bosonic $AdS_3 \times X \longleftrightarrow Sym(\mathbb{R}\times X)$

$sl(2,\mathbb{R})_3$ $\quad c=12$

Worldsheet

$c = \frac{3k}{k-2}\Big|_{k=3} = 9$

AdS_3: $\mathcal{H} = \bigoplus_{m=1}^{\infty} \oint_{-\infty}^{\infty} dp \int_0^1 \sigma^{-1}(\mathcal{H}^{\mu}_{p,j+1}) \otimes \sigma^{-1}(\mathcal{H}^{\mu}_{p,j+1})$

$sl(2,\mathbb{R})_3$ has no mass-like vector physical state condition

$L_0 \neq 0$ $n>0$ $\rightsquigarrow$ assumes 2 bosonic excitations

$L_0 \neq 4$

L_0 Virasoro $sl(2,\mathbb{R})_3 \times X$

1 bosonic lift from $sl(2,\mathbb{R})_3$

for fermions $L_0 - m J_0^3 - \frac{3}{4}m^2 + L_0^X = 1$

@ $k=3$

$C = -j(j-1)\Big|_{j=j+1} = \frac{1}{4} + p^2$

$J_0^3 = m$

Sugawara $L_0^{\mu} = \frac{C}{k-2} + N_{AdS_3}$

Marshall $\frac{1}{4} + p^2 - m\frac{m}{q} - \frac{3}{4}m^2 + h_X + N_X + N_{AdS_3} = 1$ $\quad$ (1)

Spectrum cutoff @ m spec $\quad J_0^3 = m + \frac{3}{2}w = m + \frac{3}{2}w = h^{ST}$

Solve (1) for m $\quad m = -\frac{3}{4} + p^2 + h_X + N_X + N_{AdS} - \frac{3}{4}w^2$

$m = \frac{p^2+h_X}{w} + \frac{N_X+N_{AdS}}{w} - \frac{3}{4}\frac{(w^2-1)}{w}$

$h^{ST} = \frac{p^2+h_X}{w} + \frac{N_X+N_{AdS}}{w} - \frac{3}{4}\frac{w^2-1}{w}$

Spectrum $= Sym_N(\mathbb{R}\times X)$ $\quad$ 1m

w-cycle $\quad c=12$

$h_{ST} = \frac{c}{24}\frac{w^2-1}{w} + \frac{1}{w} + \frac{N}{w}$

$c = \frac{3}{2}$

$= \frac{(\frac{3}{2})}{(\frac{3}{2})}\frac{w^2-1}{w}$

Spectrum matching

Correlation functions

worldsheet $sl(2,\mathbb{R})_3$ $\quad \langle \prod_{i=1}^{n} V^{w_i}_{p_i,h_i}(x_i,z_i)\rangle = \sum_{\gamma} \prod_{i=1}^{n} \delta^{(2)}(z_i - z_i^{\gamma}) \prod_{i=1}^{n} |z_i - z_i|^{h_i p_i} \delta(\sum p_i)$ $\quad$ for bosons $\quad$ correlation sum theory

$\left|\prod_{i=1}^{n} \frac{3}{2} \prod_{i=1}^{n} a_i^{\frac{1}{2}(w_i-1)} \cdot \lambda\right|^2$ $\quad$ covering map

$\langle\, |m\rangle$ $\quad \delta^{(2)}(\mathcal{H}_{p,j+1}) \quad h = m + \frac{3}{2}w$ $\quad \frac{3}{2}$

only belongs at ground states

$\lambda^{ST}_{-1} = J_0^+$

conformal factor

$V(x,z) = e^{xJ_0^+} V(w,z) e^{-xJ_0^+}$

$$\boxed{\begin{aligned} x_1 &= z_1 = 0 \\ x_2 &= z_2 = 1 \\ x_3 &= z_3 = \infty \end{aligned}} \leftarrow w_3$$

