Type II Seesaw leptogenesis

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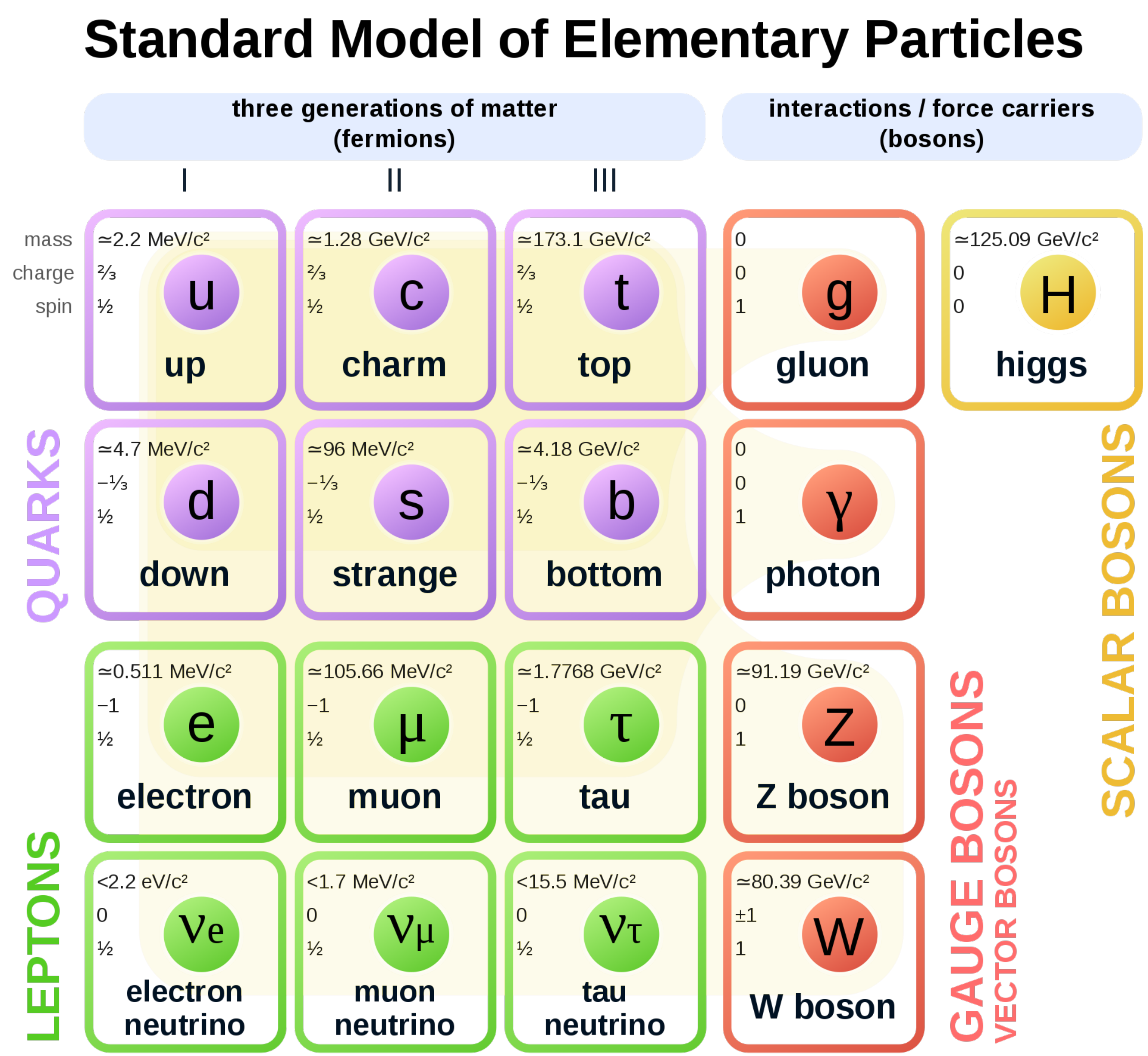
arXiv:2106.03381(Phys. Rev. Lett. 128, 141801) and
arXiv:2204.08202(JHEP 05 (2022) 160)

第十一届威海新物理研讨会
2022.8.2

Standard model



Very successful describing low energy scale physics

Observation requiring new physics

- Inflation
- Neutrino masses
- Baryon asymmetry of our universe

today's talk

- Dark matter
- Others(muon g-2?)

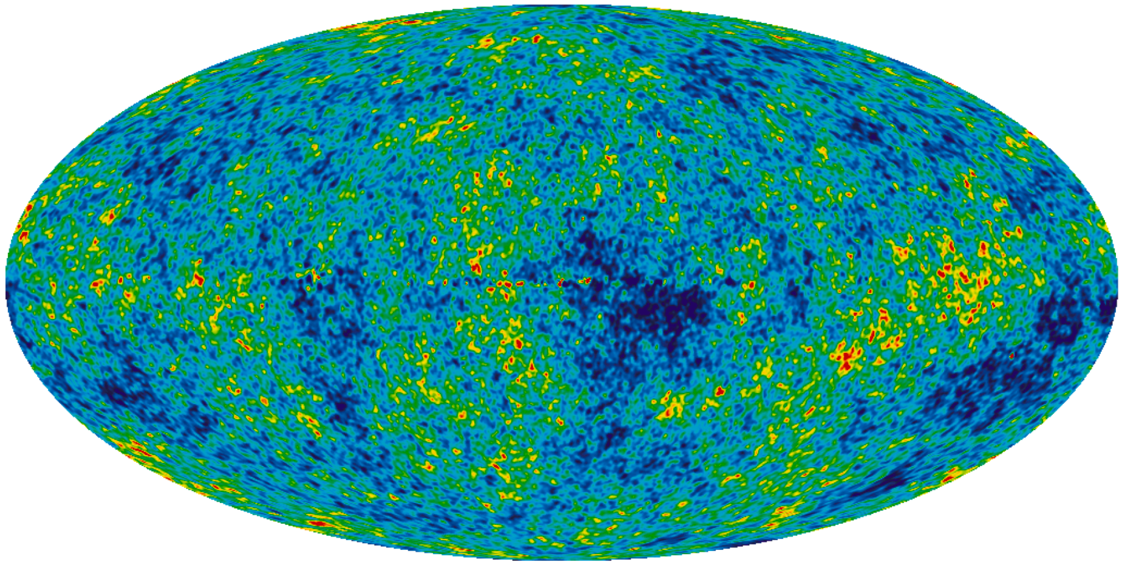
Inflation

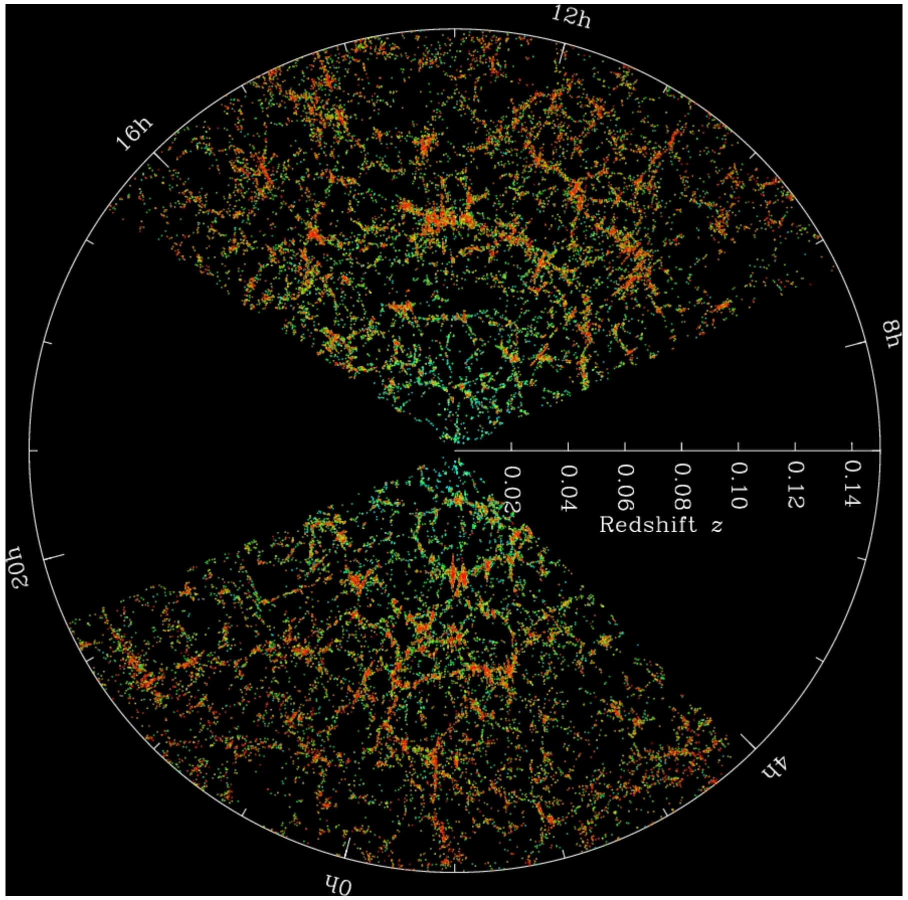
Rapid expansion of the universe in the early time

- Flatness problem
- Horizon problem
- Monopole problem?
- Seeding the primordial anisotropies in CMB

Inflation

Generating quantum fluctuations(anisotropies in CMB)





$$\frac{\delta T}{T} \sim 10^{-5}$$

Such small fluctuations finally develops the large structure of our universe

Slow-roll inflation

Assuming a scalar field, with equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

Slow roll condition $\dot{\phi}^2 \ll V(\phi) \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|\,, |V_{,\phi}|$

$$\epsilon_{\rm v}(\phi) \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta_{\rm v}(\phi) \equiv M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V} \qquad \epsilon_{\rm v}, |\eta_{\rm v}| \ll 1$$

$$H^2 \approx \frac{1}{3}V(\phi) \approx \text{const.}$$

$$\dot{\phi} \approx -\frac{V_{,\phi}}{3H}\,,$$

$\longrightarrow$ $a(t) = a_0 e^{Ht} \qquad Ht \gtrsim 60$

Daniel Baumann, TASI Lectures on Inflation

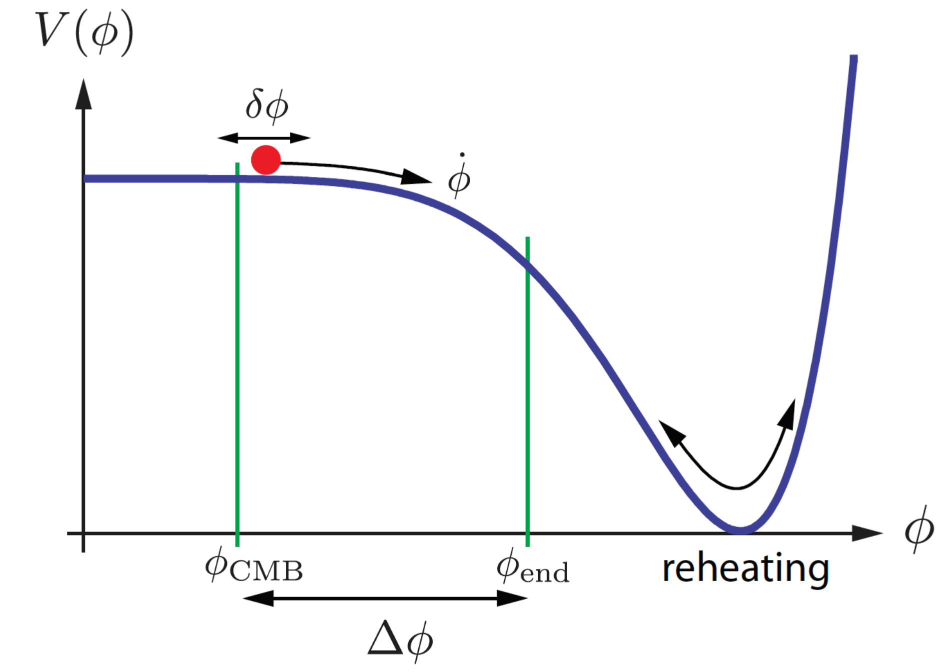
Slow-roll inflation

标量扰动

$$\Delta_s^2(k) \approx \frac{1}{24\pi^2}\frac{V}{M_{pl}^4}\frac{1}{\epsilon_V}\bigg|_{k=aH}$$

$$n_s - 1 \equiv \frac{d\ln\Delta_s^2}{d\ln k} = 2\eta_V - 6\epsilon_V$$

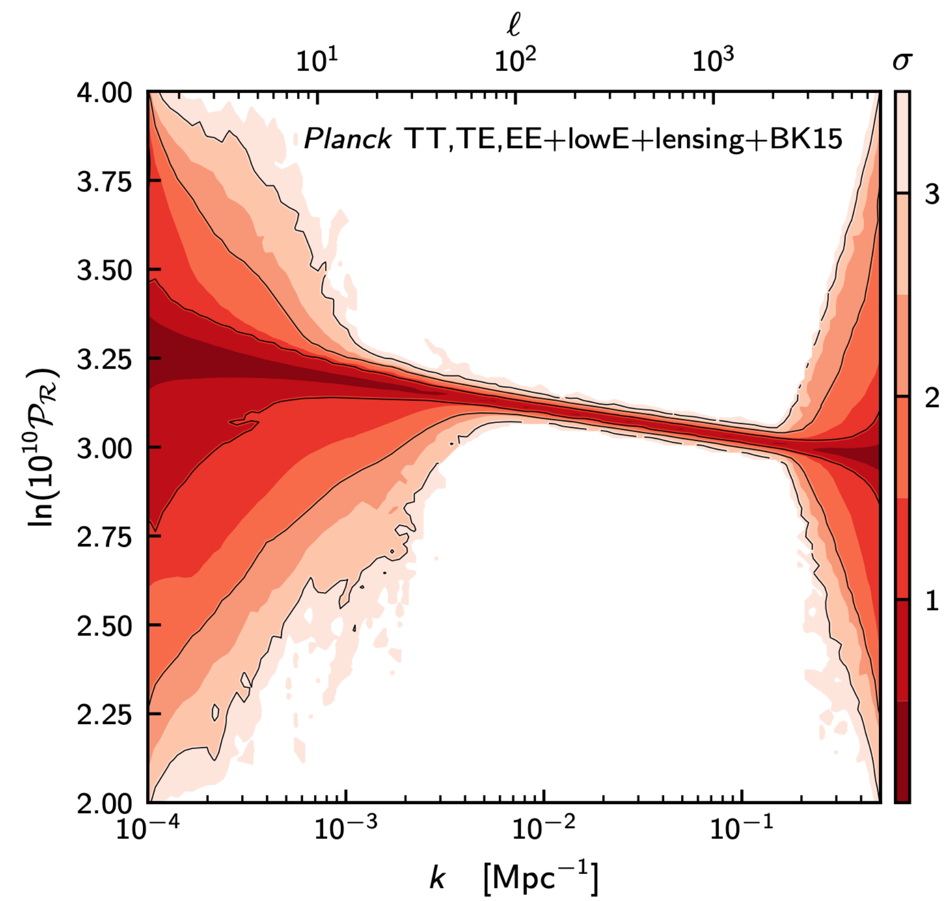
$n_s \simeq 0.965$ n=1 to be scale invariant



张量扰动

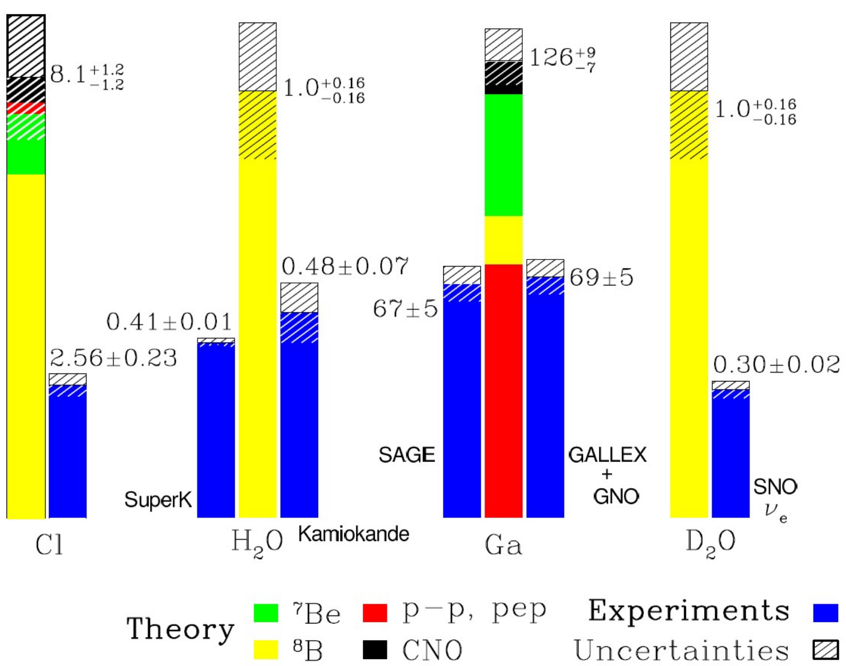
$$\Delta_t^2(k) \approx \frac{2}{3\pi^2}\frac{V}{M_{pl}^4}\bigg|_{k=aH} \qquad r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon_V$$

