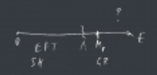
Unitarity constraints on EFTs

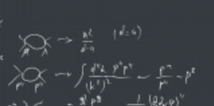


$\mathcal{L} \sim \frac{1}{2}(\partial\varphi)^2 - \frac{1}{4}\varphi^4$ — renormalizable

$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 + \frac{g_2}{\Lambda^4}(\partial_\mu\varphi)^2 - \frac{g_3}{\Lambda^6}(\partial\varphi)^2 \Big]\times$

$\mathcal{L} \sim \frac{M^2}{2}R \sim \partial_\mu\partial_\nu L^{\mu\nu}$

$g_2 \sim \frac{1}{\Lambda^2}$ [illegible]

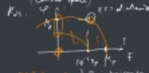


$\mathcal{L} \sim \frac{M_p^2}{2}R + \underline{R_{\mu\nu\rho\sigma}^2} + R_{\mu\nu}\Box R_{\mu\nu} + \dots$ can be reduced

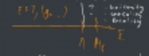
- EFT of photons $-\frac{1}{4}F_{\mu\nu}^2 + F_{\mu\nu}^4 - (F_{\mu\nu}\tilde{F}_{\mu\nu})^2 + \dots$ (imaginary mass)
- EFT of CR
- EFT of Higgs (HEFT) $\to$ SMEFT (consistent with symmetries)
- EFT of inflation $f(\varphi)\dot{\varphi} - V(\varphi)$ $\to$ EMB obtainable

(curved space)

Pot. φ — $x \to$ of inflation



has unitarizable



- boundary (symmetries)
- unitarity (consistency)
- energy theory

but all EFTs are admissible! scattering amplitudes $2 \to 2$

EFT $\to$ scattering amplitudes $\to$ UV amplitudes

$A > 0, \dots > 0$ — unitarity

EFT of scalar (collinear terms, ch. symmetry)

$\varphi \to \varphi + \text{const}$

$\mathcal{L} \sim \frac{1}{2}(\partial_\mu\varphi)^2 + \underbrace{\alpha\,\Box\varphi(\partial_\mu\varphi)^2}_{\text{infinitesimal}} + \beta(\partial_\mu\varphi)^4 + \dots$ $\to \partial\varphi^4$

$\varphi \sim \chi - \alpha(\partial_\mu\chi)^2$ $\quad \rho(\partial_\mu\varphi)^2$ $\quad \xi\varphi\Box(\partial_\mu\varphi)^2$ (not coordinate invariant)

$g_{\mu\nu} = \frac{\partial_\mu\varphi}{\Lambda}\frac{\partial_\nu\varphi}{\Lambda}$ $\to$ conformal anomaly

$\left(\frac{1}{2}\partial_\mu(\chi - \alpha(\partial_\mu\chi)^2)\right)^2 = \frac{1}{2}(\partial_\mu\chi)^2 - \alpha\,\partial_\mu\chi\,\partial_\mu(\partial_\nu\chi)^2 + \dots \chi^n$

$\alpha\,\Box\chi(\partial_\mu\chi)^2 \sim \alpha\,\Box\chi(\partial\chi)^2$ $\quad \alpha > -\alpha$

$O(\tau)$ $\quad \frac{1}{2}\left((\partial_\mu\chi)^2 + (\partial_\mu\varphi)^2\right)\delta(\vec{x}_0, \tau)$

$C_{\mu\nu} \sim x^\mu\partial_\nu x^\nu$ $\quad 2 \to 2$ amplitude $\sim$ $P(C_{\mu\nu})$

$f(\varphi)(\partial\varphi)^2$ $\quad \varphi \to \varphi - \alpha\chi$

$\hat{E}\varphi = 0$ (EOM)

$\frac{1}{2}\varphi\hat{E}\varphi + F(\varphi)\hat{E}\varphi$ — [illegible]

$\varphi \sim \chi - F(\chi)$

$\frac{1}{2}F(\chi)\hat{E}\chi + \frac{1}{2}\chi\hat{E}\chi - \frac{1}{2}F(\chi)\hat{E}\chi^2 - \frac{1}{2}\chi\hat{E}(F(\chi))$

