$$\mathcal{L} = \underbrace{\mathcal{L}_0}_{\text{quadratic in fields terms}} + \mathcal{L}_I$$

$\leftarrow$ higher deg terms.

$$\varphi^4: \quad \mathcal{L} = \underbrace{\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2}_{\mathcal{L}_0} \underbrace{- \frac{1}{4!}\lambda\varphi^4}_{\mathcal{L}_I}$$

$$\frac{1}{2}\varphi\underbrace{(-\partial^2 - m^2)}_{\downarrow\ \frac{i}{p^2-m^2}}\varphi$$

$-i\lambda$ 

$g-2$ – anomalous magn mom

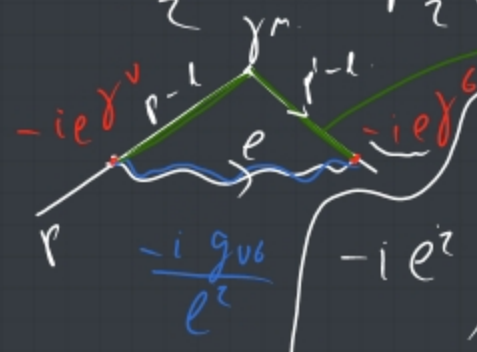
$\underbrace{j(q)}\, A(q)$ $\qquad j(q) = \bar{U}(p')\left(\underbrace{F_1(q^2)}\gamma^\mu - \underbrace{F_2(q^2)}\frac{\sigma^{\mu\nu}q_\nu}{2m}\right)U(p)$

q – momentum transfer $\qquad \hat{p} = p_\mu \gamma^\mu$

$\frac{g-2}{2} = F_2(0)$

$\bar{U}(p')\,\gamma^\mu\, U(p)$

$F_1^{(0)} = 1,\quad F_2^{(0)} = 0$



$$-ie^2\int\frac{d^4 l}{(2\pi)^4}\,\bar{U}(p')\gamma^\sigma\,\frac{\hat{p}'-\hat{l}+m}{(p'-l)^2-m^2}\,\gamma^\mu\,\frac{\hat{p}-\hat{l}+m}{(p-l)^2-m^2}\,\gamma^\nu U(p)\,\frac{g_{\nu\sigma}}{l^2}$$

$$\frac{1}{\prod_k A_k^{n_k}} = \frac{\Gamma(\sum_k n_k)}{\prod_k \Gamma(n_k)}\int\frac{\prod_k dx_k\, x_k^{n_k-1}}{\left[\sum_k A_k x_k\right]^{\sum n_k}}\,\delta\left(1-\sum_k x_k\right)$$

$$\frac{1}{(p'-l)^2-m^2}\;\frac{1}{l^2}\;\frac{1}{(p-l)^2-m^2} =$$

$\boxed{\bar{z} = 1-z}$

$$= 2\int\frac{dz\,dx\,x}{\left[l^2 - 2l(zp+\bar{z}p')\right]^3}$$

$$j^{(1)} = -2ie^2\int_0^1 dx\,x\int_0^1 dz\,\frac{d^4 k}{\left[-k^2+(zp+\bar{z}p')^2x^2\right]^3}\quad\otimes$$

$$\bar{U}(p')\Big[\underbrace{\gamma_\mu\left[2x^2(m^2+z\bar{z}q^2)-2x(2m^2-q^2)-k^2\right]}_{\gamma_\mu F_1^{(1)}} - \frac{\sigma_{\mu\nu}q^\nu}{2m}\underbrace{(4x\bar{x}m^2)}_{F_2^{(1)}}\Big]U(p)$$

$$\boxed{F_2^{(1)}(0) = \frac{\alpha}{2\pi}}$$

$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ $\qquad q = p'-p$

$\cancel{d=4}$ $\qquad d = 4-2\epsilon$ $\qquad \epsilon$ – small parameter

Dimensional regularization

Parametric repr.

$$D_k = p_k^2 - m^2$$

$$\int \frac{d^d l_1 \dots d^d l_L}{i\pi^{d/2} \dots i\pi^{d/2}} \prod_{k=1}^{N} \frac{1}{D_k^{n_k}} = \int \frac{d^d l_1 \dots d^d l_L}{i\pi^{d/2} \dots i\pi^{d/2}} \frac{\Gamma\left(\sum_k n_k\right)}{\prod_k \Gamma(n_k)} \int \frac{\prod_{k=1}^{N} dx_k\, x_k^{n_k-1} \delta(1-\sum x)}{\left[\sum x_k D_k\right]^{\sum n_k}}$$

$$\sum_k D_k x_k = A_{ij}(l_i \cdot l_j) + 2 B_i \cdot l_i + C$$

$$A_{ij} = S^T_{ik} S_{kj}$$

$$l_i \to l_i - (A^{-1})_{ij} B_j$$

$$\sum D_k x_k \to A_{ij} l_i \cdot l_j - B_i (A^{-1})_{ij} B_j + C$$

$$l_i \to (S^{-1})_{ij} l_i$$

$$d^d l = dl \cdot |l|^{d-1} \cdot \Omega_d$$

$$\boxed{\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}} \text{ – solid angle in d dimension}$$

$$\int \frac{\prod d^d l_i}{\left[\sum_i l_i^2 + C - B_i (A^{-1})_{ij} B_j\right]^{\sum n_k}}$$