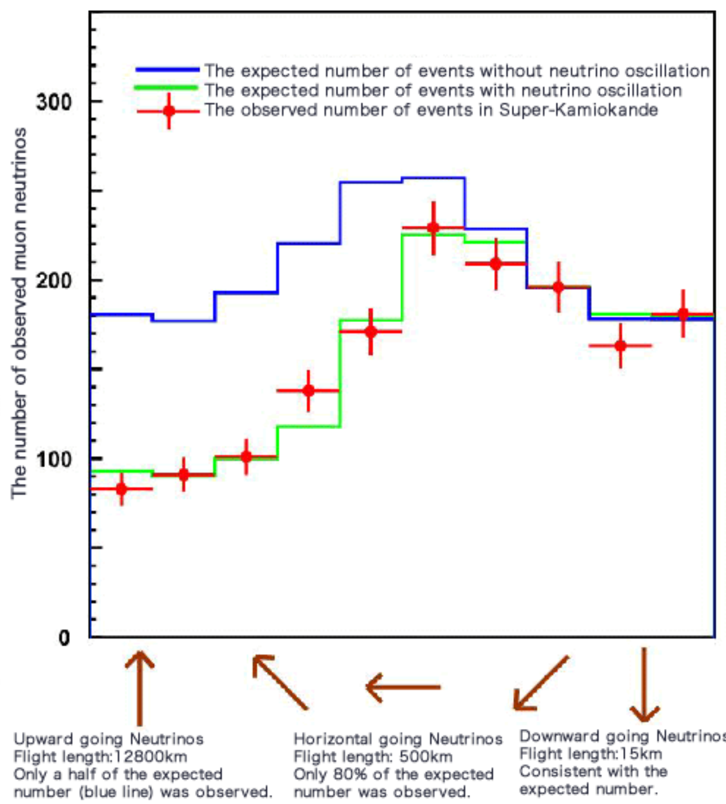
tensor-scalar ratio $r \lesssim 0.056$

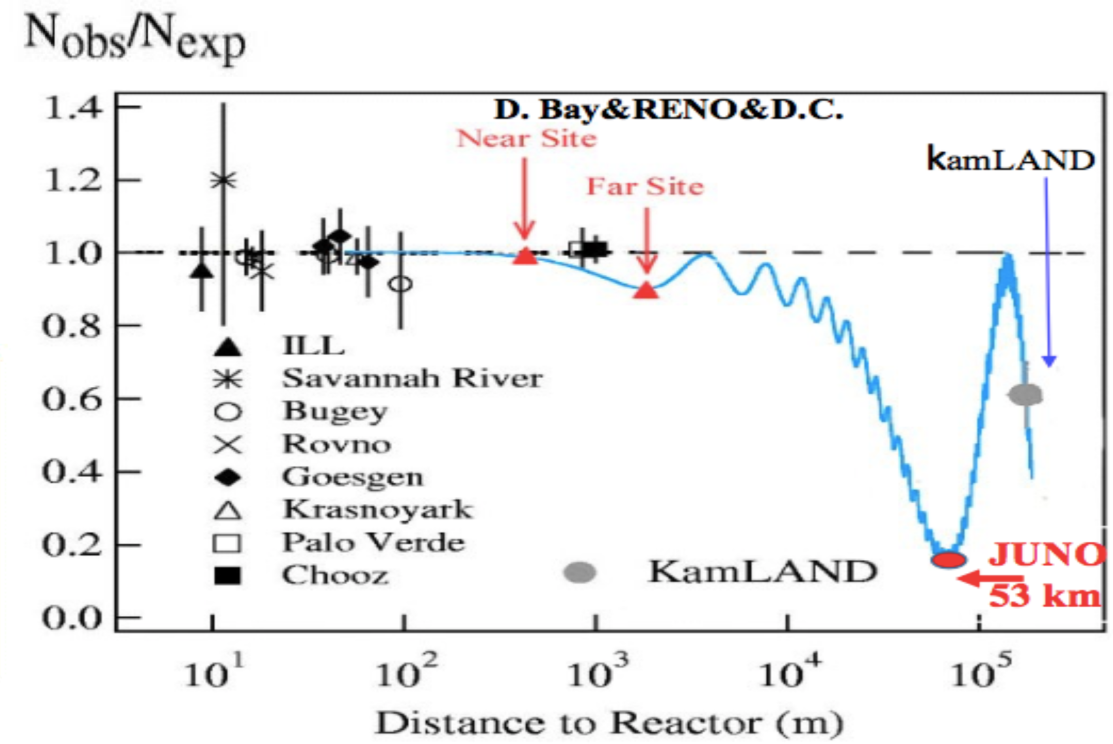


Neutrino masses

Neutrino oscillation requiring massive neutrinos







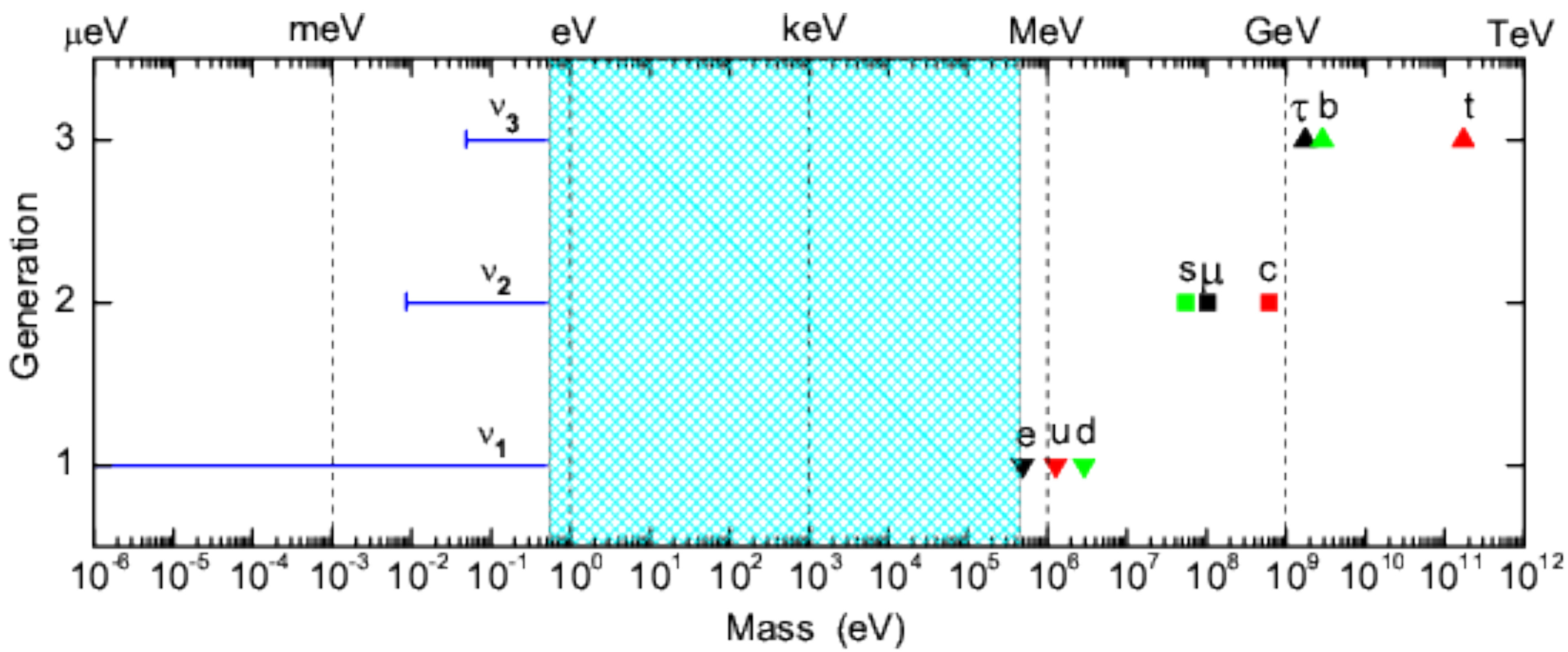
Solar Neutrino oscillations	Atmospheric Neutrino Oscillations	Reactor Neutrino Oscillations
θ_{12}	θ_{23}	θ_{13}

$$\Delta m^2_{21} \simeq 7.42 \times 10^{-5}\ \mathrm{eV}^2 \qquad |\Delta m^2_{13}| \approx |\Delta m^2_{23}| \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2$$

At least a neutrino mass larger or similar to 0.05 eV

Neutrino masses vs other fermion masses

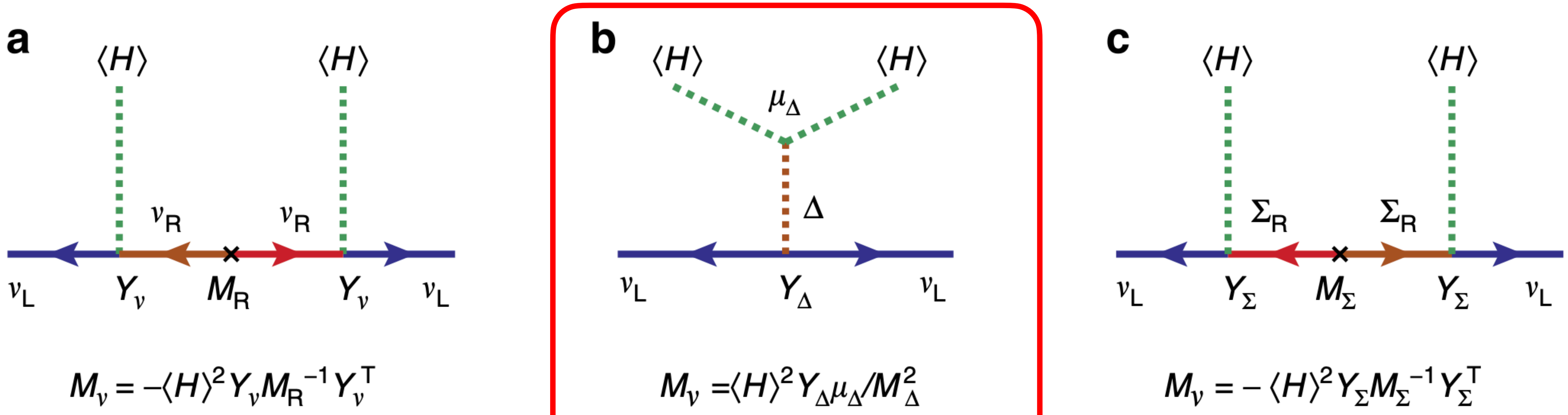
A large hierarchy comparing with other fermion masses



Origin of neutrino masses

Three types of seesaw model(tree level)

Tommy Ohlsson, Shun Zhou, Nature Commun. 5 (2014) 5153



SM + 3 singlets fermions

SM + 1 triplet Higgs

SM + 3 triplet fermions

Providing a minimal framework

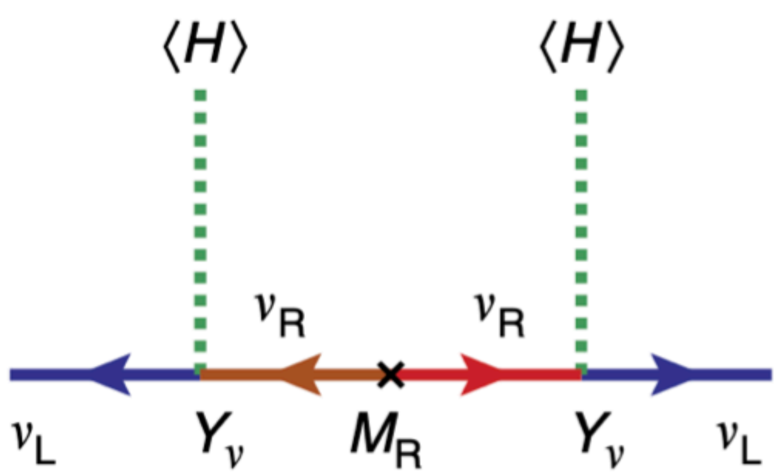
Origin of neutrino masses: type I seesaw

加入三个单态中性右手中微子 N(1, 1, 0)

$$\mathcal{L} = \mathcal{L}_{SM} + y_\nu \tilde{H} \bar{L} N - M_R \bar{N}^c N$$

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

$$m_\nu \sim \frac{m_D^2}{M_R} = \frac{1}{2} \frac{y_\nu^2 < H >^2}{M_R}$$



$$M_\nu = -\langle H \rangle^2 Y_\nu M_R^{-1} Y_\nu^T$$

中微子质量被压低！

Origin of neutrino masses: type II seesaw

引入一个希格斯三重态跟中微子直接耦合

$$H(2,1/2),\ \Delta(3,1),\ L(2,-1/2)$$

$$H = \begin{pmatrix} h^+ \\ h \end{pmatrix},\quad \Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

$$\mathcal{L}_{Yukawa} = \mathcal{L}^{\rm SM}_{\rm Yukawa} - \boxed{\frac{1}{2} y_{ij} \bar{L}_i^c \Delta L_j} + h.c.$$

$$\downarrow$$

$$\frac{1}{2} y_{ij} \Delta^0 \bar{\nu}^c \nu + h.c.$$

- Giving neutrino mass matrix with vev of Delta
- at the same time Delta get a lepton number -2

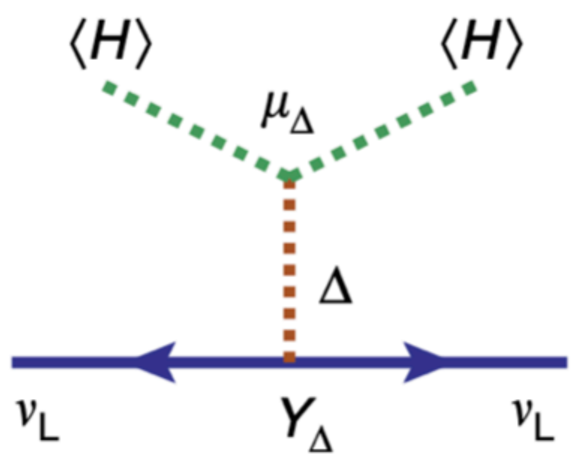
Origin of neutrino masses: type II seesaw

$$H(2,1/2),\ \Delta(3,1),\ L(2,-1/2)$$

$$\begin{aligned} V(H,\Delta) \;=\; & -m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + m_\Delta^2 \mathrm{Tr}(\Delta^\dagger \Delta) + \lambda_1 (H^\dagger H)\mathrm{Tr}(\Delta^\dagger\Delta) \\ & + \lambda_2 (\mathrm{Tr}(\Delta^\dagger\Delta))^2 + \lambda_3 \mathrm{Tr}(\Delta^\dagger\Delta)^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H \\ & + \left[\mu(H^T i\sigma^2 \Delta^\dagger H) + h.c.\right] + \ldots \end{aligned}$$

U(1)L breaking term

$$\langle \Delta^0 \rangle \simeq \frac{\mu v_{\mathrm{EW}}^2}{2 m_\Delta^2}$$



$M_\nu = \langle H \rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2$

电弱精确测量限制

$$\mathcal{O}(1)\ \mathrm{GeV} > |\langle \Delta^0 \rangle| \gtrsim 0.05\ \mathrm{eV}$$

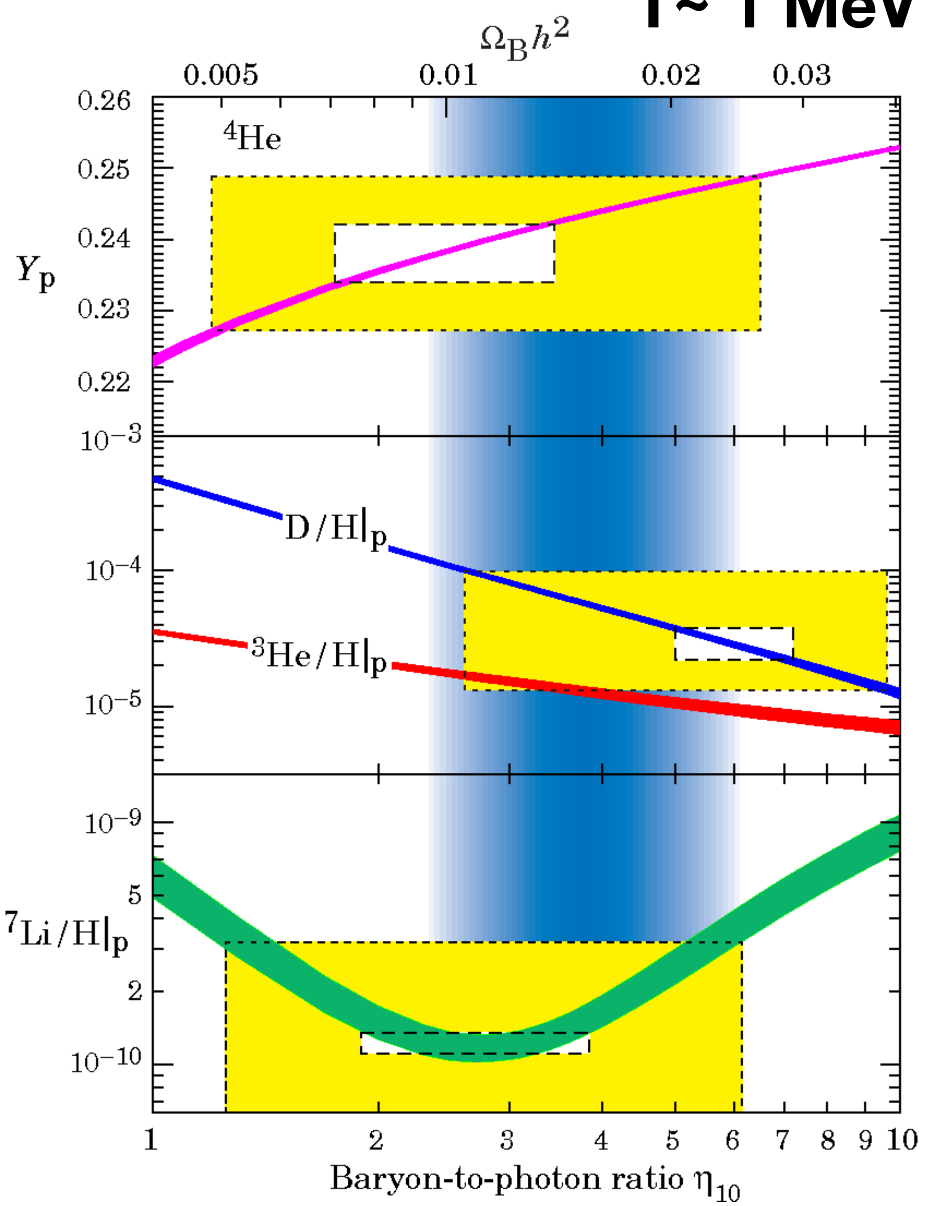
中微子质量要求

Neutrino masses connecting another important problem:

