Searching for light and ultralight dark matters

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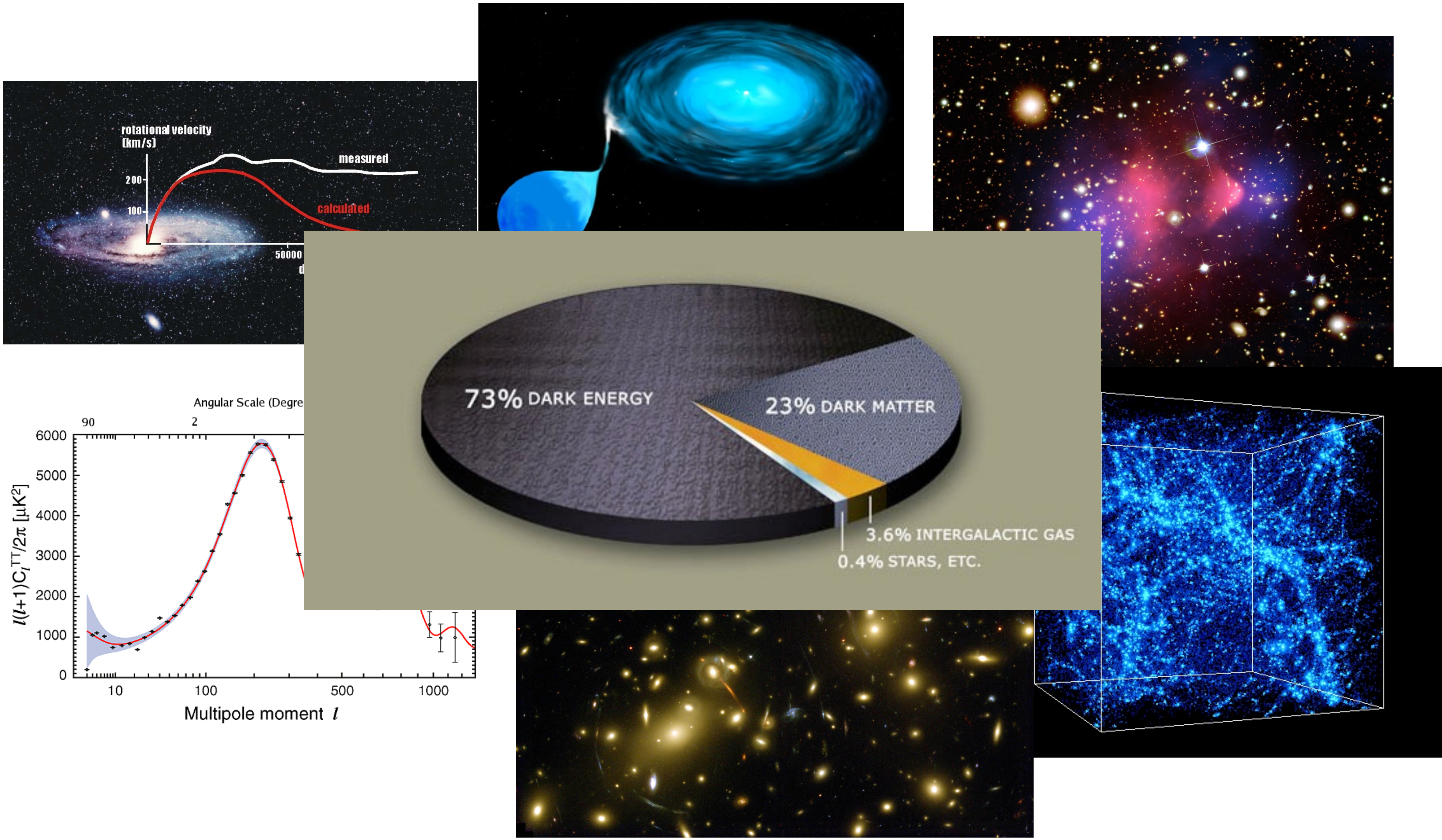
第十一届威海新物理研讨会

2022.7.31– 2022.8.7

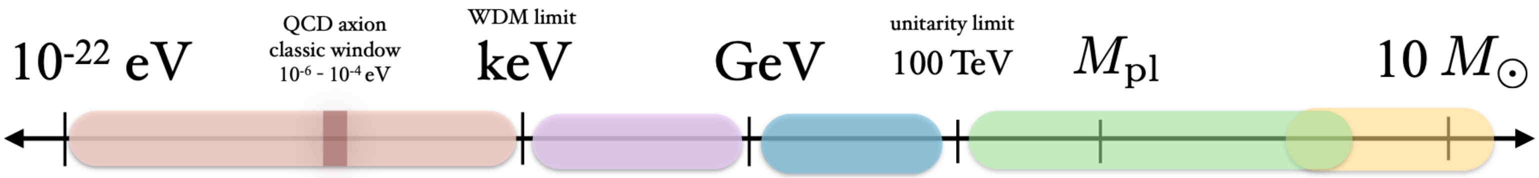
Outline

- Searching for boosted light dark matters
- Searching for ultralight dark matter with radio telescopes

Existence of DM at all scales



Searching for Dark Matter



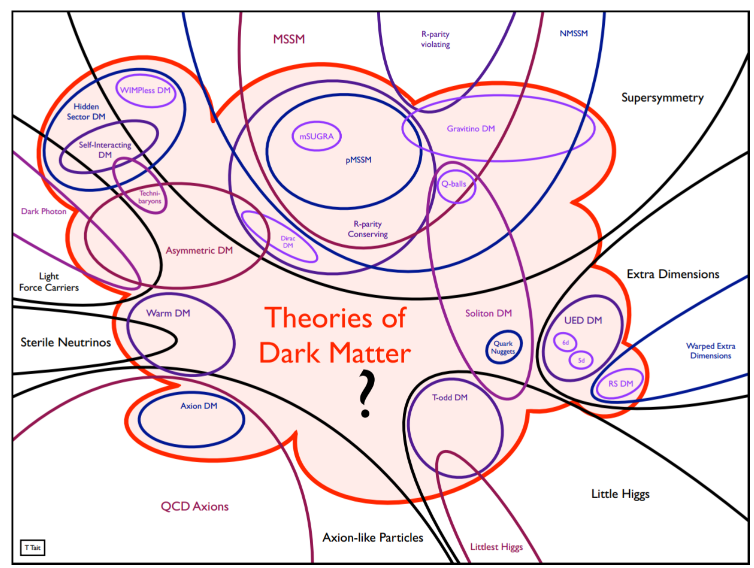
``Ultralight" DM
non-thermal
bosonic fields

``Light" DM
dark sectors
sterile ν
can be thermal

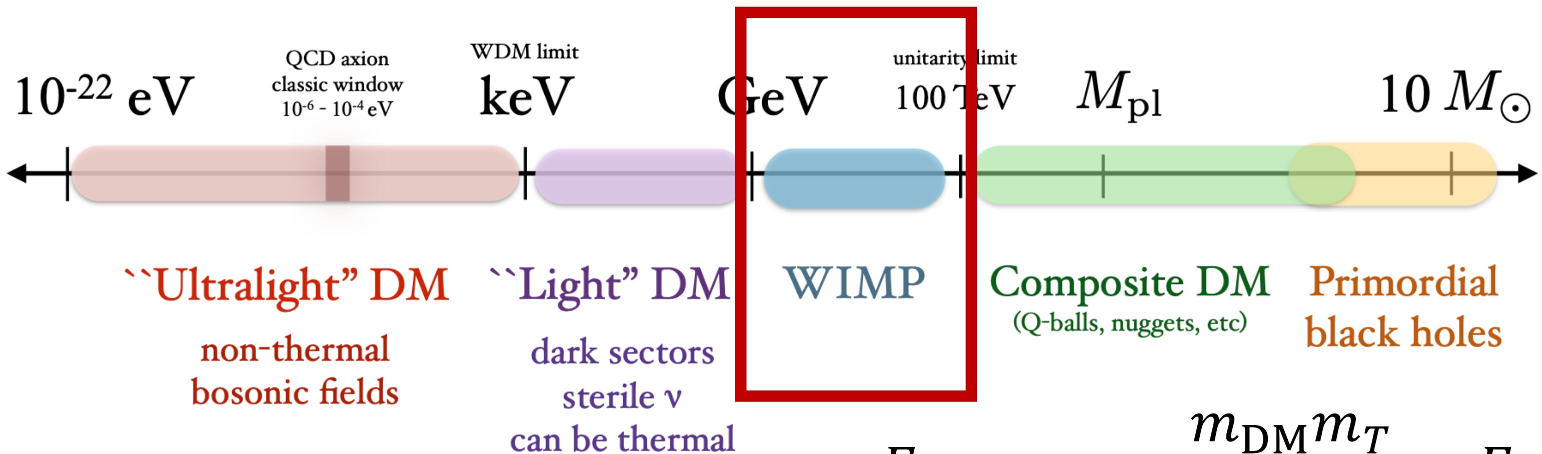
WIMP

Composite DM
(Q-balls, nuggets, etc)

