Cosmological Correlators & Wavefunction

a broad overview

see refs in the slides

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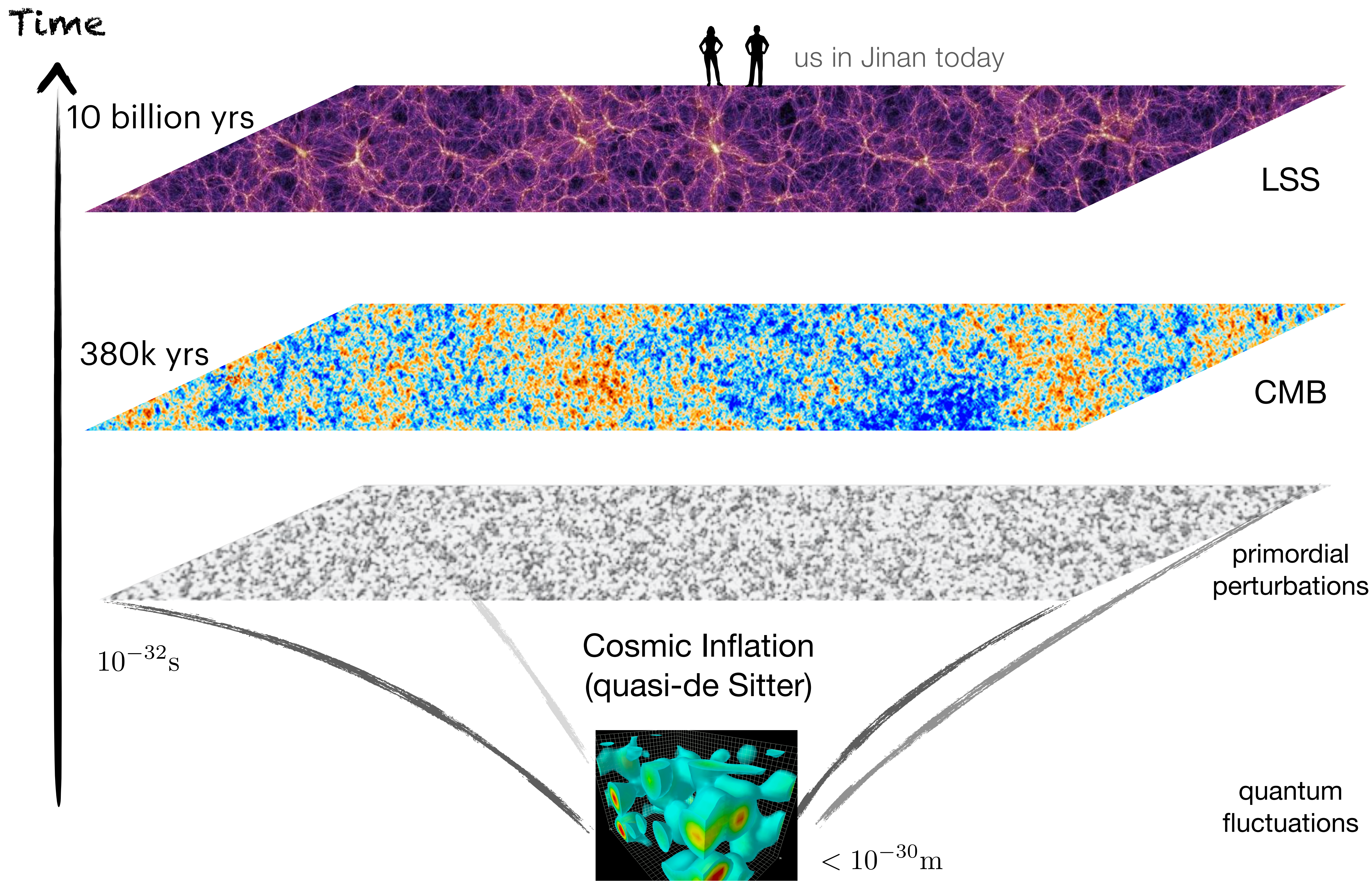
IAS HKUST & DAMTP Cambridge

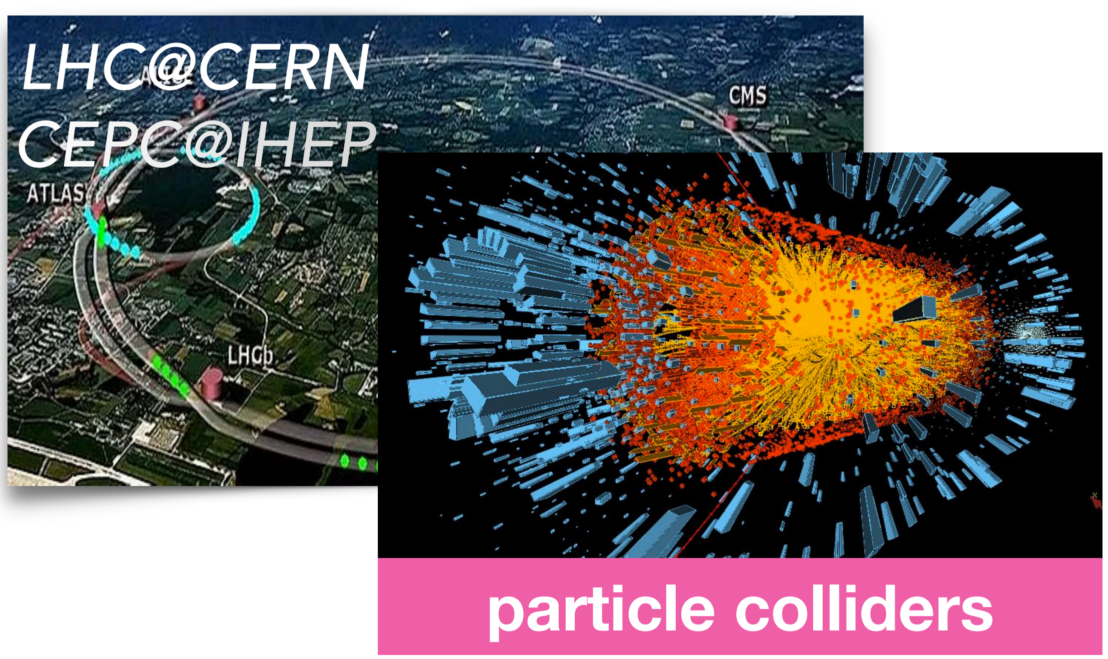
New Physics Workshop @ Jinan, July 23, 2025

Initial Condition of Our Universe

— quantum origin of cosmic structures



Cosmic Inflation: *a natural laboratory for high energy physics*



Energy

$10^{19}\mathrm{GeV}$ — M_{Pl} ***Planck Scale***

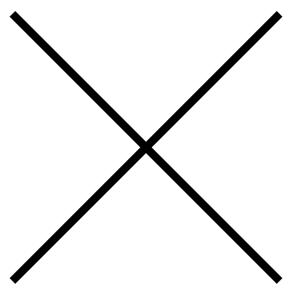
$< 10^{13}\mathrm{GeV}$ — ***Inflation Scale***

possibly the highest energy
scale accessible in nature

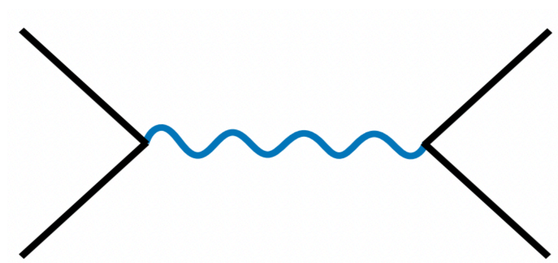
$10^{2}\mathrm{GeV}$ — ***Electroweak Scale***

Fantastic New Physics and Where to Find Them

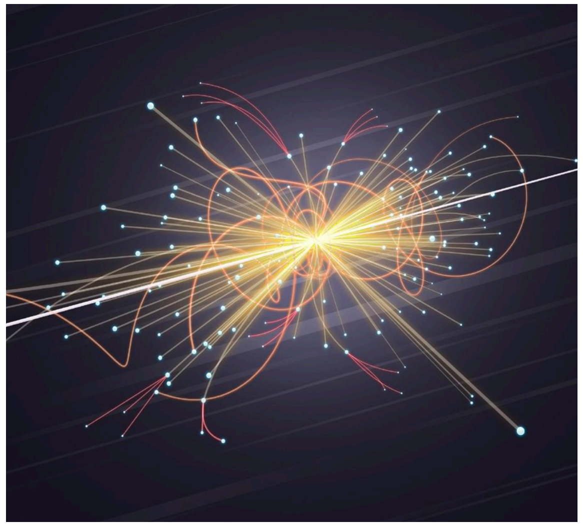
Scattering Amplitudes



Contact diagram

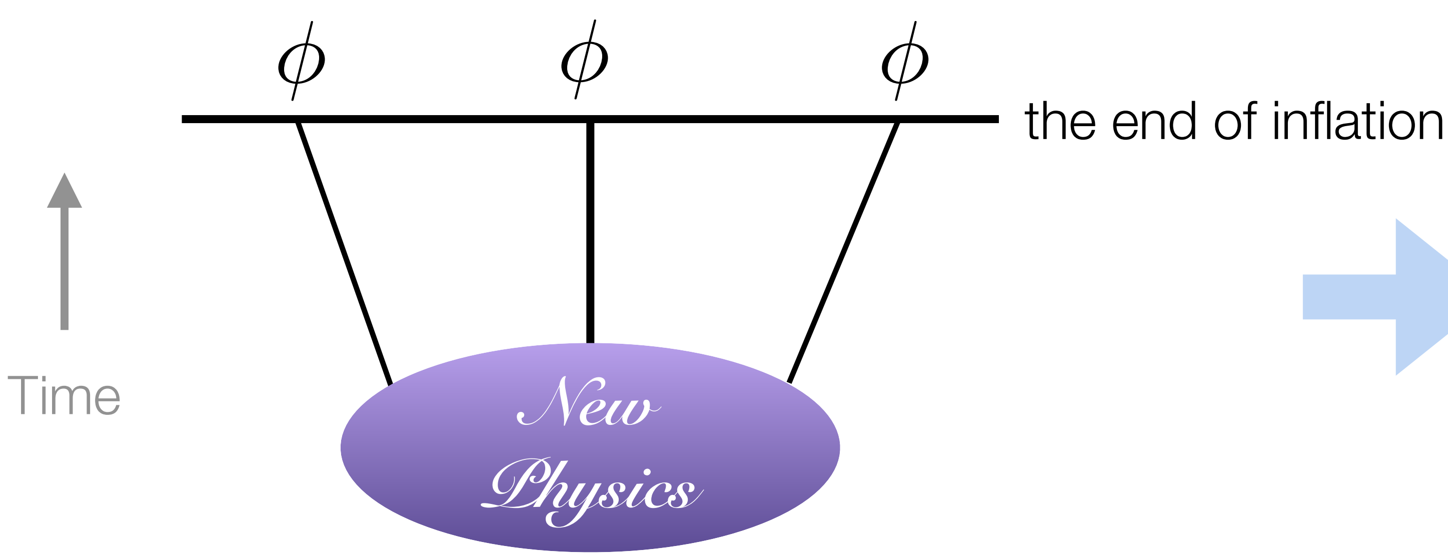


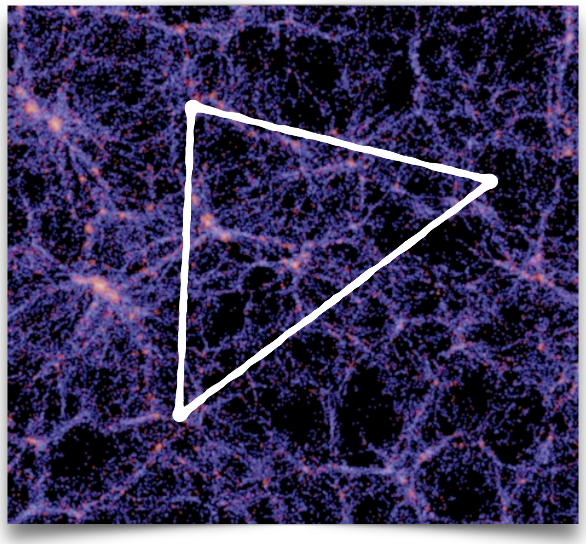
Exchange diagram



$\sim 1\text{TeV}$

Cosmological Correlators $\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle$

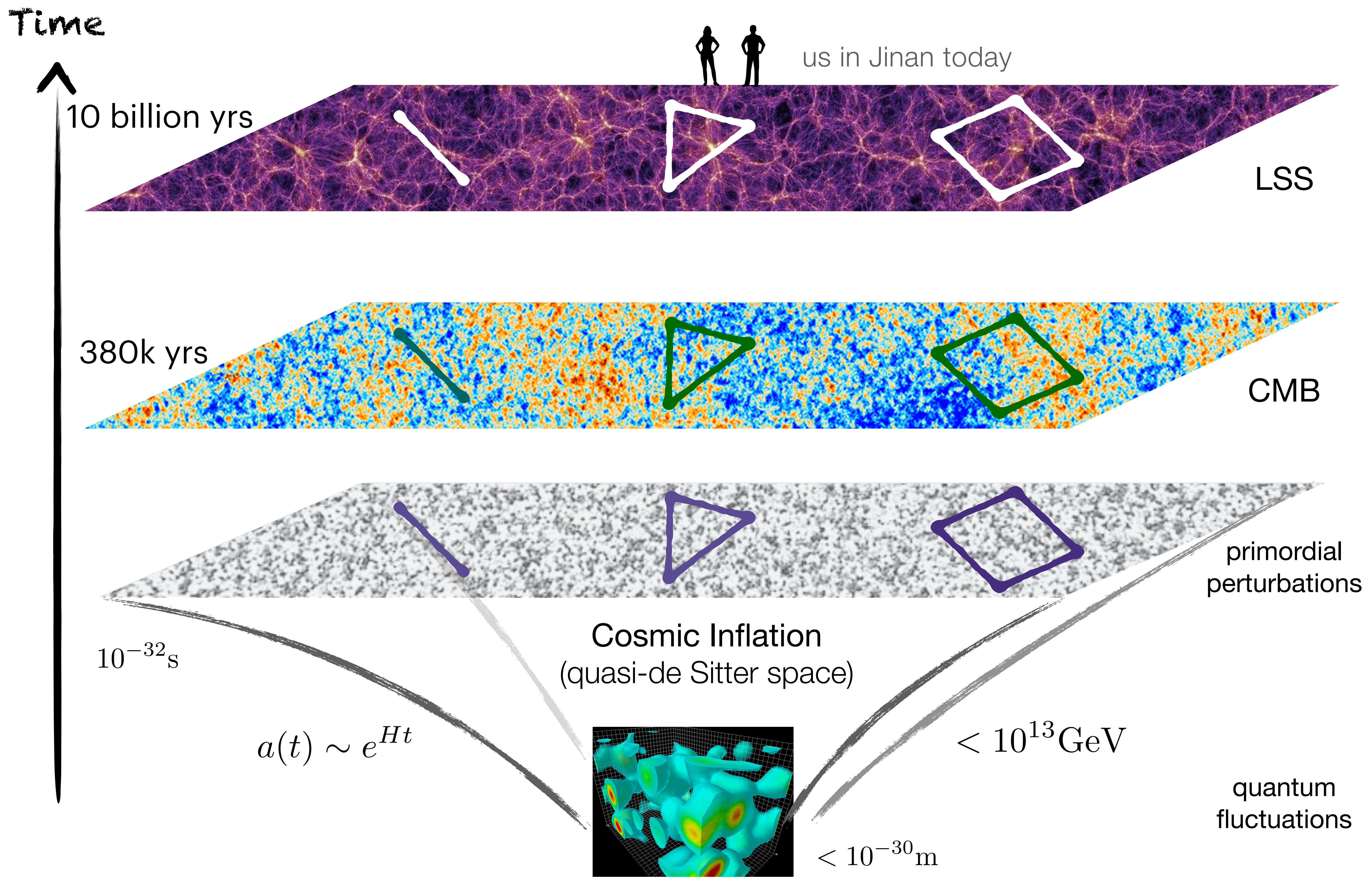




$< 10^{13}\text{GeV}$

Cosmological Correlation Matters

a unique probe for the beginning of our Universe and extremely high energy physics