Baryon asymmetry of our universe

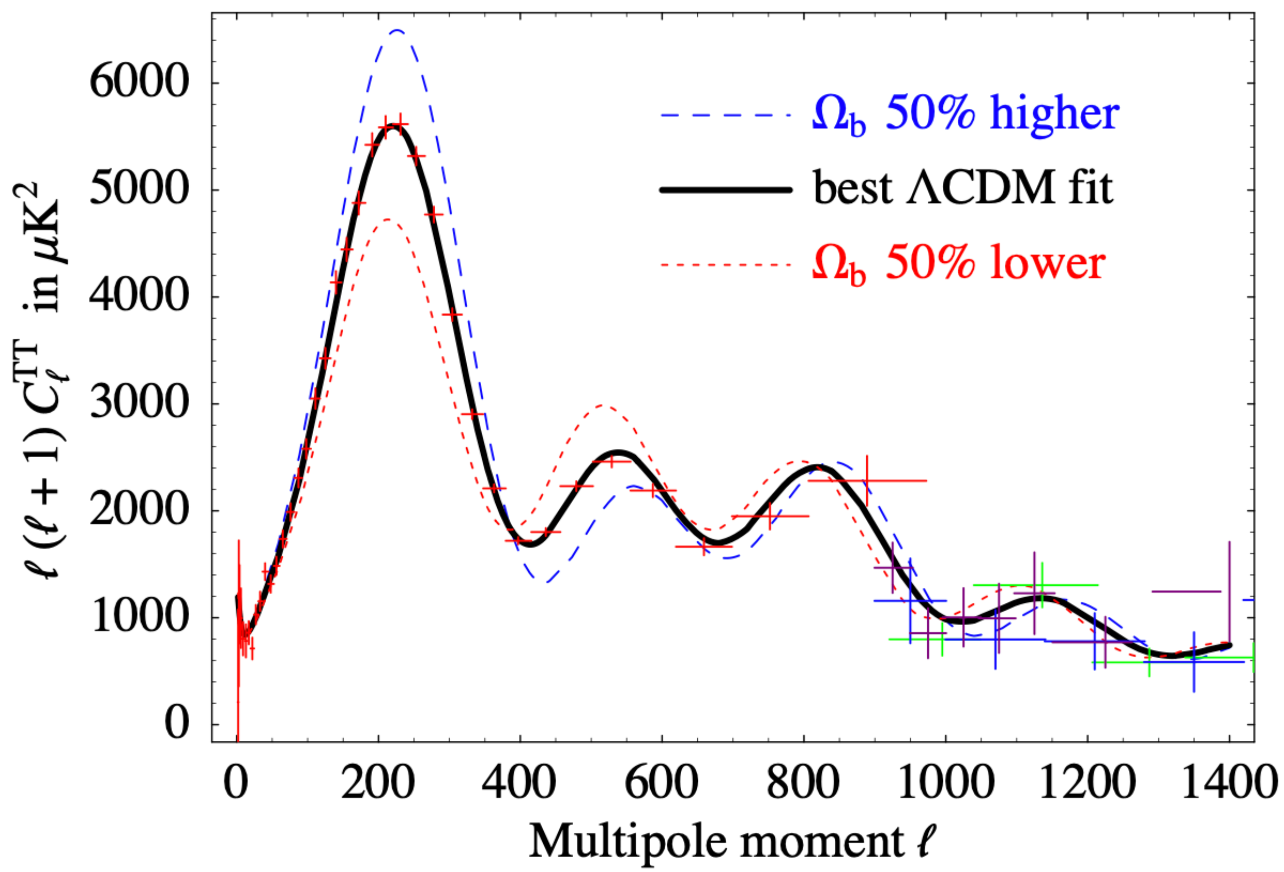
Baryon asymmetry of our universe

T~ 1 MeV



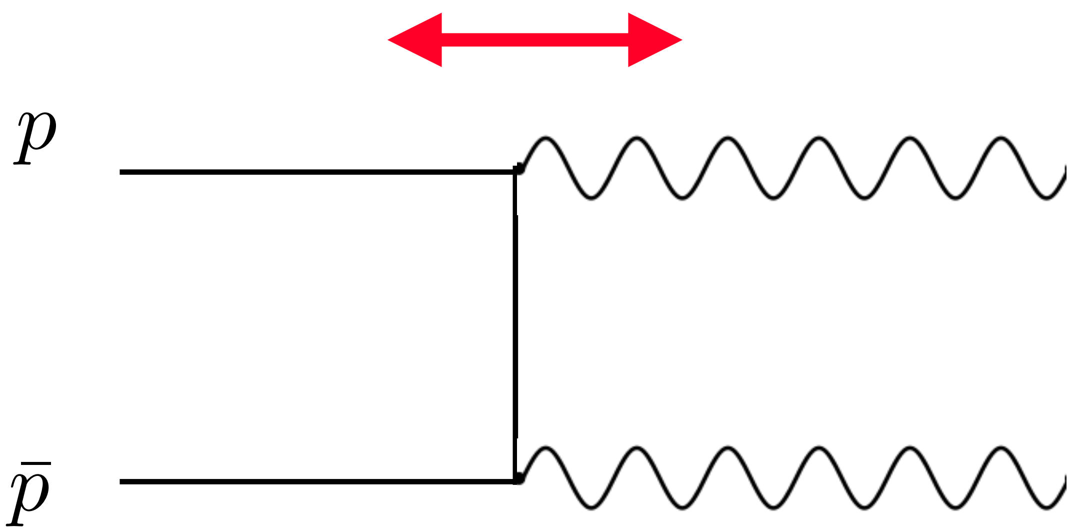
BBN

T~ 1 eV



$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10}$$

Baryon asymmetry of our universe



- T > 1 GeV, $n_b \sim n_{\bar{b}} \sim n_\gamma$
- T < 1 MeV, no baryon asymmetry $\frac{n_b}{n_\gamma} = \frac{n_{\bar{b}}}{n_\gamma} \sim 10^{-20}$
- T < 1 MeV, baryon asymmetry $\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10}$

two questions: why difference? why so small?

How to generate baryon asymmetry?

Assuming no baryon asymmetry in the beginning
(if any, diluted by inflation)

Sakharov conditions

1. B number violation
2. C and CP violation
3. Out of thermal equilibrium

SM has (1) (2) but not enough CP violation, (3) does not

Three popular ways to generate baryon asymmetry

- Electroweak baryogenesis

Rubakov and Shaposhnikov, 1996'
D. E. Morrissey and M. J. Ramsey-Musolf, 2012'

First order phase transition (adding scalars) + additional $\not{C}P$

- Baryogenesis via leptogenesis

$$n_B = \frac{28}{79}(\mathcal{B} - \mathcal{L})_i$$

sphaleron process

Fukugita and Yanagida, 1986'

Type I seesaw: SM + 3 right-handed neutrinos

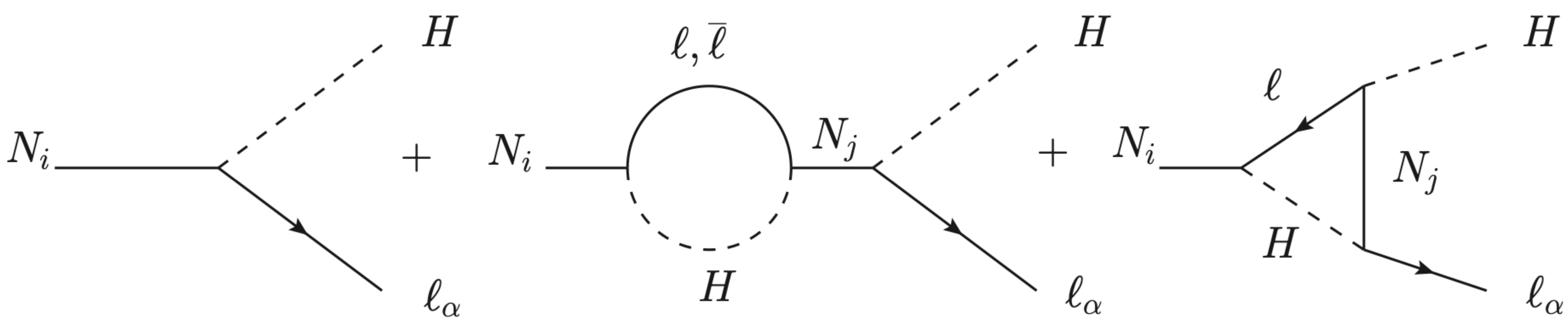
- Baryogenesis from Affleck-Dine mechanism

Affleck and Dine, 1985'

Baryogenesis via leptogenesis

Fukugita and Yanagida, 1986'

$$\mathcal{L}_I = \mathcal{L}_{SM} + i\overline{N_{R_i}}\partial\!\!\!/ N_{R_i} - \left(\frac{1}{2}M_i\overline{N^c_{R_i}}N_{R_i} + \epsilon_{ab}Y_{\alpha i}\overline{N_{R_i}}\ell^a_\alpha H^b + h.c.\right)$$



$$\epsilon_{i\alpha} = \frac{\gamma\left(N_i \to \ell_\alpha H\right) - \gamma\left(N_i \to \overline{\ell_\alpha} H^*\right)}{\sum_\alpha \gamma\left(N_i \to \ell_\alpha H\right) + \gamma\left(N_i \to \overline{\ell_\alpha} H^*\right)}$$

$$\epsilon_i \equiv \sum_\alpha \epsilon_{i\alpha} = \frac{1}{8\pi}\frac{1}{(Y^\dagger Y)_{ii}}\sum_{j\neq i}\mathrm{Im}\left[(Y^\dagger Y)^2_{ji}\right] g\left(\frac{M_j^2}{M_i^2}\right)$$

$$g(x) = \sqrt{x}\left[\frac{1}{1-x} + 1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right]$$

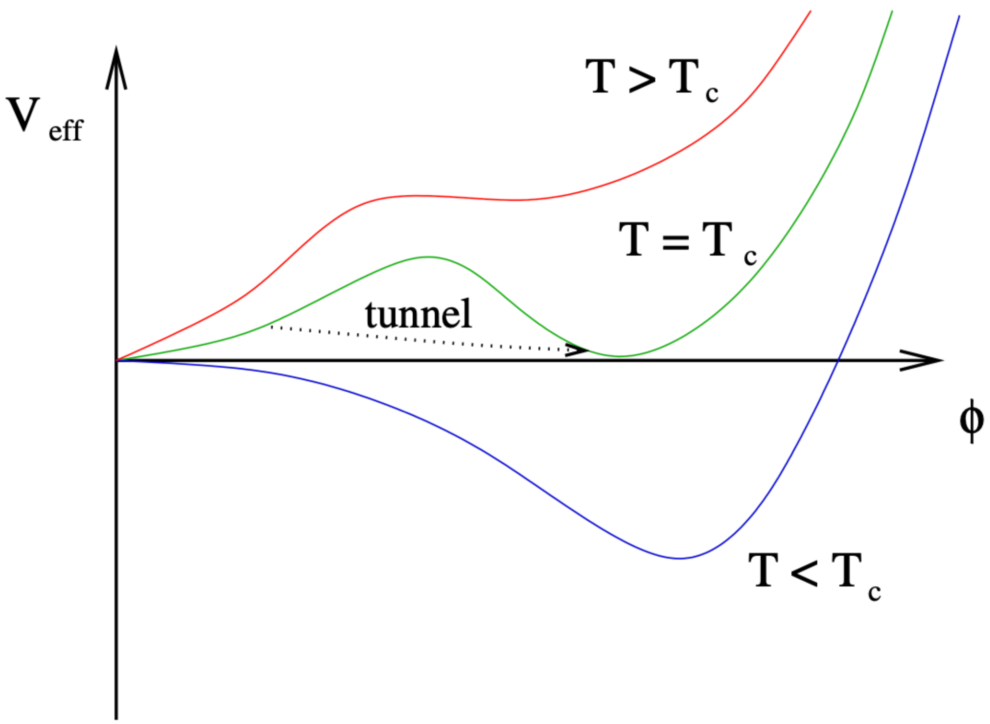
Baryogenesis via leptogenesis

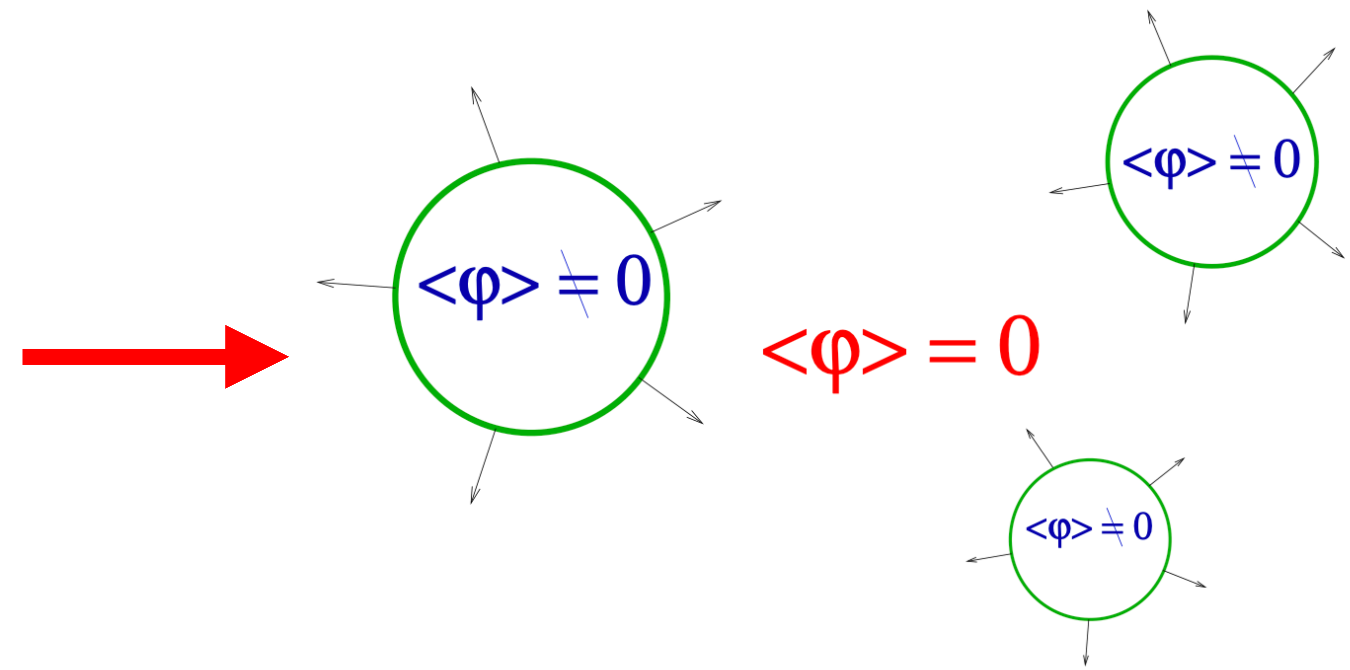
$$Y_{\Delta B} \simeq \frac{135\zeta(3)}{4\pi^4 g_*}\epsilon \times \eta \times C$$

- epsilon is the efficiency of right handed neutrino decay
- eta is the wash out factor: when the N leaves thermal equilibrium
- C is the conversion rate $\quad n_B = \frac{28}{79}(\mathcal{B} - \mathcal{L})_i$
- N mass generally > 10^8 GeV, difficult to probe

Does type II seesaw generate baryon asymmetry?

Electroweak baryogenesis





改变真空结构，产生真空泡，导致强一阶相变，可以产生引力波

要求质量希格斯三重态质量小于550 GeV, 受到实验强烈限制

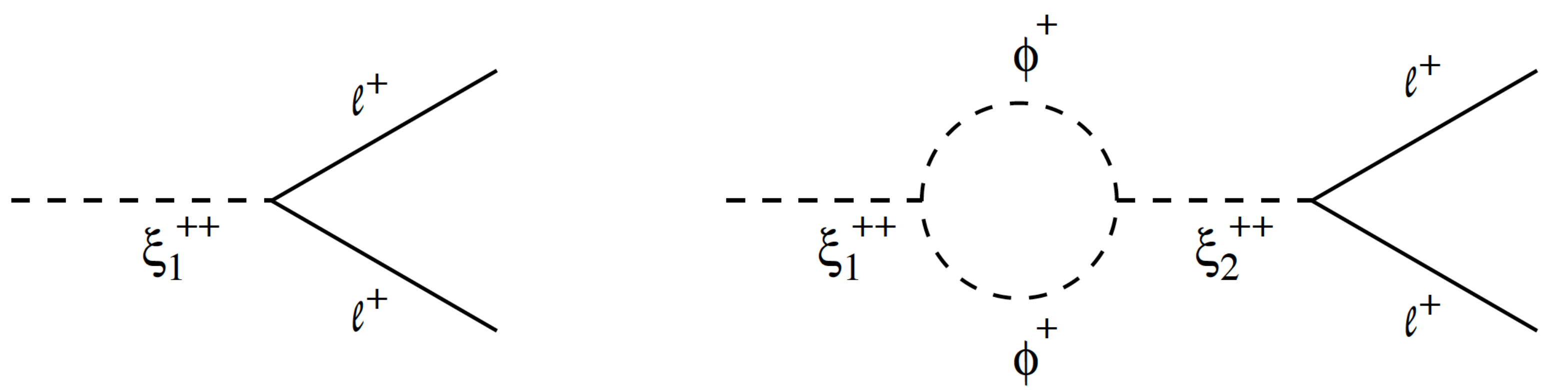
Ruiyu Zhou, Ligong Bian, Yong Du, arXiv:2203.01561

Baryogenesis via thermal leptogenesis

Type II seesaw

$M \sim 10^{13}$ GeV

Neutrino Masses and Leptogenesis with Heavy Higgs Triplets,
E. Ma, U. Sarkar, Phys.Rev.Lett. 80 (1998) 5716-5719



$$\delta_i = 2\left[B\left(\psi_i^- \to ll\right) - B\left(\psi_i^+ \to l^c l^c\right)\right]$$

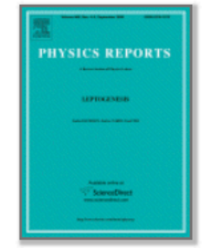
$$\delta_i = \frac{Im\left[\mu_1 \mu_2^* \sum_{k,l} y_{1kl} y_{2kl}^*\right]}{8\pi^2 (M_1^2 - M_2^2)} \left[\frac{M_i}{\Gamma_i}\right]$$

At least two triplet Higgs are needed to generate the baryon asymmetry

Baryogenesis via thermal leptogenesis

ELSEVIER

Physics Reports
Volume 466, Issues 4–5, September 2008, Pages 105-177



Leptogenesis

Sacha Davidson [a], Enrico Nardi [b, c], Yosef Nir [d, 1]

To calculate ϵ_T, one should use the Lagrangian terms given in eqn (2.15). While a single triplet is enough to produce three light massive neutrinos, there is a problem in leptogenesis if indeed this is the only source of neutrinos masses: The asymmetry is generated only at higher loops and in unacceptably small.