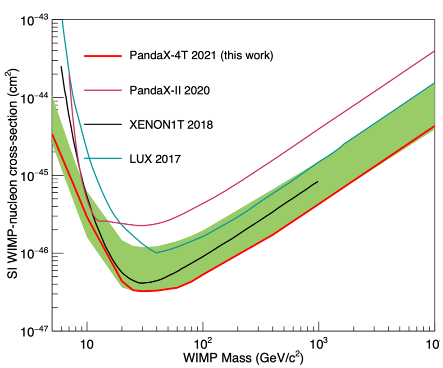
Primordial
black holes

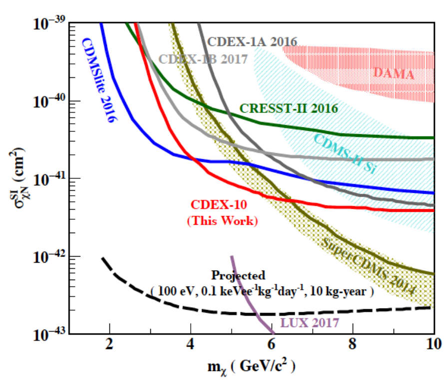


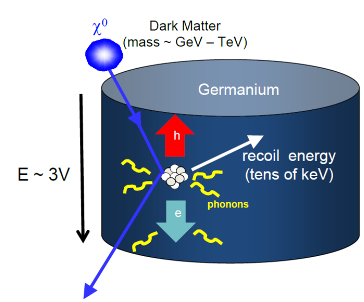
Searching for GeV-TeV DM

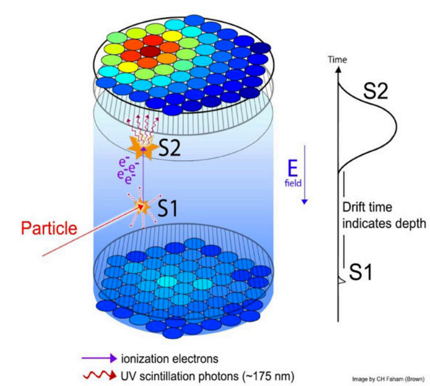


$$E_{recoil} \sim \frac{m_{\mathrm{DM}} m_T}{(m_{\mathrm{DM}} + m_T)^2} E_k$$

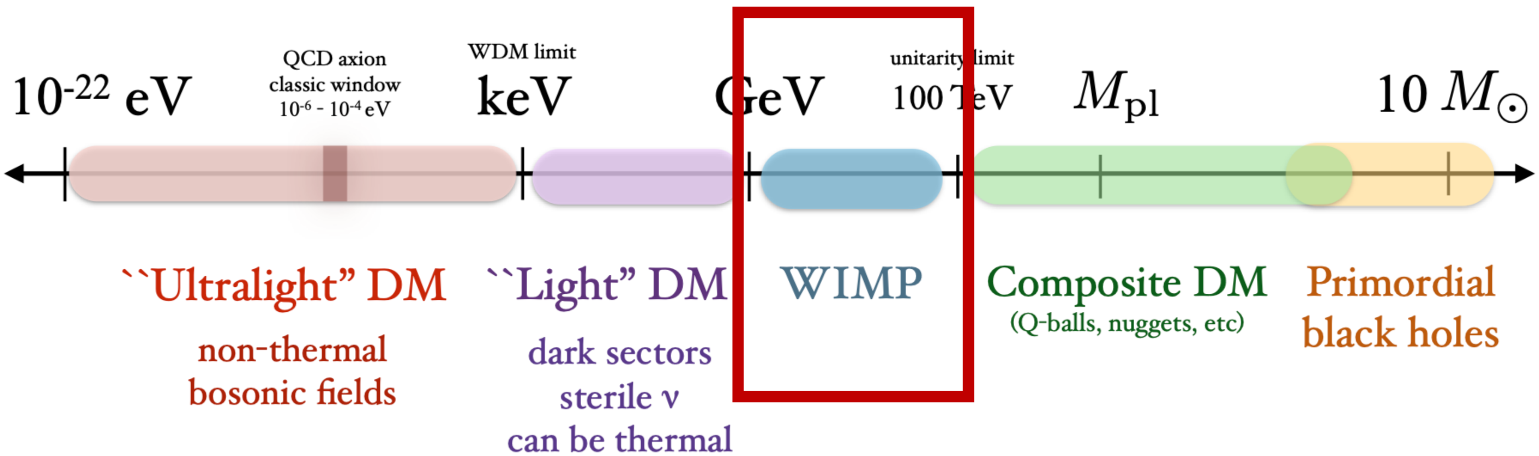


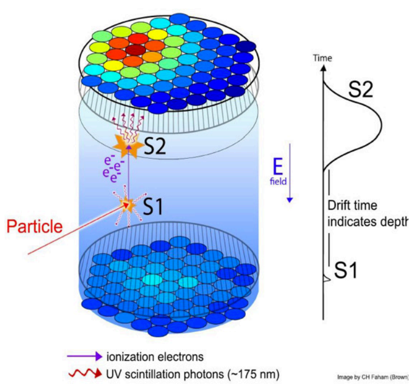


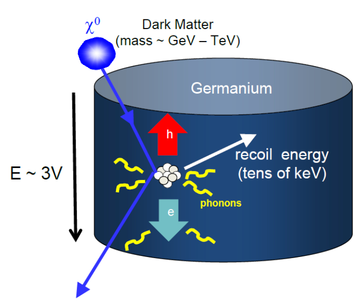


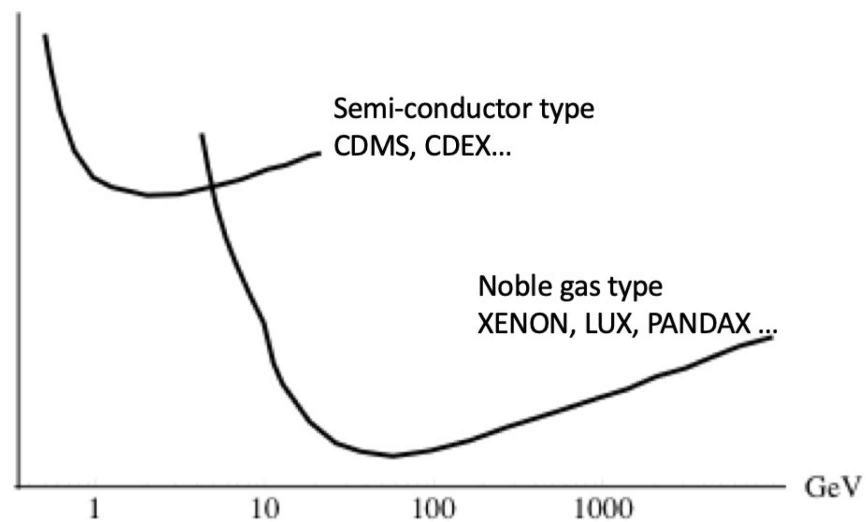


Searching for GeV-TeV DM

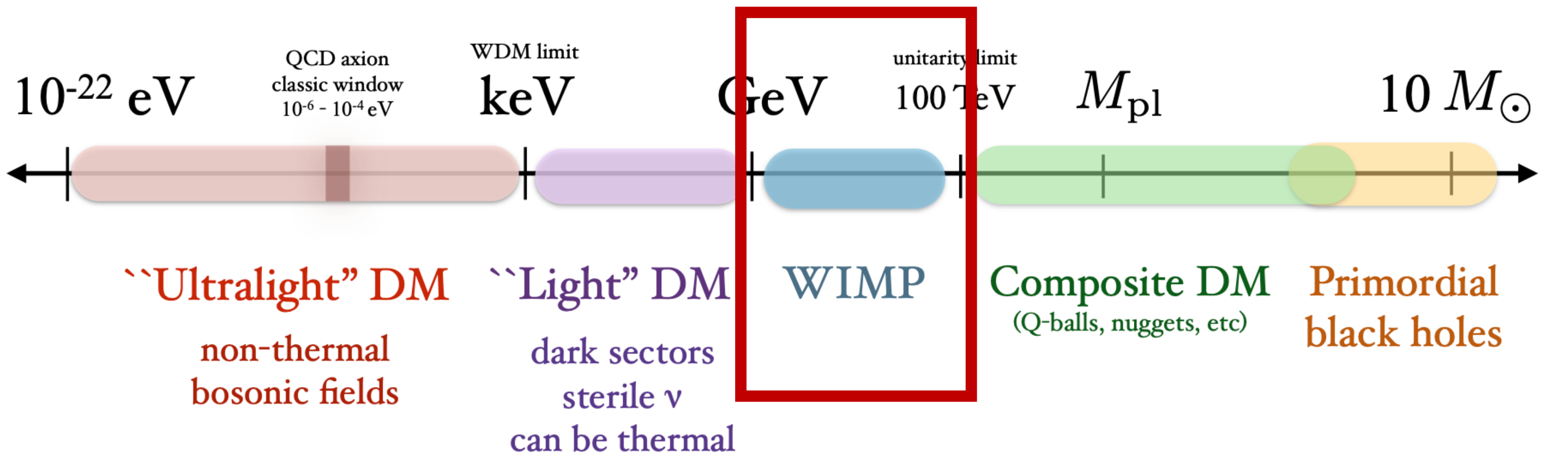


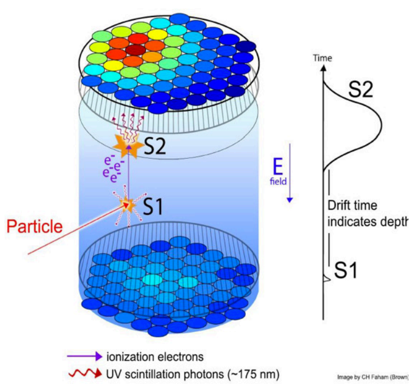


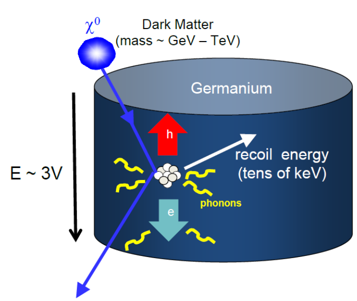


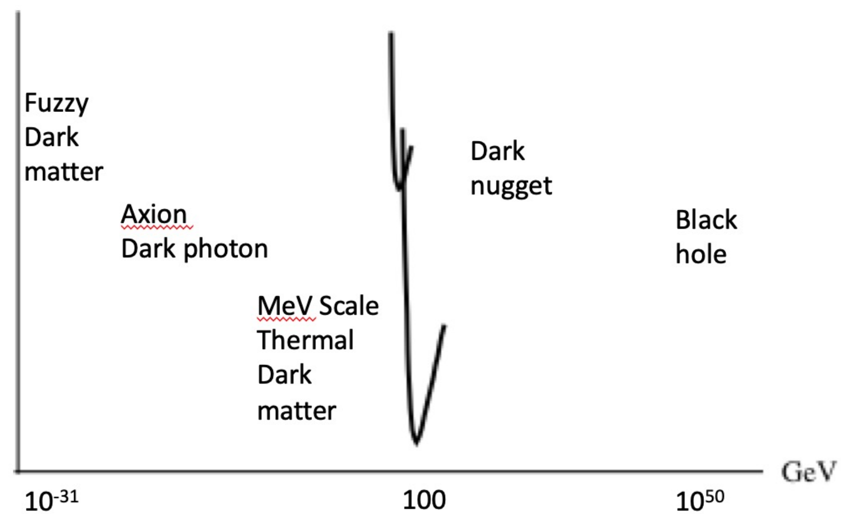


Searching for GeV-TeV DM

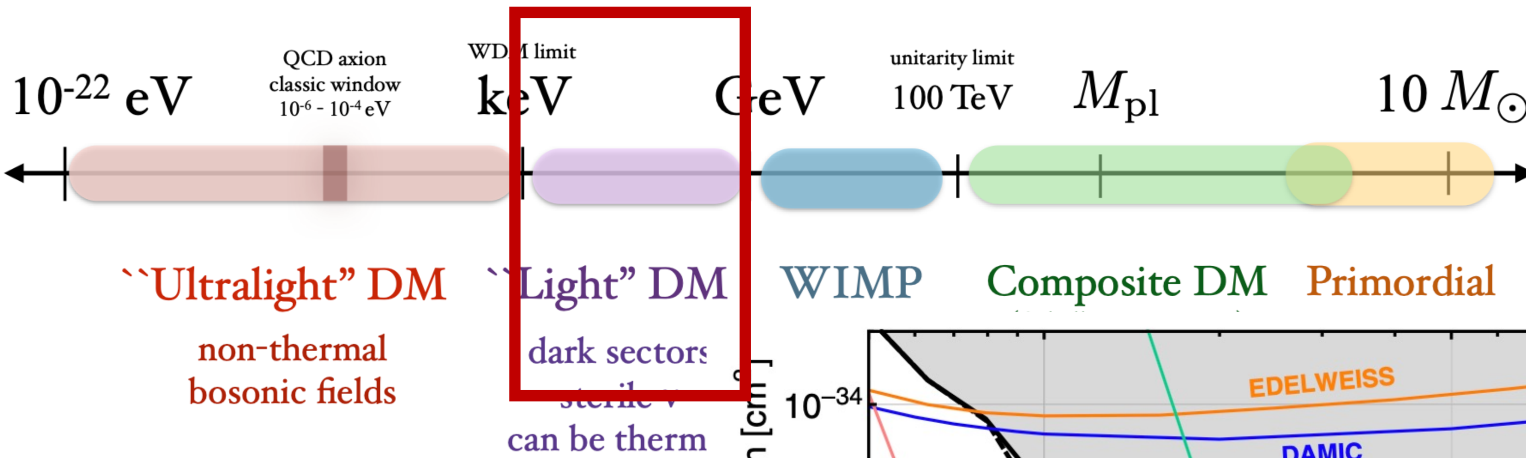






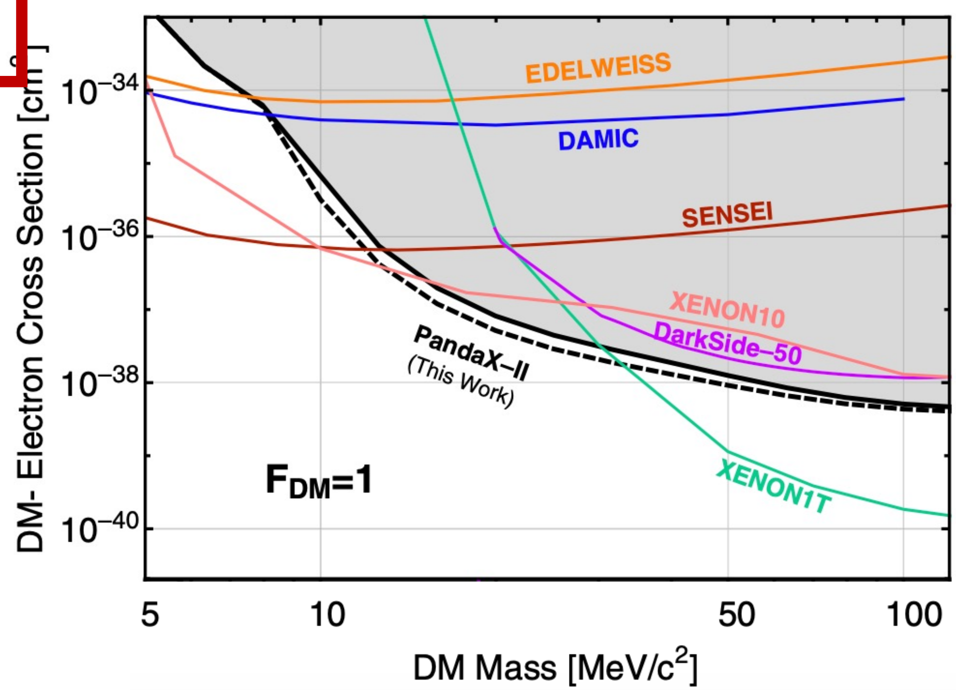


Searching for keV-GeV DM



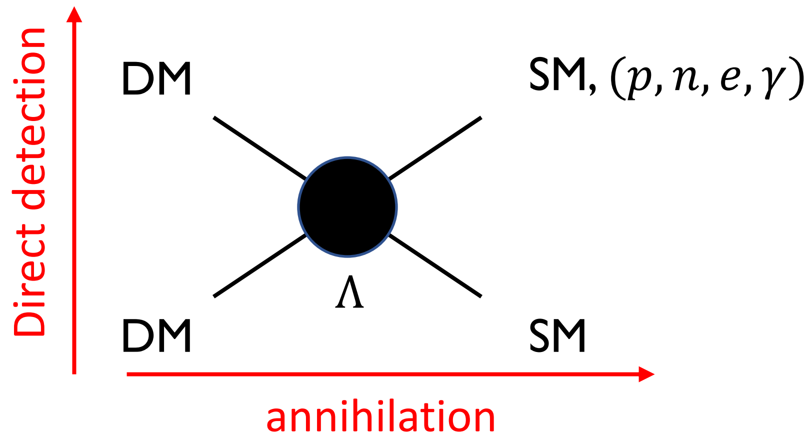
$$E_{\rm recoil} \sim \frac{m_{\rm DM} m_T}{(m_{\rm DM}+m_T)^2} E_{\rm DM}$$

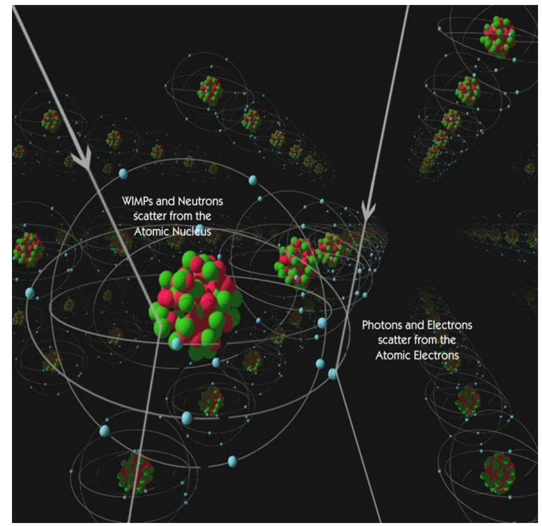
Electron recoil signal



How to produce DM?

- Thermal freeze-out





Thermal freeze out

$\Gamma_A = n_{\mathrm{DM}} < \sigma v > ,$

$\Gamma_A < \mathrm{H}$

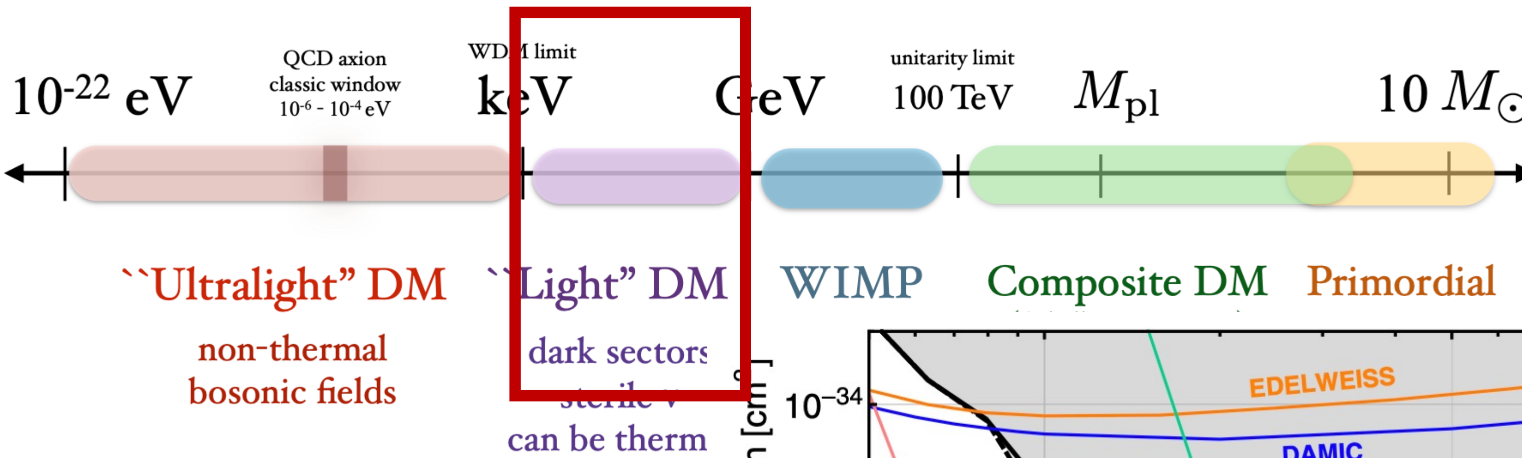
$$\langle \sigma v \rangle \approx 3 \times 10^{-26}\ \mathrm{cm}^3/\mathrm{sec}$$

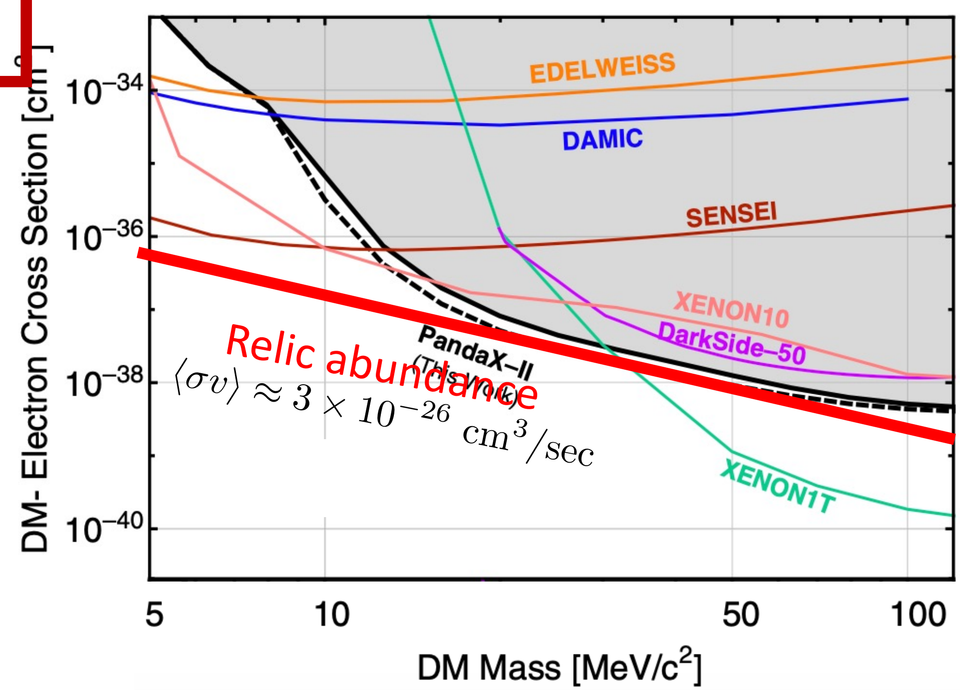
$$\sigma_{\mathrm{annihilation}} \sim \frac{m_D^2}{\Lambda^4}$$

$$\sigma_{\mathrm{scattering}} \sim \frac{\mu^2}{\Lambda^4}$$

μ: reduced mass

Searching for keV-GeV DM



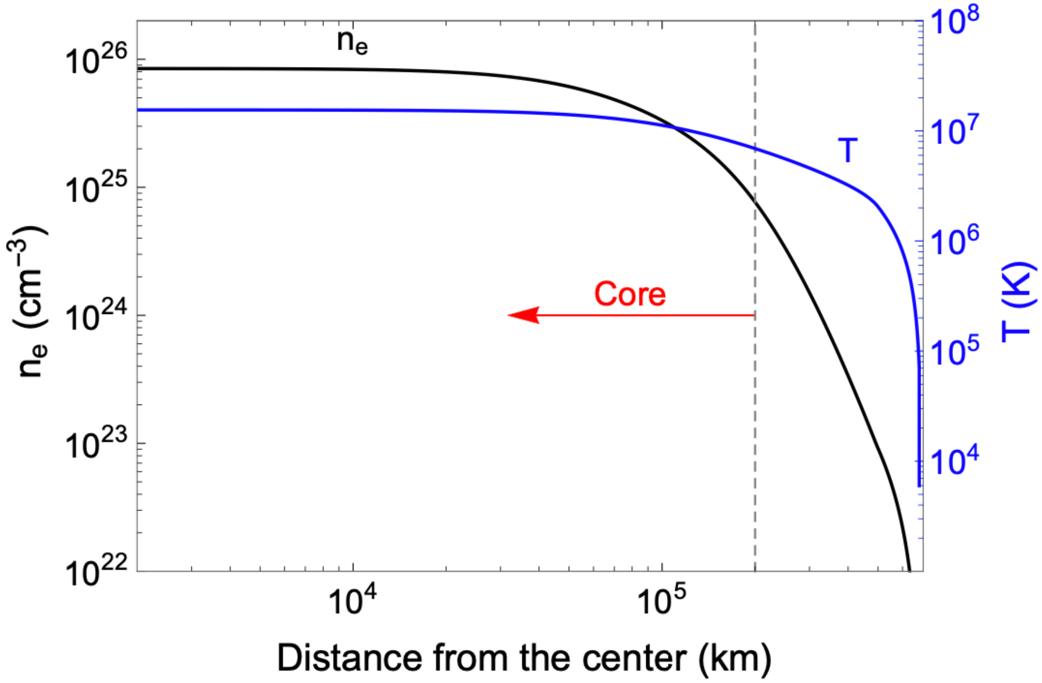


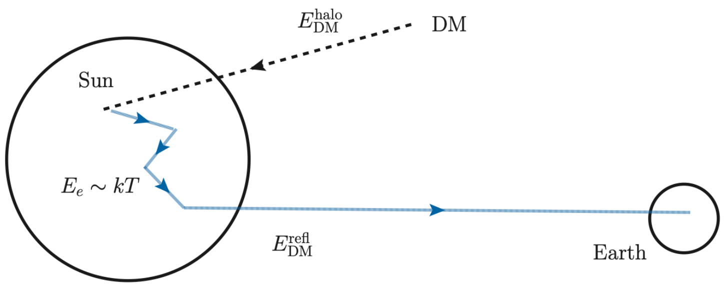
$$E_{\rm recoil} \sim \frac{m_{\rm DM} m_T}{(m_{\rm DM}+m_T)^2} E_{\rm DM}$$

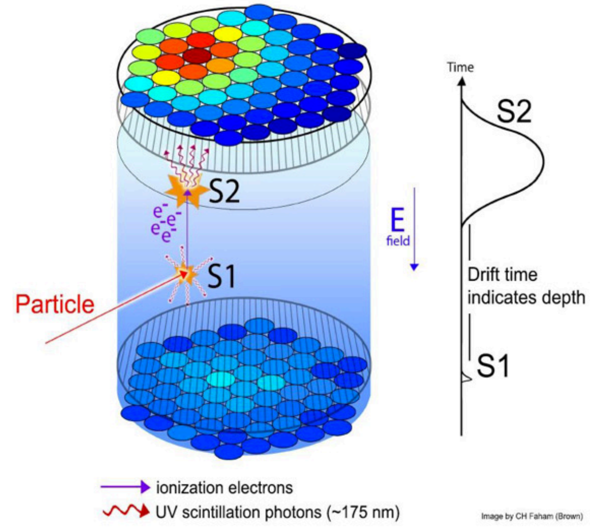
Electron recoil signal

DM accelerated inside the Sun

HA, Maxim Pospelov, Josef Pradler, Phys.Rev.Lett. 120 (2018) 141801



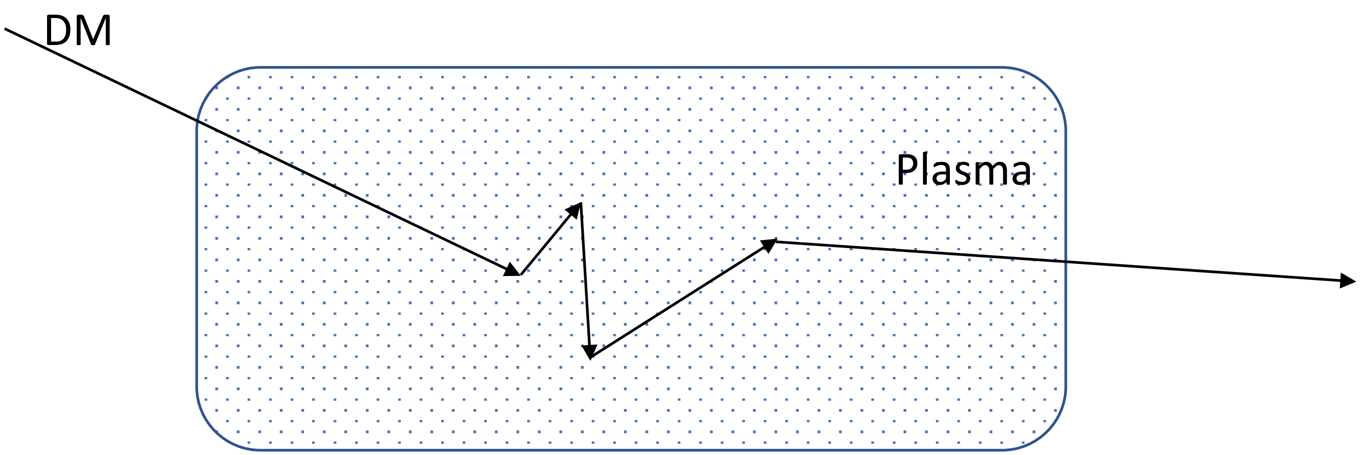




The scattering rate

$$f_{e,\text{ion}}(p,r) = n_{e,\text{ion}}(r)\left(\frac{2\pi}{m_{e,\text{ion}}T(r)}\right)^{3/2} e^{-p^2/2m_{e,\text{ion}}T(r)}$$

$$\Gamma_\chi = \frac{1}{2k_1^0}\int \frac{d^3k_2}{2k_2^0(2\pi)^3}\int \frac{d^3p_1}{2p_1^0(2\pi)^3} f(p_1)\int \frac{d^3p_2}{2p_2^0(2\pi)^3}(2\pi)^4\delta^4(k_1+p_1-k_2-p_2)\sum_{\text{spin}}|\mathcal{M}|^2$$



The contact s-wave interaction

$$x_\chi = v_\chi (m_e/T)^{1/2}$$

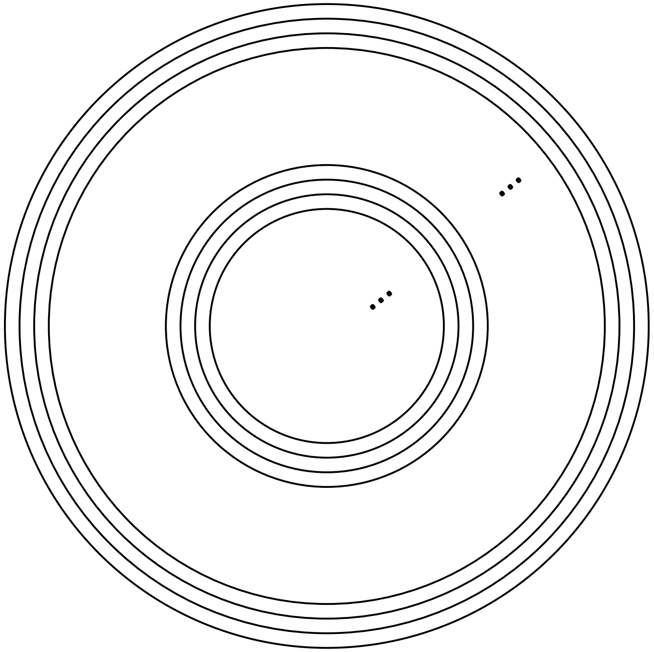
$$\Gamma_\chi = \sigma_{\rm tot} n_e \sqrt{\frac{T}{m_e}} \times \sqrt{\frac{1}{2\pi}} \left[2e^{-x_\chi^2/2} + (2\pi)^{1/2}(1+x_\chi^2)\,\mathrm{erf}\left(\frac{x_\chi}{\sqrt{2}}\right)\right]$$

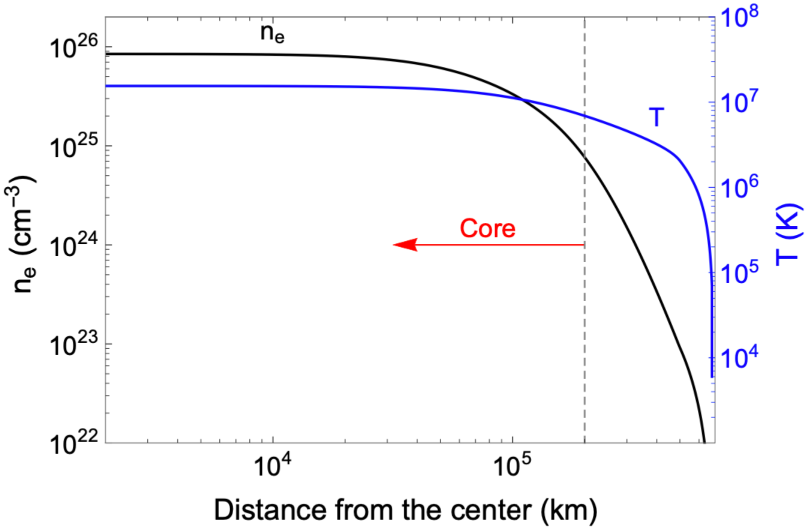
$$\sim \sigma_{\rm tot} n_e \langle v_e \rangle$$

$$\frac{d\Gamma_\chi}{dq\,d\cos\theta_q} \sim \mathcal{N} q \exp\left[-\frac{1}{2m_e T}\left(\frac{m_e}{2m_\chi}(2k_1\cos\theta_q + q) + \frac{q}{2}\right)^2\right]$$

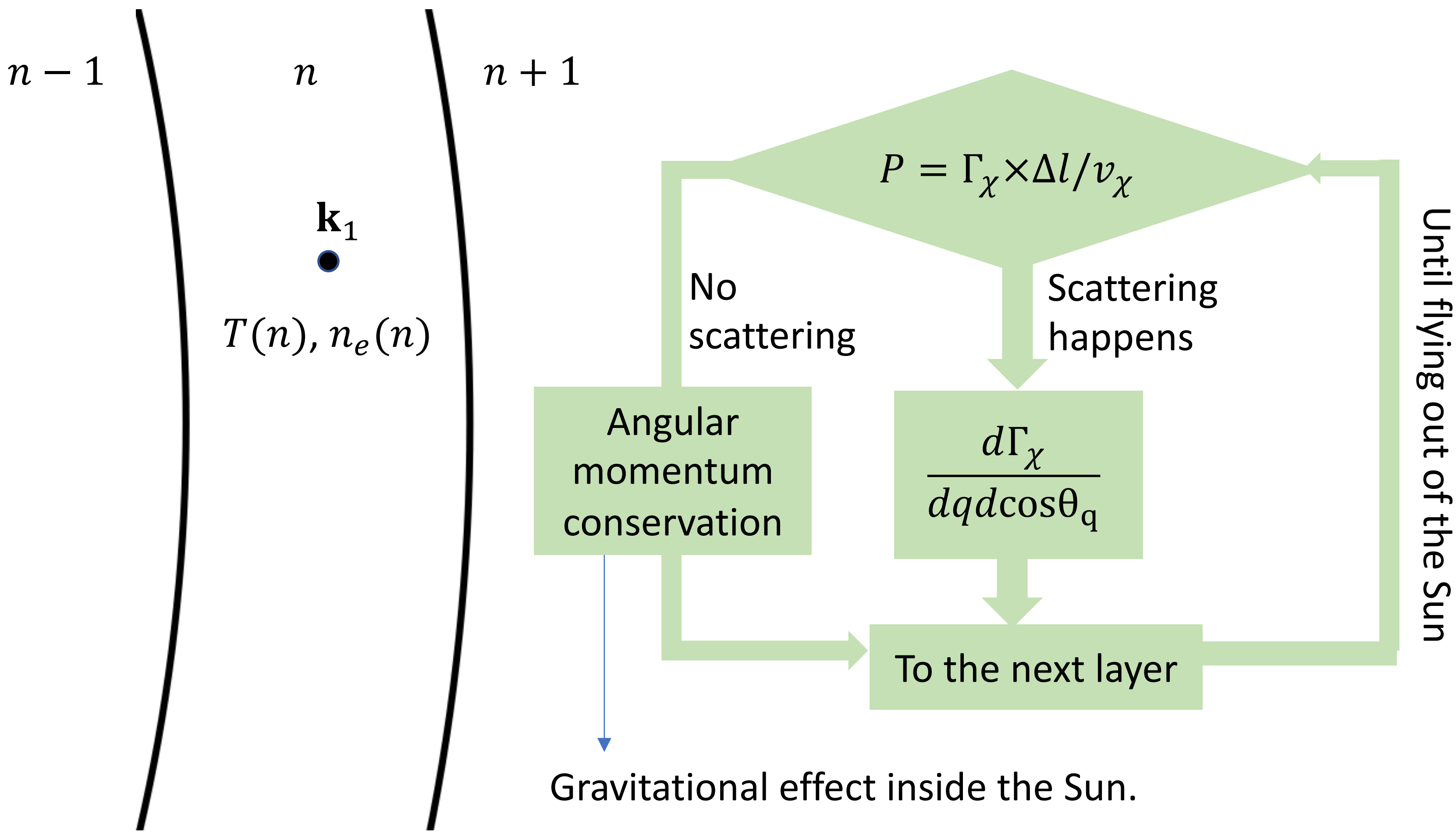
The details of the simulation

- We assume the temperature and electron density distributions are isotropic.
- We divide the Sun into slices. The collision probability in each slice is much smaller than one.

