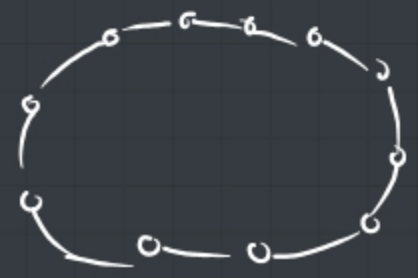
- state-operator correspondence (1984, 1985)

$$\underline{S^{d-1} \times R}$$

spatial, time

$d=2$ $\quad \underline{S}^1 \times R^1$



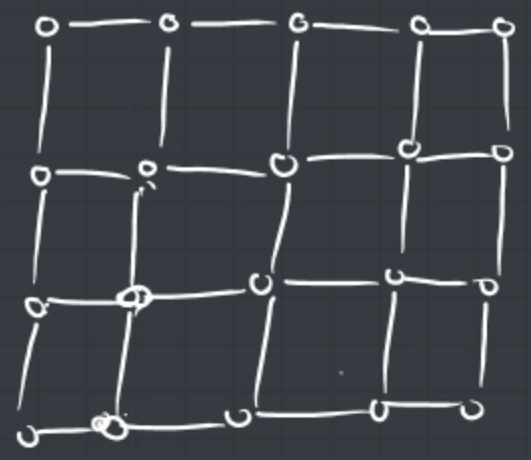
$\simeq S^1$ $\quad \{E_n - E_0\}$

Cardy 1986

$$\underline{E_n - E_0} \simeq v \cdot \underline{(\Delta_n - \Delta_0)}$$

"$d=3$"? $\quad \underline{S^2 \times R^1}$ (?)

PBC



$\simeq T^2$

$$\langle \phi(x)\, \phi(0) \rangle = \frac{1}{|x|^{2\Delta}}$$

$S^2 \longrightarrow$ fuzzy sphere

discretization

"regular" $\quad$ "irregular"

✗ $\quad$ ✓ (2013)

- Haldane (fuzzy) sphere (1983)

$$\vec{B} = \frac{2s_0\,\phi_0}{4\pi R^2}\,\hat{e}_r \qquad \phi_0 = \frac{h}{e}$$

$$\vec{A} = -\frac{2s_0\,\phi_0}{4\pi R}\cot\theta\,\hat{e}_\varphi$$

$$\nabla\times\vec{A} = \vec{B}$$

$$\nabla = \hat{e}_r\frac{\partial}{\partial r} + \hat{e}_\theta\frac{\partial}{r\,\partial\theta} + \hat{e}_\phi\frac{\partial}{r\sin\theta\,\partial\phi}$$

$$H = \frac{\hbar^2}{2mR^2}|\vec{\Lambda}|^2$$

$$\vec{\Lambda} = R\,\hat{e}_r\times(-i\nabla + e\vec{A}) \qquad \underline{\vec{\Lambda}\cdot\vec{r} = 0}$$

$$\Lambda_i = \underset{\substack{\uparrow\\ \text{angular momentum}}}{M_i} + e\,\epsilon_{ijk}r_jA_k$$

$$[\Lambda_i, \Lambda_j] = [M_i, M_j] + [M_i, r_cA_d]e\epsilon_{jcd} - [M_j, r_kA_h]e\epsilon_{ikl}$$

$$= i\hbar\epsilon_{ijk}(\Lambda_k - s_0r_k)$$

$$[\Lambda_i, r_j] = i\epsilon_{ijk}r_k$$

$$\vec{L} \equiv \vec{\Lambda} + s_0\vec{r} \Rightarrow L^2 = (\vec{\Lambda} + s_0\vec{r})^2 = |\vec{\Lambda}|^2 + \cancel{\vec{\Lambda}\cdot\vec{r} + \vec{r}\cdot\vec{\Lambda}} + s_0^2$$

$$\Rightarrow [L_i, L_j] = i\epsilon_{ijk}L_k \qquad \text{"SU(2) algebra"}$$

$\vec{L}^2$ Casimir $s(s+1)$

$$\underline{|\vec{\Lambda}|^2} = \vec{L}^2 - s_0^2 \sim s(s+1) - s_0^2$$

$$\Rightarrow \text{eigenvalue} \quad E = \underline{\frac{\hbar^2}{2mR^2}[s(s+1) - s_0^2]}$$