Plan of the Talk

- Boundary Correlators in de Sitter
  - **Cosmological Bootstrap:**

    Correlators from Symmetries, Unitarity & Locality
  - **Phenomenology:**

    Primordial Non-Gaussianity & Observations
- Cosmological Wavefunction
  - **Field-Theoretic Wavefunction**
  - **On the IR Divergences in de Sitter**

    Classical Loops & Stochastic Formalism

Cosmological Bootstrap

Correlators from Symmetry, Unitarity & Locality

The conventional approach of theoretical cosmology

a brilliant idea

Model Building:
(a Lagrangian)

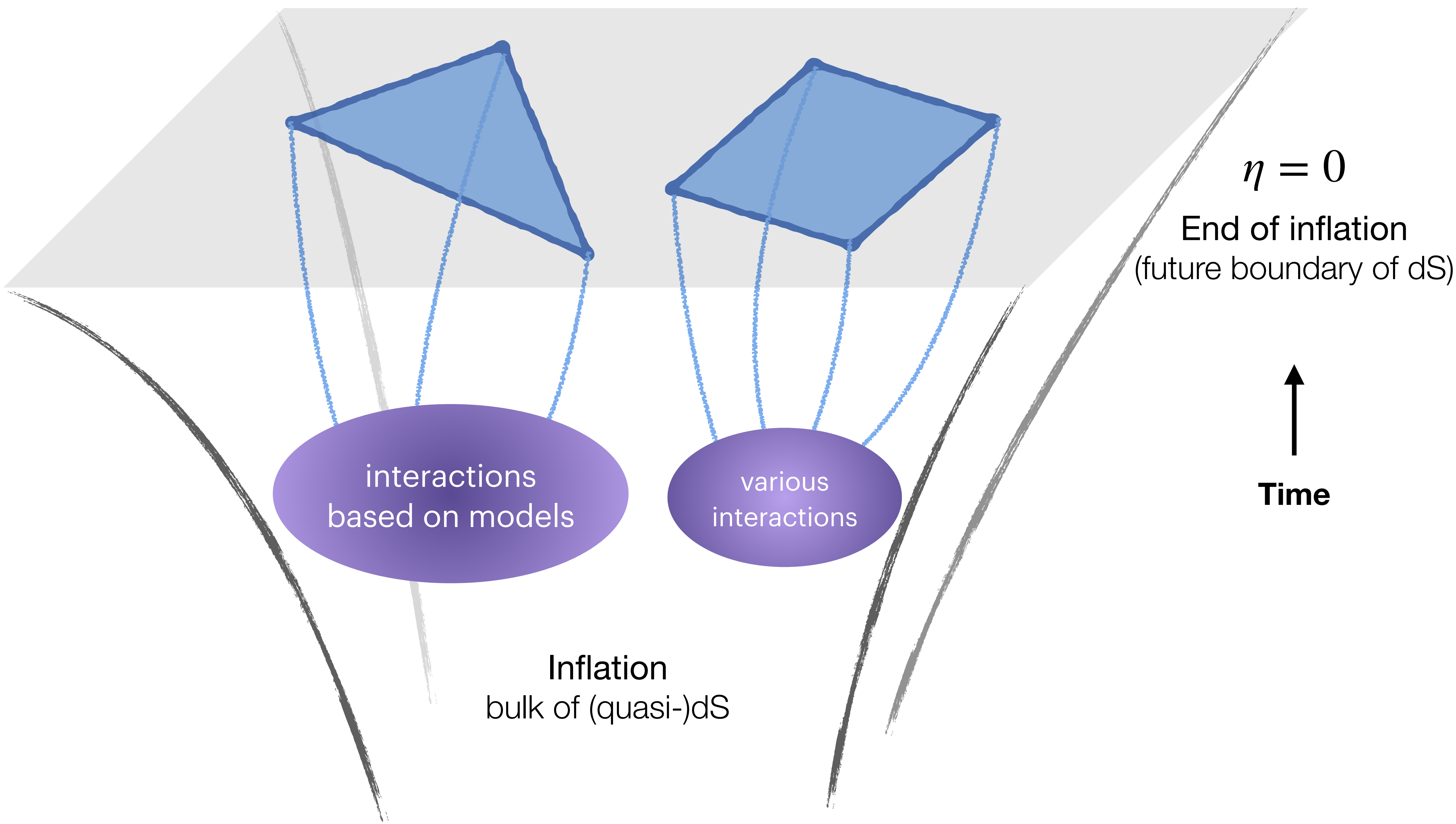
Cosmological
Perturbation Theory

predictions for
observations:

Primordial GWs   *PBHs*   *Features*   ……

The traditional approach towards cosmological correlators

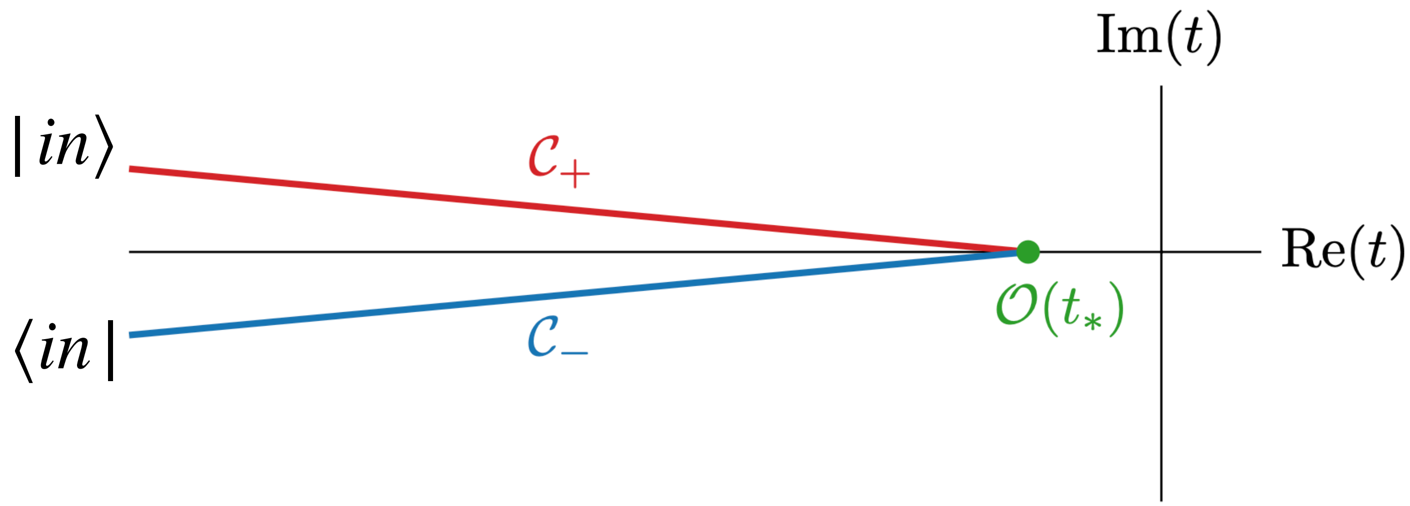
the in-in formalism (or Schwinger-Keldysh)



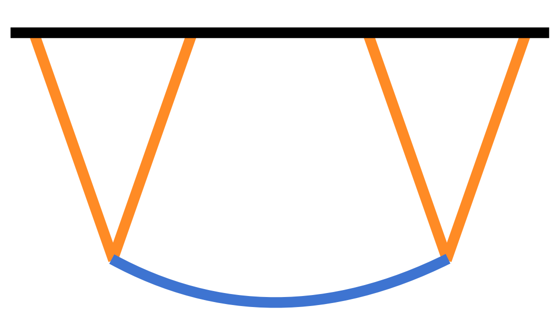
The traditional approach towards cosmological correlators

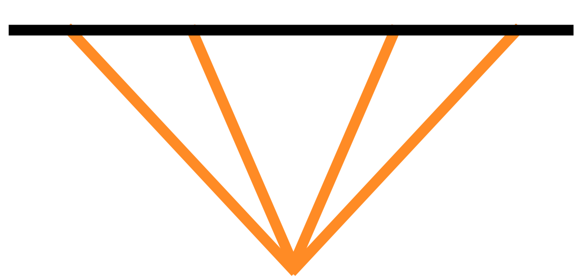
Correlators are expectation values of field operators at an equal-time slice

$$\langle \mathcal{O}(t) \rangle \equiv \langle \Omega | \mathcal{O}(t) | \Omega \rangle$$



$$\langle \mathcal{O}(t) \rangle = \langle 0 | \overline{T} e^{i \int_{-\infty^+}^{t} \mathrm{d}t' H_{\mathrm{int}}(t')} \, \mathcal{O}(t) \, T e^{-i \int_{-\infty^-}^{t} \mathrm{d}t' H_{\mathrm{int}}(t')} | 0 \rangle$$

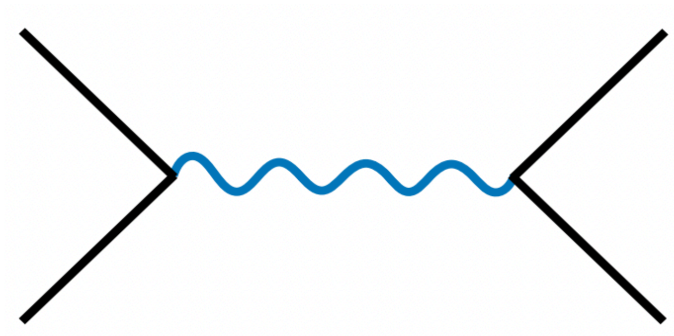




- Quite general;
- Usually model-dependent;
- Hard to compute.

Quantum Field Theory in Curved Spacetime is HARD

Scattering Amplitudes in Flat Spacetime

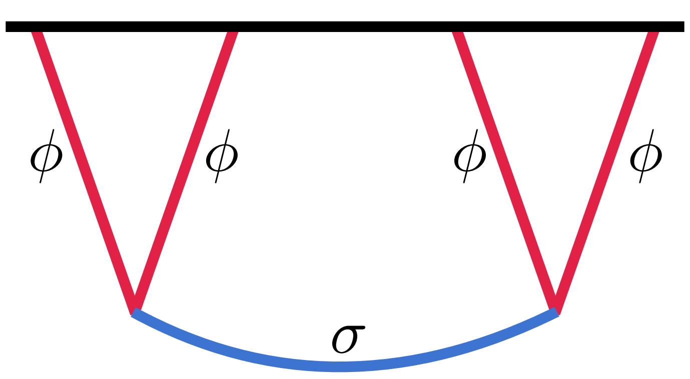


s-channel Exchange

$$= \frac{1}{s - m^2}$$

any QFT textbooks

Four-Point Exchange Correlator in de Sitter Spacetime



= ?

The Bootstrap Philosophy

fundamental principles ⟷ *observable predictions*

Symmetries
Unitarity
Locality

Scattering amplitudes
Cosmological correlators

model building X

Successful examples:
1) S-matrix bootstrap
2) conformal bootstrap

“Bootstrap”

xuē pàn
靴袢

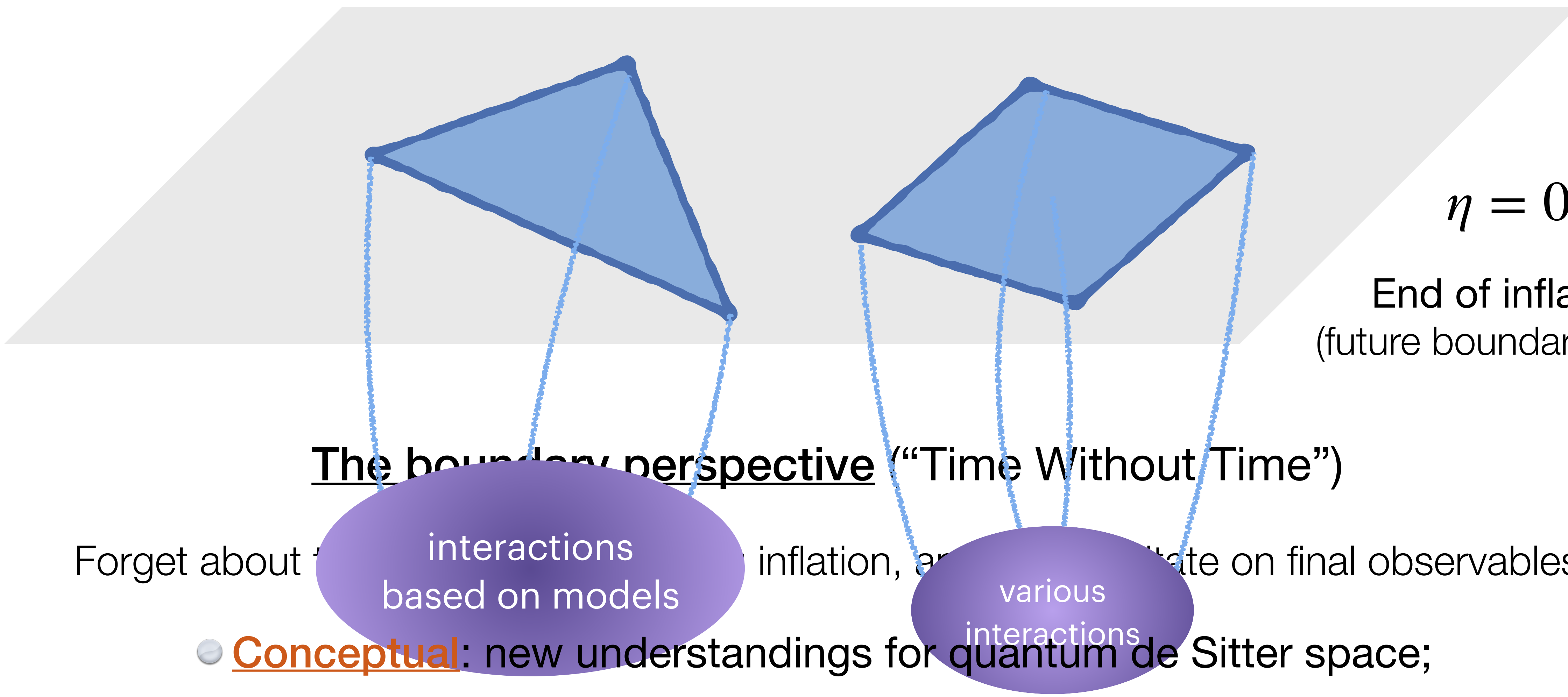
“自举/自展”

Cosmological Bootstrap:

Correlators from Symmetries, Locality & Unitarity

A new trend in theoretical cosmology since 2018:

Daniel Baumann, Enrico Pajer, Gui Pimentel, Austin Joyce, Nima Arkani-Hamed, Juan Maldacena, ...



<u>The bou</u> ... **<u>perspective</u>** (“Time Without Time”)

Forget about t ... inflation, a ... tate on final observables

- <u>Conceptual</u>: new understandings for quantum de Sitter space;

  the emergence of time, dS holography, etc

- <u>Practical</u>: A model-independent & systematical approach;

  A powerful computational tool.

