A lattice study of the $K^*(892)$ resonance in P-wave pion-kaon scattering

Qu-Zhi Li

Email: liquzhi@scu.edu.cn

College of Physics
Sichuan University, Chengdu
Seminar @ HuiZhou

Oct. 9, 2025

四川大学
SiChuan University

Contents

1. Introduction
2. Method
3. Lüscher analysis
4. Conclusions

1 Introduction

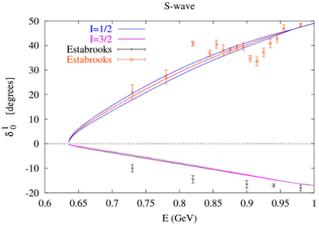
2 Method

3 Lüscher analysis

4 Conclusions

πK scattering

- The simplest scattering process involving unequal mass particles carrying strangeness.



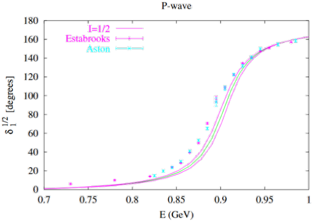


Figure 1: The figure is taken from [P. Büttiker, S. Descotes-Genon, B. Moussallam, EPJC, (2004)]

- $\kappa/\mathrm{K}_0^*(700)$ ($\mathrm{I}=1/2$ S-wave):

  PKU: $(694 \pm 53) - i(303 \pm 45)\mathrm{MeV}$

  [Z.Y. Zhou and H.Q. Zheng, Nucl.Phys.A (2006)]

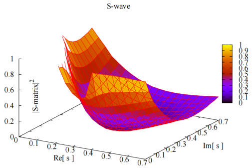
  RS Eq. : $(658 \pm 13) - i(279 \pm 12)\mathrm{MeV}$

  [S. Descotes-Genona and B. Moussallam, EPJC (2006)]

  UFD : $(651 \pm 14) - i(336 \pm 5)\mathrm{MeV}$

  CFD : $(673 \pm 13) - i(331 \pm 5)\mathrm{MeV}$

  [José R. Peláez and Arkaitz Rodas, Phys. Rept. (2022)]



- $\kappa/K_0^*(700)$ (I = 1/2 S-wave):

  PKU: $(694 \pm 53) - i(303 \pm 45)$MeV

  [Z.Y. Zhou and H.Q. Zheng, Nucl.Phys.A (2006)]

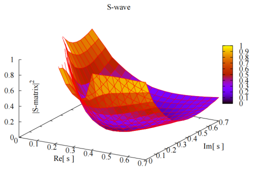
  RS Eq. : $(658 \pm 13) - i(279 \pm 12)$MeV

  [S. Descotes-Genona and B. Moussallam, EPJC (2006)]

  UFD : $(651 \pm 14) - i(336 \pm 5)$MeV

  CFD : $(673 \pm 13) - i(331 \pm 5)$MeV

  [José R. Peláez and Arkaitz Rodas, Phys. Rept. (2022)]

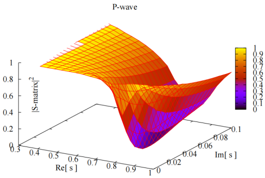


- $K^*(892)$ (I = 1/2 P-wave):

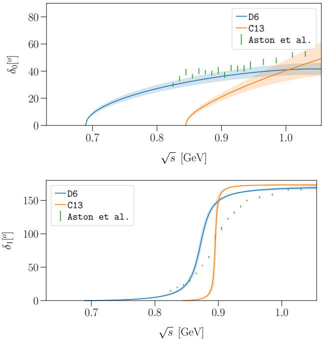
  UFD : $(892 \pm 2) - i(28.6 \pm 1.1)$MeV

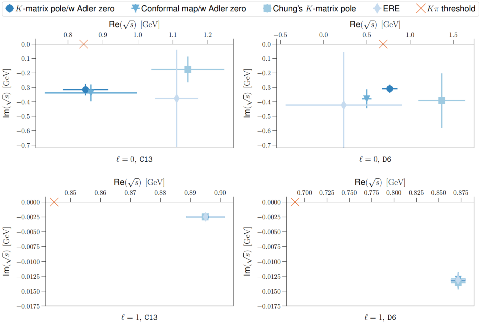
  CFD : $(891 \pm 2) - i(27 \pm 1)$MeV

  [José R. Peláez and Arkaitz Rodas, Phys. Rept. (2022)]



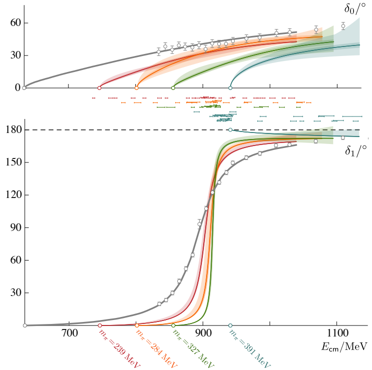
Lattice researches on πK scattering

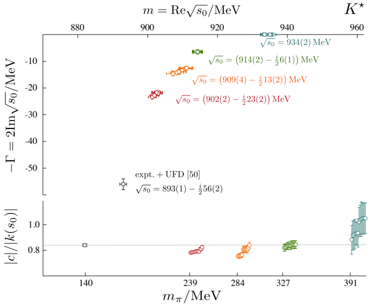




Phase shifts (left) and κ as well as $K^*(892)$ pole positions (right) obtained from lattice QCD calculation at two different quark masses [G. Rendon, et al., Phys. Rev. D (2020)].

Lattice researches on πK scattering





Phase shifts (left) and vector K*(892) pole positions (right) obtained from lattice QCD calculation at four different quark masses [D. J. Wilson, et al., Phys. Rev. Lett. (2019)].

- It is a challenge to determine the positions of broad resonances (far from the real axis), such as the σ ($\pi\pi$) , κ (πK) and N*(920) (πN) resonances.

- It is a challenge to determine the positions of broad resonances (far from the real axis), such as the σ ($\pi\pi$) , κ (πK) and N*(920) (πN) resonances.

The definition of resonances

Pole singularities in the analytic continuation of a scattering amplitude to the second Riemann sheet. The scattering amplitude should satisfy unitarity, analyticity and crossing symmetry.

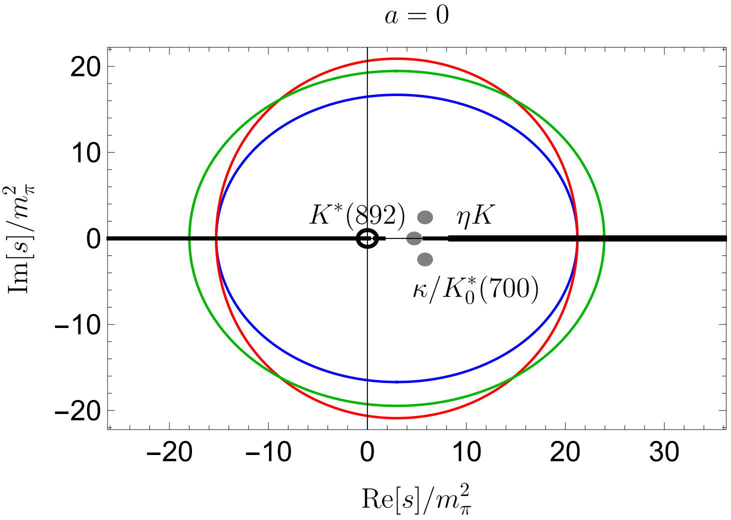
- It is a challenge to determine the positions of broad resonances (far from the real axis), such as the σ $(\pi\pi)$, κ $(\pi \mathrm{K})$ and $\mathrm{N}^*(920)$ $(\pi \mathrm{N})$ resonances.

