# Quantum search for gravitational wave of massive black hole binaries

(based on PRD 112, 083004)

Fangzhou Guo[1,2,3,4] and Jibo He[1,2,4,5]

Email: guofangzhou23@mails.ucas.ac.cn

[1]School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou

[2]University of Chinese Academy of Sciences, Beijing

[3]Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing

[4]International Centre for Theoretical Physics Asia-Pacific, UCAS, Beijing

[5]Taiji Laboratory for Gravitational Wave Universe (Beijing/Hangzhou), UCAS, Beijing

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# Contents

# Classical Matched Filtering

- In Gaussian and stationary noise environments, the optimal linear algorithm for extracting weak signals. [1]
- Works by correlating a known signal model $\mathrm{h(t)}$ (template) with the data $\mathrm{d(t)}$.
- Defining the matched-filtering SNR $\rho(\mathrm{t})$:$\rho^2(\mathrm{t}) \equiv \frac{1}{\langle \mathrm{h}|\mathrm{h}\rangle}|\langle \mathrm{d}|\mathrm{h}\rangle(\mathrm{t})|^2$, where $\langle \mathrm{d}|\mathrm{h}\rangle(\mathrm{t}) = 4\int_0^\infty \frac{\bar{\mathrm{d}}(\mathrm{f})\bar{\mathrm{h}}^*(\mathrm{f})}{\mathrm{S_n(f)}}\mathrm{e}^{2\pi \mathrm{ift}}\mathrm{df}$, $\langle \mathrm{h}|\mathrm{h}\rangle = 4\int_0^\infty \frac{\tilde{\mathrm{h}}(\mathrm{f})\bar{\mathrm{h}}^*(\mathrm{f})}{\mathrm{S_n(f)}}\mathrm{df}, \mathrm{S_n(f)}$ is noise power spectral density (one-sided).

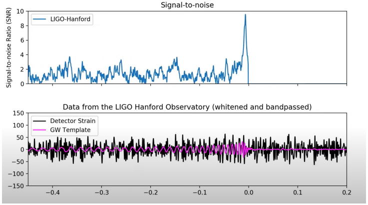



Figure 1: Matched filtering of GW151226

# Searching for MBHBs

- High-dimensional parameter space, which typically spans around 15 dimensions, depending on the inclusion of spin and orbital effects [2].
- Template bank size could reach $10^{13}$ for nonspinning MBHB searches [3, 4].
- High computational cost for large template banks when using matched filtering.
- Previous study: Grover's algorithm can be applied to matched filtering and tested on the GW150914 event [5]. But its application to MBHB signals remains unexplored.

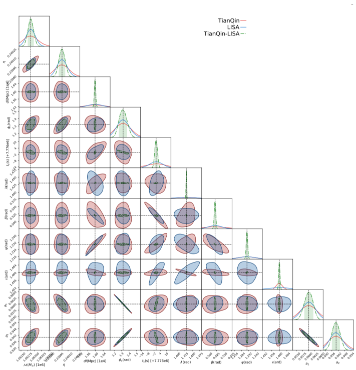



Figure 2: 11 dimension parameter space of MBHBs

# Problem Description

- Unstructured search problem: Given a search space of size N and r marked solutions such that $f(x) = 1$ (for non-solutions, $f(x) = 0$), the goal is to find one such x efficiently.
- Grover's algorithm offers a quadratic speedup with a complexity of $O(\sqrt{N})$ [6].
- Suitable for accelerating matched filtering of MBHBs.

# Geometry interpretation

- Geometrically, a rotation in the two-dimensional subspace spanned by $|w\rangle$ and $|w_\perp\rangle$ (marked and nonmarked entry).
- Preparing a uniform superposition over all basis states:

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle \,. \qquad (1)$$

- The initial state $|s\rangle$ lies at an angle $\theta$ from $|w_\perp\rangle$, where

$$\sin(\theta) = \sqrt{\frac{r}{N}}. \qquad (2)$$

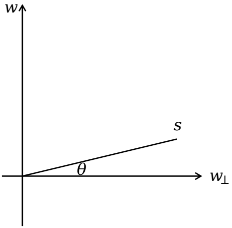



Figure 3: Initial state

# Geometry interpretation

- Apply oracle operator $U_f$ to flip the sign of the marked states:

$$U_f |x\rangle = (-1)^{f(x)} |x\rangle . \tag{3}$$

- Apply diffusion operator

$$D = 2 |s\rangle \langle s| - I. \tag{4}$$

- An oracle followed by the diffusion operator forms the Grover operator $G = D \cdot U_f$, which rotates the state vector by an angle $2\theta$ toward $|w\rangle$.

