Quantum field theory at finite temperature

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CONTENT

what is quantum field theory at finite temperature

QFT@$T = 0$

quantum mechanics+ relativity $\rightarrow$ QFT at zero temperature

QFT@$T \neq 0$

quantum mechanics+ relativity+statistics $\rightarrow$ QFT at finite temperature

why we need QFT at finite temperature:useful

Early universe in cosmology (focus on this application today)

- Warm inflation, reheating
- symmetry breaking at high energy,eg., Pecci-Quinn (axion), cosmic string...
- Electroweak phase transition/ spontaneously symmetry breaking
- dark matter production
- baryogenesis
- QCD phase transition
- .....

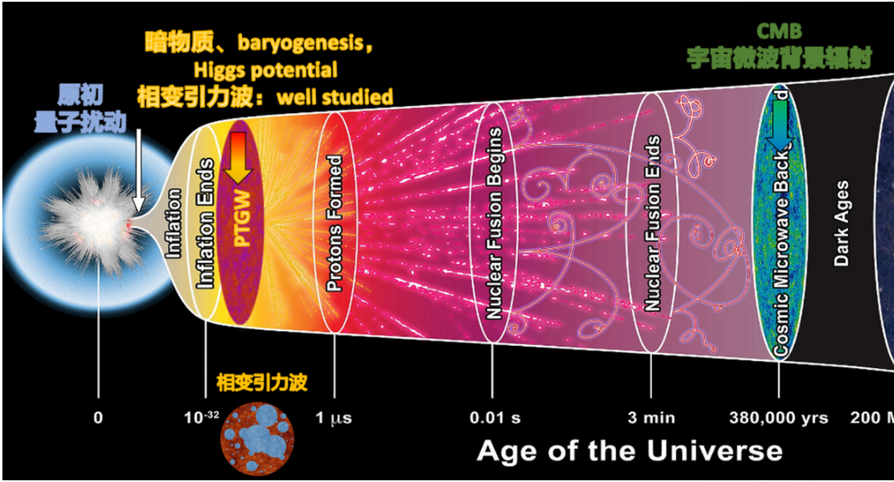
Astropysics

Heavy ion collisions

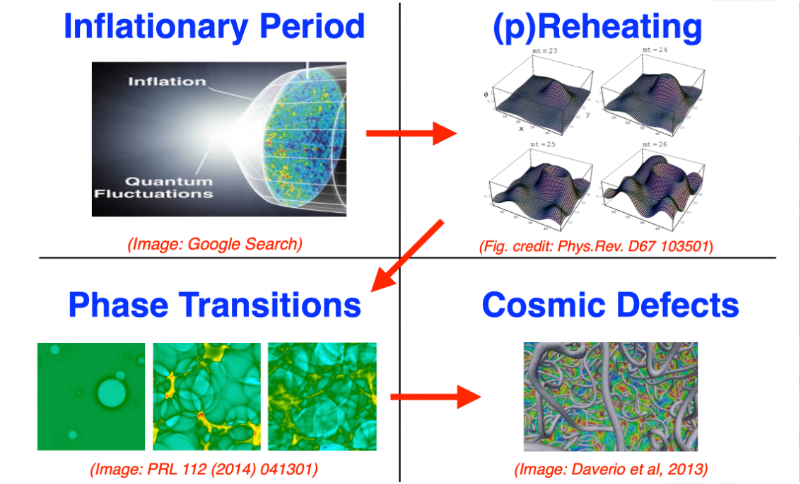
condensed matter physics

Quantum Chemistry and molecular systems

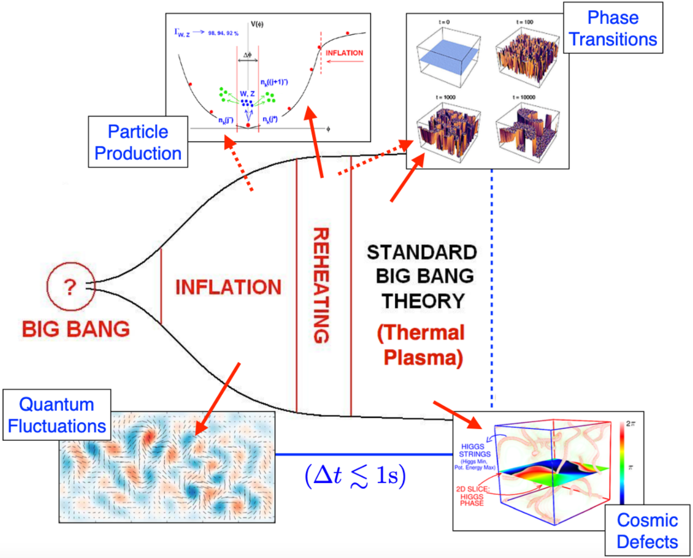
Application of QFT@T in the early universe



Application of QFT@T in the early universe



Application of QFT@T in the early universe



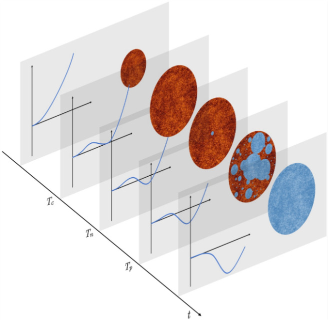
credit:D. Figueroa

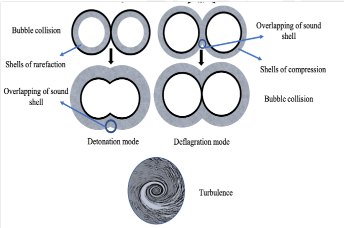
Application of QFT@T: cosmic phase transition and gravitational wave

finite-temperature effective potential using QFT@T: free energy density.

$$V_{\text{eff}}^{(1)}(\bar{\phi}) = \sum_i n_i \left[\int \frac{\mathrm{d}^D p}{(2\pi)^D} \ln\left(p^2 + m_i^2(\bar{\phi})\right) + J_{\mathrm{B,\ F}}\left(\frac{m_i^2(\bar{\phi})}{T^2}\right)\right]$$

$$S(T) = \int d^4x \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 + V_{\mathsf{eff}}(\phi, T)\right], \Gamma = \Gamma_0 e^{-S(T)}$$

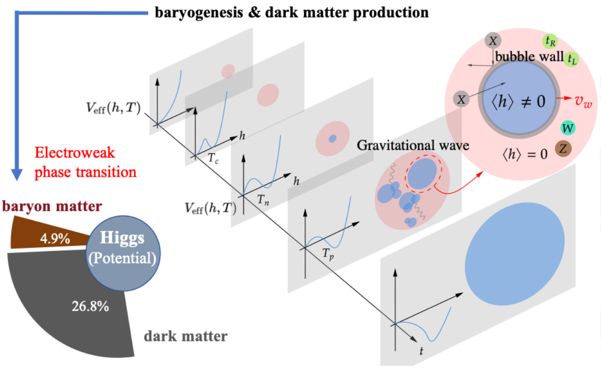




Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Application of QFT@T: dark matter and baryogenesis