It is still possible to produce the required lepton asymmetry from a single triplet scalar decays if there are additional sources for the neutrino masses, such as type I, type III, or type II contributions from

One triplet Higgs can not generate leptogenesis!
but it is enough to give neutrino masses!

Affleck-Dine mechanism

Assuming phi is a complex scalar with B/L charge and potential is "flat"

$$V(\phi) = \frac{1}{2}m^2|\phi|^2 + [c_{n,m}\phi^n(\phi^*)^m + h.c] \quad m \neq n$$

↓

(B violation)

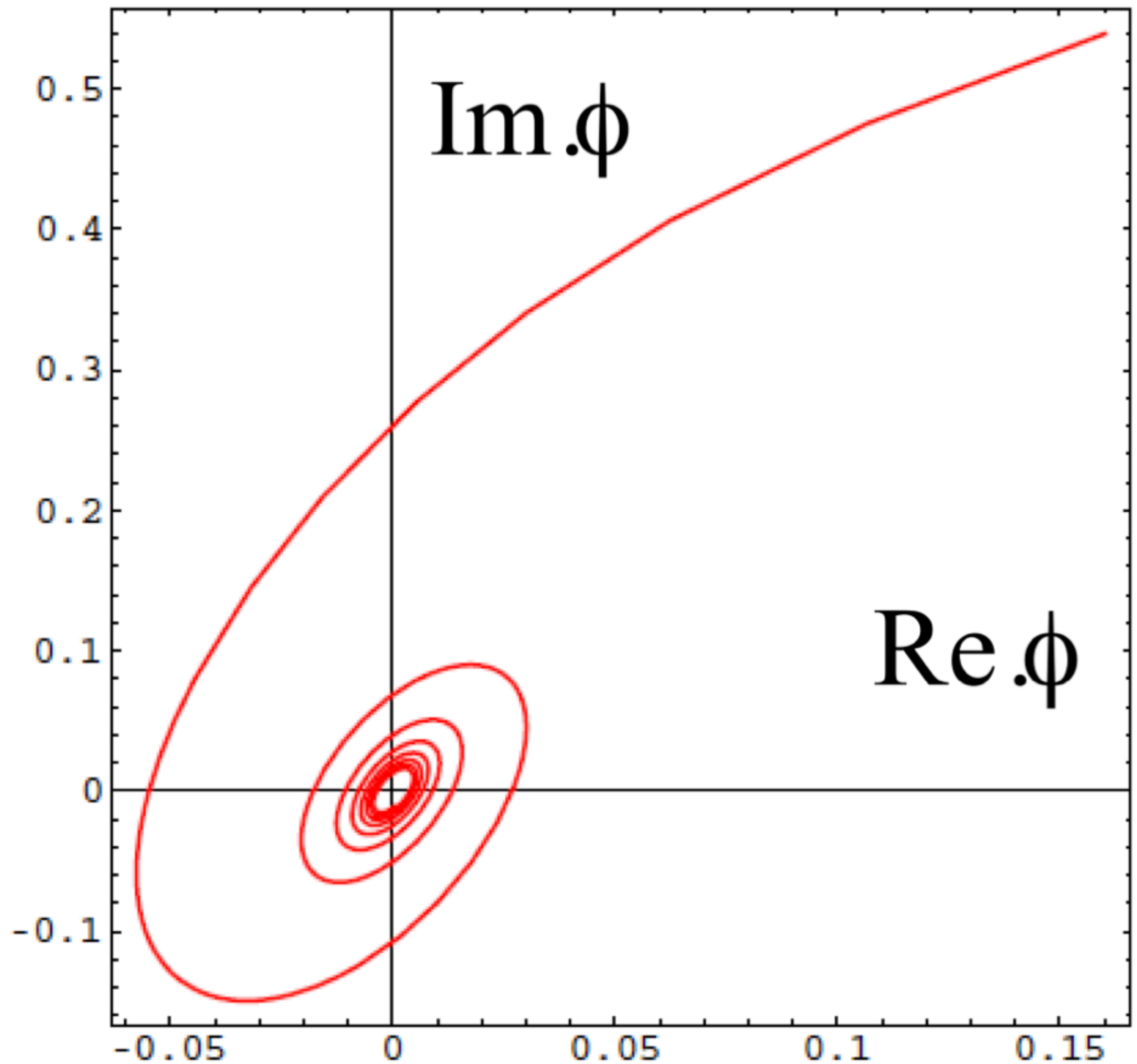
$$j_B^\mu = i(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*)$$

ϕ is spatially constant

$$n_B = i(\phi^*\dot{\phi} - \phi\dot{\phi}^*) = \rho_\phi^2\dot{\theta} \qquad \phi = \frac{1}{\sqrt{2}}\rho_\phi e^{i\theta}$$

A motion of theta will generate baryon number

Affleck-Dine mechanism



Affleck-Dine mechanism

Equation of motion in an expansion of universe

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0$$

An approximation

Only from U(1) breaking term

$$\dot{n}_B + 3Hn_B = \boxed{\mathrm{Im}\left(\phi \frac{\partial V}{\partial \phi}\right)}$$

At $t_0 = 1/H \sim 1/m$ $\quad V = \frac{1}{2}m^2|\phi|^2 + [c_n\phi^n + h.c]$

$$n_{B0} \sim \frac{nc_n\phi^n}{m} \sim \frac{nc_n\rho^n \boxed{\sin n\theta_0}}{2^{n/2}m} \qquad n_B \sim n_{B0}\left(\frac{a_0}{a}\right)^3$$

Affleck-Dine mechanism in SUSY

- Many scalars take B/L charge
- Flat directions(charge neutral)

Baryogenesis from Flat Directions of the Supersymmetric Standard Mode
M. Dine, L. Randall, S. Thomas, Nucl.Phys.B458:291-326,1996

	$B-L$
H_uH_d	0
LH_u	-1
$\bar{u}\bar{d}\bar{d}$	-1
$QL\bar{d}$	-1
$LL\bar{e}$	-1
$QQ\bar{u}\bar{d}$	0
$QQQL$	0
$QL\bar{u}\bar{e}$	0
$\bar{u}\bar{u}\bar{d}\bar{e}$	0

$$\langle\phi_i\rangle = \frac{1}{\sqrt{n}}\phi$$

$$V = m^2|\phi|^2 + \left[\frac{A}{M^{n-3}}\phi^n + h.c\right]$$

m, A term from SUSY breaking

Affleck-Dine mechanism for SUSY

For example,

$$H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad L = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}$$

$$H_u^0 = \phi \sin\alpha \qquad L^0 = \phi\cos\alpha \qquad \alpha = \frac{\pi}{4}$$

$$W = \frac{1}{M}(LH_u)^2 = \frac{1}{2M}\phi^4 \qquad M \sim 10^{15}\ \mathrm{GeV}$$

Weinberg operator in SUSY version, giving neutrino masses

Affleck-Dine mechanism for SUSY

Including the SUSY breaking (supergravity mediation)

$$V(\phi) = m^2|\phi|^2 + \boxed{(\frac{2A}{M}\phi^4 + h.c)} + \frac{4}{M^2}|\phi|^6$$

U(1)L breaking term

m, A are SUSY breaking parameters $m, A \sim m_{3/2}$

Affleck-Dine mechanism

Three conditions for Affleck-Dine mechanism

Type II seesaw

ϕ is spatially constant

- Scalar particle with initial displaced vacuum ?
- Scalar particle taking B/L charge ✓
- Small B/L violation term in the potential ✓

Importance of flat direction(scalars during inflation)

$$V(\phi) = \frac{1}{2}m^2|\phi|^2 + [c_{n,m}\phi^n(\phi^*)^m + h.c] \quad m \neq n$$

Equilibrium state of a self-interacting scalar field in the De Sitter background, Starobinsky and Yokoyama, 94'

$$m \ll H_{inf} \qquad \langle\phi^2\rangle = \frac{3H_{inf}^4}{8\pi^2 m^2}\left(1 - e^{-\frac{2m^2}{3H^2}t}\right)$$

Correlation length $\quad R_c = \frac{1}{H_{inf}} \exp\left(\frac{3}{2}\log 2\frac{H_{inf}^2}{m^2}\right)$

$m < 0.01 H_{inf}$ $\qquad \phi_0 \quad \theta_0$ as a constant in our universe

Small m is also preferred by limit of baryonic isocurvature

Importance of flat direction(scalars during inflation)

For a general model without supersymmetry

$$V(\phi) = \frac{1}{2}m^2|\phi|^2 + \lambda|\phi|^4 + [c_{n,m}\phi^n(\phi^*)^m + h.c] \quad m \neq n$$

Equilibrium state of a self-interacting scalar field in the De Sitter background, Starobinsky and Yokoyama, 94'

$$\langle\phi^2\rangle = 0.13\lambda^{-1/2}H_{inf}^2$$

$$R_c \approx \frac{1}{H_{inf}} \exp\left(3.8\lambda^{-1/2}\right)$$

$\lambda \sim 0.1$ θ_0 is random distributed

Different patches of universe different baryon number, average close 0

Combing the idea of inflation with baryogenesis

If the scalar plays the role of inflation

- Scalar particle with initial displaced vacuum ✓
- Scalar particle taking B/L charge ✓
- Small B/L violation term in the potential ✓

Potential for SM+Type II seesaw

$$H = \begin{pmatrix} h^+ \\ h \end{pmatrix}, \quad \Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

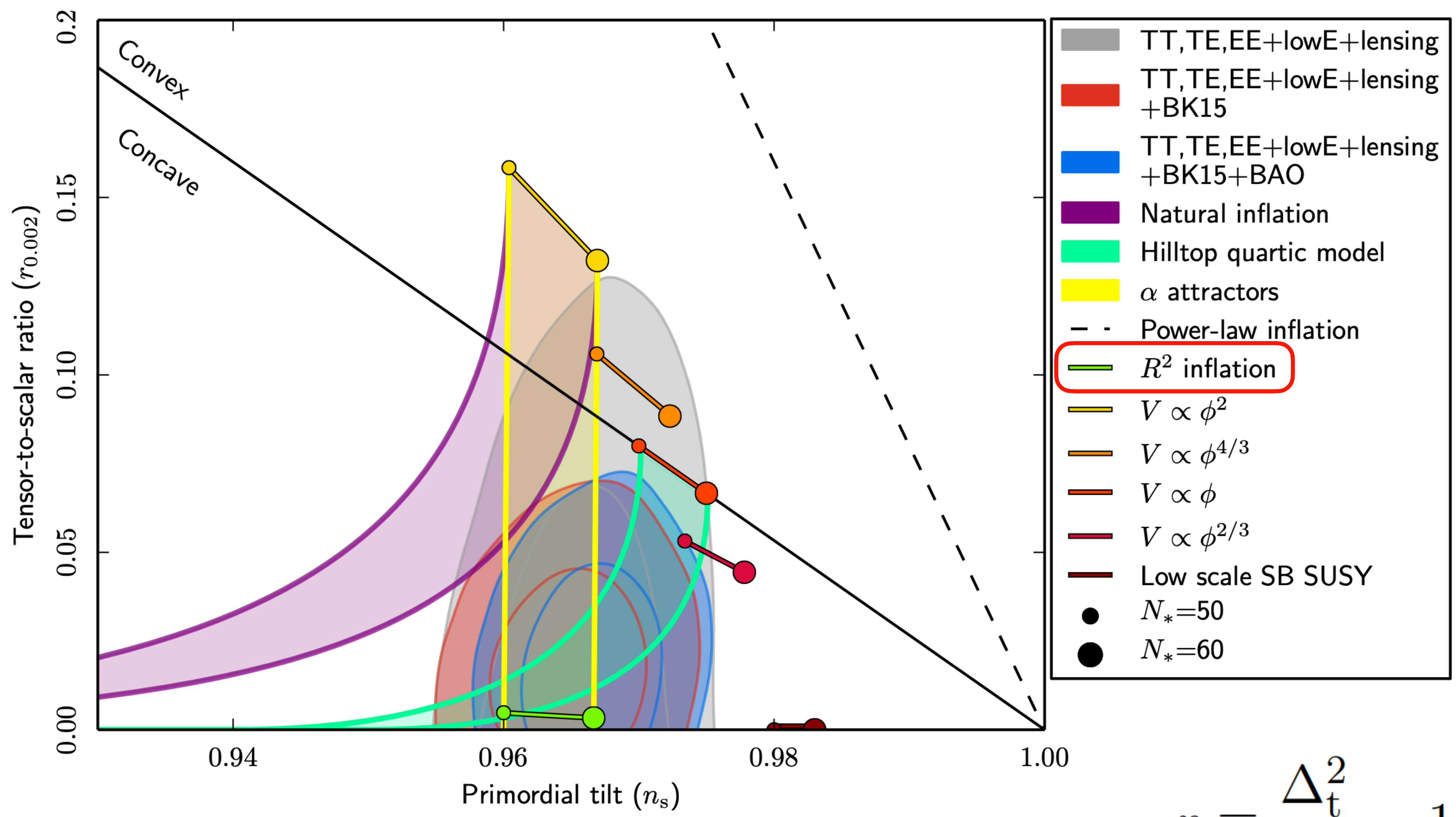
To simplify the analysis, we choose a direction

$$\begin{aligned} V(h,\Delta^0) &= -m_H^2|h|^2 + m_\Delta^2|\Delta^0|^2 + \lambda_H|h|^4 + \lambda_\Delta|\Delta^0|^4 + \lambda_{H\Delta}|h|^2|\Delta^0|^2 \\ &- \left(\mu h^2\Delta^{0*} + \frac{\lambda_5}{M_p}|h|^2h^2\Delta^{0*} + \frac{\lambda_5'}{M_p}|\Delta^0|^2h^2\Delta^{0*} + h.c.\right) + \ldots\,, \end{aligned}$$

U(1)L breaking term

- U(1)L breaking only when both <h> <delta> non-vanishing
- High dimension operator should be considered because sub-Planck
- Quartic term dominate the potential

Problem with inflation



$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon_V$$

$V(\phi) \propto \phi^n$ seems not consistent with observation

too large r due to the non-flat of the potential

Adding non-minimal coupling

Idea of Higgs inflation   Bezrukov and Shaposhnikov, Phys.Lett.B 659 (2008) 703-706

$$S_J = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(1 + \frac{\xi \phi^2}{M_P^2} \right) R_J - \frac{1}{2} |\partial_\mu \phi|^2 - V_J(\phi) \right]$$

Weyl transformation

$$g_{\mu\nu} = \Omega(\phi)^2 g_{J\mu\nu} \qquad \Omega^2 = 1 + \frac{\xi \phi^2}{M_P^2}$$

Adding non-minimal coupling

$$\frac{d\chi}{d\phi} = \left(\frac{1+\xi(1+6\xi)\phi^2/M_P^2}{(1+\xi\phi^2/M_P^2)^2}\right)^{1/2}$$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)\right]$$

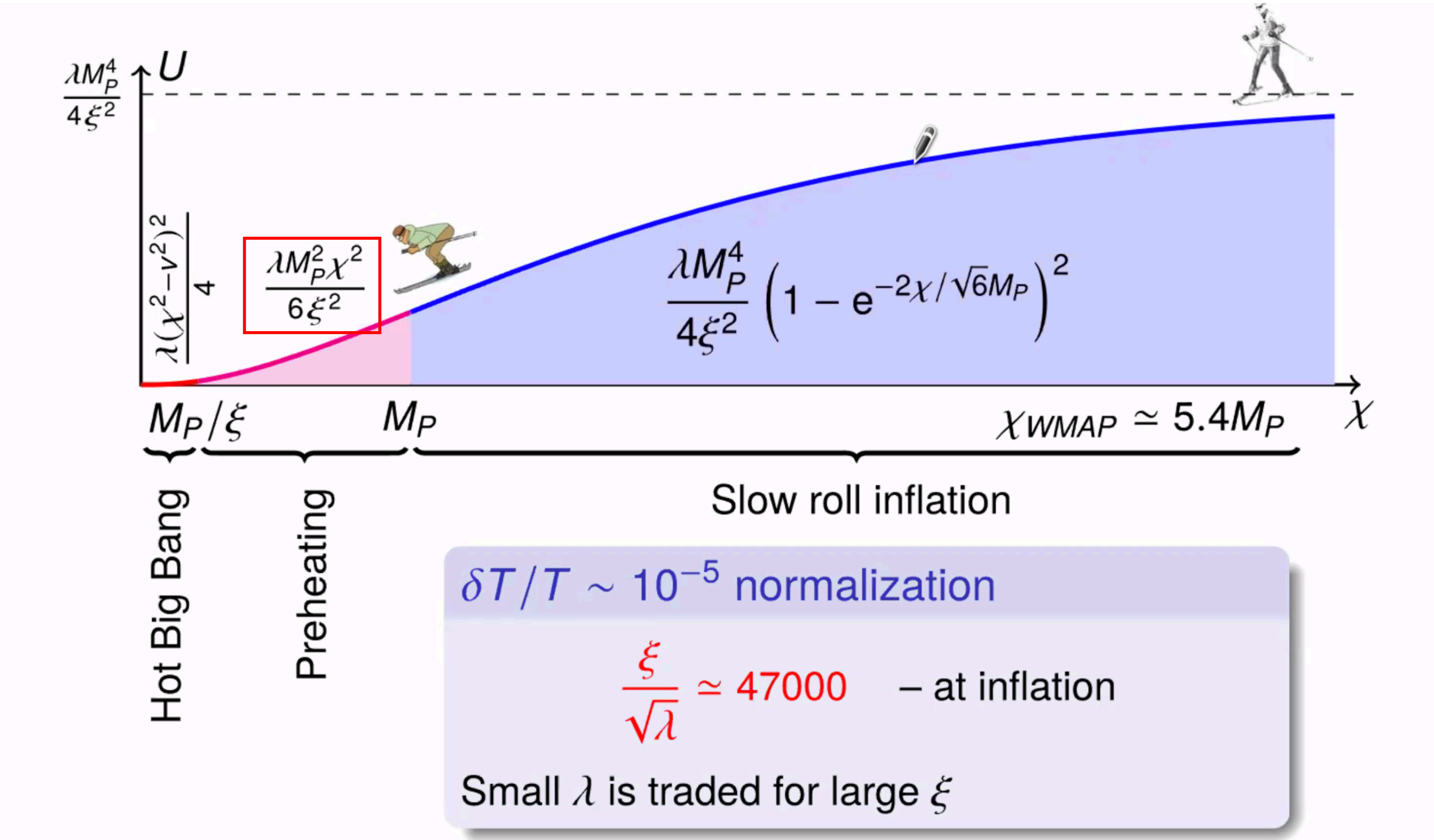
$$V(\chi) \equiv V_J(\phi(\chi))/\Omega^4(\phi(\chi)) \qquad \Omega^2 = 1 + \frac{\xi\phi^2}{M_P^2}$$

$$V_J = \frac{\lambda}{4}\phi^4 \quad \xrightarrow{\phi \gg M_p/\sqrt{\xi}} \quad V = \frac{\lambda}{4\xi^2}M_p^4$$

