真空衰变的数值模拟与研究

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报告内容

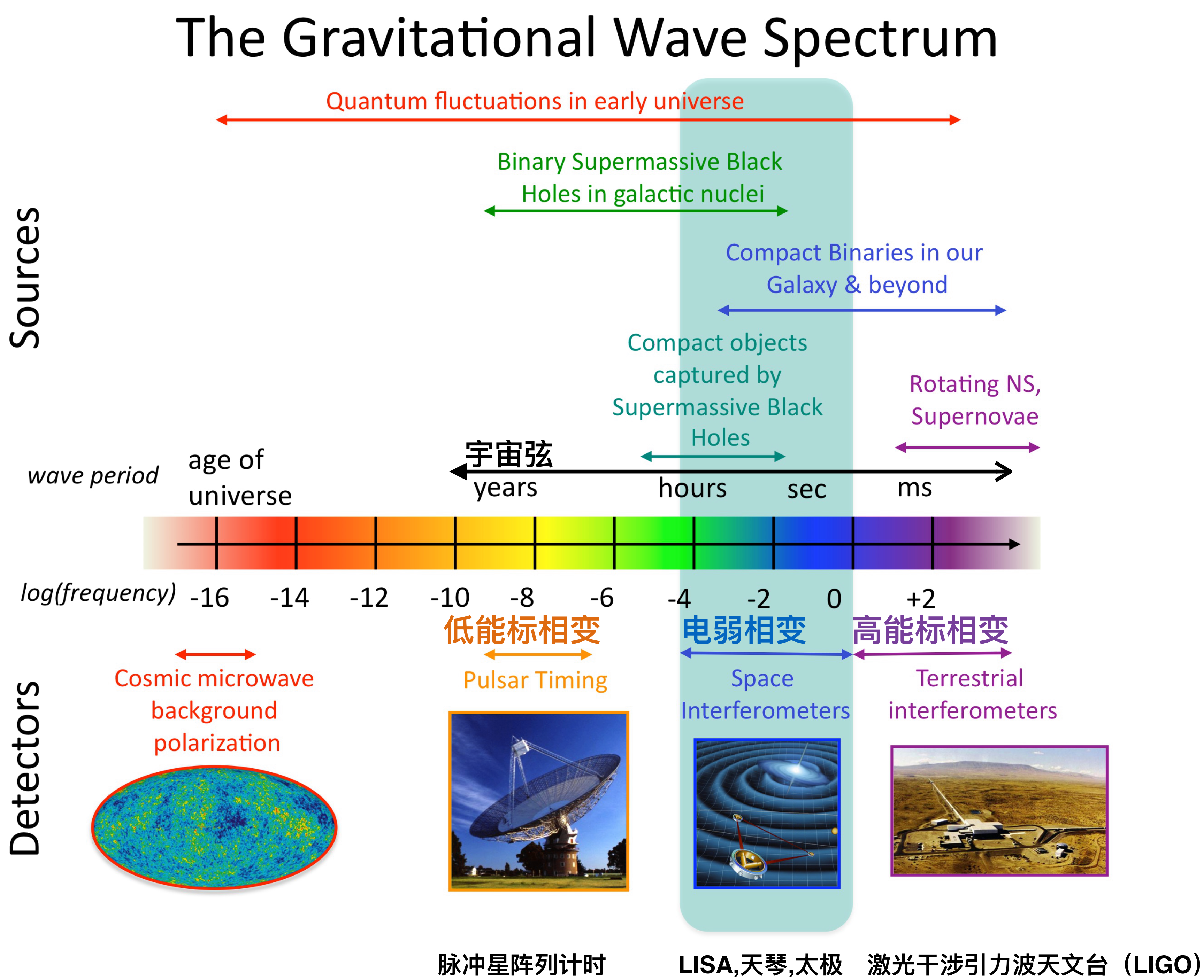
一 背景与动机

二 一阶相变与引力波

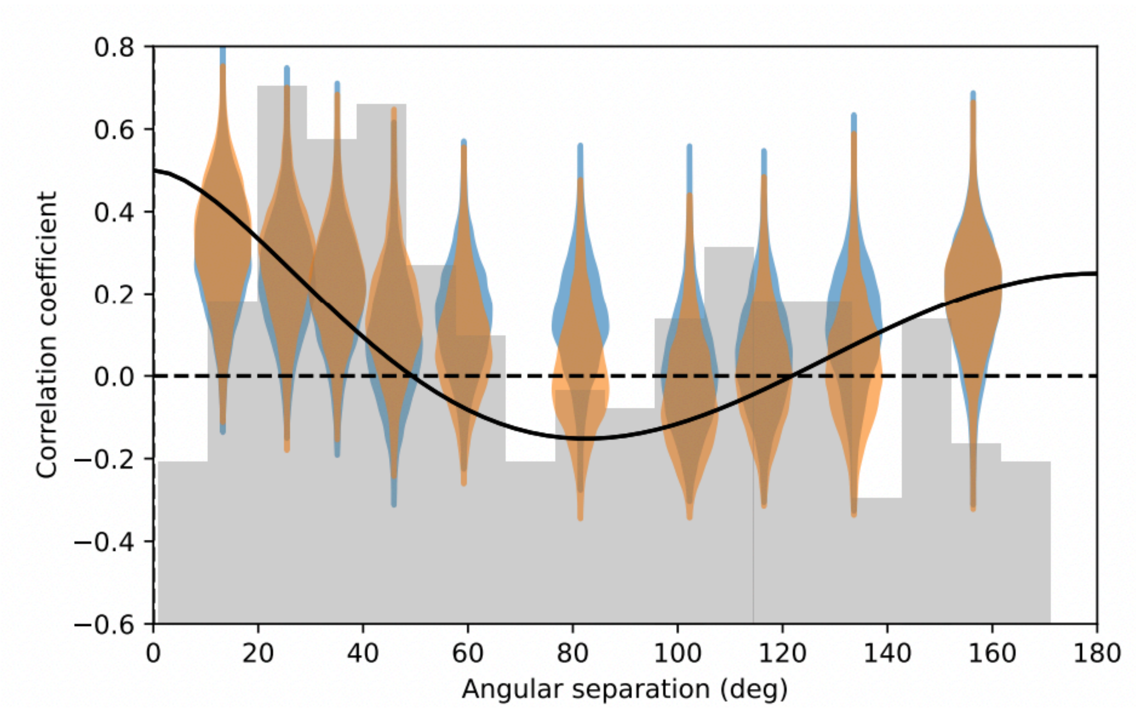
三 热涨落一阶相变

四 总结与展望

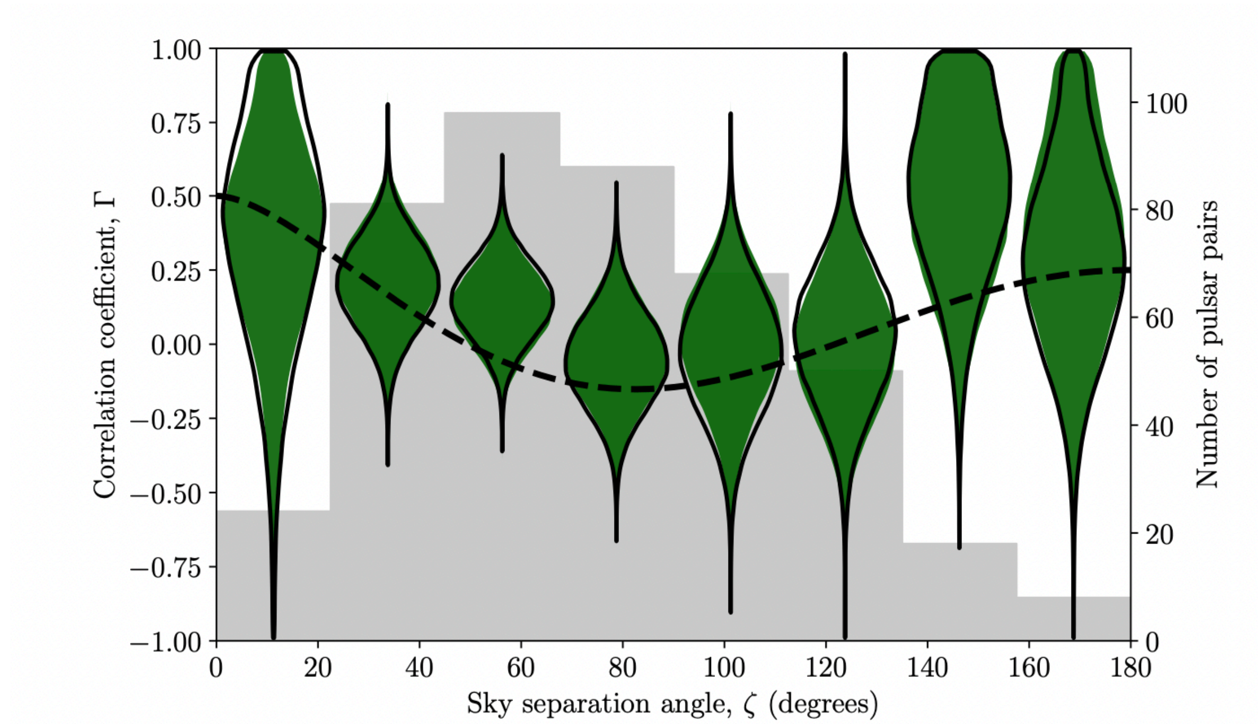
随机引力波探测开启了探索早期宇宙背后基础物理的一个新的窗口



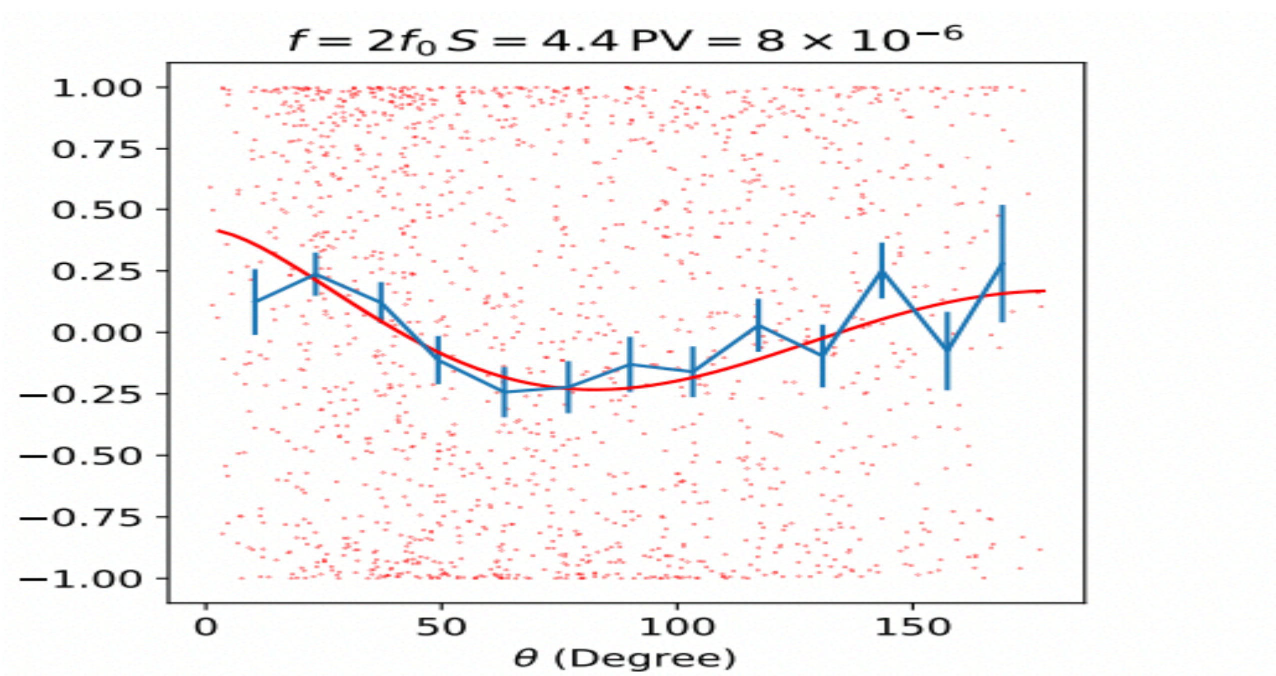
New dataset from PTAs



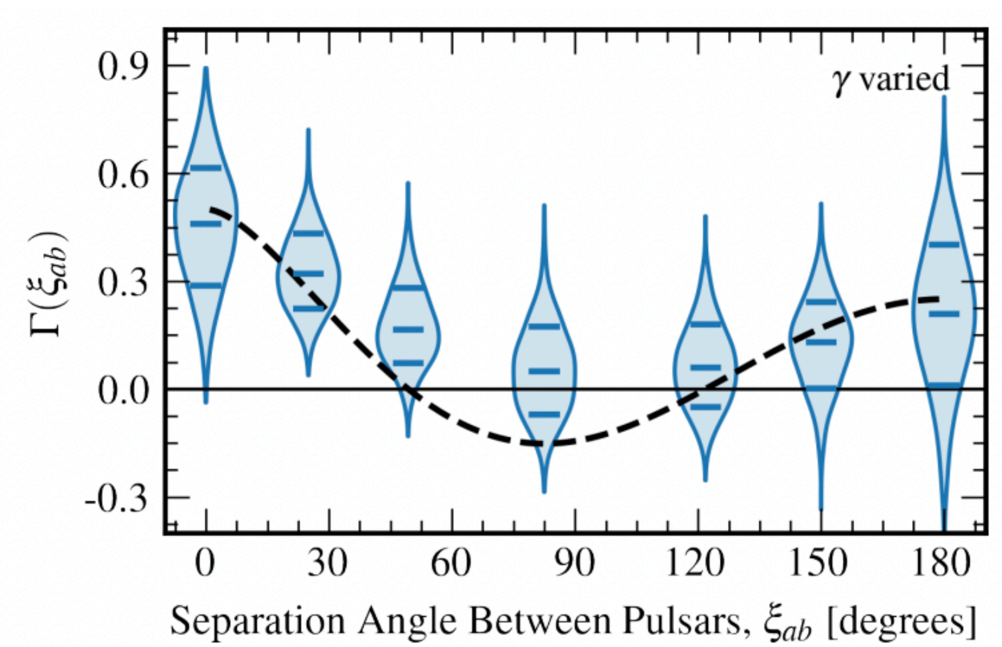
EPTA,2306.16214



PPTA,2306.16215

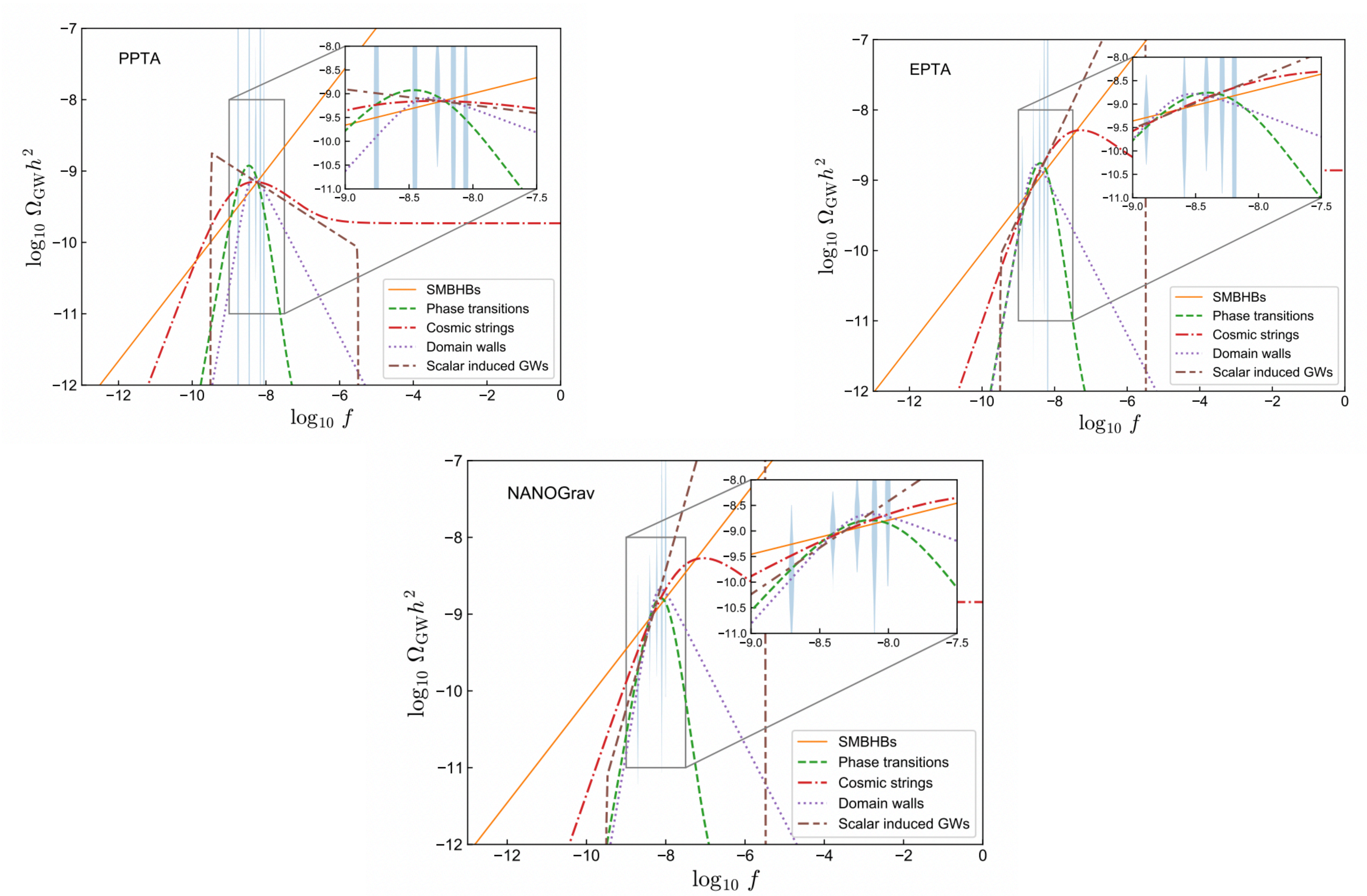


CPTA ,2306.16216



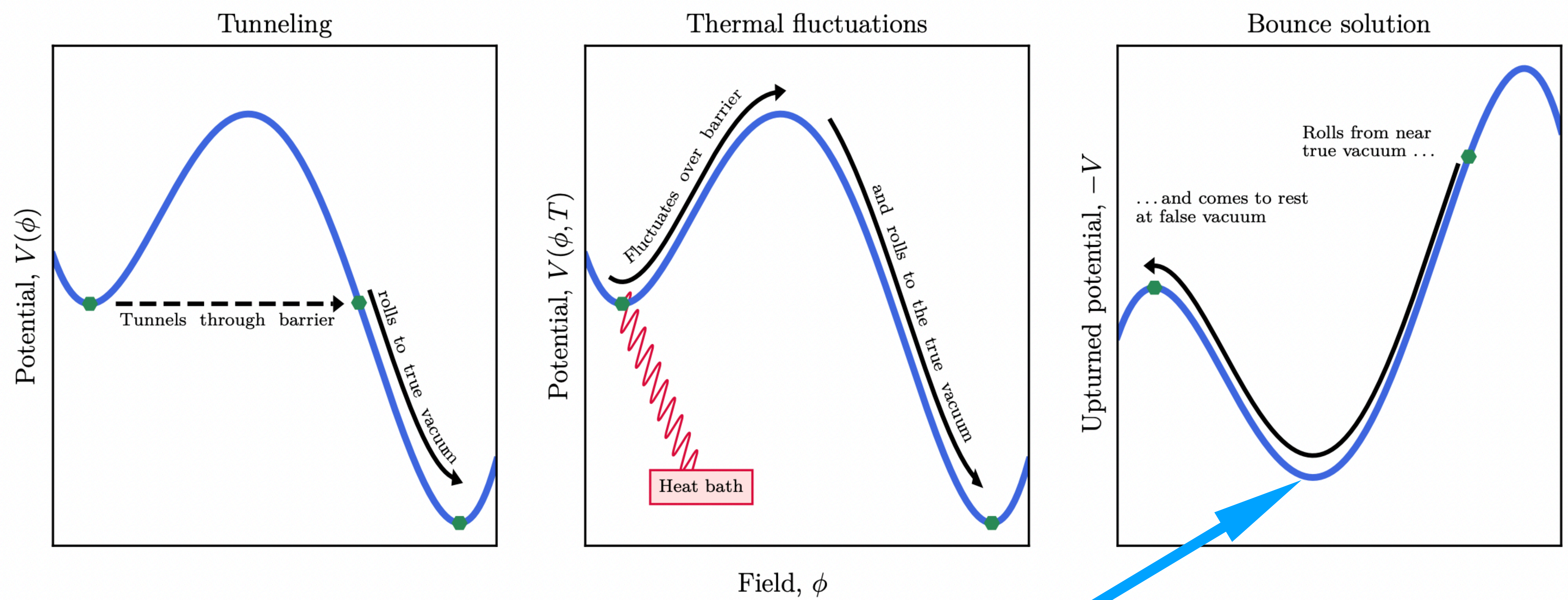
NANOGrav,2306.16213

Gravitational wave sources for Pulsar Timing Arrays



Bian,Ge,Shu,Wang,Yang,Zong,2307.02376

有限温度场论里面的真空衰变



$$p(t;T) \equiv \Gamma/V = |A(T)|e^{-B(T)/T}$$

$$A(T) \simeq T^4 \left(\frac{S_3[\bar{\phi}(T)]}{2\pi T}\right)^{\frac{3}{2}}$$

Peter Athron1 , Csaba Balázs, Andrew Fowlie, Lachlan Morris , Lei Wu, 2305.02357

GW parameters and FOPT

Bounce solution

$$S_3(T) = \int 4\pi r^2 dr \left[\frac{1}{2}\left(\frac{d\phi_b}{dr}\right)^2 + V(\phi_b, T)\right]$$

$$\lim_{r\to\infty} \phi_b = 0 \ , \qquad \frac{d\phi_b}{dr}\Big|_{r=0} = 0$$

Bubble nucleation

$$\Gamma \approx A(T)e^{-S_3/T} \sim 1$$

PT strength

$$\alpha \equiv \frac{1}{\rho_r}\left(\Delta V_{\rm eff}(\phi, T) - \frac{T}{4}\Delta\frac{\partial V_{\rm eff}(\phi, T)}{\partial T}\right)$$

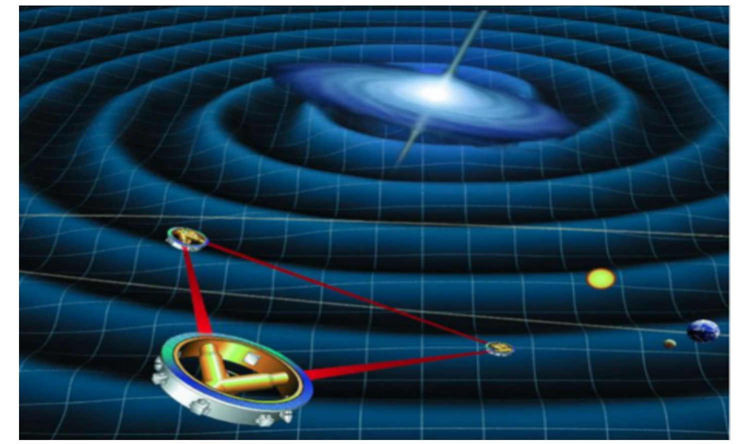
Phase transition inverse duration

$$\frac{\beta}{H_n} = T\frac{d(S_3(T)/T)}{dT}\Big|_{T=T_n}$$

新物理&相变引力波

重要的引力波源，主要科学目标之一

PTA,LIGO,LISA,天琴,太极,…



超出粒子物理标准模型
新物理

相变参数

Effective action → β, H_*

Energy budget → α, $\kappa(\alpha, v_w)$

Bubble wall dynamics → v_w

引力波功率谱

Numerical simulations →
$h^2\Omega_{GW}(f; H_*, \alpha, \beta, v_w)$

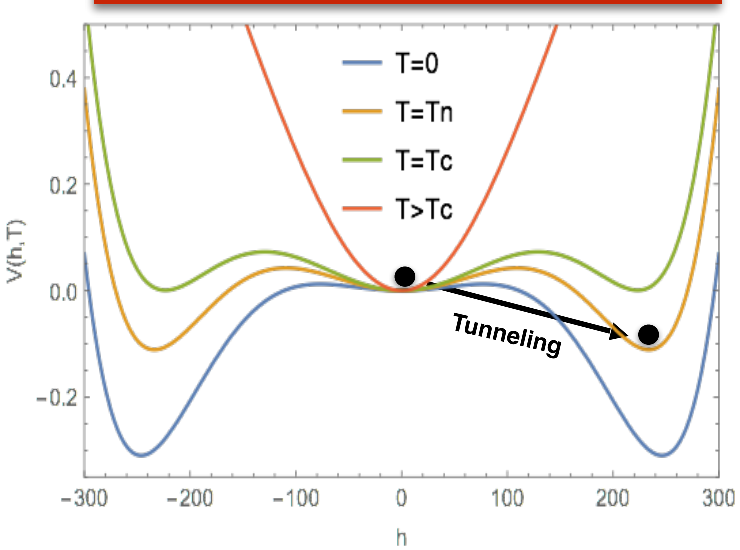
实验探测

Configuration + noise level →