The definition of resonances

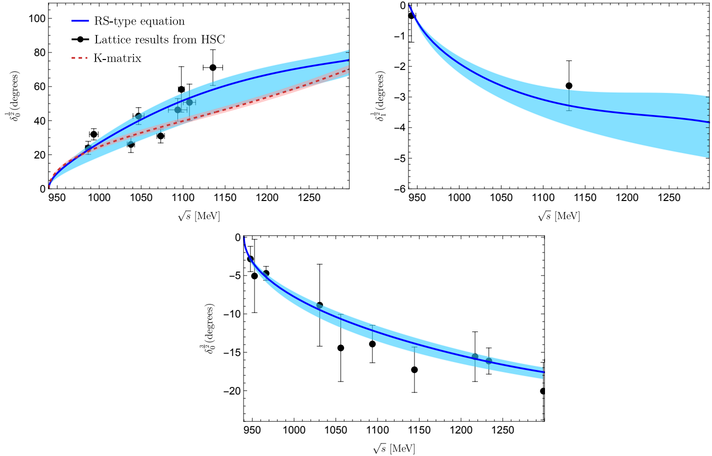
Pole singularities in the analytic continuation of a scattering amplitude to the second Riemann sheet. The scattering amplitude should satisfy unitarity, analyticity and crossing symmetry.

- Roy-steiner equations

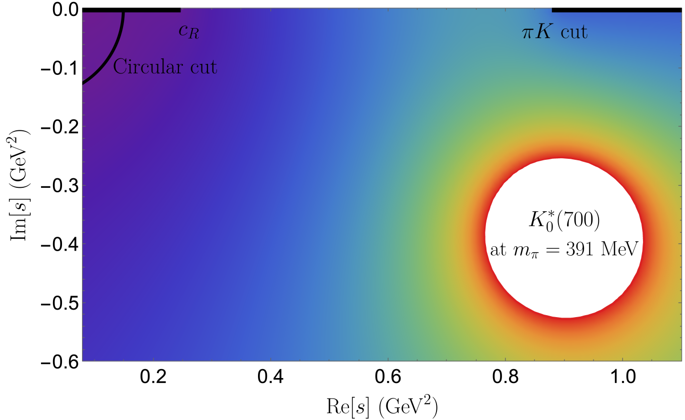
$$
\begin{aligned}
\mathrm{f}_{\mathrm{J}}^{\mathrm{I}}(\mathrm{s}) = \frac{1}{\pi}\int_{\mathrm{m}_{+}^{2}}^{\mathrm{s_m}} \mathrm{d}\mathrm{s}' \Big\{ & \mathrm{K}_{\mathrm{J},0}^{\mathrm{I},\frac{1}{2}}\left(\mathrm{s},\mathrm{s}'\right) \mathrm{Im}\, \mathrm{f}_{0}^{\frac{1}{2}}\left(\mathrm{s}'\right) \\
& + \mathrm{K}_{\mathrm{J},1}^{\mathrm{I},\frac{1}{2}}\left(\mathrm{s},\mathrm{s}'\right) \mathrm{Im}\, \mathrm{f}_{1}^{\frac{1}{2}}\left(\mathrm{s}'\right) + \mathrm{K}_{\mathrm{J},0}^{\mathrm{I},\frac{3}{2}}\left(\mathrm{s},\mathrm{s}'\right) \mathrm{Im}\, \mathrm{f}_{0}^{\frac{3}{2}}\left(\mathrm{s}'\right) \Big\} \\
& + \mathrm{ST}_{\mathrm{J}}^{\mathrm{I}}(\mathrm{s}) + \mathrm{PT}_{\mathrm{J}}^{\mathrm{I}}(\mathrm{s}) + \mathrm{sDT}_{\mathrm{J}}^{\mathrm{I}}(\mathrm{s}) + \mathrm{tDT}_{\mathrm{J}}^{\mathrm{I}}(\mathrm{s}),
\end{aligned}
$$



The figure from [X.H. Cao, F.K. Guo, Z.H. Guo and QZL, Phys.Rev.D 112 (2025)]



The figure from [X.H. Cao, F.K. Guo, Z.H. Guo and QZL, Phys.Rev.D 112 (2025)]



The figure from [X.H. Cao, F.K. Guo, Z.H. Guo and QZL, Phys.Rev.D 112 (2025)]

Motivations

- Systematically investigate the quark-mass dependence of κ and $K^*(892)$

	C24P34	C24P29	C32P29	C32P23	C48P23	C48P14	F32P30	F48P30	F32P21	F48P21	H48P32
$\tilde{L}^3 \times \tilde{T}$	$24^3 \times 64$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 96$	$32^3 \times 96$	$48^3 \times 96$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 144$
$\hat{\beta}$	6.20						6.41				6.72
a (fm)	0.10530(18)						0.07746(18)				0.05187(26)
$\tilde{m}_l^b$	-0.2770	-0.2770	-0.2770	-0.2790	-0.2790	-0.2825	-0.2295	-0.2295	-0.2320	-0.2320	-0.1850
$\tilde{m}_s^b$	-0.2310	-0.2400	-0.2400	-0.2400	-0.2400	-0.2310	-0.2050	-0.2050	-0.2050	-0.2050	-0.1700
m_l^R (MeV)	22.90(19)	16.94(12)	17.35(11)	10.55(11)	10.27(10)	3.638(83)	18.54(12)	18.511(92)	8.58(16)	8.59(08)	19.42(05)
m_s^R (MeV)	111.41(16)	87.46(10)	88.16(10)	84.48(07)	84.79(04)	103.15(05)	93.23(11)	93.05(08)	89.75(10)	90.43(08)	95.61(04)
m_π (MeV)	341.1(1.8)	292.7(1.2)	292.4(1.1)	228.0(1.2)	225.6(0.9)	135.5(1.6)	303.2(1.3)	303.4(0.9)	210.9(2.2)	207.2(1.1)	317.2(0.9)
m_K (MeV)	582.7(1.6)	509.4(1.1)	509.0(1.1)	484.1(1.0)	484.1(1.3)	510.0(1.0)	524.6(1.8)	523.6(1.4)	492.0(1.7)	493.0(1.4)	536.1(3.0)

The table from [CLQCD, Phys.Rev.D 109 (2024)]

Motivations

- Systematically investigate the quark-mass dependence of κ and $K^*(892)$

	C24P34	C24P29	C32P29	C32P23	C48P23	C48P14	F32P30	F48P30	F32P21	F48P21	H48P32
$\tilde{L}^3 \times \tilde{T}$	$24^3 \times 64$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 96$	$32^3 \times 96$	$48^3 \times 96$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 144$
$\hat{\beta}$	6.20						6.41				6.72
a (fm)	0.10530(18)						0.07746(18)				0.05187(26)
$\tilde{m}_l^b$	-0.2770	-0.2770	-0.2770	-0.2790	-0.2790	-0.2825	-0.2295	-0.2295	-0.2320	-0.2320	-0.1850
$\tilde{m}_s^b$	-0.2310	-0.2400	-0.2400	-0.2400	-0.2400	-0.2310	-0.2050	-0.2050	-0.2050	-0.2050	-0.1700
m_l^R (MeV)	22.90(19)	16.94(12)	17.35(11)	10.55(11)	10.27(10)	3.638(83)	18.54(12)	18.511(92)	8.58(16)	8.59(08)	19.42(05)
m_s^R (MeV)	111.41(16)	87.46(10)	88.16(10)	84.48(07)	84.79(04)	103.15(05)	93.23(11)	93.05(08)	89.75(10)	90.43(08)	95.61(04)
m_π (MeV)	341.1(1.8)	292.7(1.2)	292.4(1.1)	228.0(1.2)	225.6(0.9)	135.5(1.6)	303.2(1.3)	303.4(0.9)	210.9(2.2)	207.2(1.1)	317.2(0.9)
m_K (MeV)	582.7(1.6)	509.4(1.1)	509.0(1.1)	484.1(1.0)	484.1(1.3)	510.0(1.0)	524.6(1.8)	523.6(1.4)	492.0(1.7)	493.0(1.4)	536.1(3.0)

The table from [CLQCD, Phys.Rev.D 109 (2024)]