$\Gamma \begin{cases} x_1 \to z \\ x_4, \dots, x_n \end{cases}$ $\quad w_i$

γ covering map

$\Gamma(z) = x_i + a_i (z - z_i)^{w_i} + \dots \quad z \approx z_i$

γ covering map must have poles

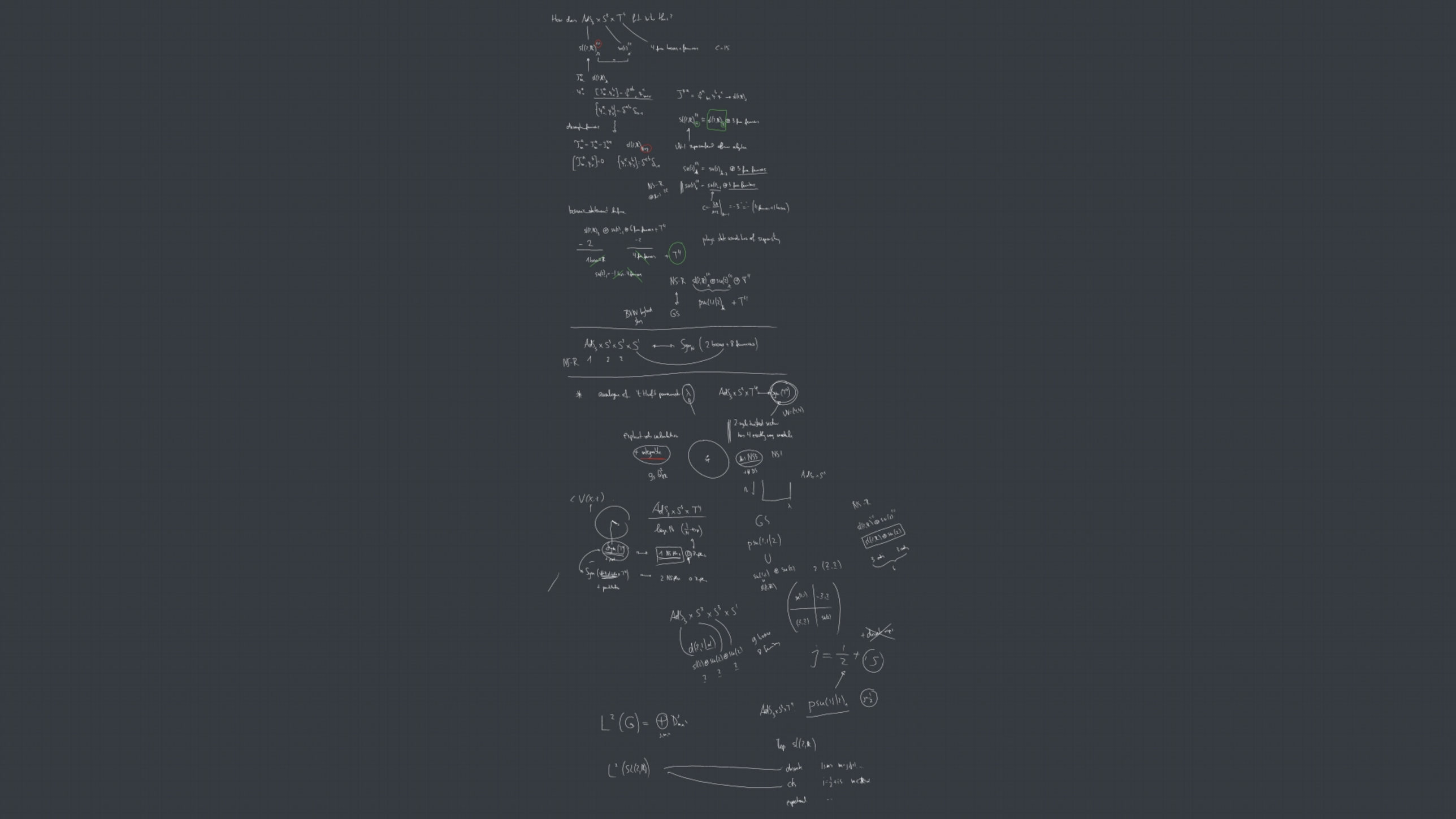
degree L $\quad \gamma(z) \sim \frac{\Pi_a}{z - z_a}$