Bootstrap from (*Spacetime*) *Symmetries*

- De Sitter Bootstrap
- Boostless Bootstrap

Bootstrap from Unitarity and Locality

- Unitarity:

  Cosmological Optical Theorem & Cutting Rules:

- Manifest Locality:

  Manifestly Local Test

De Sitter Bootstrap

Arkani-Hamed, Baumann, Lee, Pimentel 2018
Baumann, Duaso Pueyo, Joyce, Lee, Pimentel 2019, 2020
DGW, Pimentel, Achucarro 2022

Symmetries of dS spacetime: $\mathrm{d}s^2 = \frac{1}{H^2\eta^2}\left(-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2\right)$

spatial translation $P_i = \partial_i$

spatial rotation $J_{ij} = x_i\partial_j - x_j\partial_i$

→ kinematics variables $u \equiv \frac{|\mathbf{k}_1 + \mathbf{k}_2|}{|\mathbf{k}_1| + |\mathbf{k}_2|}$ $v \equiv \frac{|\mathbf{k}_3 + \mathbf{k}_4|}{|\mathbf{k}_3| + |\mathbf{k}_4|}$

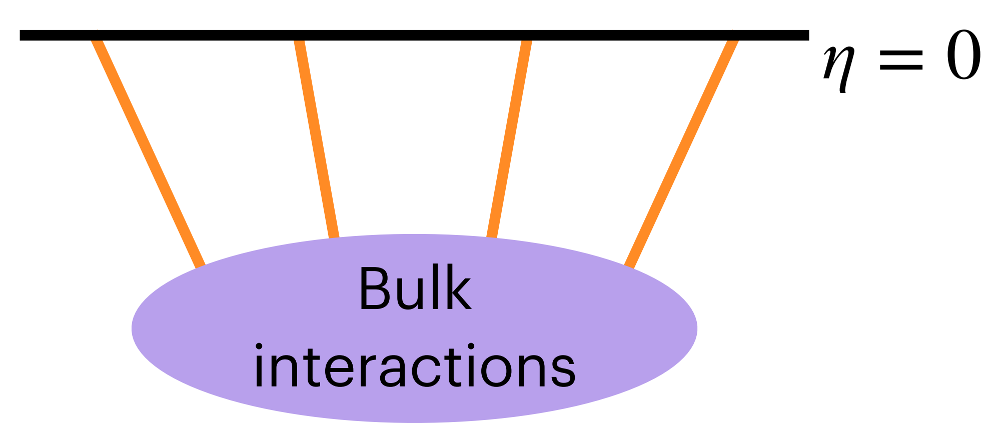
dS dilation $D = -\eta\partial_\eta - x^i\partial_i$ → scale invariance

dS boosts $K_i = 2x_i\eta\partial_\eta + \left(2x^j x_i + (\eta^2 - x^2)\delta_i^j\right)\partial_j$

→ ***Conformal Ward identities*** on the de Sitter boundary (η->0)

$$(\Delta_u + M^2)\,F = C_n\,,$$
$$(\Delta_v + M^2)\,F = C_n\,,$$

$\langle\varphi^4\rangle \sim F(u,v)$



Fully fix the analytical solutions of correlators

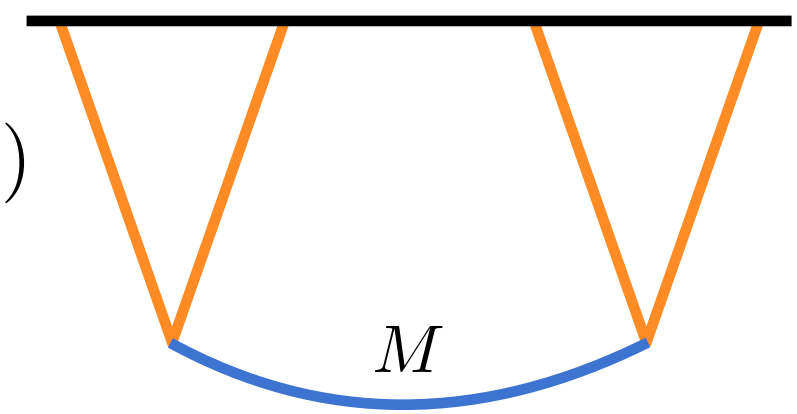
De Sitter Bootstrap as an example

Arkani-Hamed, Baumann, Lee, Pimentel 2018
Baumann, Duaso Pueyo, Joyce, Lee, Pimentel 2019, 2020

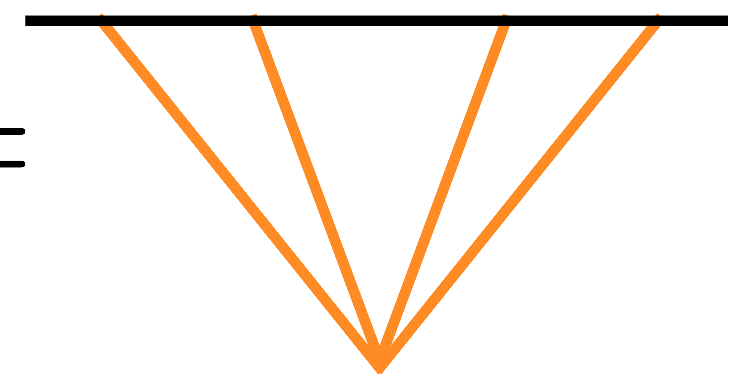
Boundary Differential Equations (from Conformal Ward Identities)

$$\langle \varphi^4 \rangle \sim F(u,v) \qquad\qquad C(u,v) \sim \frac{uv}{u+v}$$

$$(\Delta_u + M^2)$$



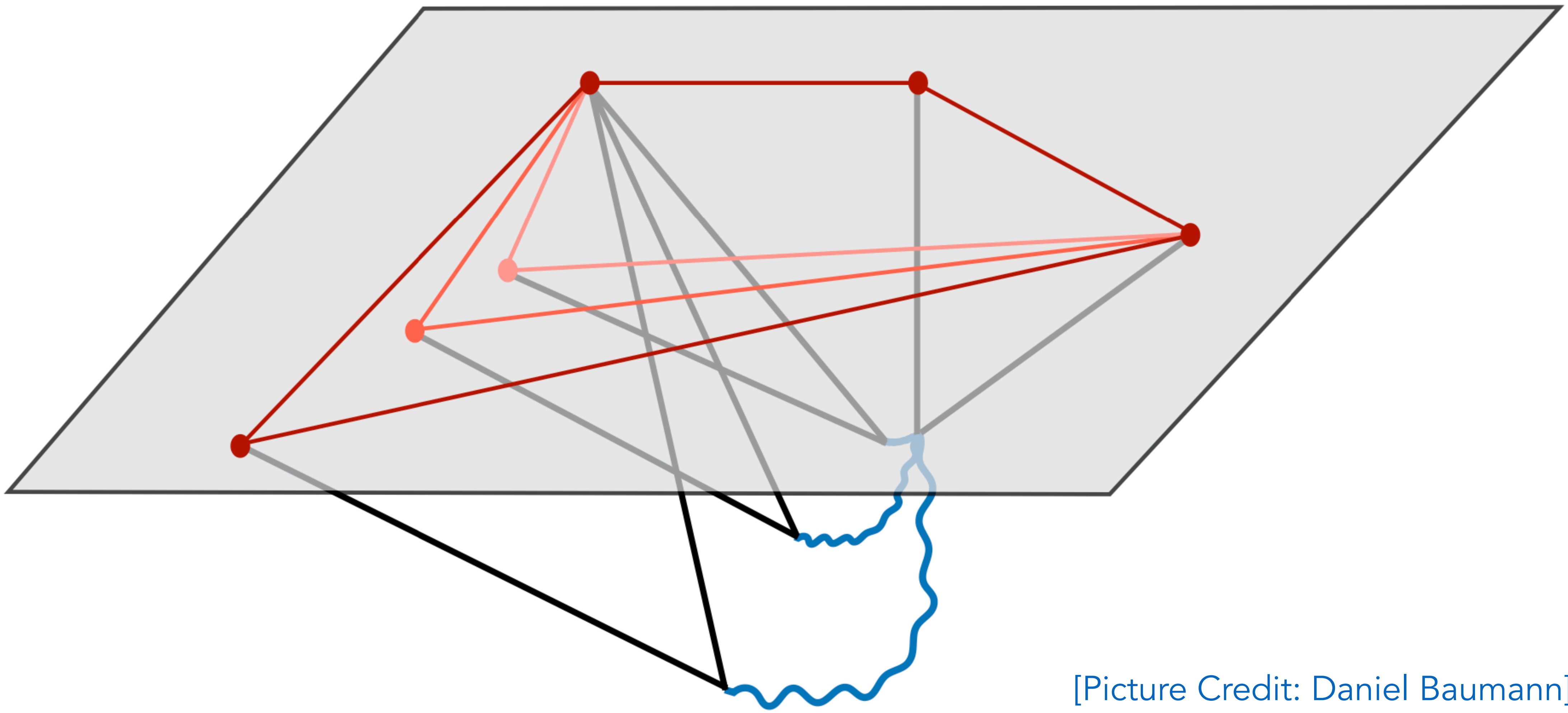
$$=$$



$$\Delta_u = u^2(1-u^2)\partial_u^2 + 2u^3\partial_u \qquad u \equiv \frac{|\mathbf{k}_1 + \mathbf{k}_2|}{|\mathbf{k}_1| + |\mathbf{k}_2|} \qquad v \equiv \frac{|\mathbf{k}_3 + \mathbf{k}_4|}{|\mathbf{k}_3| + |\mathbf{k}_4|}$$

a "timeless" description for boundary correlators

Conceptual Picture: **The Emergence of Time**



[Picture Credit: Daniel Baumann]

The **pattern** of correlators on the ***boundary*** traces the **dynamics** in the ***bulk***.

Quantum Field Theory in Curved Spacetime is HARD

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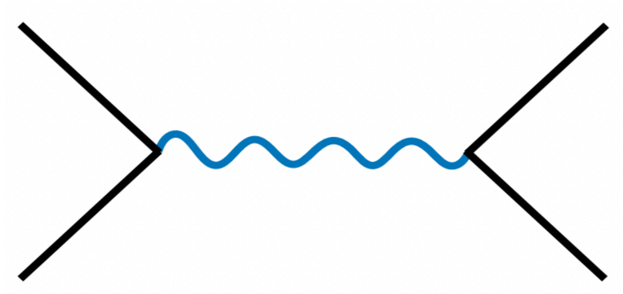

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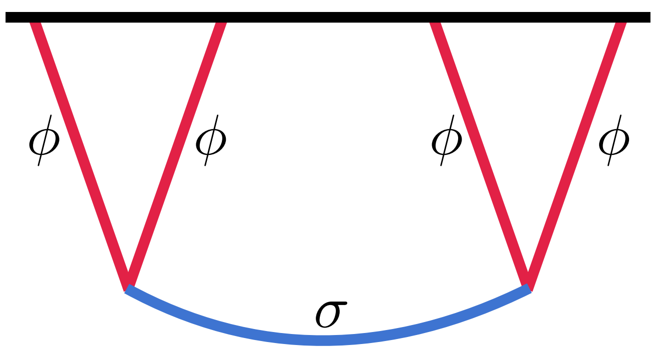
Four-Point Exchange Correlator in de Sitter Spacetime



= Hypergeometric Functions
+ Power Series

Arkani-Hamed, Maldacena 2015
Arkani-Hamed, Baumann, Lee, Pimentel 2018
DGW, Pimentel, Achucarro 2022

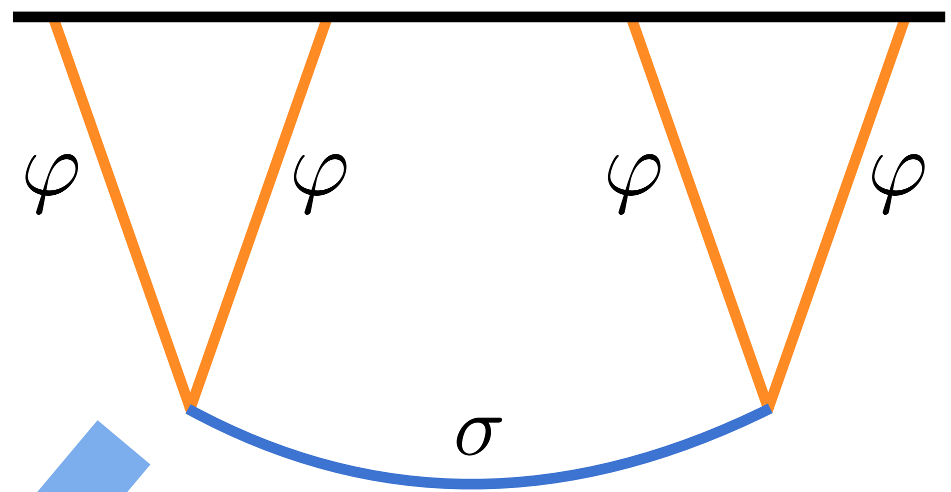
De Sitter Bootstrap

Arkani-Hamed, Baumann, Lee, Pimentel 2018
Baumann, Duaso Pueyo, Joyce, Lee, Pimentel 2019, 2020

Four-point scalar seed: *F*

conformally coupled scalar

$m^2 = 2H^2$



$m > 0, s = 0$

dS Boosts =>

$$\left(\Delta_u + M^2\right) F = C_n \,,$$
$$\left(\Delta_v + M^2\right) F = C_n \,,$$

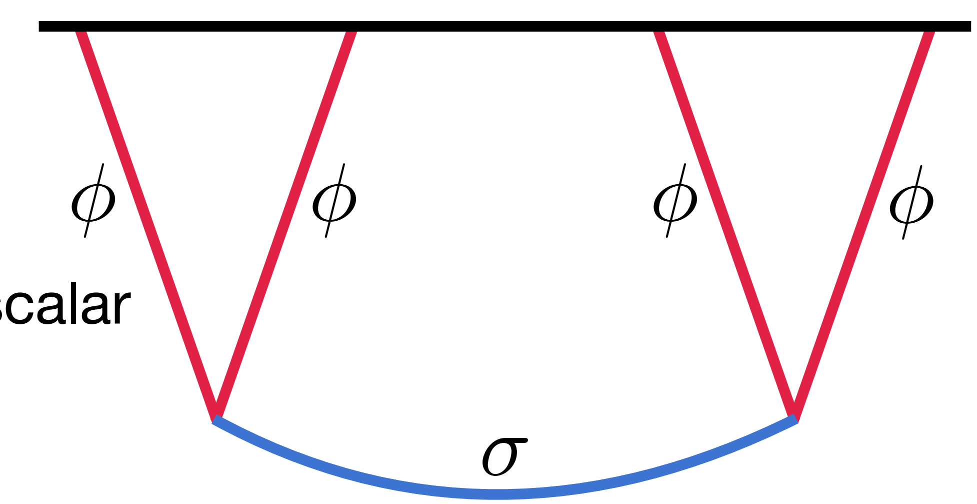
A set of 2nd order ODEs

Weight-Shifting
& Spin-Raising

Inflaton four-point function

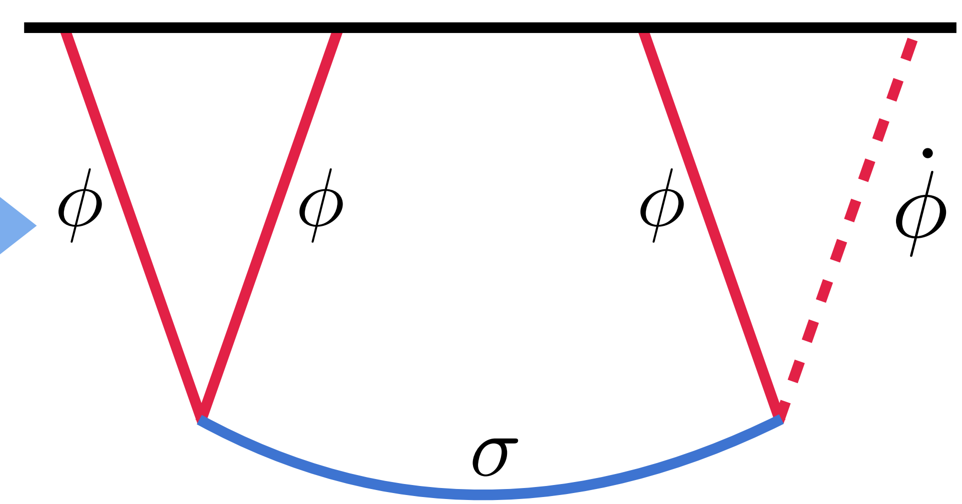
$m^2 = 0$

massless scalar



$m > 0, s \geq 0$

Inflaton Bispectra (~ slow-roll)