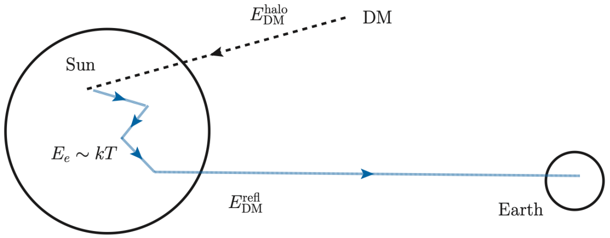


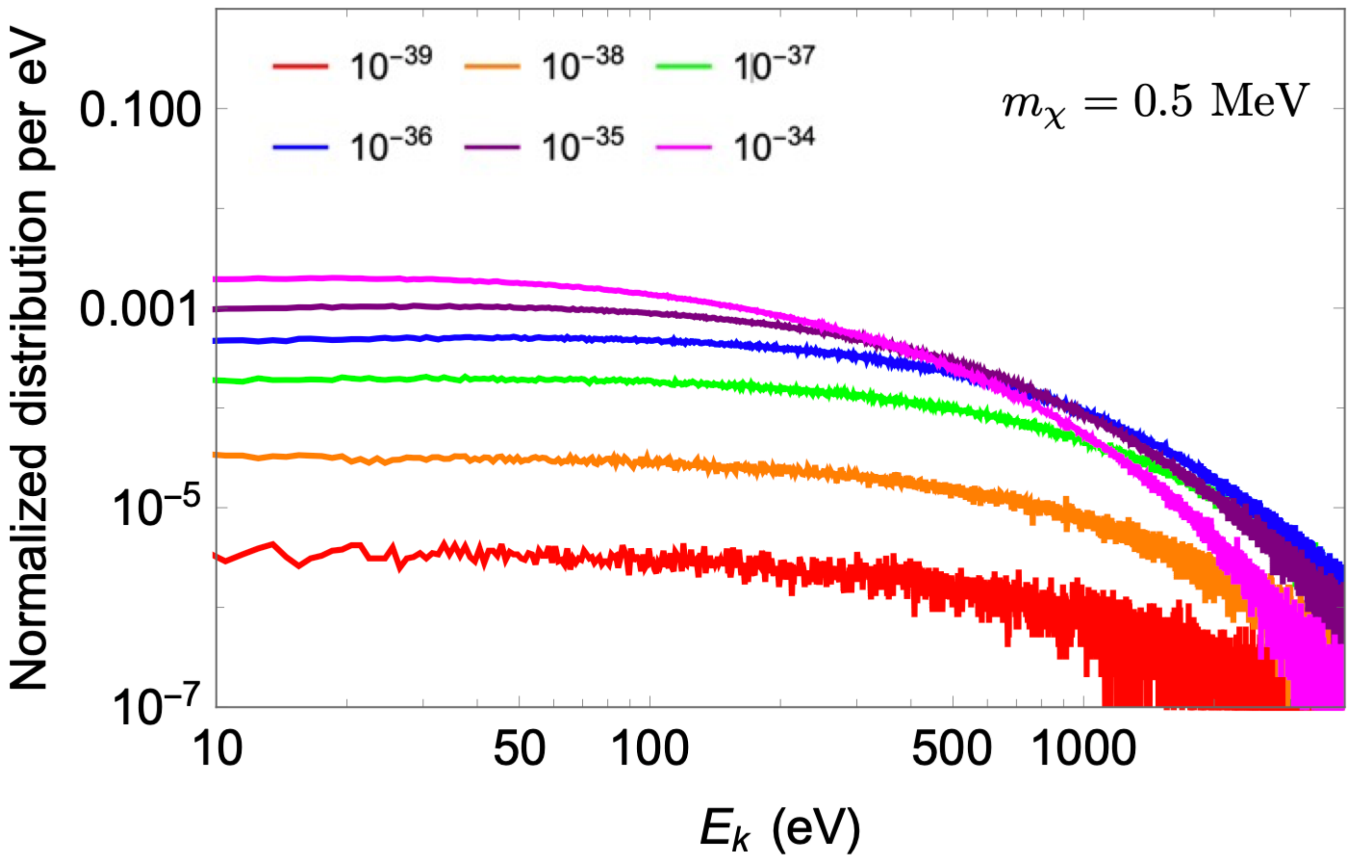


The details of the simulation



The reflected flux

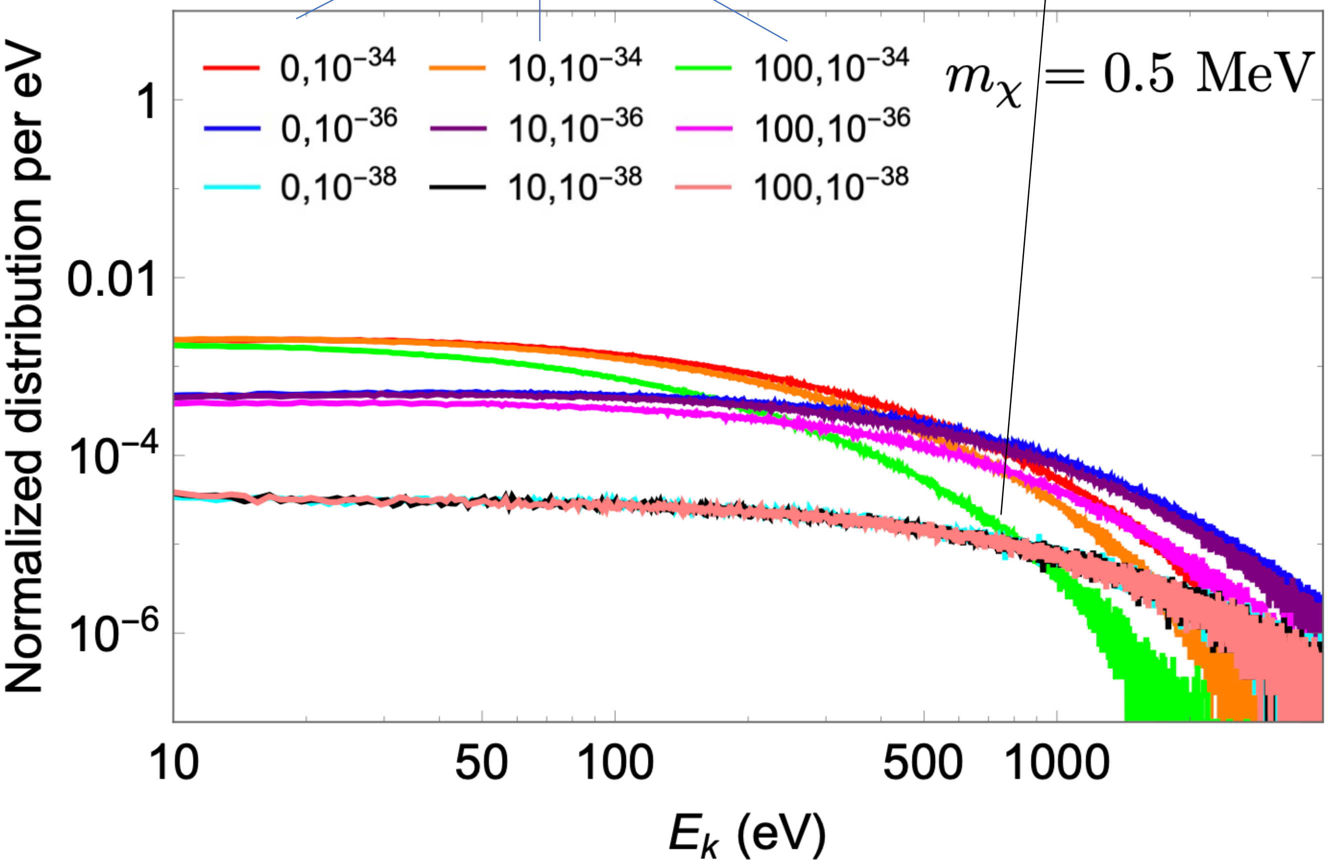




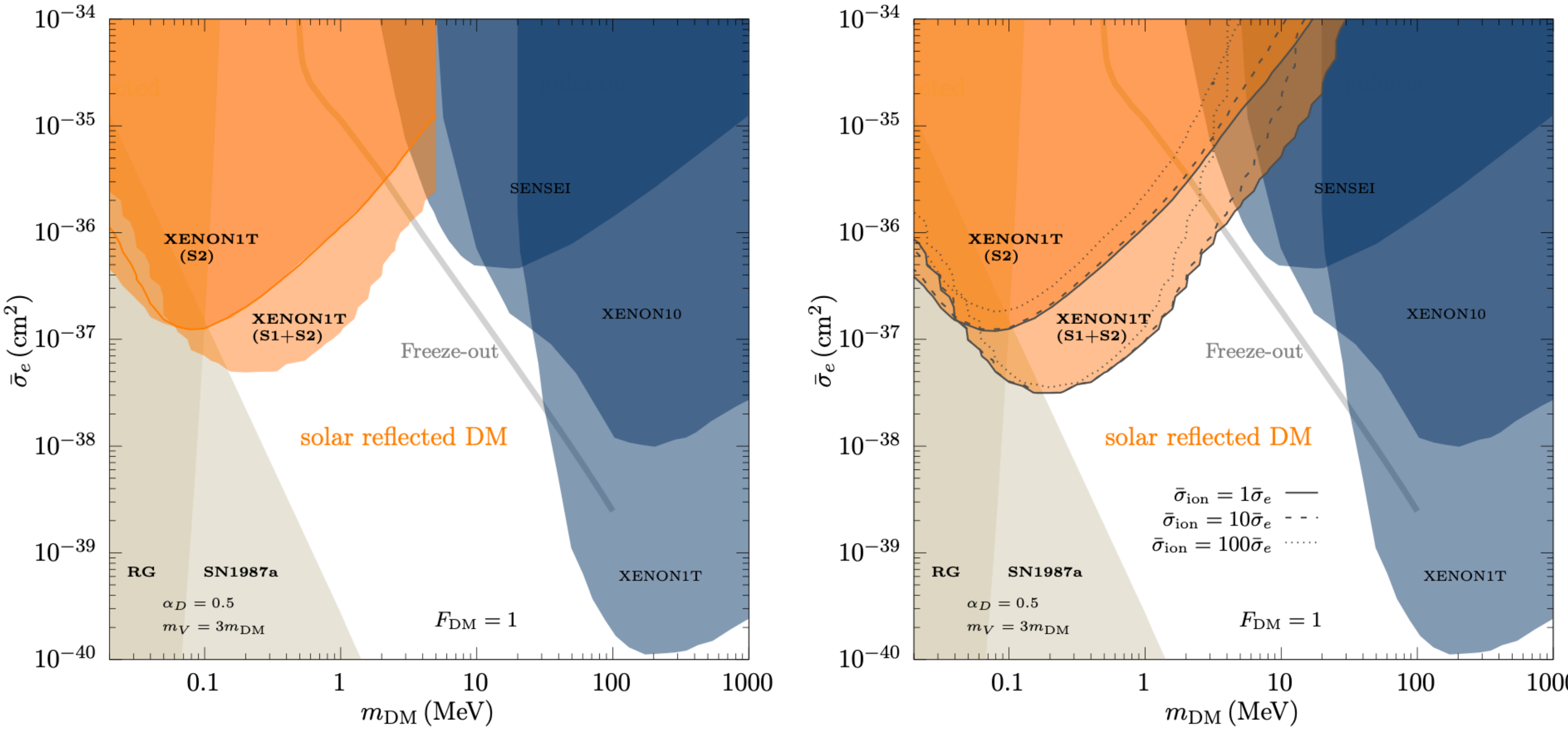
Effect of ions

$$\frac{\sigma_{\chi-\mathrm{ion}}}{\sigma_{\chi-e}}$$

The ions blocks the DM from going into the center.



Results



HA, Haoming Nie, Maxim Pospelov, Josef Pradler, Adam Ritz, Phys.Rev.D 104 (2021) 103026

Light Mediator Case

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\kappa}{2}F_{\mu\nu}V^{\mu\nu} - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{1}{2}m_V^2 V_\mu V^\mu + V_\mu J_\chi^\mu + A_\mu J^\mu$$

Kinetic mixing

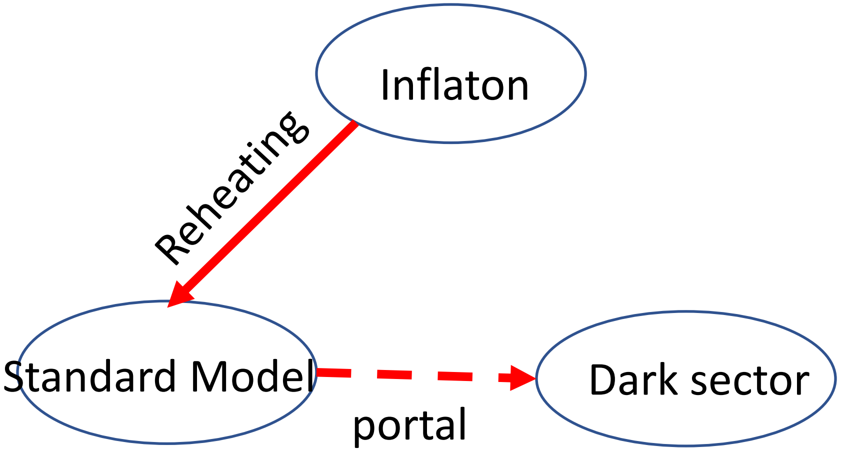
Dark photon

Dark photon mass

$e_D \bar{\chi}\gamma^\mu \chi$

EM current

IR freeze-in

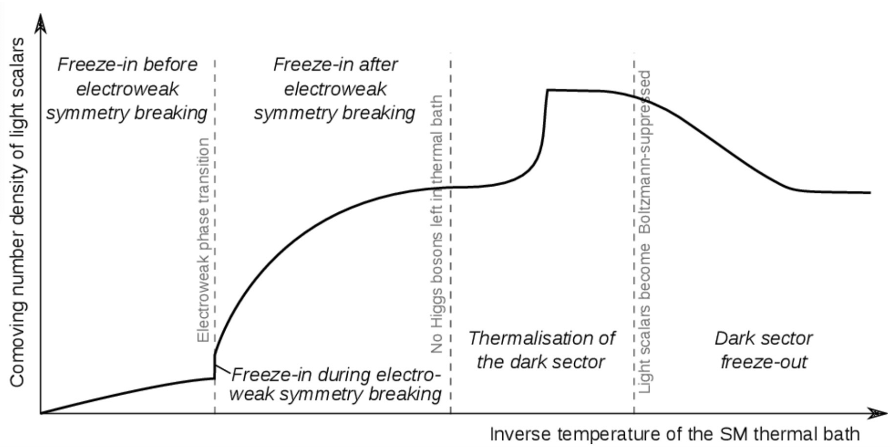


For freeze-in to produce enough DM density: $\kappa e_D \gtrsim 10^{-11}$

For DM not to annihilate away:

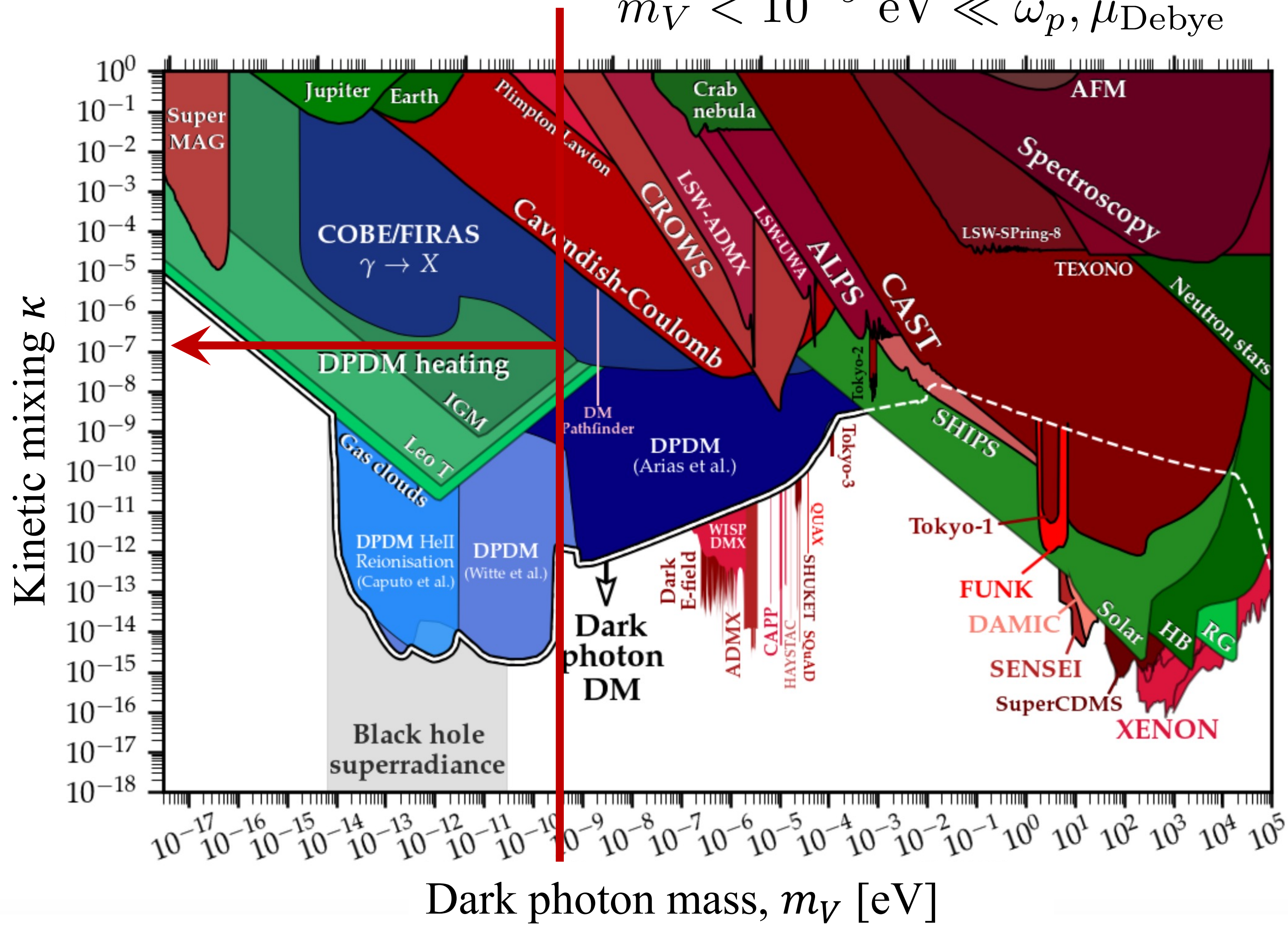
$$e_D \lesssim 1.5 \times 10^{-4}$$

$$\Rightarrow \quad \kappa \gtrsim 10^{-7}$$



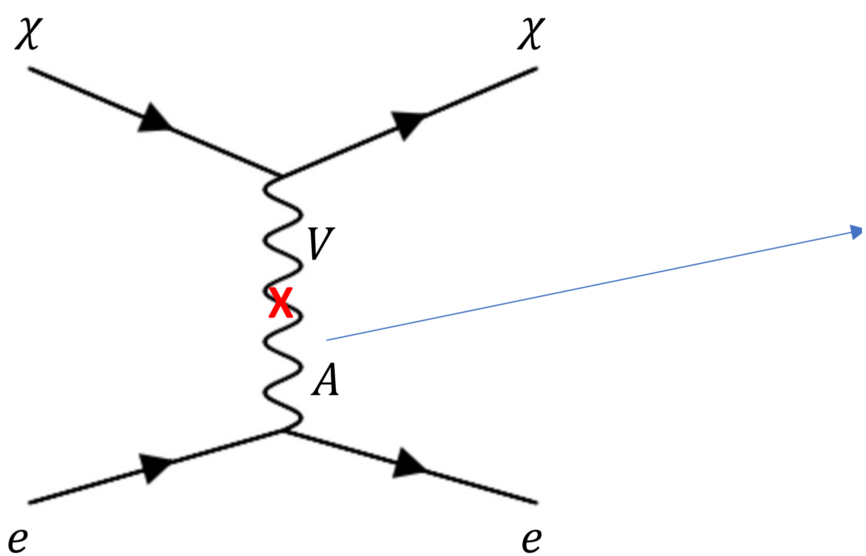
Current Constraints Dark photon

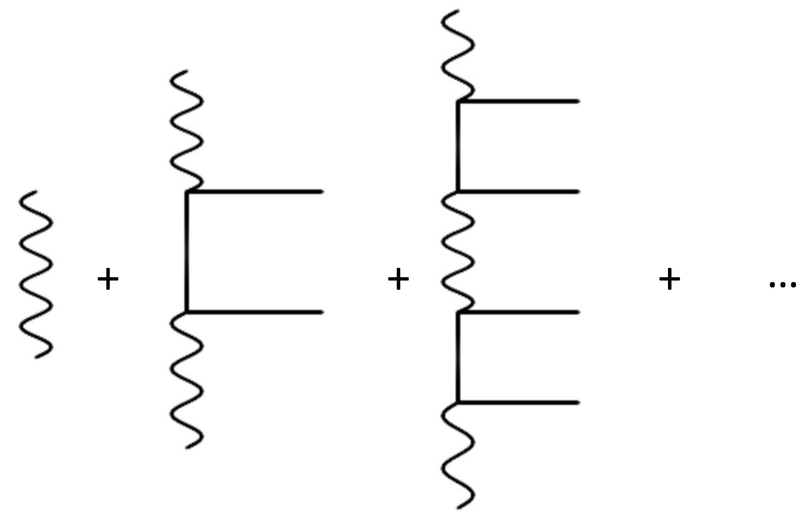
$$m_V < 10^{-9}\ \text{eV} \ll \omega_p, \mu_{\text{Debye}}$$