- Push the way to further Roy-Steiner equation investigations

1 Introduction

2 Method

3 Lüscher analysis

4 Conclusions

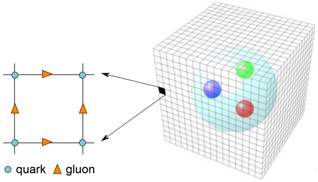
lattice QCD

- QCD Lagrangian

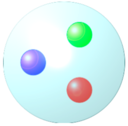
$$\mathcal{L} = \frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}_{\mu\nu} + \sum_{\mathrm{q=u,d,s,c,b,t}} \bar{\mathrm{q}}\left[\gamma_\mu(\partial_\mu - \mathrm{igA}_\mu) + \mathrm{m_q}\right]\mathrm{q}$$

Lattice QCD Key Features:

- Non-perturbative approach to QCD



meson



baryon

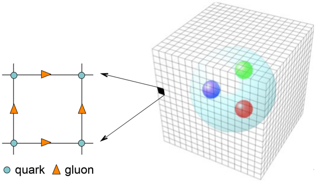
lattice QCD

- QCD Lagrangian

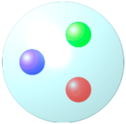
$$\mathcal{L} = \frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}_{\mu\nu} + \sum_{\mathrm{q=u,d,s,c,b,t}} \bar{\mathrm{q}}\left[\gamma_\mu(\partial_\mu - \mathrm{igA}_\mu) + \mathrm{m_q}\right]\mathrm{q}$$

Lattice QCD Key Features:

- Non-perturbative approach to QCD
- Discrete space-time lattice



meson



baryon

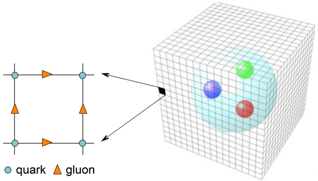
lattice QCD

- QCD Lagrangian

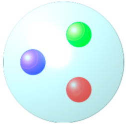
$$\mathcal{L} = \frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}_{\mu\nu} + \sum_{\mathrm{q=u,d,s,c,b,t}} \bar{\mathrm{q}}\left[\gamma_\mu(\partial_\mu - \mathrm{igA}_\mu) + \mathrm{m_q}\right]\mathrm{q}$$

Lattice QCD Key Features:

- Non-perturbative approach to QCD
- Discrete space-time lattice
- Quarks on sites, gluons on links



meson



baryon

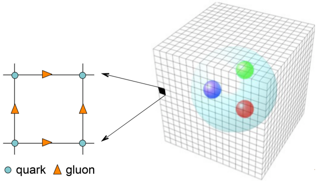
lattice QCD

- QCD Lagrangian

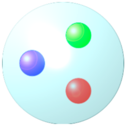
$$\mathcal{L} = \frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}_{\mu\nu} + \sum_{\mathrm{q=u,d,s,c,b,t}} \bar{\mathrm{q}}\left[\gamma_\mu(\partial_\mu - \mathrm{i}\mathrm{g}\mathrm{A}_\mu) + \mathrm{m_q}\right]\mathrm{q}$$

Lattice QCD Key Features:

- Non-perturbative approach to QCD
- Discrete space-time lattice
- Quarks on sites, gluons on links
- Numerical simulations via Monte Carlo



meson

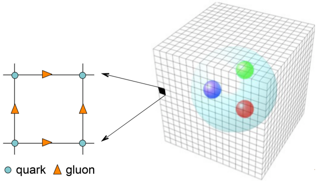


baryon

lattice QCD

- QCD Lagrangian

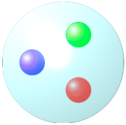
$$\mathcal{L} = \frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}_{\mu\nu} + \sum_{\mathrm{q=u,d,s,c,b,t}} \bar{\mathrm{q}}\left[\gamma_\mu(\partial_\mu - \mathrm{i}\mathrm{g}\mathrm{A}_\mu) + \mathrm{m_q}\right]\mathrm{q}$$



Lattice QCD Key Features:

- Non-perturbative approach to QCD
- Discrete space-time lattice
- Quarks on sites, gluons on links
- Numerical simulations via Monte Carlo
- Computes hadron masses, decay constants, etc.

meson



baryon

- Path integral on 4-dim. lattice

$$\langle \mathcal{O}[\mathrm{U}_\mu, \mathrm{q}, \bar{\mathrm{q}}] \rangle = \frac{1}{\mathrm{Z}} \int \prod_{\mathrm{n},\mu} \mathrm{dU}_\mu \mathrm{dq} \mathrm{d}\bar{\mathrm{q}} \ \mathcal{O}[\mathrm{U}_\mu, \mathrm{q}, \bar{\mathrm{q}}] \ \exp\left\{ -\sum_{\mathrm{n}} \mathcal{L}_{\mathrm{latt}}[\mathrm{U}_\mu, \mathrm{q}, \bar{\mathrm{q}}] \right\}$$

- Path integral on 4-dim. lattice

$$\langle \mathcal{O}[\mathrm{U}_\mu, \mathrm{q}, \bar{\mathrm{q}}] \rangle = \frac{1}{\mathrm{Z}} \int \prod_{\mathrm{n},\mu} \mathrm{d}\mathrm{U}_\mu \mathrm{d}\mathrm{q} \mathrm{d}\bar{\mathrm{q}}\ \mathcal{O}[\mathrm{U}_\mu, \mathrm{q}, \bar{\mathrm{q}}]\ \exp\left\{ -\sum_{\mathrm{n}} \mathcal{L}_{\mathrm{latt}}[\mathrm{U}_\mu, \mathrm{q}, \bar{\mathrm{q}}] \right\}$$

- quark fields are Grassmann numbers $\Longrightarrow$ analytically integratied

$$\langle \bar{\mathrm{O}}[\mathrm{U}_\mu] \rangle = \frac{1}{\mathrm{Z}} \int \prod_{\mathrm{n},\mu} \mathrm{d}\mathrm{U}_\mu\ \bar{\mathrm{O}}[\mathrm{U}_\mu]\ \exp\left\{ -\mathrm{S}^{\mathrm{eff}}_{\mathrm{latt}}[\mathrm{U}_\mu] \right\}$$

- Path integral on 4-dim. lattice

$$\langle \mathcal{O}[U_\mu, q, \bar{q}] \rangle = \frac{1}{Z} \int \prod_{n,\mu} dU_\mu dq d\bar{q} \; \mathcal{O}[U_\mu, q, \bar{q}] \; \exp\left\{ -\sum_{n} \mathcal{L}_{latt}[U_\mu, q, \bar{q}] \right\}$$

- quark fields are Grassmann numbers $\Longrightarrow$ analytically integratied

$$\langle \bar{O}[U_\mu] \rangle = \frac{1}{Z} \int \prod_{n,\mu} dU_\mu \; \bar{O}[U_\mu] \; \exp\left\{ -S^{eff}_{latt}[U_\mu] \right\}$$

- Average over the configurations:

$$\langle \bar{\mathcal{O}}[U_\mu] \rangle = \frac{1}{N} \sum_{i=1}^{N} \bar{\mathcal{O}}[U_\mu^{(i)}] + \underline{O\left(\frac{1}{\sqrt{N}}\right)} \quad \text{statistical error}$$

- Path integral on 4-dim. lattice

$$\langle \mathcal{O}[\mathrm{U}_\mu, \mathrm{q}, \bar{\mathrm{q}}] \rangle = \frac{1}{\mathrm{Z}} \int \prod_{\mathrm{n},\mu} \mathrm{d}\mathrm{U}_\mu \mathrm{d}\mathrm{q} \mathrm{d}\bar{\mathrm{q}} \; \mathcal{O}[\mathrm{U}_\mu, \mathrm{q}, \bar{\mathrm{q}}] \; \exp\left\{ -\sum_{\mathrm{n}} \mathcal{L}_{\mathrm{latt}}[\mathrm{U}_\mu, \mathrm{q}, \bar{\mathrm{q}}] \right\}$$

- quark fields are Grassmann numbers $\Longrightarrow$ analytically integratied

$$\langle \bar{\mathrm{O}}[\mathrm{U}_\mu] \rangle = \frac{1}{\mathrm{Z}} \int \prod_{\mathrm{n},\mu} \mathrm{d}\mathrm{U}_\mu \; \bar{\mathrm{O}}[\mathrm{U}_\mu] \; \exp\left\{ -\mathrm{S}^{\mathrm{eff}}_{\mathrm{latt}}[\mathrm{U}_\mu] \right\}$$