$h^2\Omega_{sens}(f)$

有限温场论计算

格点场论模拟建立理论和实验的桥梁

信噪比



Chiara Caprini *et al* JCAP03(2020)024

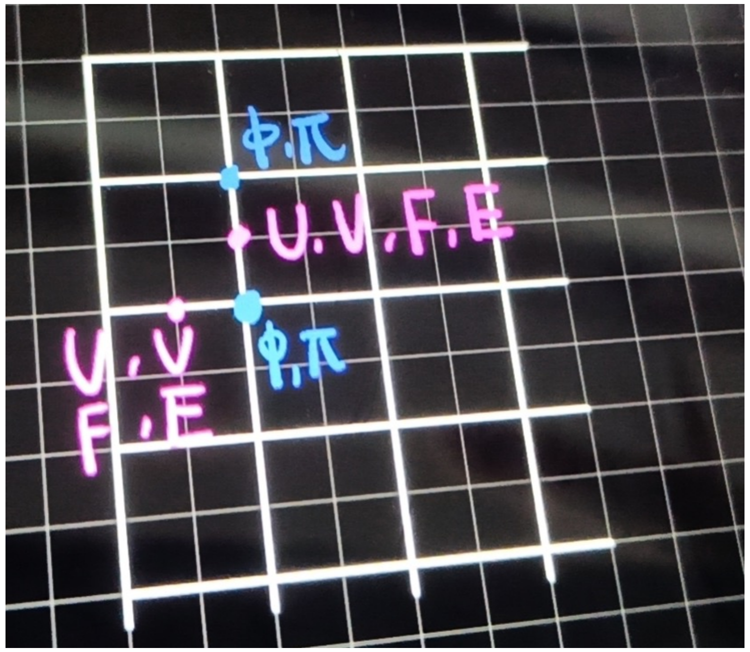
格点电弱理论

$\Phi(t, x)$： Higgs field doublet defined on sites;

$U_i(t, x)$ and $V_i(t, x)$： SU(2) and U(1) link fields, defined on the link between the neighboring sites x and x + i， $\Phi(t, x)$, $U_i(t, x)$ and $V_i(t, x)$ are defined at time steps $t + \Delta t$, $t + 2\Delta t$, . . .;

Conjugate momentum fields： $\Pi(t+\Delta t/2, x)$, $F(t+\Delta t/2, x)$ and $E(t+\Delta t/2, x)$, are defined at time steps $t + \Delta t/2$, $t + 3\Delta t/2$.

$$U_i(t,x) = \exp\Big(-\frac{i}{2} g \Delta x \sigma^a W_i^a\Big)$$
$$U_0(t,x) = \exp\Big(-\frac{i}{2} g \Delta t \sigma^a W_0^a\Big)$$
$$V_i(t,x) = \exp\Big(-\frac{i}{2} g \Delta x B_i\Big)$$
$$V_0(t,x) = \exp\Big(-\frac{i}{2} g \Delta t B_0\Big).$$



$$D_i\Phi = \frac{1}{\Delta x}\left[U_i(t,x)V_i(t,x)\Phi(t,x+i) - \Phi(t,x)\right]$$
$$D_0\Phi = \frac{1}{\Delta t}\left[U_0(t,x)V_0(t,x)\Phi(t+\Delta t,x) - \Phi(t,x)\right].$$

$$\Phi(t+\Delta t, x) = \Phi(t,x) + \Delta t \Pi(t+\Delta t/2, x)$$
$$V_i(t+\Delta t, x) = \frac{1}{2} g' \Delta x \Delta t E_i(t+\Delta t/2, x) V_i(t,x)$$
$$U_i(t+\Delta t, x) = g \Delta x \Delta t F_i(t+\Delta t/2, x) U_i(t,x),$$

Temporal gauge
$U_0(t, x) = I_2$, $V_0(t, x) = 1$

leapfrog

Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

一阶电弱相变模拟-量子隧穿

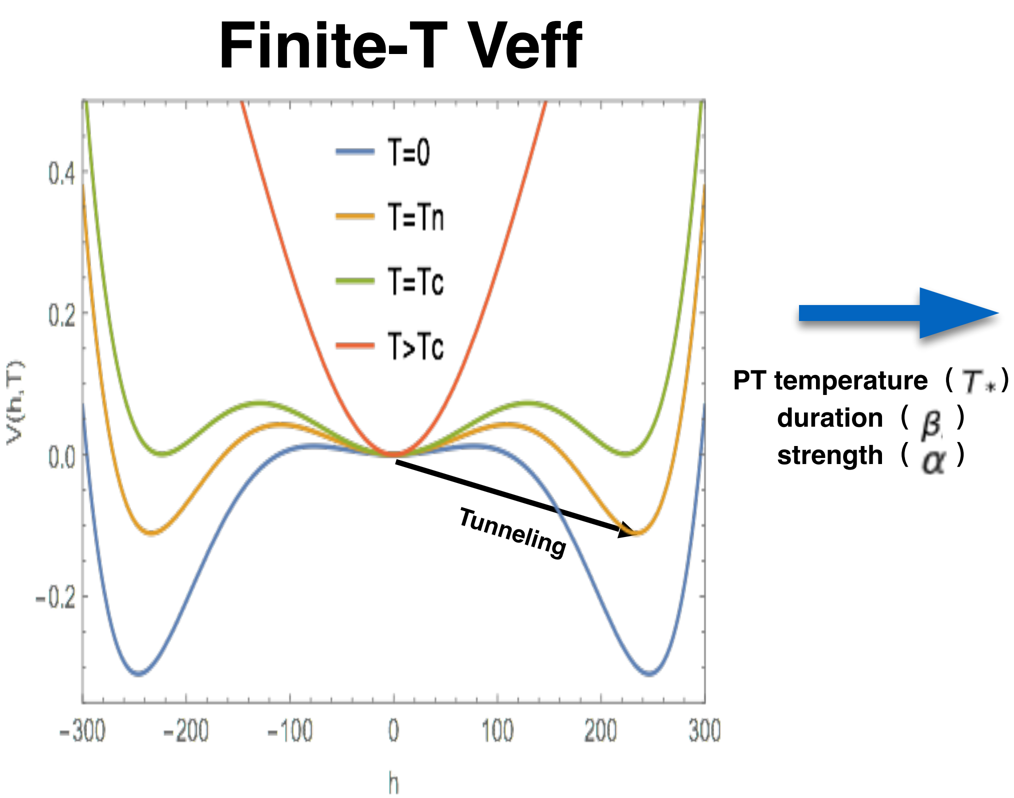
Field basis equation of motion

$$
\begin{aligned}
\partial_0^2\Phi &= D_i D_i \Phi - \frac{dV(\Phi)}{d\Phi}, \\
\partial_0^2 B_i &= -\partial_j B_{ij} + g' \,\mathrm{Im}[\Phi^\dagger D_i \Phi], \\
\partial_0^2 W_i^a &= -\partial_k W_{ik}^a - g\,\epsilon^{abc} W_k^b W_{ik}^c + g\,\mathrm{Im}[\Phi^\dagger \sigma^a D_i \Phi]\,,
\end{aligned}
$$