Detailed calculation

$$\Gamma_\chi = \frac{1}{2k_1^0}\int \frac{d^3k_2}{2k_2^0(2\pi)^3}\int \frac{d^3p_1}{2p_1^0(2\pi)^3} f(p_1)\int \frac{d^3p_2}{2p_2^0(2\pi)^3}(2\pi)^4\delta^4(k_1+p_1-k_2-p_2)\sum_{\text{spin}}|\mathcal{M}|^2$$

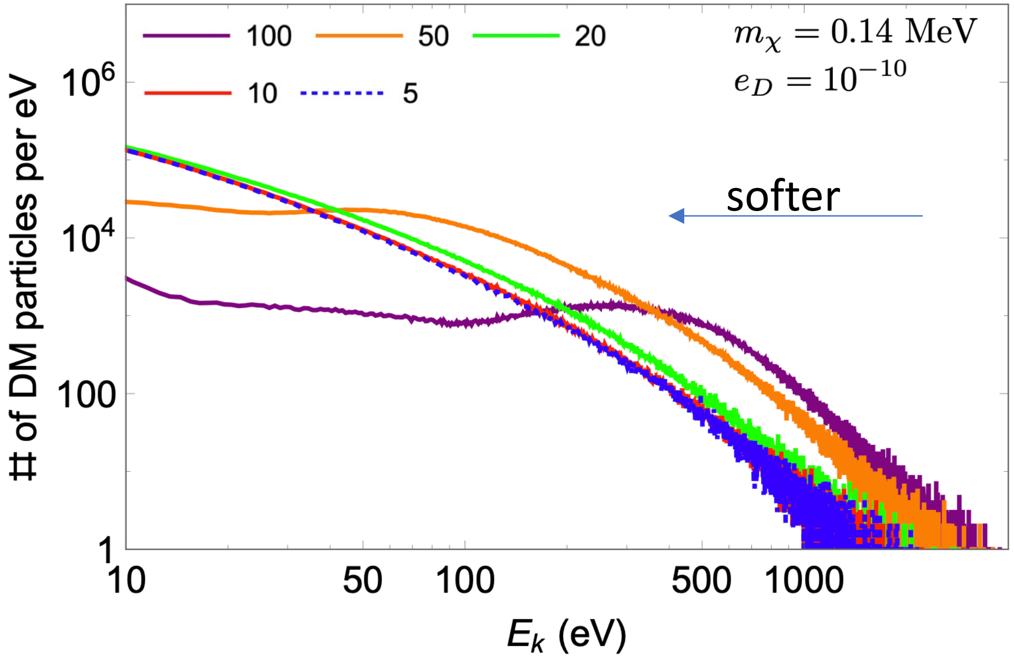




$$\frac{1}{\mathbf{q}^2} \to \frac{1}{\mathbf{q}^2+\mu^2_{\text{Debye}}}$$

Numerical Simulation

- Reflected spectrum with different choices of ζ.
- $\zeta = 0.5$ in our results.



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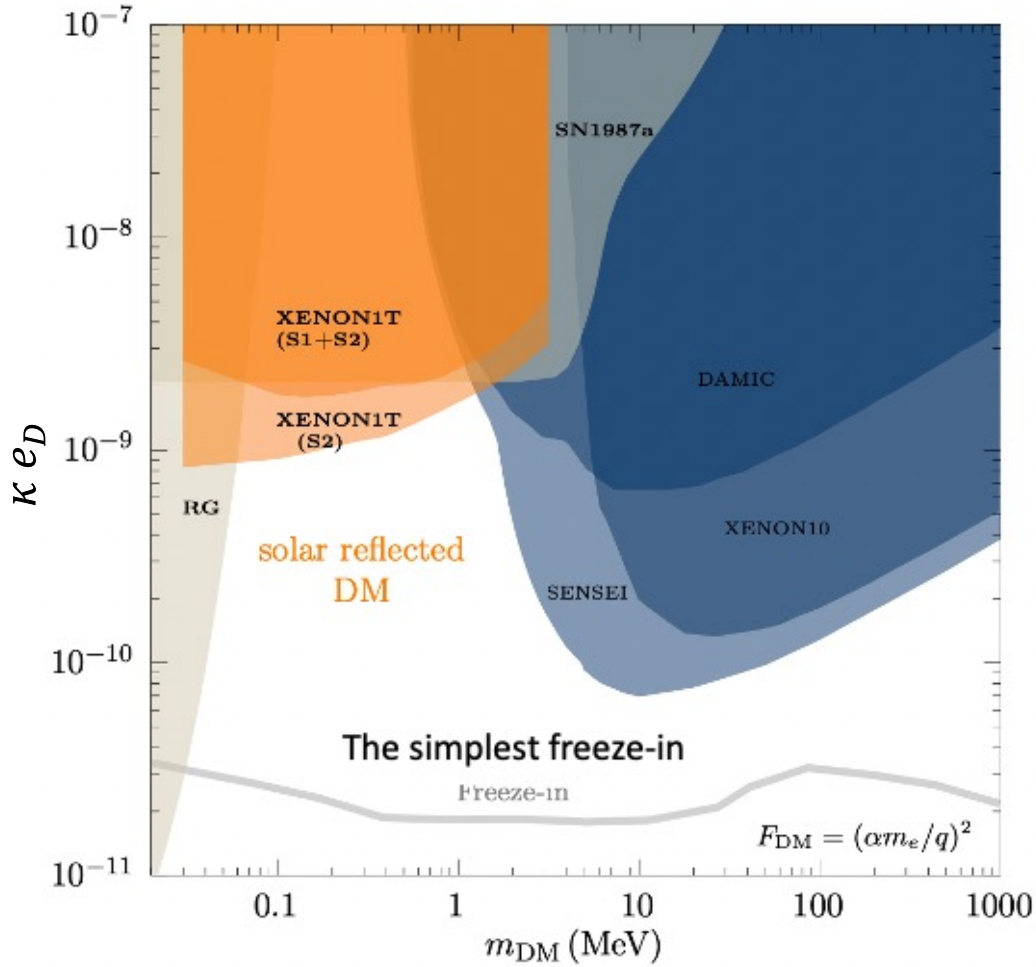

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Ultralight dark photon DM

Dark photon dark matter

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\kappa}{2}F_{\mu}V^{\mu\nu} - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{1}{2}m_V^2 V_\mu V^\mu + eA_\mu J^\mu$$

- Dark photon can decay through the three-photon channel and neutrino channel. Both are highly suppressed!

$$\Gamma_{V\to 3\gamma} \sim \frac{\kappa^2 m_V^9}{m_e^8}$$

$$\Gamma_{V\to \nu\nu} \sim \frac{\kappa^2 m_V^5}{m_Z^4}$$

Ultralight dark photon DM

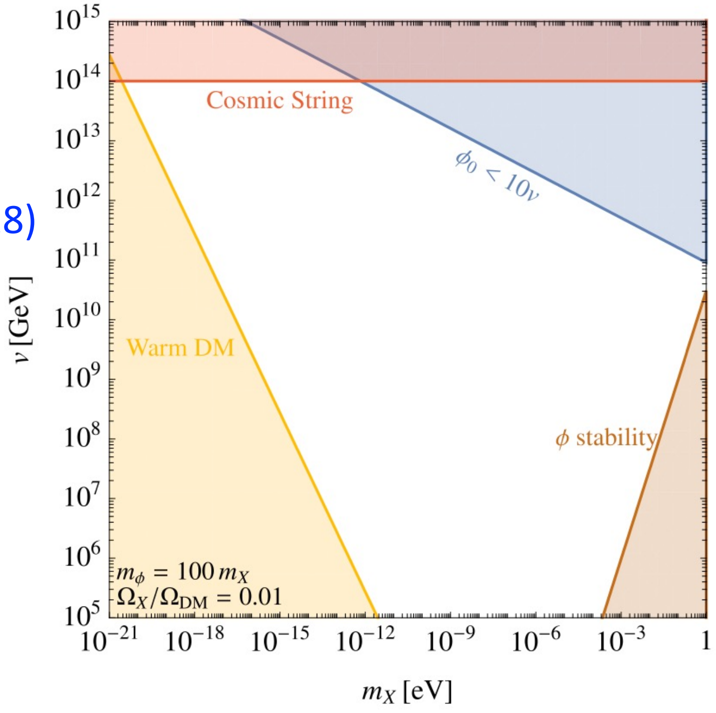
- From quantum fluctuation during inflation

  Graham, Mardon, Majendra (2015)

$$m_{A'} = 10^{-5}\ \text{eV} \times \left(\frac{10^{14}\text{GeV}}{H_I}\right)^4$$

- From parametric resonant production

  Co, Pierce, Zhang, Zhao (2018)
  Dror, Harigaya, Narayan (2018)
  Bastero-Gil, Santiago, Ubaldi, Vega-Morales (2018)
  Agrawal, Kitajima, Reece, Sekiguchi, Takahashi (2018)



- From decay of cosmic string

  Long, Wang (2019)

- …

The field configuration

- $$\mathbf{E}_D(t,\mathbf{x}) = \mathbf{E}_D^{(0)} \cos(\omega_D t - \mathbf{k}_D \cdot \mathbf{x})$$

$$\omega_D \approx m_V + \frac{\mathbf{k}_D^2}{2m_V}$$

$$\frac{k_D}{m_V} \approx v_D \sim 10^{-3}$$

$$\lambda_D \sim \frac{2\pi}{k_D} \approx \frac{2\pi}{m_V v_D} \approx 10^3 \times \frac{2\pi}{m_V}$$

Photon Dark Photon Oscillation

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\kappa}{2}F_{\mu\nu}V^{\mu\nu} - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{1}{2}m_V^2 V_\mu V^\mu + eA_\mu J^\mu$$

$$\Big\downarrow \quad A \to A - \kappa V$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{1}{2}m_V^2 V_\mu V^\mu + eA_\mu J^\mu - \kappa e V_\mu J^\mu$$

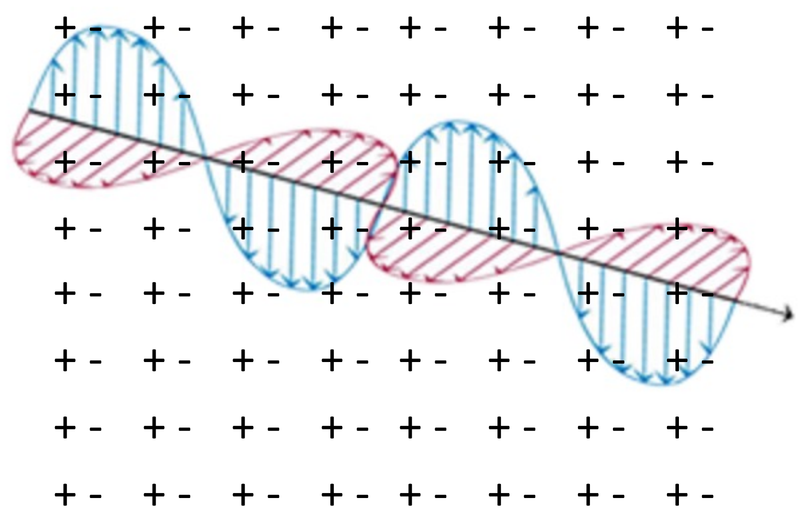
V_μ and A_μ are in mass eigenstate.

Photon Dark Photon Oscillation

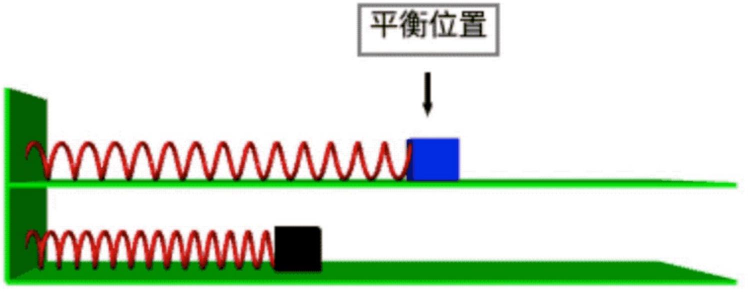
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{1}{2}m_V^2 V_\mu V^\mu + eA_\mu J^\mu - \kappa e V_\mu J^\mu$$

V_μ and A_μ are in mass eigenstate.

- In the vacuum, V cannot be converted into A, no interaction
- In the plasma, (1) a mixing between V and A is generated. (2) a mass for A is also generated.



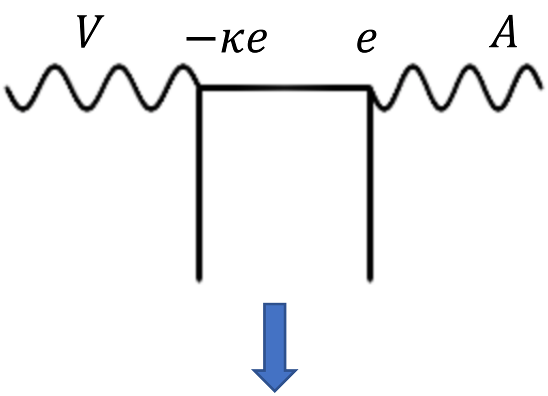
Photon Dark Photon Oscillation



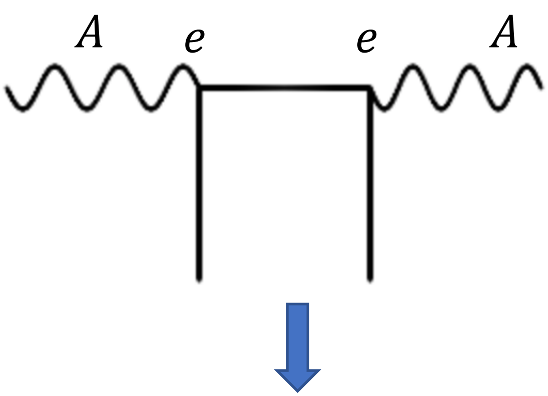
- When $\omega_p = m_{A'}$, photon and dark photon resonantly convert into each other.

Photon Dark Photon Oscillation

- Projecting onto the transverse modes



$$-\kappa\omega_p^2 A_i^T V_i^T$$



$$\omega_p^2 A_i^T A_i^T$$

- One to one transition matrix element

$$\mathcal{M}_{V_T \to A_T} = -\kappa\omega_p^2 \epsilon_A \cdot \epsilon_V$$

Polarization vectors

Photon dark photon oscillation

- The transition probability

$$\omega_p^2 = m_V^2$$

$$P_{V\to A} = \int dt \Gamma_{V\to A} = \int \frac{dt}{2\omega}\frac{d^3p}{(2\pi)^3 2\omega}(2\pi)^4\delta^4(p_V - p_A)\frac{1}{3}\sum_{\rm pol}|\mathcal{M}|^2$$

$$= \frac{2}{3}\pi\kappa^2 m_V v_r^{-1}\left|\frac{\partial \log \omega_p^2}{\partial r}\right|^{-1}_{r=r_c}$$

Resonant region

Average of polarization

If v_r=0 the DM stays at the resonance region forever.

Searching for ultralight DM with radio telescopes

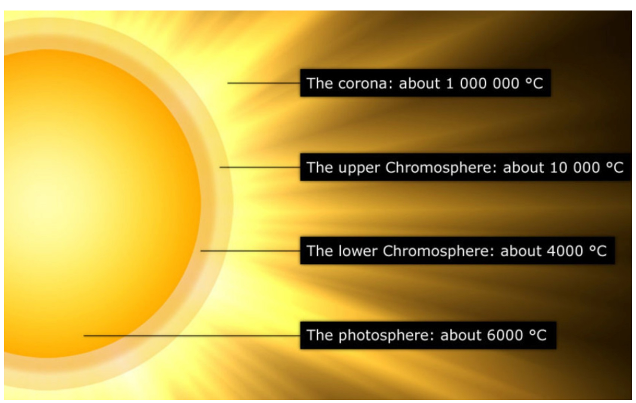
- For dark photon:

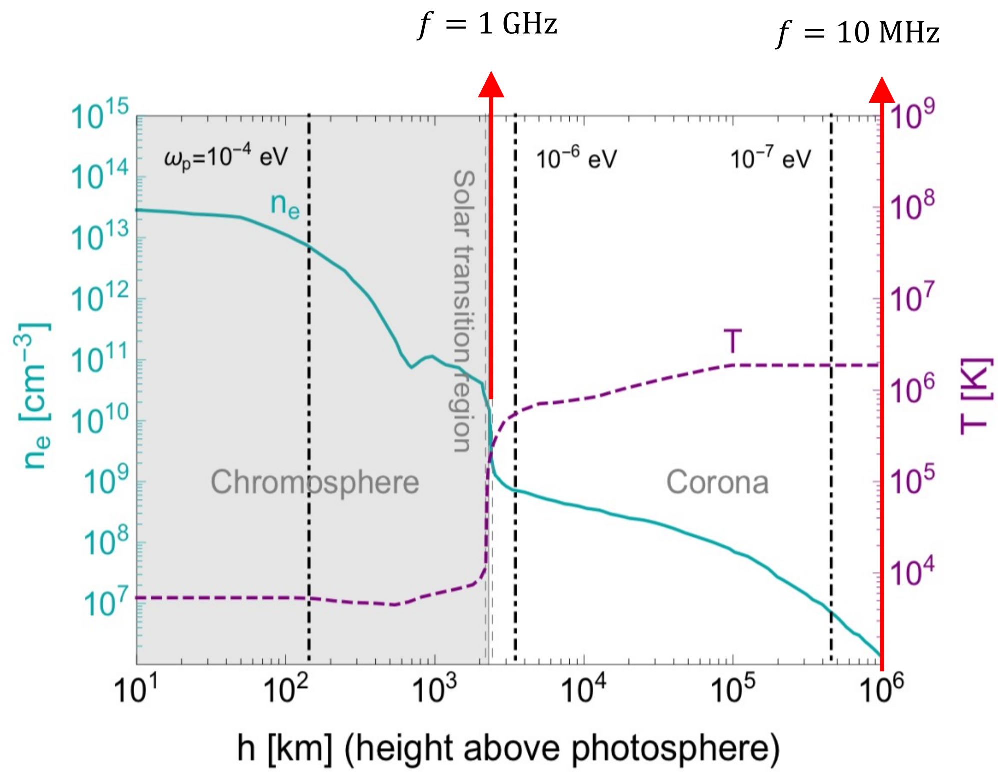
$$\omega^2 - k^2 = m^2$$

- For photon in plasma:

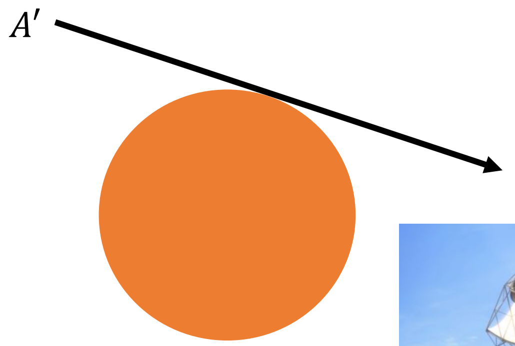
$$\omega^2 - k^2 = \omega_p^2$$

- We need plasma.





