Aspects of generalized Symmetries in QFT

Outline:

- Motivation
- Symmetries and generalizations
- $\mathbb{C}P^1$ in 4d
- $\mathbb{C}P^1$ in 3d
- $\mathbb{C}P^1$ in 2d

A central question in QFT : understanding RG flow.

UV

(model on Lattice, CFT w/ relevant perturbation, free field theory)

- strongly coupled
- interacting
- non-SUSY
- non-integrable

RG

IR

Well understood in many Examples:

- weakly coupled (e.g. ϕ^4)
- free field theory (e.g. free fermion + mass)
- Supersymmetric field theories. ($\mathcal{N}=2$ SYM)
- Integrable systems. (tricritical Ising + σ')

— New semiclassical limit.

— numerical simulation

$$UV \xrightarrow{RG} IR \quad \rightarrow \mathcal{I} : \text{Sym \& Anom}$$

what constraints (can we put on RG flow)

$$\mathcal{I}_{UV} = \mathcal{I}_{IR}$$

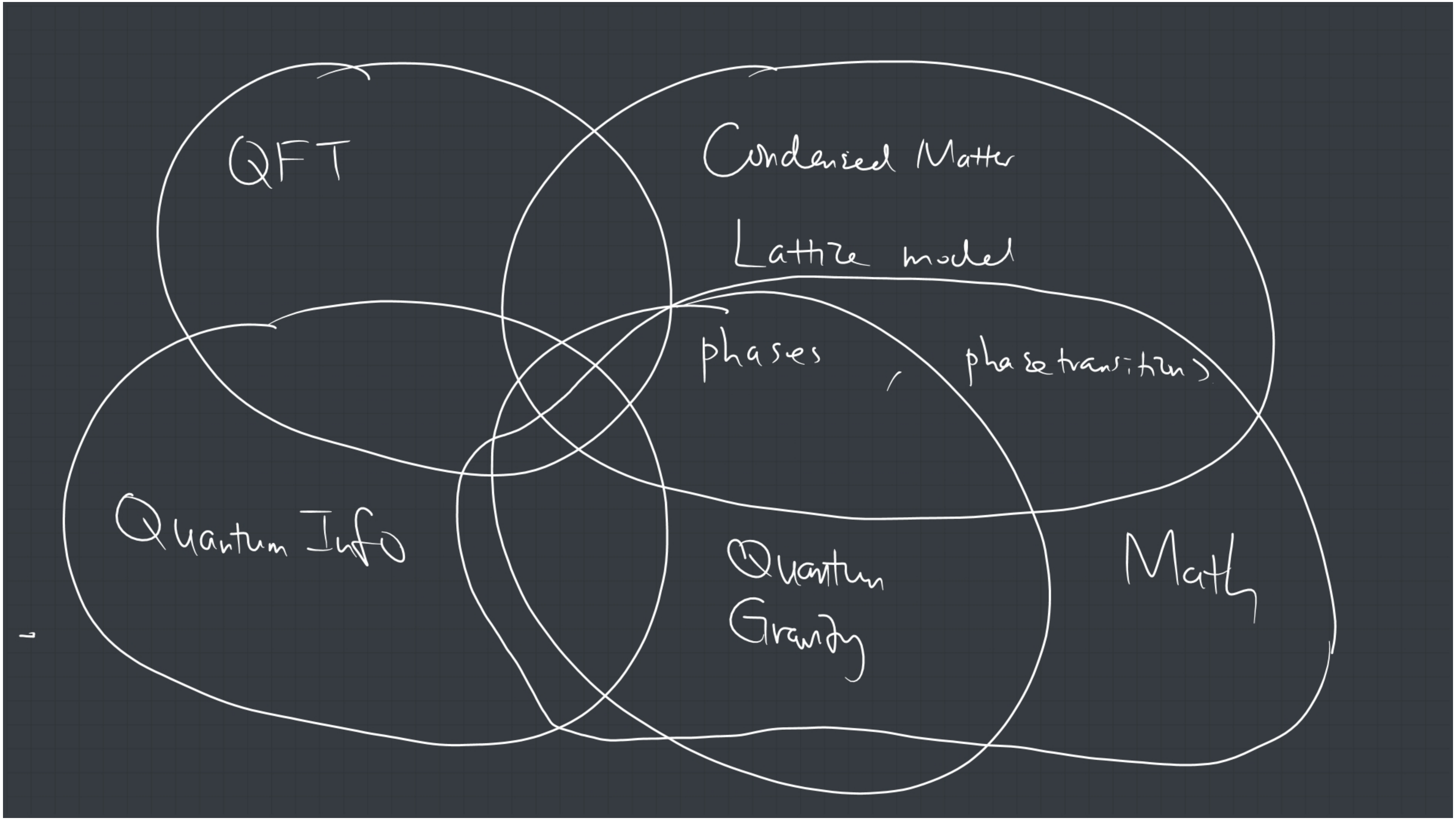
Love (Seiberg 19 Strings)