$\frac{M_p^2}{2}R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ $\quad \frac{1}{2}\chi\hat{E}(F(\chi)\hat{E}\chi)$

$d\sqrt{g}R$ $\quad \frac{\alpha}{\Lambda^2}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}R_{\alpha\beta}^{\mu\nu} + \dots$

$\begin{cases} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0 & \text{EOM} \\ R = 0 \end{cases}$ $\quad d=4$ $\quad GB = R^2 - 4R_{\mu\nu}R^{\mu\nu} + \underbrace{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}}_{} = \partial(\dots)$

$R_{\mu\nu\rho\sigma}$, $\nabla_\rho R_{\mu\nu\rho\sigma}$ $\quad R_{\mu\nu\rho\sigma} + R_{\mu\rho\sigma\nu} + R_{\mu\sigma\nu\rho} = 0$

$\nabla_\mu\nabla_\nu R_{\mu\nu\rho\sigma}$ $\quad \nabla_\lambda R_{\mu\nu\rho\sigma} + \nabla_\mu R_{\nu\lambda\rho\sigma} + \nabla_\nu R_{\lambda\mu\rho\sigma} = 0$

$\nabla^\mu R_{\mu\nu\rho\sigma} = 0$. $\quad \Box R_{\mu\nu\rho\sigma} \sim \left(k_{\mu\nu\rho\sigma}^2\right)$ $\quad (\Box R_{\mu\nu})^2 \to$ higher poles

$S \sim M_p^{d-2}\int d^dx\sqrt{-g}\left\{\frac{R}{2} + \frac{\alpha_1}{\Lambda^2}(GB) + \frac{\alpha_2}{\Lambda^2}R^2 + \frac{\alpha_3}{\Lambda^4}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})^2 + \right.$

$\left. + \frac{\alpha_4}{\Lambda^2}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}) \right\}$

$\Box R_{\mu\nu\rho\sigma}\Box R^{\mu\nu\rho\sigma}$ $\quad R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ $\quad R_{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}R_{\alpha\beta}{}^{\rho\sigma}$

$+ \frac{1}{\Lambda^2}\alpha_3(\dots) + \frac{\alpha_4}{\Lambda^2}(\dots$

$t \to 0$

$\Lambda(1^+1^-1^+1^-)$ $\quad t_{\mu\nu} = \text{const}$

$s \to 0$ $M_p^{d-2}A_{1111}$ $\quad \frac{s^2}{t} - \frac{s(d-4)}{d-2}\frac{\alpha_1}{\Lambda^2}s^2 - 12\frac{\alpha_2}{\Lambda^2}s^2\left(\frac{d-4}{d-2}s^2 + 2tu + 2t^2\right)$

$s \to 0$ $M_p^{d-2}A_{1111}$ $\quad \frac{s^2}{\Lambda^2}\left(\alpha_1 - \frac{s(d-4)}{d-2}\frac{\alpha_1}{\Lambda^2}\right)(p_1 \cdot p_2 \cdot p_3) - \Lambda(1^+1^-1^+1^-) - (1^+1^-)$

$s \to 0$ $M_p^{d-2}A_{1111}$ $\quad \frac{c}{\Lambda^2} \quad \Lambda(1^+1^-) + \gamma(1^+1^-)^2 + \dots$ $\quad$ Lie group

$(\partial\varphi)^4 + (\partial_\mu\varphi)^2(\partial_\mu\partial_\nu\varphi)^2 + \dots$ $\quad \frac{1}{\Box} \sim \frac{1}{p^2 - m^2}e^{ik\cdot x}e^{ik\cdot x}e^{ik\cdot x}e^{ik\cdot x}$ $\quad$ Lie group

$A = g_s(s^2 + t^2 + u^2) + g_{s\,1}s(t+u) + g_{s\,2}(s^2 + t^2 + u^2) + \dots$ $\quad$ 1-loop unitarity for EFT

$s = -(p_1 + p_2)^2$

$t = -(p_2 + p_3)^2$

$u = -(p_1 + p_3)^2$

$\chi \sim \frac{\chi - \chi\varphi}{\varphi + \varphi^2 + \varphi^3 + \dots}$ $\quad \sum g_{mn}x^m y^n$