$m > 0, s \geq 0$

Boostless Bootstrap

Pajer 2020
Jazayeri, Pajer, Stefanyszyn 2021
Bonifacio, Pajer, DGW 2021
Cabass, Pajer, Stefanyszyn, Supel 2021

Boostless theories:

- translation, rotations, and dilation are still respected;
- dS boosts are broken;
- *Examples: P(X) theory, EFT of inflation* $\dot\phi^3 \quad \dot\phi(\partial_i\phi)^2 \quad \dot\phi^4 \quad \dot\phi^2(\partial_i\phi)^2$

Bootstrap Rules for single field inflation:

scalar three-point function
(inflaton bispectrum) as an example

$$\langle \phi_{k_1}\phi_{k_2}\phi_{k_3}\rangle \sim \frac{\psi_3}{k_1^3 k_2^3 k_3^3}$$

$$\psi_3 = \frac{\mathrm{Poly}_{3+p}(k_T, e_2, e_3)}{k_T^p}$$

Scale invariance

Bose symmetry

$k_T \equiv k_1 + k_2 + k_3$

$e_2 \equiv k_1k_2 + k_2k_3 + k_1k_3$

$e_3 \equiv k_1k_2k_3$

tree level in dS

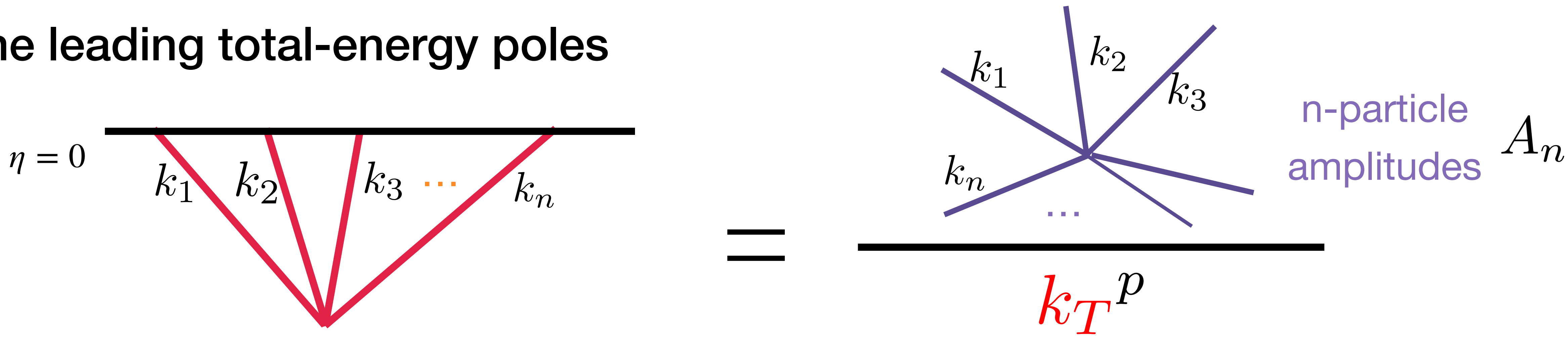
Bunch Davies vacuum

Total-Energy Poles

the only allowed singularity in *contact correlators*

$$k_T = k_1 + k_2 + \ldots + k_n \to 0$$

The leading total-energy poles



n-point contact correlators $\langle \phi_{k_1} \phi_{k_2} \ldots \phi_{k_n} \rangle$

Maldacena, Pimentel 2011
Raju 2012

The sub-leading total-energy poles

fixed by the requirement of "**manifest locality**" of contact interactions

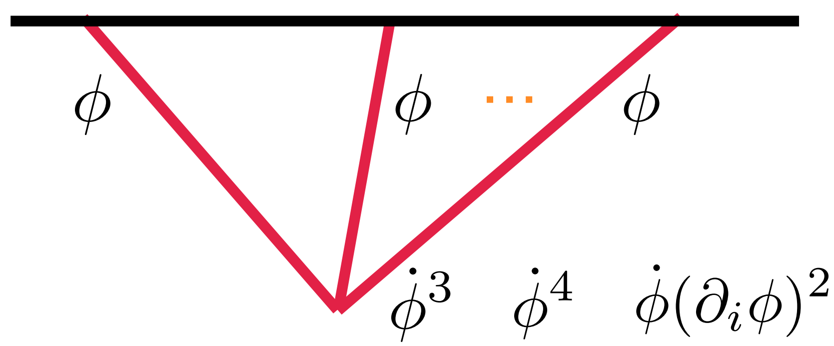
$$\partial_{k_1} \psi_n |_{k_1=0} = 0$$

Jazayeri, Pajer, Stefanyszyn 2021

Boostless Bootstrap

Pajer 2020; Jazayeri, Pajer, Stefanyszyn 2021;
Bonifacio, Pajer, DGW 2021; ….

Contact



de Sitter boosts are broken

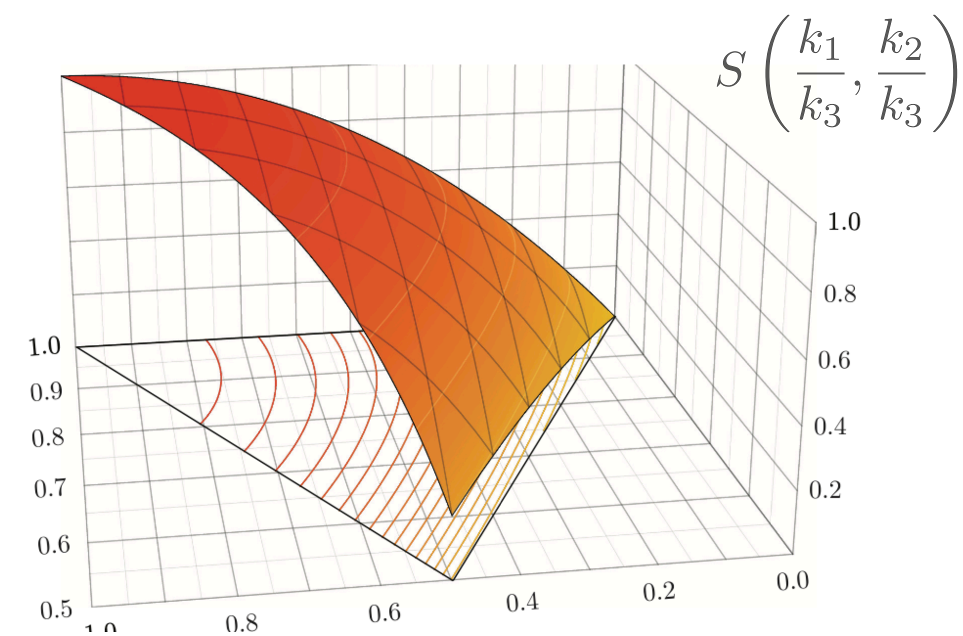
- write an ***ansatz*** for correlators of a single massless scalar

$$S^{\rm eq}(k_1,k_2,k_3) = \frac{\mathrm{Poly}_{p+3}(k_T,e_2,e_3)}{k_1k_2k_3k_T^p}$$

- keep imposing constraints from unbroken symmetries, locality, unitarity, …

Single Field Inflation

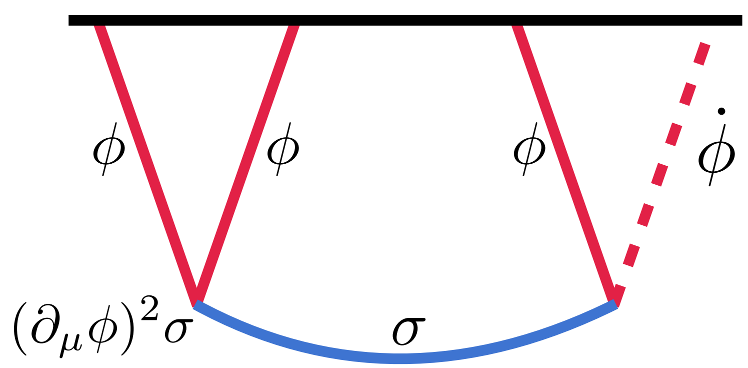
captures models such as P(X), DBI, and all single field EFT of inflation with small sound speed & derivative couplings



Equilateral-type non-Gaussianity

De Sitter Bootstrap *v.s.* Boostless Bootstrap

in analog with slow-roll inflation



Pros

dS symmetries are nicely manifested;

Fully analytical control for correlators;

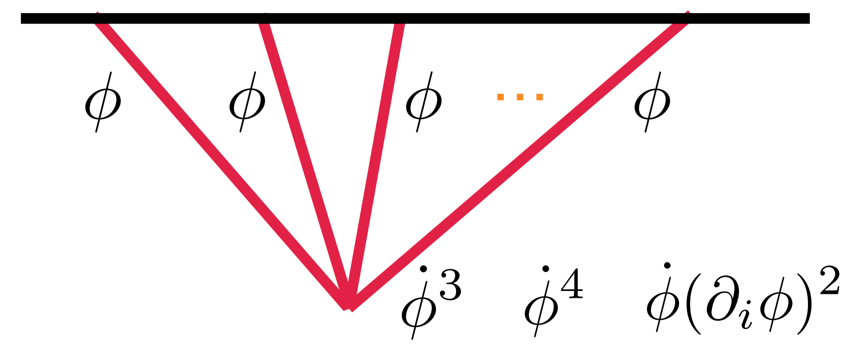
Cons

Non-Gaussianity signals are ***very small***;

Need 4-pt first, before computing 3-pt.

in analog with P(X), DBI inflation

with **small sound speed**



Pros

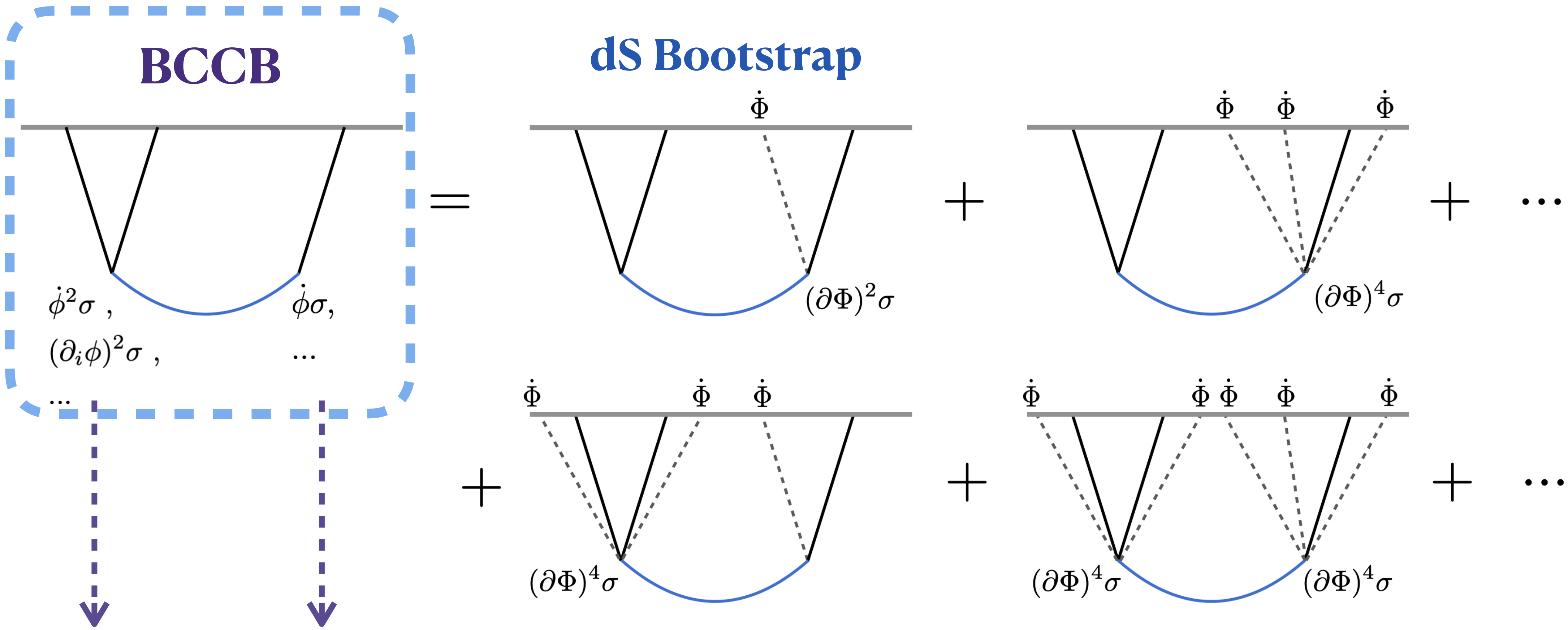
Large signals are possible;

A complete set of *single-field* correlators.

Cons

Only for correlators from single massless field interactions

Signals are boosted in strong boost-breaking scenarios



Cubic & quadratic interactions from the EFT of inflation

Baumann, Lee, Pimentel 2016
Bordin, Creminelli, et al 2018
Pimentel, DGW 2022

The EFT of Cosmo Collider

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle \sim f_{\rm NL} S(k_1, k_2, k_3) P_\zeta^2$$

size

shape

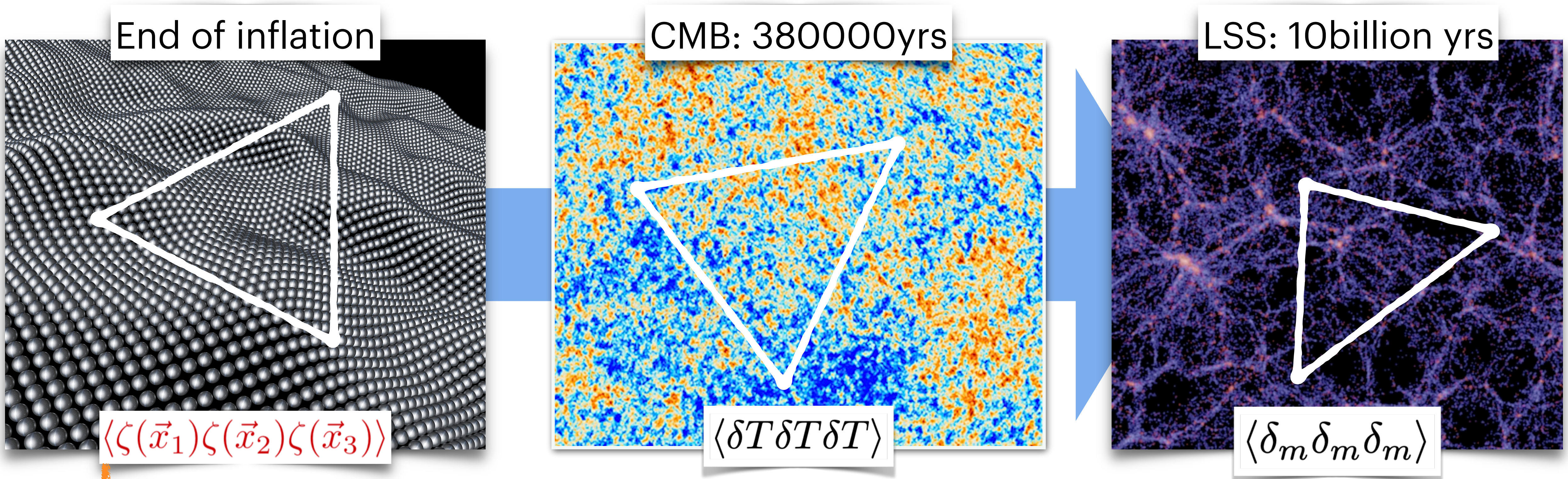
$f_{\rm NL} \lesssim \mathcal{O}(10)$

Phenomenology

Primordial Non-Gaussianity & Observations

Triangles in the Sky

a.k.a primordial non-Gaussianity



→ **primordial bispectrum**: the Fourier transf. of the 3pt correlation function

$$\langle\zeta_{\mathbf{k_1}}\zeta_{\mathbf{k_2}}\zeta_{\mathbf{k_3}}\rangle \sim f_{\rm NL}S(k_1,k_2,k_3)P_\zeta^2$$

How easy/hard to be detected ← *size* ($f_{\rm NL}$)

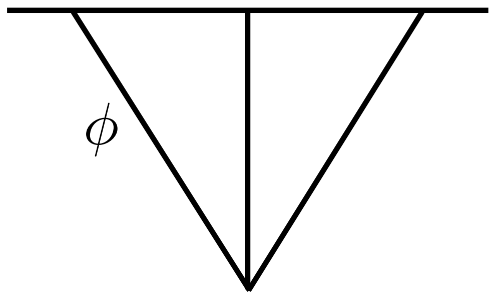
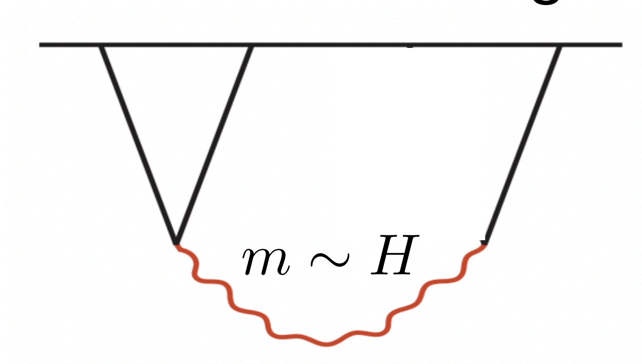
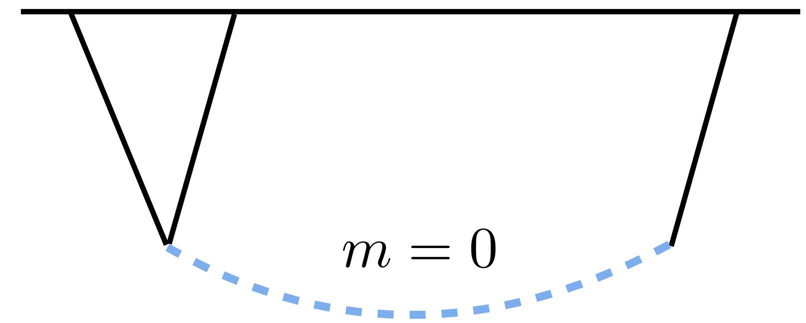
shape ($S(k_1,k_2,k_3)$)

Lots of information;

Main focus of this talk

Major target of CMB, LSS, 21cm..

