CFT data $\{\Delta_i, f_{ijk}\}$

$$\langle \phi_i(x_1)\phi_j(x_2)\phi_k(x_3)\rangle = \frac{f_{ijk}}{x_{12}^{\Delta_i+\Delta_j-\Delta_k}\, x_{13}^{\Delta_i+\Delta_k-\Delta_j}\, x_{23}^{\Delta_j+\Delta_k-\Delta_i}}$$

$x_1 = +\infty$
$x_3 = 0$
$x_{12} = +\infty$
$x_{13} = +\infty$
$x_{23} = x_2$

state-operator correspondence

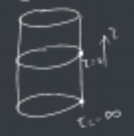
$$|\phi\rangle = \lim_{r\to 0} \phi(r)|0\rangle \qquad \langle\phi| = \lim_{r\to\infty} r^{2\Delta}\langle 0|\phi(r)$$

R^d plane

$$\langle 0|\phi_{pl}(r)\phi_{pl}(r=0)|0\rangle = \frac{1}{r^{2\Delta}}$$

$\tau = R\ln r$

$S^{d-1}\times R$



$$\phi_{cyl}(\tau,\Omega) = \Lambda^{\Delta/2}\,\phi_{pl}(r,\Omega) \qquad \Lambda = R^{-2}e^{\frac{2\tau}{R}}$$

$$\langle 0|\phi_{cyl}(\tau)\phi_{cyl}(-\infty)|0\rangle = \Lambda^{\Delta/2}\frac{1}{e^{\frac{2\tau}{R}}} \overset{\tau=0}{=\!=} \frac{1}{R^{+\Delta}}$$

$$\Downarrow$$

$$\langle 0|\phi_{cyl}(\tau)|\phi\rangle = \frac{1}{R^{+\Delta}} \quad \checkmark$$

R^d $r_1 = +\infty$

$$\langle 0|\phi_{pl,i}(r_1)\phi_{pl,j}(r_2)\phi_{pl,k}(r_3)|0\rangle = \frac{f_{ijk}}{r_j^{-\Delta_i+\Delta_j+\Delta_k}}$$

$$\Downarrow$$

$$\langle\phi_i|\phi_{pl,j}(r_2)|\phi_k\rangle = \frac{f_{ijk}}{r_j^{-\Delta_i+\Delta_j+\Delta_k}}$$

$$\downarrow$$

$\tau = R\ln r$

$$\langle\phi_i|\phi_{cyl,j}(\tau)|\phi_k\rangle \underset{r=1}{\overset{\tau=0}{=\!=}} f_{ijk}\,R^{-\Delta_j} \quad \checkmark$$

$$\Rightarrow \quad f_{ijk} = \frac{\langle\phi_i|\phi_j(\tau=0)|\phi_k\rangle}{\langle 0|\phi_j(\tau=0)|\phi_j\rangle}$$

$$H \longrightarrow \{E_n,\ |\phi_n\rangle\}$$

$$f_{\epsilon\epsilon\epsilon} = \frac{\langle\epsilon|\phi_\epsilon|\epsilon\rangle}{\langle 0|\phi_\epsilon|\epsilon\rangle}$$

$$f_{\sigma\epsilon\sigma} = f_{\sigma\sigma\epsilon} = \cdots = \frac{\langle\sigma|\phi_\epsilon|\sigma\rangle}{\langle 0|\phi_\epsilon|\epsilon\rangle}$$

$$\phi^{CFT}(x) = \sum_n c_n\,\phi^{micro}(x)$$

$$\phi_\sigma \simeq c_1 n^z(\theta,\varphi) + c_2\, n^z(\theta,\varphi)n^x(\theta,\varphi) + \cdots$$

$$\phi_\epsilon \simeq c_1 n^x(\theta,\varphi) + \cdots$$

$$f_{\epsilon'\epsilon\epsilon'} \sim \langle\epsilon'|\phi_\epsilon|\epsilon'\rangle$$

$$\phi^{micro}(x) = \sum_n \hat{c}_n\,\phi_n^{CFT}(x)$$

$$h^z(\theta,\varphi) \simeq \sigma(x) + \cdots$$

$$h^x(\theta,\varphi) \simeq d_1\epsilon(x) + d_2\epsilon'(x)$$

$$O(\theta,\varphi) \simeq e_1\epsilon(x) + e_2\epsilon'(x) + \cdots$$

$$\alpha h^x + \beta O_0 = \epsilon(x) + \cdots$$

Applications

- more transitions / universality class

XY transtion $O(2)$

Heisenberg transiton $O(3)$.

