$

$$\bar{T}^{00} = a^{2\alpha+4}T^{00},$$

$$\bar{T}^{0i} = a^{\alpha+5}T^{0i},$$

$$\bar{T}^{ij} = a^6 T^{ij}.$$

$$\partial_\eta \bar{T}^{00} + a^{\alpha-1}\partial_i \bar{T}^{0i} + a^4(3p-\rho)\frac{a'}{a} = 0,$$

$$\partial_\eta \bar{T}^{0i} + a^{\alpha-1}\partial_j \bar{T}^{ij} = 0.$$

$$\bar{T}^{ij} = \frac{\bar{T}^{0i}\bar{T}^{0j}}{\bar{T}^{00} + a^4 p} + a^4 p\delta^{ij}$$

Equations of Motion

- Spacetime:

$$\mathcal{H}^2 = \left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{3m_p^2}\langle E\rangle$$

$$\frac{a''}{a} = \frac{a^{2\alpha}}{6m_p^2}\langle(2\alpha-1)E-3P\rangle$$

$$E = a^{2\alpha}T^{00} = \bar{T}^{00}/a^4,$$

$$P = a^2\sum_i T^{ii}/3 = \sum_i \bar{T}^{ii}/(3a^4).$$

- GW:

$$u''_{ij} + (3-\alpha)\mathcal{H}u'_{ij} - a^{2\alpha-2}\nabla^2 u_{ij} = 2a^{2\alpha-2}\Pi^{\mathrm{eff}}_{ij}$$

$$\Pi^{\mathrm{eff}}_{ij} = a^2(\rho+p)\gamma^2 u_i u_j = a^2(\rho+p)\gamma^2 u^i u^j = (\bar{T}^{ij} - a^4 p\delta^{ij})/a^2$$

$$\tilde{\eta} = \omega_*\eta, \quad \tilde{x} = \omega_* x, \quad \tilde{T}_{\mu\nu} = \frac{\bar{T}_{\mu\nu}}{f_*^2\omega_*^2}, \quad \tilde{u}_{ij} = \left(\frac{m_p}{f_*}\right)^2 u_{ij},$$

$f_* = 1$ and $\omega_* = 1/r_h$

Outline

- Introduction
- Sound Shell Model
- Simulation Setup
- Numerical Results
- Conclusions and Discussion