Classification of Primordial Non-Gaussianity

Contact	Massive Exchange	Massless Exchange
		
single field inflation	cosmological collider	multi-field inflation

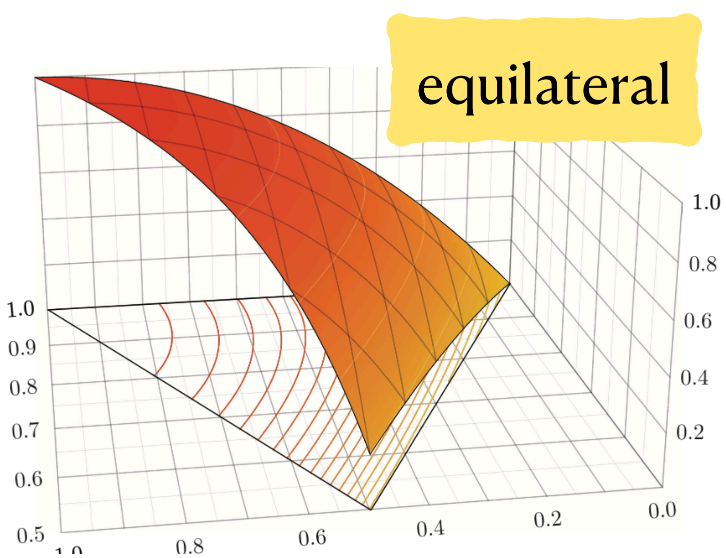
$m \gg H$ $\quad$ H $\quad$ $m \ll H$

mass ←

integrate-out

=>EFT

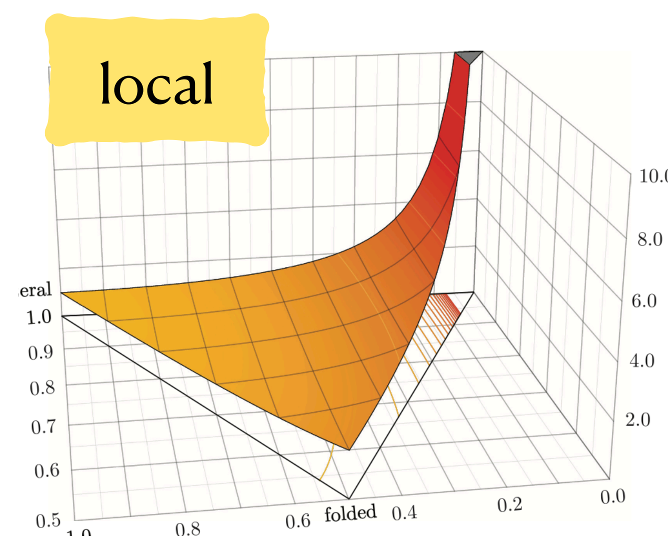
isocurvature modes



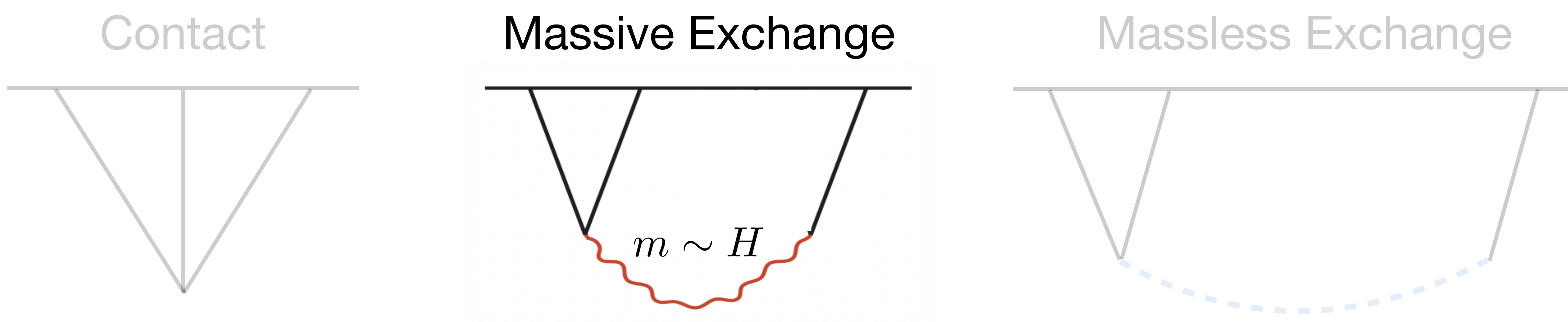
Assuming:

1) (nearly) scale-invariant

2) weakly coupled (tree-level leading order)



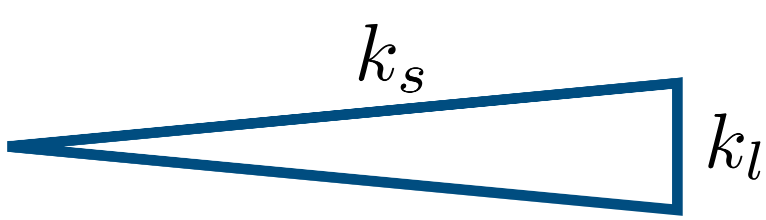
Classification of Primordial Non-Gaussianity



Cosmological Colliders

Squeezed limit of the inflationary bispectrum

Arkani-Hamed, Maldacena 2015
Chen, Wang 2009
Baumann, Green 2010
Noumi et al 2012
……



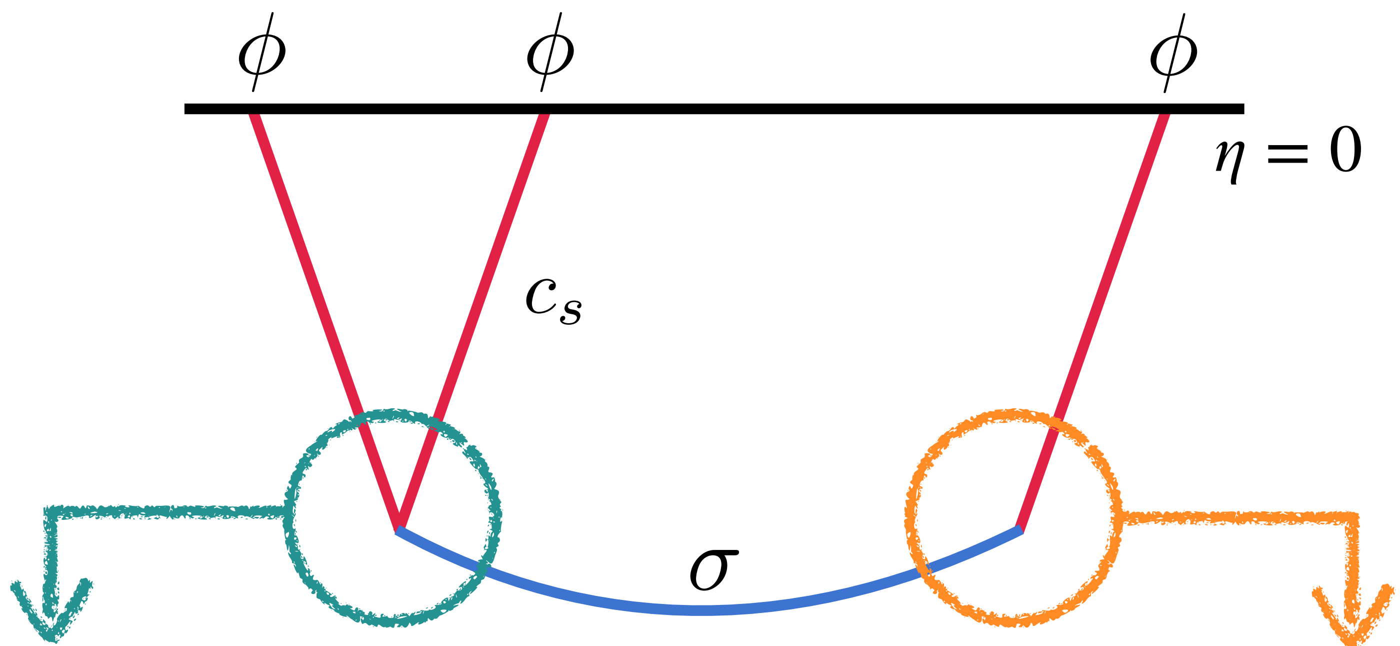
contains information of heavy particles in the high energy environment of inflation

$$\lim_{k_l \to 0} \langle \zeta_{\mathbf{k_l}} \zeta_{\mathbf{k_s}} \zeta_{\mathbf{k_s}} \rangle \propto \left(\frac{k_l}{k_s}\right)^{3/2} \cos\left[\frac{m}{H} \ln\left(\frac{k_l}{k_s}\right) + \delta\right] P_s(\mathbf{k_l} \cdot \mathbf{k_s})$$

oscillation measures mass, **angular** dependence measures spin

Boostless Cosmological Collider Bootstrap

Pimentel, DGW 2022



All possible $\phi\phi\sigma$ cubic interactions

$m > 0, s \geq 0$

c_σ

All possible $\phi\sigma$ quadratic interactions

$\dot{\phi}^2\sigma,\ (\partial_i\phi)^2\sigma,\ \ddot{\phi}^2\sigma, \ldots$	scalar exchange	$\dot{\phi}\sigma,\ \partial^2\phi\sigma,\ \ddot{\phi}\sigma, \ldots$
$\dot{\phi}\partial_{ij..}\dot{\phi}\sigma_{ij..}, \ldots$	spinning exchange	$\partial_{ij..}\dot{\phi}\sigma_{ij..}, \ldots$

One Formula to Find Them All: $\langle \phi_{k_1}\phi_{k_2}\phi_{k_3} \rangle \sim P_s(\cos\theta)\ \mathcal{W} \cdot \mathcal{U} \cdot \hat{\mathcal{I}}^{(n)}$

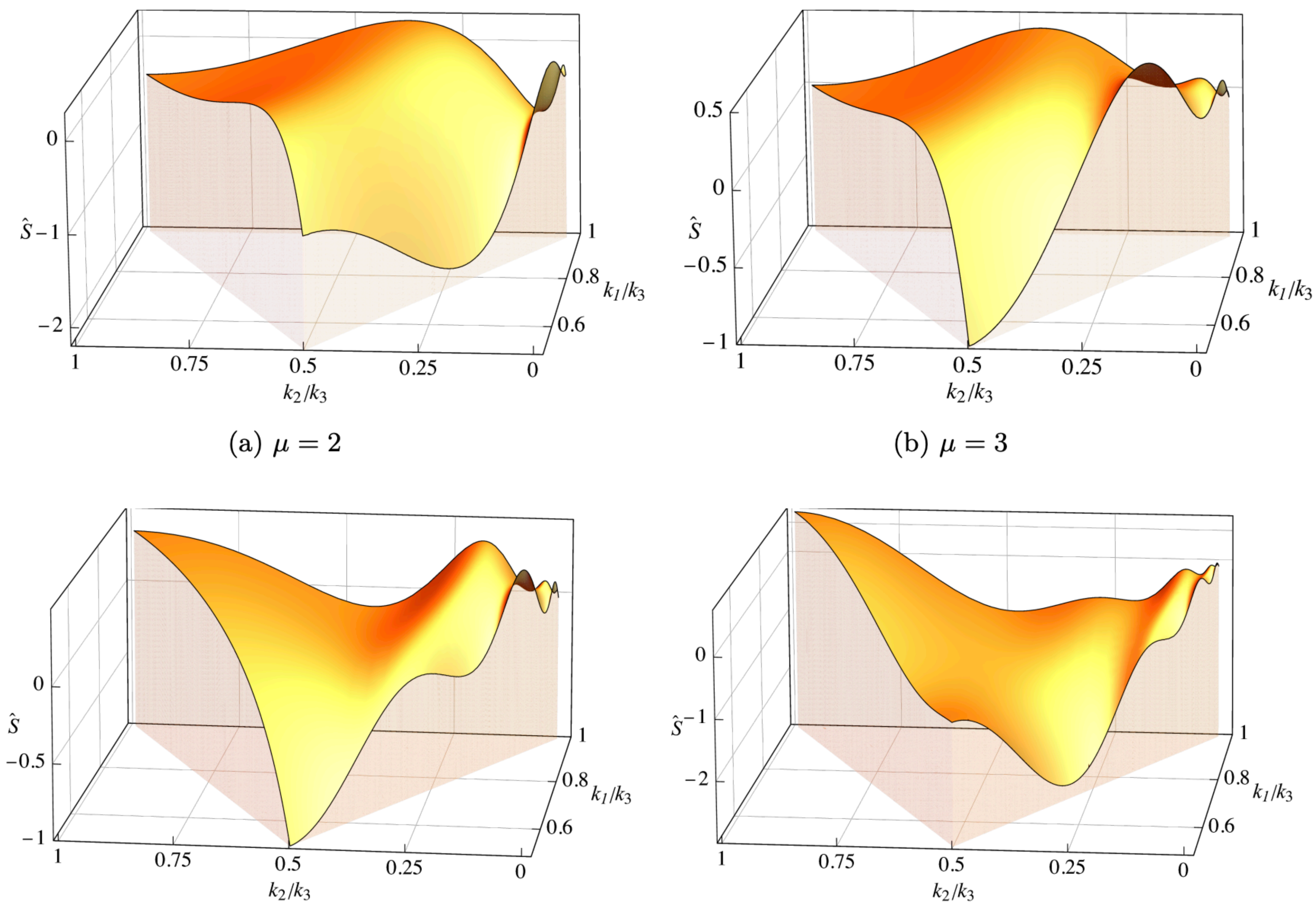
Weight-shifting ($\mathcal{W}$)
Spin-raising ($\mathcal{U}$)
Scalar Seeds ($\hat{\mathcal{I}}^{(n)}$)