Dark photon dark matter converted at the Sun's atmosphere



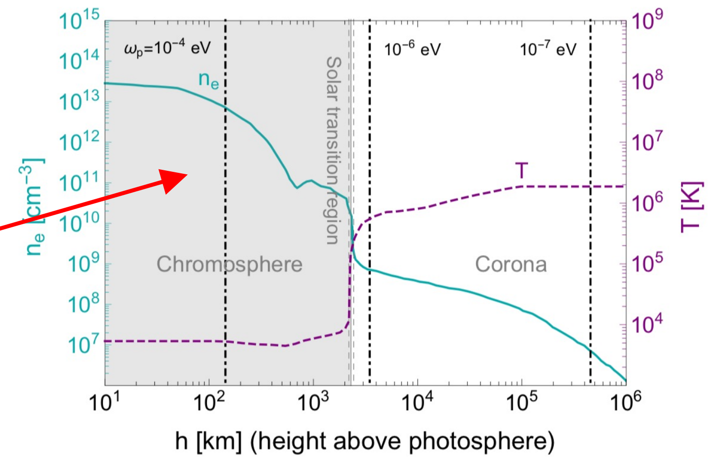
- Resonant conversion
  - $\omega_p = m_{A'}$
- Inside the dark matter halo
  - $v_{A'} \sim 10^{-3}$
- The frequency of the converted photon
  - $\omega \approx m_{A'}$ with the dispersion $\sim 10^{-6}$.
- The signal is a sharp peak in the solar spectrum

Absorption of the converted photon during propagation

- Inverse bremsstrahlung absorption

$$\Gamma_{\rm inv} \approx \frac{8\pi n_e n_N \alpha^3}{3\omega^3 m_e^2}\left(\frac{2\pi m_e}{T}\right)^{1/2}\log\left(\frac{2T^2}{\omega_p^2}\right)\left(1-e^{-\omega/T}\right)$$

Photon converted in chromosphere cannot fly out.



- Compton scattering
  - Compton scattering can shift the frequency of the converted photon.
- $\Gamma_{att} = \Gamma_{inv} + \Gamma_{com}$

Searching for the converted photon with radio telescopes

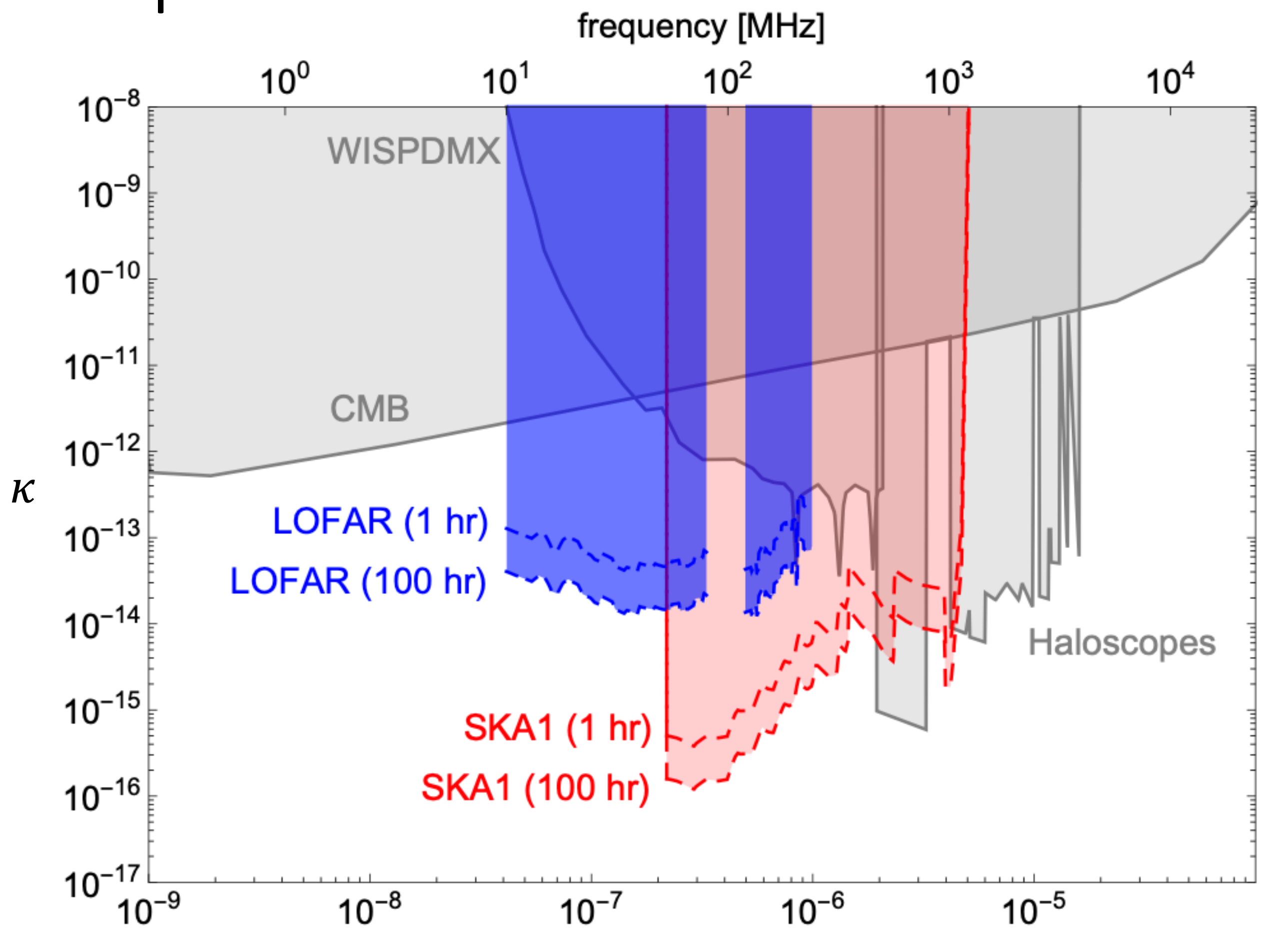
- The minimal detectable flux $$S_{\rm min} = \frac{\rm SEFD}{\eta_s \sqrt{n_{\rm pol}\, \mathcal{B}\ t_{\rm obs}}}$$ $$\mathrm{SEFD} = 2k_B \frac{T_{\rm sys} + T_{\odot}^{\rm nos}}{A_{\rm eff}}$$

Name	f [MHz]	$B_{\rm res}$ [kHz]	$\langle T_{\rm sys}\rangle$ [K]	$\langle A_{\rm eff}\rangle$ [m^2]
SKA1-Low	(50, 350)	1	680	2.2×10^5
SKA1-Mid B1	(350, 1050)	3.9	28	2.7×10^4
SKA1-Mid B2	(950, 1760)	3.9	20	3.5×10^4
LOFAR	(10, 80)	195	28,110	1,830
LOFAR	(120, 240)	195	1,770	1,530


West Australia


Netherland

Searching for ultralight DM with radio telescopes

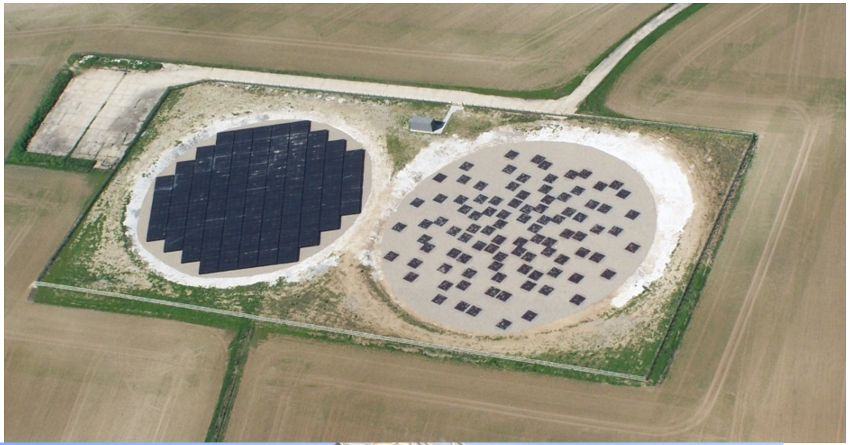


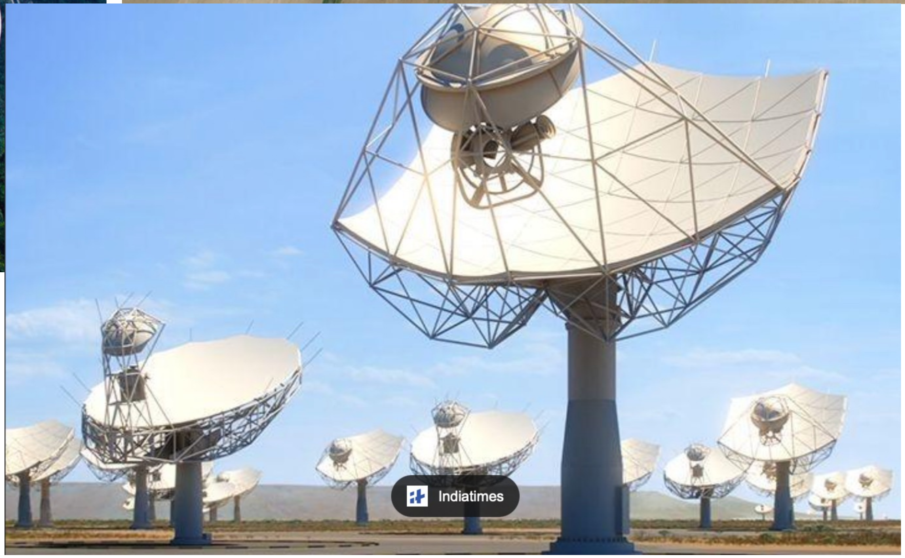
Dark photon dark matter m_V [eV]

HA, F.P. Huang, J.Liu, W.Xue, Phys.Rev.Lett. 126 (2021) 181102

Searching for dark photon dark matter directly with radio telescopes

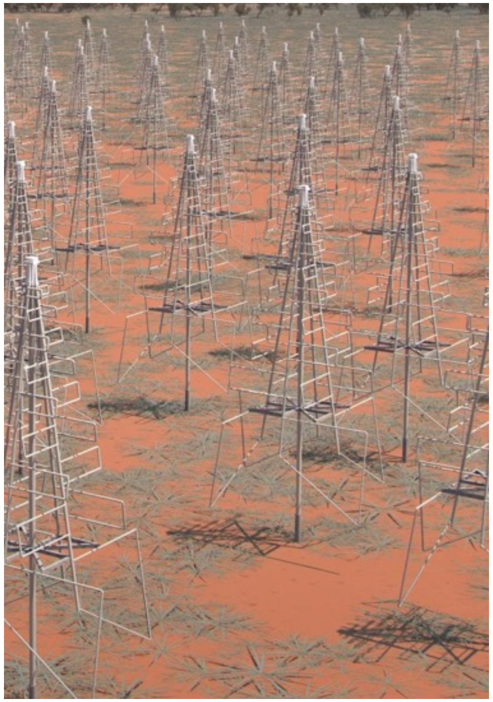
- Large scale radio telescopes

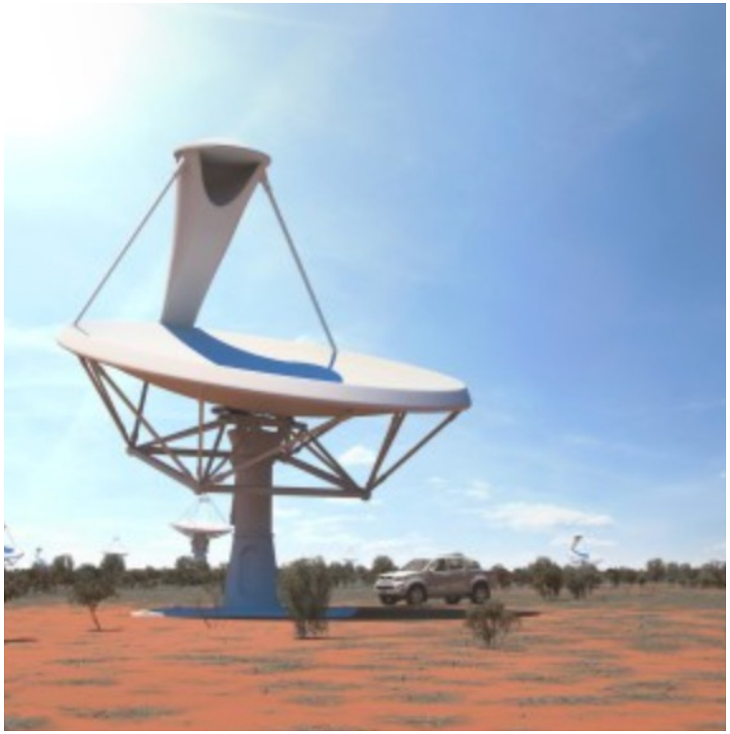




Searching for dark photon dark matter directly with radio telescopes

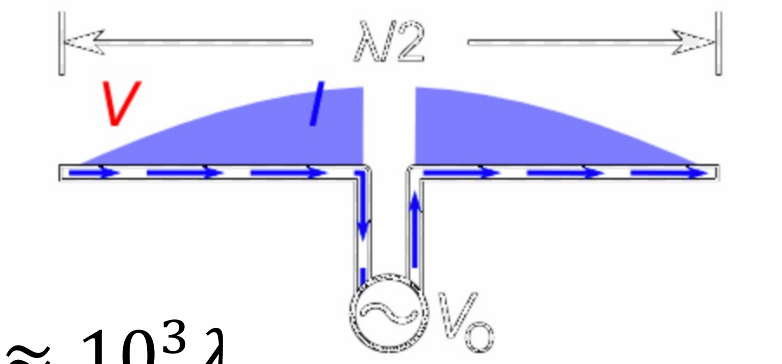
- The dark photon dark matter has an interaction with the electric current, $\kappa e V_\mu J^\mu$ (although suppressed)





Dipole antennas

- Usually $\ell \leq \frac{\lambda}{2}$
- For photon, $\lambda = \frac{1}{f}$
- For dark photon, $\lambda_D = \frac{1}{f \times v_D} \approx 10^3 \lambda$
- Equivalent electric signal:



$$E_{\rm EM}^{\rm eqv} = \kappa E_D^{(0)} \cos(2\pi f t)$$

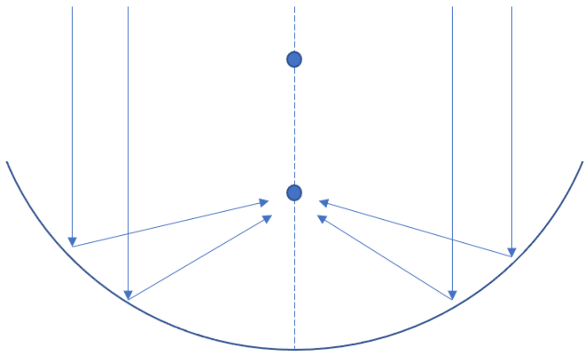
$$I_{\rm dipole}^{\rm eqv} = \mathcal{C} \kappa^2 \rho_{\rm DM}$$

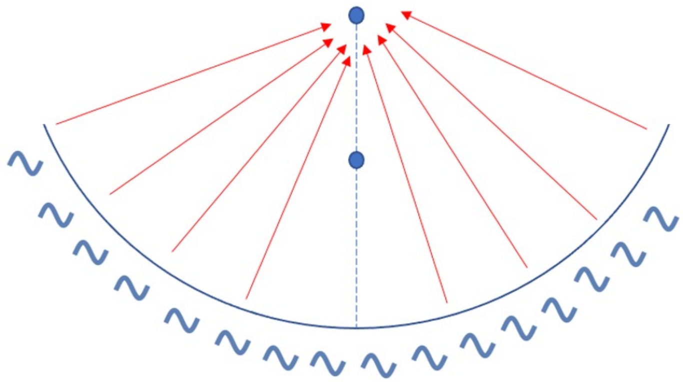
$\rho_{\rm DM}$ → 0.4 GeV/cm^3

$\mathcal{C}$ → Order one parameter, determined by the detailed shape of the antenna

Dish antennas

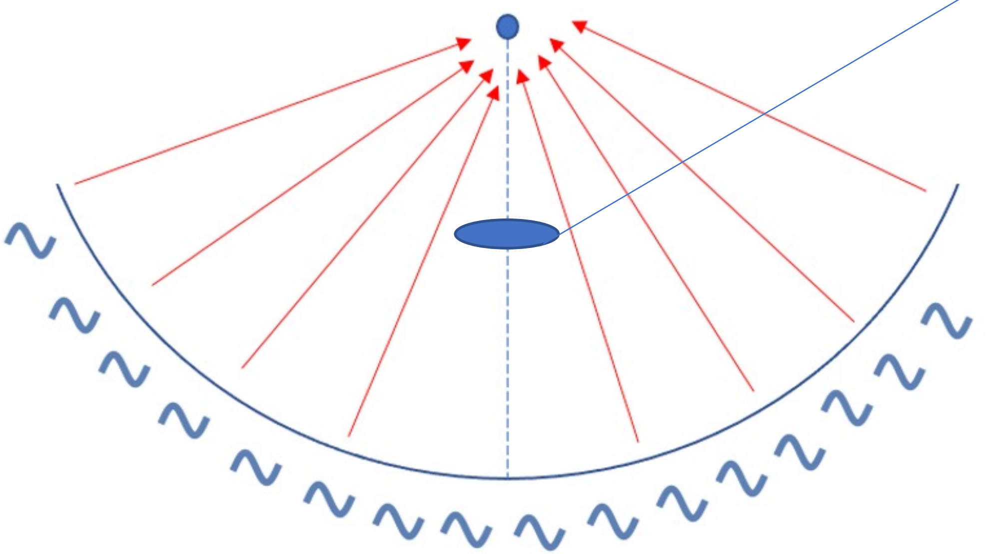
- For dish antennas, the oscillation of the dark photon field induces the oscillation of the electrons in the reflector plate, and produces EM waves, which can be detected by the feed.





Dish antennas

- The size of the feed $\sim \lambda$



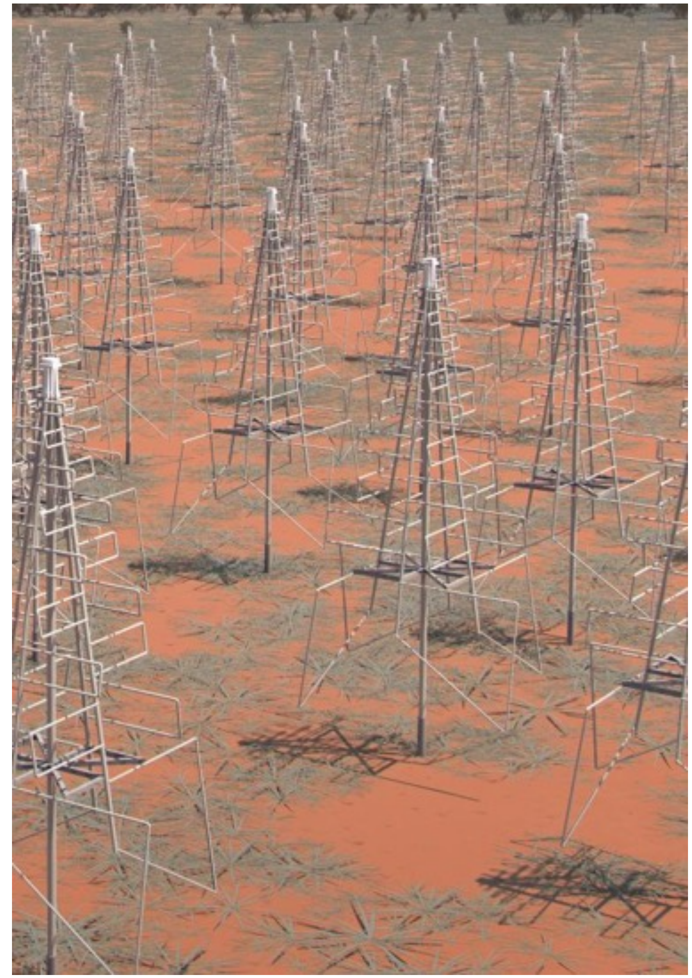
$$I_{\rm dish}^{\rm eqv} = \mathcal{C}\kappa^2\rho_{\rm DM} \times \frac{\lambda^2}{\mathcal{A}}$$

Area of the telescope

Antenna arrays

- $\lambda_D \sim 10^3\lambda$
  - $\lambda_D \approx 4$ km for $f = 70$ MHz
  - $\lambda_D \approx 150$m for $f = 2$ GHz
- Interferometry techniques can be used.
- Correlation suppressed when the distance of two antennas is larger than λ_D.

$$\mathcal{S}_{mn} = \exp(-m_{A'}^2 \sigma_v^2 d_{mn}^2/4)$$



Limits from antenna arrays

- The signal is a peak,

$$f_{\rm signal} = m_V/2\pi \qquad \Delta f_{\rm signal} \approx 10^{-6} f$$

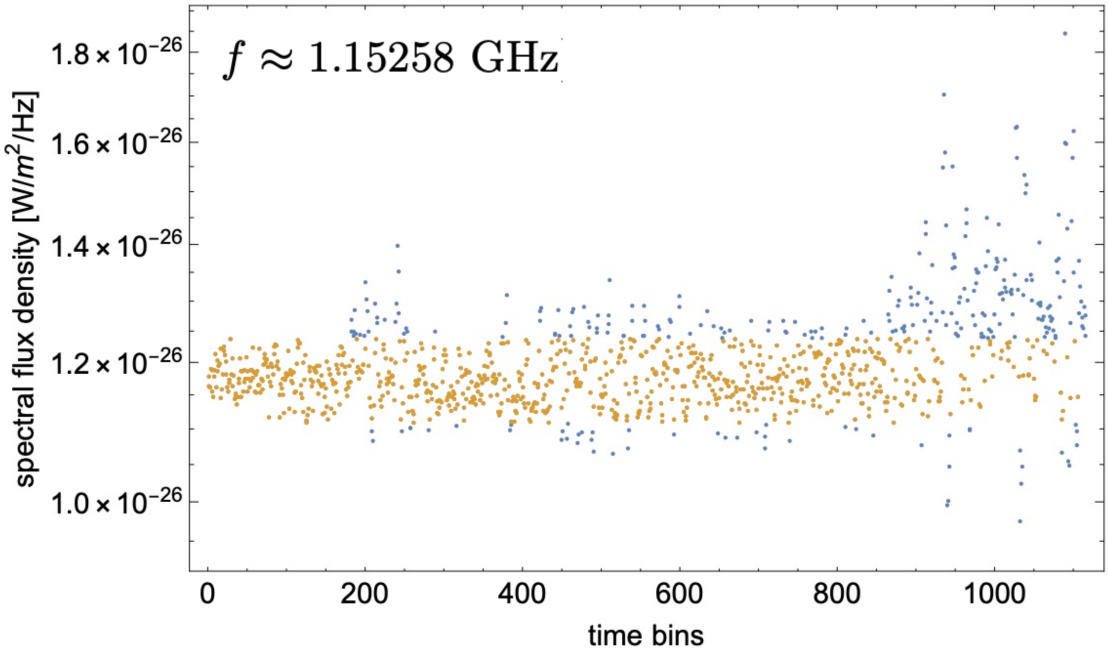
- Minimum detectable spectral flux

$$S_{\rm min} = \frac{\rm SEFD}{\eta_s \sqrt{n_{\rm pol} \mathcal{B} t_{\rm obs}}} \qquad {\rm SEFD} = \frac{2 k_B T_{\rm sys}}{A_{\rm eff}}$$

- We require $I^{\rm eqv}_{\rm array}/\mathcal{B} > S_{\rm min}$ to calculate the sensitivities of the antenna arrays.