Representative Slices

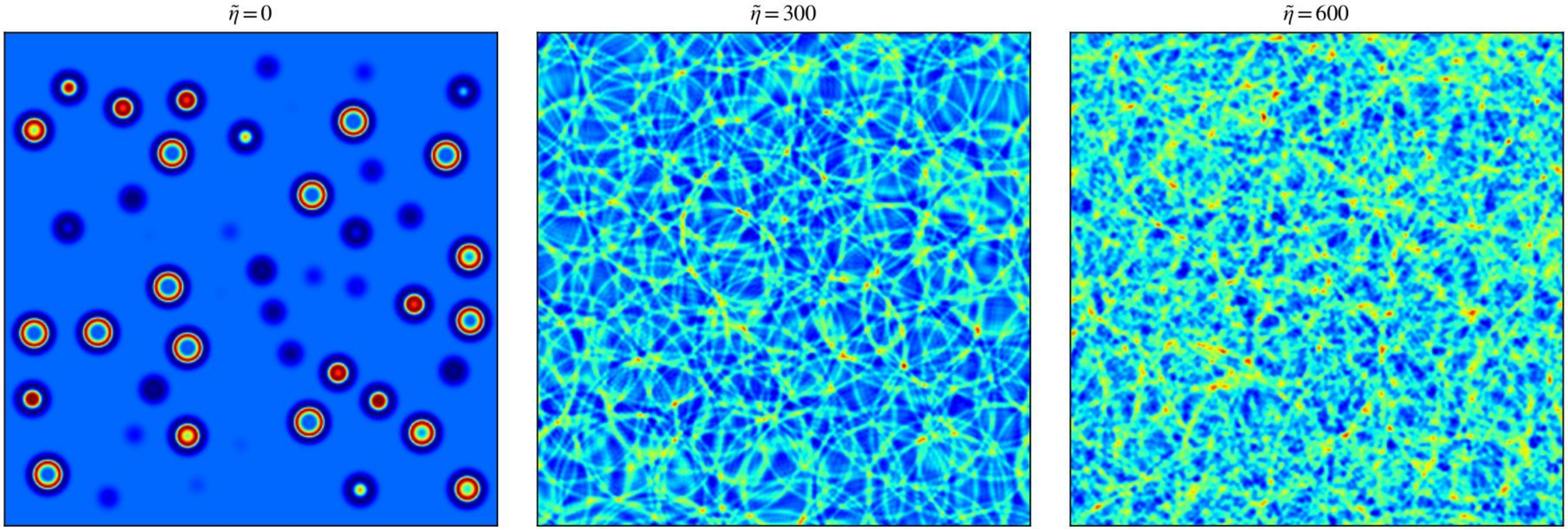
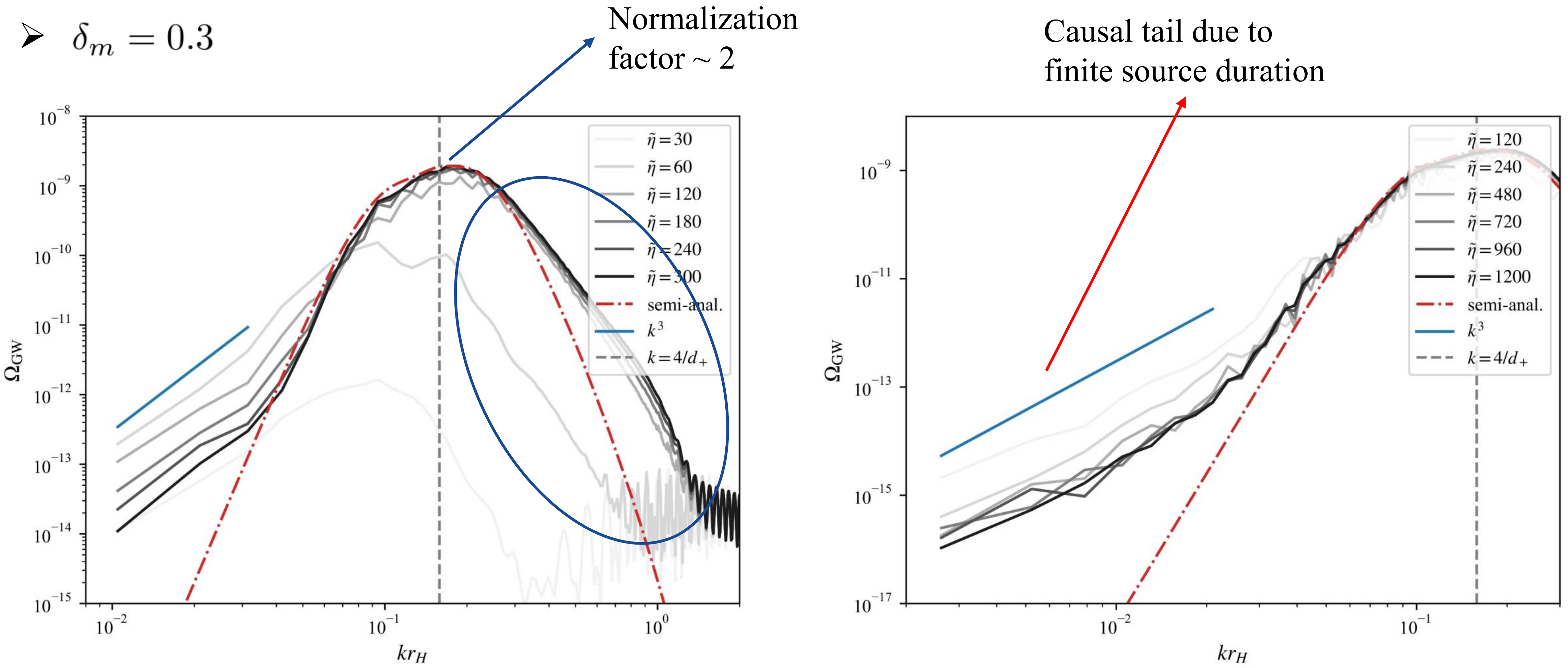