- Average over the configurations:

$$\langle \bar{\mathcal{O}}[\mathrm{U}_\mu] \rangle = \frac{1}{\mathrm{N}} \sum_{\mathrm{i}=1}^{\mathrm{N}} \bar{\mathcal{O}}[\mathrm{U}_\mu^{(\mathrm{i})}] + \underline{\mathrm{O}\left(\frac{1}{\sqrt{\mathrm{N}}}\right)} \quad \text{statistical error}$$

- Hadron correlators in terms of quark fields

$$\langle \mathcal{O}_{\mathrm{h}}(\mathrm{t}) \mathcal{O}_{\mathrm{h}}^{\dagger}(0) \rangle \overset{\mathrm{t} \gg 0}{\approx} \mathrm{C} \exp(-\mathrm{m}_{\mathrm{h}} \mathrm{t})$$

Hadron spectrum

- Interpolators
  - Single-hadron operators:

$$K_i^{*+}(t,p) = \sum_x e^{ip\cdot x}\bar{s}(t,x)\gamma_i u(t,x), \quad \pi^+(t,p) = \sum_x e^{ip\cdot x}\bar{d}(t,x)\gamma_5 u(t,x)$$

Hadron spectrum

- Interpolators
  - * Single-hadron operators:

$$K_i^{*+}(t,p)=\sum_x e^{ip\cdot x}\bar{s}(t,x)\gamma_i u(t,x),\quad \pi^+(t,p)=\sum_x e^{ip\cdot x}\bar{d}(t,x)\gamma_5 u(t,x)$$

  - * double-hadron operator:

$$O_{K\pi}^{I=1/2}(t,p_1,p_2)=\sqrt{\frac{2}{3}}\pi^+(t,p_1)K^0(t,p_2)-\sqrt{\frac{1}{3}}\pi^0(t,p_1)K^+(t,p_2)$$

with

$$\pi^0(t,p_1)=\sum_x e^{ip_1\cdot x}\frac{1}{\sqrt{2}}\left[\bar{d}(t,x)\gamma_5 d(t,x)-\bar{u}(t,x)\gamma_5 u(t,x)\right]$$

$$K^+(t,p_2)=\sum_x e^{ip_2\cdot x}\bar{s}(t,x)\gamma_5 u(t,x),\quad K^0(t,p_2)=\sum_x e^{ip_2\cdot x}\bar{s}(t,x)\gamma_5 d(t,x)$$

- Projection method

$$\hat{P}_r^{\Lambda} = \frac{l}{\hbar} \sum_{R \in LG(d)} D^{\Lambda *}(\hat{R})_{rr} \hat{R}, \quad d = \frac{L}{2\pi} P$$

$$LG(d) = \{g | gdg^{-1} = d, g \in O_h\}$$

- single-hadron operators:

$$(\hat{P}_r^{\Lambda} \hat{K}^{*+}(t, P))_i = \frac{l}{\hbar} \sum_{\hat{R} \in LG(d)} D^{\Lambda}(\hat{R})_{rr} \hat{R}_{ij} K_j^{*+}(t, P)$$

d	LG(d)	irrep Λ	SO(3)	operators $O^{\Lambda}(P)$
(0, 0, 0)	O_h	A_1^+	$J = 0, 4 \cdots$	$K_0^+, \pi_0 K_0, \pi_1 K_1, \pi_2 K_2, \pi_3 K_3$
		T_1^-	$J = 1, 3 \cdots$	$K_i^{*+}, \pi_1 K_1, \pi_2 K_2, \pi_3 K_3$
(0, 0, 1)	C_{4v}	A_1	$J = 0, 1 \cdots$	$K_0^+, K_i^{*+}, \pi_0 K_1, \pi_1 K_0, \pi_1 K_2, \pi_2 K_1, \pi_3 K_2, \pi_2 K_3$
		E	$J = 1, 2 \cdots$	$K_i^{*+}, \pi_1 K_2, \pi_2 K_1, \pi_3 K_2, \pi_2 K_3$
(0, 1, 1)	C_{2v}	A_1	$J = 0, 1 \cdots$	$K_0^+, K_i^{*+}, \pi_0 K_2, \pi_2 K_0, \pi_1 K_1, \pi_2 K_2, \pi_3 K_1, \pi_3 K_1$
		B_2	$J = 1, 2 \cdots$	$K_i^{*+}, \pi_2 K_2, \pi_3 K_1, \pi_3 K_1$
		B_1	$J = 1, 2 \cdots$	$K_i^{*+}, \pi_1 K_1, \pi_2 K_2$
(1, 1, 1)	C_{3v}	A_1	$J = 0, 1, \cdots$	$K_0^+, K_i^{*+}, \pi_0 K_3, \pi_3 K_0, \pi_1 K_2, \pi_2 K_1$
		E	$J = 1, 2 \cdots$	$K_i^{*+}, \pi_2 K_1, \pi_2 K_1$

- Projection method

$$\hat{P}^{\Lambda}_{r} = \frac{l}{\hbar} \sum_{R \in LG(d)} D^{\Lambda *}(\hat{R})_{rr} \hat{R}, \quad d = \frac{L}{2\pi} P$$

$$LG(d) = \{g | g d g^{-1} = d, g \in O_h\}$$

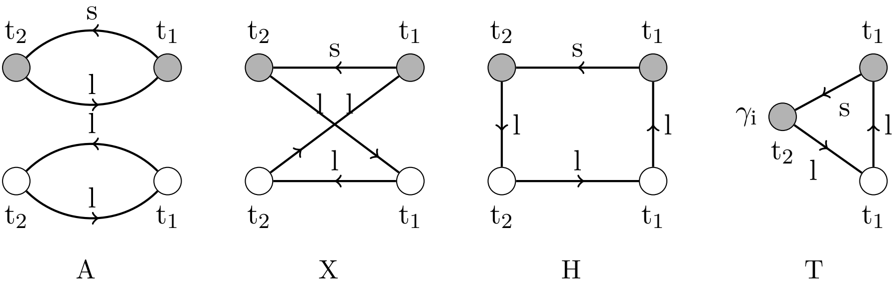
- double-hadron operators:

$$\hat{P}^{\Lambda}_{k} \pi(t, p_1) K(t, p_2) = \frac{l}{\hbar} \sum_{\hat{R} \in LG(d)} D^{\Lambda}(\hat{R})_{kk} \pi(t, \hat{R} p_1 \hat{R}^{-1}) K(t, \hat{R} p_2 \hat{R}^{-1})$$

d	LG(d)	irrep Λ	SO(3)	operators $O^{\Lambda}(P)$
(0, 0, 0)	O_h	A_1^+	$J = 0, 4 \cdots$	$K_0^+, \pi_0 K_0, \pi_1 K_1, \pi_2 K_2, \pi_3 K_3$
		T_1^-	$J = 1, 3 \cdots$	$K_i^{*+}, \pi_1 K_1, \pi_2 K_2, \pi_3 K_3$
(0, 0, 1)	C_{4v}	A_1	$J = 0, 1 \cdots$	$K_0^+, K_i^{*+}, \pi_0 K_1, \pi_1 K_0, \pi_1 K_2, \pi_2 K_1, \pi_3 K_2, \pi_2 K_3$
		E	$J = 1, 2 \cdots$	$K_i^{*+}, \pi_1 K_2, \pi_2 K_1, \pi_3 K_2, \pi_2 K_3$
(0, 1, 1)	C_{2v}	A_1	$J = 0, 1 \cdots$	$K_0^+, K_i^{*+}, \pi_0 K_2, \pi_2 K_0, \pi_1 K_1, \pi_2 K_2, \pi_3 K_1, \pi_3 K_1$
		B_2	$J = 1, 2 \cdots$	$K_i^{*+}, \pi_2 K_2, \pi_3 K_1, \pi_3 K_1$
		B_1	$J = 1, 2 \cdots$	$K_i^{*+}, \pi_1 K_1, \pi_2 K_2$
(1, 1, 1)	C_{3v}	A_1	$J = 0, 1, \cdots$	$K_0^+, K_i^{*+}, \pi_0 K_3, \pi_3 K_0, \pi_1 K_2, \pi_2 K_1$
		E	$J = 1, 2 \cdots$	$K_i^{*+}, \pi_2 K_1, \pi_2 K_1$