Potential becomes flat when chi(phi) becomes large

Adding non-minimal coupling

Plot borrowed from Bezrukov



Adding non-minimal coupling

Prediction of the model

$$n_s \simeq 1 - \frac{2}{N_*} \;, \quad \text{and} \quad r \simeq \frac{12}{N_*^2}$$

$$0.96 \lesssim n_s \lesssim 9.667$$
$$0.0033 \lesssim r \lesssim 0.0048$$

Current observation

$$n_s = 0.9649 \pm 0042 \quad (68\%\text{C.L.})$$
$$r_{0.002} < 0.056 \quad (95\%\text{C.L.})$$

SM+Type II seesaw

To be consistent with inflation, we need add non-minimal couplings

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}M_P^2 R - f(H,\Delta)R - g^{\mu\nu}(D_\mu H)^\dagger(D_\nu H)$$
$$-g^{\mu\nu}(D_\mu\Delta)^\dagger(D_\nu\Delta) - V(H,\Delta) + \mathcal{L}_{\text{Yukawa}}$$

$$h \equiv \frac{1}{\sqrt{2}}\rho_H e^{i\eta} \qquad \Delta^0 \equiv \frac{1}{\sqrt{2}}\rho_\Delta e^{i\theta}$$

$$F(H,\Delta) = \xi_H|h|^2 + \xi_\Delta|\Delta^0|^2 = \frac{1}{2}\xi_H\rho_H^2 + \frac{1}{2}\xi_\Delta\rho_\Delta^2$$

SM+Type II seesaw

During inflation(Oleg Lebedev and Hyun Min Lee, arXiv:1105.2284)

$$\frac{\rho_H}{\rho_\Delta} \equiv \tan\alpha = \sqrt{\frac{2\lambda_\Delta \xi_H - \lambda_{H\Delta}\xi_\Delta}{2\lambda_H \xi_\Delta - \lambda_{H\Delta}\xi_H}}$$

$$\rho_H = \varphi \sin\alpha, \;\; \rho_\Delta = \varphi\cos\alpha$$

$$\xi \equiv \xi_H \sin^2\alpha + \xi_\Delta \cos^2\alpha$$

Similar to SUSY case, but mixing with a general angle

SM+Type II seesaw

Finally the model can be simplified as

$$
\begin{aligned}
\frac{\mathcal{L}}{\sqrt{-g}} \quad = \quad & -\frac{M_p^2}{2}R - \frac{\xi}{2}\varphi^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \\
& -\frac{1}{2}\varphi^2\cos^2\alpha \; g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - V(\varphi,\theta)
\end{aligned}
$$

$$
V(\varphi,\theta) = \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4}\varphi^4 + 2\varphi^3\left(\tilde{\mu} + \frac{\tilde{\lambda}_5}{M_p}\varphi^2\right)\cos\theta
$$

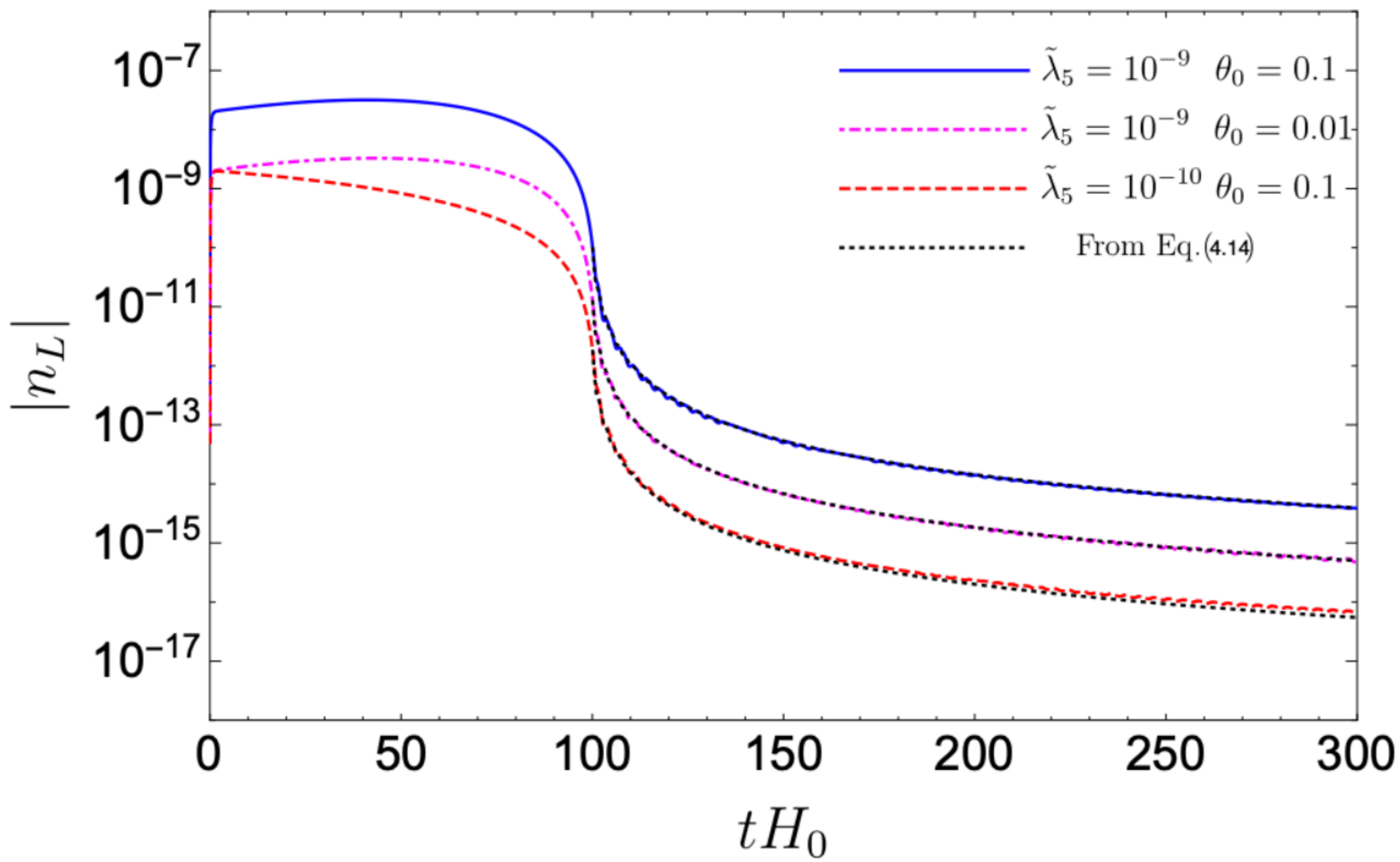
We need keep the theta term, because

$$
n_L = Q_L\varphi^2\dot{\theta}\cos^2\alpha
$$

Lepton number generation

$\xi = 300,\ \lambda = 4.5 \cdot 10^{-5}$

$\chi_0 = 6.0 M_p,\ \dot{\chi}_0 = 0$, and $\theta_0 = 0$



- 暴胀开始轻子数为0
- 轻子数在暴胀过程中产生
- 暴胀结束后轻子数守恒

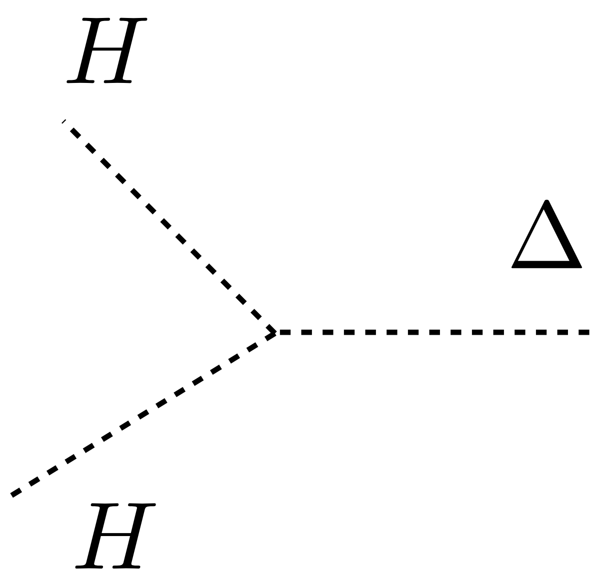
SM+Type II seesaw

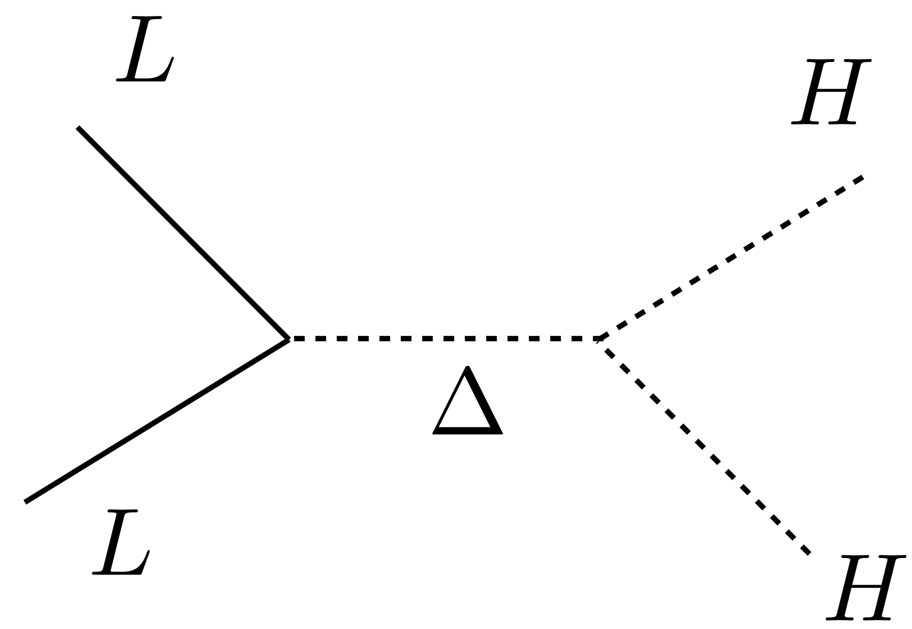
$T_{\rm reh} \approx 2.2 \cdot 10^{14}$ GeV

$$\eta_B = \frac{n_B}{s}\bigg|_{\rm reh} = \eta_B^{\rm obs} \left(\frac{|n_{L{\rm end}}|/M_p^3}{1.3 \cdot 10^{-16}} \right) \left(\frac{g_*}{112.75} \right)^{-\frac{1}{4}}$$