$\Pi = w_3^{-w_3-1} \prod_{a=1}^{N-w_3} \Pi_a$

In string theory $\int d\mu(z_i) \langle V^{w_i}(x_i,z_i)\rangle = \sum_{\gamma} (\text{conf fct})(\gamma)$ correlation of sums $\quad \hat{\Pi} = \prod_{i=1}^{n} w_i^{\frac{1}{2}(w_i+1)} \Pi$

n-point fct sphere

How de


How does $AdS_3 \times S^3 \times T^4$ fit into this?
$sl(2,\mathbb{R})$
$su(1,1)^{(2)}$
4 free bosons + fermions
$c = 15$
$\mathcal{I}_n^a$ $sl(2,\mathbb{R})_k$
$\mathcal{H}^a$ $[\mathcal{H}^a_m, \mathcal{H}^b_n] = k\eta^{ab} m\delta_{m+n}$
$\{\psi^a_r, \psi^b_s\} = \delta^{ab}\delta_{r+s}$
$J^{ab} = \epsilon^{abc}\psi^b\psi^c \to sl(2,\mathbb{R})_2$
$sl(2,\mathbb{R})^{(1)}_{k+2} = sl(2,\mathbb{R})_k \oplus$ 3 free fermions
$sl(2,\mathbb{R})^{(1)}_2$ supersymmetric affine algebra
$sl(2,\mathbb{R})^{(1)}_k = su(1,1)^{(1)}_{k-2} \oplus$ 3 free fermions
$su(1,1)^{(1)}_{k-2} = sl(2,\mathbb{R})^{(1)}_k \oplus$ 3 free fermions
$c = \frac{3k}{k+2}\Big|_{k=1} = -3 \cdot \frac{1}{3}$ (4 bosons + 4 fermions)
$sl(2,\mathbb{R})_1 \oplus su(1,1)_1 \oplus$ 6 free fermions + T^4
-2 1 boson
-2 4 fermions
$+T^4$
$su(1,1)_{-1}$ 4 fermions
physic state conditions of superstring
NS-R $sl(2,\mathbb{R})^{(1)} \oplus su(1,1)^{(1)} \oplus 8^v$
$psu(1,1|2)_1 + T^4$
BRST helped
GS
$AdS_3 \times S^3 \times S^3 \times S^1$ ⟷ Sym^N (2 bosons + 8 fermions)
NS-R 1 2 2
* analogue of 't Hooft parameter
$AdS_3 \times S^3 \times T^4$
Sym$^N(T^4)$
$\mathcal{U}=(1,1)$
2 right-handed sectors
has 4 exactly any symmetries
explicit orbifold realization
+ orientifold
$g_s \to 0$ free
$\mathcal{C}$
for NS5
NS1
#D5
$AdS_3 \times S^3$
n
λ
$V(x,\tau)$
$AdS_3 \times S^3 \times T^3$
loop M (M-theory)
Sym$^N(T^3)$
1 NS5 + 1 KK
2 NS5s + 0 KK
GS
$psu(1,1|2)$
U
$su(1,1) \oplus su(2)$
$sl(2,\mathbb{R})$
$2\ (2,2)$
NS-R
$sl(2,\mathbb{R})^{(1)} \oplus su(2)^{(1)}$
$sl(2,\mathbb{R})^{(1)} \oplus su(2)$
3 susy
3 susy
4
$AdS_3 \times S^3 \times S^3 \times S^1$
$sl(2,\mathbb{R}) \oplus su(2) \oplus su(2)$
2 2 2
9 bosons
8 fermions
$su(1,1)$ $-3,2$
$(2,2)$ $su(1)$
$\hat{J} = \frac{1}{2} \times$ (S)
+ charged (S)
$AdS_3 \times S^3 \times T^4$ $psu(1,1|2)_1$
$L^2(G) = \bigoplus D_{j,m,\bar{m}}$
Top $sl(2,\mathbb{R})$
$L^2(SL(2,\mathbb{R}))$
discrete
cts
expected
lines $m = \frac{1}{2}(j+1) - n$
$1 - \frac{1}{2}$ is weird
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