Lec 2 Thermal two-point function. Multiscale information preserved spectral form factor

AdS/CFT

BH $\leftrightarrow$ high T

Lorentzian BH

$\sum_n e^{-\beta E_n}$

$\sum_{E_i, E_j \in [E, E+\delta E]} e^{-\beta(E_i+E_j)}$

In holog CFT. $\langle O(t_1) O(t_2) \rangle_\beta = \mathrm{Tr}\left[e^{-\beta H} e^{iHt_1} O e^{-iHt_2} O \right]$

Thermalization

$\langle O(t) O(0) \rangle_\beta$ should decay as function of t

Quantization of ϕ free field $\int \sqrt{g} \left((\partial\phi)^2 - m^2 \phi^2 \right)$

$\partial^2 \phi - m^2 \phi = 0$

metric $ds^2 = \frac{dz^2}{f(z)} - f(z) dt^2 + r^2 d\vec{x}_{d-1}^2$

$\phi = \phi_k(z) e^{-i\omega t + i \vec{k} \cdot \vec{x}} Y(\Omega)$

second order diff eqn for $\phi_k(z)$

Two independent solns

Near the horizon $f(z) \sim 4\pi T - \beta' z + \cdots$ $\quad \beta' = r^2 \frac{df}{dr}$

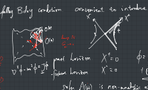
$\phi = A(\omega) z^{\frac{i\omega}{4\pi T}} + \cdots = A(\omega) z^{\frac{i\omega}{\beta'}} + \cdots$

Ruling out ref. ingoing unphysical

$\frac{B(\omega)}{A(\omega)}$ determined

softly Ricky condition corresponds to retarded $\frac{X^+ + \beta p e^{\frac{\beta\omega}{2}}}{X^+ + \beta p}$

$X^+ X^- = \beta^2$



past horizon $X^+ = 0$

future horizon $X^- = 0$

$\phi \sim A(\omega) \left(X^+ \right)^{-i\omega\beta}$

$\sim A(\omega) \left(\beta e^{\frac{\beta\omega}{2}} \right)^{-i\omega\beta} e^{-i\omega t}$

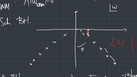
$\sim B(\omega) e^{-i\omega t}$

note $A(\omega)$ is non-analytic at future horizon

Infalling bdy cond $\rightarrow$ $A(\omega) = 0$

Generically on real axis for stable BHs, poles in lower half complex plane of ω

Upon $A(\omega_n) = 0$



In AdS, Sch. BH

Fingerprint of BH

Ringdown of BHs

$\langle O(t) O(0) \rangle \sim \sum_{\omega_n} d_{\omega_n} e^{-i\omega_n t}$ $\quad t \gg 0$

exponentially decay (w oscillations)

Q: Are there constraints on $\{\omega_n\}$?

CFT $\langle O(t) O(0) \rangle_\beta = \frac{1}{Z_\beta} \mathrm{Tr}\left[e^{-\beta H} e^{iHt} O e^{-iHt} O \right]$

Thermal averaging

Energy eigenstates

2-pt func decay $\longleftrightarrow$ pert "BH background"

(Multiscale) Periodic

This is not the full story

In the CFT $\langle O(t) O(0) \rangle_\beta = \frac{1}{Z_\beta} \mathrm{Tr}\left[e^{-\beta H} e^{iHt} O e^{-iHt} O \right]$

$= \frac{1}{Z_\beta} \sum_{n,m} e^{-\beta E_n} e^{i(E_n - E_m)t} |\langle n|O|m \rangle|^2$

phase

Not to wonder: treat $e^{i\theta_n}$ that $\theta_n \in [0, 2\pi)$ random phase

also are independent

treat Q_{nm} as also random variables

$\overline{\langle Q_{nm} \rangle} = e^{-S}$

value $\sim \frac{1}{e^S} \sum_{n,m} e^{\frac{\beta(E_n+E_m)}{2}} |Q_{nm}| \sim O(1)$

$\frac{1}{e^S} e^{\frac{\beta E_n}{2}} e^S \sim O(1)$

Look at 2-pt func at $t \neq 0$

2-pt $= \frac{1}{e^S} \sum_{n,m} e^{\frac{\beta E_n}{2}} e^{\frac{\beta E_m}{2}} |Q_{nm}|^2$

Stationary of $\{E_n\}$

On average, this is zero

$\overline{|2\text{-pt}|^2} = \frac{1}{e^{2S}} \sum_{n,m} \overline{|e^{\cdots}|} \overline{|e^{\cdots}|} \overline{|Q_{nm}|^2}$

$= \frac{1}{e^{2S}} e^{2S} e^{-2S} \sim e^{-2S}$

How to understand these fluctuations from gravity?

Maldacena's info puzzle

Can

- exact 2-pt function
- averaged properties of late time 2-pt func

Are there similar puzzles also only involve $\{E_i, \ldots\}$ or $\{Q_{nm}\}$?

Spectral form factor $\{E_n\}$

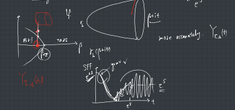
$\mathrm{SFF} \equiv \left| Z(\beta + it) \right|^2$

$Z(\beta) = \sum_n e^{-\beta E_n}$ partition function

Sometimes microcanonical version $\left| Y_{E,\Omega}(t) \right|^2$

$Y_{E,\Omega}(t) = \sum_{E_i \in [E, E+\delta E]} e^{-iE_i t} Y_{E_i}(\Omega)$

$\overline{\sum_{n,m} c_n c_m} = \sum_n |c_n|^2$

$\sim e^S$

In general how to compute $Z(\beta + it)$