- Correlation matrix

$$C_{i,j}^{\Lambda,P}(t_2 - t_1) = \langle O_i^{\Lambda}(t_2, P) O_j^{\Lambda\dagger}(t_1, P)\rangle \ .$$



- matrix elements: $\langle (K\pi)(K\pi)^\dagger\rangle = A + \frac{1}{2}X - \frac{3}{2}H$, $\langle K_i (K\pi)^\dagger\rangle = -\sqrt{\frac{3}{2}}T$

- Generalized Eigenvalue Problems

$$C^{\Lambda,P}(t)v_n = \lambda_n^{\Lambda,P}(t)C^{\Lambda,P}(t_0)v_n$$

t_0: a chosen reference time

- Generalized Eigenvalue Problems

$$C^{\Lambda,P}(t)v_n = \lambda_n^{\Lambda,P}(t)C^{\Lambda,P}(t_0)v_n$$

t_0: a chosen reference time

- $\lambda_n^{\Lambda,P}(t)$ decay exponentially:

$$\lambda_n^{\Lambda,P}(t) = (1 - A_n)e^{-E_n^{\Lambda,P}(t-t_0)} + A_n e^{-E_n'^{\Lambda,P}(t-t_0)} \ ,$$

- Generalized Eigenvalue Problems

$$C^{\Lambda,P}(t)v_n = \lambda_n^{\Lambda,P}(t)C^{\Lambda,P}(t_0)v_n$$

t_0: a chosen reference time

- $\lambda_n^{\Lambda,P}(t)$ decay exponentially:

$$\lambda_n^{\Lambda,P}(t) = (1 - A_n)e^{-E_n^{\Lambda,P}(t-t_0)} + A_n e^{-E_n'^{\Lambda,P}(t-t_0)} ,$$

- fit windows:

$$[t^l_{min}, t^l_{max}] \in [t_0 + 1, t_e] , \quad t^l_{max} - t^l_{min} \geq 4 ,$$

t_e: signal of effective mass disappearing

- Generalized Eigenvalue Problems

$$C^{\Lambda,P}(t)v_n = \lambda_n^{\Lambda,P}(t)C^{\Lambda,P}(t_0)v_n$$

t_0: a chosen reference time

- $\lambda_n^{\Lambda,P}(t)$ decay exponentially:

$$\lambda_n^{\Lambda,P}(t) = (1 - A_n)e^{-E_n^{\Lambda,P}(t-t_0)} + A_n e^{-E_n'^{\Lambda,P}(t-t_0)} ,$$

- fit windows:

$$[t_{min}^l, t_{max}^l] \in [t_0 + 1, t_e] , \quad t_{max}^l - t_{min}^l \geq 4 ,$$

t_e: signal of effective mass disappearing

- minimizing the correlated chi-square

$$\chi_l^2 \equiv \sum_{t,t' \in [t_{min}^l, t_{max}^l]} \left[\lambda_n^{\Lambda,P}(t) - \tilde{\lambda}_{n,b=0}^{\Lambda,P}(t)\right] \left(\Sigma^{-1}\right)_{tt'} \left[\lambda_n^{\Lambda,P}(t') - \tilde{\lambda}_{n,b=0}^{\Lambda,P}(t')\right]$$

Uncertainties from fit windows

Akaike information criterion[P. Boyle, et al., Phys. Rev. Lett. (2025)]

$$\mathrm{AIC}^{\mathrm{l}} = \chi^2_{\mathrm{l}} + 2\mathrm{n}^{\mathrm{para}} - \mathrm{n}^{\mathrm{l}}_{\mathrm{data}}$$

- fit result weighted by

$$\omega^{\mathrm{l}} \propto \mathrm{e}^{-\frac{1}{2}\mathrm{AIC}^{\mathrm{l}}}$$

Uncertainties from fit windows

Akaike information criterion[P. Boyle, et al., Phys. Rev. Lett. (2025)]

$$\mathrm{AIC}^{\mathrm{l}} = \chi^2_{\mathrm{l}} + 2\mathrm{n}^{\mathrm{para}} - \mathrm{n}^{\mathrm{l}}_{\mathrm{data}}$$

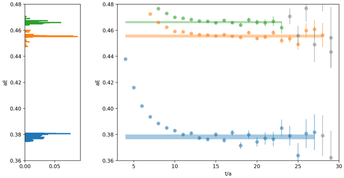
- fit result weighted by

$$\omega^{\mathrm{l}} \propto \mathrm{e}^{-\frac{1}{2}\mathrm{AIC}^{\mathrm{l}}}$$

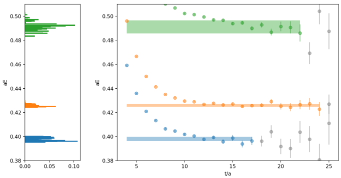
- system uncertainties:

$$\sigma^{\Lambda,\mathrm{P}}_{\mathrm{n,sys}} = \sqrt{\sum_{\mathrm{l}} \omega^{\Lambda,\mathrm{l}}_{\mathrm{n}} (\mathrm{E}^{\Lambda,\mathrm{P}}_{\mathrm{n,l}} - \bar{\mathrm{E}}^{\Lambda,\mathrm{P}}_{\mathrm{n}})^2}, \quad \bar{\mathrm{E}}^{\Lambda,\mathrm{P}}_{\mathrm{n}} = \sum_{\mathrm{l}} \omega^{\Lambda,\mathrm{l}}_{\mathrm{n}} \mathrm{E}^{\Lambda,\mathrm{P}}_{\mathrm{n,l}}.$$

- F48P30:

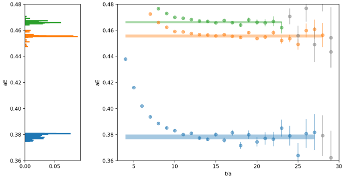


$\mathrm{LG(d)} = \mathrm{C_{4v}},\ \Lambda = \mathrm{E}$

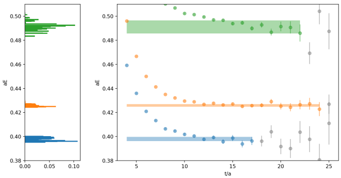


$\mathrm{LG(d)} = \mathrm{C_{2v}},\ \Lambda = \mathrm{B_1}$

- F48P30:

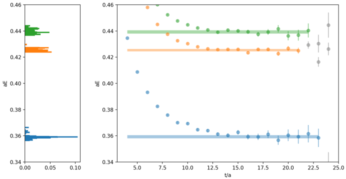


$\mathrm{LG(d)} = \mathrm{C_{4v}},\ \Lambda = \mathrm{E}$

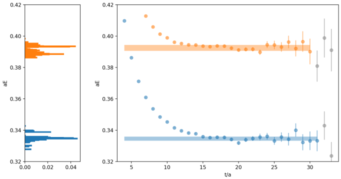


$\mathrm{LG(d)} = \mathrm{C_{2v}},\ \Lambda = \mathrm{B_1}$

- F48P21:



$\mathrm{LG(d)} = \mathrm{C_{4v}},\ \Lambda = \mathrm{E}$



$\mathrm{LG(d)} = \mathrm{O_h},\ \Lambda = \mathrm{T_1^-}$

- Statistics uncertainties: bootstrap resample method

$$\sigma^{\Lambda}_{\mathrm{n,stat}} = \sqrt{\frac{1}{\mathrm{N_{boot}} - 1} \sum_{\mathrm{b}=1}^{\mathrm{N_{boot}}} \left(\mathrm{E}^{\Lambda}_{\mathrm{b,n}} - \bar{\mathrm{E}}^{\Lambda}_{\mathrm{n}} \right)^2}, \quad \bar{\mathrm{E}}^{\Lambda}_{\mathrm{n}} = \frac{1}{\mathrm{N_{boot}}} \sum_{\mathrm{b}=1}^{\mathrm{N_{boot}}} \mathrm{E}^{\Lambda}_{\mathrm{b,n}}$$