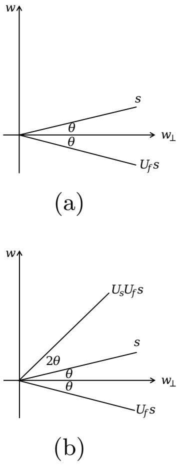



Figure 4: Rotation process

# Geometry interpretation & Quantum circuit

- After approximately

$$k \approx \left\lfloor \frac{\pi/2-\theta}{2\theta} \right\rfloor \quad (5)$$

$$\approx \left\lfloor \frac{\pi}{4}\sqrt{\frac{N}{r}} - \frac{1}{2} \right\rfloor \quad (6)$$

iterations, the state vector is close to $|w\rangle$, and a measurement yields a correct result with high probability, completing the quantum search in $O(\sqrt{N})$ steps.

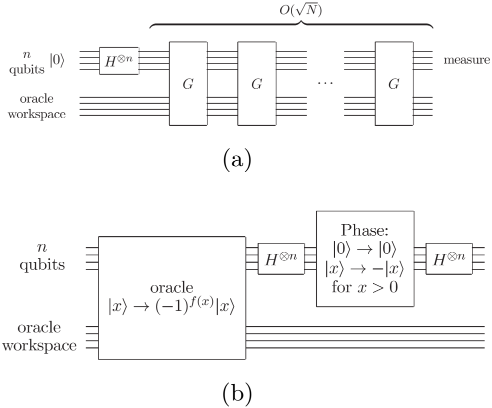



Figure 5: Quantum circuit of Grover's algorithm and operator G [7].

# Quantum counting I

- Quantum counting: estimate the unknown number of solutions r or angle $\theta$ from $|w_\perp\rangle$ using quantum phase estimation.

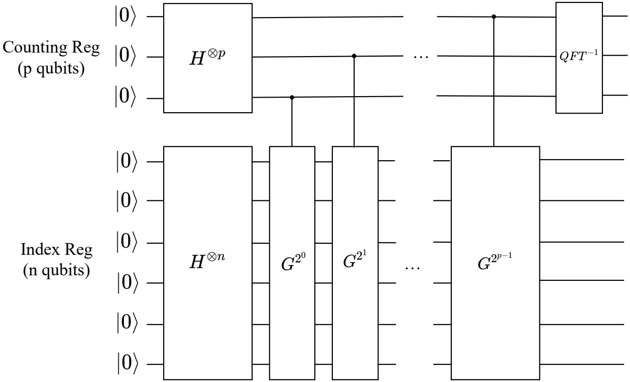



Figure 6: Quantum counting circuit [7].

# Quantum counting II

Let b be the measured outcome, the estimated angle is

$$\theta_* = \begin{cases} \frac{b\pi}{2^p}, & b \leq 2^{p-1}, \\ \pi - \frac{b\pi}{2^p}, & b > 2^{p-1}. \end{cases} \tag{7}$$

Estimation error: For a desired precision of m bits and error probability $\epsilon$, the required number of qubits is [7]:

$$p = m + \log\left(2 + \frac{1}{2\epsilon}\right). \tag{8}$$

If the target precision for $\theta$ is $O(1/\sqrt{N})$, then $m = O(\frac{1}{2}\log N)$ and hence $p = O(\log N)$. The number of applications of G is $2^p - 1 = O(\sqrt{N})$, so the total complexity remains $O(\sqrt{N})$.

# Oracle construction

**Algorithm 1** Grover's Gate
Complexity: $O(M \log M + \log N)$ [5]

**function** Grover's Search algorithm($N$, $|D\rangle$, $\rho_{thr}$)
  **procedure** Oracle Construction
    Creating templates:
    **for all** $i < N$ **do**
      $|i\rangle |0\rangle \leftarrow |i\rangle |T_i\rangle$
    Comparison with the data:
    $|i\rangle |D\rangle |T_i\rangle |0\rangle \leftarrow |i\rangle |D\rangle |T_i\rangle |\rho(i)\rangle$
    **if** $\rho(i) < \rho_{thr}$ **then**
      $f(i) = 0$
    **else**
      $f(i) = 1$
    $|i\rangle |D\rangle |T_i\rangle |\rho(i)\rangle \leftarrow (-1)^{f(i)} |i\rangle |D\rangle |T_i\rangle |\rho(i)\rangle$
    Dis-entangling registers:
    $(-1)^{f(i)} |i\rangle |D\rangle |T_i\rangle |\rho(i)\rangle \leftarrow (-1)^{f(i)} |i\rangle |D\rangle |T_i\rangle |0\rangle$
    $(-1)^{f(i)} |i\rangle |D\rangle |T_i\rangle |0\rangle \leftarrow (-1)^{f(i)} |i\rangle |D\rangle |0\rangle |0\rangle$
  **procedure** Diffusion Operator
    $\sum(-1)^{f(i)} |i\rangle \leftarrow \sum(2 |i\rangle \langle i| - \hat{I})(-1)^{f(i)} |i\rangle$

