WZW model

- $J = dg g^{-1}$ holomorphic $\bar{\partial} J = 0$ $J^a(z) = \sum_n J^a_n z^{-n}$

$[J^a_n, J^b_m] = f^{abc} J^c_{n+m} + k n \delta^{ab} \delta_{n+m}$

- sugaw $L_n = \frac{1}{2(k+2)} \sum_a : J^a J^a :_n$ central charge $c = \frac{k \dim G}{k+2}$

$[L_n, J^a_m] = -m J^a_{n+m} \longrightarrow [L_n, J^a(z)] = z^{n+1} \partial_z J^a(z)$

- $[L_n, L_m] = \ldots L_{n+m}$
- $L_n |\phi\rangle = \left(\sum_{k \geq 1} \ldots \right) |\phi\rangle$

AdS$_3$ WZW model $SL(2,\mathbb{R})$ $\boxed{AdS_3 = SL(2,\mathbb{R})}$ Maldacena 1997

$g = e^{\gamma_+ \sigma^+} e^{\phi \sigma_3} e^{\gamma_- \sigma^-}$ $\sigma_1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}$ $\sigma_3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

$= \begin{pmatrix} e^{\phi} + \gamma_+ \gamma_- e^{-\phi} & \gamma_+ e^{-\phi} \\ \gamma_- e^{-\phi} & e^{-\phi} \end{pmatrix}$ $u = \{1, 0, 0\}$ $v = \{0, 1, 0\}$

$ds^2 = -d\phi^2 + e^{2\phi} d\gamma_+ d\gamma_-$

$SL(2,\mathbb{R})$ 1 parabolic 2 elliptic 3 hyperbolic

$\widetilde{SL(2,\mathbb{R})}$ elliptic 1 not periodic

$\longrightarrow$ $\widetilde{SL(2,\mathbb{R})}$ J^+, J^-

$[J^+_n, J^-_m] = 2 J^3_{n+m}$
$[J^3_n, J^\pm_m] = \pm J^\pm_{n+m}$
$[J^3_n, J^3_m] = -\frac{k}{2} n \delta_{n+m}$

Poincaré coordinates

$SL(2,\mathbb{R})$ J^+, J^-

$\Downarrow$

$\boxed{J^+ J^- \quad J^+ L^0 \quad J^- L^0}$

$L_n = \frac{1}{(k+2)} \sum_a : J^a J^a :_n$ modified

phys state $\begin{cases} L_n |\phi\rangle = 0 \quad n > 0 \\ L_0 |\phi\rangle = \bar{h} |\phi\rangle \end{cases}$

$[L_n, J^\pm_m] = -m J^\pm_{n+m}$ $[L_n, J^3_m] = 0$

$L_n |\phi\rangle$ physical

$\alpha^\pm_n = \alpha^0_n$

$L_n \alpha^\pm_n |\phi\rangle = \alpha^\pm_n L_n |\phi\rangle$ $p^\pm = 0$

$L_n \alpha^\pm_n |\phi\rangle = \alpha^\pm_n L_n |\phi\rangle$ $p^\pm = 1$

What are the Fock spaces of $SL(2,\mathbb{R})$

$J^\pm_n \ldots J^\pm_n |\phi\rangle$ $J^\pm_n |\phi\rangle = 0 \quad n > 0$

$\Downarrow$ (unitarity of $SL(2,\mathbb{R})$)

$C |\phi\rangle = (\gamma_1 \ldots \gamma_1) |\phi\rangle$

$\sum |\phi\rangle = n |\phi\rangle$

Fock space $\mathcal{H} = \oplus_{\phi} \oplus_{\gamma_1} \mathcal{H}_{\phi, \gamma_1}$

ϕ γ_1 γ_2 γ_3

Good basis for $SL(2,\mathbb{R})$

$|\gamma\rangle$ — continuous $p \geq 1/2$ $(\gamma = 2 + ip)$

$\int^\infty$

$\gamma \in$ unitary

$SL(2,\mathbb{R})$ $\mathcal{H} = \oint dz \oint d\bar{z} \, \mathcal{O}(h, \bar{h}) \, \mathcal{O}^\dagger(h, \bar{h})$ — what does it mean "h, $\bar{h}$ bounded"?