Collider Signals in the Bispectrum

- ☑ Oscillations in the squeezed limit (massive scalar exchange)
- ☑ Angular dependence from spinning exchange

Spin-2 Template

$$S^{\text{spin}-s}_{\text{col.}}(k_1,k_2,k_3) = P_s(\hat{\mathbf{k}}_2\cdot\hat{\mathbf{k}}_3)k_2^{s-1}k_3^{-s}\ \mathcal{W}^s_{12}\ \mathcal{D}^{(s)}_{23}\ \left[k_3\hat{\mathcal{I}}^{(s)}\right] + 5 \text{ perms.}$$



(a) $\mu = 2$ (b) $\mu = 3$

(c) $\mu = 4$ (d) $\mu = 5$

Collider Signals in the Bispectrum

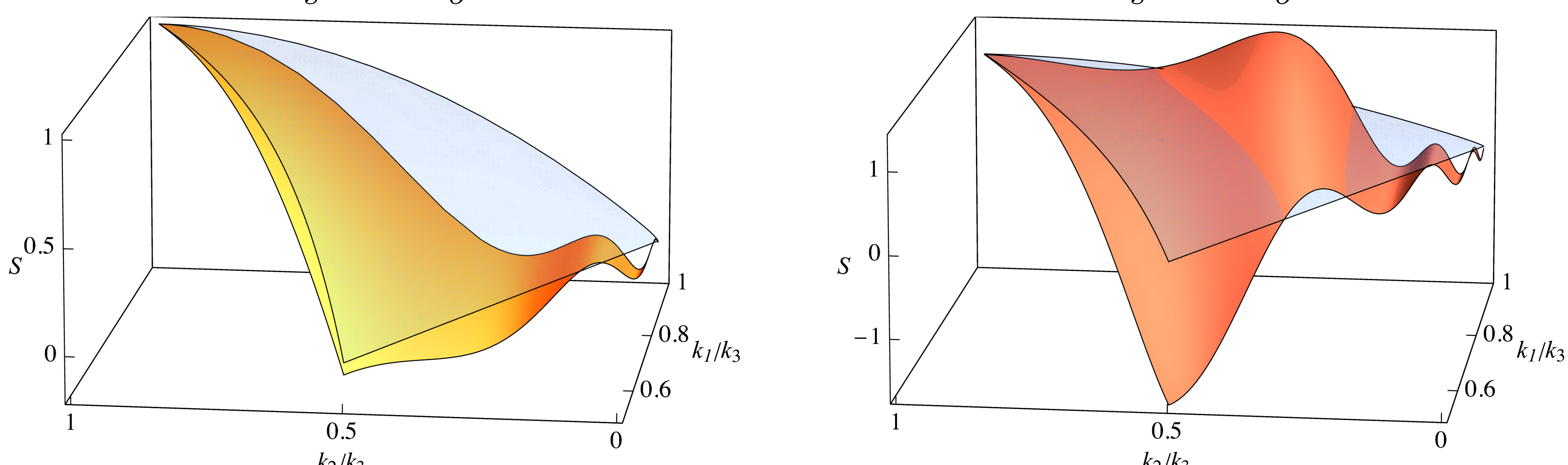
- ☑ Oscillations in the squeezed limit (massive scalar exchange)
- ☑ Angular dependence from spinning exchange
- ☑ New pheno from nontrivial sound speeds

For a special case $c_\sigma \ll c_s$: Collider signals outside of squeezed limit.

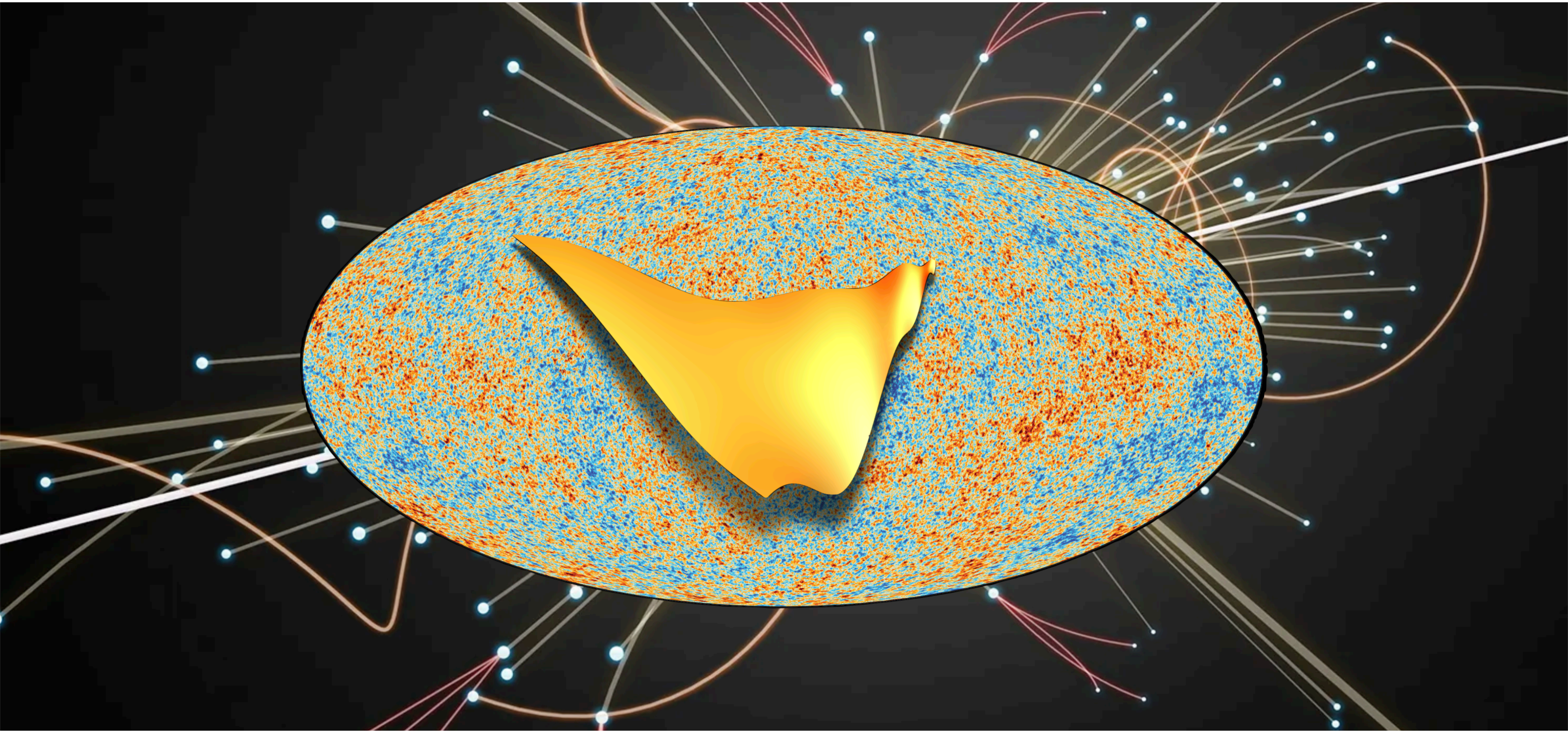
The Equilateral Collider Shape

$$S^{\text{eq.col.}}(k_1, k_2, k_3) = \frac{k_1 k_2}{(k_1+k_2)^2} \left(\frac{k_3}{k_1+k_2} \right)^{1/2} \cos \left[\mu \log \left(\frac{c_\sigma k_3}{2c_s(k_1+k_2)} \right) + \delta \right] + \text{perms.}$$

$c_s = 10 c_\sigma$ $\qquad$ $c_s = 20 c_\sigma$



Searching for Cosmological Collider in the Planck CMB data

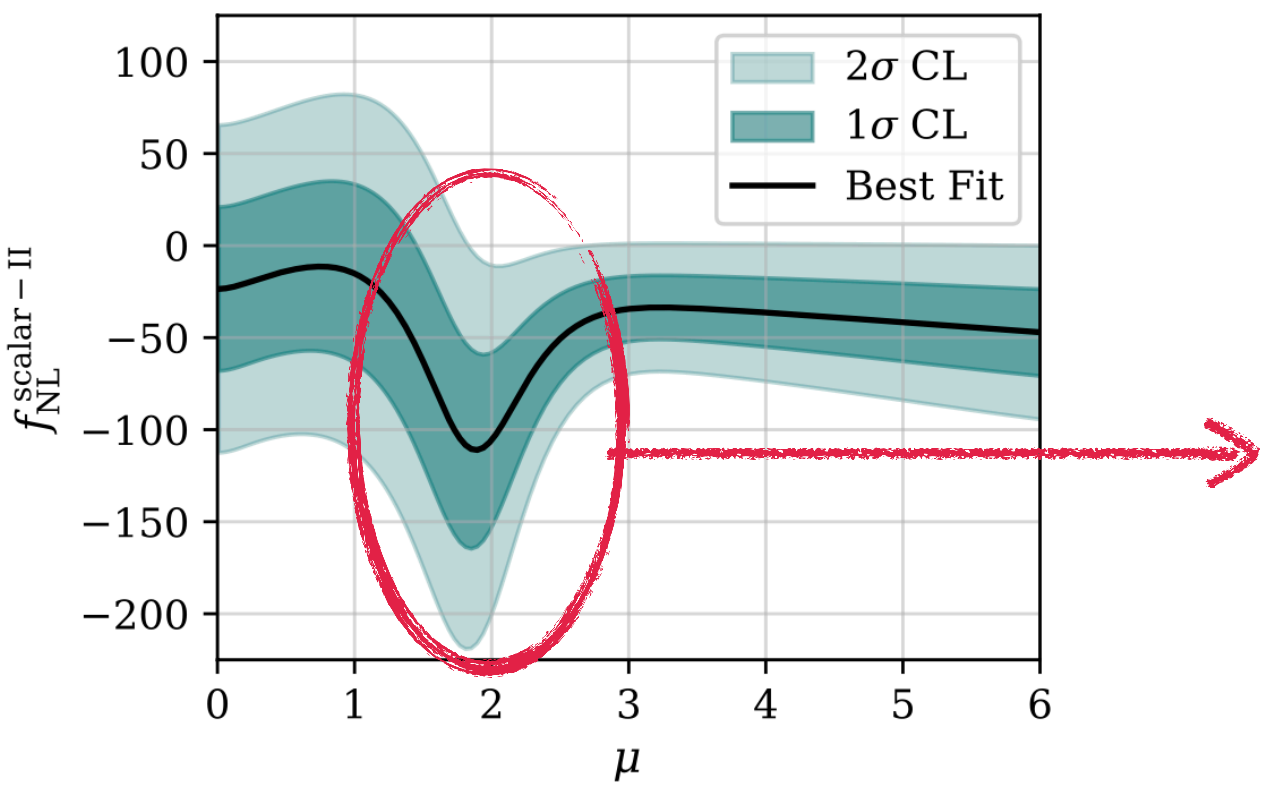


Sohn, DGW, Fergusson, Shellard

arXiv:2404.07302

The First Observational Test of Collider Signals

- **Oscillations in the squeezed limit** — massive scalar exchange



maximum significance

$$f_{\rm NL} = -91 \pm 40$$
$$\sigma = 2.3 \text{ for } \mu = 2.13$$

(adjusted significance $\sigma = 1.8$)

For comparison:

Planck constraint on the orthogonal shape

$$f_{\rm NL}^{\rm ortho} = -38 \pm 24$$
$$\sigma = 1.34$$

We have improved the constraints.

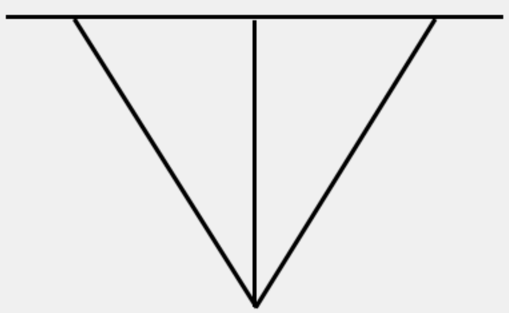
Stay Tuned!

Suman, DGW, Sohn, Fergusson, Shellard
2508.xxxxx

Bootstrapped, and Tested in Real Data!

☑ single field inflation

Contact



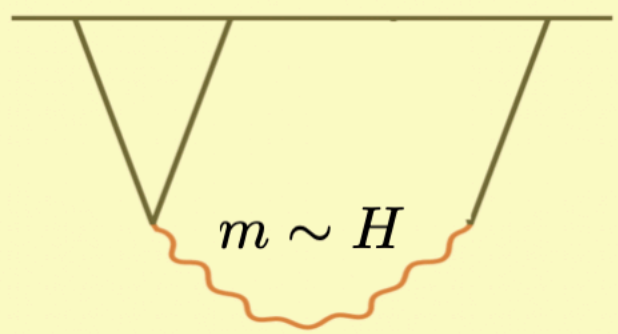
self-interaction of the inflaton

Boostless Bootstrap

Pajer 2020; Bonifacio, Pajer, DGW 2021

☑ cosmological colliders

Massive Exchange



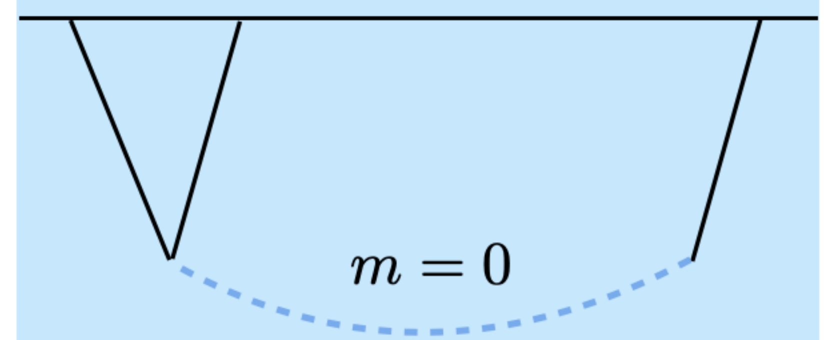
intermediate massive particles

Boostless Cosmological Collider Bootstrap

Pimentel, DGW 2022

☑ multi-field inflation

Massless Exchange



additional light scalars

multi-field Bootstrap

DGW, Pimentel, Achucarro 2022

The Field-Theoretic Wavefunction

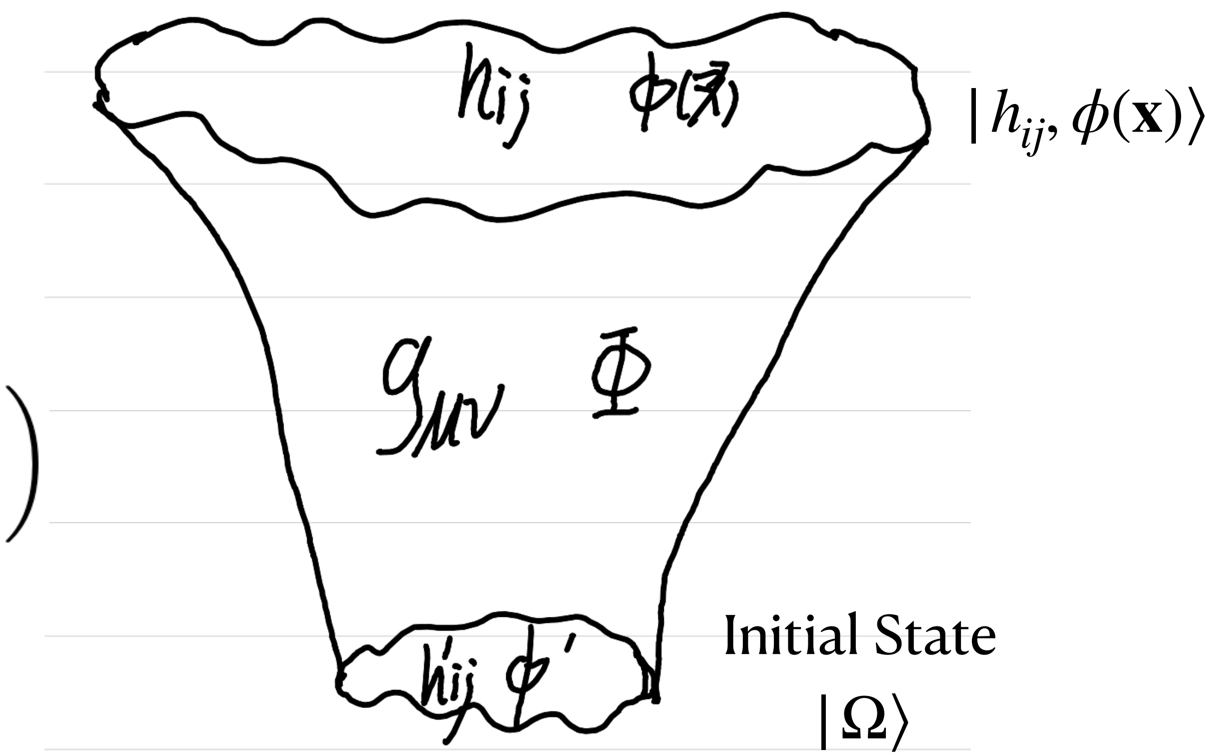
Scattering Amplitudes in de Sitter

The Wave Function of the Universe

Wheeler, De Witt
Hartle, Hawking
Vilenkin
...

$$\Psi[h_{ij}, \phi] = \langle h_{ij}, \phi | \Omega \rangle$$

$$\equiv \int \mathcal{D}g_{\mu\nu} \mathcal{D}\Phi \ \exp\left(\frac{i}{\hbar} S[g_{\mu\nu}, \Phi] \right)$$



extensively studied in *quantum cosmology, no-boundary proposal, Euclidean quantum gravity, holography, etc*.

