AdS_3/CFT_2 duality

$AdS_5 \times S^5 \longleftrightarrow \mathcal{N}=4$ SYM 4D $SU(N)$

$\left(\frac{R}{\ell_p}\right)^4 \sim N$ $\quad g_s \sim g_{YM}^2 \quad \left(\frac{R}{\ell_s}\right)^4 \sim g_{YM}^2 N = \lambda_{t'Hooft}$

large

· Sugra is good (large)

CFT perturbation good (small)

① AdS_3/CFT_2

string theoretic description $\longleftrightarrow$ tensionless string (ℓ_s large) $\longleftrightarrow$ small

- AdS_3: Maldacena-Ooguri $\boxed{\text{WZW-model descr.}}$ strings on AdS_3

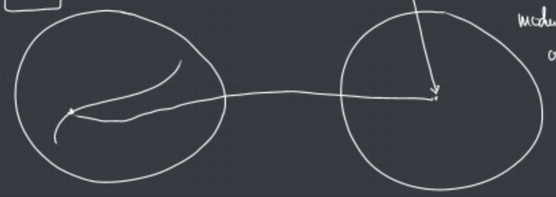
$SL(2,\mathbb{R})_k$ worldsheet

- dual CFT $(Sym_N(T^4))$

$AdS_3 \times S^3 \times T^4 \longleftrightarrow Sym_N(T^4)$

$\boxed{k=1}$

moduli space of string backgr.



moduli space of 2d CFTs

$\sqrt{k} \sim \frac{R}{\ell_s}$

bosonic $\boxed{\underset{\substack{SL(2,\mathbb{R})_3 \\ c=9}}{AdS_3} \times \underset{c=17}{X} \longleftrightarrow Sym_N(\mathbb{R} \times X)}$

① WZW models

$\mathbb{R}^2 \equiv \mathbb{C}$ $\quad z \longmapsto f(z)$ holomorphic

$L_n \quad z \longmapsto z + \epsilon z^{n+1} \qquad [L_m, L_n] = (m-n) L_{m+n}$ Witt algebra

$L_n = -z^{n+1}\partial_z$

normal ordering $\quad [L_m, L_n] = (m-n) L_{m+n} + c\,\delta_{m,n}$ (central term)

$\frac{c}{12}m(m^2-1)\delta_{m,-n}$ Virasoro algebra

$M_{p,q}$ $\quad \boxed{so(p+1, q+1)}$

d-pq $\quad$ $\mathbb{R}$ $\quad so(1,1)$

d=2 conformal gp $\quad so(1,1) = so(2) \oplus so(2)$

$z \to \boxed{f(z)}$ $\quad \bar{z}$

$L_n \quad \bar{L}_n$

* WZW models G

$g(z) \in G \quad |z| = 1$

string theory with target space G

$\boxed{g: S^2 \to G}$

$$S_0 = \frac{1}{4\lambda^2}\int_{S^2} d^2z \, \mathrm{tr}\left(g^{-1}\partial_\mu g \, g^{-1}\partial^\mu g\right) \quad \text{sigma model}$$

$$\delta S = \frac{1}{2\lambda^2}\int_S d^2z \, \mathrm{tr}\left(\left(-g^{-1}\delta g g^{-1}\partial_\mu g + g^{-1}\partial_\mu \delta g\right)\left(g^{-1}\partial^\mu g\right)\right)$$

$$= \frac{1}{2\lambda^2}\int_S d^2z \, \mathrm{tr}\left(\partial_\mu(g^{-1}\delta g) \, g^{-1}\partial^\mu g\right)$$

$-g^{-1}\partial_\mu g \, g^{-1}\delta g + g^{-1}\partial_\mu \delta g$

$$= -\frac{1}{2\lambda^2}\int d^2z \, \mathrm{tr}\left(g^{-1}\delta g \, \underbrace{\partial_\mu(g^{-1}\partial^\mu g)}_{0}\right)$$

$g \to g g_L$

$$\boxed{\begin{aligned} &J^\mu = g^{-1}\partial^\mu g \\ &\partial_\mu J^\mu = 0 \end{aligned}}$$

$$\boxed{\partial_z J_{\bar z} + \partial_{\bar z} J_z = 0} \qquad \boxed{\begin{aligned} &J_{\bar z} = g^{-1}\bar\partial g \\ &J_z = g^{-1}\partial g \end{aligned}}$$

$\partial \bar J = 0 \quad \bar\partial \bar J = 0$

WZW $\quad S = S_0 + k\,\Gamma[g] \qquad g: S^2 \to G$

WZ action

$\tilde g: B \to G \qquad \partial B = S^2$

$$\Gamma[g] = -\frac{i}{12\pi}\int_B d^3y \; \epsilon_{abc} \, \mathrm{tr}\left(g^{-1}\partial_a g \, g^{-1}\partial_b g \, g^{-1}\partial_c g\right)$$

3d volume

volume ambiguity based up $2\pi \mathbb{Z}$

$$\left(1 + \frac{\lambda^2 k}{\pi}\right)\partial(g^{-1}\bar\partial g) + \left(1 - \frac{\lambda^2 k}{\pi}\right)\bar\partial(g^{-1}\partial g) = 0$$

$\boxed{\lambda^2 = \frac{\pi}{k}}$ $\qquad 0$

$\bar J = g^{-1}\bar\partial g$ is anti-holomorphic $\Leftrightarrow$ $J = \partial g g^{-1}$ is holomorphic

$$\bar\partial J - \bar\partial(\partial g g^{-1}) = \bar\partial\partial g g^{-1} - \partial g g^{-1}\bar\partial g g^{-1}$$

$$\partial(g^{-1}\bar\partial g) = g^{-1}\partial\bar\partial g - g^{-1}\partial g g^{-1}\bar\partial g$$

$g \qquad g^{-1}$

$J(z) = \sum_a J^a(z) t^a$ $\quad$ basis of g $\qquad J^a(z) = \sum_n J^a_n z^{-n-1}$ $\qquad f^{ab}{}_c f^{cd}{}_a = -\frac{1}{2}\delta^{bd}\delta^{ab}$

$$\boxed{[J^a_m, J^b_n] = i f^{ab}{}_c J^c_{m+n} + k m \delta^{ab}\delta_{m,-n}}$$

Sugawara $\quad \boxed{L_n = \frac{1}{2(k+h^\vee)}\sum_m J^a_m J^a_{n-m}}$ $\qquad \boxed{C = \frac{k \dim g}{k + h^\vee}}$ $\quad$ $R^{(adj)} = N$

dual coxeter number