$0 < \mathrm{Im}\, \ell_q \text{ if } ?$ $(d=4)$ 2204.07140

$g_\ell = \frac{1}{\pi^2}\int \frac{A(\ell)}{p^2} = \int_0^\infty \frac{2\,dp}{\pi}\, \mathrm{disc}\, A =$

$= \int_0^\infty \frac{2\,dp}{\pi} \tilde{\ell}(\ell(\ell)) P_\ell\left(1+\frac{2p^2}{s}\right) \cdot \ell_q$

$\tau < \Lambda \quad \left(\frac{1}{\Lambda^2}\right) \cdot g_\ell = \left[\frac{1}{p^2}\right] = \sum_{\ell=0}^{\infty} \sum_{q} 16(2\ell+1) \frac{\mathrm{Im}\, \ell_q}{p^2}$ 4)

$A(s,t) = g_0 s^2 + g_1 s\, 6t + g_2 (t^2+u^2+u^2)^2 + \ldots$ (valid up to Λ)

bounds on g_0, g_1, g_2 1) perturbative bounds $\frac{g_0}{g_1} \ll \frac{g_1}{g_2}$ $(\mathrm{Im}\, \ell_q > 0)$

2) de-projected EFT bounds $g_2 < \frac{g_1}{\Lambda^2}$ $(\mathrm{Im}\, \ell_q < 2)$

$\lim \left[\frac{A(\ell,\ell)(\ell(\ell,\ell)-\ell)}{p^2}\right] = 0 = \sum 16(2\ell+1)(\ell(\ell,\ell))(\ell(\ell,\ell)-\ell) \int \frac{\mathrm{Im}\, \ell_q(\ell)}{p^2} dp = 0$ 3)

$A_{\text{imp}} = b_1 s^2 \log(-s) + b_2 s^2 \ln \log(-s) + (s \to t, u)$

L-moment problem $a_n = \int_0^1 f(z) z^{n-1} dz$ $0 < f(z) < L$ $(L=2)$

solution:

lower bound $f(z) \geq \theta(m-z)$ upper bound $f(z) \leq L\,\theta(z-m)$

$\lim\left(16(2\ell+1) z \frac{m^2}{2},\ z(2\ell+1)\left(z^2+\ell\right)\left(z^2+\ell-2\right)\frac{m^2}{4}\right)$

$\left(16(2\ell+1) z \frac{L-m^2}{2},\ z(2\ell+1)\left(z^2+\ell-2\right)\left(z^2+\ell\right)\frac{L-m^2}{4}\right)$

$\ell=0$ $(16 \cdot m^2, 0)$, $(16 m^2, 0)$

$\ell=2$ $(16 \cdot 5 m^2, 2 \cdot 5 (5-2) \cdot 5 \frac{m^4}{4})$

Nachkowski sum

$x = y(x)$ $m = x'(1)$ -2

$x = y(x)$ $y = y(x'(1))$ $y = y(x)$

$A = \frac{1}{2\pi^2}\left(\frac{s^2}{t} + \frac{s^2}{u} + \frac{t^2+u^2}{s}\right)$ $1 + 10 \cdot \frac{s^2}{t}$

$A < s^2$ (Froggatt Nielsen bound) $(d>4)$ 2202.07230

$\frac{1}{t} + \frac{1}{t} \sum \ell^n = 2 P_\ell\left(1+\frac{2t}{s}\right)$

$A = \frac{1}{2} \sum_{\ell=0}^{\infty} n(\ell,d)\, \ell_\ell(s)\, P_\ell^{(d)}\left(1+\frac{2t}{s-4m^2}\right)$

$n(\ell,d) = \frac{(4\pi)^{\frac{d}{2}}(d+2\ell-3)\,\Gamma(d+\ell-3)}{\pi\,\Gamma\left(\frac{d-1}{2}\right)\Gamma(\ell+1)}$ $2\,\mathrm{Im}\, \ell_\ell(s) \geq \frac{(s-4m^2)^{\frac{d-3}{2}}}{\sqrt{s}} |\ell(s)|^2$

$f_\ell(s) = \frac{\sqrt{s}}{(s-4m^2)^{\frac{d-3}{2}}} \cdot \left(1-e^{2i\delta_\ell(s)}\right)$

$A(s,\ell)$ $\beta = \frac{2\ell}{\sqrt{s}}$

$\mathrm{Im}\, \delta_\ell = 0$

$\ell_\ell(s) \to f(s,\ell) = \frac{\Gamma\left(\frac{d-4}{2}\right) C_d\, s}{4^{\frac{d-4}{2}} \ell^{d-4}}$

$\ell_\ell(s)\Big|_{\text{low}} \sim \frac{s}{\ell_q}(-)$

$\delta_\ell = \ell_\ell\Big|_{\text{low}}$

$\frac{1}{\ell^{d-3}} n(\ell,d) P_\ell^{(d)}\left(1-\frac{\beta^2\ell^2}{2\ell^2}\right) \approx 2^{\frac{d}{2}} (2\pi)^{\frac{d-2}{2}} (\ell q)^{\frac{2-d}{2}} J_{\frac{d-2}{2}}(\ell q)$

$\ell \gg \ell$ $\sum_\ell \sim \frac{\sqrt{s}}{\ell} \int d\ell$

$A_{\text{improved}} = 2i s (2\pi)^{\frac{d-2}{2}} \int_{\mathcal{C}} d\ell\, \ell^{d-3} (\ell q)^{\frac{2-d}{2}} J_{\frac{d-2}{2}}(\ell q)\left(1-e^{2i\delta(s,\ell)}\right) + A_{\text{low}\,\ell}$ $1 - \frac{s}{\ell^2}$

$\ell > \ell^2 - s^{\frac{1}{d-3}}$ (subdominant $|\ell_\ell| < 1$)

(Amati, Ciafaloni, Veneziano 1988)

$A\Big|_{\text{bound}} < s^2$

$\int g(\ell) e^{i f(\ell)} d\ell = g(\ell_0) e^{i f(\ell_0)} e^{i\frac{\pi}{4}\mathrm{sgn}(f''(\ell_0))} \sqrt{\frac{2\pi}{f''(\ell_0)}}$

$f'(\ell_0) = 0$

$J_{\frac{d-2}{2}} = \sqrt{\frac{2}{\pi q}} \cos(\ell q + \alpha)$

$2\,\mathrm{Im}\, \ell_\ell = \ell |\ell|^2$ $\frac{\pi}{N}$ $(\ell \to N)$

$\frac{\partial}{\partial \ell}\left(\pm \ell q + f_{\text{low}}(s,\ell)\right) = 0$

$\pm q - (d-4)\, \alpha\, s\, \ell^{2-d} = 0$

$\ell_\ell \sim s^{\frac{1}{d-3}}$

$A \sim s^{2 - \frac{d-2}{2(d-3)}} q^{\frac{(d-2)^2}{2(d-3)}} e^{\frac{i s^{\frac{1}{d-3}}}{q}}$

$d=5$ $\frac{1}{2-3}$ $q^2 = -t$ $e^{i s^{\frac{1}{2}}}$

s^{1-i}

$\frac{\mathrm{Im}\, a}{s^2} \quad \frac{\mathrm{Im}\, a = \mathbb{I}}{\partial N} \Rightarrow \mathrm{Im}\, a = 2 s^2 b$ $\frac{2 s^2}{s^2}$

$G_N^{-1} s^2 \log t$