# Signal Detection

**Algorithm 2** Signal Detection $\quad O\left((M\log M + \log N)\cdot\sqrt{N}\right)$ [5]

procedure Quantum Counting($p$, $N$, $|D\rangle$, $\rho_{thr}$)
    Creating the counting register :
    $|i\rangle \leftarrow |0\rangle^{p}\,|i\rangle$
    $|0\rangle^{p}\,|i\rangle \leftarrow \frac{1}{2^{p/2}}(|0\rangle + |1\rangle)^{p} \otimes |i\rangle$
    Controlled Grover's gate:
    for all $j < 2^{p}$ do
        $a \leftarrow j$
        repeat
            Algorithm 1 Grover's Gate($N$, $|D\rangle$, $\rho_{thr}$), $a--$
        until $a == 0$
    $\frac{1}{2^{p/2}}(|0\rangle + |1\rangle)^{n} \otimes |i\rangle \leftarrow \frac{1}{2^{(p+1)/2}}\sum(e^{2i\theta j}|j\rangle \otimes |s_{+}\rangle + e^{-2i\theta j}|j\rangle \otimes |s_{-}\rangle)$
    Inverse Quantum Fourier Transform:
    $\frac{1}{2^{(p+1)/2}}\sum(e^{2i\theta j}|j\rangle \otimes |s_{+}\rangle + e^{-2i\theta j}|j\rangle \otimes |s_{-}\rangle) \leftarrow \frac{1}{2^{p+1/2}}\sum\sum(e^{i2\pi j(\frac{\theta}{\pi}-\frac{1}{2^{p}})}|l\rangle \otimes |s_{+}\rangle + e^{i2\pi j(\frac{\pi-\theta}{\pi}-\frac{1}{2^{p}})}|l\rangle \otimes |s_{-}\rangle)$
    Measurement ($b$):
    if $b = 0$ then
        return 'There is no match.'
    else $r_{*} \leftarrow$ Round $\left[N\sin\left(\frac{b}{2^{p}}\pi\right)^{2}\right]$

# Template retrieval

**Algorithm 3** Template retrieval
Complexity: $\mathrm{O}\left((\mathrm{M}\log\mathrm{M} + \log\mathrm{N})\cdot\sqrt{\mathrm{N}}\right)$[5]

```
N ← number of templates
i ← index of templates
ρ_thr ←  threshold
|0⟩ ← Data |D⟩
r_* ← number of matched templates
Calculating the number of repetitions:
k_* ← Round [ (π/4) √(N/r_*) − 1/2 ]
procedure Retrieve one template
    repeat
        Algorithm 1 Grover's Gate(N, |D⟩, ρ_thr), k_* − −
    until k_* == 0
    Output:
    i_correct
```

# Resource estimation

Table 1: Estimated quantum resources for simulated case.

| Resource | Symbol | Estimate / Requirement |
|---|---|---|
| Template number (library size) | $N$ | $2^{17} = 131072$ |
| Data length | $M$ | 2592000 |
| Sampling rate | $f_s$ | 0.1 Hz |
| Data register qubits | $n_d$ | $64M \approx 20MB$ |
| Counting register precision | $p$ | 11 qubits |
| Index register qubits | $n_i$ | $\log_2 N = 17$ |
| Total qubits (logical) | $n_{tot}$ | $n_d + n_i + p \approx 20$ MB |
| Circuit depth (order) | $D$ | $O(\sqrt{N/r})$ Grover iterations |

# Signal Detection I

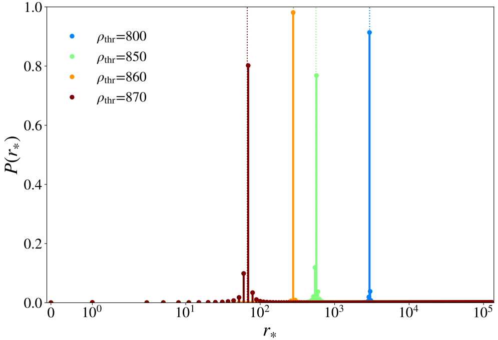



Figure 7: The probability distributions of outcomes from measuring the counting register transformed to estimates on the number of matching templates $r_*$ for each of the different cases of $\rho_{\text{thr}}$. The distributions are compared to the true number of matching templates r (dotted).