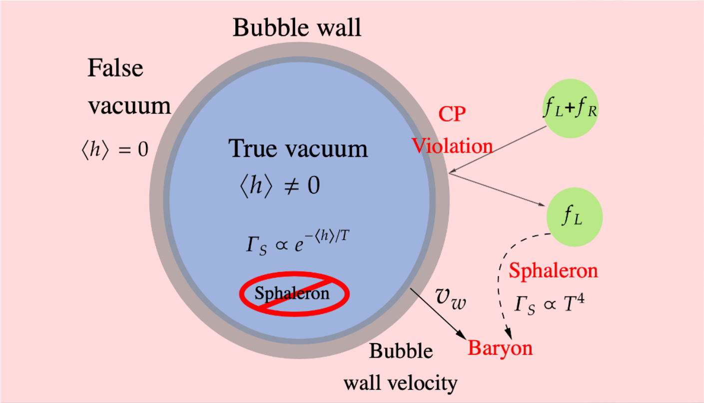
The observation of Higgs@LHC and GW@LIGO initiates a new era of exploring DM by GW. SFOPT by Higgs could provide a new approach for DM production. Higgs' deep connections to cosmology, such as EW baryogenesis, DM testable by GW signals.



The First Particles, FPH, arXiv: 2501.15543

Application of QFT@T: dark matter and baryogenesis

Bubble wall is a natural filter for baryon and DM production through particles scattering and diffusion in high temperature plasma around the boundary.



The First Particles, FPH, arXiv: 2501.15543

Basic of QFT@T:KMS relation

Kubo-Martin-Schwinger (KMS) relation:

$$\begin{aligned}
\left\langle A_H(t) B_H\left(t'\right)\right\rangle_\beta &= Z^{-1}(\beta) \operatorname{Tr} e^{-\beta \hat{H}} A_H(t) e^{\beta \hat{H}} e^{-\beta \hat{H}} B_H\left(t'\right) \\
&= Z^{-1}(\beta) \operatorname{Tr} A_H(t+i\beta) e^{-\beta \hat{H}} B_H\left(t'\right) \\
&= Z^{-1}(\beta) \operatorname{Tr} e^{-\beta \hat{H}} B_H\left(t'\right) A_H(t+i\beta) \\
&= \left\langle B_H\left(t'\right) A_H(t+i\beta)\right\rangle_\beta
\end{aligned}$$

KMS mixes the temperature and the imaginary time. Similar behavior occurs when considering the Hawking radiation of black holes.

Ensemble Average

$$\langle A\rangle = \frac{\mathrm{Tr}(A e^{-\beta H})}{\mathrm{Tr} e^{-\beta H}}$$

All thermodynamic quantities can be calculated from the partition function Z.

Partition Function in Equilibrium

Canonical Ensemble

$$Z = \mathrm{Tr}\, e^{-\beta H}$$

Grand Canonical Ensemble

$$Z = \mathrm{Tr}\, e^{-\beta(H - \mu_i Q_i)}$$

Ensemble Average

$$\langle A \rangle = \frac{\mathrm{Tr}(A e^{-\beta H})}{\mathrm{Tr} e^{-\beta H}}$$

All thermodynamic quantities can be calculated from Z.

Green's function at finite temperature

$$G\left(t,t';\vec{x},\vec{x'}\right)_\beta \equiv \langle\phi(x)\phi\left(x'\right)\rangle_\beta \qquad A_H(t)\to\phi(x)$$

$$B_H\left(t'\right)\to\phi\left(x'\right)$$

KMS:

$$G\left(t,t';\vec{x}-\vec{x'}\right)_\beta = G\left(t+i\beta,t';\vec{x}-\vec{x'}\right)_\beta$$

"Imaginary time":

$$t\longrightarrow i\tau$$

Imaginary time Greens functions:

$$\mathcal{G}_\beta\left(\tau;\vec{x}-\vec{x'}\right)\equiv G\left(0,i\tau;\vec{x}-\vec{x'}\right)_\beta$$

KMS:

$$\mathcal{G}_\beta(\tau;\vec{r}) = \mathcal{G}_\beta(\tau+\beta;\vec{r})$$

Propagator at finite temperature

Equations of motion:

$$\left(\Box + m^2\right) G\left(t, t'; \vec{x} - \vec{x'}\right) = -\delta^4(x)$$

Klein-Gordon operator

$$\left(\frac{\partial^2}{\partial \tau^2} + \nabla^2 - m^2\right) \mathcal{G}_\beta(\tau; \vec{r}) = -\delta^3(r)\delta(\tau)$$

$$\vec{r} = \vec{x} - \vec{x}'$$

$$\mathcal{G}_\beta(\tau; \vec{r}) = \mathcal{G}_\beta(\tau + \beta; \vec{r})$$

Solutions:

$$\mathcal{G}_\beta(\tau; \vec{r}) = \frac{1}{\beta} \sum_n \int \frac{d^3 k}{(2\pi)^3} e^{-i\left(\omega_n \tau - \vec{k}\cdot\vec{r}\right)} \mathcal{G}_\beta\left(\vec{k}, \omega_n\right)$$

$$\mathcal{G}_\beta\left(\vec{k}, \omega_n\right) = \frac{1}{\omega_n^2 + \vec{k}^2 + m^2}$$

Thermal Green Functions and Propagators

$$\mathcal{G}_\beta(\tau;\vec{r}) = \frac{1}{\beta}\sum_n \int \frac{d^3k}{(2\pi)^3} e^{-i\left(\omega_n\tau - \vec{k}\cdot\vec{r}\right)} \mathcal{G}_\beta\left(\vec{k},\omega_n\right)$$

$$\mathcal{G}_\beta\left(\vec{k},\omega_n\right) = \frac{1}{\omega_n^2 + \vec{k}^2 + m^2}$$

"Matsubara modes"

$$\omega_n = \begin{cases} \frac{2n\pi}{\beta}, & \text{bosons} \longleftarrow \\ \frac{(2n+1)\pi}{\beta}, & \text{fermions} \longleftarrow \end{cases}$$

bosons ⟵	Periodic boundary conditions
fermions ⟵	Anti-periodic boundary conditions

Quantum correction:Sum integral

Sum integrals

$$\int \frac{d^4k}{(2\pi)^4} \longrightarrow \frac{1}{\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}$$