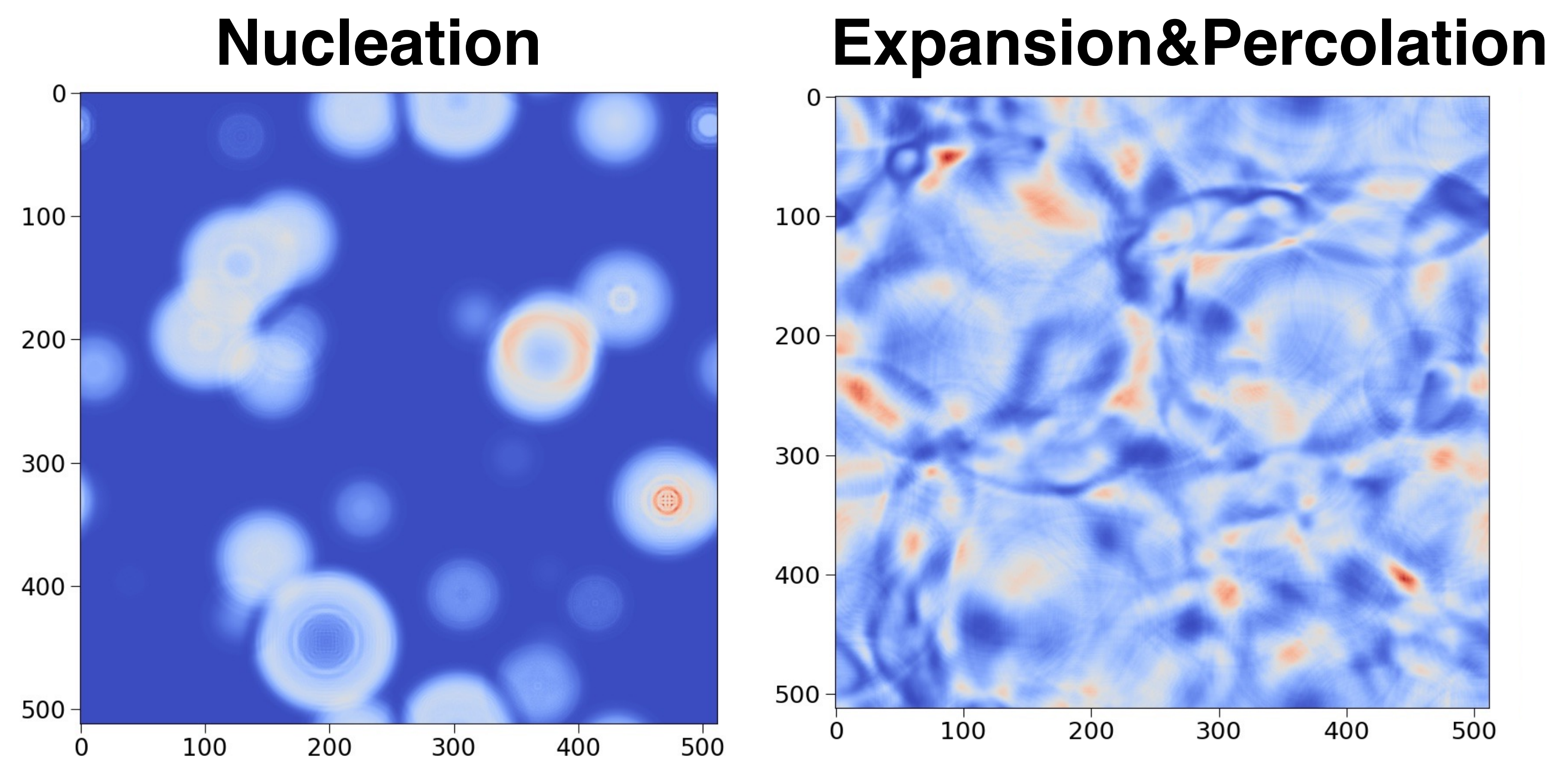
$$
\begin{aligned}
&\partial_0\partial_j B_j - g' \,\mathrm{Im}[\Phi^\dagger \partial_0 \Phi] = 0, \\
&\partial_0\partial_j W_j^a + g\,\epsilon^{abc} W_j^b \partial_0 W_j^c - g\,\mathrm{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] = 0.
\end{aligned}
$$

Lattice implementation

$$
\begin{aligned}
\Pi(t+\Delta t/2, x) =& \Pi(t-\Delta t/2, x) + \Delta t\Big\{ \frac{1}{\Delta x^2} \sum_i \big[U_i(t,x)V_i(t,x)\Phi(t,x+i) \\
& - 2\Phi(t,x) + U_i^\dagger(t,x-i)V_i^\dagger(t,x-i)\Phi(t,x-i)\big] - \frac{\partial U}{\partial \Phi^\dagger}\Big\} \\
\mathrm{Im}[E_k(t+\Delta t/2, x)] =& \mathrm{Im}[E_k(t-\Delta t/2, x)] + \Delta t\Big\{ \frac{g'}{\Delta x}\mathrm{Im}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)\Phi(t,x)] \\
& - \frac{2}{g'\Delta x^3} \sum_i \mathrm{Im}[V_k(t,x)V_i(t,x+k)V_k^\dagger(t,x+i)V_i^\dagger(t,x) \\
& + V_i(t,x-i)V_k(t,x)V_i^\dagger(t,x+k-i)V_k^\dagger(t,x-i)]\Big\} \\
\mathrm{Tr}[i\sigma^m F_k(t+\Delta t/2, x)] =& \mathrm{Tr}[i\sigma^m F_k(t-\Delta t/2, x)] + \Delta t\Big\{ \frac{g}{\Delta x}\mathrm{Re}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x) i\sigma^m \Phi(t,x)] \\
& - \frac{1}{g\Delta x^3} \sum_i \mathrm{Tr}[i\sigma^m U_k(t,x)U_i(t,x+k)U_k^\dagger(t,x+i)U_i^\dagger(t,x) \\
& + i\sigma^m U_k(t,x)U_i^\dagger(t,x+k-i)U_k^\dagger(t,x-i)U_i(t,x-i)]\Big\},
\end{aligned}
$$



Finite-T calculation



Lattice Simulation

Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

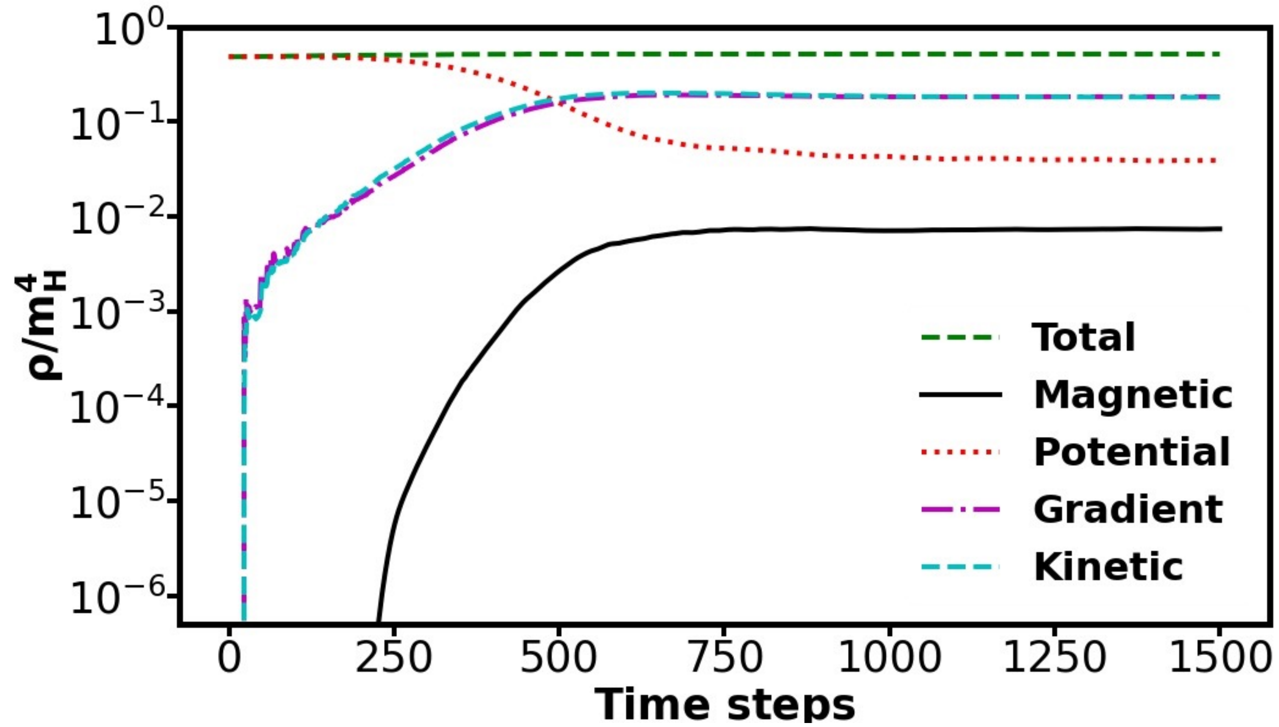
一阶电弱相变真空泡碰撞、合并产生引力波

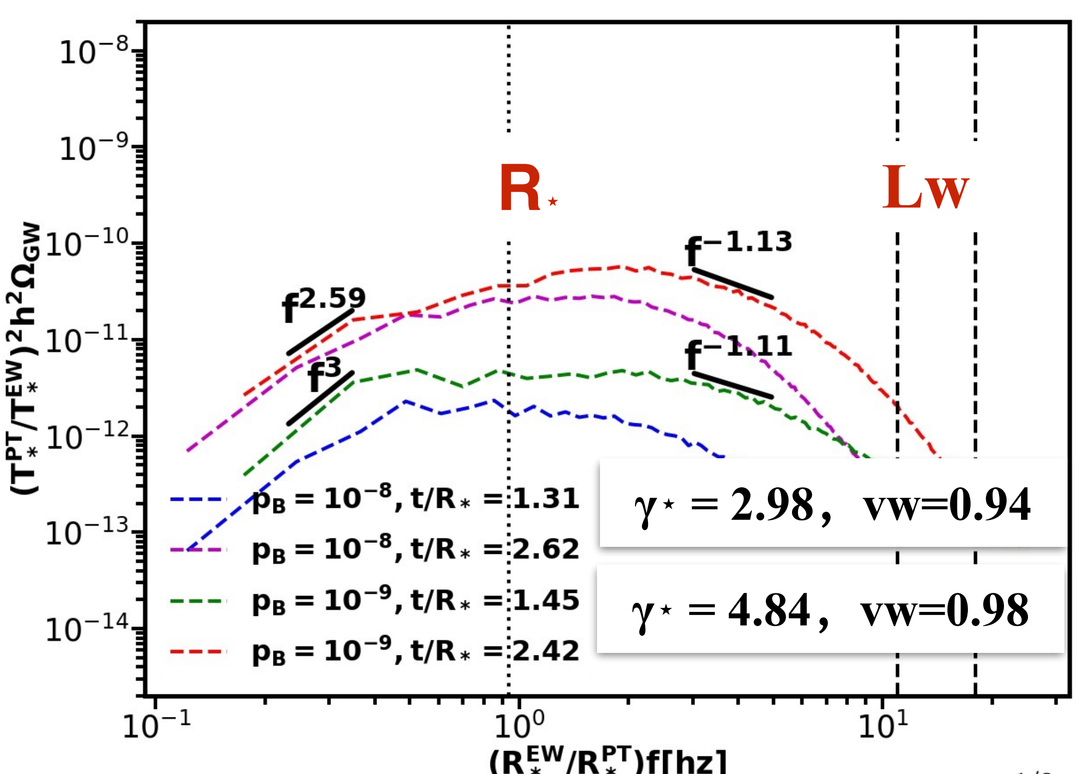
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

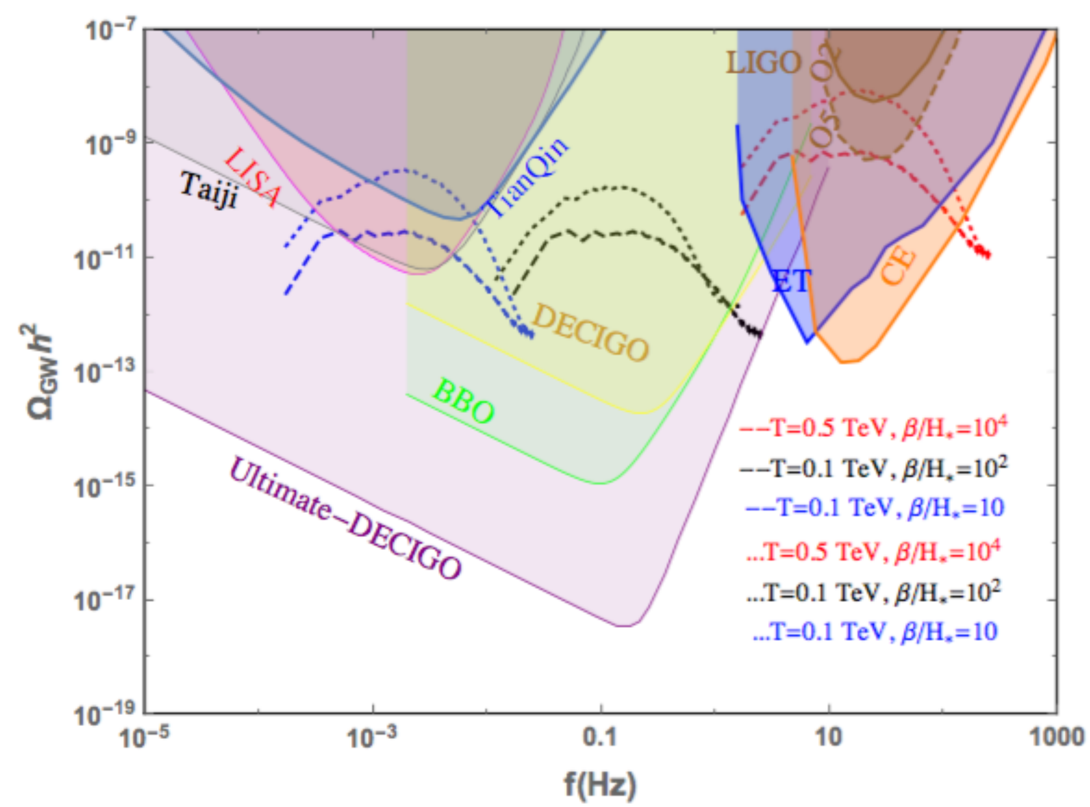
$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu}\frac{1}{2}\mathrm{Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

$$\langle \dot{h}_{ij}^{TT}(\mathbf{k},t)\dot{h}_{ij}^{TT}(\mathbf{k}',t)\rangle = P_{\dot{h}}(\mathbf{k},t)(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')$$

$$\frac{d\Omega_{\mathrm{gw}}}{d\ln(k)} = \frac{1}{32\pi G\rho_c}\frac{k^3}{2\pi^2}P_{\dot{h}}(\mathbf{k},t)$$