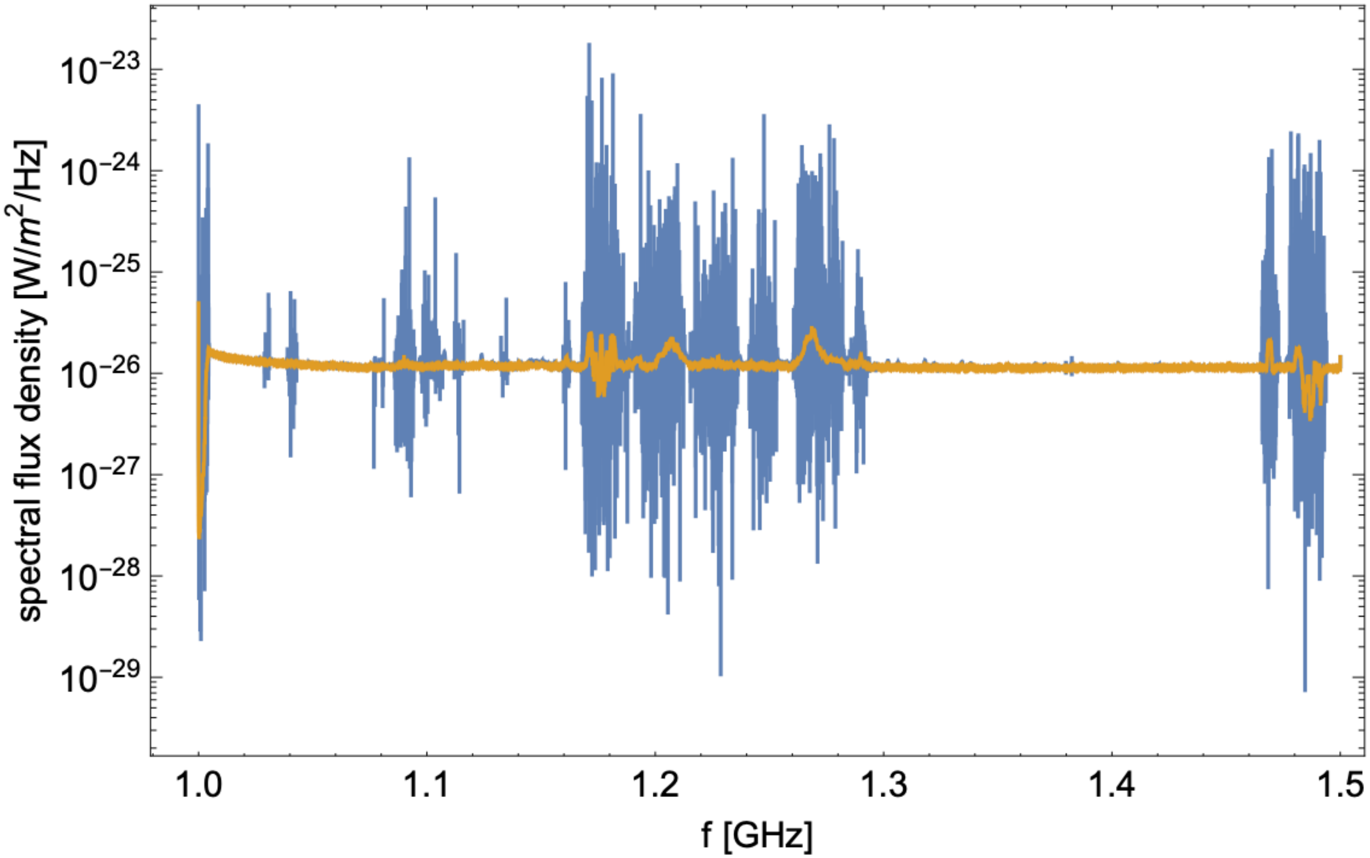
FAST data

- 1– 1.5 GHz, Band width = 7.63 kHz, data observed on Dec 14, 2020.
- The signal is constant, we remove data with large variation in time.



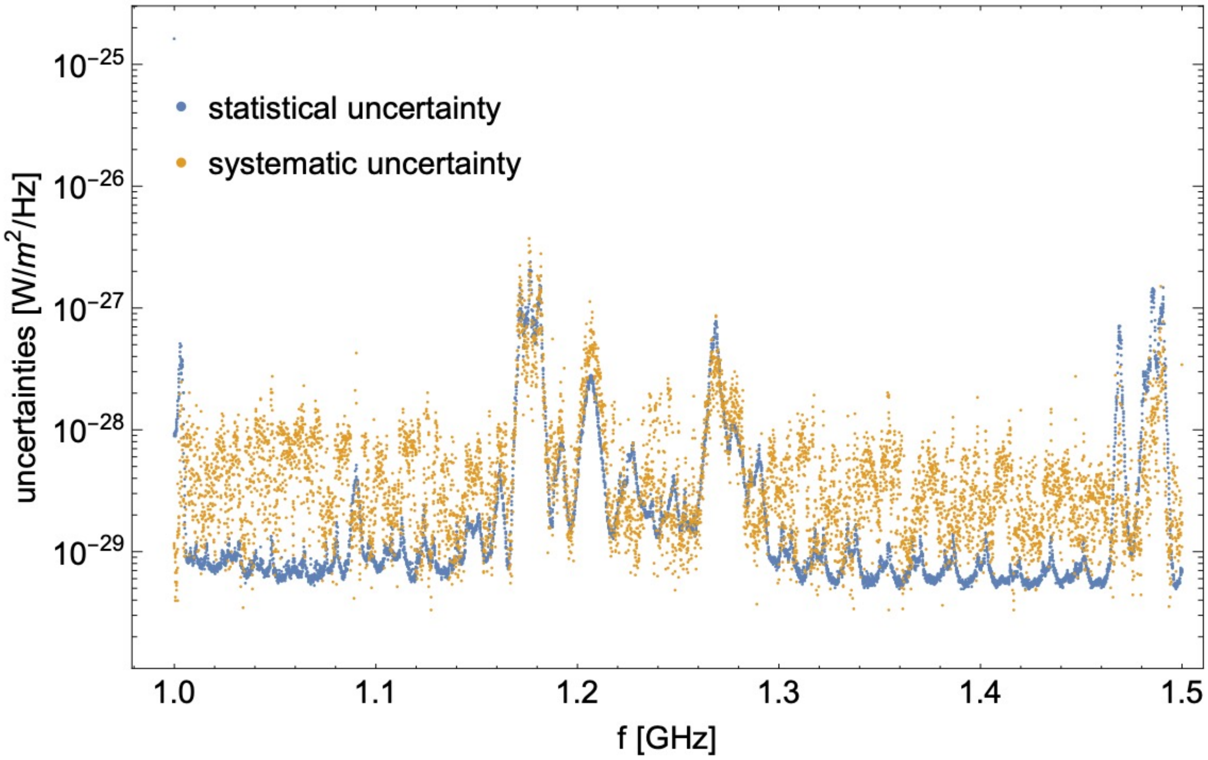
FAST data

- Spectrum after data cleasing

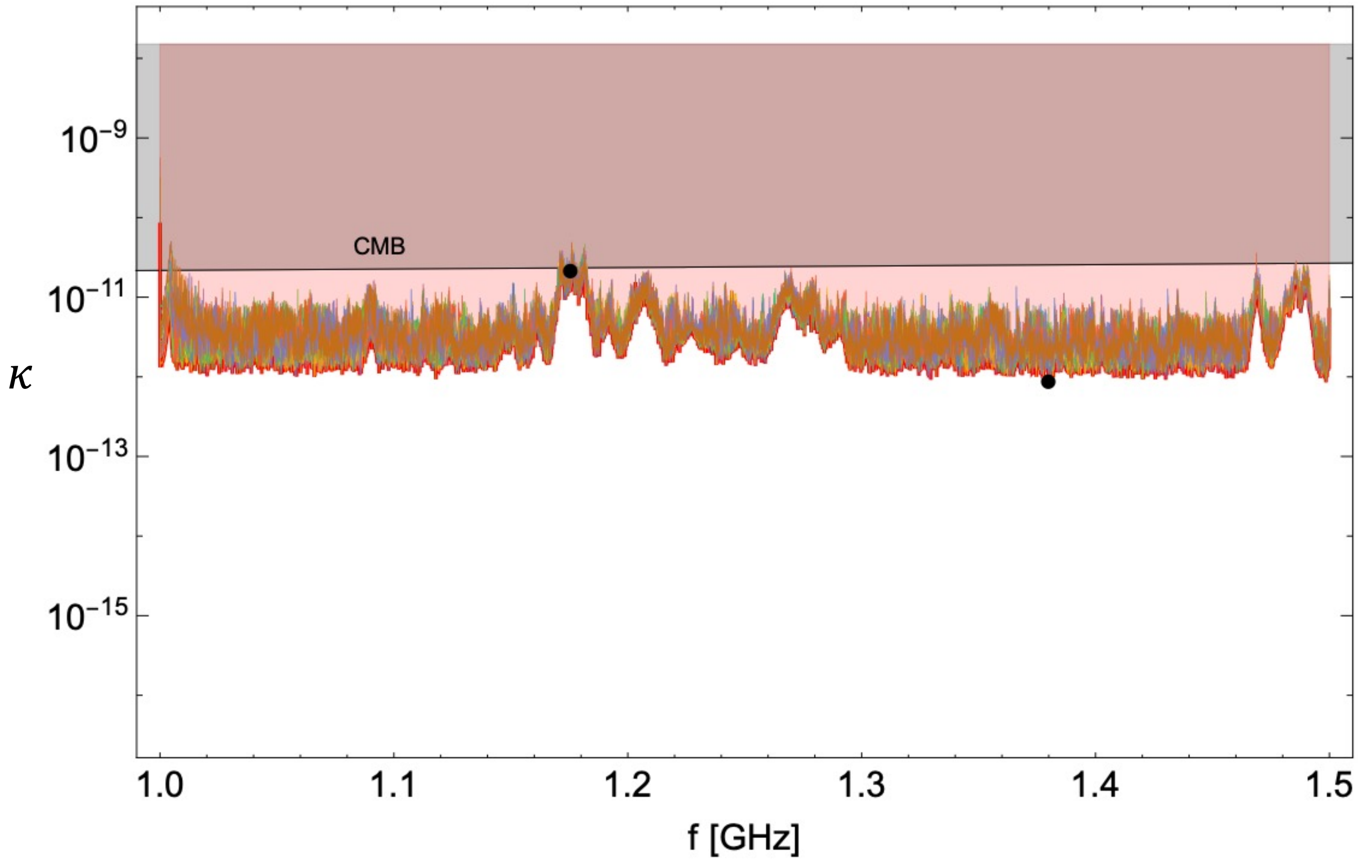


FAST data

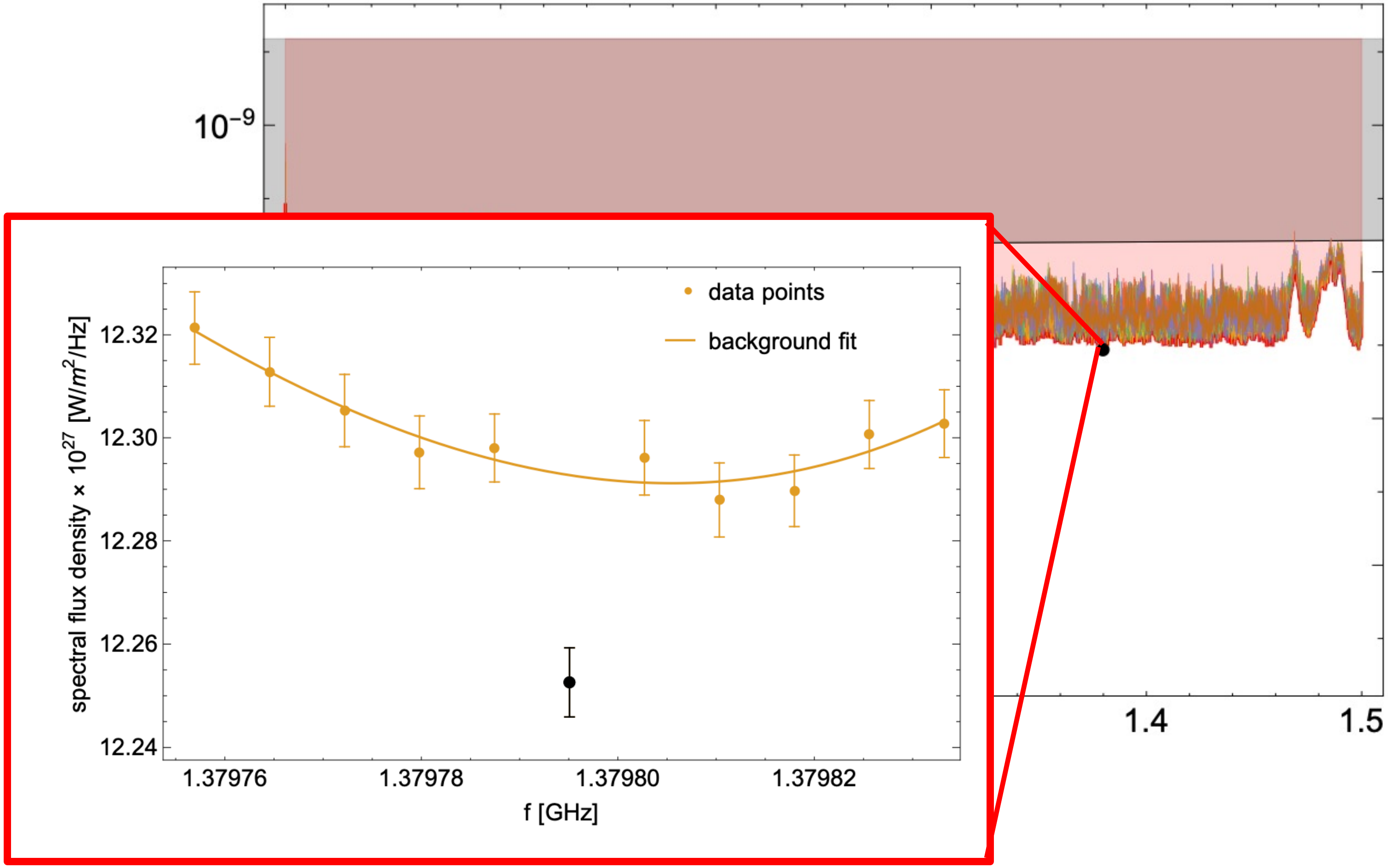
- We calculate the uncertainties from the fluctuations of the data.



Constraint FAST data



Constraint FAST data



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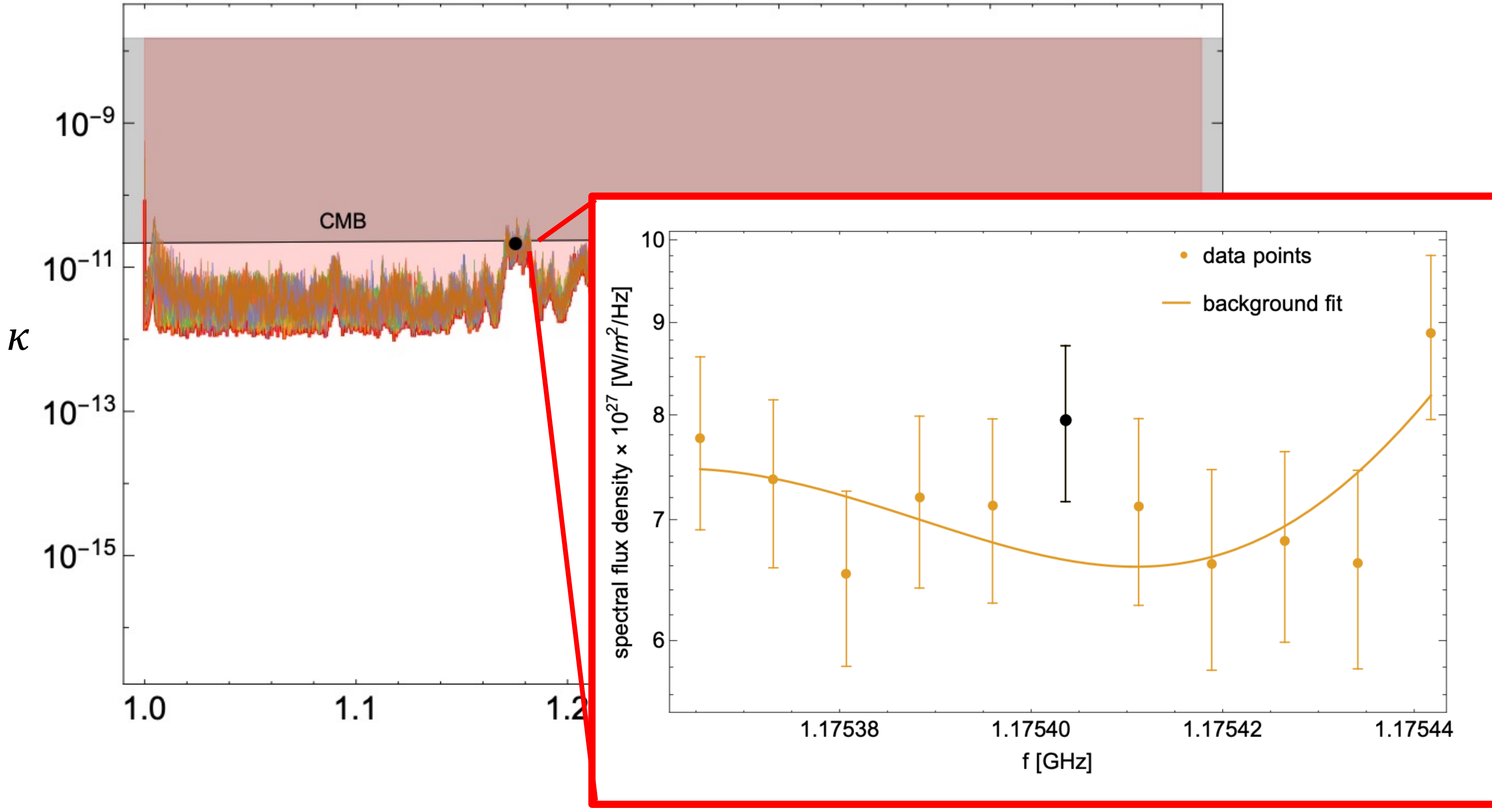
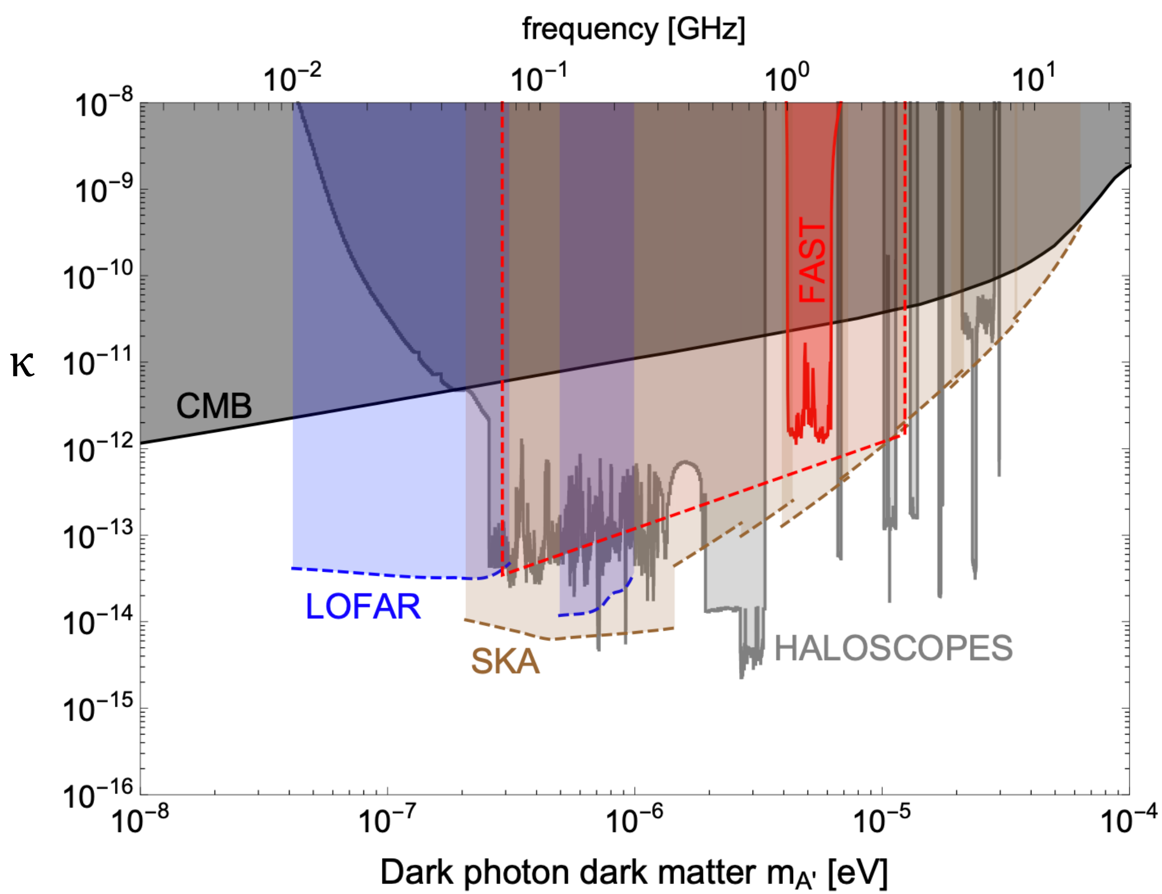