Zero vs Finite Temperature

"Zero Temperature"	Finite Temperature
Green Functions $G(x_1, x_2) = \langle 0 \| T\phi(x_1)\phi(x_2) \| 0\rangle$ (vacuum expectation value)	**thermal Green Functions** $G_\beta(x_1, x_2) \propto \sum_n e^{-\beta E_n} \langle n \| T\phi(x_1)\phi(x_2) \| n\rangle$ (ensemble averaged expectation)
Generating Functional for Green functions $Z[J] = \int \mathcal{D}\phi\, e^{i \int d^4x\, (\mathcal{L}+J\phi)}$ (real time)	**Generating Functional for Green functions** $Z[\beta, J] = \int \mathcal{D}\phi\, e^{-\int_0^\beta d\tau \int d^3x\, (\mathcal{L}_E+J\phi)}$ (imaginary time Green functions) $Z[\beta, J, \bar{J}] = \int \mathcal{D}\phi\, e^{\int_0^\beta d\tau \int d^3x\, (\mathcal{L}_E+J\phi+\bar{J}\bar{\phi})}$ (complex time Green functions)
S-matrix and Scattering Amplitudes (from the Green functions)	Used more like 'correlation functions' in the statistical mechanics

Path Integral for Partition Function

- Zero Temperature:

$$Z = \int \mathcal{D}\phi \, \exp\left(i \int d^4 x \mathcal{L}\right)$$

- Finite Temperature (Canonical):

$$Z(\beta) = \int_{\phi(0)=\phi(\beta)} \mathcal{D}\phi(\tau, \vec{x}) \, \exp\left(-\int_0^\beta d\tau \int d^3\vec{x} \mathcal{L}_E\right)$$

Perturbation Theory

Use Feynman diagrams as in zero temperature. Consider:

- Periodic boundary condition $\phi(0) = \phi(\beta)$
- Euclidean Lagrangian $\mathcal{L}_E$
- Derive path integral with the identity for transition amplitudes:

$$\langle \phi_b(\vec{x}) | \, e^{-iHT} \, | \phi_a(\vec{x}) \rangle \propto \int_{\phi(\vec{x},0)=\phi_a(\vec{x})}^{\phi(\vec{x},T)=\phi_b(\vec{x})} \mathcal{D}\phi(\vec{x}, t) \, e^{i \int_0^T dt \int d^3\vec{x} \mathcal{L}(\phi)}$$

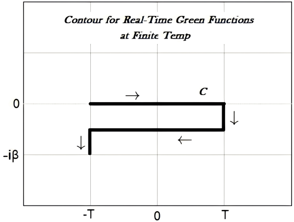
Thermal Green Functions

Real-Time Green Function:

$$G_\beta(x_1, x_2) = \frac{1}{Z}\mathrm{Tr}\left(e^{-\beta H} T_c \phi(x_1)\phi(x_2)\right)$$

$$Z[\beta, J] = \int \mathcal{D}\phi_c \, e^{i\int_c dt \int d^3\vec{x}(\mathcal{L}+J\phi)}$$

- Describes non-equilibrium time evolution
- Reduces to zero-temperature Green functions at $\beta \to \infty$

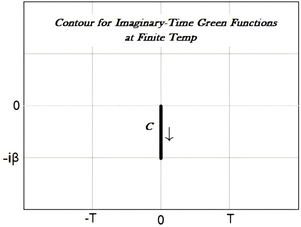


Imaginary-Time Green Function:

$$G_\beta(x_1, x_2) = \frac{1}{Z}\mathrm{Tr}\left(e^{-\beta H} T_c \phi(x_1)\phi(x_2)\right)$$

$$Z[\beta, J] = \int \mathcal{D}\phi \, e^{-\int_0^\beta d\tau \int d^3\vec{x}(\mathcal{L}_E+J\phi)}$$

- Simpler for equilibrium calculations
- Periodic in imaginary time $\tau \to \tau + \beta$



Free Boson Field at Thermal Equilibrium

Euclidean Lagrangian

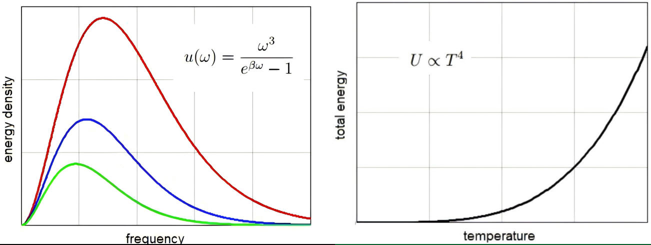
$$\mathcal{L}_E = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}|\nabla\phi|^2 + \frac{1}{2}m^2\phi^2$$

Partition Function

$$Z[\beta] = N\left(\det\left[-\partial_0^2 - \nabla^2 + m^2\right]\right)^{-1/2}$$

$$\ln Z = V\int\frac{d^3k}{(2\pi)^3}\left(-\frac{\beta\omega_k}{2} - \ln\left(1 - e^{-\beta\omega_k}\right)\right) + \text{const}$$

where $\omega_k = \sqrt{k^2 + m^2}$.



Energy Density Calculation

$$U \equiv \frac{\langle H \rangle - E_0}{V}, \quad \langle H \rangle = -\frac{\partial \ln Z}{\partial \beta}$$

- For $m = 0$:

$$U \propto \int_0^\infty d\omega \frac{\omega^3}{e^{\beta\omega} - 1} \propto T^4$$

- Frequency sum:

$$\omega_n = \frac{2\pi n}{\beta} \quad \text{(discrete Matsubara frequencies)}$$

Generating Functional with Source

$$Z[\beta, J] = NZ(\beta) \exp\left(\frac{1}{2}\int_0^\beta d^4x d^4y\, J(x)\Delta(x-y)J(y)\right)$$

where $\Delta(z)$ satisfies $(-\partial_0^2 - \nabla^2 + m^2)\Delta(z) = \delta(z)$.

Interacting Lagrangian

$$\mathcal{L}_E = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}|\nabla\phi|^2 + \frac{1}{2}m^2\phi^2 + V(\phi)$$

Perturbative Expansion

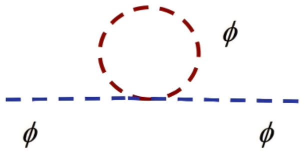
$$Z[\beta, J] = \exp\left(-\int_0^\beta d^4x\, V\left(-i\frac{\delta}{\delta J}\right)\right) Z_F[\beta, J]$$

- Similar to zero temperature Feynman diagrams
- Differences: Euclidean propagator, discrete frequency sums

One-loop Self Energy at Finite Temperature

$\lambda\phi^4$ Theory Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$



$$\Delta m^2 = \frac{\lambda}{2\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}\left[\omega_n^2 + \omega_k^2\right]^{-1}$$
$$= \frac{\lambda}{2\beta}\left(\frac{\beta}{2\pi}\right)^2 \sum_n \int \frac{d^3k}{(2\pi)^3}\left[n^2 + \left(\frac{\beta\omega_k}{2\pi}\right)^2\right]^{-1}$$

where $\omega_k^2 = \vec{k}^2 + m^2$.