Figure 3. Slices of $\tilde{T}^{00}$ from a representative simulation. The initial condition is constructed by embedding the 1D sound-shell profile with $\delta_m = 0.3$ at $t_i = 50t_m$; the box length is $\tilde{L} = 1200$ and the number of sound shells is $N_s = 400$. The three slices correspond to $\tilde{\eta} = 0$ (left), $\tilde{\eta} = 300$ (middle), and $\tilde{\eta} = 600$ (right).

Shape of the GW Energy Spectrum

- $\delta_m = 0.3$

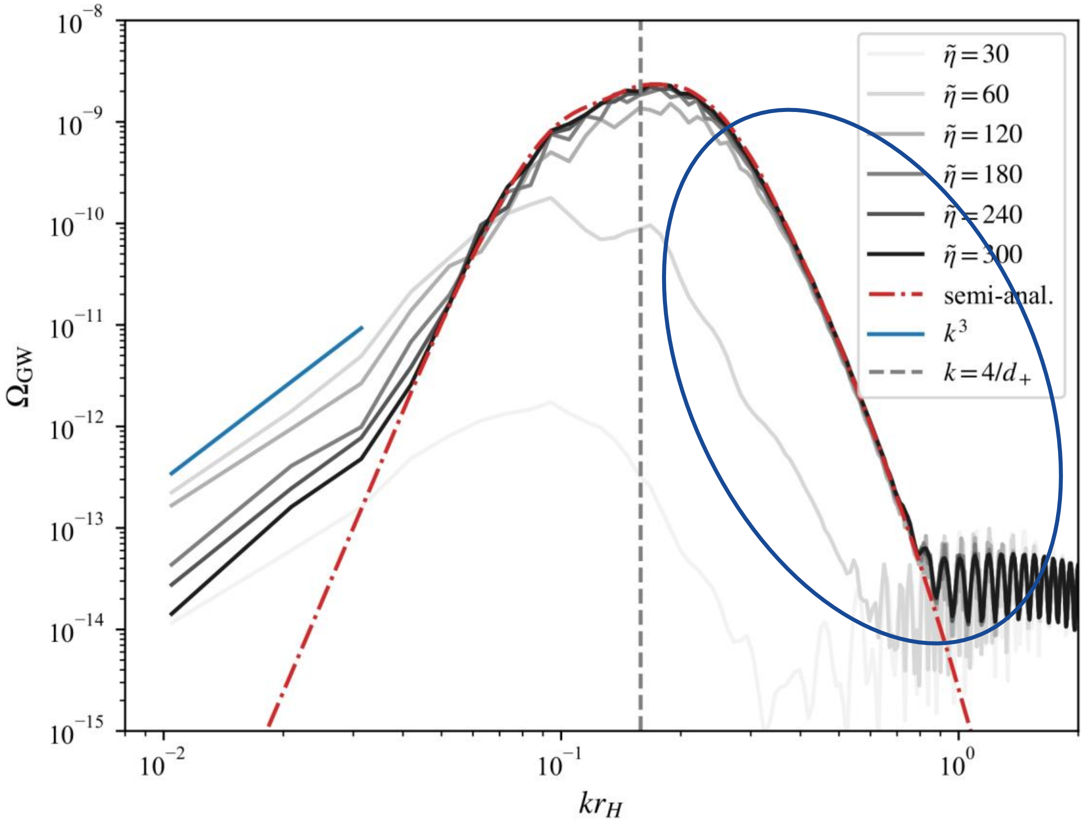


$\tilde{L} = 600,\ N_s = 50$ $\quad$ $\tilde{L} = 1200,\ N_s = 400$

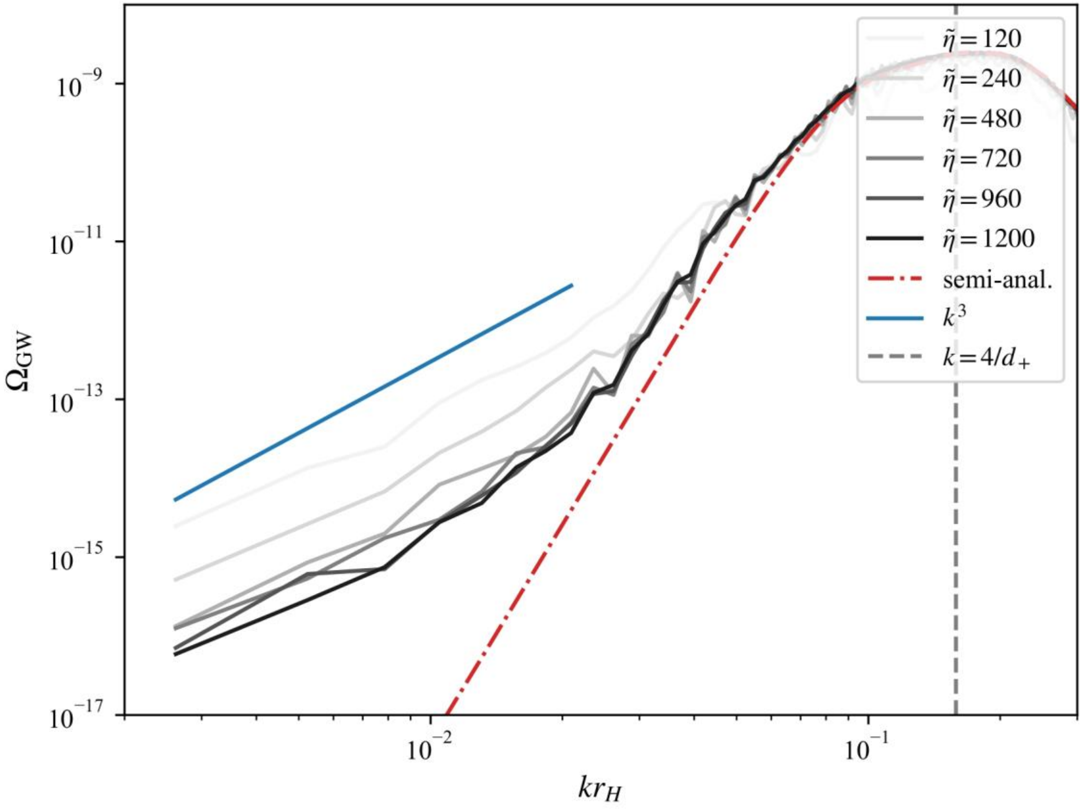
Shape of the GW Energy Spectrum