$\gamma_\star = R_\star/(2Rc)$

$R_* = \left(\frac{\mathcal{V}}{N_b}\right)^{1/3}$

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Bubble dynamics of FOPT

From QFT $$p(t;T) \equiv \Gamma/V = |A(T)|e^{-B(T)/T}$$

Tunneling

$$A(T) = T\left(\frac{S_3[\bar\phi(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det'[-\nabla^2+V''(\bar\phi)]}{\det[-\nabla^2+V''(\phi_f)]}\right)^{-\frac{1}{2}}$$

Fluctuation

$$\begin{aligned}A(T) &= \frac{\sqrt{-\lambda_-}}{2\pi}\left(\frac{S_3[\bar\phi(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det'[-\nabla^2+V''(\bar\phi)]}{\det[-\nabla^2+V''(\phi_f)]}\right)^{-\frac{1}{2}}\\ &= \frac{1}{2\pi}\left(\frac{S_3[\bar\phi(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det^+[-\nabla^2+V''(\bar\phi)]}{\det[-\nabla^2+V''(\phi_f)]}\right)^{-\frac{1}{2}},\end{aligned}$$

2305.02357

False vacuum fraction considering bubbles expansion

Bubble Volume

$$h(t) = \exp\left[-\int_{t_c}^{t} dt' \frac{4\pi}{3}v_w^3(t-t')^3\frac{\Gamma(t')}{\mathcal{V}}\right]$$

$$\frac{\Gamma}{\mathcal{V}} = \frac{\Gamma_f}{\mathcal{V}}e^{\beta(t-t_f)} \qquad \beta \equiv \frac{d}{dt}\log\left(\frac{\Gamma(t)}{\mathcal{V}}\right)\Big|_{t=t_f}$$

$$\begin{aligned}-\log h(t) &\simeq \int^{t} dt'\frac{4\pi}{3}v_w^3(t-t')^3\frac{\Gamma_f}{\mathcal{V}}e^{\beta(t'-t_f)}\\ &= \frac{4\pi}{3}v_w^3\frac{\Gamma_f}{\mathcal{V}}\frac{3!}{\beta^4}e^{\beta(t-t_f)}.\end{aligned}$$

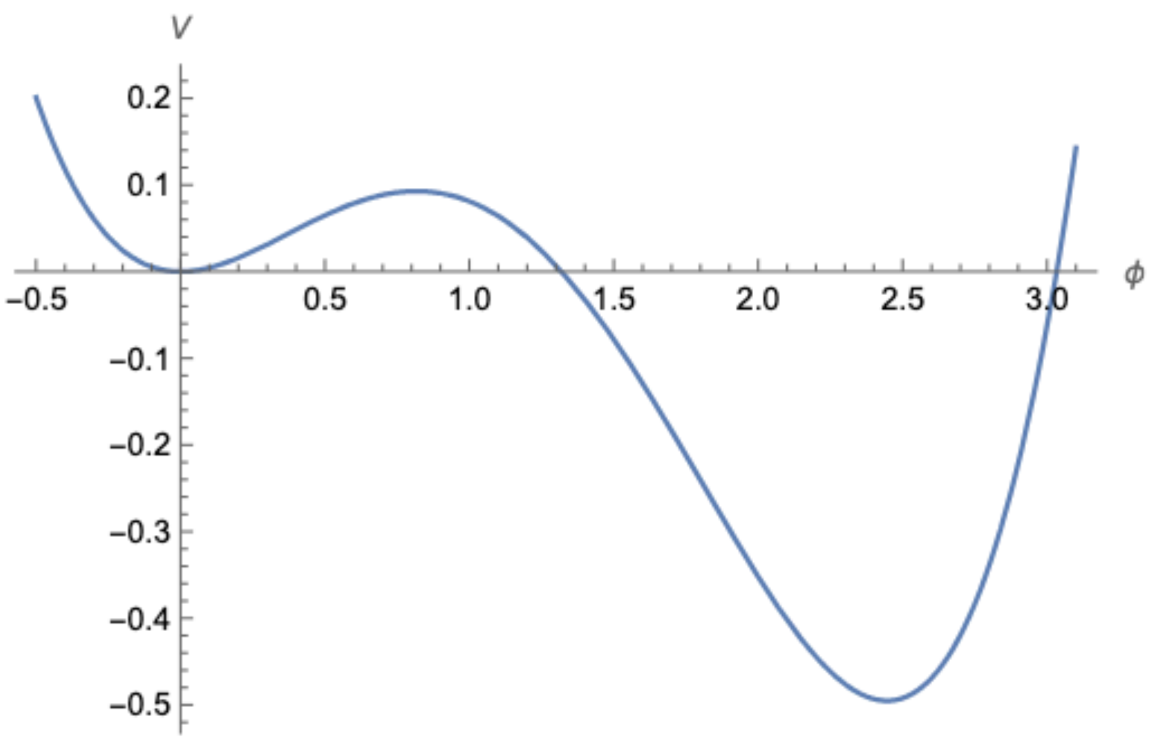
h(tf)=1/e

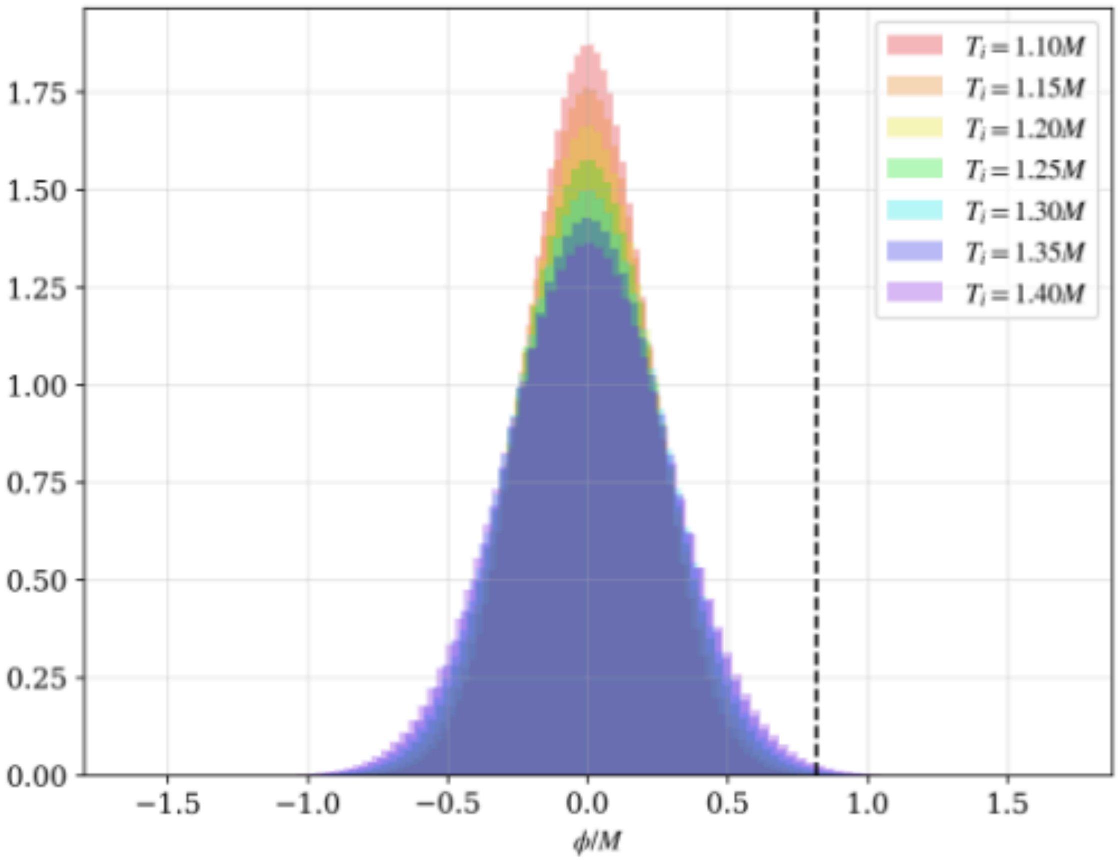
$$h(t) = \exp\left[-e^{\beta(t-t_f)}\right]$$

$$8\pi\frac{v_w^3}{\beta^4}\frac{\Gamma_f}{\mathcal{V}} = 1$$

2008.09136

一阶电弱相变模拟3D-热涨落





$$V(\phi) = \frac{1}{2}M^2\phi^2 + \frac{1}{3}\delta\phi^3 + \frac{1}{4}\lambda\phi^4$$

$$\tilde{\phi} = \phi/f_*, \quad \mathrm{d}\tilde{t} = w_*\mathrm{d}t, \quad \mathrm{d}\tilde{x}^i = w_*\mathrm{d}x^i,$$

$$\ddot{\tilde{\phi}} - \tilde{\nabla}^2\tilde{\phi} + \tilde{V}_{,\tilde{\phi}} = 0$$

$$\mathcal{P}_\phi(k) = \frac{n_k}{w_k} = \frac{1}{w_k}\frac{1}{e^{w_k/T}-1}, \quad \mathcal{P}_{\dot{\phi}}(k) = n_k w_k = \frac{w_k}{e^{w_k/T}-1},$$

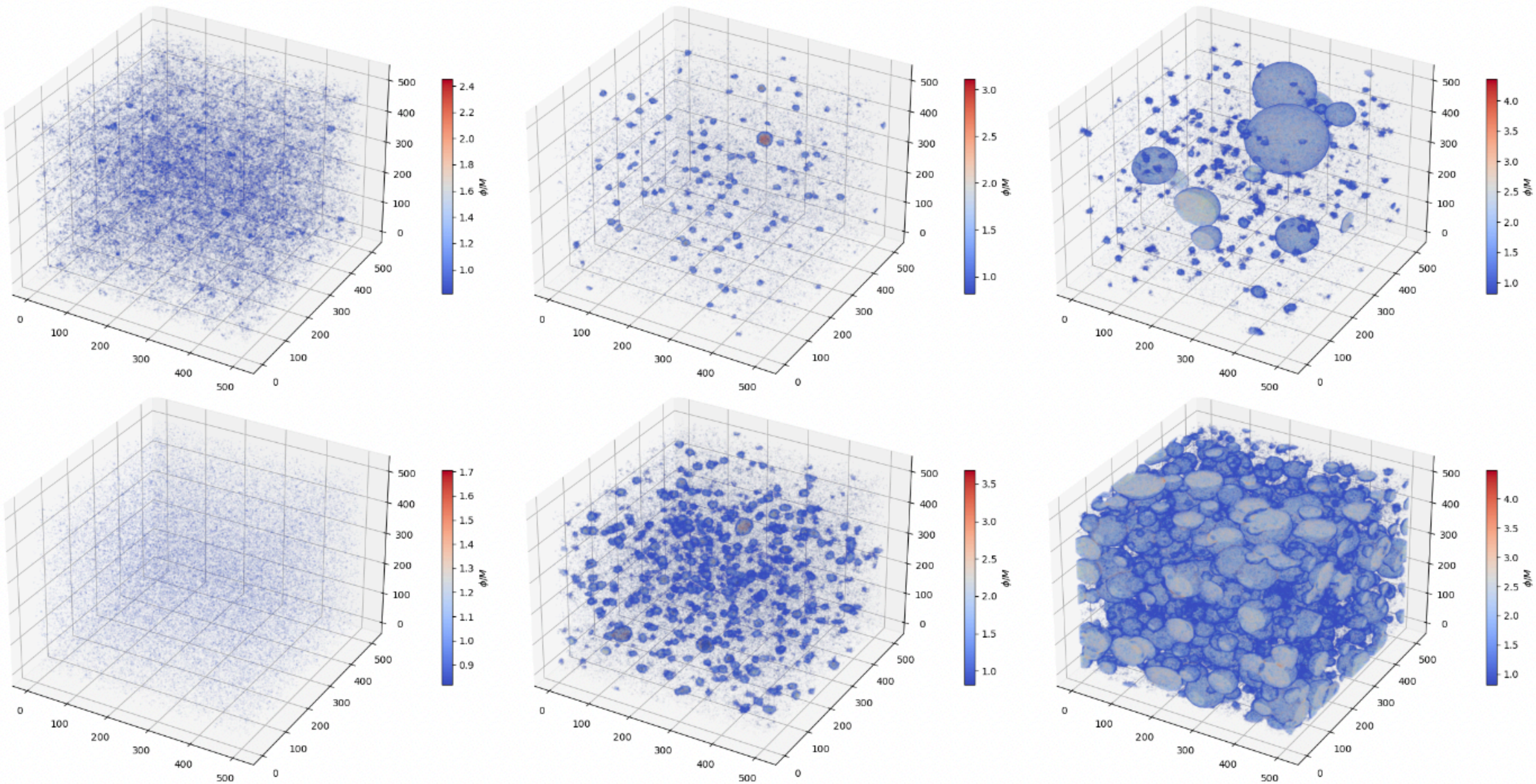
$$\langle\phi(\mathbf{k})\phi(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_\phi(k)\delta(\mathbf{k}-\mathbf{k}'),$$
$$\langle\dot{\phi}(\mathbf{k})\dot{\phi}(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_{\dot{\phi}}(k)\delta(\mathbf{k}-\mathbf{k}'),$$
$$\langle\phi(\mathbf{k})\dot{\phi}(\mathbf{k}')\rangle = 0\,.$$

$$\langle|\phi(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_\phi(k), \qquad \langle\phi(\mathbf{k})\rangle = 0,$$
$$\langle|\dot{\phi}(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\dot{\phi}}(k), \qquad \langle\dot{\phi}(\mathbf{k})\rangle = 0,$$