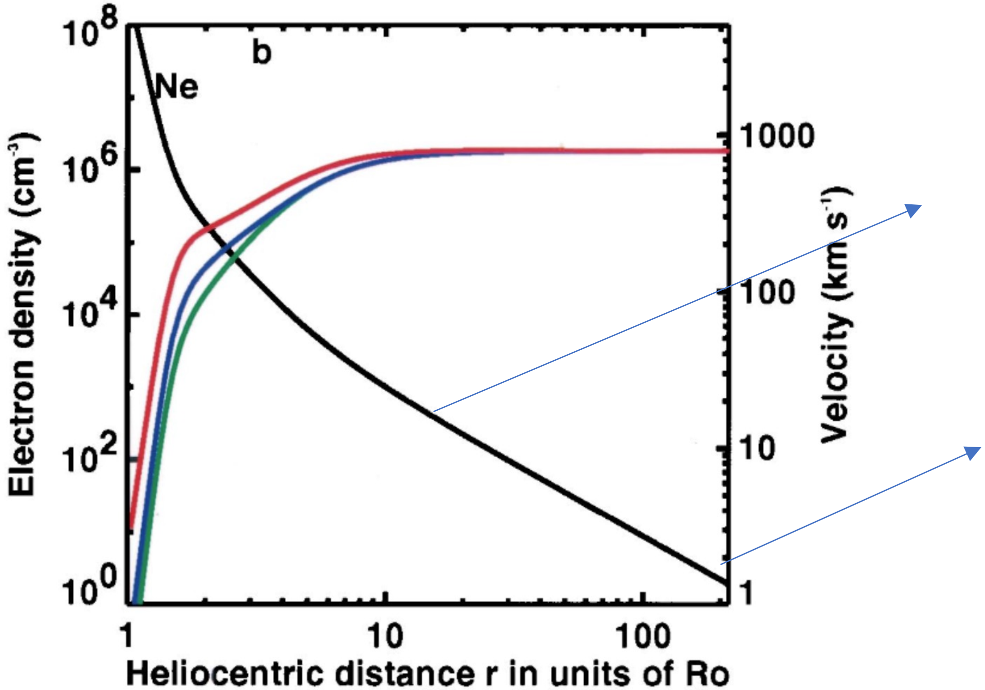
Direct detection of dark photon dark matter with radio telescopes



HA, S Ge, W-Q Guo, X Huang, J Liu, Z Lu, 2207.05767

For dark photon dark matter with even smaller mass

- No terrestrial telescopes can cover $f < 10$ MHz.
- Go to outer space.
- Free electrons between Earth and Sun



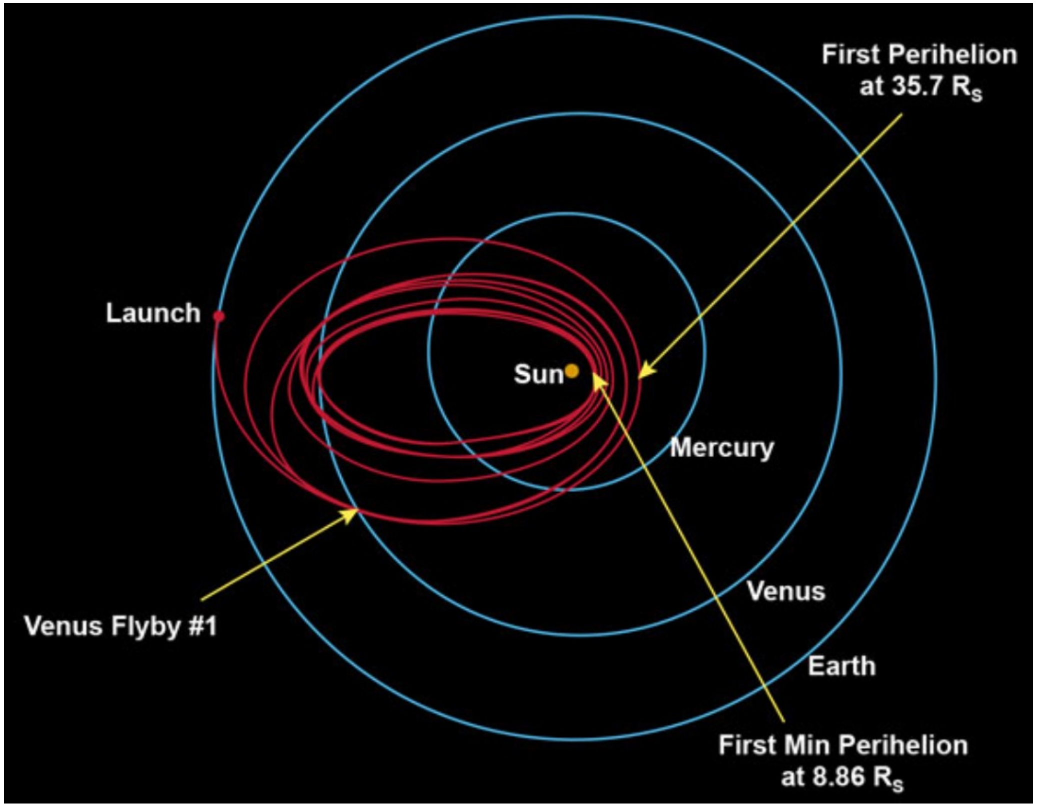
$$\frac{\omega_p}{2\pi} = 1\ \mathrm{MHz}$$

$$\frac{\omega_p}{2\pi} = 20\ \mathrm{kHz}$$

For dark photon dark matter with even smaller mass

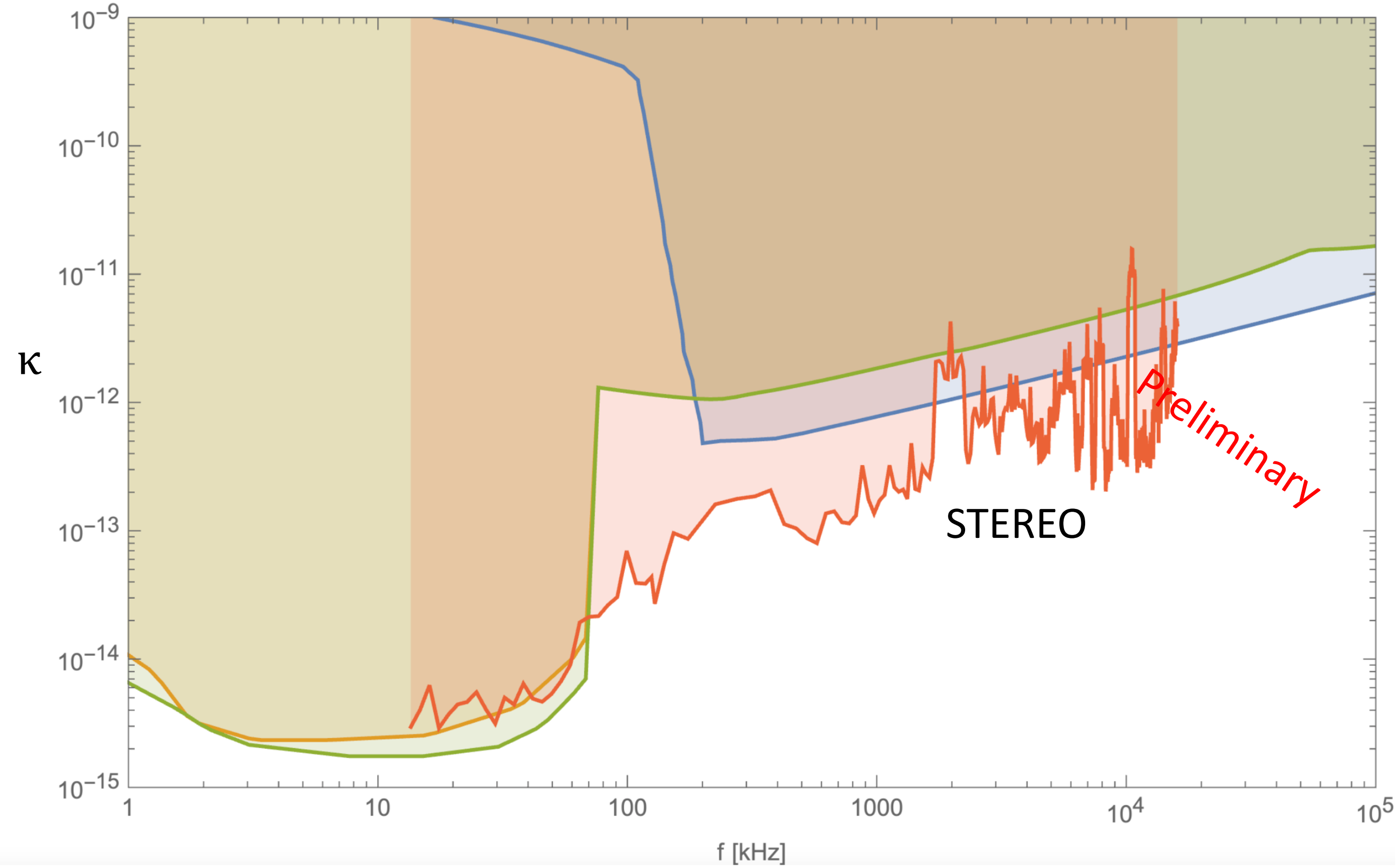
- STEREO A/B
- Parker Solar Probe
- 鸿蒙（lunar satellite chain）

Work in progress with Jia Liu and Shuailiang Ge

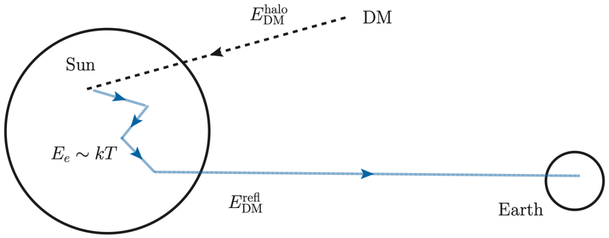


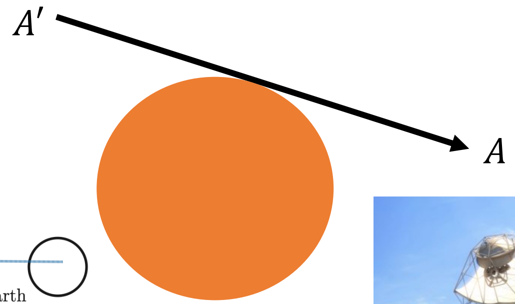
For dark photon dark matter with even smaller mass

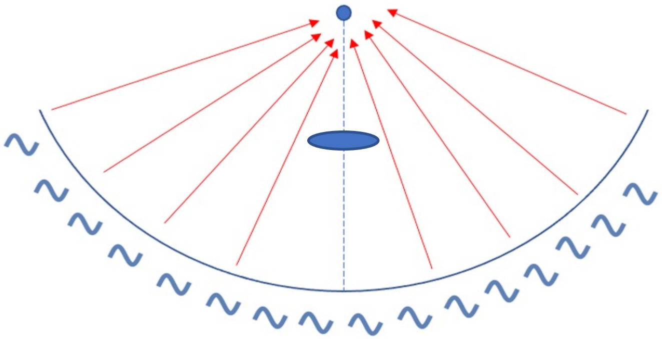
Work in progress with Jia Liu and Shuailiang Ge



Summary

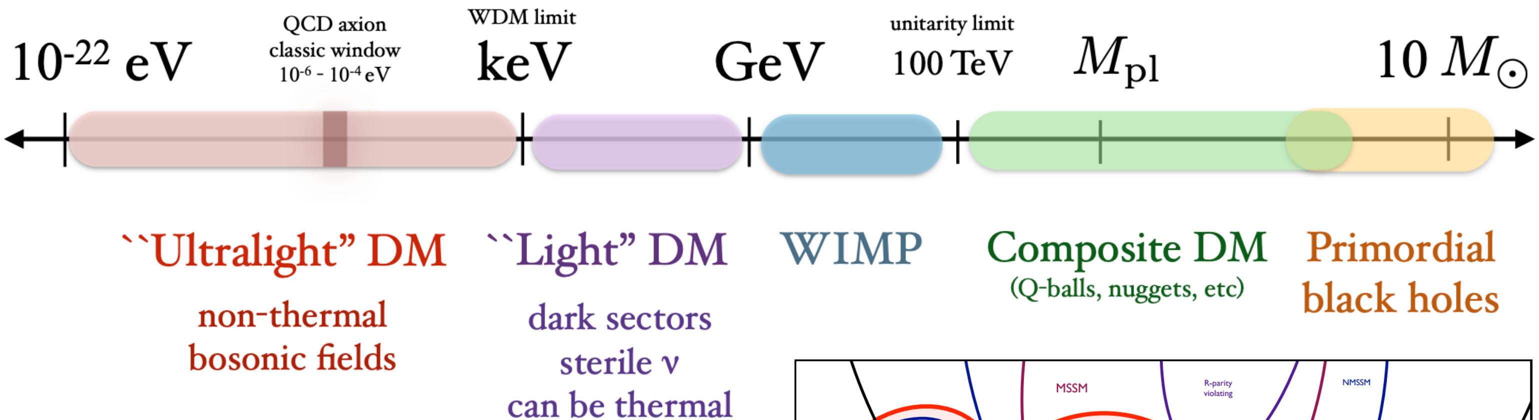


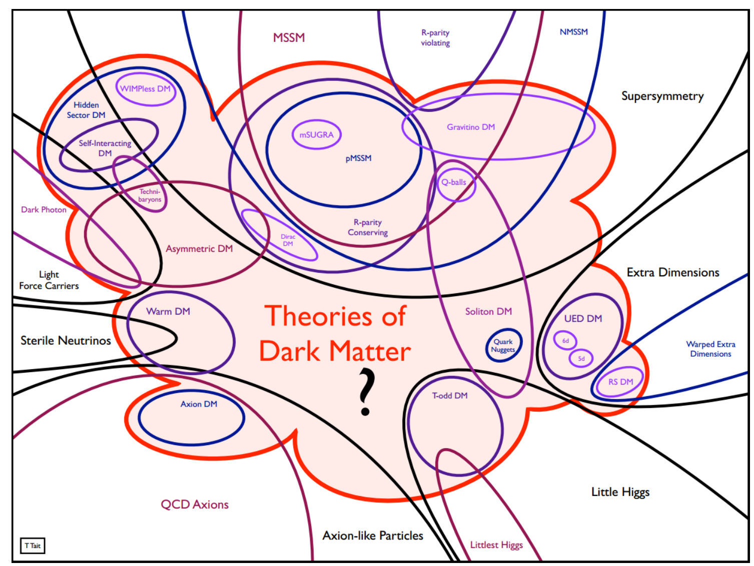




- The Sun as our own star, does not only give us light.
- It may also help us search for the dark side of the Universe.
- We convert all photon detectors to dark photon detectors.

Searching for Dark Matter





Where do we start from?

- We know almost nothing about dark matter except for:
  - Equation of state   Non-relativistic particles
  - Total energy density
    - 23% of the total energy density
    - About five times of the energy density of baryons
  - Its velocity around the earth
    - About 200 km/sec
  - Energy density around the earth
    - 0.4 GeV/cm^3 22.4 mol/L ~ 1Pa

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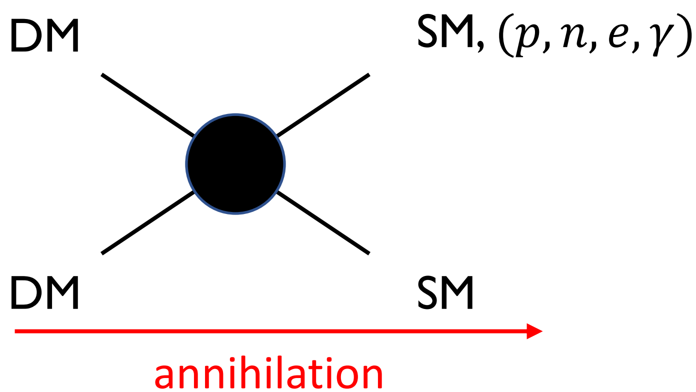
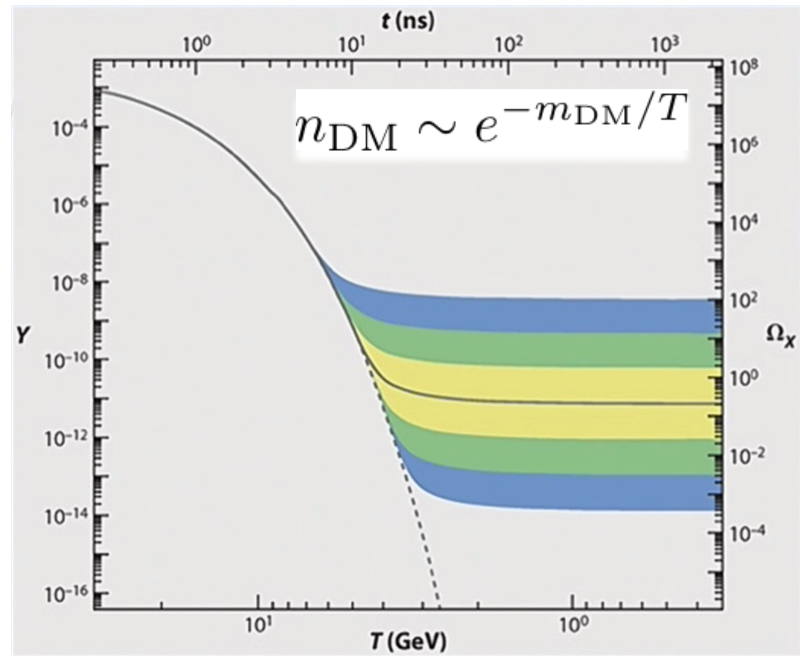
How to produce DM?

- Thermal freeze-out





Thermal freeze out

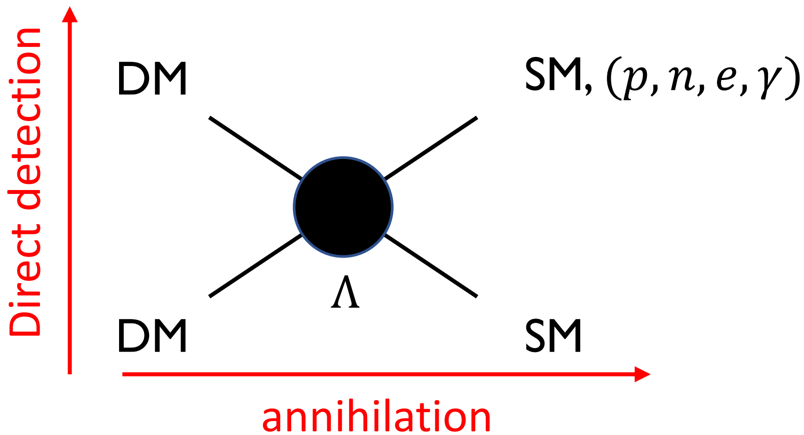
$\Gamma_A = n_{\mathrm{DM}} < \sigma v >$,

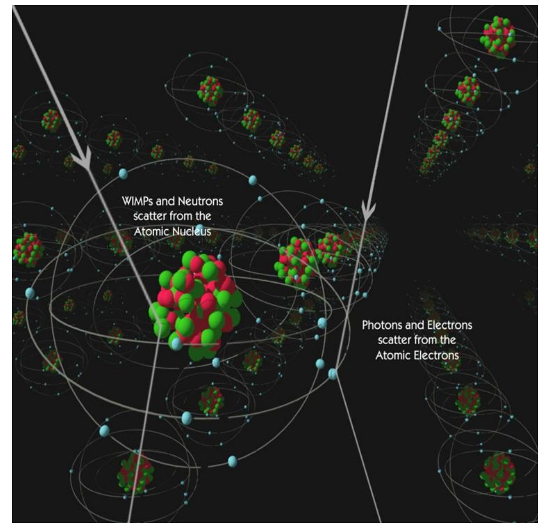
$\Gamma_A < \mathrm{H}$

$\langle \sigma v \rangle \approx 3 \times 10^{-26}\ \mathrm{cm}^3/\mathrm{sec}$

How to produce DM?