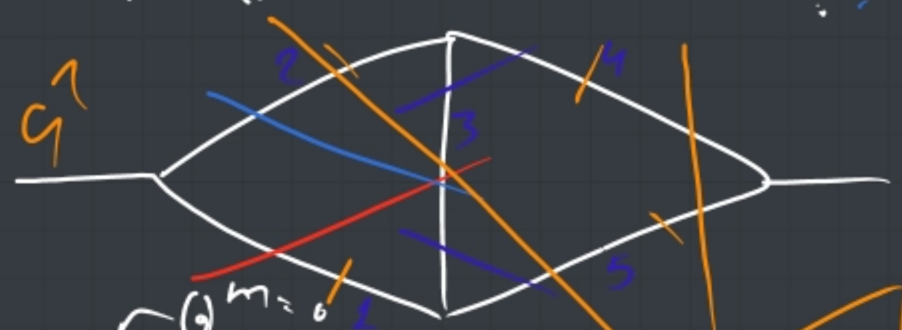
$\Omega_4 = 2\pi^2$

$\Omega_3 = 4\pi$, $\Omega_2 = 2$, $\Omega_1 = 2$

Feynman ~~parametrization~~ representation

$$\int \frac{dl_1 \dots dl_L}{(i\pi^{d/2})^L} \frac{1}{D_1^{n_1}\dots D_N^{n_N}} = \Gamma\left(\sum_k n_k - L\frac{d}{2}\right) \int_0^\infty \prod_k \frac{dx_k\, x_k^{n_k-1}}{\Gamma(n_k)} \frac{F^{L\frac{d}{2}-\sum n_k}}{U^{(L+1)\frac{d}{2}-\sum n_k}} \delta\left(1-\sum x_k\right)$$

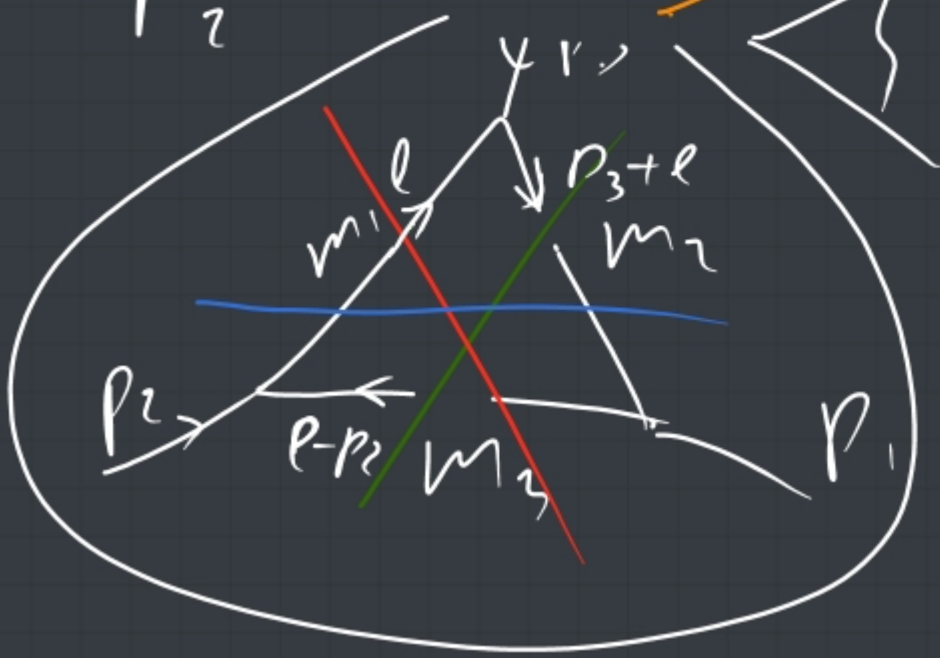
$$\boxed{U = \det A, \quad F = \left(C - B_i (A^{-1})_{ij} B_j\right) U}$$



$F_2^{(m=0)} = 0$

$U = x_1x_3 + x_2x_3 + x_3x_5 + x_3x_4 + x_1x_4 + x_2x_5$

Symanzik polys



$L=1$

$U = x_2 + x_1 + x_3$

$F = M\cdot U - V$

$x_2x_3x_5 q^2$

$M = \sum_k m_k^2 x_k = x_1m_1^2 + x_2m_2^2 + x_3m_3^2$

$V = x_2x_3\, p_1^2 + x_1x_3\, p_2^2 + x_1x_2\, p_3^2$

$D_1 = l^2 - m_1^2$