Field-Theoretic Wavefunction ($G_N \to 0$)

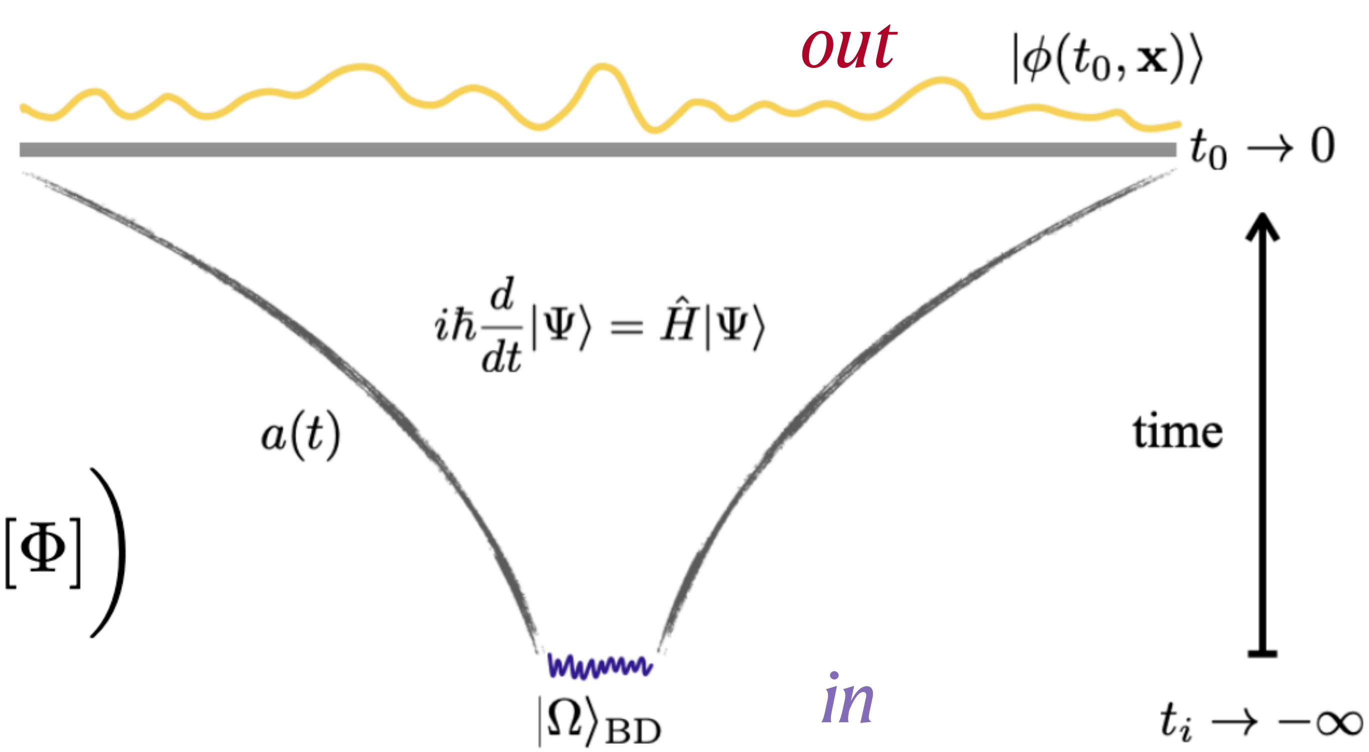
Quantum field theory in a fixed spacetime background

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \frac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{H^2\eta^2}$$

out ← → in

$$\Psi[\phi(\mathbf{x}), t_0] \equiv \langle \phi(\mathbf{x}) | \Omega \rangle$$

$$= \int_{\substack{\Phi(t_0)=\phi \\ \Phi(-\infty)=0}} \mathcal{D}\Phi \exp\left(\frac{i}{\hbar} S[\Phi]\right)$$



Wavefunction Coefficients

$$\Psi[\phi] = \exp\left[\frac{1}{2}\int_{\mathbf{k}_1,\mathbf{k}_2} \underbrace{\psi_2(\mathbf{k}_1,\mathbf{k}_2)}_{\text{Free}} \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2} + \sum_{n=3}^{\infty}\frac{1}{n!}\int_{\mathbf{k}_1,\ldots,\mathbf{k}_n} \underbrace{\psi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)}_{\text{Interaction}} \phi_{\mathbf{k}_1}\cdots\phi_{\mathbf{k}_n}\right]$$

Field-Theoretic Wavefunction: perturbation theory

Classical Way:

- Saddle-point approximation

$$\Psi[\phi] \simeq \exp\left(\frac{i}{\hbar} S[\Phi_{\rm cl}]\right)$$

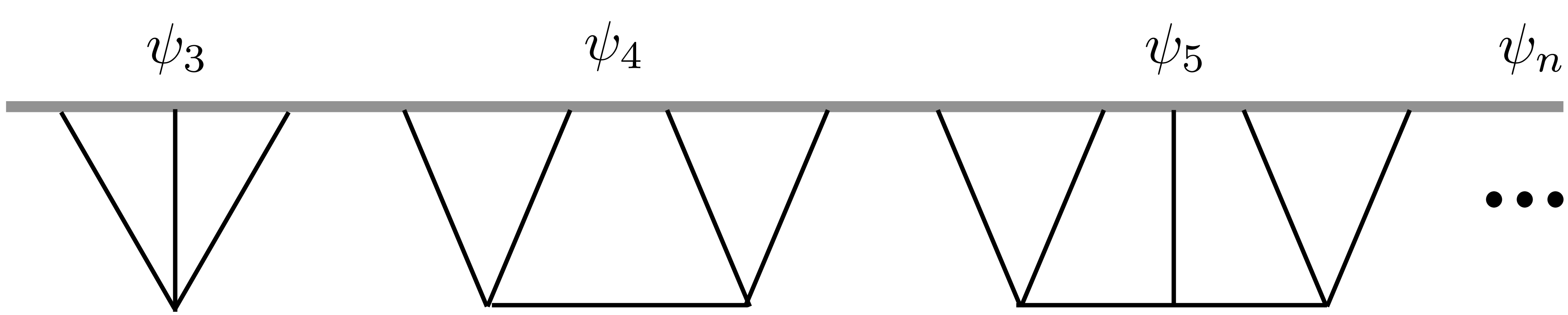
On-shell condition

bulk-to-boundary

$$(\Box - m^2)\Phi = -\frac{1}{\sqrt{-g}}\frac{\delta S_{\rm int}}{\delta \Phi} \Rightarrow \Phi_{\rm cl}(\eta, \mathbf{k}) = \phi_{\mathbf{k}} K(k,\eta) + \frac{i}{\hbar}\int d\eta' G(k;\eta,\eta') \frac{\delta S_{\rm int}}{\delta \Phi_{\mathbf{k}}(\eta')}\bigg|_{\Phi=\Phi_{\rm cl}}$$

bulk-to-bulk

All the tree-level wavefunction coefficients:



$\hbar^{-1}$

(semi-)classical wavefunction coefficients

Field-Theoretic Wavefunction: perturbation theory

Cespedes, Davis, DGW 2023

Quantum Way:

- Functional Quantization

$$\Psi[\phi(\mathbf{x})] = \int_{\substack{\Phi(t_0)=\phi \\ \Phi(-\infty)=0}} \mathcal{D}\Phi \exp\left(\frac{i}{\hbar}S[\Phi]\right)$$

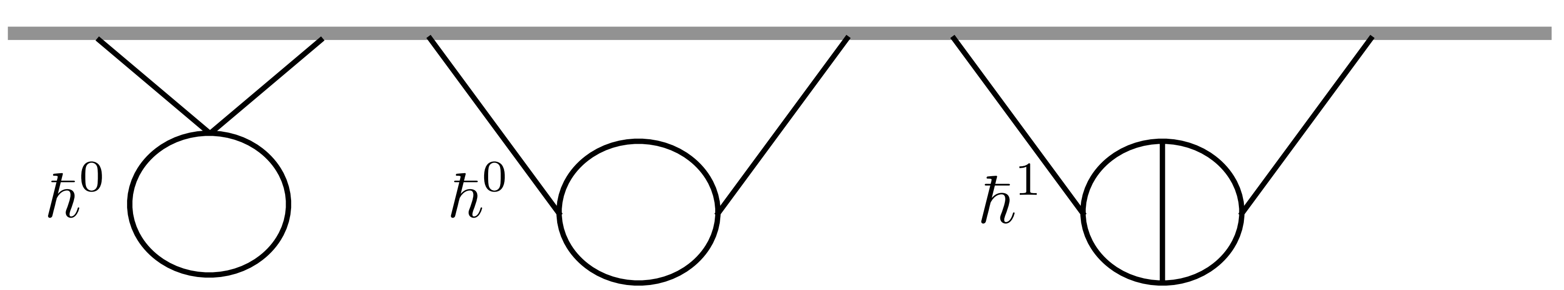
- Propagators

$$\langle \Phi_{\mathbf{k}}(\eta)\Phi_{\mathbf{k}'}(\eta')\rangle \equiv \frac{\int \mathcal{D}\Phi \Phi_{\mathbf{k}}(\eta)\Phi_{\mathbf{k}'}(\eta') \exp\left(\frac{i}{\hbar}S_0[\Phi]\right)}{\int \mathcal{D}\Phi \exp\left(\frac{i}{\hbar}S_0[\Phi]\right)} = G(k,\eta,\eta')(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

$$\langle \Pi_{\mathbf{k}}(\eta_0)\Phi_{\mathbf{k}'}(\eta)\rangle = 2i\hbar K(k,\eta)(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

Both tree-level & loop-level wavefunction coefficients:

$$\psi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n) \equiv \left.\frac{\delta^n\Psi[\phi]}{\delta\phi_{\mathbf{k}_1}\ldots\delta\phi_{\mathbf{k}_n}}\right|_{\phi=0} = \frac{(i/2\hbar)^n}{\Psi[0]}\int \mathcal{D}\Phi \Pi_{\mathbf{k}_1}(\eta_0)\ldots\Pi_{\mathbf{k}_n}(\eta_0)\exp\left(\frac{i}{\hbar}S[\Phi]\right)$$



quantum wavefunction coefficients

From Wavefunction to Correlators

Born rule:

$$\langle \phi_{\mathbf{k}_1} \cdots \phi_{\mathbf{k}_n} \rangle = \frac{\int \mathcal{D}\phi \;\; \phi_{\mathbf{k}_1} \cdots \phi_{\mathbf{k}_n} \left|\Psi[\phi, \eta_0]\right|^2}{\int \mathcal{D}\phi \;\; \left|\Psi[\phi, \eta_0]\right|^2}$$

Tree-level examples:

$$\langle \phi_{\mathbf{k}} \phi_{-\mathbf{k}} \rangle' = -\frac{1}{2\,\mathrm{Re}\,\psi_2'(k)}\,,$$

$$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3} \rangle' = -\frac{\mathrm{Re}\,\psi_3'(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)}{4\mathrm{Re}\,\psi_2'(k_1)\;\mathrm{Re}\,\psi_2'(k_2)\;\mathrm{Re}\,\psi_2'(k_3)}\,.$$

$$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3} \phi_{\mathbf{k}_4} \rangle' = \frac{\mathrm{Re}\,\psi_4'(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)}{8\prod_{a=1}^4 \mathrm{Re}\,\psi_2'(k_a)} - \frac{1}{8\prod_{a=1}^4 \mathrm{Re}\,\psi_2'(k_a)}\left[\frac{\mathrm{Re}\,\psi_3'(\mathbf{k}_1, \mathbf{k}_2, \mathbf{s})\,\mathrm{Re}\,\psi_3'(-\mathbf{s}, \mathbf{k}_3, \mathbf{k}_4)}{\mathrm{Re}\,\psi_2'(s)} + ..\right]$$

Analogy to Flat Spacetime

Wavefunction Coefficients ↔ Scattering Amplitudes

Cosmological Correlators ↔ Cross Section

A simpler object to look at..

$$\Psi[\phi] = \exp\left[\frac{1}{2}\int_{\mathbf{k}_1,\mathbf{k}_2} \psi_2(\mathbf{k}_1,\mathbf{k}_2)\ \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2} + \sum_{n=3}^{\infty}\frac{1}{n!}\int_{\mathbf{k}_1,\ldots,\mathbf{k}_n} \psi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)\ \phi_{\mathbf{k}_1}\cdots\phi_{\mathbf{k}_n}\right]$$

- dS/CFT

Maldacena 2002
Harlow, Stanford 2011

$$\psi_n \quad \leftrightarrow \langle \mathcal{O}_{\Delta_1} \ldots \mathcal{O}_{\Delta_n} \rangle$$ (3d Euclidean CFT Correlators)

- Unitarity: Cosmological Optical Theorem & Cutting Rules

Goodhew, Jazayeri, Pajer 2020
Melville, Pajer 2021

- Kinematic Flow / Differential Equation

Arkani-Hamed, Baumann, Joyce, Lee, Pimentel 2023

- Parity: Cosmological Reality & CPT Theorem

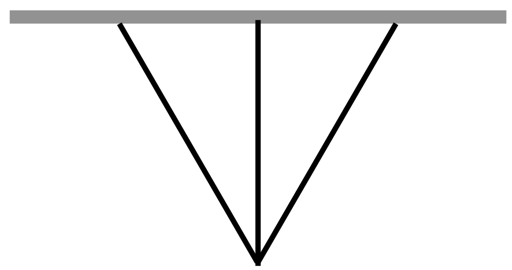
Goodhew, Thavengsan, Wall 2024
Stefanyszyn, Tong, Zhu 2023, 2024

- Infrared Divergences in de Sitter

Gorbenko, Senatore 2019
Cespedes, Davis, DGW 2023

IR Divergences in de Sitter Space

Case study: massless Φ^3 interaction in perturbation theory

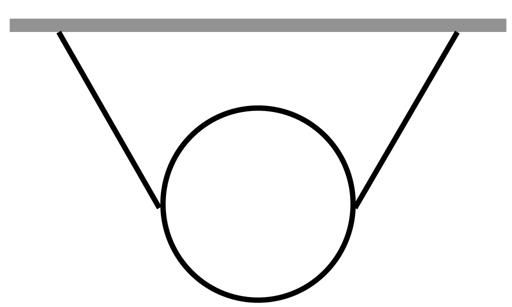


massless modes freeze after horizon-exit

cumulative interactions => logarithmic secular growth

$$\langle \phi_{k_1}\phi_{k_2}\phi_{k_3}\rangle' \sim i\int_{-\infty}^{\eta_0} \frac{\mathrm{d}\eta}{\eta^4}\left[(1-ik_1\eta)(1-ik_2\eta)(1-ik_3\eta)e^{ik_T\eta} - c.c.\right] + \text{perm.}$$

$$= \frac{H^2}{2k_1^3k_2^3k_3^3}\left[(k_1^3+k_2^3+k_3^3)\left(\gamma_E - 1 + \log(-k_t\eta_0)\right) + 4e_3 - e_2k_t\right]$$



$$\langle \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\rangle'_{1-\text{loop}} \sim \log(kL)\log(-2k\eta_0)^2$$

Are they really divergent? Do we need to worry about higher loops?