$\int^\infty$ dg g^{-1} hol

$\int^\infty$ g dg $\bar{\partial}$ hol

$\boxed{g(z) = g_L(z) g_R(\bar{z})}$ $g_L(e^{2\pi i} z) = g_L(z) \begin{pmatrix} e^{i\theta} & \\ & e^{-i\theta} \end{pmatrix}$

$g_R(e^{2\pi i} \bar{z}) = \begin{pmatrix} e^{i\theta} & \\ & e^{-i\theta} \end{pmatrix} g_R(\bar{z})$

$SL(2,\mathbb{R})$ $g^{-1} \partial_z g = e^{\phi \sigma_3} \partial_z e^{-\phi \sigma_3}$ $g(z)$

$g^{-1} \partial_{\bar{z}} g = g(z) \partial_{\bar{z}} e^{-\phi \sigma_3}$

$\boxed{\theta = \pi + i\phi}$ $\boxed{\theta = \{1, 0, 0\} + i\{0, 1, 0\}}$

$g = e^{\gamma_+ \sigma^+} e^{\phi \sigma_3} e^{\gamma_- \sigma^-}$

$\theta = \theta + \frac{1}{2} \ln(\gamma_+ \gamma_-) + i \frac{1}{2} \ln(\gamma_+ \gamma_-)$

$\phi = \phi + \frac{1}{2} \ln(\gamma_+ \gamma_-) + i \frac{1}{2} \ln(\gamma_+ \gamma_-)$

$\gamma_+ = \gamma_-$

$\boxed{\gamma_+ = \gamma_-}$

$\gamma_+ = \gamma_-$ $\gamma_+ = \gamma_-$

$J = dg g^{-1}$ $g = e^{\int \partial_z \phi} g_L$ $g = e^{\int \partial_{\bar{z}} \phi} g_R$

$J_{\mu\nu} = \mathrm{Tr}(e^{-\phi \sigma_3} \sigma_1)(g_L)^{-1} e^{\phi \sigma_3} = \begin{pmatrix} \cos\theta & \\ & \cos\theta \end{pmatrix} e^{i\theta \sigma_3} J_{\mu\nu}^{-1} e^{-i\theta \sigma_3}$

$J^a(z) = \oint (\partial_z g g^{-1})^a = \sum_n J^a_n(z) z^{-n}$

$\boxed{\begin{aligned} \bar{J}^a_n &= J^a_n + k n \delta_{n+m} \\ \bar{J}^a_n &= J^a_{n+m} \end{aligned}}$ canonical of $SL(2,\mathbb{R})$

$\sigma^\mu(H)$ $\psi(z)$ $\mathcal{H}: \{\psi\}^{\sigma^\mu(H)} = \{\sigma^\mu(H)\psi\}^{\sigma^\mu(H)}$

holomorphic $J^a_n = \sigma^a(\psi)$ $J^a_n |\psi\rangle = 0 \quad n > 0 \quad n = \frac{1}{2}$

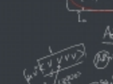
$\sigma^\mu(H)$ $J^a_n [|\psi\rangle] = \{\sigma(J^a_n) |\psi\rangle\}$

$J^a_n [|\psi\rangle] = [J^a_n |\psi\rangle] = 0 \quad n \geq 1/2$

$\boxed{\sigma^\mu(L_n) = L_n + n J^a_n - \frac{1}{2} n^2 \delta_{n+m}}$ $[L_n, J^a_n] = -n J^a_{n+m}$

L_n

AdS$_3$ WZW $SL(2,\mathbb{R})$ $\boxed{\mathcal{H} = \oplus_{\phi} \oint dh \oint d\bar{h} \, \mathcal{O}(h, \bar{h}) \mathcal{O}^\dagger(h, \bar{h})}$



σ^μ σ^μ σ^μ σ^μ