- Statistics uncertainties: bootstrap resample method

$$\sigma^{\Lambda}_{\mathrm{n,stat}} = \sqrt{\frac{1}{\mathrm{N_{boot}} - 1}\sum_{\mathrm{b}=1}^{\mathrm{N_{boot}}}\left(\mathrm{E}^{\Lambda}_{\mathrm{b,n}} - \bar{\mathrm{E}}^{\Lambda}_{\mathrm{n}}\right)^2}, \quad \bar{\mathrm{E}}^{\Lambda}_{\mathrm{n}} = \frac{1}{\mathrm{N_{boot}}}\sum_{\mathrm{b}=1}^{\mathrm{N_{boot}}}\mathrm{E}^{\Lambda}_{\mathrm{b,n}}$$

- total uncertainties

$$\sigma^{\Lambda}_{\mathrm{n,tot}} = \sqrt{\left(\sigma^{\Lambda}_{\mathrm{n,sys}}\right)^2 + \left(\sigma^{\Lambda}_{\mathrm{n,stat}}\right)^2}$$

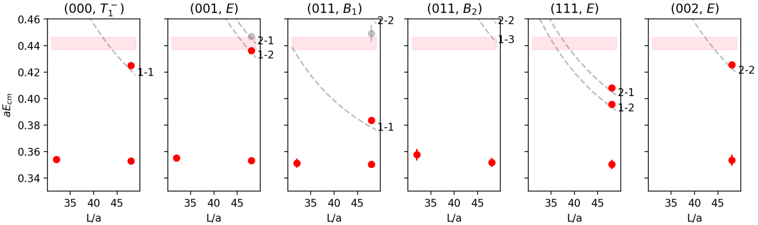
- Statistics uncertainties: bootstrap resample method

$$\sigma^{\Lambda}_{\mathrm{n,stat}} = \sqrt{\frac{1}{\mathrm{N_{boot}}-1}\sum_{\mathrm{b}=1}^{\mathrm{N_{boot}}}\left(\mathrm{E}^{\Lambda}_{\mathrm{b,n}} - \bar{\mathrm{E}}^{\Lambda}_{\mathrm{n}}\right)^2}, \quad \bar{\mathrm{E}}^{\Lambda}_{\mathrm{n}} = \frac{1}{\mathrm{N_{boot}}}\sum_{\mathrm{b}=1}^{\mathrm{N_{boot}}}\mathrm{E}^{\Lambda}_{\mathrm{b,n}}$$

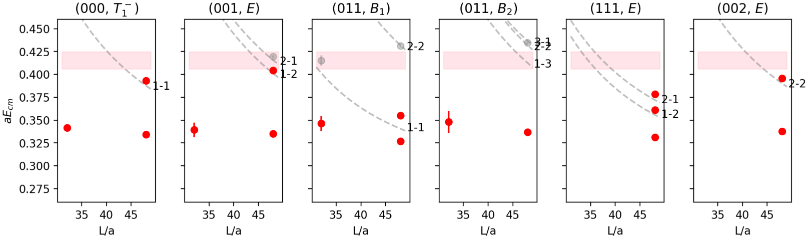
- total uncertainties

$$\sigma^{\Lambda}_{\mathrm{n,tot}} = \sqrt{\left(\sigma^{\Lambda}_{\mathrm{n,sys}}\right)^2 + \left(\sigma^{\Lambda}_{\mathrm{n,stat}}\right)^2}$$

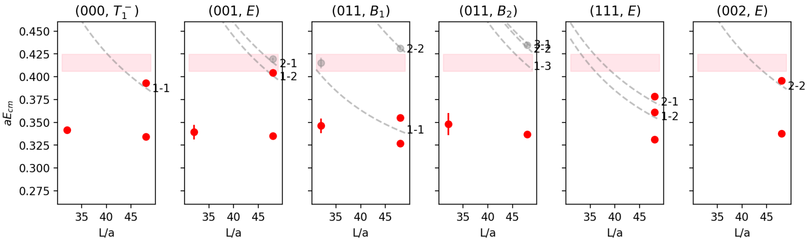
- finite-volume spectrum



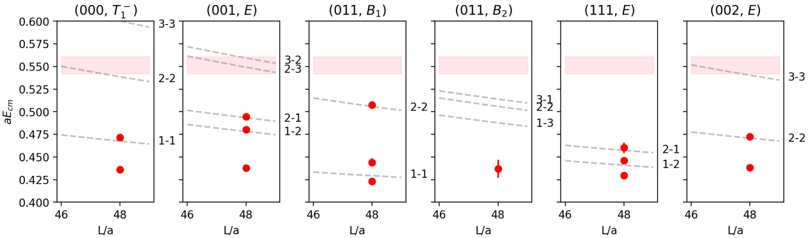
F48P30 && F32P30



F48P21 && F32P21



F48P21 && F32P21



C48P23

1 Introduction

2 Method

3 Lüscher analysis

4 Conclusions

- Infinite volume $\Leftrightarrow$ finite volume: boundary conditions and space symmetries

Quantization conditions [M. Luscher and U. Wolff, Nucl. Phys. B (1990); S. He, X. Feng and C. Liu, JHEP 07 (2005); Z. Fu, Phys. Rev. D (2012)]

$$\det\left(1 + \mathrm{i}\mathcal{T}\left(1 + \mathrm{i}\mathcal{M}^{\mathrm{P}}\right)\right) = 0$$

- $\mathcal{M}^{\mathrm{P}}$ for $\mathrm{l}, \mathrm{l}' \leq 1$

$$\left(\mathcal{M}^{\mathrm{P}}_{\ell\mathrm{m},\ell'\mathrm{m}'}\right) = \begin{pmatrix} \mathrm{w}_{00} & \mathrm{i}\sqrt{3}\mathrm{w}_{10} & \mathrm{i}\sqrt{3}\mathrm{w}_{11} & \mathrm{i}\sqrt{3}\mathrm{w}_{1-1} \\ -\mathrm{i}\sqrt{3}\mathrm{w}_{10} & \mathrm{w}_{00} + 2\mathrm{w}_{20} & \sqrt{3}\mathrm{w}_{21} & \sqrt{3}\mathrm{w}_{2-1} \\ \mathrm{i}\sqrt{3}\mathrm{w}_{1-1} & -\sqrt{3}\mathrm{w}_{2-1} & \mathrm{w}_{00} - \mathrm{w}_{20} & -\sqrt{6}\mathrm{w}_{2-2} \\ \mathrm{i}\sqrt{3}\mathrm{w}_{11} & -\sqrt{3}\mathrm{w}_{21} & -\sqrt{6}\mathrm{w}_{22} & \mathrm{w}_{00} - \mathrm{w}_{20} \end{pmatrix}$$

- $\mathcal{T}$ matrix is diagonal in the $\ell\mathrm{m}$ span space:

$$\mathcal{T}_{\ell\mathrm{m},\ell'\mathrm{m}'} = \mathrm{T}^{\ell}\delta_{\ell,\ell'}\delta_{\mathrm{m},\mathrm{m}'}\ .$$