Ligong Bian,Yuefeng Di, Yongtao Jia,Yang Li,Kehao Zeng， arXiv:2505.15360

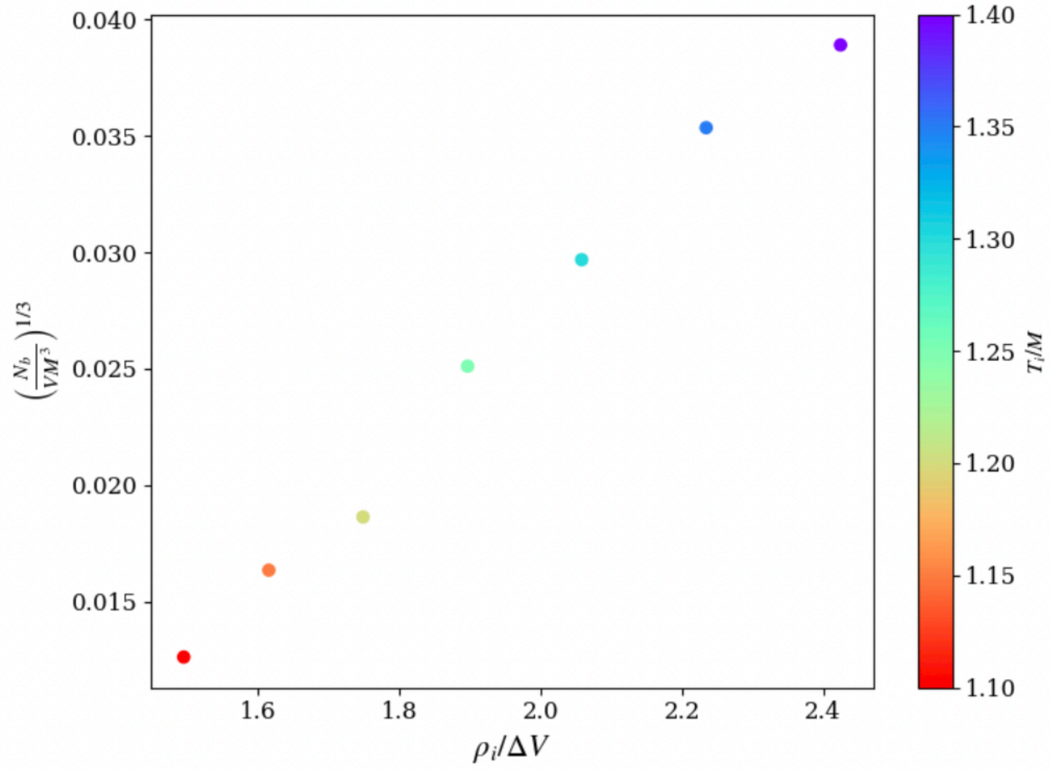
一阶电弱相变模拟3D-热涨落

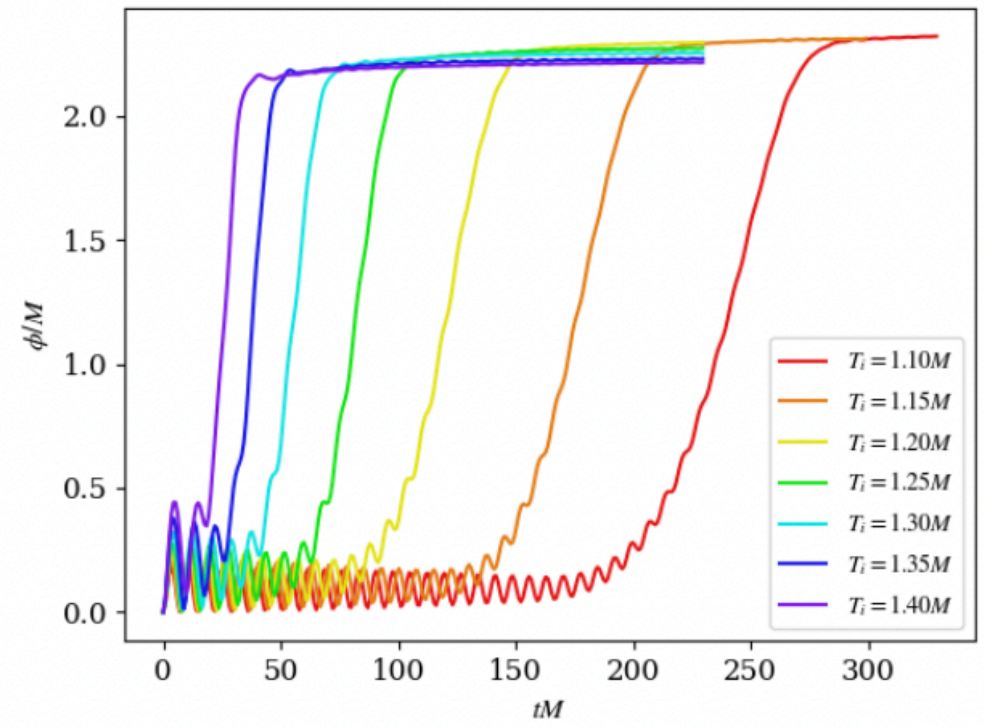
Ti = 1.15M

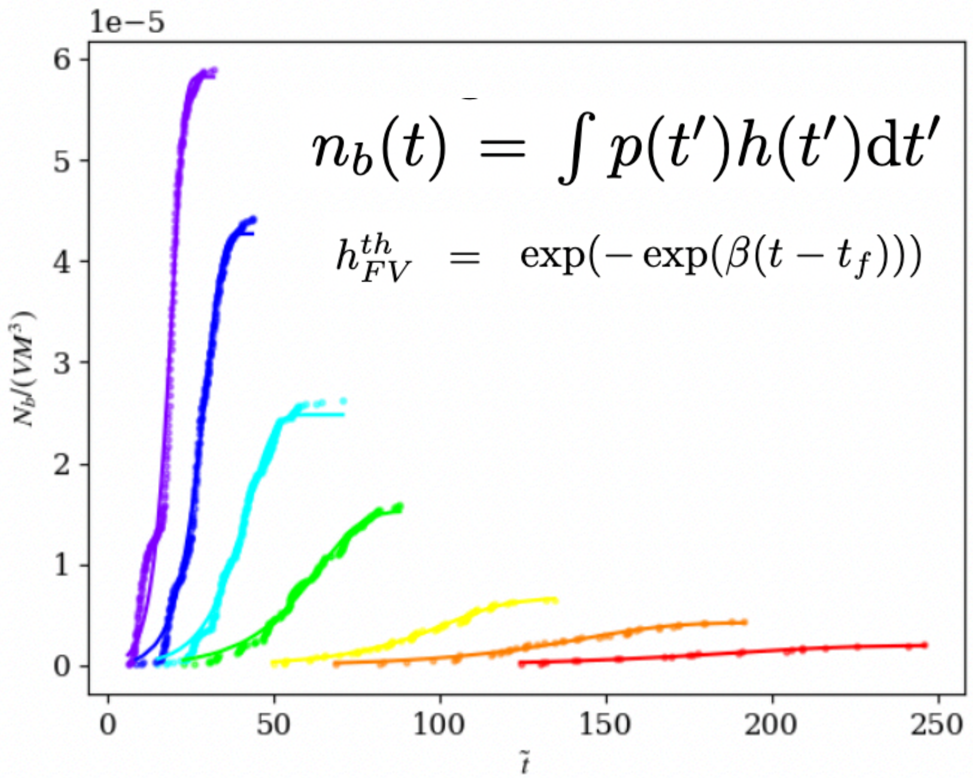


Ti = 1.35M

一阶电弱相变模拟3D-热涨落





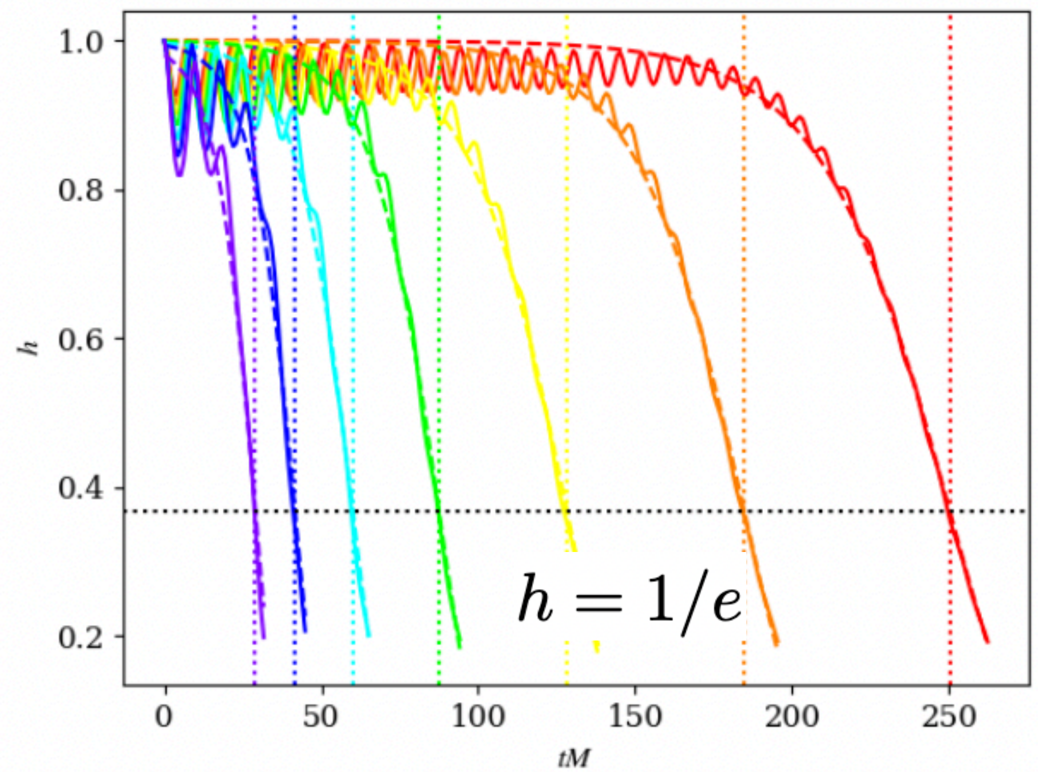


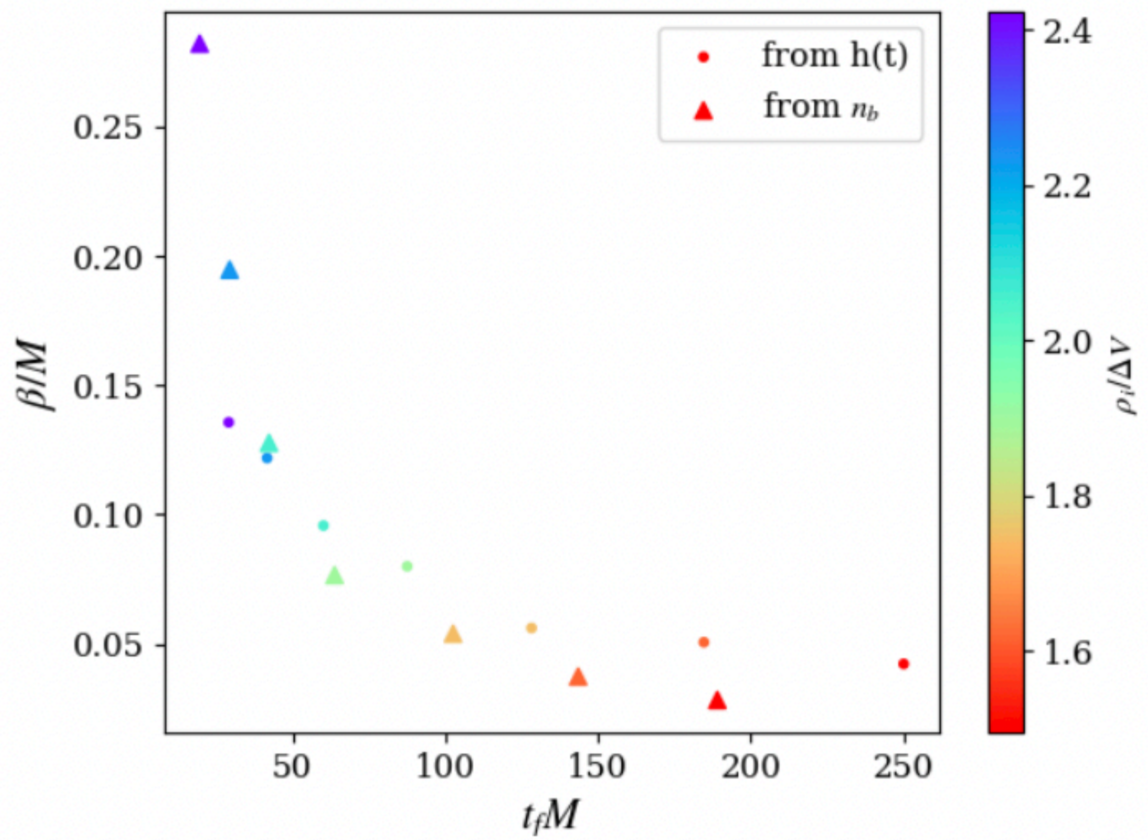
p(t) = pf exp[β(t – tf)]

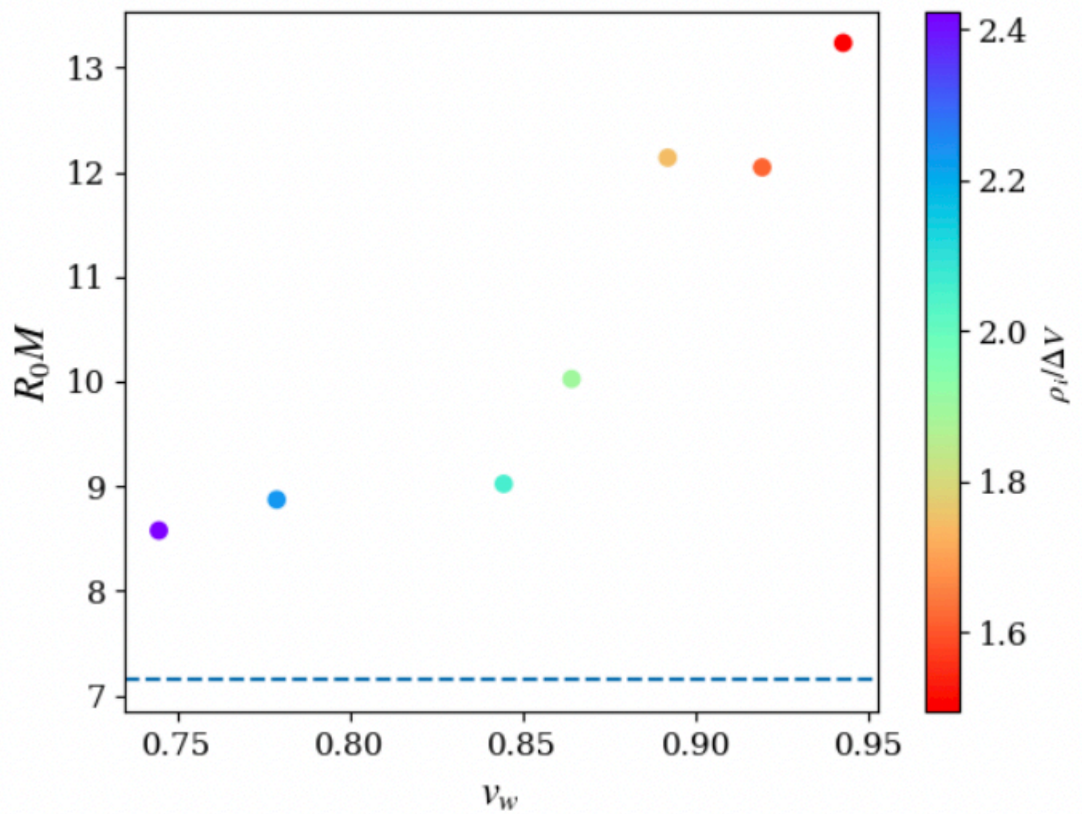
T_i/M	p_f/M^4	β/M	$t_f M$
1.10	5.56×10^{-8}	0.028	189.10
1.15	1.58×10^{-7}	0.037	143.53
1.20	3.57×10^{-7}	0.054	102.54
1.25	1.17×10^{-6}	0.077	63.72
1.30	3.18×10^{-6}	0.128	42.25
1.35	8.34×10^{-6}	0.195	29.35
1.40	1.65×10^{-5}	0.282	19.53

一阶电弱相变模拟3D-假真空比例

$$h_{FV}^{sim} = 1 - \frac{\langle\phi\rangle}{\phi_v} \qquad h_{FV}^{th} = \exp(-\exp(\beta(t-t_f)))$$