Thermal Mass at Finite Temperature

At finite temperatures, particles can acquire an effective "thermal mass" due to interactions with the surrounding thermal bath.

$$\Delta m^2(T) = \frac{\lambda}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k} \frac{1}{e^{\beta\omega_k} - 1}$$

- At high T limit:

$$\Delta m^2(T) \approx \frac{\lambda T^2}{24} + O(m/T)$$

- Total mass:

$$\Delta m^2 = \Delta m_0^2 + \Delta m^2(T)$$

where Δm_0^2 is the $T = 0$ (divergent) part requiring renormalization, and $\Delta m^2(T)$ is finite.

Thermodynamic Potentials

- Partition Function Decomposition:

$$Z[\beta, J] = NZ_0(\beta)\, S(\beta, J)$$

- Free Energy:

$$W[\beta, J] = -\frac{1}{\beta}\ln Z[\beta, J] = W_0(\beta) - \frac{1}{\beta}\ln S(\beta, J)$$

$$\frac{\delta\, W[\beta, J]}{\delta\, J(x)} = \frac{1}{\beta}\overline{\phi}(x)$$

- Effective Action:

$$\Gamma[\beta, \overline{\phi}] = W[\beta, J] - \frac{1}{\beta}\int J\overline{\phi}$$

- Equilibrium Condition:

$$\left.\frac{\delta\Gamma[\beta, \overline{\phi}]}{\delta\overline{\phi}}\right|_{\text{equilibrium}} = 0$$

Effective action and effective potential at finite temperature

$$\Gamma[\phi_c] = S_{\rm cl}(\phi_c) + \frac{i\hbar}{2}\,{\rm Tr}\ln G^{-1} + \mathcal{O}(\hbar^2)$$

$$S_{\rm cl}[\phi] = \int d^4x \left[\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V_0(\phi)\right]$$

$$V_{\rm eff}(\phi_c) = V_0(\phi_c) - \frac{i\hbar}{2}\Omega^{-1}\,{\rm Tr}\ln G^{-1}$$

$$V_1(\phi_c, T) = \frac{\hbar}{2\beta}\sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left[\omega_n^2 + \vec{k}^2 + m^2(\phi_c)\right]$$

$$V_1(\phi_c, T) = \int \frac{d^3k}{(2\pi)^3}\tilde{I}[m(\phi_c)]$$

with

$$\tilde{I}[m(\phi_c)] = \frac{\omega}{2} + \frac{1}{\beta}\ln(1 - e^{-\beta\omega}), \quad \omega^2 = \vec{k}^2 + m^2(\phi_c)$$

- $T = 0$: Coleman-Weinberg contribution (divergent)
- $T > 0$: Temperature-dependent finite part

High-T Expansion

$$V_{\rm eff}(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\bar{\lambda}}{4}\phi^4 + \ldots$$

Standard Model Effective Potential and Electroweak Symmetry Restoration

$$V_{\text{eff}}(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\overline{\lambda}}{4}\phi^4 + \dots$$

- Coefficient D:

$$D = \frac{1}{32}\left(g_1^2 + 3g_2^2 + 4y_t^2 + 8\lambda\right)$$

- Critical temperature relation:

$$T_0^2 = \frac{\mu^2}{2D}$$

- Electroweak phase transition: First order (non-analytic in ϕ_c if Higgs mass is lighter than 75 GeV)

- Kapusta, *Finite Temperature Field Theory* (1989)
- Le Bellac, *Thermal Field Theory* (1996)
- Kirzhnits & Linde (1967), Niemi & Semenoff (1984)

Recap

- Finite temperature field theory generalizes zero-temperature methods using:
  - Partition functions with statistical ensembles
  - Imaginary/real-time Green functions
  - Periodic boundary conditions in imaginary time
- Thermodynamic quantities derive from $Z(\beta)$ via path integrals.
- Spontaneous symmetry breaking at finite T is analyzed via effective potential minimization.

"The universe is a grand thermodynamics machine."

Computational Challenges

Unique challenges at finite temperature

Perturbation Theory and Resummation

Infrared Problems

Other similar challenges as in the zero-temperature QFT

Gauge dependence problem of the effective potential

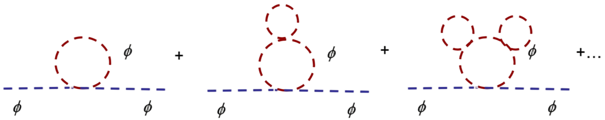
Nielsen identities ensure gauge independence of T_C (H. Patel & MJRM: 1101.4665 [hep-ph]), JHEP 11 (2022) 047,JHEP 07 (2022) 135,Phys.Rev.Lett. 130 (2023) 25, 251801

Non-perturbative effects

Higher loop corrections

See M. Laine, P. Schicho's works; Cosmological phase transitions at three loops: The final verdict on perturbation theory, Phys.Rev.D 110 (2024) 9, 096006

Higher correction: Daisy diagram



- Thermal mass parameter:

$$\kappa = \frac{T^2}{m^2(\phi_c)}$$

- Corrections:

$$\Delta m^2(T) \sim \frac{\lambda T^2}{24}, \quad \sim \frac{\lambda T^2}{24}\kappa, \quad \sim \frac{\lambda T^2}{24}\kappa^2$$

Ring Diagram Contribution: Arnold-Espinosa (AE) Method

$$V_1(\phi_c, T) \to V_1(\phi_c, T) + \Delta V_{\text{ring}}(\phi_c, T)$$

where

$$\Delta V_{\text{ring}}(\phi_c, T) = -\frac{T}{12\pi}\sum_k n_k \left\{ [m_k^2(\phi_c, T)]^{3/2} - [m_k^2(\phi_c)]^{3/2} \right\}$$

Resummation method

Truncated Full Dressing Methods (rely on high-temperature approximation)

- **Arnold-Espinosa (AE) Method**: inserts thermal mass into the cubic term
- **Parwani Method**: replaces tree-level masses with thermal masses globally in V_{CW} and V_T
- ...

Full-Dressing (FD) Methods

Substitutes the field independent thermal mass to the effective potential

$$V_{\text{eff}}^{\text{FD}} = V_{\text{eff}}\left(M^2(\phi, T)\right)$$

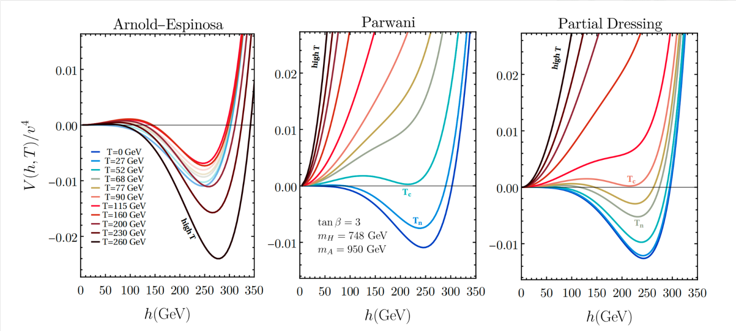
Partial-Dressing (PD) Methods

Applies the substitution to the first derivative of the effective potential

$$V_{\text{eff}}^{\text{PD}} = \int d\phi \left(\frac{\partial V_{\text{eff}}\left(m_i^2(\phi), T\right)}{\partial \phi} \right)_{m_i^2(\phi) \to M_i^2(\phi, T)}$$