$$3\delta' + 4\partial_i u^i = 0,$$
$$4u^{i\prime} + \partial_i \delta = 0,$$

- $\delta_m = 0.3$ using the linearized hydrodynamical equations

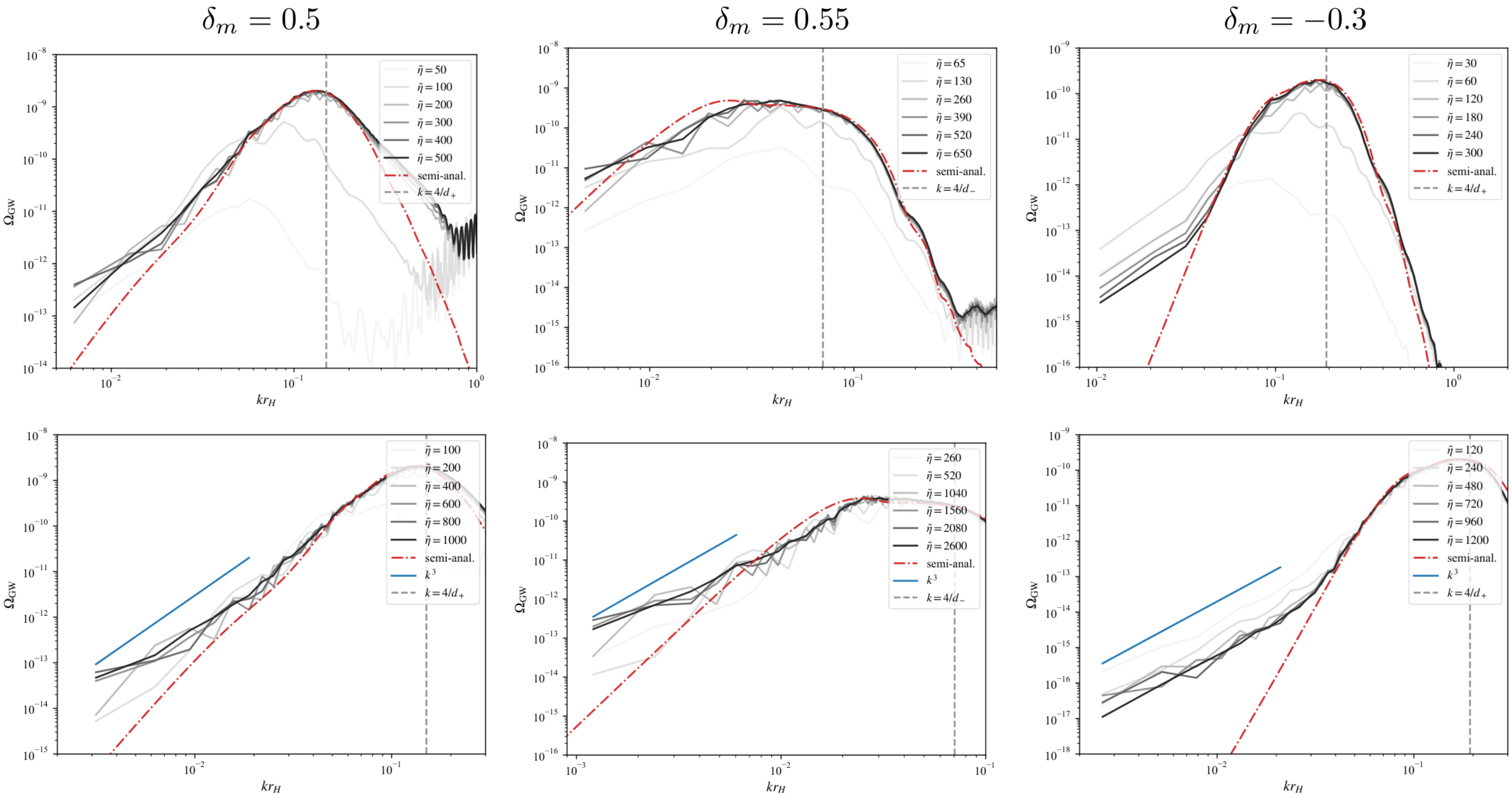


$\tilde{L} = 600,\ N_s = 50$



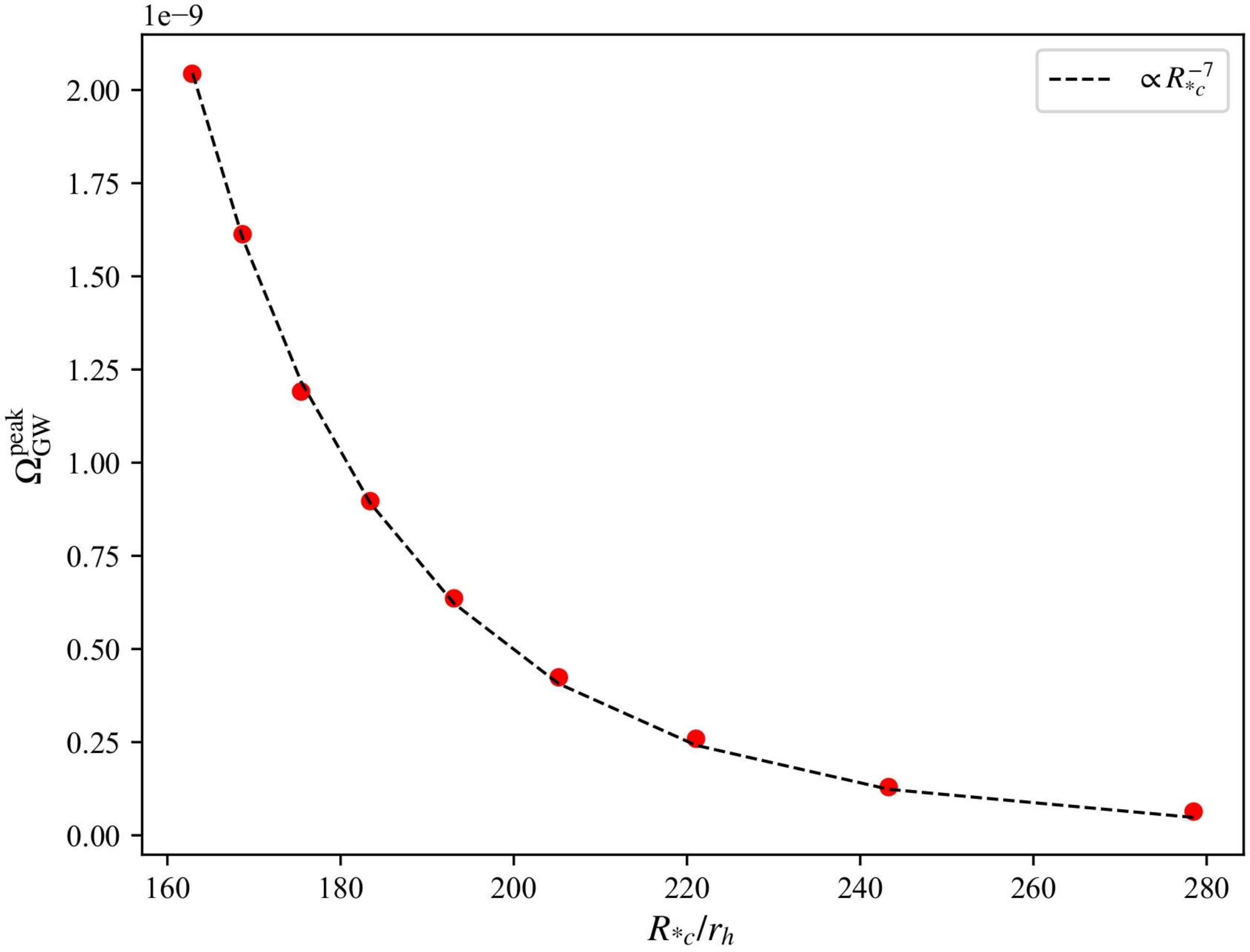
$\tilde{L} = 1200,\ N_s = 400$