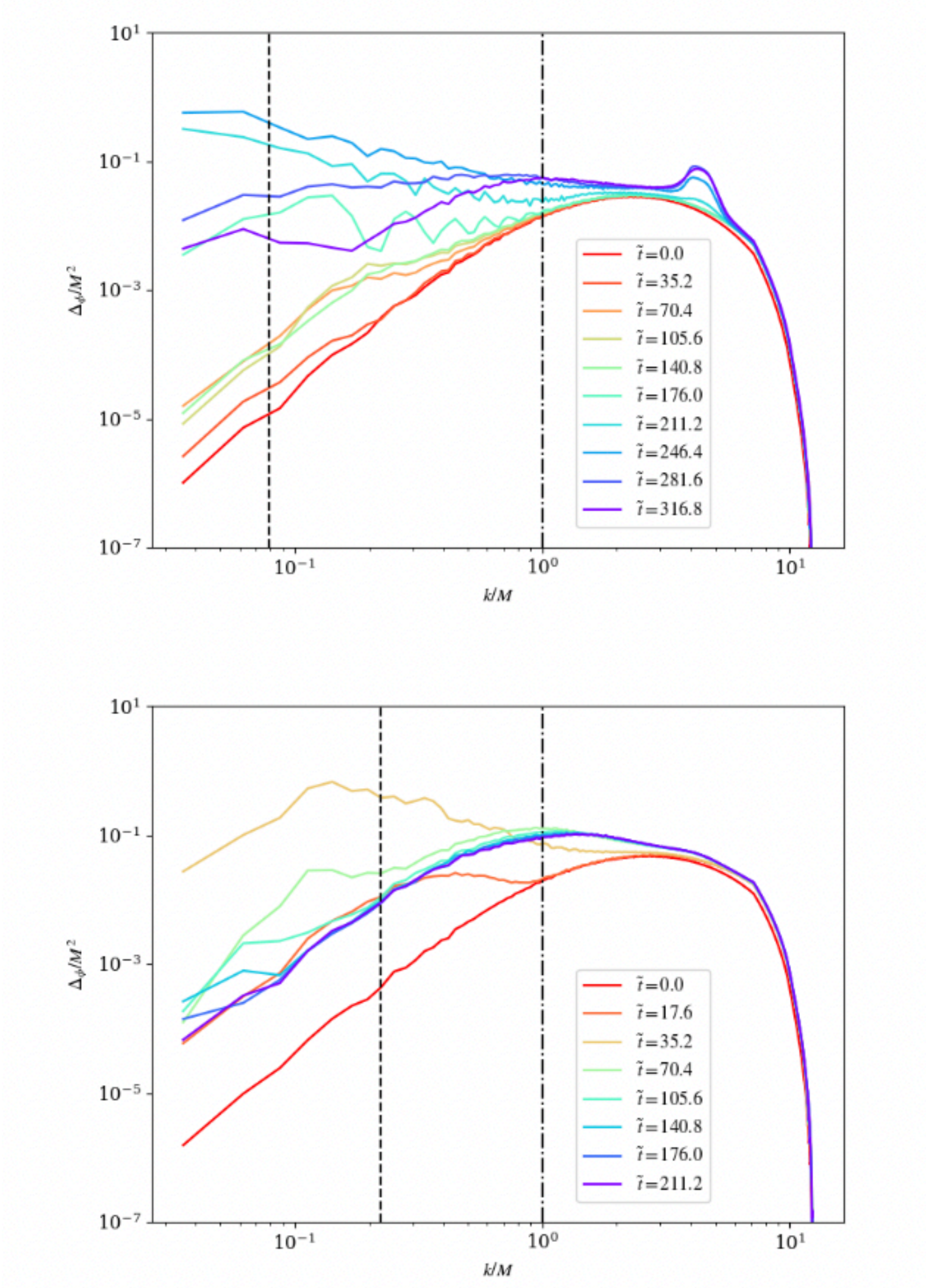
$$v_w \approx \sqrt{\Delta\rho_k/\Delta\rho_g}$$

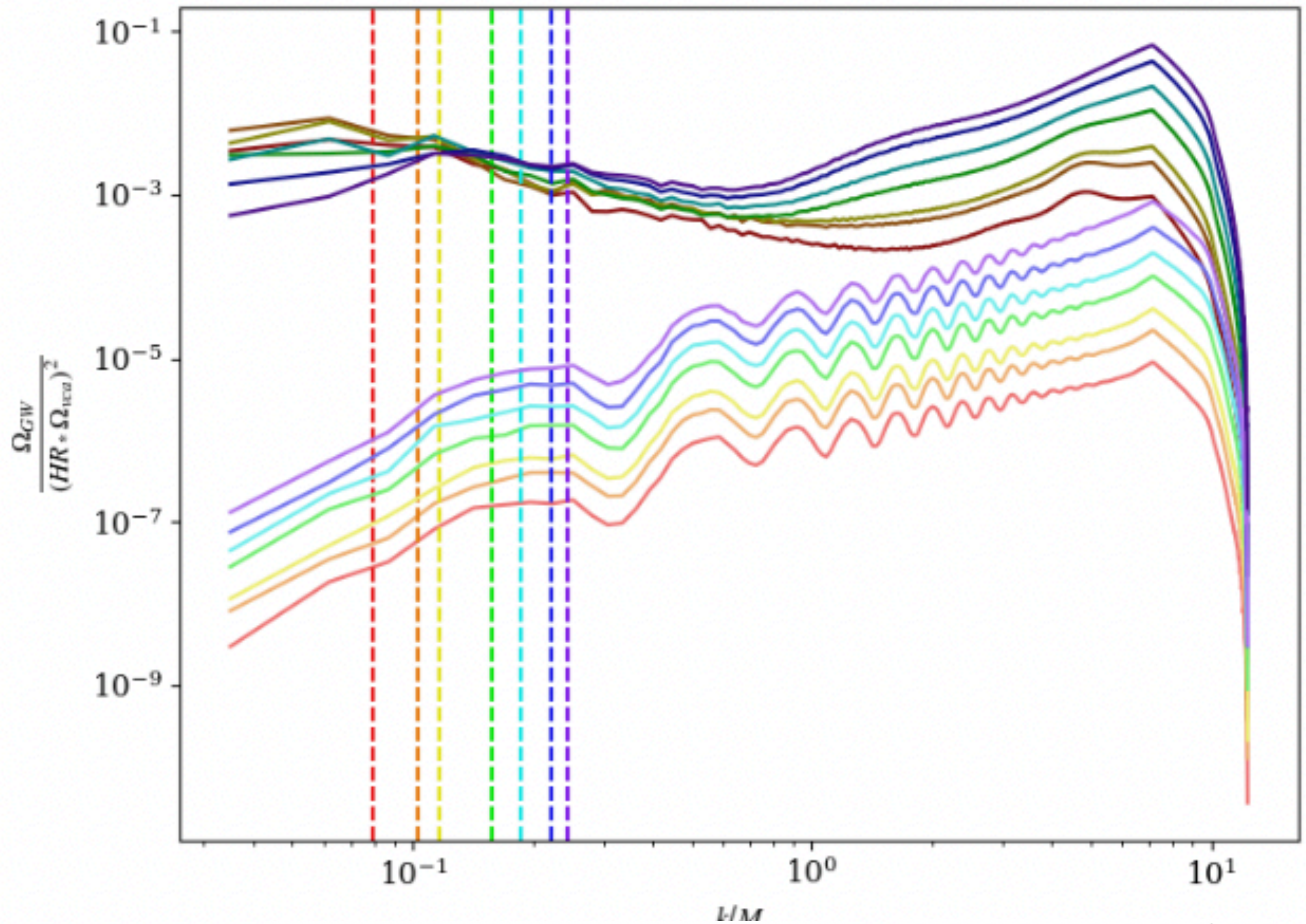
$$\frac{1}{2}\dot{\phi}^2 \qquad \frac{1}{2}(\nabla\phi)^2$$

$$R_0 = R_*\sqrt{1-v_w^2}/2$$

$$R_* = (V/N_b)^{1/3}$$

一阶电弱相变模拟3D-引力波





$\Omega_{GW} \approx \mathcal{O}(10^{-3})(HR_*\Omega_{vac})$ $\Omega_{vac} = \Delta V/\rho_r$

一阶电弱相变模拟-热涨落

场论中的Wigner函数

$$P_R(T) \equiv \int_R dx |\psi(x,T)|^2 \quad \Longrightarrow \quad P_R(t) \equiv \int_R \mathcal{D}\phi |\Psi(\phi,t)|^2$$

（量子力学中Wigner函数：(q, p)相空间中准概率密度性质）

$$\Psi(\phi,t) = \langle \phi(x)|\Psi(t)\rangle = \int \mathcal{D}\phi_i \langle \phi | \hat{U}(t|t_0)|\phi_i\rangle\langle\phi_i(\mathbf{x})|\Psi(t_0)\rangle$$

$$W(q,p;t) \sim |\Psi(x,t)|^2$$

$$W(q,p;t) = \int du e^{-\frac{i}{\hbar}pu} \left\langle q + \frac{u}{2}\right|\hat{\rho}(t)\left|q - \frac{u}{2}\right\rangle \quad \Longrightarrow \quad W[\phi(x),\Pi(x);t] = \int \mathcal{D}\varphi(x) \exp\left[-\frac{i}{\hbar}\int dx \Pi(x)\varphi(x)\right] \times \left\langle \phi(x) + \frac{\varphi(x)}{2}\right|\hat{\rho}(t)\left|\phi(x) - \frac{\varphi(x)}{2}\right\rangle$$

量子场论中Wigner函数：(ϕ, Π)相空间中准概率密度性质

$$\hat{\rho}(0) = \frac{1}{Z}e^{-\beta\hat{H}} \qquad W[\phi(x),\Pi(x);t] \gtrless 0$$

$$W[\phi(x),\Pi(x);0] = \frac{1}{Z}\int \mathcal{D}\varphi(x) \exp\left[-\frac{i}{\hbar}\int dx \Pi(x)\varphi(x)\right] \times \left\langle \phi(x) + \frac{\varphi(x)}{2}\right| e^{-\beta\hat{H}} \left|\phi(x) - \frac{\varphi(x)}{2}\right\rangle$$

$$\approx \frac{1}{Z} exp\left[-\beta \int dx [\frac{1}{2}\Pi^2(x) + \frac{1}{2}(\nabla\phi(x))^2 + V(\phi)]\right]$$

$$Z \equiv Tr\hat{\rho} = \int \mathcal{D}\phi(x) \frac{\mathcal{D}\Pi(x)}{2\pi} W[\phi,\Pi;t]$$

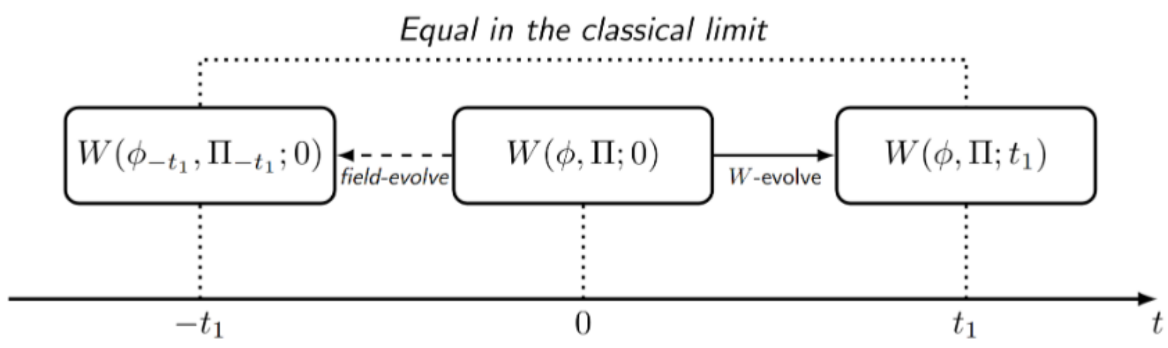
$\phi(x)$ is the real scale field

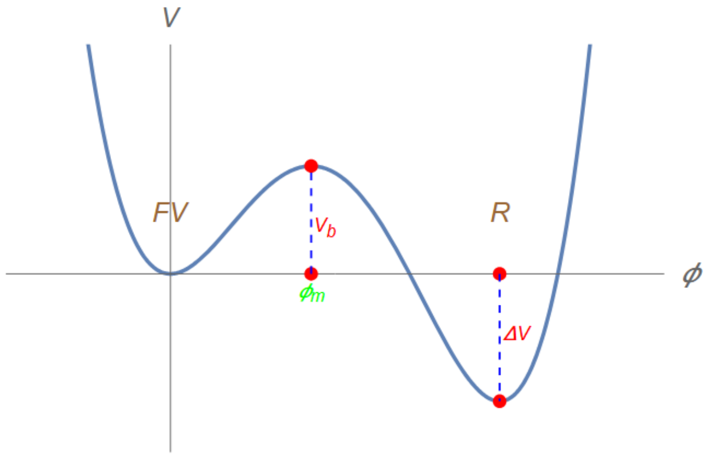
$\Pi(x) \equiv \frac{\delta\mathcal{L}}{\delta\dot{\phi}(x)}$

$\hat{\rho}(t) = |\Psi(t)\rangle\langle\Psi(t)|$

Stanisław Mrówczyński and Berndt Müller, Phys. Rev. D, 50:7542–7552, Dec 1994.

Wigner函数的运动方程





$$\hat{L}=\int d^3x(\frac{1}{2}\partial^\mu\hat{\phi}\partial_\mu\hat{\phi}-V(\hat{\phi}))\qquad \hat{H}=\int d^3x\hat{\mathcal{H}}=\int d^3x[\frac{\hat{\Pi}^2}{2}+\frac{(\nabla\hat{\phi})^2}{2}+V(\hat{\phi})]$$

$$i\hbar\frac{\partial}{\partial t}\hat{\rho}(t)=[\hat{H},\hat{\rho}(t)]\qquad \frac{\partial}{\partial t}W[\phi,\pi;t]=-2H\frac{1}{i\hbar}\sin(\frac{i\hbar}{2}\Lambda)W[\phi,\pi;t]$$

$\Lambda=\frac{\overleftarrow{\delta}}{\delta\Pi}\frac{\overrightarrow{\delta}}{\delta\phi}-\frac{\overleftarrow{\delta}}{\delta\Phi}\frac{\overrightarrow{\delta}}{\delta\Pi}$ is the Poisson bracket operator

忽略 $O(\hbar^2)$

$$\left[\frac{\partial}{\partial t}+\int d^3x(\frac{\delta H}{\delta\Pi}\frac{\delta}{\delta\phi}-\frac{\delta H}{\delta\phi}\frac{\delta}{\delta\Pi})\right]W[\phi,\Pi;t]=0$$

$$W[\phi,\Pi;t]=W[\phi_{-t},\Pi_{-t};0]$$

$$\begin{cases}\frac{\delta H}{\delta\Pi}=\frac{d\phi}{dt}=\Pi\\ -\frac{\delta H}{\delta\phi}=\frac{d\Phi}{dt}=\nabla^2\phi-\frac{\delta V(\phi)}{\delta\phi}\end{cases}$$

$$P_{\rm FV}(T)\sim e^{-\Gamma T}\ \Longrightarrow\ \Gamma=-\frac{1}{P_{\rm FV}}\frac{d}{dT}P_{\rm FV}$$

$$P_{FV}(t)\equiv\int_{FV}\mathcal{D}\phi|\Psi(\phi,t)|^2=\frac{1}{Z}\int_{FV}\mathcal{D}\phi\int\frac{\mathcal{D}\Pi}{2\pi}W[\phi,\Pi;t]$$

$$\begin{aligned}P_{FV}(t)&=\frac{1}{Z}\int_{FV}D\phi\frac{D\Pi}{2\pi}W(\phi,\Pi;t)\\&=\frac{1}{Z}\int_{FV}D\phi\frac{D\Pi}{2\pi}W(\phi_{-t},\Pi_{-t};0)\\&=\frac{1}{Z}\int_{FV}D\phi_{-t}\frac{D\Pi_{-t}}{2\pi}J(\phi_{-t},\Pi_{-t})W(\phi_{-t},\Pi_{-t};0)\end{aligned}$$

$$J(\phi_{-t},\Pi_{-t})=\det\begin{vmatrix}\frac{\partial\phi}{\partial\phi_{-t}} & \frac{\partial\phi}{\partial\Pi_{-t}}\\ \frac{\partial\Pi}{\partial\phi_{-t}} & \frac{\partial\Pi}{\partial\Pi_{-t}}\end{vmatrix}=1$$

