- PROPERTIES OF CFTs $\rightarrow$ CLASSIFY THEM

$\downarrow$

· RECONSTRUCT ALL CORR

$\hookrightarrow$ n point correlators

2-pt fn $\leftarrow$ $\{\Delta, f_{ijk}\}$ $\rightarrow$ 3pt fnc

$\uparrow$ conf dim

ALL PRIMARY OPERATORS

- OPE

- 4 pt function $\longrightarrow$ 2 different ways of taking the OPE

$$\langle \underbrace{O(x_1)\,O(x_2)}\,\underbrace{O(x_3)\,O(x_4)}\rangle = \langle O(x_1)\,\underbrace{O(x_2)\,O(x_3)}\,O(x_4)\rangle$$

Δ_0

Associativity of the OPE

$$g(u,v) = \left(\frac{u}{v}\right)^{\Delta_0} g(v,u)$$

$$\boxed{O \times O \propto \sum_{\Delta,\ell} f_{OOO_{\Delta,\ell}} (O_{\Delta,\ell} + \text{desc})}$$

BOOTSTRAP EQUATION
$$\sum_{\Delta,\ell} f^2_{OOO_{\Delta,\ell}}\, G_{\Delta,\ell}(u,v) = \left(\frac{u}{v}\right)^{\Delta_0} \sum_{\Delta,\ell} f^2_{OOO_{\Delta,\ell}}\, G_{\Delta,\ell}(v,u)$$

Aim "compute" Δ, $f^2_{OOO_{\Delta,\ell}}$

$u = z\bar{z}$ $\quad v = (1-z)(1-\bar{z})$ $\quad$ Lorentzian regime

double light cone limit : $u \to 0$, $v \to 0$

$z \to 0$, $\bar{z} \to 1$ with $z \ll 1-\bar{z} \ll 1$

1) $\tau = \Delta - \ell$ $\rightarrow$ conformal twist

$$(*) \quad G_{\Delta,\ell}(z,\bar{z}) = (z\bar{z})^{\frac{\tau}{2}}\, \tilde{G}_{\Delta,\ell}(z,\bar{z})$$

$$G_{\Delta,\ell}(z,\bar{z}) \xrightarrow[u \to 0]{} (z\bar{z})^{\tau/2} \left(1 - (1-z)(1-\bar{z})\right)^{\ell} {}_2F_1\left(\frac{\tau}{2}+\ell, \frac{\tau}{2}+\ell, \tau+2\ell, \ldots\right)$$

$$G_{\Delta,\ell}(z,\bar{z}) \xrightarrow[\substack{v \to 0 \\ \bar{z} \to 1}]{} \underbrace{a(u,v)\,\log v + b(u,v)}_{\substack{\text{functions of } u \text{ and } v \text{ which are not} \\ \text{singular in the limit } u \to 0,\ v \to 0}}$$

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1) 4 pt functions of identical operators

$f^2_{OO1} = 1$ $\qquad O \times O = 1 + \ldots$ $\quad (\Delta = 0, \ell = 0)$

$$\sum_{\tau,\ell} f^2_{OOO_{\tau,\ell}}\, u^{\tau/2}\, \tilde{G}_{\tau,\ell}(u,v) = \left(\frac{u}{v}\right)^{\Delta_0} \left(\sum_{\tau',\ell'} f^2_{OOO_{\tau',\ell'}}\, v^{\tau'/2}\, \tilde{G}_{\tau',\ell'}(v,u)\right)$$

$f_{OO1} = 1$

$$1 + \sum_{\tau,\ell} f^2_{OOO_{\tau,\ell}}\, u^{\tau/2}\, \tilde{G}_{\tau,\ell}(u,v) = \left(\frac{u}{v}\right)^{\Delta_0} \left(\sum_{\tau',\ell'} f^2_{OOO_{\tau',\ell'}}\, v^{\tau'/2}\, \tilde{G}_{\tau',\ell'}(v,u)\right)$$

- Limit $u \to 0$

$$1 \sim \left(\frac{u}{v}\right)^{\Delta_0} \sum_{\tau',\ell'} f^2_{OOO_{\tau',\ell'}}\, v^{\tau'/2}\, \tilde{G}_{\tau',\ell'}(v,u)$$

$$\boxed{\frac{1}{u^{\Delta_0}}} \sim \frac{1}{v^{\Delta_0}} \boxed{\sum_{\tau',\ell'}} f^2_{OOO_{\tau',\ell'}}\, v^{\tau'/2}\, \underbrace{\tilde{G}_{\tau',\ell'}(v,u)}_{\sim \log u}$$

- lhs : power law div. $u^{-\Delta_0}$

rhs : single block $\rightarrow$ $\log u$

how can we reconcile lhs and rhs?