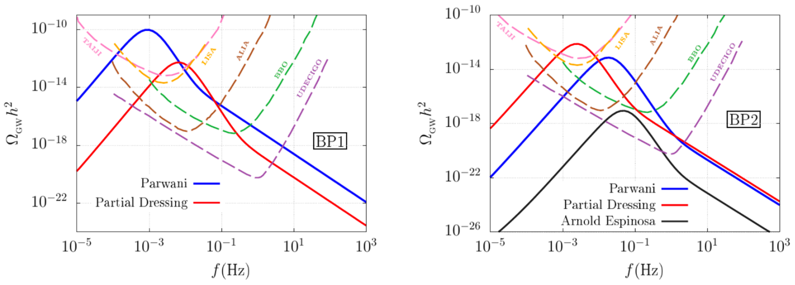
Example on different resummation schemes

Results on the Electroweak phase transition in the 2HDM (2504.02024).
High-temperature behavior: Non-restoration vs. restoration



Example on different resummation schemes

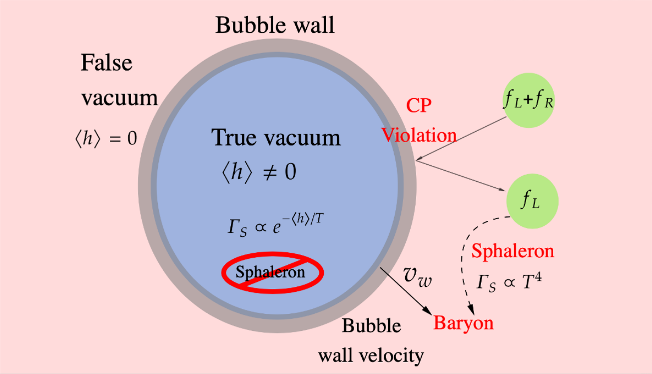
Impact of thermal resummation on gravitational waves predictions



The most typical application of QFT@T: electroweak baryogenesis

Sakharov conditions and baryogenesis

- Baryon number (B) violation
- Charge (C) and charge-parity (CP) violation
- Departure from thermal equilibrium

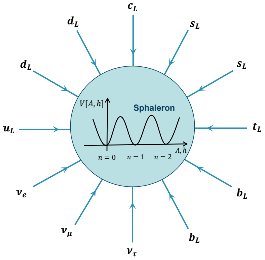


Each condition in EW baryogenesis needs QFT@T. FPH, arXiv: 2501.15543

The most typical application of QFT@T: electroweak baryogenesis

Baryon number violation from EW sphaleron process at high temperature

- At zero temperature, the baryon number violation (sphaleron) rate is negligible $\Gamma_\mathrm{S}(T=0) \sim \exp(-2S_\mathrm{E}) \sim 10^{-170}$
- However, when the temperature exceeds the electroweak scale (roughly corresponding to $T > \mathcal{O}(100)\mathrm{GeV}$), the processes breaking the baryon number will be in thermal equilibrium. At high temperatures, the sphaleron rate increases $\Gamma_\mathrm{S}(T) = \mu \left(\frac{M_\mathrm{W}}{\alpha_\mathrm{W} T}\right)^3 M_\mathrm{W}^4 \exp\left(-\frac{E_\mathrm{sph}(T)}{T}\right)$.
- In the false vacuum, the sphaleron rate becomes large at high temperature $\Gamma_\mathrm{S}(T) = \kappa' \alpha_\mathrm{W} (\alpha_\mathrm{W} T)^4 \quad \kappa' \sim 30$



The most typical application of QFT@T: electroweak baryogenesis

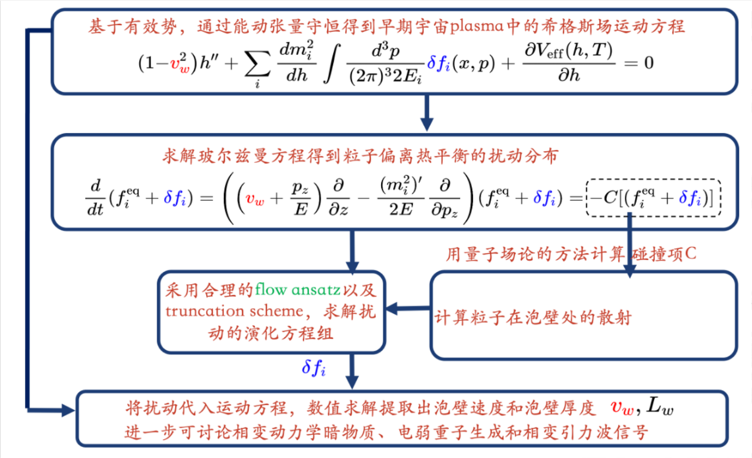
Baryon asymmetry of universe

$$\eta_{\rm B} = \frac{405\Gamma_{\rm S}}{4\pi^2\gamma_{\rm w} v_{\rm w} g_* T}\int dz \mu_{B_{\rm L}} f_{\rm sph}\, e^{-45\Gamma_{\rm s}|z|/4\gamma_{\rm w} v_{\rm w}}$$

Bubble wall velocity from QFT@T

- Theory: The most important and difficult phase transition parameter for GW, dynamical DM, baryogenesis is bubble wall velocity v_w.
- Experiment: GW experiment is most sensitive to bubble wall velocity v_w.

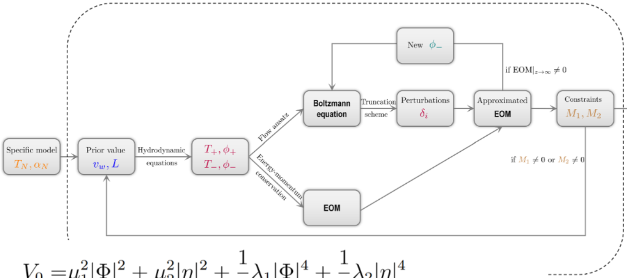
The most typical application of QFT@T: electroweak baryogenesis



Siyu Jiang, FPH, Xiao Wang, Phys.Rev.D 107 (2023) no.9, 095005, Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smitt, arXiv:2009.14295, Xiao Wang, FPH, Xinmin Zhang, arXiv:2011.12903

The most typical application of QFT@T: electroweak baryogenesis

A simple DM Model: **Bubble wall velocity in inert doublet model**



← More accurate processes

$$V_0 = \mu_1^2|\Phi|^2 + \mu_2^2|\eta|^2 + \frac{1}{2}\lambda_1|\Phi|^4 + \frac{1}{2}\lambda_2|\eta|^4 + \lambda_3|\Phi|^2|\eta|^2 + \lambda_4|\Phi^\dagger\eta|^2 + \frac{1}{2}\{\lambda_5(\Phi^\dagger\eta)^2 + \text{H.c.}\}\ ,$$