- Thermal freeze-out





Thermal freeze out

$$\Gamma_A = n_{\rm DM} < \sigma v > ,$$

$$\Gamma_A < \mathrm{H}$$

$$\langle \sigma v \rangle \approx 3 \times 10^{-26} \ \mathrm{cm}^3/\mathrm{sec}$$

$$\sigma_{\rm annihilation} \sim \frac{m_D^2}{\Lambda^4}$$

$$\sigma_{\rm scattering} \sim \frac{\mu^2}{\Lambda^4}$$

μ: reduced mass

The contact s-wave interaction

$$f_{e,\mathrm{ion}}(p,r) = n_{e,\mathrm{ion}}(r)\left(\frac{2\pi}{m_{e,\mathrm{ion}}T(r)}\right)^{3/2} e^{-p^2/2m_{e,\mathrm{ion}}T(r)}$$

$$\Gamma_\chi = \frac{1}{2k_1^0}\int \frac{d^3k_2}{2k_2^0(2\pi)^3}\int \frac{d^3p_1}{2p_1^0(2\pi)^3} f(p_1)\int \frac{d^3p_2}{2p_2^0(2\pi)^3}(2\pi)^4\delta^4(k_1+p_1-k_2-p_2)\sum_{\mathrm{spin}}|\mathcal{M}|^2$$

$$= \frac{(m_\chi+m_e)^2\pi\sigma_{\mathrm{tot}}}{m_e^2 m_\chi^2}\int \frac{d^3p_1}{(2\pi)^3} f_e(p_1)\frac{d^3q}{(2\pi)^3}\delta\left(q^2\left(\frac{1}{2m_\chi}+\frac{1}{2m_e}\right)+\frac{\mathbf{k}_1\cdot\mathbf{q}}{m_\chi}-\frac{\mathbf{p}_1\cdot\mathbf{q}}{m_e}\right)$$

$$\overline{|\mathcal{M}|^2} = 16\pi(m_\chi+m_e)^2\sigma_{\mathrm{tot}}$$

$$q = k_2 - k_1$$

The contact s-wave interaction

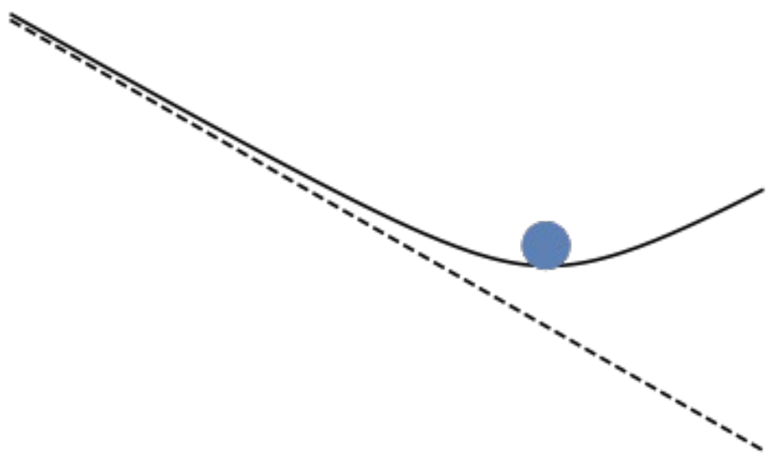
$$x_\chi = v_\chi (m_e/T)^{1/2}$$

$$\Gamma_\chi = \sigma_{\text{tot}} n_e \sqrt{\frac{T}{m_e}} \times \sqrt{\frac{1}{2\pi}} \left[2e^{-x_\chi^2/2} + (2\pi)^{1/2}(1 + x_\chi^2)\text{erf}\left(\frac{x_\chi}{\sqrt{2}}\right)\right]$$

$$\frac{d\Gamma_\chi}{dq\, d\cos\theta_q} \sim \mathcal{N} q \exp\left[-\frac{1}{2m_e T}\left(\frac{m_e}{2m_\chi}(2k_1\cos\theta_q + q) + \frac{q}{2}\right)^2\right]$$

The details of the simulation

- Outside the Sun



- We randomly shoot DM with impact from 0 to $4R_{sun}$ to include the focusing effect.

$$\frac{1}{2}v_{\rm DM}^2 = -\frac{G_N M_\odot}{R_\odot} + \frac{1}{2}v_{\rm DM}'^2$$

$$v_{\rm DM} R_0 = v'_{\rm DM} R_\odot$$

$$\Longrightarrow \frac{R_0^2}{R_\odot^2} = 1 + \frac{2G_N M_\odot}{R_\odot v_{\rm DM}^2}$$

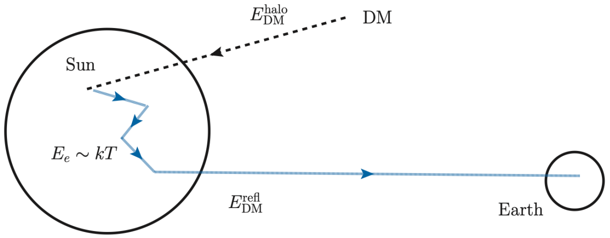
$$\frac{2G_N M_\odot}{R_\odot} = v_{\rm esc}^2 \approx (620~{\rm km/sec})^2$$

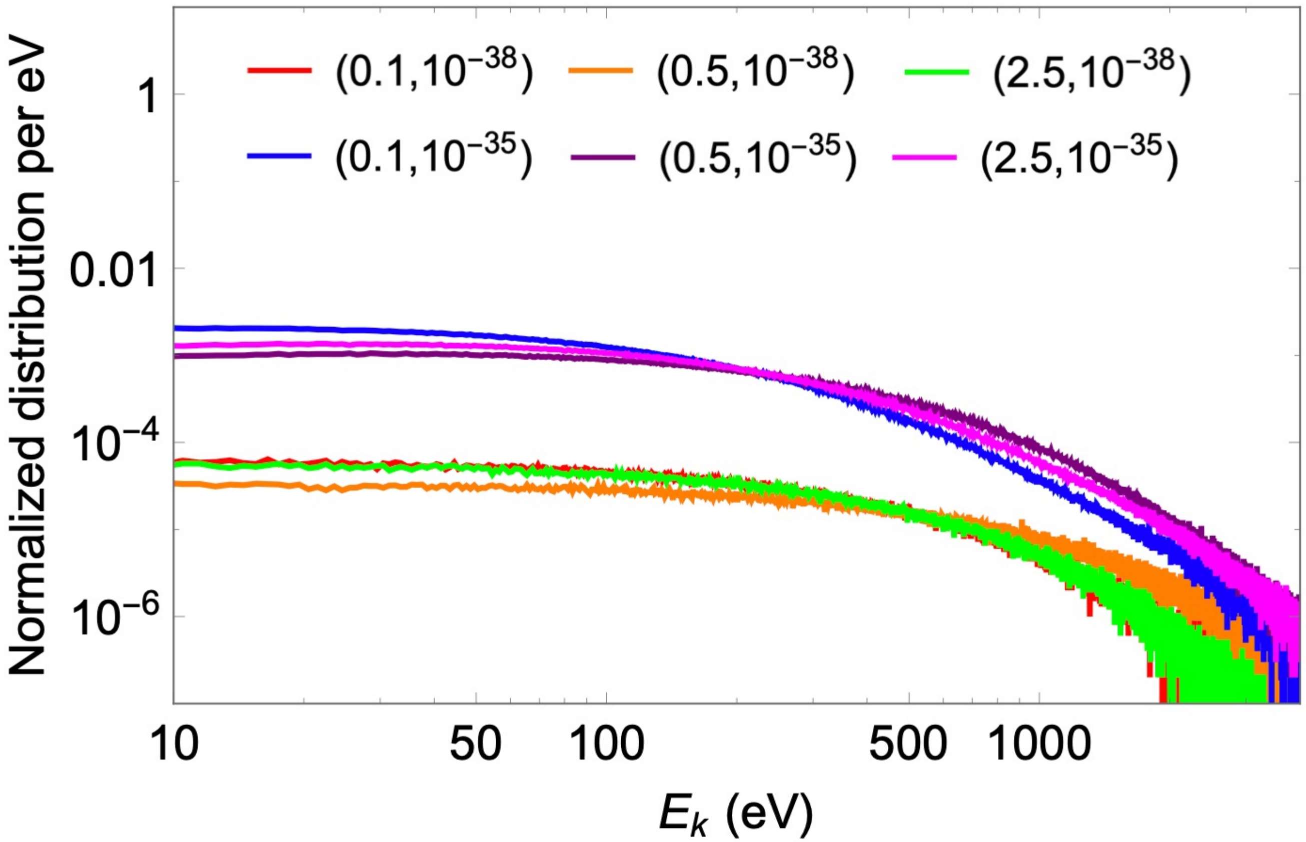
$$v_{\rm DM} \approx 220~{\rm km/sec}$$

$$\Longrightarrow \frac{R_0^2}{R_\odot^2} \approx 10$$

Solar focusing effect.

The reflected flux



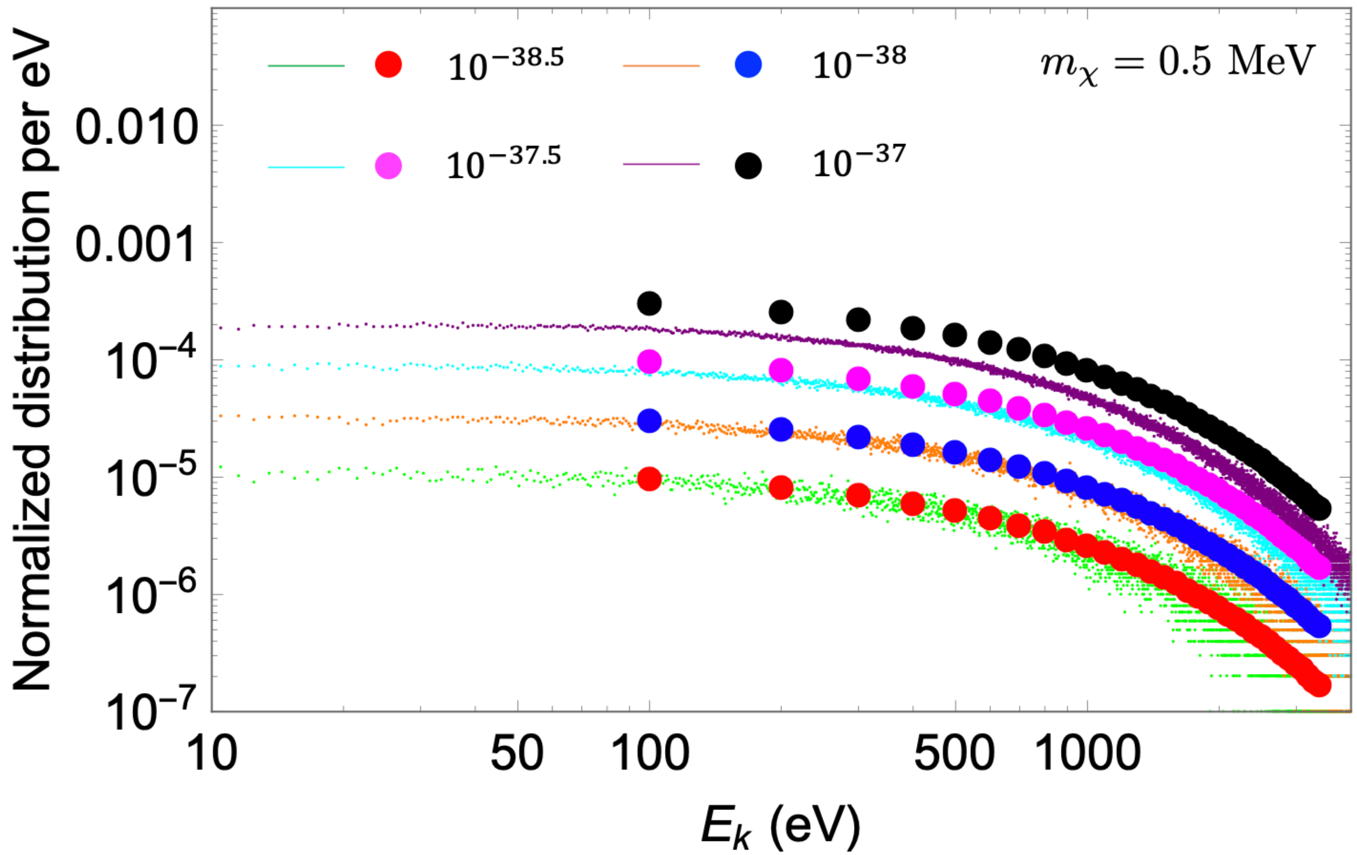


The one scattering limit

- When $\sigma_{\text{tot}} \ll 10^{-38}\text{cm}^2$, $P_{\text{tot}} \ll 1$.
  - We simulate the trajectory of DM considering only the gravity.
  - $$\left.\frac{dP}{dE_2}\right|_{\text{trajectory}} = \int_{\text{trajectory}} \frac{dl}{v_\chi} \frac{d\Gamma_\chi}{dE_2}(k_1(r(l)), T(r(l)), n_e(r(l)))$$
  - For the high energy tail, $k_2 \gg k_1$, $k_2 \approx q$

$$\frac{d\Gamma}{dE_2} = \frac{\sigma_{\text{tot}} n_e}{(2\pi m_e T)^{1/2}} \frac{(m_e + m_\chi)^2}{m_e m_\chi} \exp\left[-\frac{E_2}{T} \frac{(m_e + m_\chi)^2}{4 m_e m_\chi}\right]$$

The one scattering limit



Effect of ions

- Energy transfer $$E_{\rm recoil} \sim \frac{m_{\rm DM} m_T}{(m_{\rm DM} + m_T)^2} E_{\rm DM}$$

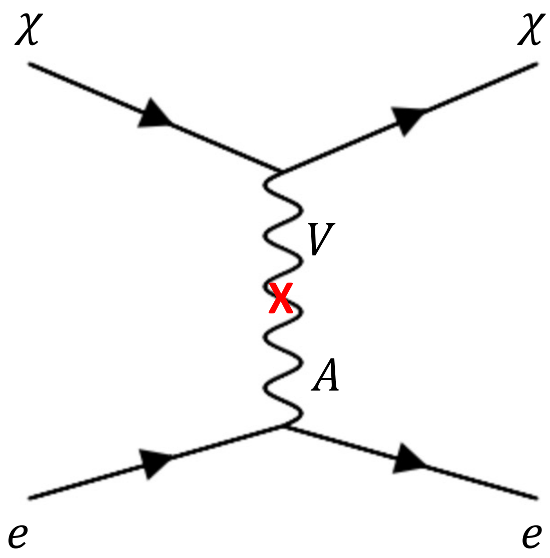
→
- The energy transifer from ions can be neglected.
- The ions can only change the direction.

- $\Gamma_\chi \sim n\langle \sigma v \rangle \to n\sigma \langle v \rangle$
  - For χ-e scattering, $v \sim v_e \sim 0.03 \gg v_\chi$
  - For χ-ion scattering, $v \sim v_\chi \approx 10^{-3}$.

$$\frac{\Gamma_{\chi-e}}{\sigma_{\chi-e}} \gg \frac{\Gamma_{\chi-\rm ion}}{\sigma_{\chi-\rm ion}}$$

Detailed calculation

$$\Gamma_\chi = \frac{1}{2k_1^0} \int \frac{d^3k_2}{2k_2^0(2\pi)^3} \int \frac{d^3p_1}{2p_1^0(2\pi)^3} f(p_1) \int \frac{d^3p_2}{2p_2^0(2\pi)^3} (2\pi)^4 \delta^4(k_1 + p_1 - k_2 - p_2) \sum_{\text{spin}} |\mathcal{M}|^2$$



Momentum dependent Debye screening

- The general form of the photon polarization tensor in plasma

$$\Pi^{\mu\nu} = \frac{1}{\varepsilon_L(q^{0^2} - q^2)}\epsilon_L^\mu \epsilon_L^\nu + \frac{1}{\varepsilon_T q^{0^2} - q^2}\sum_r \epsilon_{Tr}^\mu \epsilon_{Tr}^\nu + \frac{q^\mu q^\nu}{\xi(q^{0^2} - q^2)^2}$$

- The general form of the matrix element

$$\begin{aligned}\mathcal{M} &= \langle J_\chi^0\rangle \frac{1}{q^2 - m_V^2}\kappa q^2 \frac{\epsilon_L^{0\,2}}{\varepsilon_L(q^{0^2} - q^2)}\langle J^0\rangle \\ &= \frac{\kappa q^2}{\varepsilon_L q^2(q^2 - m_V^2)}\langle J_\chi^0\rangle\langle J^0\rangle\ . \\ &\approx \frac{\kappa\langle J_\chi^0\rangle\langle J^0\rangle}{\varepsilon_L(q^2 + m_V^2)}\end{aligned}$$

Momentum dependent Debye screening

- The general form of the matrix element

$$\mathcal{M} \approx \frac{\kappa \langle J^0_\chi \rangle \langle J^0 \rangle}{\varepsilon_L q^2}$$

- We use linear response method to calculate ε_L in NR plasma.

$$\varepsilon_L = 1 + \frac{e^2 n_e}{T q^2} F_1(A)$$

$$F_1(A) = 1 - \left(\frac{\pi}{2}\right)^{1/2} A e^{-A^2/2} \left[\mathrm{erfi}(A/\sqrt{2}) - i\right]$$

$$A = \frac{q^0}{q}\left(\frac{m_e}{T}\right)^{1/2}$$

- A is $O(1)$ in our case.

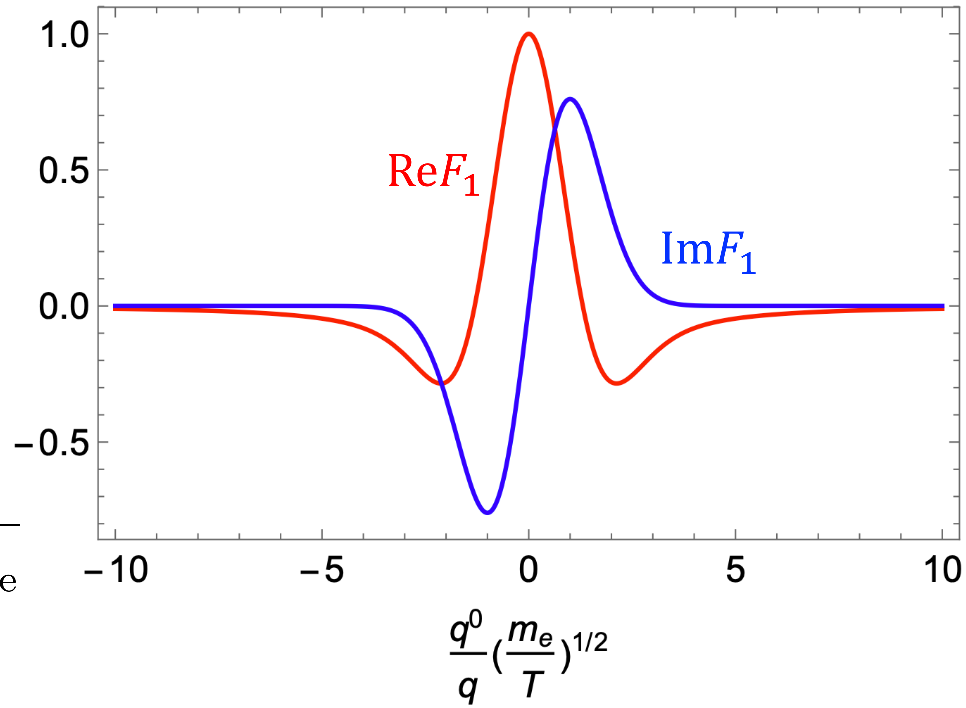
HA, Haoming Nie, Maxim Pospelov, Josef Pradler, Adam Ritz, 2108.10332

Momentum dependent Debye screening

$$\mathcal{M} \approx \frac{\kappa \langle J^0_\chi \rangle \langle J^0 \rangle}{\varepsilon_L q^2}$$

$$\varepsilon_L = 1 + \frac{e^2 n_e}{T q^2} F_1(A)$$

$$\frac{1}{\varepsilon_L q^2} \xrightarrow{q^0 \to 0} \frac{1}{q^2 + \mu^2_{\rm Debye}}$$



Numerical Simulation

- The scattering rate blows up even with the screening. We need to put an IR cut-off for the momentum transfer.

$$\Gamma_{\chi-e} = \frac{n_e \kappa^2 e^2 e_D^2}{(2\pi T)^{3/2} m_e^{1/2}} \int dx_q dc_q \exp\left[-\frac{1}{2}\left(\frac{1}{2}x_q + A_e\right)^2\right] \frac{1}{|x_q^2 + \frac{\mu_{\text{Debye}}^2}{m_e T} F_1(A_e)|^2}$$

c_q → $\cos\theta_q$ angle between $\mathbf{q}$ and $\mathbf{k_1}$

$$x_q = \frac{q}{\sqrt{m_e T}} = \frac{\Delta v_1 m_\chi}{\sqrt{m_e T}} > \frac{\zeta v_1 m_\chi}{\sqrt{m_e T}} = \zeta \frac{m_\chi}{m_e} v_1 \sqrt{\frac{m_e}{T}}$$

v_1: velocity of DM in each layer before scattering