$$\tilde{\lambda}_5 = 7 \cdot 10^{-15} \text{ for } \theta_0 = 0.1$$

Wash out process



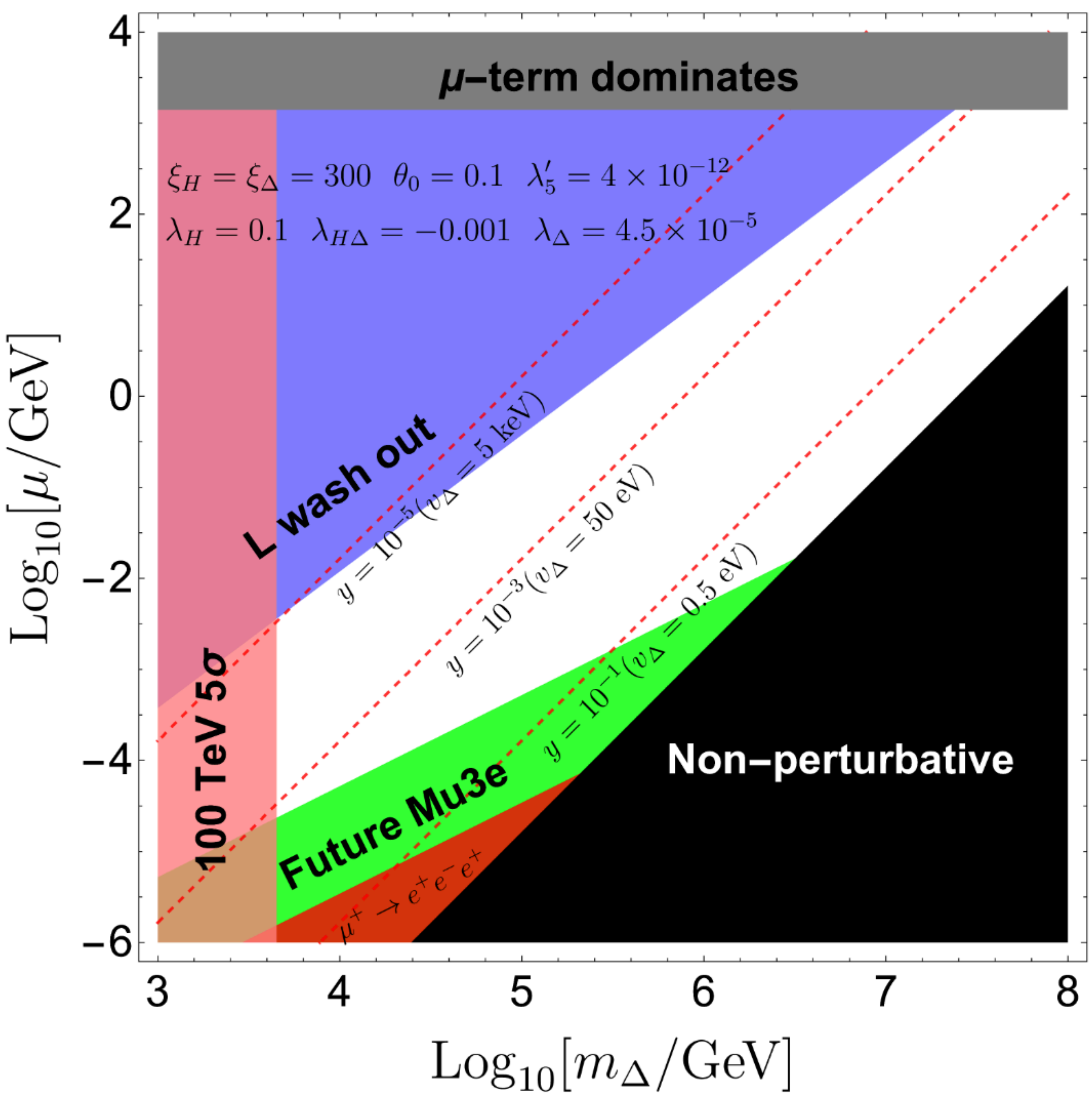


$$\frac{\mu^2}{8\pi m_\Delta} < H(m) = \frac{m_\Delta^2}{M_P}$$

$$m_\Delta < 10^{12}\ \mathrm{GeV}$$

A small mu term is preferred

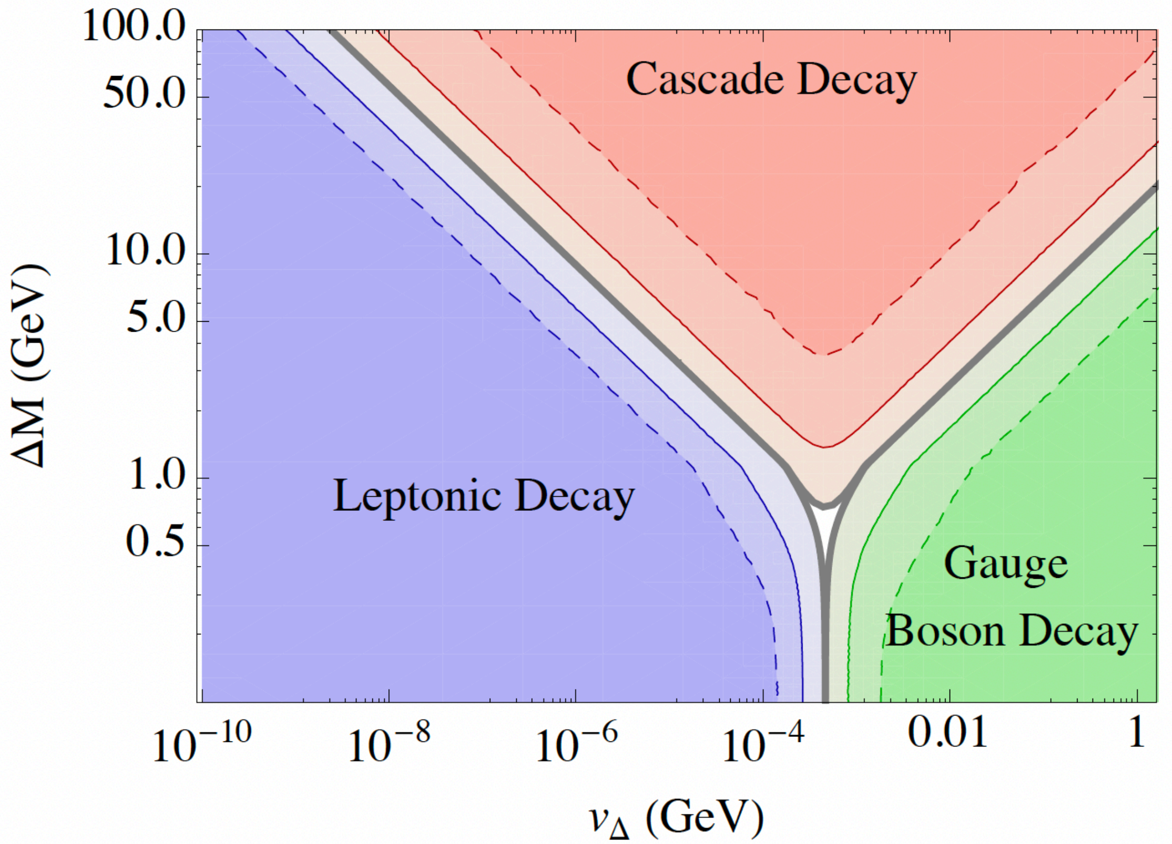
SM+Type II seesaw



Phenomenology implications I: collider physics

Decay of the doubly-charged Higgs

$$\Delta M = m_{\Delta^{++}} - m_{\Delta^{+}}$$



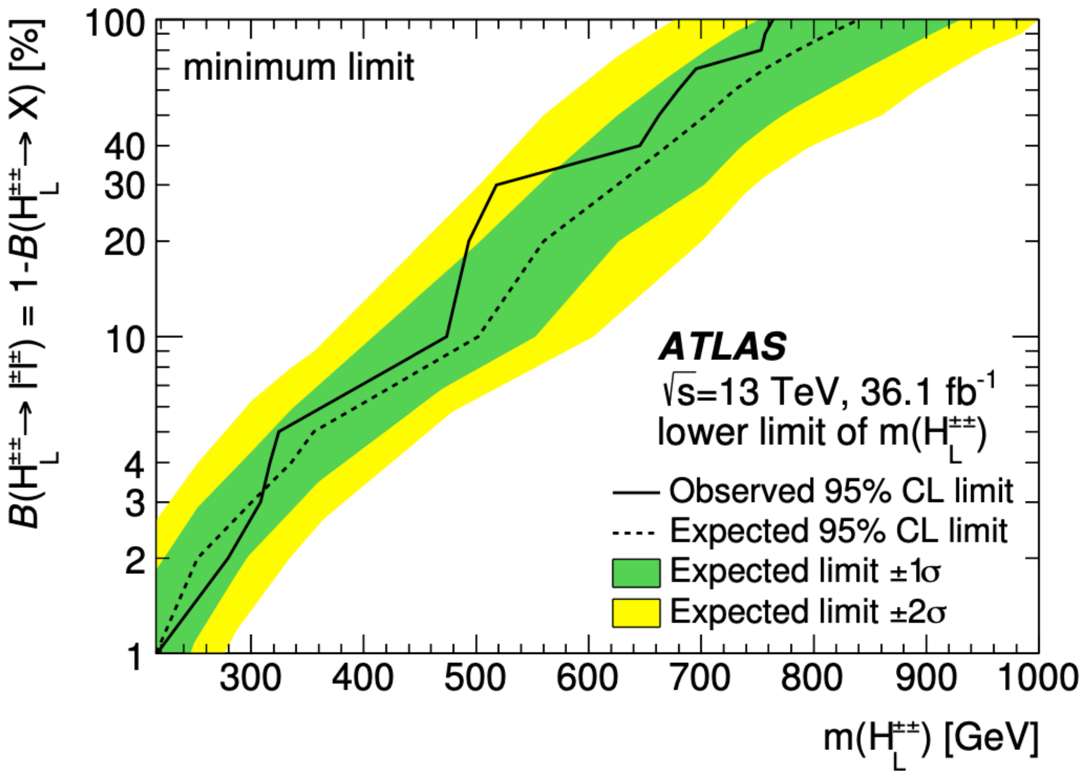
For v > 1 MeV, mainly decay gauge bosons

For v < 0.1 MeV, mainly decay leptons

Phenomenology implications I: collider physics

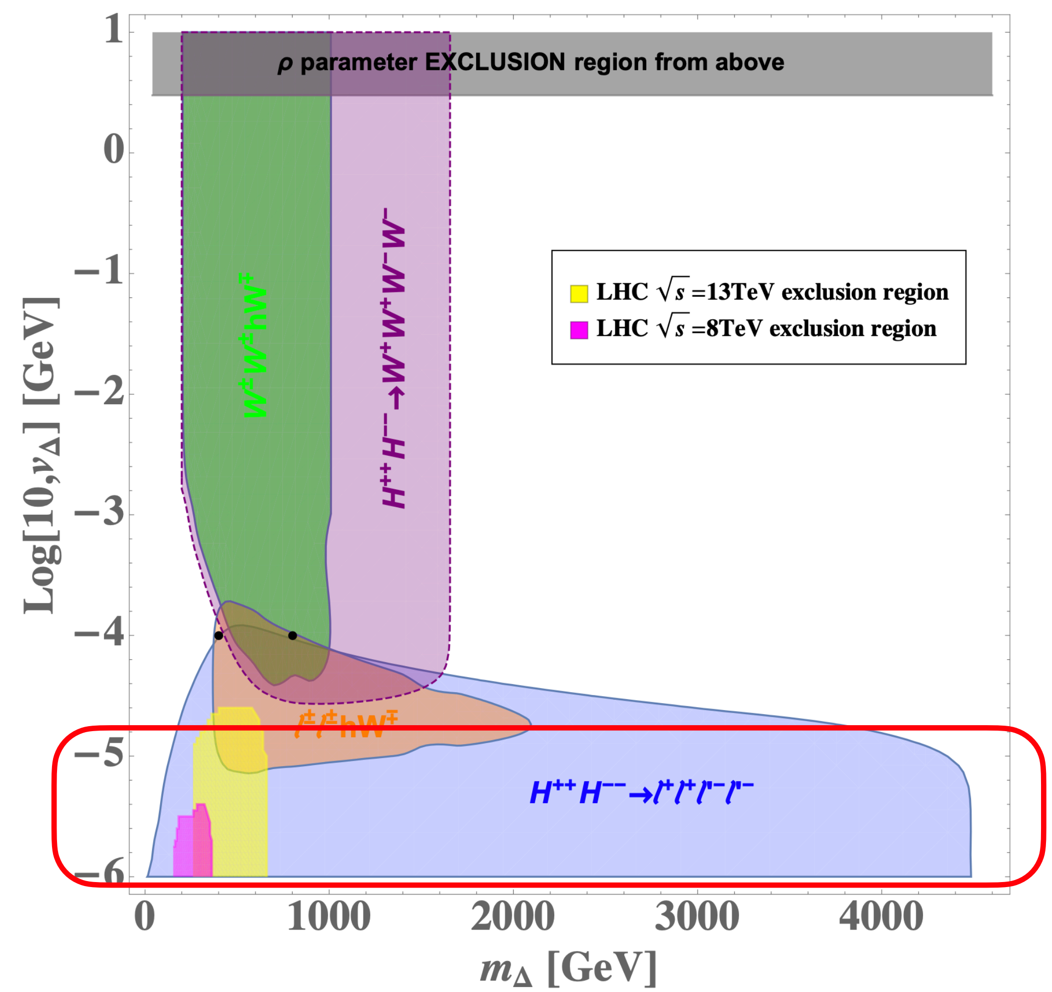
Current limit from LHC

ATLAS, Eur. Phys. J. C 78 (2018) 199



Future reach

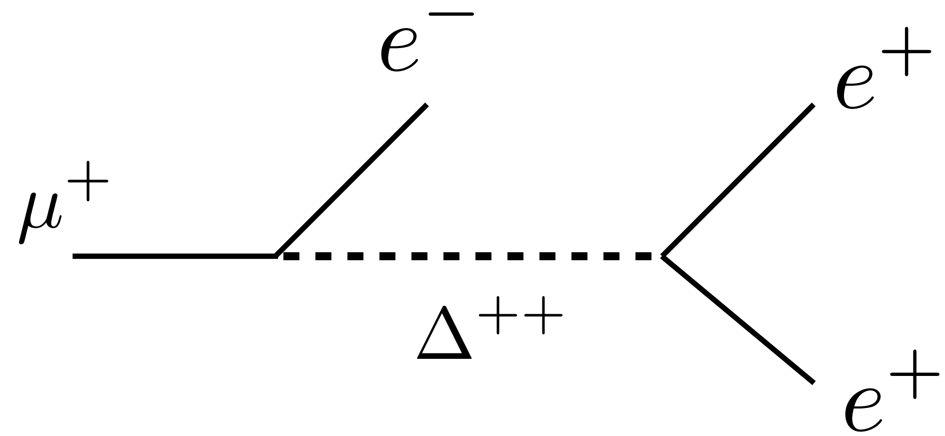
Y. Du, A. Dunbrack, M. J. Ramsey-Musolf, J. Yu, JHEP01(2019)101



5 sigma discover region @100 TeV collider

Smoking gun: observing doubly-charged Higgs from leptonic channel

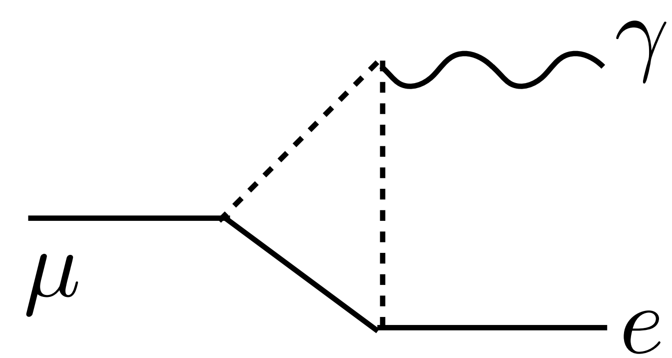
$$\mathcal{B}(\mu^+ \to e^+ e^- e^+) = \frac{|y_{\mu e} y^\dagger_{ee}|^2}{16 G_F^2 m^4_{\Delta^{++}}}$$



$$\mathcal{B}(\mu^+ \to e^+ e^- e^+) \leqslant 1.0 \times 10^{-12}$$

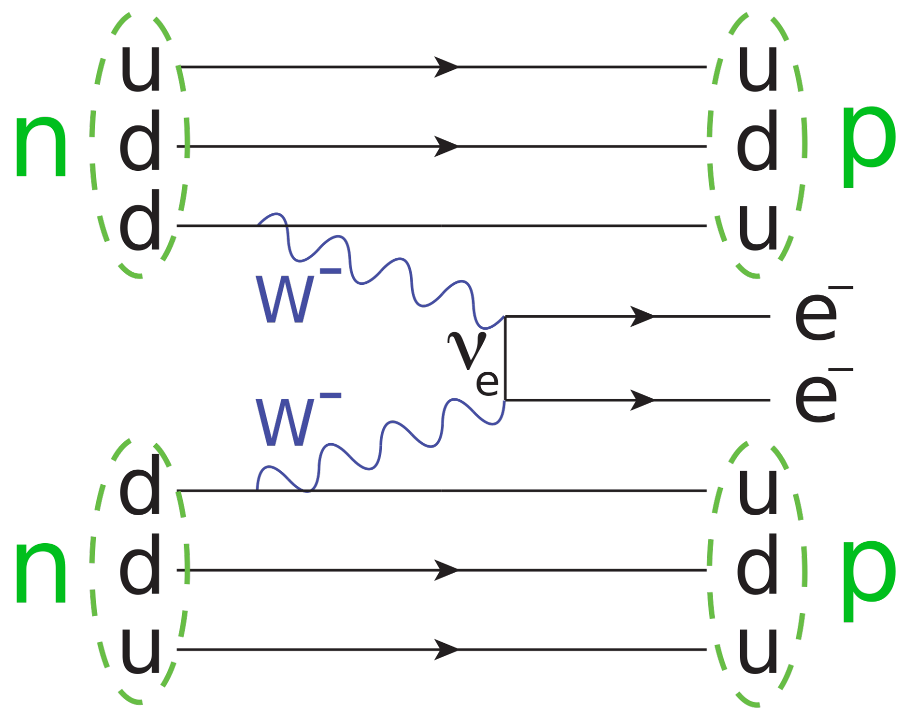
$$\mathcal{B}(\mu \to e\gamma) \simeq \frac{\alpha}{3072\pi} \frac{\left|(y^\dagger y)_{e\mu}\right|^2}{G_F^2} \left(\frac{1}{m^2_{\Delta^+}} + \frac{8}{m^2_{\Delta^{++}}} \right)^2$$