Siyu Jiang, **FPH**, Xiao Wang, Phys.Rev.D 107 (2023) no.9, 095005

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Siyu Jiang, FPH, Xiao Wang, Phys.Rev.D 107 (2023) no.9, 095005,

The most typical application of QFT@T: electroweak baryogenesis

Collision terms (Monte Carlo integration)

$$\Gamma_{\mu 1,t} \simeq \left(5.0 \times 10^{-4} g_s^4 + 5.8 \times 10^{-4} g_s^2 y_t^2\right) T \,,$$

$$\Gamma_{T1,t} \simeq \Gamma_{\mu 2,t} \simeq \left(1.1 \times 10^{-3} g_s^4 + 1.3 \times 10^{-3} g_s^2 y_t^2\right) T \,,$$

$$\Gamma_{T2,t} \simeq \left(1.1 \times 10^{-2} g_s^4 + 4.0 \times 10^{-3} g_s^2 y_t^2\right) T \,,$$

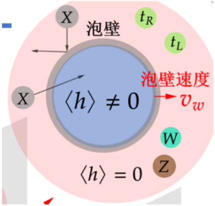
$$\Gamma_{v,t} \simeq \left(2.0 \times 10^{-2} g_s^4 + 1.8 \times 10^{-3} g_s^2 y_t^2\right) T \,,$$

$$\Gamma_{\mu 1,W} \simeq \left(2.3 \times 10^{-3} g_s^2 g_w^2 + 2.0 \times 10^{-3} g_w^4\right) T \,,$$

$$\Gamma_{T1,W} \simeq \Gamma_{\mu 2,W} \simeq \left(4.7 \times 10^{-3} g_s^2 g_w^2 + 4.1 \times 10^{-3} g_w^4\right) T$$

$$\Gamma_{T2,W} \simeq \left(1.5 \times 10^{-2} g_s^2 g_w^2 + 1.5 \times 10^{-2} g_w^4\right) T \,,$$

$$\Gamma_{v,W} \simeq \left(5.7 \times 10^{-2} g_s^2 g_w^2 + 1.5 \times 10^{-2} g_w^4\right) T \,,$$



$$\Gamma_{\mu 1,A} \simeq 1.0 \times 10^{-2} \lambda_3^4 T \,,$$

$$\Gamma_{T1,A} \simeq \Gamma_{\mu 2,A} \simeq 4.9 \times 10^{-3} \lambda_3^4 T \,,$$

$$\Gamma_{T2,A} \simeq 5.1 \times 10^{-3} \lambda_3^4 T \,,$$

$$\Gamma_{v,A} \simeq 1.8 \times 10^{-3} \lambda_3^4 T \,.$$

Siyu Jiang, FPH, Xiao Wang, Phys.Rev.D 107 (2023) no.9, 095005,
More precise calculations based on finite-temperature quantum field theory are needed for the collision terms.

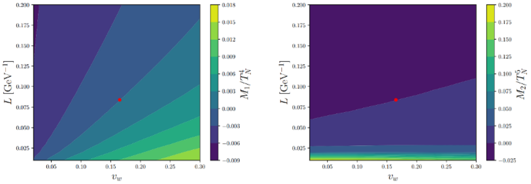
The most typical application of QFT@T: electroweak baryogenesis

Solving the EOM: bubble wall pressure difference is 0; bubble wall thickness fixed

$$S_{\rm EOM} \equiv (1-v_w^2)\,\phi'' + \frac{\partial V_{\rm eff}(\phi, T_+)}{\partial \phi} + \frac{N_t T_+}{2}\frac{dm_t^2}{d\phi} \times \left(c_1^t \mu_t + c_2^t(\delta T_t + \delta T_{bg})\right)$$

$$+ \sum_b \frac{N_b T_+}{2}\frac{dm_b^2}{d\phi}\left(c_1^b \mu_b + c_2^b(\delta T_b + \delta T_{bg})\right) = 0 \ ,$$

$$M_1 = \int S_{\rm EOM}\phi' dz = 0, \quad M_2 = \int S_{\rm EOM}(2\phi - \phi_-)\phi' dz = 0 \ .$$



In the allowed parameter spaces, the wall velocity is around 0.165. The basic procedure in this work can also be used for any other SFOPT and dynamical DM model.

Siyu Jiang, FPH, Xiao Wang, Phys.Rev.D 107 (2023) no.9, 095005,
More efforts are needed to improve the prediction of bubble wall velocity through quantum field theory at finite temperature.

The most typical application of QFT@T: dark matter

Renaissance of quark nugget DM idea by E. Witten.

Recently, dynamical DM formed by phase transition has became a new idea for heavy. Bubble wall in FOPT can be the "filter" to obtain the needed heavy DM when avoiding the unitarity constraints.

E. Krylov, A. Levin, V. Rubakov, Phys.Rev.D 87 (2013) 8, 083528
FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028
arXiv:1912.04238, Dongjin Chway, Tae Hyun Jung, Chang Sub Shin
Phys.Rev.Lett. 125 (2020) 15, 151102 , M. J. Baker, J. Kopp,and A. J. Long
arXiv:2101.05721, Aleksandr Azatov, Miguel Vanvlasselaer, Wen Yin
arXiv:2103.09827, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov
arXiv:2103.09822, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov
Siyu Jiang, FPH, Chong Sheng Li, arXiv:2305.02218
Siyu Jiang, FPH, Pyungwon Ko, arXiv:2404.16509
more than 100 papers in recent 5 years

FOPT in the early universe	Coffee making process
Bubble wall	filter
Case I:(gauged) Q-ball DM	Large coffee beans
Case II: filtered DM	Coffee
Phase transition GW	Aroma

The most typical application of QFT@T: dark matter

Case I: Q-ball DM

What is Q-ball?