- $\sum_{\tau',\ell'}$ is not convergent for all real u

$\sqrt{u} < 0$, the sum diverges $\longrightarrow$ analitically continue the sum to the region of convergence, it can be seen that there is a powerlaw behaviour $u^{-\Delta_0}$

- Do we have any parameter controlling the divergence? the limit ℓ' large and τ' fixed

$\boxed{\text{rhs}}$
$$\sum_{\ell',\tau'} (-1)^{\ell'} f^2_{\tau',\ell'}\, v^{\frac{\tau'}{2}-\Delta_0} (1-u)^{\ell'}\, {}_2F_1\left(\ell'+\frac{\tau'}{2}, \ell'+\frac{\tau'}{2}, 2\ell'+\tau', 1-u\right)$$

$\boxed{\substack{\ell' \to \infty \\ \tau' \text{ fixed}}}$

$$v^{\tau'/2}\, u^{-\Delta_0} \left(A(\tau', \Delta_0) \right)$$

lhs

$$\sim \frac{1}{u^{\Delta_0}} \qquad \frac{\tau}{2} - \Delta_0 = 0 \rightarrow \boxed{\tau' = 2\Delta_0}$$

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1) observation : the identity operator always appears in the OPE of 2 identical operators

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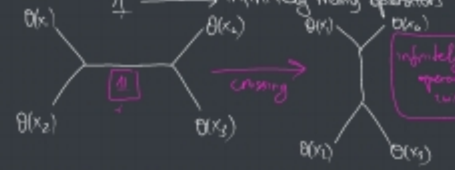
$u \to 0$, $v \to 0$

LHS $\longrightarrow$ RHS

$1 \longrightarrow$ infinitely many operators



Any CFT

infinitely many ℓ' operators with $\ell' \to \infty$ twist $\tau' = 2\Delta_0$

Lorentzian Inversion Formula

$$\underbrace{g(u,v)} = \sum_{\Delta,\ell} \underline{f^2_{OOO_{\Delta,\ell}}}\, G_{\Delta,\ell}(u,v)$$

- If there is any way that allow us to extrac (Δ,ℓ) and $f^2_{OOO_{\Delta,\ell}}$ from $g(u,v)$?

Yes (Caron Huot)

$$\langle O(x_1) O(x_2) O(x_3) O(x_4) \rangle$$

t-channel, u-channel, s-channel

$$\boxed{d=4}$$

spectral function

$$C_{\ell,\Delta} = C^{(t)}_{\ell,\Delta} + (-1)^{\ell}\, C^{(u)}_{\ell,\Delta}$$

(Δ,ℓ,f^2)

$$\boxed{C^{(t)}_{\ell,\Delta}} = \frac{1}{4} K_{\frac{\Delta+\ell}{2}} \int_0^1 dz\, d\bar{z} \left(\frac{z-\bar{z}}{z\bar{z}}\right)^2 \frac{G_{\ell+3,\Delta-3}(z,\bar{z})}{z\bar{z}} \times \boxed{\mathrm{dDisc}[g(u,v)]}$$

$$\mathrm{dDisc}[g(z,\bar{z})] = g_{eucl}(z,\bar{z}) \underbrace{- \frac{1}{2} g^{\circlearrowright}(z,\bar{z}) - \frac{1}{2} g^{\circlearrowleft}(z,\bar{z})}_{\text{analytic cont. around } \bar{z}=1}$$

$\downarrow$

singularities of $g(u,v)$ around $\bar{z}=1$ $(v=0)$

$$\boxed{C_{\ell,\Delta}} \xrightarrow[\Delta\to\Delta_k]{} \boxed{\frac{f^2_{OOO_{\Delta,\ell}}}{\Delta-\Delta_k}}$$

it has poles at the position of the exchanged operators, residues corresponding f^2.

Singularities around $\bar{z}=1$ of $g(z,\bar{z})$ $\Longrightarrow$ CFT data Δ_k, ℓ_k, $f^2_{OOO_{\Delta,k}}$