$$\mathcal{B}(\mu \to e\gamma) < 4.2 \times 10^{-13}$$

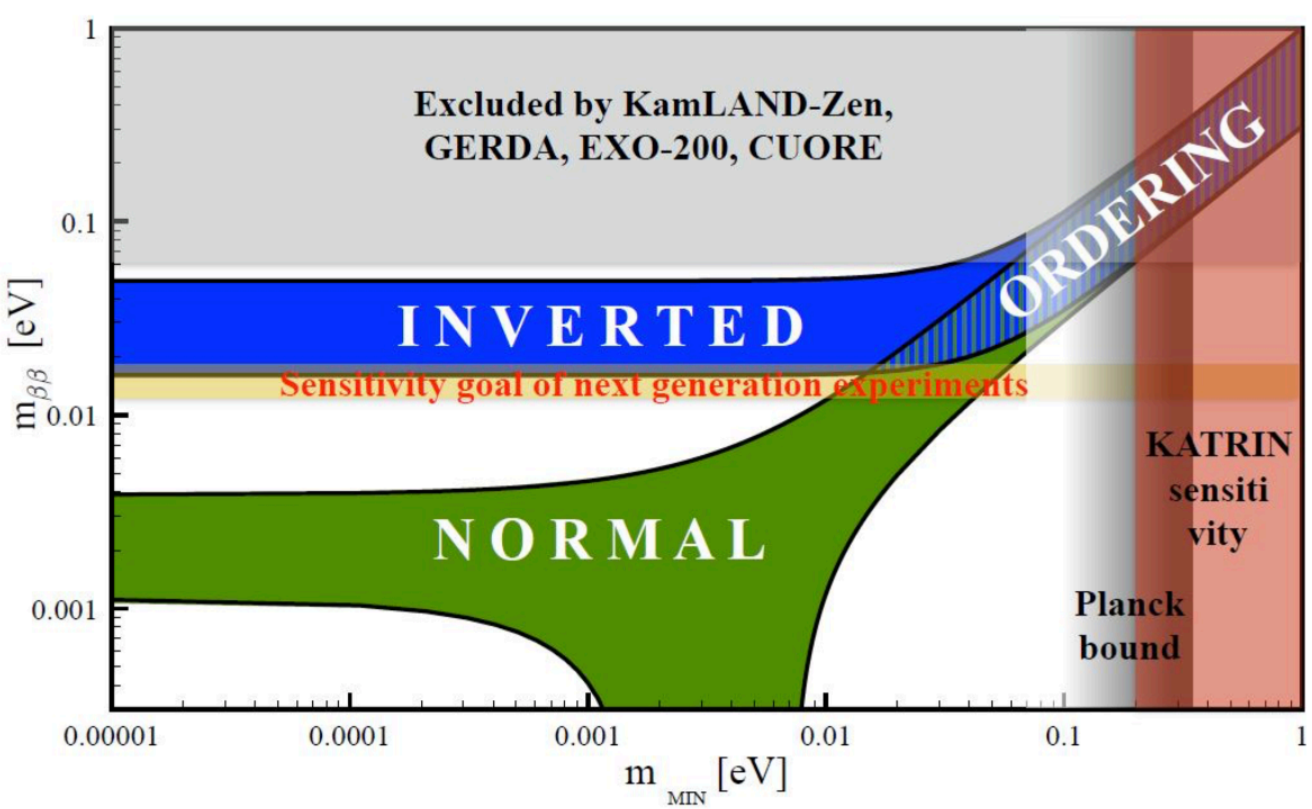


Phenomenology implications III: neutrino physics

- Neutrino must be majorana type



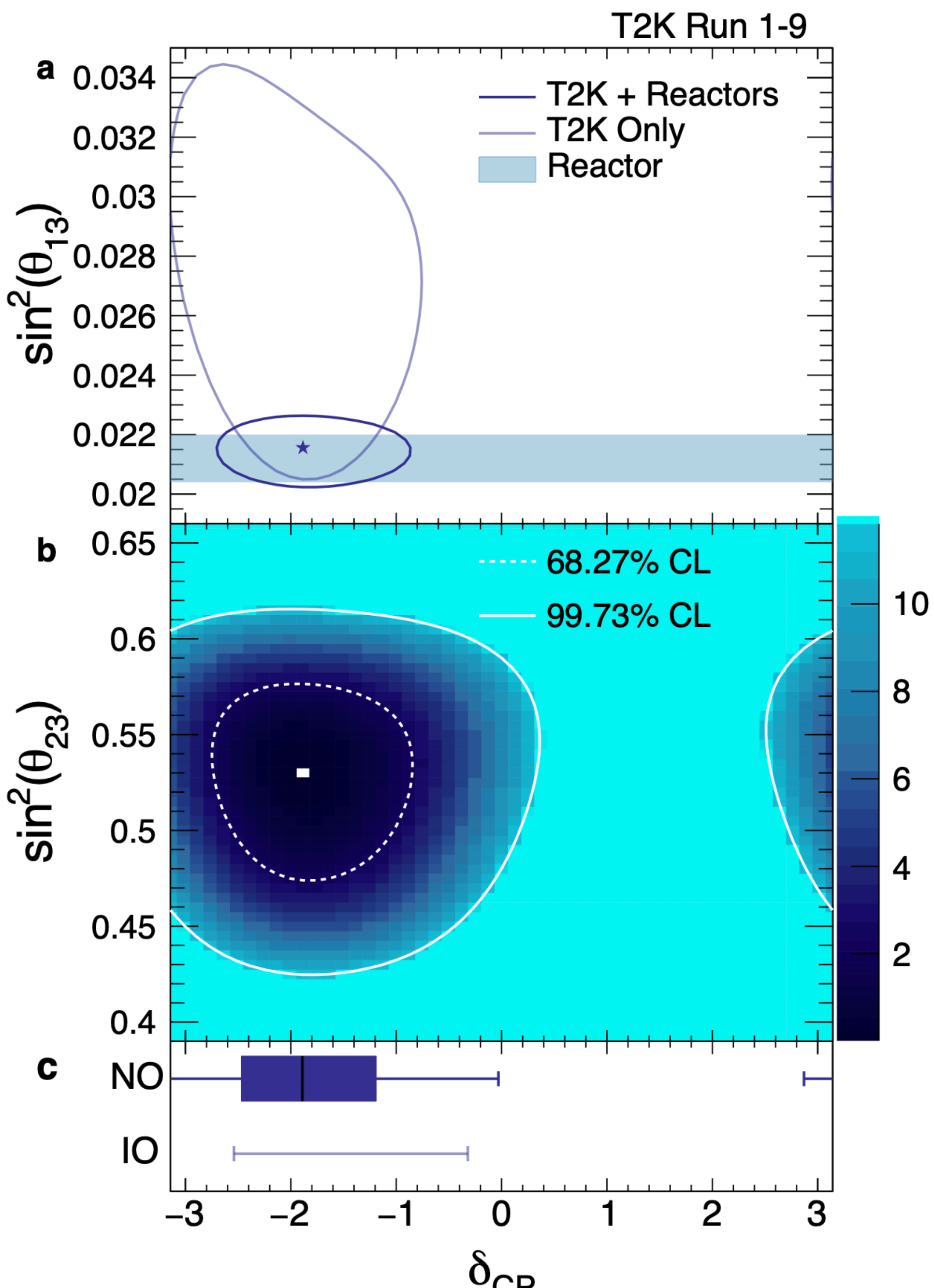
Neutrinoless double beta decay



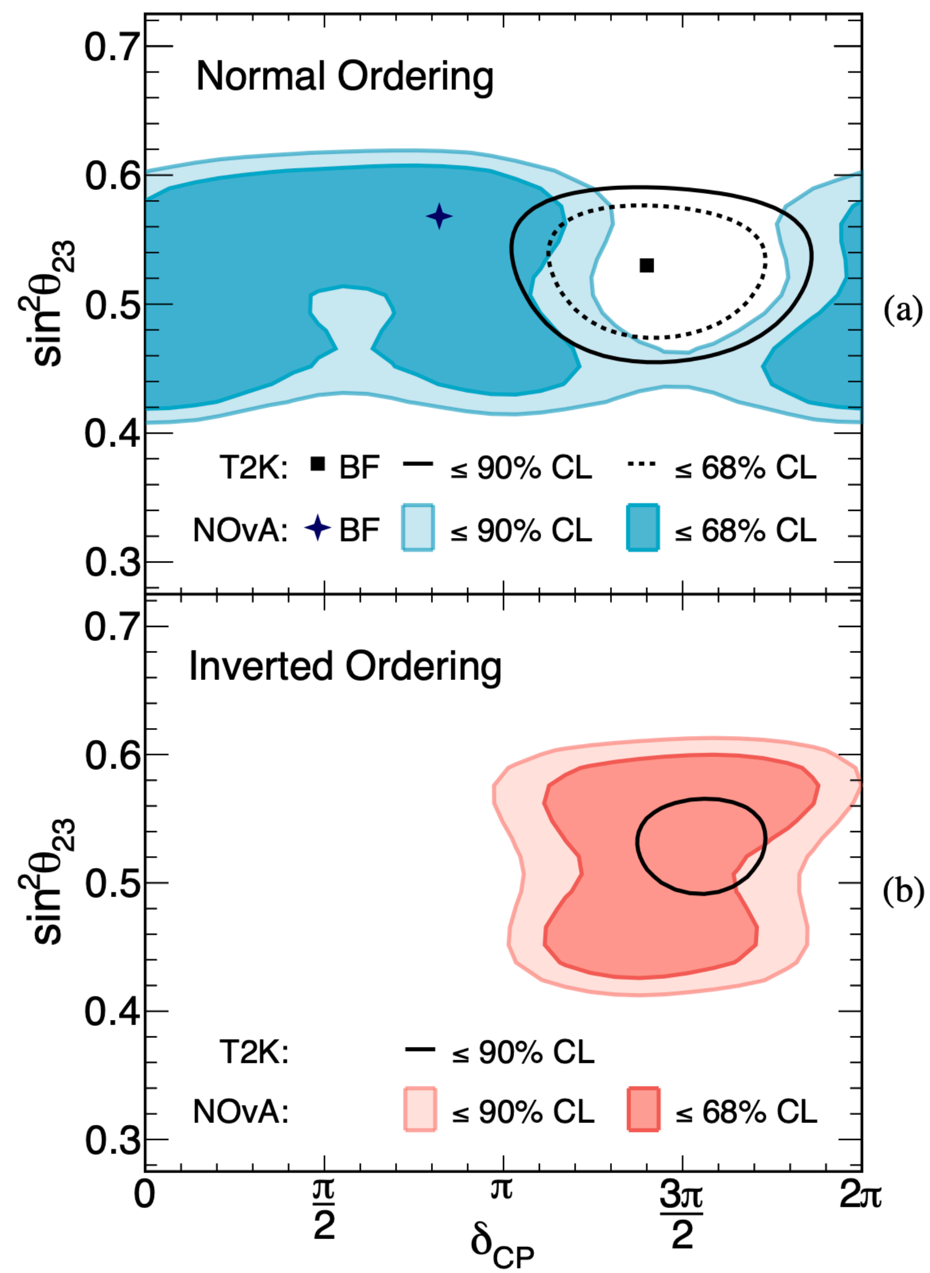
Phenomenology implications III: neutrino physics

- CP violation in neutrino sector

T2K, 19'



Nova, 21'



Leptogenesis even without CP violation

Phenomenology implications IV: cosmological signal

- Tensor to scalar ratio, within the future reach of LiteBIRD

$$0.0033 < r < 0.0048$$

- Non-Gaussian signature, model dependent

- Imprint of isocurvature signature from baryon matter

- Gravitational wave from preheating

More work need to do!

Summary

- One simple extension of SM, three problems can be solved: inflation, baryogenesis and neutrino masses

- Unique signatures at collider, LFV violation, neutrino experiments and astronomy observations

- More studies in future

Thanks

Back up

Slow-roll inflation

Assuming a scalar field, with equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

Slow roll condition $\dot{\phi}^2 \ll V(\phi)$ $\quad |\ddot{\phi}| \ll |3H\dot{\phi}|\,,\ |V_{,\phi}|$

$$\epsilon_{\rm v}(\phi) \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta_{\rm v}(\phi) \equiv M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V} \qquad \epsilon_{\rm v}, |\eta_{\rm v}| \ll 1$$

$$H^2 \approx \frac{1}{3}V(\phi) \approx \text{const.}$$

$$\dot{\phi} \approx -\frac{V_{,\phi}}{3H}\,,$$

$\longrightarrow$ $a(t) = a_0 e^{Ht}$ $\quad Ht \gtrsim 60$

Daniel Baumann, TASI Lectures on Inflation

Slow-roll inflation

标量扰动

$$\Delta_{\rm s}^2(k) \approx \frac{1}{24\pi^2}\frac{V}{M_{\rm pl}^4}\frac{1}{\epsilon_{\rm V}}\bigg|_{k=aH}$$

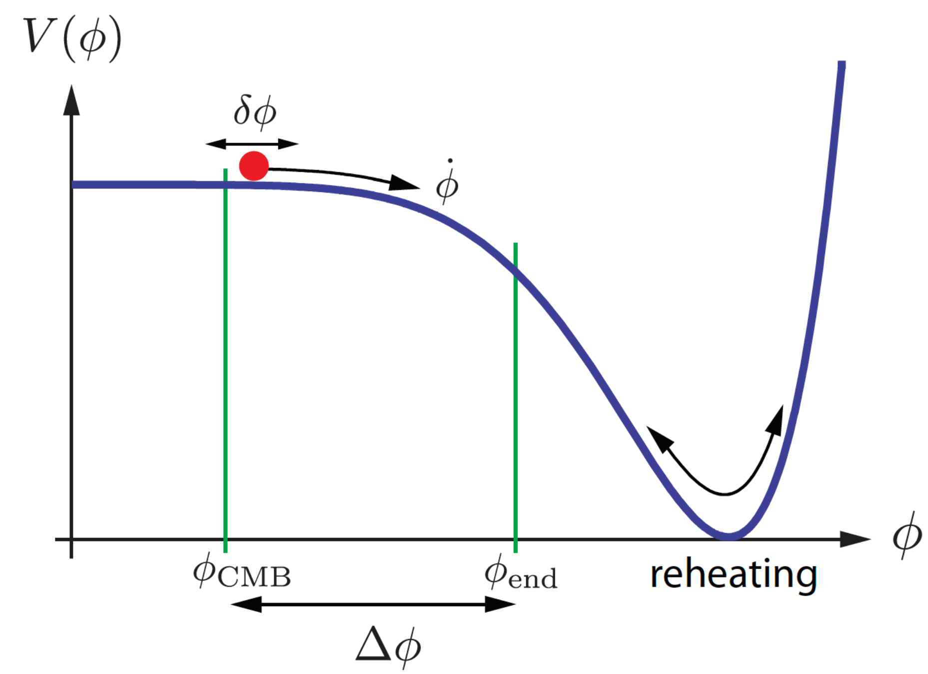
$$n_{\rm s} - 1 \equiv \frac{d\ln\Delta_{\rm s}^2}{d\ln k} = 2\eta_{\rm V} - 6\epsilon_{\rm V}$$

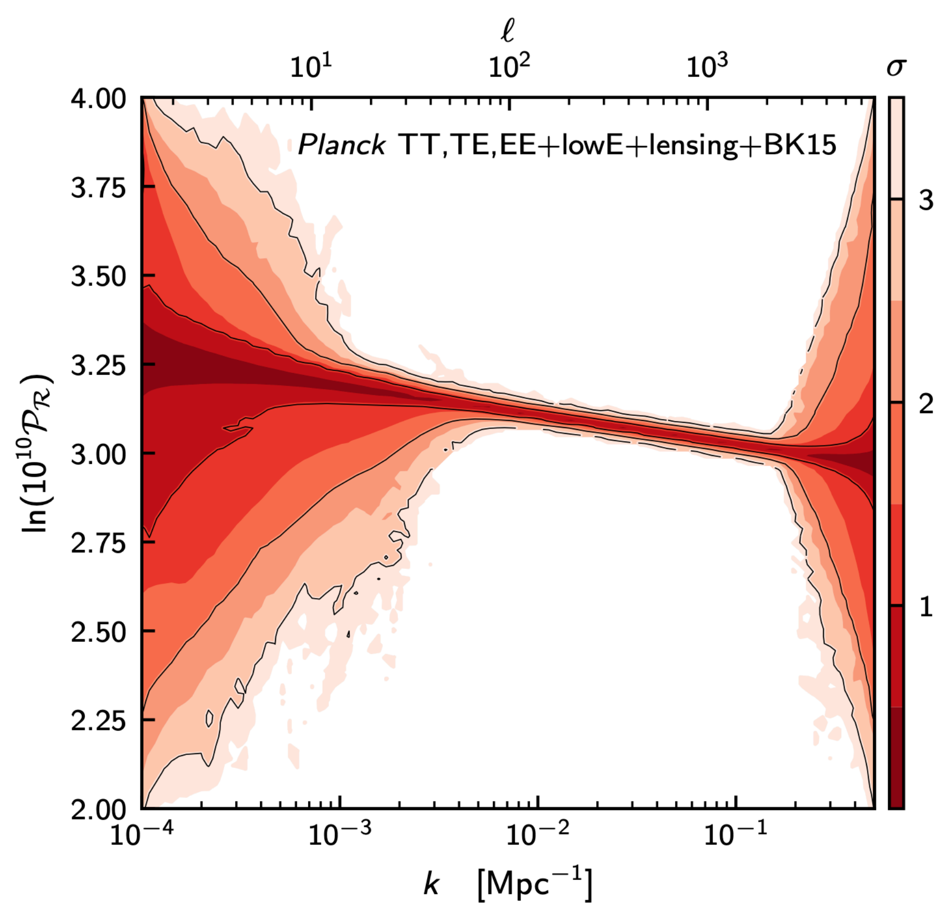
$n_s \simeq 0.965$ **n=1 to be scale invariant**

张量扰动

$$\Delta_{\rm t}^2(k) \approx \frac{2}{3\pi^2}\frac{V}{M_{\rm pl}^4}\bigg|_{k=aH} \qquad r \equiv \frac{\Delta_{\rm t}^2}{\Delta_{\rm s}^2} = 16\epsilon_{\rm V}$$

tensor-scalar ratio $r \lesssim 0.056$

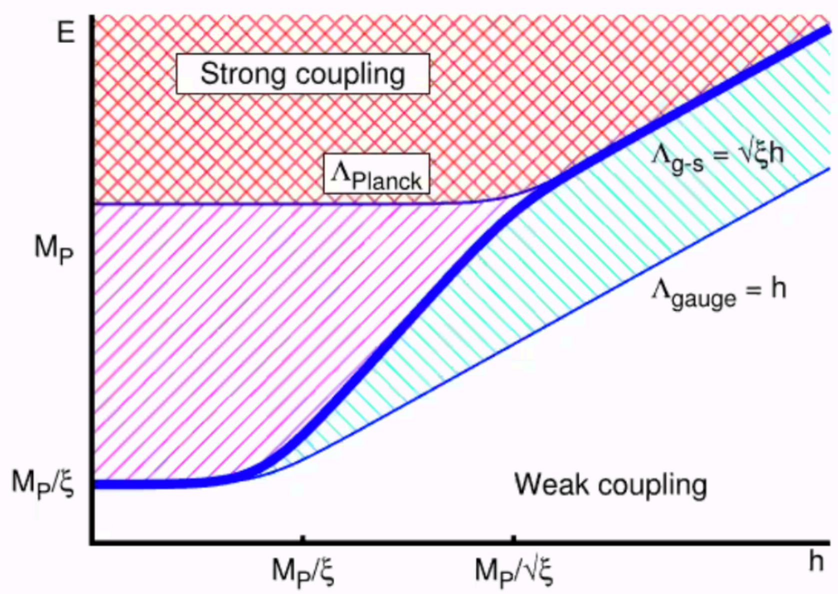




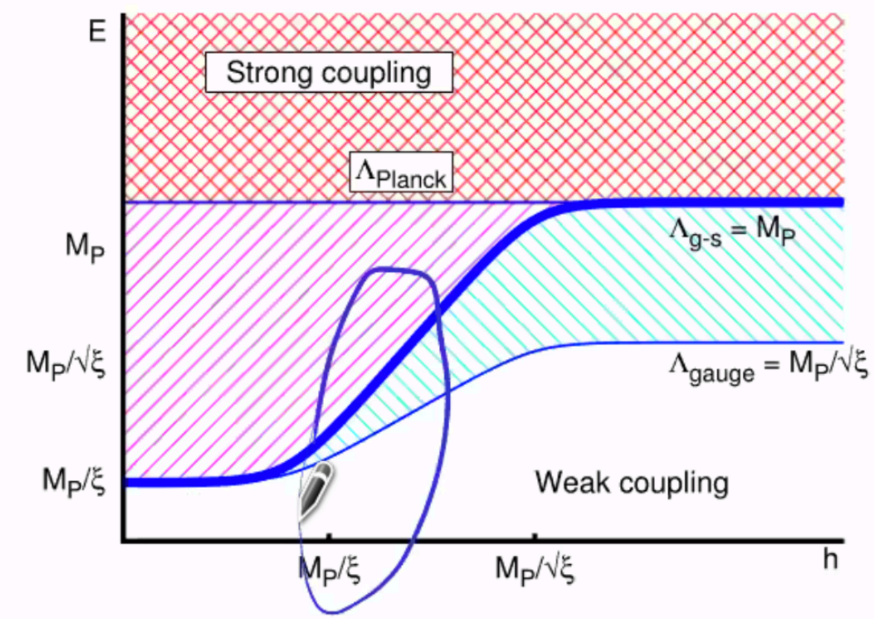
Unitary problem for Higgs inflation

Cut-off grows with the field background

Jordan frame



Einstein frame



Relevant scales at inflation

Hubble scale $H \sim \lambda^{1/2}\frac{M_P}{\xi}$

Energy density at inflation

$$V^{1/4} \sim \lambda^{1/4}\frac{M_P}{\sqrt{\xi}}$$

Relation between cut-offs in different frames:

$$\Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}}\Omega$$

Reheating temperature $M_P/\xi < T_{\text{reheating}} < M_P/\sqrt{\xi}$

Problems during reheating

SM+Type II seesaw

$$\chi = \sqrt{\frac{3}{2}} M_p \log\left(1 + \frac{\xi_H |h|^2}{M_p^2} + \frac{\xi_\Delta (|\Delta^0|)^2}{M_p^2}\right) \quad \text{and} \quad \kappa = \frac{\rho_H}{\rho_\Delta}$$

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\left(1 + \frac{1}{6}\frac{\kappa^2 + 1}{\xi_H \kappa^2 + \xi_\Delta}\right)(\partial_\mu \chi)^2 + \frac{M_p}{\sqrt{6}} \frac{(\xi_\Delta - \xi_H)\kappa}{(\xi_H \kappa^2 + \xi_\Delta)^2}(\partial_\mu \chi)(\partial^\mu \kappa) + \frac{M_p^2}{2} \frac{\xi_H^2 \kappa^2 + \xi_\Delta^2}{(\xi_H \kappa^2 + \xi_\Delta)^3}(\partial_\mu \kappa)^2 \, .$$

$$\xi \equiv \xi_H + \xi_\Delta \gg 1$$

$$\mathcal{L}_{\rm kin} = \frac{1}{2}(\partial_\mu \chi)^2 + \frac{M_p^2}{2} \frac{\xi_H^2 \kappa^2 + \xi_\Delta^2}{(\xi_H \kappa^2 + \xi_\Delta)^3}(\partial_\mu \kappa)^2$$