Shape of the GW Energy Spectrum



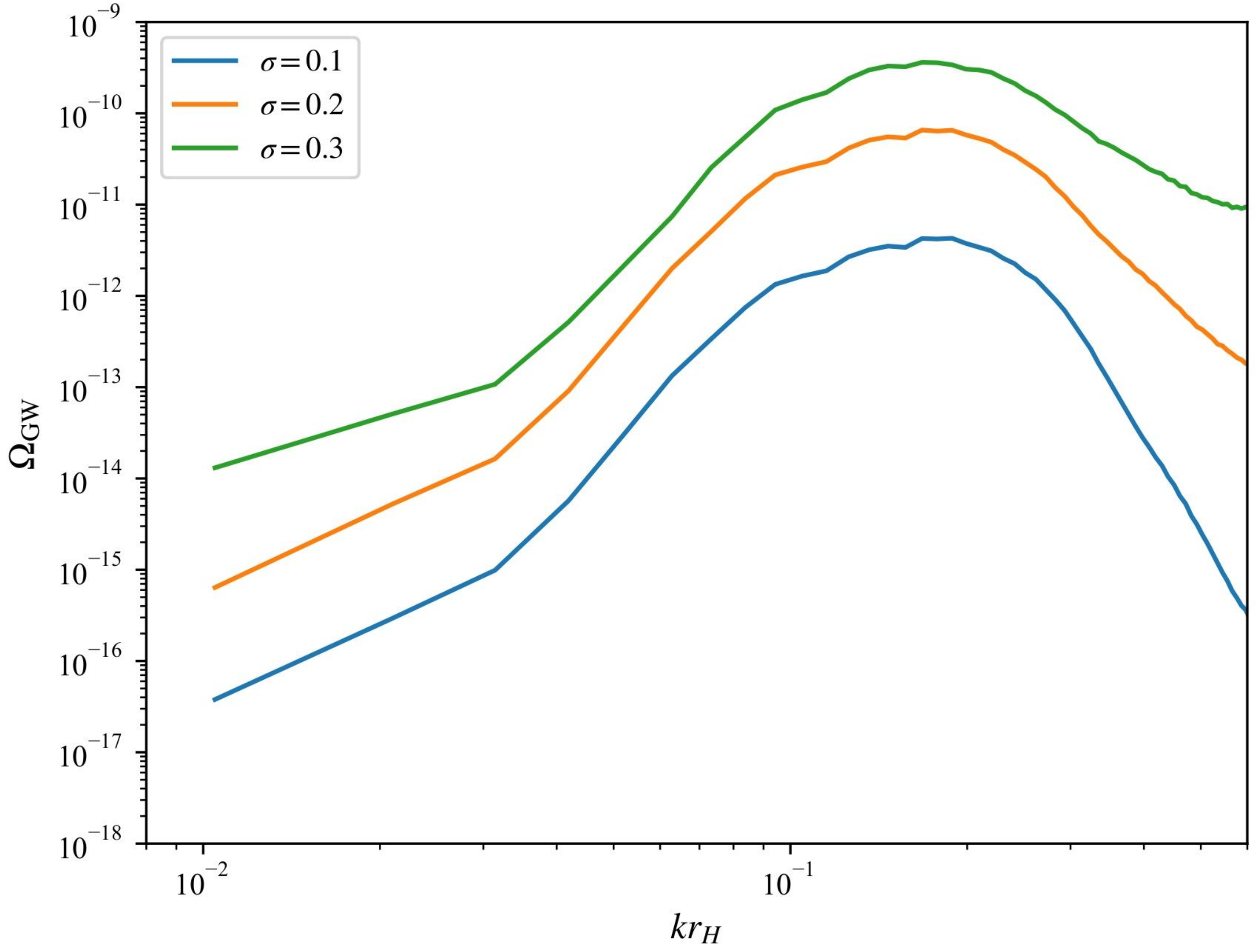
Effect of the Mean Comoving Separation

- R_{*c}^{-7} scaling



Effect of the Perturbation-amplitude Distribution

- Gaussian probability density with standard deviation σ: $-0.49 \leq \delta_m \leq 0.49$



Outline

Conclusions and Discussion

- GW peak frequency is controlled by the shell thickness
- GW peak amplitude is extremely sensitive to R_{*c}
- Causality sets the IR scaling
- Nonlinear hydrodynamics determines the UV scaling

- More realistic treatment?
- Fully GR simulations?
- Quantitative comparison with SIGWs?

Thank you for your attention!