# Signal Detection II

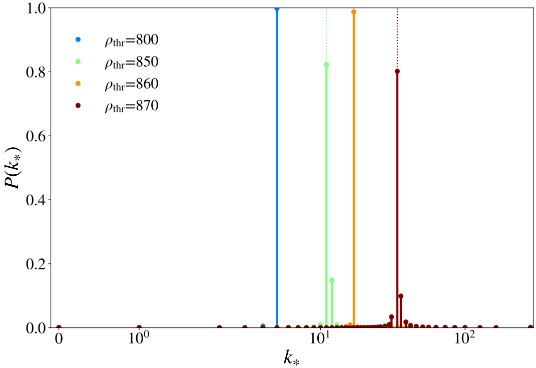
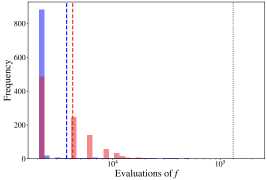
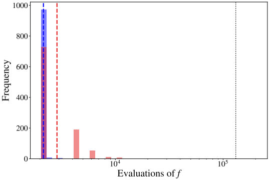



Figure 8: The probability distributions of outcomes from measuring the counting register transformed to estimates on the optimal number of Grover's applications $k_*$ for each of the different cases of $\rho_{\text{thr}}$. The probabilities are compared to the true k (dotted) for each case.

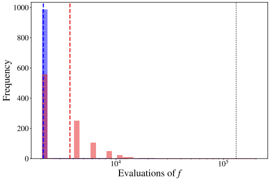
# Template retrieval



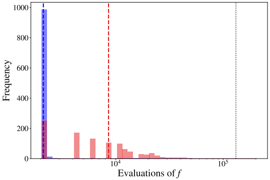

(a) $\rho_{\mathrm{thr}} = 800$




(b) $\rho_{\mathrm{thr}} = 850$



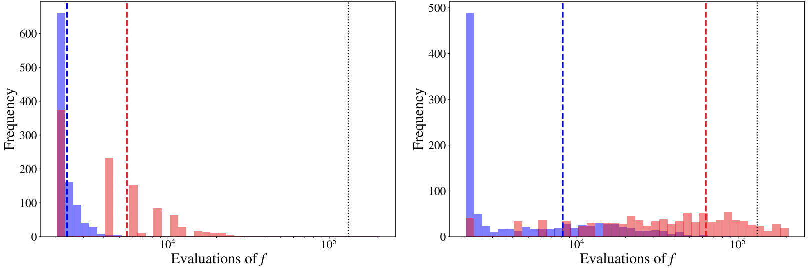

(c) $\rho_{\mathrm{thr}} = 860$




(d) $\rho_{\mathrm{thr}} = 870$

Figure 9: Number of evaluations of the oracle function f required to retrieve a matching template in 1,000 simulations at different threshold levels.

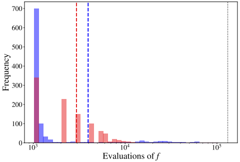
# Tempalte retrieval (GW150914)



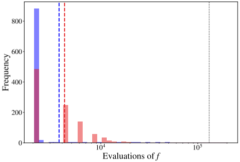

(a) $\rho_{\mathrm{thr}} = 18$ (b) $\rho_{\mathrm{thr}} = 16$

Figure 10: Comparison of algorithm efficiency for detection thresholds $\rho_{\mathrm{thr}} = 16$ and $\rho_{\mathrm{thr}} = 18$ using 1000 simulations given the GW150914 example. The $\rho_{\mathrm{thr}} = 18$ case reproduces the results reported in Ref. [5], while the $\rho_{\mathrm{thr}} = 16$ case shows the results when lowering the threshold.

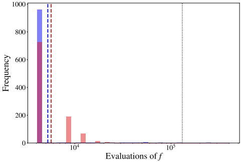
# Impact of precision p



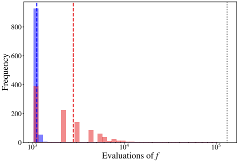

(a) $\mathrm{p} = 10$



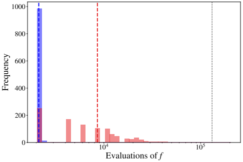

(b) $\mathrm{p} = 11$



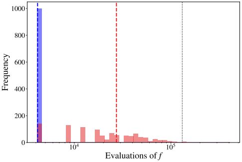

(c) $\mathrm{p} = 12$




(d) $\mathrm{p} = 10$




(e) $\mathrm{p} = 11$




(f) $\mathrm{p} = 12$

Figure 11: $\rho_{\mathrm{thr}} = 800$ (a-c) and $\rho_{\mathrm{thr}} = 870$ (d-f).

# Conclusion

- Apply Grover-based quantum search to MBHBs.
- Quadratic speedup over classical searches theoretically, reducing the cost of searching large template banks.
- Shows potential for future application in space-based GW data analysis as quantum hardware matures
- Performance sensitive to matched-filter thresholds and precision p; robust algorithmic designs are needed.

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Please feel free to contact us:

guofangzhou23@mails.ucas.ac.cn

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