$\mathcal{I}$ = Global sym & Anomalies

— If IR is gapped : $\mathcal{I}$ often determines in IR TQFT

— If IR is gapless : 1) sym & Conf bootstrap

2) CFT


QFT
Condensed Matter
Lattice model
phases
phase transitions
Quantum Info
Quantum Gravity
Math

Overview of global symmetries & generalizations

example: $U(1) \supset \mathbb{Z}_N$ charge conservation

j_μ conserved current. $\partial_\mu j^\mu = 0$ $\quad$ $d * j = 0$

✓ conserved charge $Q = \int_{\Sigma_{d-1}} * j \in \mathbb{Z}$

✓ symmetry operator topological $U_\alpha = e^{i\alpha Q}$

$$Q - Q' = \int_{\Sigma_{d-1} - \Sigma'_{d-1}} * j = \int_D d * j = 0$$

Symmetry $\longrightarrow$ topological operator
$\longleftarrow$

Ordinary sym. ① $\exists$ inverse.

② Σ_{d-1} Codim - 1

Crucial step in relaxing ② : [Gaiotto. Kapustin. Seiberg. Willett. 14']

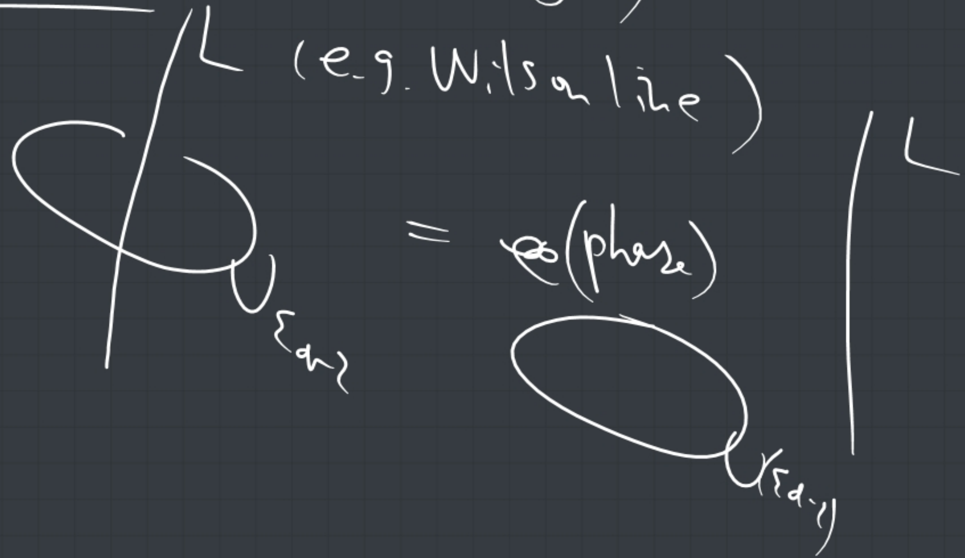
① : [Bhardwaj. Tachikawa. 17']
[Chang. Lin. Shao. Wang. Yin 18']