Quantization conditions [M. Luscher and U. Wolff, Nucl. Phys. B (1990); S. He, X. Feng and C. Liu, JHEP 07 (2005); Z. Fu, Phys. Rev. D (2012)]

$$\det\left(1 + \mathrm{i}\mathcal{T}\left(1 + \mathrm{i}\mathcal{M}^{\mathrm{P}}\right)\right) = 0$$

d	LG(d)	irrep	QCs
$(0,0,0)$	$\mathrm{O_h}$	A_1^+	$\cot\delta_0 = \omega_{00}$
		T_1^-	$\cot\delta_1 = \omega_{00}$
$(0,0,\mathrm{n})$	$\mathrm{C_{4v}}$	A_1	$(\cot\delta_0 - \omega_{00})(\cot\delta_1 - \omega_{00} - 2\omega_{20}) - 3\omega_{10}^2 = 0$
		E	$\cot\delta_1 = \omega_{00} - \omega_{20}$
$(\mathrm{n},\mathrm{n},0)$	$\mathrm{C_{2v}}$	A_1	$(\cot\delta_0 - \omega_{00})\left(\cot\delta_1 - \omega_{00} + \omega_{20} + \mathrm{i}\sqrt{6}\omega_{22}\right) = 0$
		B_2	$\cot\delta_1 = \omega_{00} + 2\omega_{20}$
		B_1	$\cot\delta_1 = \omega_{00} - \omega_{20} - \sqrt{6}\,\mathrm{Im}\left[\omega_{22}\right]$
$(\mathrm{n},\mathrm{n},\mathrm{n})$	$\mathrm{C_{3v}}$	A_1	$(\cot\delta_0 - \omega_{00})\left(-\cot\delta_1 + \omega_{00} - 2\mathrm{i}\sqrt{6}\omega_{22}\right) + 9\omega_{10}^2 = 0$
		E	$\cot\delta_1 = \omega_{00} + \mathrm{i}\sqrt{6}\omega_{22}$

- phase shifts δ_ℓ:

$$\mathrm{k}^{2\ell+1}\cot\delta_\ell = \frac{\mathrm{k}^{2\ell}\sqrt{\mathrm{s}}}{2}\,\mathrm{Re}\left(\frac{1}{\mathrm{T}^\ell(\mathrm{s})}\right), \quad \mathrm{k}: \text{center-mass momentum}$$

Parameterizations for T^{ℓ}

- K-matrix method:

$$\mathrm{T}_{\ell}^{-1}(\mathrm{s}) = \frac{1}{(2\mathrm{k})^{2\ell}}\mathrm{K}_{\ell}^{-1}(\mathrm{s}) - \mathrm{i}\rho(\mathrm{s}) \ , \quad \rho : \text{two-body phase space}$$

Parameterizations for T^ℓ

- K-matrix method:

$$T_\ell^{-1}(s) = \frac{1}{(2k)^{2\ell}} K_\ell^{-1}(s) - i\rho(s)\ , \quad \rho : \text{two-body phase space}$$

or

$$k^{2k+1} \cot \delta_\ell(s) = \frac{\sqrt{s}}{2^{2\ell+1}} K^{-1}(s)$$

Parameterizations for T^ℓ

- K-matrix method:

$$T_\ell^{-1}(s) = \frac{1}{(2k)^{2\ell}} K_\ell^{-1}(s) - i\rho(s) \ , \quad \rho : \text{two-body phase space}$$

or

$$k^{2k+1} \cot \delta_\ell(s) = \frac{\sqrt{s}}{2^{2\ell+1}} K^{-1}(s)$$

  - A single pole term:

$$K_\ell(s) = \frac{g_\ell^2}{m_\ell^2 - s}$$

Parameterizations for T^ℓ

- K-matrix method:

$$T_\ell^{-1}(s) = \frac{1}{(2k)^{2\ell}} K_\ell^{-1}(s) - i\rho(s)\ , \quad \rho : \text{two-body phase space}$$

or

$$k^{2k+1} \cot \delta_\ell(s) = \frac{\sqrt{s}}{2^{2\ell+1}} K^{-1}(s)$$

  - A single pole term:

$$K_\ell(s) = \frac{g_\ell^2}{m_\ell^2 - s}$$

  - effective range expansion:

$$k^{2\ell+1} \cot \delta_\ell(s) = \frac{1}{a_\ell} + \frac{r_\ell}{2} k^2\ , \quad K_\ell^{-1}(s) = \frac{2^{2\ell+1}}{\sqrt{s}} \left(\frac{1}{a_\ell} + \frac{r_\ell}{2} k^2 \right)$$

- Production Representation (PKU representation)[Z.Y. Zhou and H.Q. Zheng, JHEP (2005)]

$$\mathrm{S(s)} = \Pi_{\mathrm{i}} \mathrm{S}^{\mathrm{p_i}}(\mathrm{s}) \cdot \mathrm{S}^{\mathrm{cut}}(\mathrm{s}) \ , \quad \mathrm{S(s)} = 1 + 2\mathrm{i}\rho(\mathrm{s})\mathrm{T(s)}$$

- Production Representation (PKU representation)[Z.Y. Zhou and H.Q. Zheng, JHEP (2005)]

$$\mathrm{S(s)} = \Pi_\mathrm{i} \mathrm{S}^{\mathrm{p_i}}(\mathrm{s}) \cdot \mathrm{S}^{\mathrm{cut}}(\mathrm{s}) \ , \quad \mathrm{S(s)} = 1 + 2\mathrm{i}\rho(\mathrm{s})\mathrm{T(s)}$$

  - bound state $\mathrm{s_0}$: $\mathrm{S(s)} = \frac{1 - \mathrm{i}\rho(\mathrm{s}) \frac{\mathrm{s}}{\mathrm{s}-\mathrm{s_L}} \sqrt{\frac{\mathrm{s_0}-\mathrm{s_L}}{\mathrm{s_R}-\mathrm{s_0}}}}{1 + \mathrm{i}\rho(\mathrm{s}) \frac{\mathrm{s}}{\mathrm{s}-\mathrm{s_L}} \sqrt{\frac{\mathrm{s_0}-\mathrm{s_L}}{\mathrm{s_R}-\mathrm{s_0}}}}, \quad \mathrm{s_L} < \mathrm{s_0} < \mathrm{s_R}$

- Production Representation (PKU representation)[Z.Y. Zhou and H.Q. Zheng, JHEP (2005)]

$$\mathrm{S(s)} = \Pi_i \mathrm{S}^{p_i}(\mathrm{s}) \cdot \mathrm{S}^{\mathrm{cut}}(\mathrm{s}) \ , \quad \mathrm{S(s)} = 1 + 2i\rho(\mathrm{s})\mathrm{T(s)}$$

- bound state s_0: $S(s) = \frac{1-i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_0-s_L}{s_R-s_0}}}{1+i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_0-s_L}{s_R-s_0}}}, \quad s_L < s_0 < s_R$
- virtual state s_v: $S(s) = \frac{1+i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_v-s_L}{s_R-s_0}}}{1-i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_v-s_L}{s_R-s_0}}}, \quad s_L < s_0 < s_R$

- Production Representation (PKU representation)[Z.Y. Zhou and H.Q. Zheng, JHEP (2005)]

$$\mathrm{S(s) = \Pi_i S^{p_i}(s) \cdot S^{cut}(s)\ , \quad S(s) = 1 + 2i\rho(s)T(s)}$$

- bound state $\mathrm{s_0}$: $\mathrm{S(s) = \dfrac{1 - i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_0-s_L}{s_R-s_0}}}{1 + i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_0-s_L}{s_R-s_0}}}}$, $\quad \mathrm{s_L < s_0 < s_R}$
- virtual state $\mathrm{s_v}$: $\mathrm{S(s) = \dfrac{1 + i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_v-s_L}{s_R-s_0}}}{1 - i\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_v-s_L}{s_R-s_0}}}}$, $\quad \mathrm{s_L < s_0 < s_R}$
- resonance $\mathrm{z_0}$: $\mathrm{S(s) = \frac{M^2(z_0) - s + i\rho(s)sG}{M^2(z_0) - s - i\rho(s)sG}}$, with

$$\mathrm{G = \frac{Im[z_0\rho(z_0)]}{Re[z_0\rho(z_0)]}, \quad M^2(z_0) = Re[z_0] + \frac{Im[z_0]Im[z_0\rho(z_0)]}{Re[z_0\rho(z_0)]}.} \tag{1}$$

- the left-hand cuts contributions $\mathrm{S}^{\mathrm{cut}}$: $\mathrm{S}^{\mathrm{cut}} = \exp[2\mathrm{i}\rho(\mathrm{s})\mathrm{f}(\mathrm{s})]$

- the left-hand cuts contributions S^{cut} : $S^{cut} = \exp[2i\rho(s)f(s)]$
  - physical pion mass: f(s) obtained from χPT