$$\vec{L}^2\,Y^{(s_0)}_{s,m} = s(s+1)\,Y^{(s_0)}_{s,m}$$

$$L_z\,Y^{(s_0)}_{s,m} = m\,Y^{(s_0)}_{s,m}$$

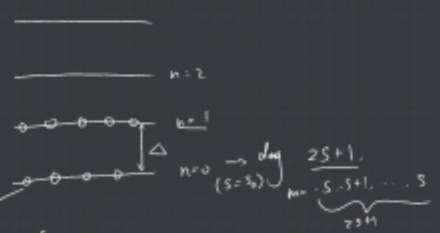
$$\boxed{\begin{array}{c} Y^{(s_0)}_{s,m}(\theta,\varphi) \\ \text{"monopole harmonics"} \\ \text{Wu \& Yang 1975, 1976} \end{array}}$$

$$\begin{array}{c}(s=s_0)\\ n=0\end{array} \quad Y^{(s_0)}_{s,m}(\theta,\varphi) = \sqrt{\frac{(2s+1)!}{4\pi(s-m)!(s+m)!}}\,u^{s+m}\,v^{s-m} \qquad \underline{m = -s, -s+1, \cdots, s}$$

$$u = \cos\frac{\theta}{2}e^{i\phi/2}, \quad v = \sin\frac{\theta}{2}e^{-i\phi/2}$$

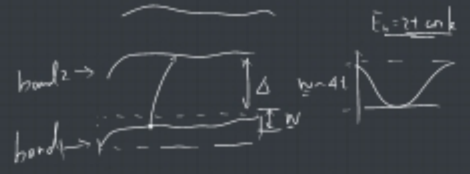
$s = s_0 + n$

Landau level



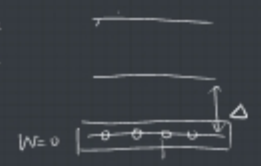
$\Delta > 0$

Landau level projection (LLL)



condition

$$\underline{W \ll U \ll \Delta}$$



$$\underline{W=0 \ll U \ll \Delta}$$

$$H = \frac{1}{2}\int dr \int dr' \, \bar{\psi}(r)\bar{\psi}(r')\, V(r-r')\, \psi(r')\psi(r) \leftarrow \text{real space}$$

LLL:

$$\bar{\psi}(r) \simeq \sum_{m=-S}^{S} \underbrace{Y^{(S_0)}_{S,m}(\theta,\varphi)}_{\varphi_k(r)} \cdot C^{+}_{m}$$

orbital space

$$H = \frac{1}{2}\sum_{m_1 m_2 m_3 m_4} C^{+}_{m_1} C^{+}_{m_2} C_{m_3} C_{m_4} \langle m_1, m_2 | V | m_3, m_4 \rangle$$

$$V(r_1 - r_2) = \sum_{k=0}^{\infty} U_k \, P_k(\cos\theta_{12}) \quad , \quad P_k(\cos\theta_{12}) \equiv \sum_{m=-l}^{l} Y^{*}_{lm}(\theta_a,\varphi_a) Y_{lm}(\theta_b,\varphi_b)$$

① $$V_1(r_1-r_2) = \frac{1}{R^2}\delta(|r_1-r_2|) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} Y^{*}_{lm}(\theta_a,\varphi_a)\, Y_{l,m}(\theta_b,\varphi_b)$$

② $$V_2(r_1-r_2) = \frac{1}{R}\nabla^2\delta(|r_1-r_2|) = \sum_{l}\sum_{m}(-l(l+1))\, Y^{*}(\theta,\varphi)\, Y(\theta,\varphi)$$

③ $$V_3(r_1-r_2) = \frac{1}{|r_1-r_2|} \qquad \Omega = (\theta,\varphi)$$

$$\langle m_1, m_2 | V | m_3, m_4 \rangle = \int d\Omega_1 d\Omega_2 \, \bar{Y}^{(S_0)}_{S m_1}(\Omega_1)\, \bar{Y}^{(S_0)}_{S m_2}(\Omega_2) \left(\sum_{k} U_k \sum_{m=-k}^{k} Y_{lm}(\Omega_1) Y_{l,m}(\Omega_2)\right) Y^{(S_0)}_{S m_3}(\Omega_2)\, Y^{(S_0)}_{S m_4}(\Omega_1)$$

Wu - Yang (1976)

$$s = S_0 = Q_i$$

$$\int d\Omega \, Y^{(Q_1)}_{S_1,m_1}(\Omega)\, Y^{(Q_2)}_{S_2,m_2}(\Omega)\, Y^{(Q_3)}_{S_3,m_3}(\Omega) = (-)^{S_1+S_2+S_3}\left[\frac{(2S_1+1)(2S_2+1)(2S_3+1)}{4\pi}\right]^{1/2} \begin{pmatrix} S_1 & S_2 & S_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} S_1 & S_2 & S_3 \\ Q_1 & Q_2 & Q_3 \end{pmatrix}$$