Modern point of view:

Topological operator = (Generalized) Symmetries,

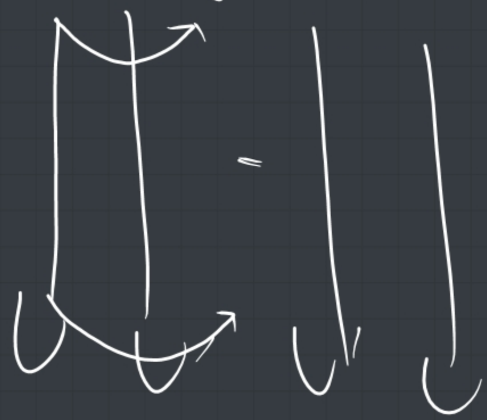
<u>Higher form symmetries</u> (1-form sym)

$U_{\Sigma_{d-2}}$



General feature:

Higher form sym are always abelian



Example ① Quantum Hall / Chern-Simons. $\mathcal{L} = \frac{3}{4\pi} b\,db$

$U(1)_3$

$\nu = \frac{1}{3}$ FQHE

Abelian TQFT

I a a^2

$\to$ 1-form sym.

G_k

Wilson line G

Example: Yang-Mills. $SU(2)$

$\mathbb{Z}_2^{(1)}$ 1-form sym.

$R^{-1}FR^{-1}$

$= FR^{-1}F$

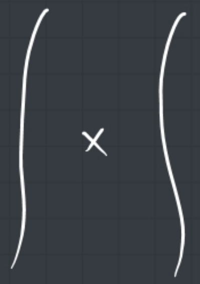
Non-invertible symmetry $\underline{x} \otimes N_{ab}^{e} \neq 0$

e.g. $N_{ab}^{e} = 2 \Rightarrow x = 1, 2$

1+1d QFT

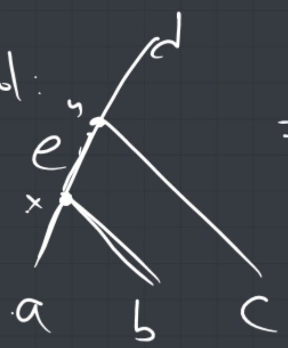
RFR

$= FRF$

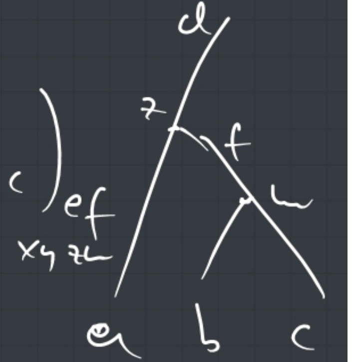


$$= \sum_c N_{ab}^{c}$$

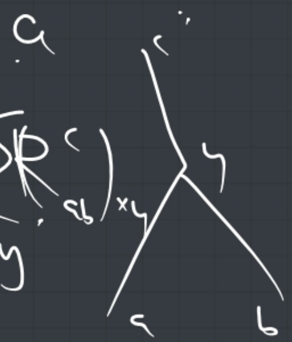
F-symbol:



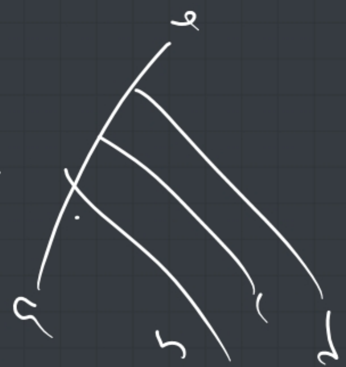
$$= \sum_f \left(F^{d}_{abc}\right)_{ef}$$



$$= \sum_y \left(R^{c}_{ab}\right)_{xy}$$



$$= \sum_c$$



$FFF = FF$

Pentagon identity