IR Divergences & the Wavefunction of the Universe

$$P = |\Psi|^2$$

Stochastic Formalism

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V_\phi}{3H} P\right) + \frac{H^3}{8\pi^2}\left(\frac{\partial^2 P}{\partial \phi^2}\right)$$

- Fokker-Planck eq.
- non-perturbative
- equilibrium behaviour

Cosmological Bootstrap

$$\Psi[\phi] = \exp\left[\sum_{n=2}^{\infty} \frac{1}{n!} \int_{\mathbf{k}_1,\ldots,\mathbf{k}_n} \psi_n(\mathbf{k}_1, \ldots, \mathbf{k}_n)\, \phi_{\mathbf{k}_1} \cdots \phi_{\mathbf{k}_n}\right]$$

- related to correlators
- perturbative
- secular growth

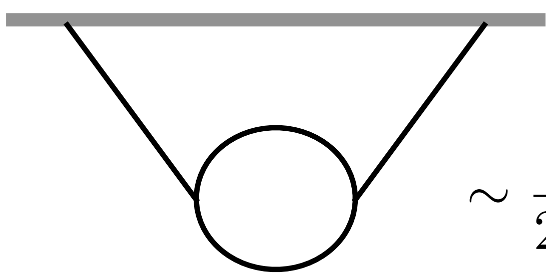
?

Cespedes, Davis, DGW 2023
Cespedes, DGW, *Phys. Rep.* 202x

Classical Loops: leading IR logs in correlators

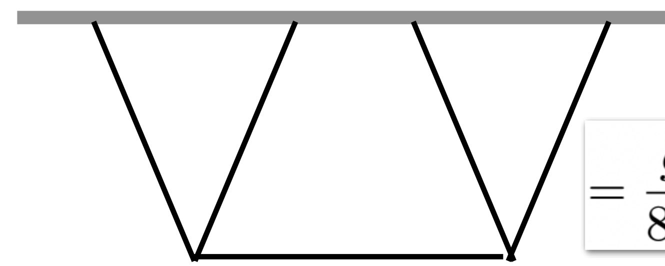
From Wavefunction to Correlators: One-Loop Example for $\frac{g}{3!}\Phi^3$

$\psi_2^{1-\text{loop}}$



$$\sim \frac{g^2}{2k^3}\log(-2k\eta_0)$$

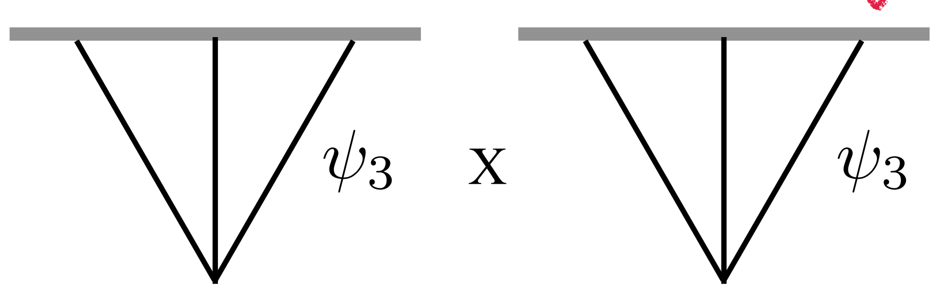
ψ_4



$$= \frac{g^2}{8k^3}\frac{-5}{18\pi^2}\log(kL)\log(-2k\eta_0)^2$$

$$\langle \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\rangle'_{1-\text{loop}} = \frac{\mathrm{Re}\,\psi'^{1-\text{loop}}_{\mathbf{k}_1\mathbf{k}_2}}{2\mathrm{Re}\,\psi'_2(k_1)\mathrm{Re}\,\psi'_2(k_2)} - \frac{1}{8\mathrm{Re}\,\psi'_2(k_1)\mathrm{Re}\,\psi'_2(k_2)}\int_{\mathbf{p}}\frac{\mathrm{Re}\,\psi'_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{p},-\mathbf{p})}{\mathrm{Re}\,\psi'_2(p)}$$
$$+\frac{1}{8\mathrm{Re}\,\psi'_2(k_1)\mathrm{Re}\,\psi'_2(k_2)}\int_{\mathbf{p}}\left[\frac{\mathrm{Re}\,\psi'_3(\mathbf{k}_1,\mathbf{p},-\mathbf{p}-\mathbf{k}_1)\mathrm{Re}\,\psi'_3(\mathbf{k}_2,-\mathbf{p},\mathbf{p}+\mathbf{k}_1)}{\mathrm{Re}\,\psi'_2(p)\mathrm{Re}\,\psi'_2(|\mathbf{p}+\mathbf{k}_1|)} + \frac{\mathrm{Re}\,\psi'_3(\mathbf{k}_1,\mathbf{k}_2,-\mathbf{k}_1-\mathbf{k}_2)\mathrm{Re}\,\psi'_3(\mathbf{k}_1+\mathbf{k}_2,\mathbf{p},-\mathbf{p})}{\mathrm{Re}\,\psi'_2(p)\mathrm{Re}\,\psi'_2(|\mathbf{k}_1+\mathbf{k}_2|)}\right]$$

Classical Loops



ψ_3 X ψ_3

$$= \frac{g^2}{8k^3}\frac{1}{3\pi^2}\log(kL)\log(-2k\eta_0)^2$$

IR Loops: Wavefunction is IR safe

Gorbenko, Senatore 2019

L-loop *n*-point wavefunction coefficient:

$$\psi_n^{L-\text{loop}} \sim \int d\eta_1 ... d\eta_m a(\eta_1)^4 ... a(\eta_m)^4 K(k_1, \eta_1) ... K(k_n, \eta_m)$$
$$\int_{\mathbf{p}_1,...,\mathbf{p}_L} G(p_1, \eta_a, \eta_b) ... G(p_L, \eta_c, \eta_d) G(|\mathbf{p}_\mathbf{x} + \mathbf{k}_\mathbf{y}|, \eta_e, \eta_f) ...$$

Bulk-to-bulk propagator at IR:

$$\lim_{p \to 0} G(p, \eta, \eta') = -\frac{i}{6} H^2 (\eta^3 + \eta'^3) + \mathcal{O}(p)$$

Momentum integration from loops:

$$\int_\mathbf{p} \frac{1}{p^n} = \frac{1}{(2\pi)^3} \int_{1/L}^{\Lambda} 4\pi p^{2-n} dp \xrightarrow{L \to \infty} \begin{cases} \text{IR-finite} \ , & n < 3 \\ \frac{1}{2\pi^2} \log(kL) \ , & n = 3 \ . \end{cases}$$

only secular divergence from time integrals,

no IR divergences from loop integrals

Classical Loops: leading IR logs in correlators

L-loop *n*-point correlator (with *V* vertices):

$$\langle\phi^n\rangle_{L-\text{loop}} \sim \frac{1}{(\text{Re}\psi_2)^n}\left[\text{Re}\ \psi_n^{L-\text{loop}} + \int_{\mathbf{p_1}} \frac{\text{Re}\psi_{n+2}^{(L-1)-\text{loop}}}{\text{Re}\psi_2(p_1)} + \ldots + \int_{\mathbf{p_1},\ldots,\mathbf{p_{L-1}}} \frac{\text{Re}\psi_{n+2(L-1)}^{1-\text{loop}}}{(\text{Re}\psi_2(p_1)\ldots\text{Re}\psi_2(p_{L-1}))}\right.$$
$$\left. + \int_{\mathbf{p_1},\ldots,\mathbf{p_L}} \frac{1}{\text{Re}\psi_2(p_1)\ldots\text{Re}\psi_2(p_L)}\left(\text{Re}\psi_{n+2L}^{\text{ex}} + \frac{\text{Re}\psi_{n+2L-1}^{\text{ex}}\text{Re}\ \psi_3}{\text{Re}\ \psi_2} + \ldots + \frac{(\text{Re}\ \psi_3)^V}{(\text{Re}\ \psi_2)^{3V-2L-n}}\right)\right]$$

$$\propto \lambda^V \log(kL_{\text{IR}})^L \log(-k\eta_0)^V$$

in agreement with
Baumgart, Sundrum 2019

IR-divergent correlators are always dominated by Classical loops

IR divergence ⟷ **Superhorizon physics** ⟷ **Semi-classical effect**

A tempting thought

saddle-point approx. ⟷ Fokker-Planck equation

$$\Psi[\phi] \simeq \exp\left(\frac{i}{\hbar} S[\Phi_{\rm cl}]\right)$$

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V_\phi}{3H} P\right) + \frac{H^3}{8\pi^2}\left(\frac{\partial^2 P}{\partial \phi^2}\right)$$

First, screen out the short wavelength modes:

$$P[\phi_l] = \int \mathcal{D}\phi \;\; \delta\left(\phi_l - \int \frac{d^3k}{(2\pi)^3} \Omega_k \phi_k\right) |\Psi(\phi_k)|^2$$

Then, let's check the perturbative regime of $\lambda\phi^4$, where the equilibrium has not been reached.

$$\text{coupling} \times \log^2 < 1$$

- controllable playground to explicitly match two computations
- could be interesting for pheno

Correlators from stochastic formalism

$$\langle\phi^n\rangle = \int d\phi\,\phi^n P(\phi) \qquad \frac{d}{dt}\langle\phi^n\rangle = \int d\phi\,\phi^n \frac{P(\phi,t)}{dt} = -\frac{n}{3H}\langle\phi^{n-1}V_\phi\rangle + \frac{n(n-1)H^3}{8\pi^2}\langle\phi^{n-2}\rangle$$

a set of diff. eqs. that can be solved perturbatively

$$\begin{cases} \frac{d}{dt}\langle\phi^2\rangle = -\frac{1\lambda}{9H}\langle\phi^4\rangle + \frac{H^3}{4\pi^2} \\ \frac{d}{dt}\langle\phi^4\rangle = -\frac{2\lambda}{9H}\langle\phi^6\rangle + \frac{3H^3}{4\pi^2}\langle\phi^2\rangle \\ \vdots \\ \frac{d}{dt}\langle\phi^n\rangle = -\frac{n\lambda}{18H}\langle\phi^{n+2}\rangle + \frac{n(n-1)H^3}{8\pi^2}\langle\phi^{n-2}\rangle \end{cases}$$

$$\langle\phi^2\rangle = \frac{H^2}{4\pi^2}\log a - \frac{\lambda H^4}{144\pi^4}(\log a)^3 + \frac{\lambda^2 H^6}{2880\pi^6}(\log a)^5 + \mathcal{O}(\lambda^3(\log a)^7)$$

Free theory — one-loop — two-loop — ......

Gaussian variance

classical loops from tree-level wavefunction coefficients

quantum loops are absent

Beyond perturbation theory

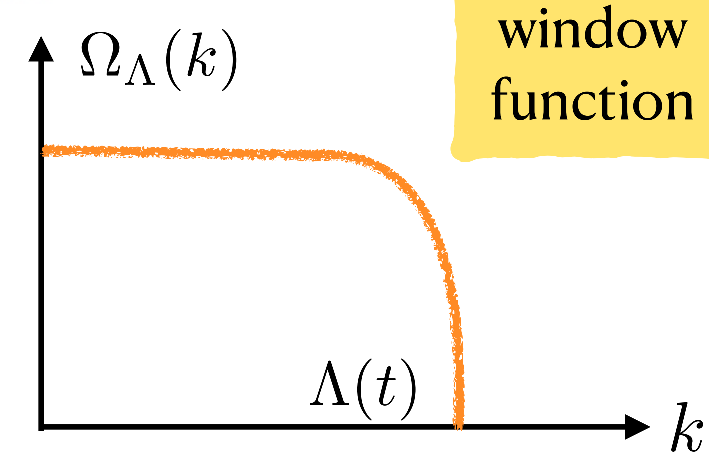
Cespedes, Davis, DGW 2023

Coarse-grained probability distribution

$$P_\Lambda[\phi, t] = |\Psi_\Lambda[\phi, t]|^2 = e^{W_0[\phi] + W_I[\phi]}$$

free part $$W_0 = \int_{\mathbf{k}} \mathrm{Re}\ \psi_2(k)\Omega_\Lambda^{-1}(k)\ \phi_{\mathbf{k}}\phi_{-\mathbf{k}}\ ,$$

interacting part $$W_I = \sum_{n=3}^{\infty} \frac{1}{n!} \int_{\mathbf{k}_1,\ldots,\mathbf{k}_n} 2\mathrm{Re}\ \psi_n^\Lambda(\mathbf{k}_1, \ldots, \mathbf{k}_n, t)\ \phi_{\mathbf{k}_1} \cdots \phi_{\mathbf{k}_n}\ .$$



window function

$$\frac{d}{dt} P_\Lambda[\phi, t] = \frac{\partial}{\partial t} P_\Lambda[\phi, t] + \dot{\Lambda} \frac{\partial}{\partial \Lambda} P_\Lambda[\phi, t]$$

Schrodinger Eq.

RG flow?

drift term $$\frac{\partial}{\partial \phi}\left(\frac{V'}{3H} P_\Lambda\right)$$

?

Fokker-Planck = Schrodinger + Polchinski

Cespedes, Davis, DGW 2023

The probability conservation should be unaffected by varying the artificial cutoff

$$\frac{d}{d\ln\Lambda}\int\mathcal{D}\phi\ P_\Lambda[\phi,t]=\int\mathcal{D}\phi\ \left[\frac{dW_0}{d\ln\Lambda}e^{W_I}+\frac{de^{W_I}}{d\ln\Lambda}\right]e^{W_0}=0$$

Polchinski's equation of Exact RG (modified version for *semi-classical wavefunction*)

$$e^{W_0}\frac{de^{W_I}}{d\ln\Lambda}=\frac{1}{4}\int_{\mathbf{k}}\frac{d\Omega_\Lambda}{d\ln\Lambda}\frac{1}{\mathrm{Re}\psi_2}\left[\left(\frac{\delta^2}{\delta\phi_{\mathbf{k}}\delta\phi_{-\mathbf{k}}}e^{W_I}\right)e^{W_0}-2\frac{\delta}{\delta\phi_{\mathbf{k}}}\left(e^{W_0}\frac{\delta e^{W_I}}{\delta\phi_{-\mathbf{k}}}\right)\right]$$

diffusion term

$$\dot{\Lambda}\frac{\partial}{\partial\Lambda}P_\Lambda[\phi,t]=\frac{H^3}{8\pi^2}\frac{\delta^2 P_\Lambda}{\delta\phi_l^2}$$

Fokker-Planck equation

$$\frac{dP_\Lambda}{dt}=\frac{\partial}{\partial\phi}\left(\frac{V'}{3H}P_\Lambda\right)+\frac{H^3}{8\pi^2}\frac{\partial^2 P_\Lambda}{\partial\phi^2}$$

drift: time flow **diffusion: RG flow**

Bridge the Gap!

High Energy Physics

- scattering amplitudes
- holography
- CFT
- positivity
- Non-pert. QFT
- strings
- …

New theoretical methods

- Bootstrap
- Wavefunctional

Primordial Cosmology

- Physics of Inflation
- Observational tests of new physics

A lot to be done.

We just get started…