PHYSICS REPORTS (Review Section of Physics Letters) 221, Nos. 5 & 6 (1992) 251–350. North-Holland PHYSICS REPORTS

Nontopological solitons*

T.D. Lee
Department of Physics, Columbia University, New York, NY 10027, USA

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Received May 1992; editor: D.N. Schramm

Nuclear Physics B262 (1985) 263-283
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Q-BALLS*

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Q-ball is the most typical non-topological soliton, initially proposed by Prof. Tsung-Dao Lee and Sidney Coleman. In quantum field theory, a spherically symmetric extended body that forms a non-topological soliton structure with a conserved global quantum number Q is called a Q-ball.

$$\phi = (\phi_R + i\phi_I)/\sqrt{2} \qquad Q = \int j^0 dx = \int \left(\phi_I \dot\phi_R - \phi_R \dot\phi_I\right) dx.$$

$$\delta(E - \omega Q) = 0$$

$$\downarrow$$

$$E = \int \left\{ \frac{1}{2}\left[\dot\phi_R^2 + \dot\phi_I^2 + (\nabla\phi_R)^2 + (\nabla\phi_I)^2\right] + U\left[\frac{1}{2}(\phi_R^2 + \phi_I^2)\right] \right\} dx$$

$$\phi = f(r) e^{-i\omega t}$$

The most typical application of QFT@T: dark matter

Q-ball production mechanism

Q-ball production:

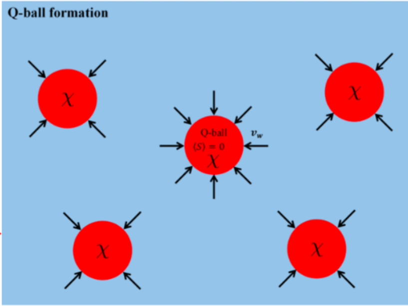
(1) produce the charge asymmetry (i.e. locally produce lots of particles with the same charge to form Q-ball)

(2) and packet the same sign charge in the small size after overcoming the Coulomb repulsive interaction.

1. Supersymmetry? Affleck-Dine mechanism.

We do not observe the supersymmetry until now!

2. Q-ball formation based on FOPT.
This talk

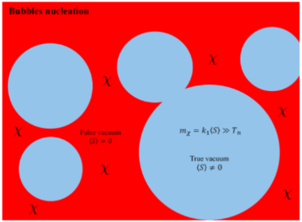


The most typical application of QFT@T: dark matter

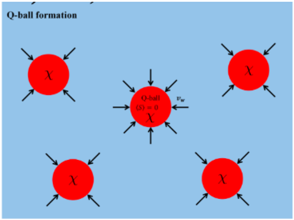
Case I: Q-ball DM

Global Q-ball DM: The cosmic phase transition with Q-balls production can explain baryogenesis and DM simultaneously.

$$\rho_{DM}^4 v_w^{3/4} = 73.5(2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$



(a) Bubble nucleation:χ particles trapped in the false vacuum due to Boltzmann suppression



(b) Q-ball formation:After the formation of Q-balls, they should be squeezed by the true vacuum

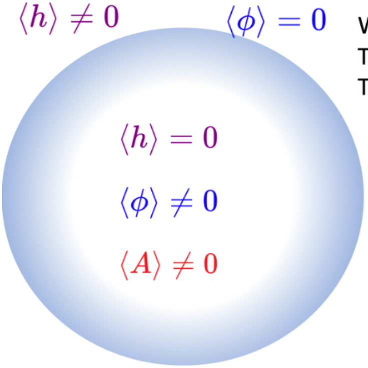
New DM production scenario by the bubbles.
The global Q-ball model proposed by T.D. Lee

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;

R. Friedberg, T.D. Lee and A. Sirlin, Rev. D 13 (1976) 2739

The most typical application of QFT@T: dark matter

Case I: Gauged Q-ball DM



When the conserved U(1) symmetry is local,
This introduces an extra gauge field A.
The minimal model achieving

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) + \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{4}\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} - V(\phi, h)$$

$$V(\phi, h) = \frac{\lambda_{\phi h}}{2} h^2 |\phi|^2 + \frac{\lambda_h}{4}\left(h^2 - v_0^2\right)^2$$

Interestingly, this portal coupling also naturally induces a strong FOPT.

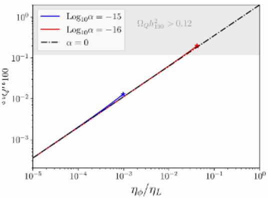
$$J_\mu = i\left(\phi^\dagger \overleftrightarrow{\partial}_\mu \phi + 2i\tilde{g}\tilde{A}_\mu |\phi|^2\right) \qquad Q = \int d^3x J^0$$

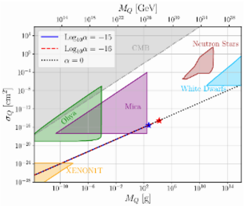
Siyu Jiang, FPH, Pyungwon Ko, JHEP 07 (2024) 053

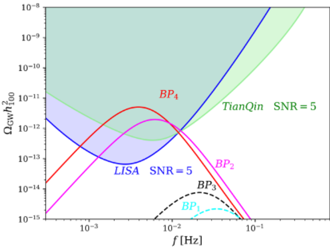
Conserved charge

The most typical application of QFT@T: dark matter

Gauged Q-ball DM from a FOPT







$$\Omega_Q h^2_{100} \simeq 2.81 \times \left(\frac{s_0 h^2_{100}}{\rho_c}\right)\left(\frac{\Gamma(T_\star)}{v_w}\right)^{3/16} s_\star^{-1/4}(F_\phi^{\rm trap}\eta_\phi)^{3/4}\lambda_h^{1/4} v_0 \left(1+\frac{108^{1/4}\tilde{g}^2 F_\phi^{\rm trap}\eta_\phi s_\star v_w^{3/4}}{5.4\pi^{7/4}\Gamma(T_\star)^{3/4}}\right)$$

	$\lambda_{\phi h}$	T_p [GeV]	α_p	β/H_p	v_w	$F_\phi^{\rm trap}$	η_ϕ/η_L	$\delta\sigma_{Zh}$	GW
BP_1	6.8	69.8	0.12	540	0.1	0.932	0.48	-0.36%	● (cyan)
BP_2	6.8	70.4	0.12	578	0.6	0.805	3.0	-0.36%	● (magenta)
BP_3	7.0	63.0	0.15	372	0.1	0.965	3.4	-0.37%	● (black)
BP_4	7.0	63.9	0.15	403	0.6	0.858	20.8	-0.37%	● (red)

F_ϕ^{trap}: The fraction of particles trapped into the false vacuum. It is determined by the phase transition dynamics.

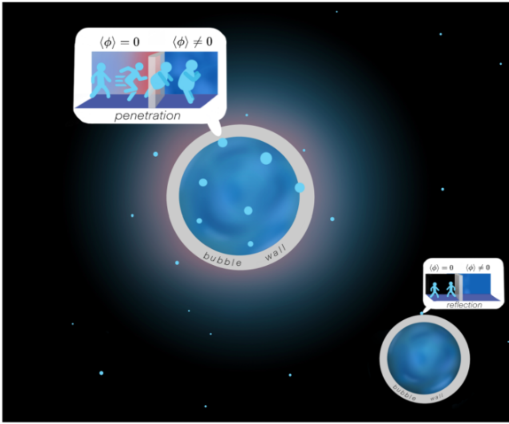
Siyu Jiang, FPH, Pyungwon Ko, JHEP 07 (2024) 053

The most typical application of QFT@T: dark matter

Case II: filtered DM from a FOPT

Bubble wall dynamics plays an essential role in the filtered DM mechanism.

