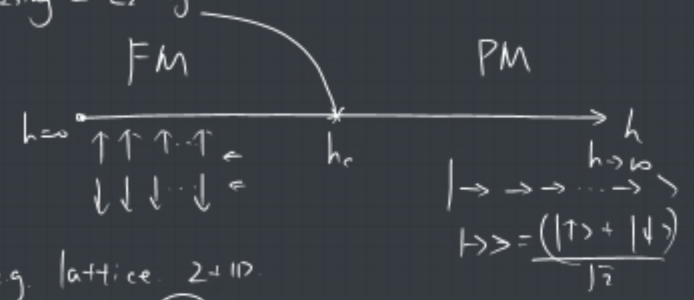
2+1D transverse Ising model on sphere geometry

Ising = Z_2 sym. bre

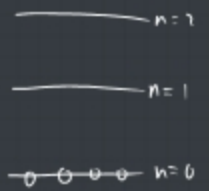


e.g. lattice 2+1D

$$H = -\sum_{\langle ij\rangle} \sigma^z_i \sigma^z_j + h \sum_i \sigma^x_i$$

$N_x \times N_y = L^2$

$S^2 \times R$



$N_\phi = 2S+1$

$\boxed{\nu = 1}$

$\nu = \frac{N_e}{N_\phi} = 1$

$$H = \int d\Omega_a d\Omega_b\, U(\Omega_a - \Omega_b)\left(n^0(\Omega_a) n^0(\Omega_b) - n^z(\Omega_a) n^z(\Omega_b)\right) + h \int d\Omega\, n^x(\Omega)$$

$$= \int d\Omega_a d\Omega_b\, U(\Omega_a - \Omega_b)\left(\underbrace{n^\uparrow(\Omega_a)}_{\psi^\dagger_\uparrow \psi_\uparrow} \underbrace{n^\downarrow(\Omega_b)}_{\psi^\dagger_\downarrow \psi_\downarrow}\right) + h \int d\Omega\, n^x(\Omega)$$

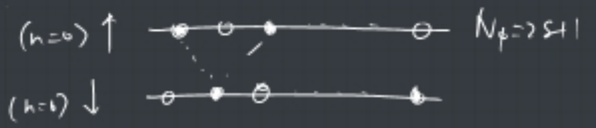
$\Omega = (\theta, \varphi)$

$$n^\alpha(\theta,\varphi) = \bar{\psi}(\theta,\varphi)\, \sigma^\alpha\, \psi(\theta,\varphi)$$

Pauli matrix

$\alpha = 0, x, y, z$

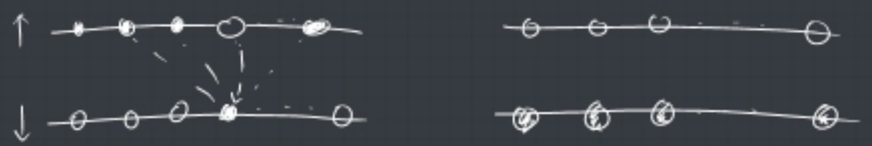
$N_e = 2S+1$



$$\begin{cases} n^0 = n^\uparrow + n^\downarrow \\ n^z = n^\uparrow - n^\downarrow \end{cases}$$

inter ($h=0$)

$E = 0$



$$\prod_{m=-S}^{S} c^\dagger_{m\uparrow} |0\rangle \qquad \prod_{m=-S}^{S} c^\dagger_{m\downarrow} |0\rangle$$

$$U(\Omega_a - \Omega_b) = g_0\, \delta(\Omega_a - \Omega_b) + g_1 \nabla^2 \delta(\Omega_a - \Omega_b)$$

$h \to \infty$ $\quad \prod_{m=-S}^{S} \frac{1}{\sqrt{2}} \left(c^\dagger_{m\uparrow} + c^\dagger_{m\downarrow}\right) |0\rangle$

n^0

quantization
(LLL projection)
$\downarrow$

$$\psi_{\alpha=\uparrow,\downarrow}(\theta,\varphi)=\sum_{m=-s}^{s} Y^{(s_0)}_{s,m}(\theta,\varphi)\, c_{m,\alpha=\uparrow,\downarrow}$$

$$H=\sum_{m_1,m_2,m_3,m_4} V_{m_1m_2m_3m_4}\, c^\dagger_{m_1\uparrow}c^\dagger_{m_2\downarrow}c_{m_3\downarrow}c_{m_4\uparrow}$$

$$\langle 0| M_{\mu\nu}^2 |\phi\rangle=\ell(\ell+1)$$

State-operator correspondence.

$$\boxed{E_n-E_0\sim v(\Delta_n-\underset{=0}{\Delta_0})}$$

rot ℓ

Z_2 $c^\dagger_{m\uparrow}\leftrightarrow c^\dagger_{m\downarrow}$

Z_2 particle-hole sym. $c_m\to i\sigma^y c_m^\dagger$, $i\to -i$

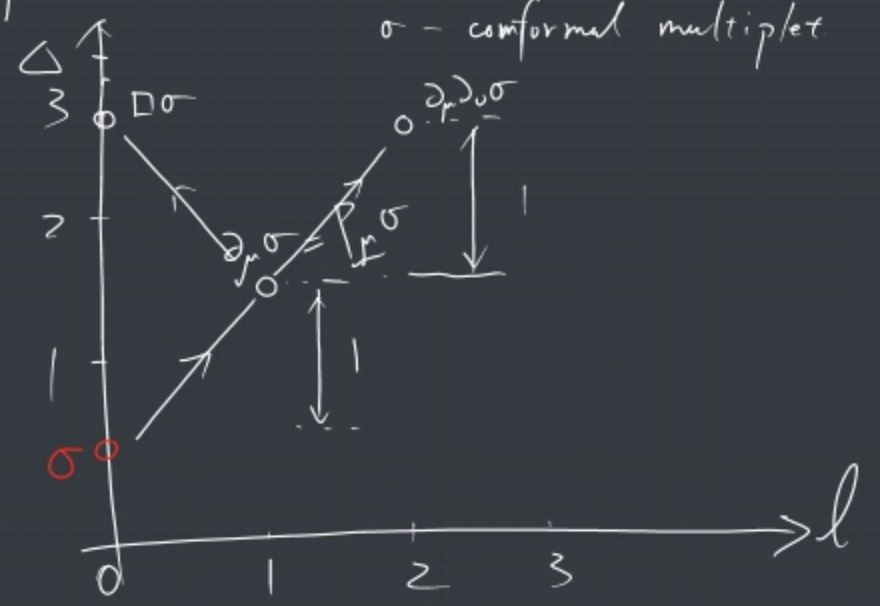
improper Z_2 $\det(T_{ph})=-1$

$\Updownarrow$

3D space-time parity sym.

Ising $Z_2=-1$

parity $Z_2=+1$



$$\partial_\mu T_{\mu\nu}=0$$

$$P_\mu T_{\mu\nu}=0$$