SM+Type II seesaw

$$U = \frac{\lambda_H \kappa^4 + \lambda_{h\Delta}\kappa^2 + \lambda_\Delta}{4(\xi_H \kappa^2 + \xi_\Delta)^2} M_p^4$$

$$\begin{aligned}
&(1)\ 2\lambda_H\xi_\Delta - \lambda_{h\Delta}\xi_H > 0\ ,\ 2\lambda_\Delta\xi_H - \lambda_{h\Delta}\xi_\Delta > 0\ , \quad \kappa = \sqrt{\frac{2\lambda_\Delta\xi_H - \lambda_{h\Delta}\xi_\Delta}{2\lambda_H\xi_\Delta - \lambda_{h\Delta}\xi_H}} \\
&(2)\ 2\lambda_H\xi_\Delta - \lambda_{h\Delta}\xi_H > 0\ ,\ 2\lambda_\Delta\xi_H - \lambda_{h\Delta}\xi_\Delta < 0\ , \quad \kappa = 0\ , \\
&(3)\ 2\lambda_H\xi_\Delta - \lambda_{h\Delta}\xi_H < 0\ ,\ 2\lambda_\Delta\xi_H - \lambda_{h\Delta}\xi_\Delta > 0\ , \quad \kappa = \infty\ , \\
&(4)\ 2\lambda_H\xi_\Delta - \lambda_{h\Delta}\xi_H < 0\ ,\ 2\lambda_\Delta\xi_H - \lambda_{h\Delta}\xi_\Delta < 0\ , \quad \kappa = 0, \infty\ .
\end{aligned}$$

Kappa is almost fixed during inflation

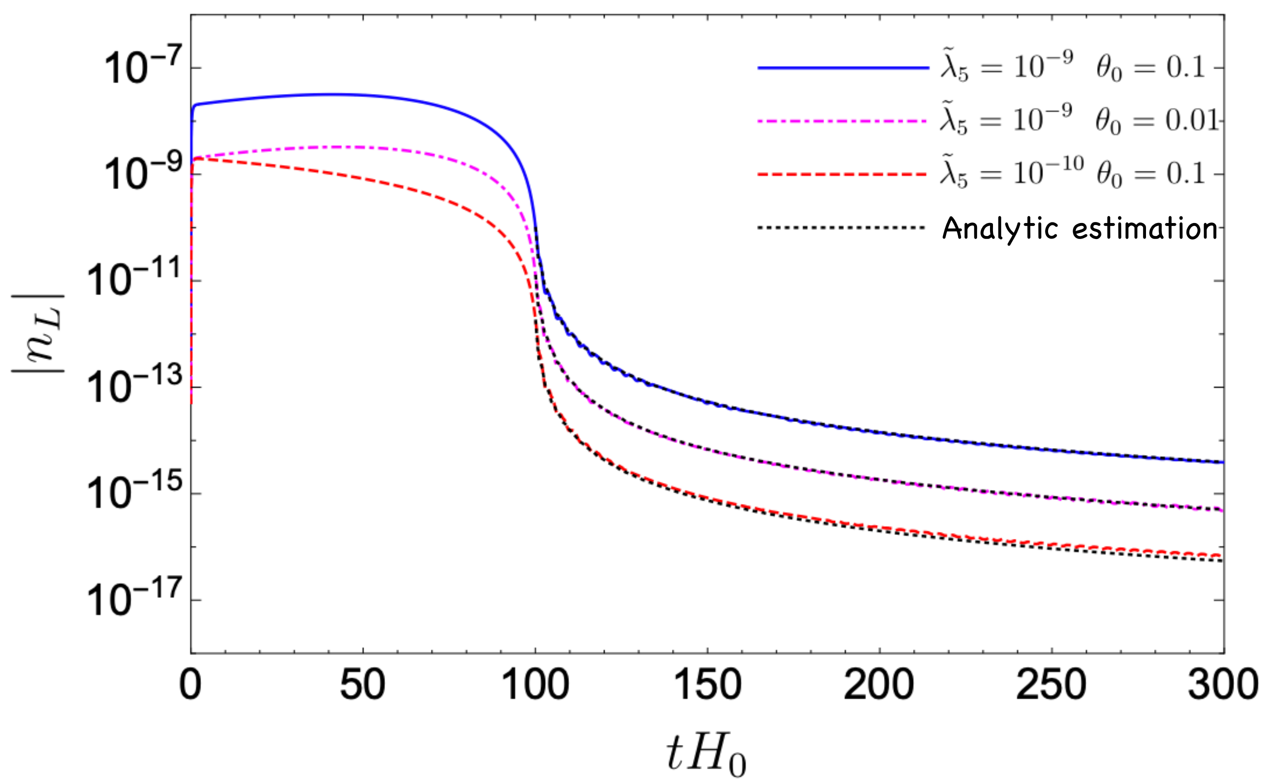
After inflation

$$\frac{d[a^3 f(\chi)\dot{\theta}]}{dt} = \frac{d[a^3 n_L/(Q_L\Omega^2)]}{dt} = a^3 U_{,\theta}$$

If U,theta red shifted faster than a^3, we can ignore the last term

$$n_L(t) = n_{L\mathrm{end}} \frac{\Omega^2(\chi)}{\Omega^2(\chi_{\mathrm{end}})} \left(\frac{a}{a_{\mathrm{end}}}\right)^{-3}$$

After inflation

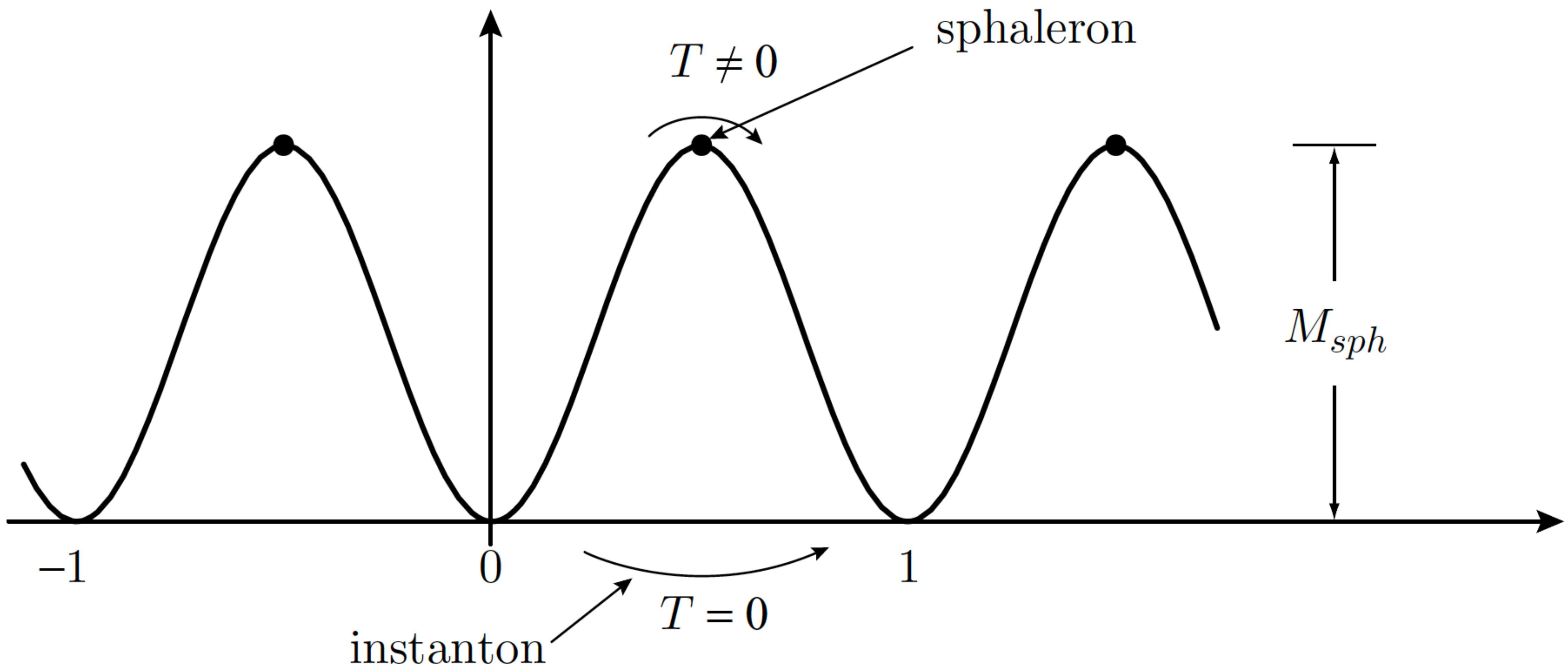


After inflation, nL is just conserved with additional factor Omega

Instanton, sphaleron process

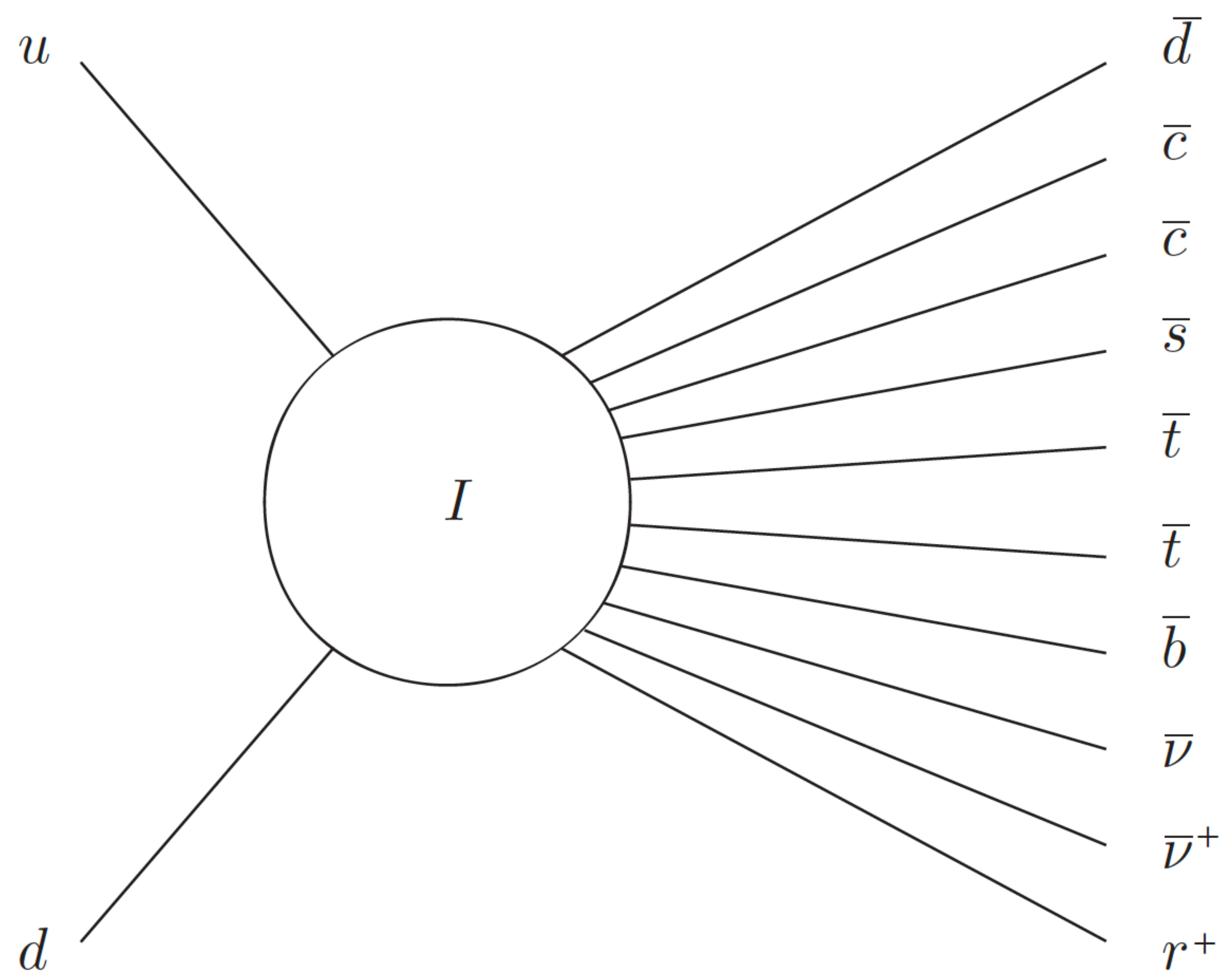
Effective for T> 100 GeV

$$\exp\left(-\frac{M_{sph}(T)}{T}\right) \sim \exp\left(-2\pi\frac{M_W(T)}{\alpha_w T}\right)$$



$$\Gamma \propto \exp\left(-\frac{4\pi}{\alpha}\right)$$

Instanton, sphaleron



Baryon asymmetry via leptogenesis

1. the sphaleron interactions themselves:

$$\sum_i (3\mu_{q_i} + \mu_{\ell_i}) = 0$$

2. a similar relation for QCD sphalerons:

$$\sum_i (2\mu_{q_i} - \mu_{u_i} - \mu_{d_i}) = 0.$$

3. vanishing of the total hypercharge of the universe:

$$\sum_i \left(\mu_{q_i} - 2\mu_{\bar{u}_i} + \mu_{\bar{d}_i} - \mu_{\ell_i} + \mu_{\bar{e}_i}\right) + \frac{2}{N}\mu_H = 0$$

4. the quark and lepton Yukawa couplings give relations:

$$\mu_{q_i} - \mu_\phi - \mu_{d_j} = 0, \quad \mu_{q_i} - \mu_\phi - \mu_{u_j} = 0, \quad \mu_{\ell_i} - \mu_\phi - \mu_{e_j} = 0.$$

$$B = \frac{8N + 4}{22N + 13} (\mathcal{B} - \mathcal{L})_i$$

SM+Type II seesaw

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \xi \frac{\varphi^2}{M_p^2}$$

$$\frac{d\chi}{d\varphi} = \frac{\sqrt{(6\xi^2\varphi^2/M_p^2) + \Omega^2}}{\Omega^2}$$

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{M_p^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}f(\chi)g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - U(\chi,\theta)$$

$$f(\chi) \equiv \frac{\varphi(\chi)^2\cos^2\alpha}{\Omega^2(\chi)}, \quad \text{and} \quad U(\chi,\theta) \equiv \frac{V(\varphi(\chi),\theta)}{\Omega^4(\chi)}$$

SM+Type II seesaw

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{M_p^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}f(\chi)g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - U(\chi,\theta)$$

$$f(\chi) \equiv \frac{\varphi(\chi)^2\cos^2\alpha}{\Omega^2(\chi)}, \quad \text{and} \quad U(\chi,\theta) \equiv \frac{V(\varphi(\chi),\theta)}{\Omega^4(\chi)}$$

Motion during inflation

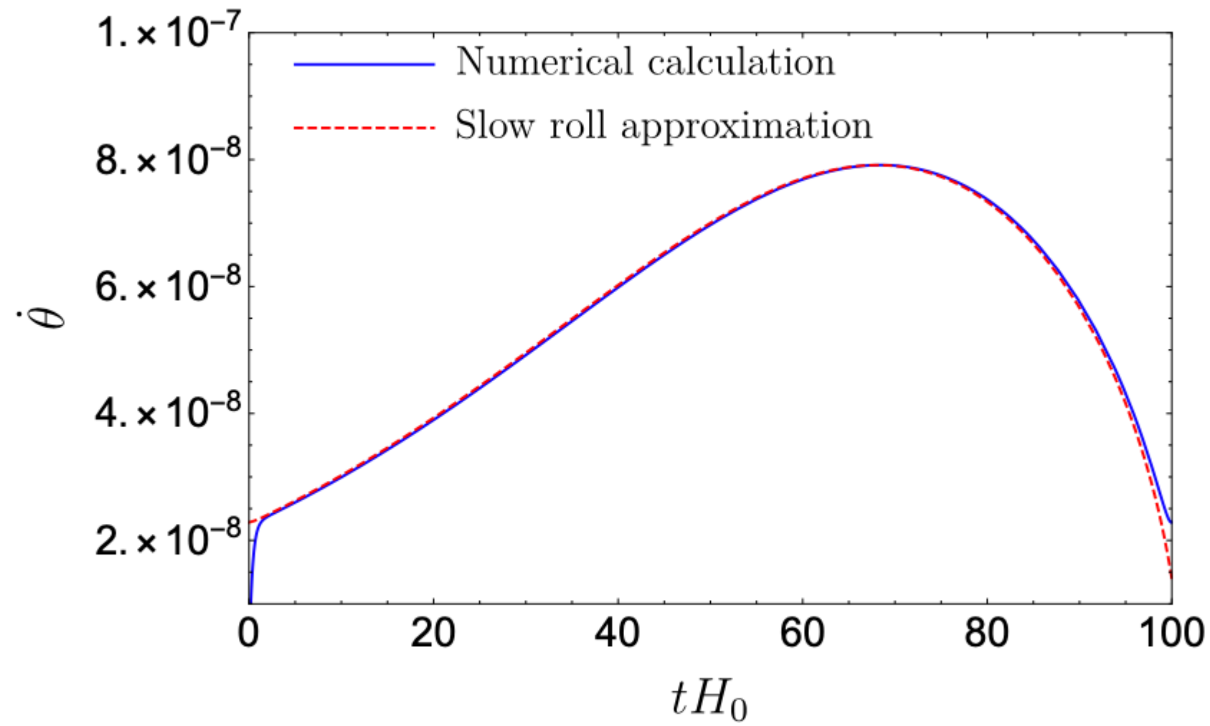
$$\ddot{\chi} - \frac{1}{2}f_{,\chi}\dot{\theta}^2 + 3H\dot{\chi} + U_{,\chi} = 0$$

$$\ddot{\theta} + \frac{f_{,\chi}}{f(\chi)}\dot{\theta}\dot{\chi} + 3H\dot{\theta} + \frac{1}{f(\chi)}U_{,\theta} = 0$$

SM+Type II seesaw

$$\xi = 300,\ \lambda = 4.5 \cdot 10^{-5},\ \ \tilde{\lambda}_5 = 10^{-9}$$
$$\chi_0 = 6.0 M_p,\ \dot{\chi}_0 = 0,\ \text{and}\ \dot{\theta}_0 = 0$$

Theta follow slow roll



$$\begin{aligned} n_{L\text{end}} &= Q_L \varphi_{\text{end}}^2 \dot{\theta}_{\text{end}} \cos^2\alpha \\ &\simeq -\mathcal{O}(1) Q_L \varphi_{\text{end}}^2 \frac{M_p U_{,\theta}}{f(\chi_{\text{end}})\sqrt{3U_{\text{end}}}} \cos^2\alpha \\ &\simeq -\mathcal{O}(1) Q_L \tilde{\lambda}_5 \varphi_{\text{end}}^3 \sin\theta_{\text{end}} / \sqrt{3\lambda} \end{aligned}$$