HaiYang Wang, Kehao Zeng, Ligong Bian, arXiv: 2506.18334

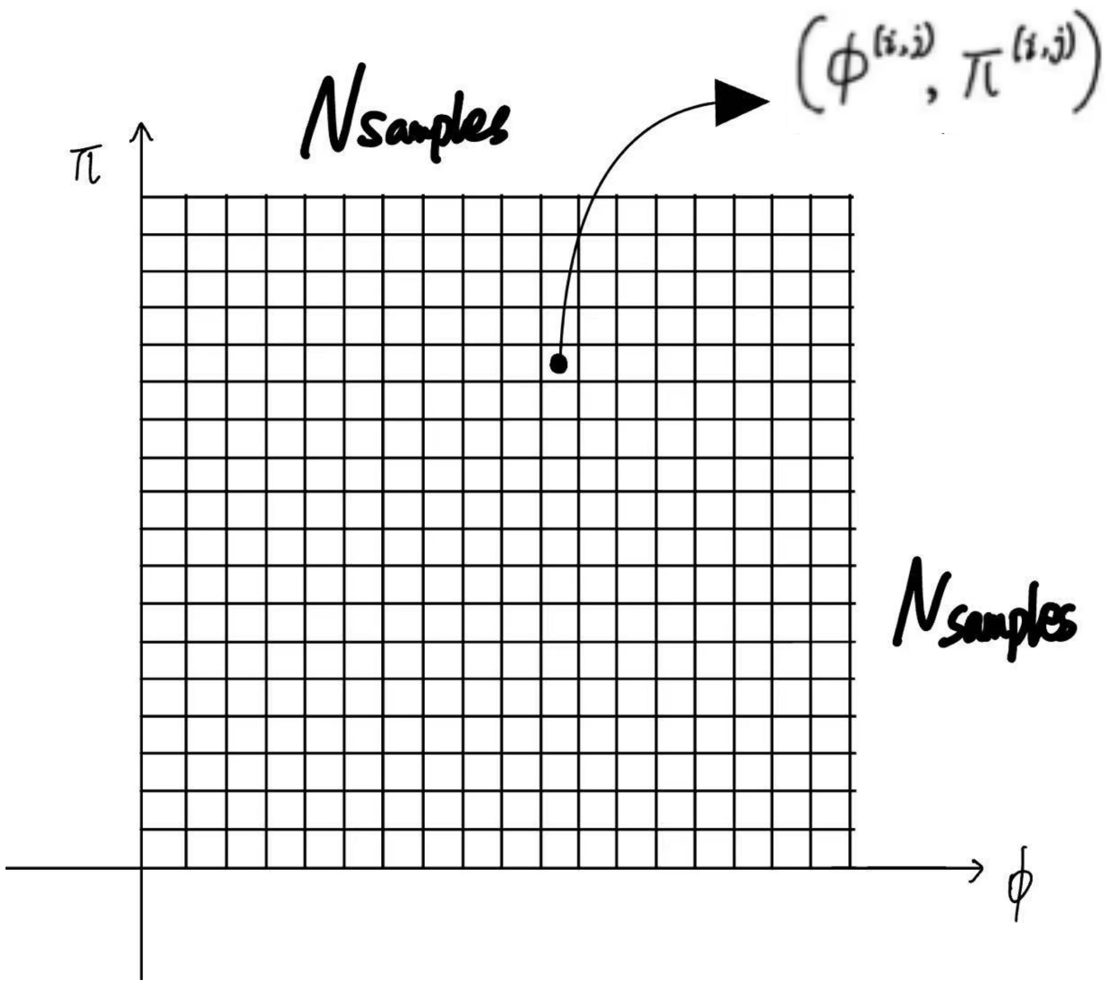
数值模拟的具体实现

$$\hat{L} = \int d^3x(\frac{1}{2}\partial^\mu\hat{\phi}\partial_\mu\hat{\phi} - V(\hat{\phi})) \quad V(\phi) = a\phi^2 + b\phi^3 + c\phi^4$$

Initial condition: $\mathcal{P}_\phi(k) = \frac{n_k}{w_k} = \frac{1}{w_k}\frac{1}{e^{w_k/T}-1}, \mathcal{P}_\Pi(k) = n_k w_k = \frac{w_k}{e^{w_k/T}-1}$

$$w_k = \sqrt{k^2/R^2 + m_{eff}^2}, m_{eff}^2 = V''(\phi_{fv})$$

生成N_samples*N_samples个独立的样本对张成离散的（phi，pi）相空间



$$P_{FV}(t) \approx \frac{\sum_{ij}\chi_{FV}(\phi_{-t}^{(i,j)}) \cdot W(\phi_{-t}^{(i,j)}, \Pi_{-t}^{(i,j)}; 0)}{\sum_{ij} W(\phi_{-t}^{(i,j)}, \Pi_{-t}^{(i,j)}; 0)}$$

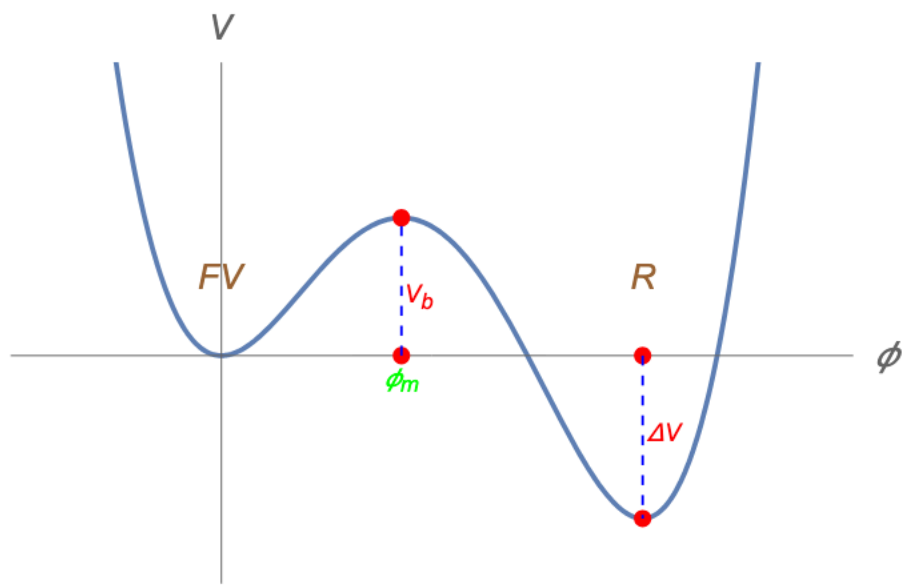
Summation range： i, j = 1,2, 3… $N_{samples}$

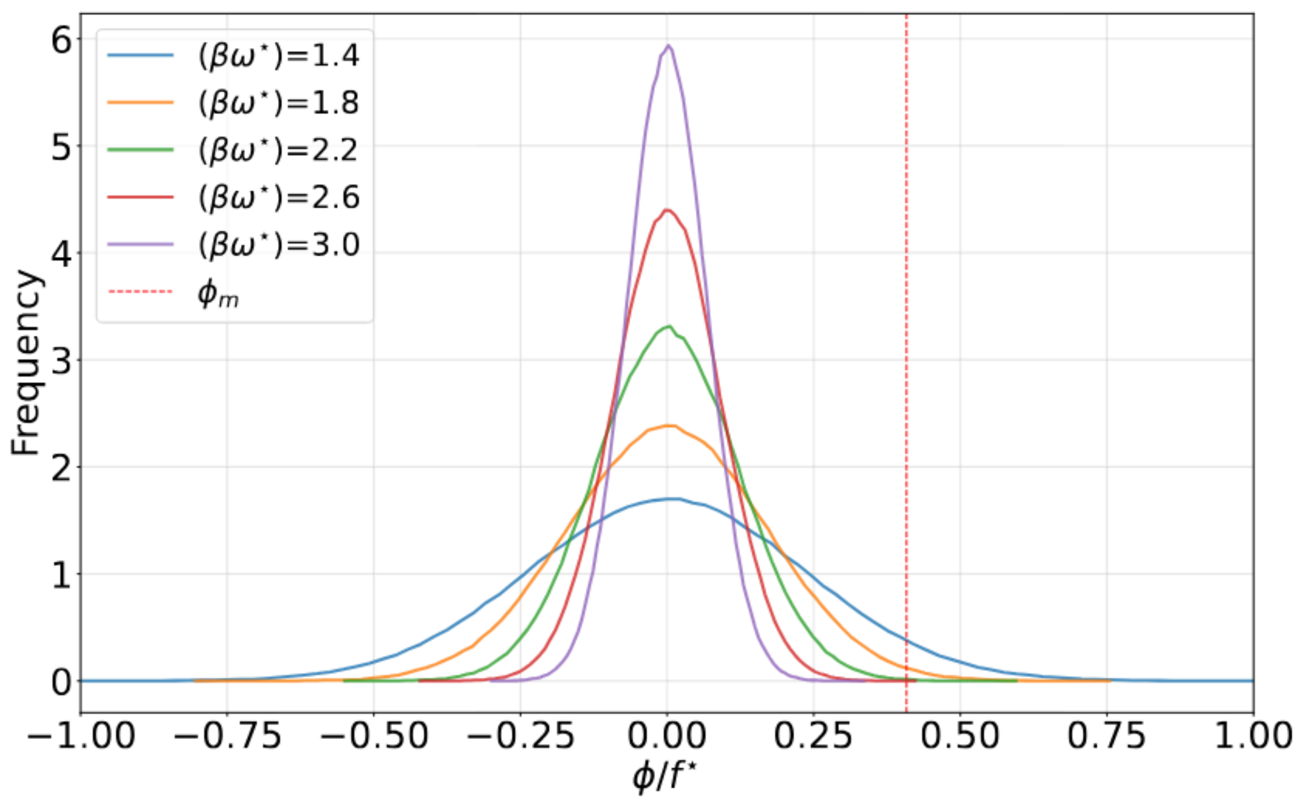
$$\chi_{FV}(\phi_{-t}^{(i,j)}) = \frac{N_x(\phi_x^{(i,j)} < \phi_m)}{N_x}$$

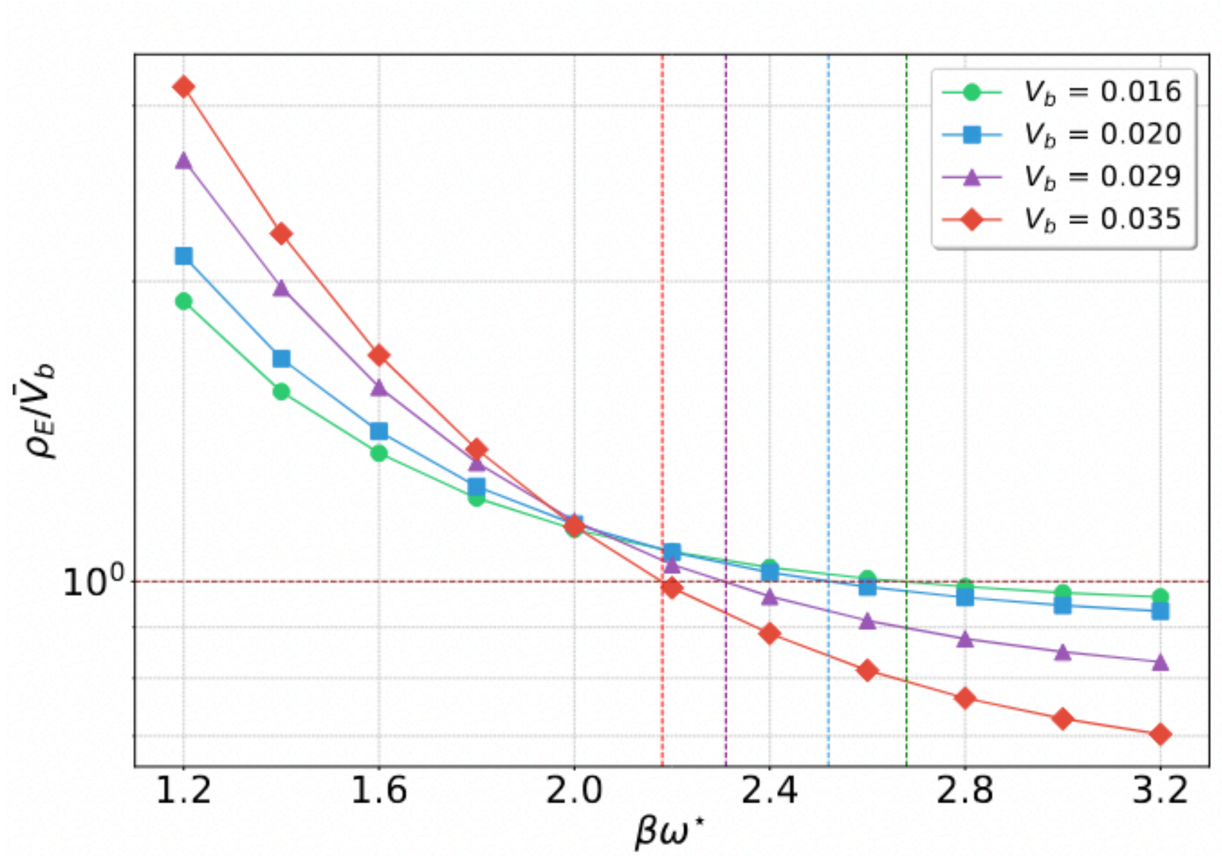
数值模拟结果

$$\phi_s = \phi/f^\star \quad t_s = \omega^\star * t \quad x_s = \omega^\star * x$$

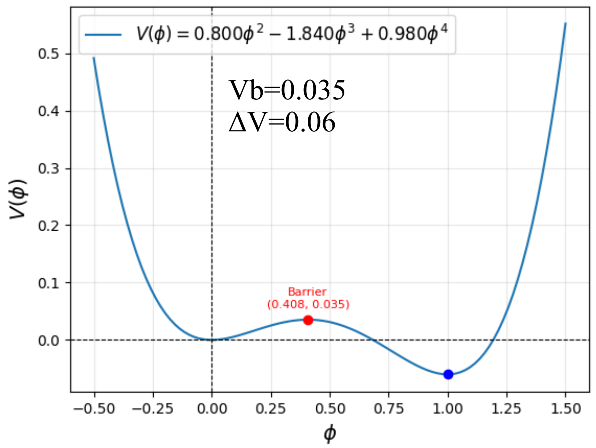
Nsamples = 1000
Δx = 0.25
Δt = 0. 01
Nx = 1024