Wilson-Fisher $O(N)$

$\underline{H}$

Lee-Yang transition $\mathcal{L} = (\nabla\phi)^2 + (h - h_c)\phi + ig\phi^3$

$$H = H_{Ising} + ig\int d^3x\, \sigma(x)$$

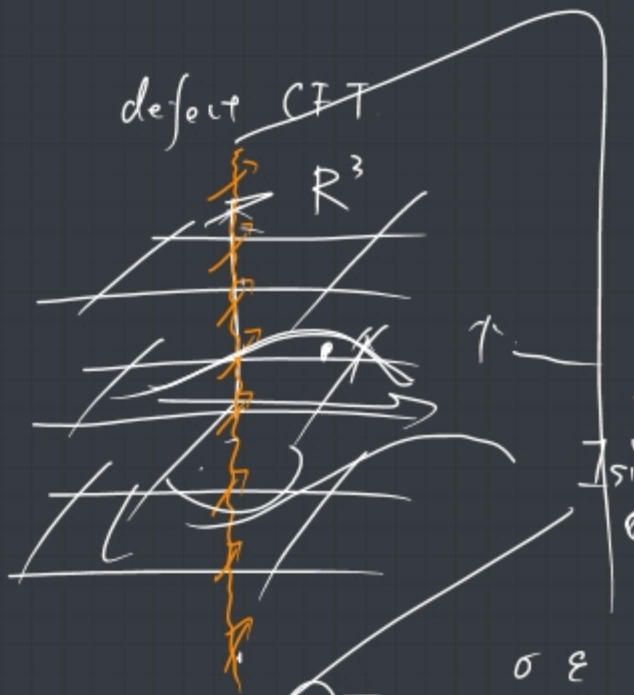
$$= \int d\Omega_a d\Omega_b\, U(\Omega_a - \Omega_b)\left[n^\uparrow(\Omega_a) n^\downarrow(\Omega_b)\right] + h\int d\Omega\, n^z(\Omega)$$

$$+ \underline{ig\int d\Omega\, n^z(\Omega)}$$

$$\boxed{n^z(\Omega) \sim \sigma(x) + \cdots\cdots}$$

$\Delta_\phi \sim 0.2$

defect CFT

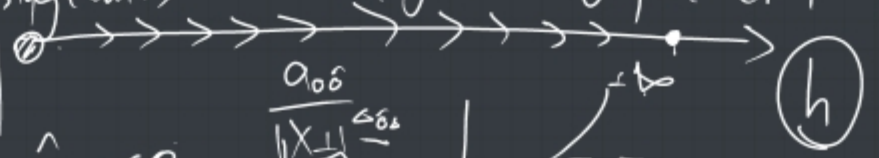


RG $\mathcal{L}(\phi^4) + \int d^1x \, \sigma(x)$

$\Delta_\sigma \approx 0.518 \ldots < 1 = d_{defect}$

1.412 >

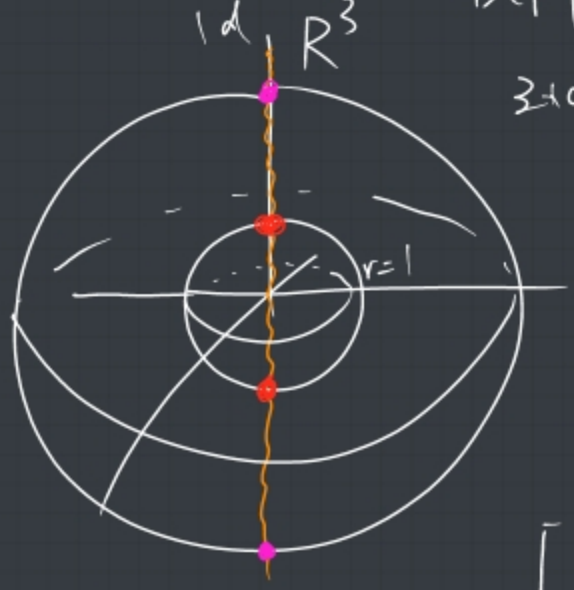
$\sim \frac{1}{(\lg x)^q}$



$\sigma \; \varepsilon \; \hat{O} \; \langle O_a \rangle$

$H = H_{Ising} + h \sum_{i \in line} S_i$

BCFT $\nabla_\mu T_{\mu\nu} = 0$

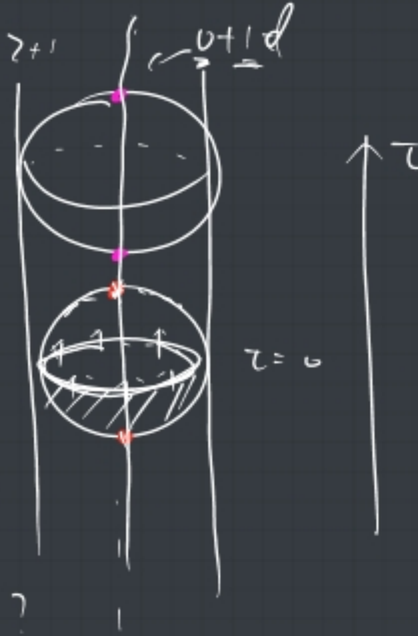


3+0

Weyl

$\tau = R \ln r$

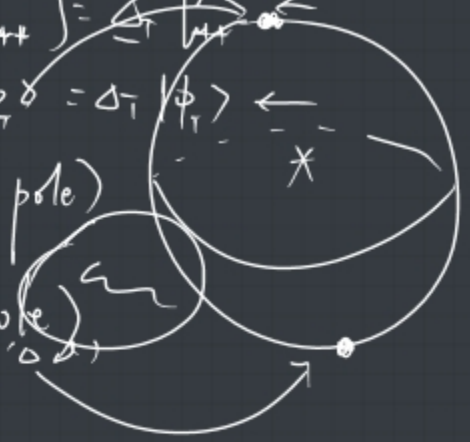
$h(r) = h(\tau) d\tau$



$M_{\mu\nu}$ $\langle \; \rangle$ $T_{\mu\nu}$

$[H, T_{\mu\nu}]$

$H|\phi_i\rangle = \Delta_i |\phi_i\rangle$



$H = H_{Ising} + h\, n^z(\text{north pole}) + h\, n^z(\text{south pole})$