DM



Siyu Jiang, FPH, Chong Sheng Li, Phys.Rev.D 108 (2023) 6, 063508

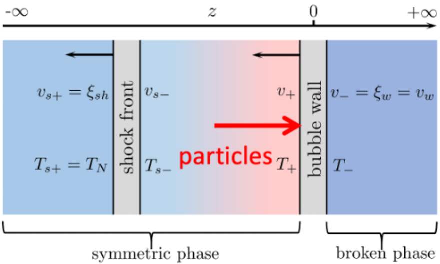
The most typical application of QFT@T: dark matter

Case II: filtered DM

Original work:

$$\tilde{v}_{\rm pl} = v_w, \quad T = T' = T_n$$

Phys.Rev.Lett. 125 (2020) 15, 151102 , M. J. Baker, J. Kopp, and A. J. Long



$$\tilde{v}_{\rm pl} = \tilde{v}_+, \quad T = T_+, \quad T' = T_- \qquad \text{(this work with hydrodynamic effects)}\ .$$

$$J_w^{\rm in} = \frac{g_\chi}{(2\pi)^2}\int_0^{-1} d\cos\theta\cos\theta\int_{-\frac{m_\chi^{\rm in}}{\cos\theta}}^{\infty} dp\frac{p^2}{e^{\tilde{\gamma}_+(1+\tilde{v}_+\cos\theta)p/T_+}} = \frac{g_\chi T_+^3\left(1+\tilde{\gamma}_+ m_\chi^{\rm in}(1-\tilde{v}_+)/T_+\right)}{4\pi^2\tilde{\gamma}_+^3(1-\tilde{v}_+)^2}e^{-\tilde{\gamma}_+ m_\chi^{\rm in}(1-\tilde{v}_+)/T_+}.$$

$$n_\chi^{\rm in} = \frac{J_w^{\rm in}}{\gamma_w v_w} \qquad \Omega_{\rm DM}^{\rm (hy)}h^2 = \frac{m_\chi^{\rm in}(n_\chi^{\rm in}+n_{\bar{\chi}}^{\rm in})}{\rho_c/h^2}\frac{g_{\star 0}T_0^3}{g_\star(T_-)T_-^3} \simeq 6.29\times 10^8\frac{m_\chi^{\rm in}}{\rm GeV}\frac{(n_\chi^{\rm in}+n_{\bar{\chi}}^{\rm in})}{g_\star(T_-)T_-^3}$$

The most typical application of QFT@T: dark matter

Case II: filtered DM

Boltzmann equation

$$\mathbf{L}[f_\chi] = \mathbf{C}[f_\chi]$$

$$f_\chi = \mathcal{A}(z,p_z)\, f^{\rm eq}_{\chi,+} = \mathcal{A}(z,p_z)\exp\left(-\frac{\tilde{\gamma}_+(E-\tilde{v}_+ p_z)}{T_+}\right)$$

$$\mathbf{L}[f_\chi] = \frac{p_z}{E}\frac{\partial f_\chi}{\partial z} - \frac{m_\chi}{E}\frac{\partial m_\chi}{\partial z}\frac{\partial f_\chi}{\partial p_z} \qquad m_\chi(z) \equiv \frac{m_\chi^{\rm in}(\phi_-)}{2}\left(1+\tanh\frac{2z}{L_w}\right)$$

$$g_\chi \int \frac{dp_x dp_y}{(2\pi)^2}\mathbf{L}[f_\chi] \approx \left[\left(\frac{p_z}{m_\chi}\frac{\partial}{\partial z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\partial}{\partial p_z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\tilde{\gamma}_+\tilde{v}_+}{T_+}\right)\mathcal{A}(z,p_z)\right]\frac{g_\chi m_\chi T_+}{2\pi\tilde{\gamma}_+}e^{\tilde{\gamma}_+\left(\tilde{v}_+ p_z - \sqrt{m_\chi^2+p_z^2}\right)/T_+}$$

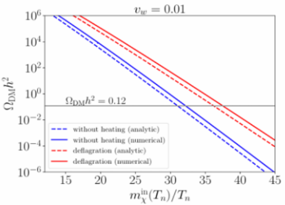
$$\text{including}\,\chi\bar{\chi}\leftrightarrow\phi\phi,\chi\phi\leftrightarrow\chi\phi,\chi\chi\leftrightarrow\chi\chi,\chi\bar{\chi}\leftrightarrow\chi\bar{\chi},\dots$$

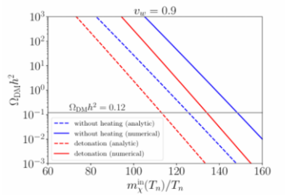
$$\begin{aligned} g_\chi \int \frac{dp_x dp_y}{(2\pi)^2}\mathbf{C}[f_\chi] &= -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar{\chi}\to\phi\phi}\left[f_{\chi_p} f^{\rm eq}_{\bar{\chi}_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar{\chi}_q}\right] \\ &= -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar{\chi}\to\phi\phi}\left[\mathcal{A} f^{\rm eq}_{\chi_p,+} f^{\rm eq}_{\bar{\chi}_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar{\chi}_q}\right] \\ &\equiv \Gamma_{\rm P}(z,p_z)\mathcal{A}(z,p_z) - \Gamma_{\rm I}(z,p_z)\ , \end{aligned}$$

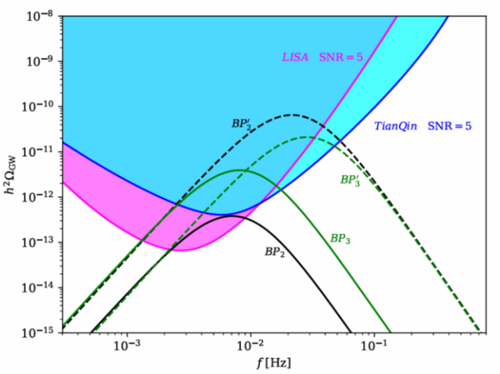
The most typical application of QFT@T: dark matter

Case II: filtered DM

$$n_\chi^{\rm in} = \frac{T_+}{\gamma_w \tilde{\gamma}_+} \int_0^\infty \frac{dp_z}{(2\pi)^2} \mathcal{A}(z \gg L_w, p_z) \exp\left[\tilde{\gamma}_+ \left(\tilde{v}_+ p_z - \sqrt{p_z^2 + (m_\chi^{\rm in})^2}\right)/T_+\right] \left(\sqrt{p_z^2 + (m_\chi^{\rm in})^2} + \frac{T_+}{\tilde{\gamma}_+}\right)$$







Summary and outlook

1. Quantum field theory at finite temperature is an essential tool to understand our early universe, such as reheating, baryogenesis, and dark matter.
2. Meanwhile, the relics of our early universe (GW) might be the unique signals to test the quantum field theory at finite temperature experimentally.
3. For future experimental detection, precise prediction in the framework of finite-temperature field theory is necessary.
4. Various new developments in zero-temperature quantum field theory may help to solve the difficult problems in finite-temperature field theory, such IR problem and resummation.

Thanks for your attention.