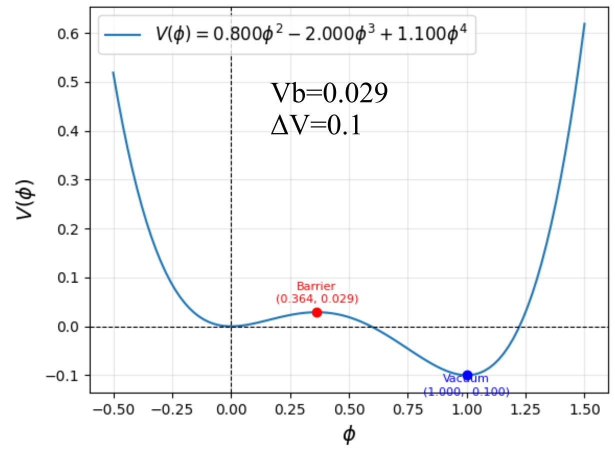


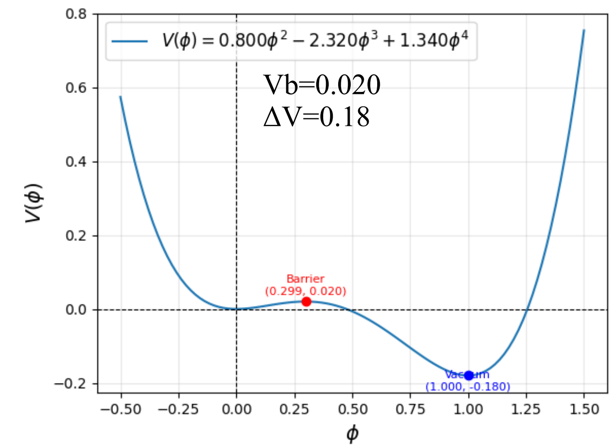


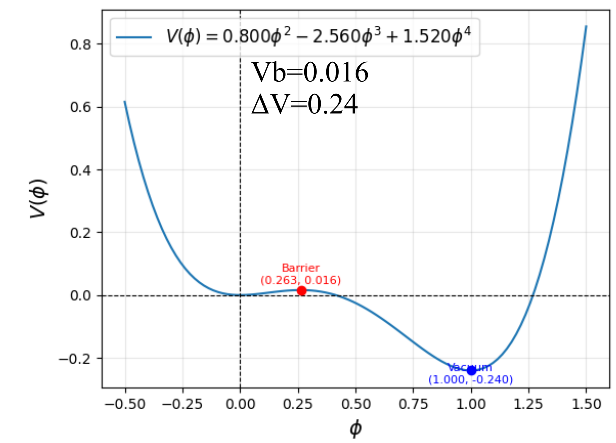


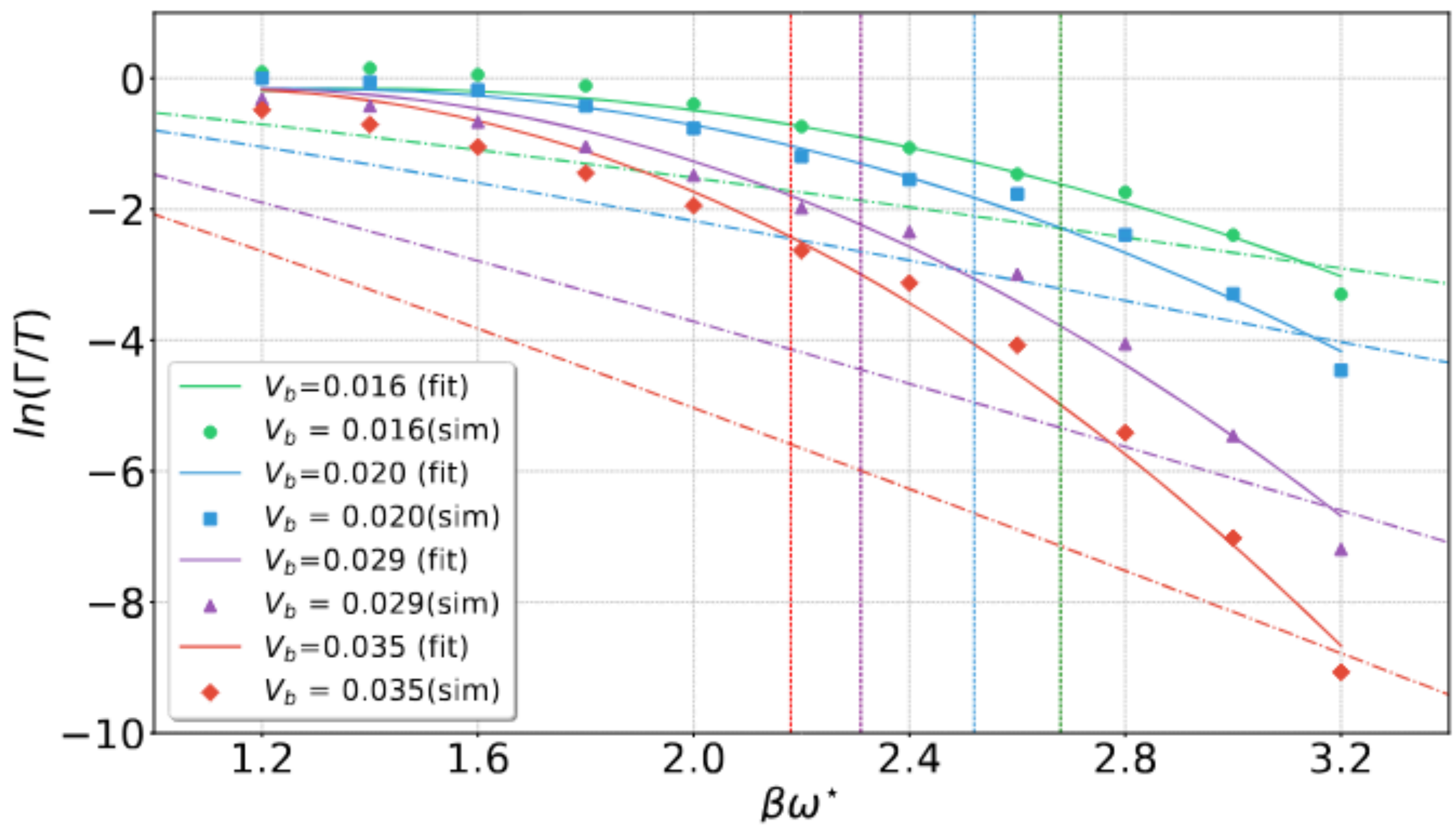
数值模拟结果


$V(\phi) = 0.800\phi^2 - 1.840\phi^3 + 0.980\phi^4$
Vb=0.035
ΔV=0.06
Barrier
(0.408, 0.035)
$V(\phi)$
ϕ


$V(\phi) = 0.800\phi^2 - 2.000\phi^3 + 1.100\phi^4$
Vb=0.029
ΔV=0.1
Barrier
(0.364, 0.029)
(1.000, -0.100)
$V(\phi)$
ϕ


$V(\phi) = 0.800\phi^2 - 2.320\phi^3 + 1.340\phi^4$
Vb=0.020
ΔV=0.18
Barrier
(0.299, 0.020)
(1.000, -0.180)
$V(\phi)$
ϕ

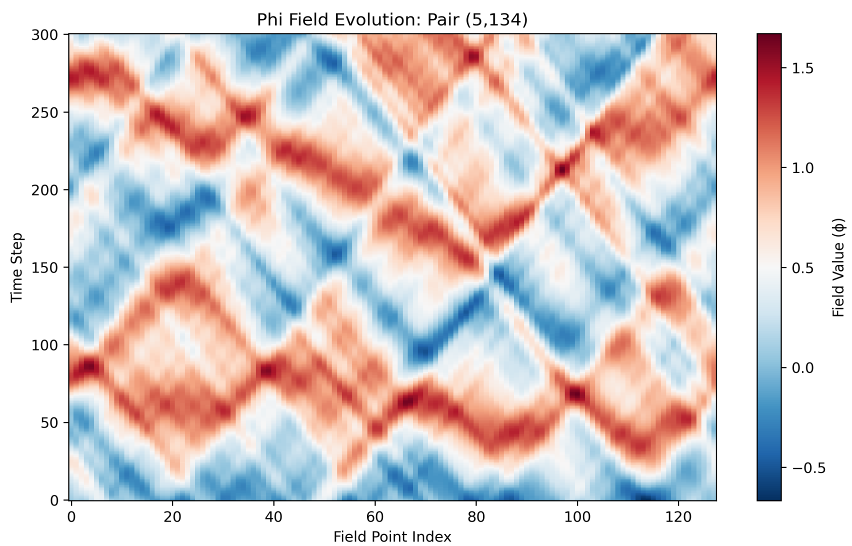

$V(\phi) = 0.800\phi^2 - 2.560\phi^3 + 1.520\phi^4$
Vb=0.016
ΔV=0.24
Barrier
(0.263, 0.016)
(1.000, -0.240)
$V(\phi)$
ϕ


V_b=0.016 (fit)
V_b = 0.016(sim)
V_b=0.020 (fit)
V_b = 0.020(sim)
V_b=0.029 (fit)
V_b = 0.029(sim)
V_b=0.035 (fit)
V_b = 0.035(sim)
$ln(\Gamma/T)$
$\beta\omega^*$

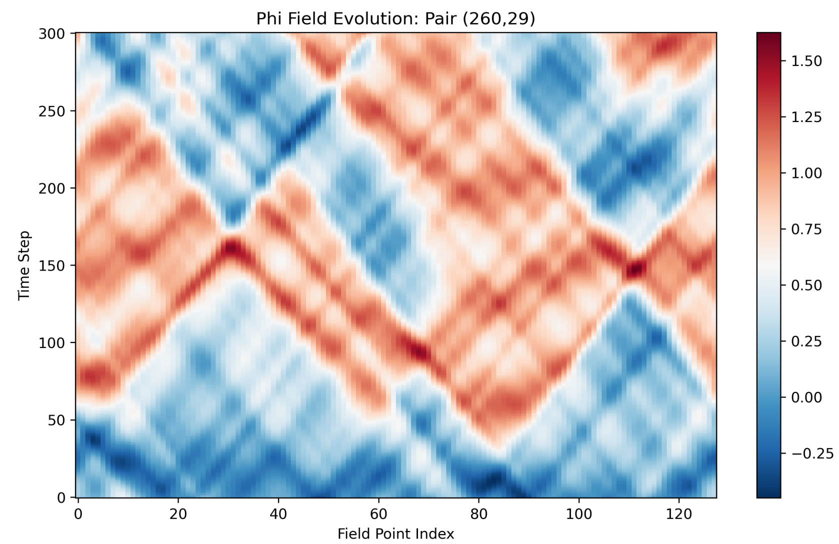
数值模拟结果

1D场演化图

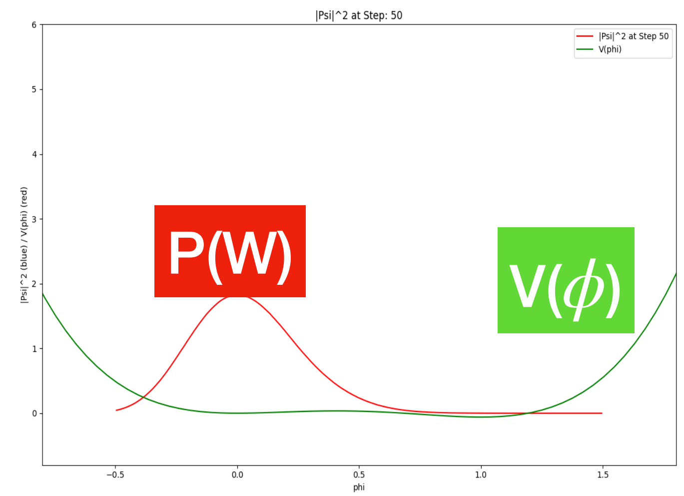
Wave function evolution

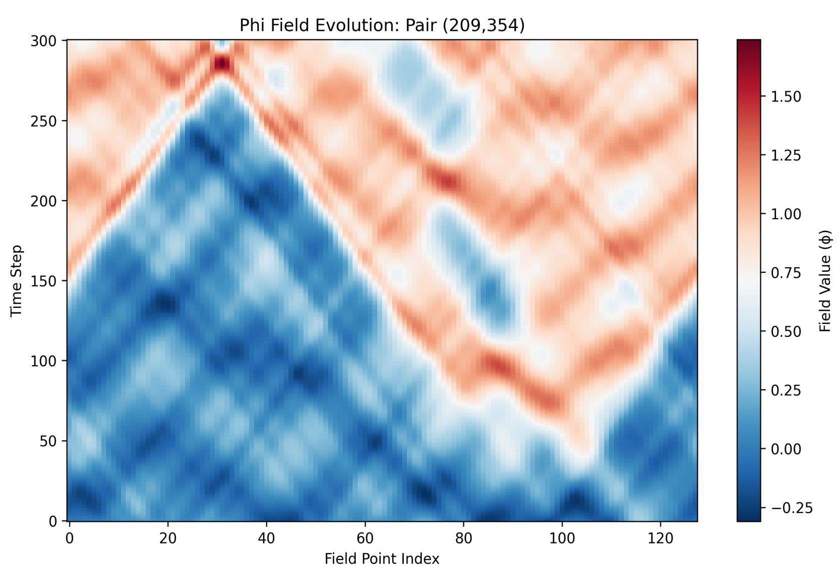


β=0.8

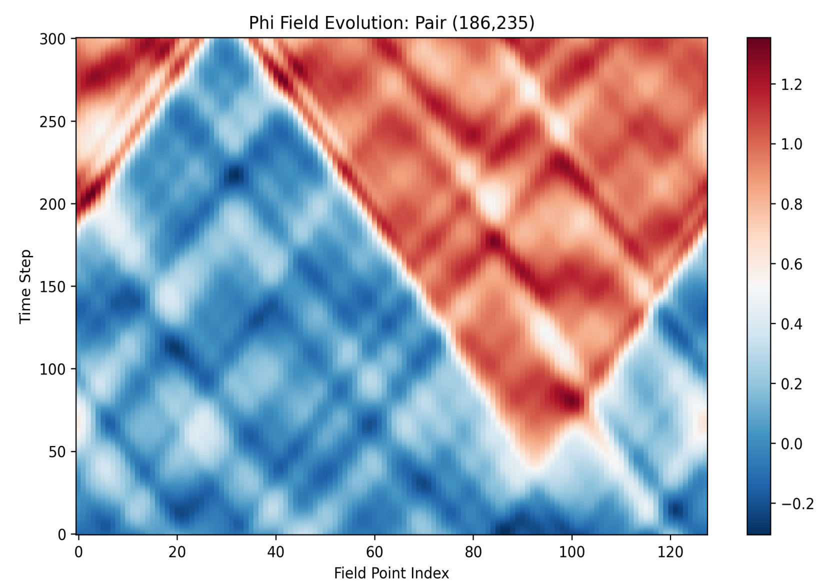


β=1.0

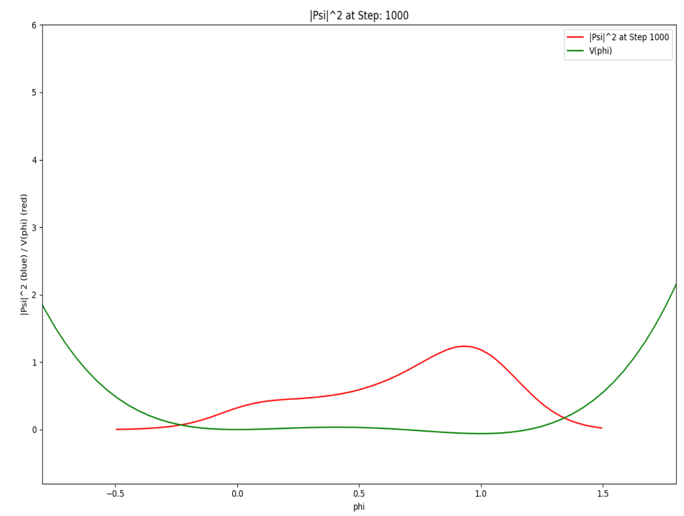




β=1.2



β=1.4



$\beta\omega^* = 1$

总结和展望

Gravitational waves provide a new window to probe/constrain beyond standard model physics

❖Lattice simulation

- **Nucleation/Sphaleron rate simulations**
- **PT-GW simulation**
- **Topological defects: Magnetic monopoles, cosmic strings, domain walls, string-wall**

❖Pheno

1. Baryon Asymmetry of the Universe and GW from FOPT

- **Sphaleron process, bubble velocity （local equilibrium? ）**

2. DM and GW from FOPT

- **DM and high/low-scale PT, DM out-of-equilibrium & FOPT, PBH DM&FOPT**

谢谢！