- the left-hand cuts contributions $\mathrm{S^{cut}}$: $\mathrm{S^{cut}} = \exp[2i\rho(s)f(s)]$
  - physical pion mass: f(s) obtained from χPT
  - conformal mapping[W. R. Frazer, Phys. Rev. (1961)]:

$$f(s) = \sum_n C_n \omega^n(s)$$

with conformal mapping variables $\omega(s)$

$$\omega(s) = -\frac{\left(\sqrt{s} - \sqrt{s_E}\right)\left(\sqrt{s}\sqrt{s_E} + s_-\right)}{\left(\sqrt{s} + \sqrt{s_E}\right)\left(\sqrt{s}\sqrt{s_E} - s_-\right)}, \quad s_- = m_2^2 - m_1^2$$

- the left-hand cuts contributions S^{cut} : $S^{cut} = \exp[2i\rho(s)f(s)]$
  - physical pion mass: f(s) obtained from χPT
  - conformal mapping[W. R. Frazer, Phys. Rev. (1961)]:

$$f(s) = \sum_n C_n \omega^n(s)$$

with conformal mapping variables $\omega(s)$

$$\omega(s) = -\frac{\left(\sqrt{s} - \sqrt{s_E}\right)\left(\sqrt{s}\sqrt{s_E} + s_-\right)}{\left(\sqrt{s} + \sqrt{s_E}\right)\left(\sqrt{s}\sqrt{s_E} - s_-\right)}, \quad s_- = m_2^2 - m_1^2$$

  - mapping the left-hand cut plane onto the unit circle.

- the left-hand cuts contributions $\mathrm{S}^{\mathrm{cut}}$: $\mathrm{S}^{\mathrm{cut}} = \exp[2\mathrm{i}\rho(\mathrm{s})\mathrm{f}(\mathrm{s})]$
  - physical pion mass: f(s) obtained from χPT
  - conformal mapping[W. R. Frazer, Phys. Rev. (1961)]:

$$\mathrm{f}(\mathrm{s}) = \sum_{\mathrm{n}} \mathrm{C}_{\mathrm{n}} \omega^{\mathrm{n}}(\mathrm{s})$$

with conformal mapping variables $\omega(\mathrm{s})$

$$\omega(\mathrm{s}) = -\frac{\left(\sqrt{\mathrm{s}} - \sqrt{\mathrm{s}_{\mathrm{E}}}\right)\left(\sqrt{\mathrm{s}}\sqrt{\mathrm{s}_{\mathrm{E}}} + \mathrm{s}_{-}\right)}{\left(\sqrt{\mathrm{s}} + \sqrt{\mathrm{s}_{\mathrm{E}}}\right)\left(\sqrt{\mathrm{s}}\sqrt{\mathrm{s}_{\mathrm{E}}} - \mathrm{s}_{-}\right)}, \quad \mathrm{s}_{-} = \mathrm{m}_2^2 - \mathrm{m}_1^2$$

  - mapping the left-hand cut plane onto the unit circle.
  - converging more rapidly than the ordinal power series of s.

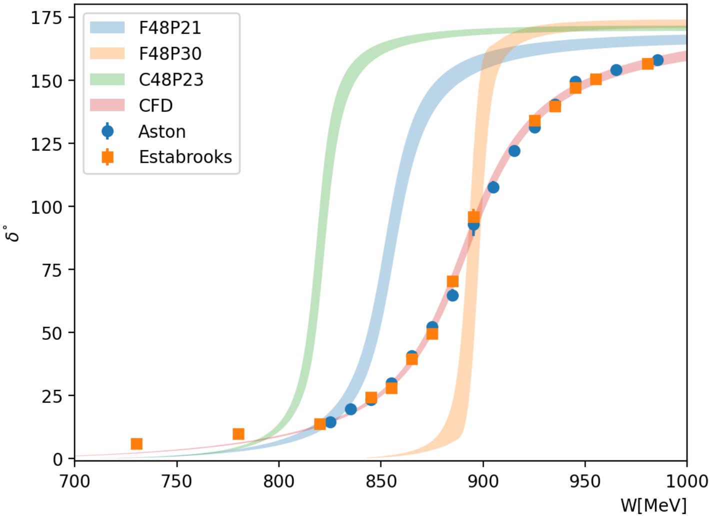
d	LG(d)	irrep	QCs
$(0,0,0)$	$\mathrm{O_h}$	$\mathrm{A_1^+}$	$\cot\delta_0 = \omega_{00}$
		$\mathrm{T_1^-}$	$\cot\delta_1 = \omega_{00}$
$(0,0,\mathrm{n})$	$\mathrm{C_{4v}}$	$\mathrm{A_1}$	$(\cot\delta_0 - \omega_{00})(\cot\delta_1 - \omega_{00} - 2\omega_{20}) - 3\omega_{10}^2 = 0$
		E	$\cot\delta_1 = \omega_{00} - \omega_{20}$
$(\mathrm{n},\mathrm{n},0)$	$\mathrm{C_{2v}}$	$\mathrm{A_1}$	$(\cot\delta_0 - \omega_{00})\left(\cot\delta_1 - \omega_{00} + \omega_{20} + \mathrm{i}\sqrt{6}\omega_{22}\right) = 0$
		$\mathrm{B_2}$	$\cot\delta_1 = \omega_{00} + 2\omega_{20}$
		$\mathrm{B_1}$	$\cot\delta_1 = \omega_{00} - \omega_{20} - \sqrt{6}\,\mathrm{Im}\left[\omega_{22}\right]$
$(\mathrm{n},\mathrm{n},\mathrm{n})$	$\mathrm{C_{3v}}$	$\mathrm{A_1}$	$(\cot\delta_0 - \omega_{00})\left(-\cot\delta_1 + \omega_{00} - 2\mathrm{i}\sqrt{6}\omega_{22}\right) + 9\omega_{10}^2 = 0$
		E	$\cot\delta_1 = \omega_{00} + \mathrm{i}\sqrt{6}\omega_{22}$

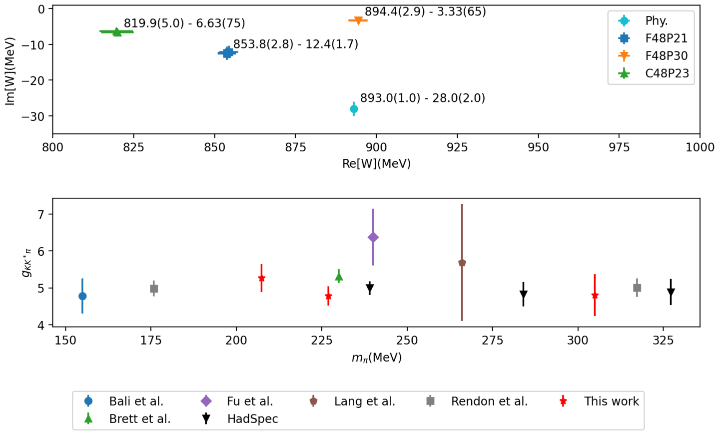
- Inverting the QCs to obtain finite-volume spectrum $\{\tilde{\mathrm{E}}_{\mathrm{n}}^{\Lambda,\mathrm{P},\mathrm{L}}(\boldsymbol{\alpha})\}$, then minimize the χ^2

$$\chi^2 = \sum_{\mathrm{L}} \left(\sum_{\Lambda,\mathrm{n},\mathrm{P}} \sum_{\Lambda',\mathrm{n}',\mathrm{P}'} \left(\mathrm{E}_{\mathrm{n}}^{\Lambda,\mathrm{P},\mathrm{L}} - \tilde{\mathrm{E}}_{\mathrm{n}}^{\Lambda,\mathrm{P},\mathrm{L}}\right) \left[\mathrm{C}^{-1}(\mathrm{L})\right]_{\Lambda,\mathrm{n},\mathrm{P};\Lambda',\mathrm{n}',\mathrm{P}'} \left(\mathrm{E}_{\mathrm{n}'}^{\Lambda',\mathrm{P}',\mathrm{L}} - \tilde{\mathrm{E}}_{\mathrm{n}'}^{\Lambda',\mathrm{P}',\mathrm{L}}\right) \right)$$

Numerical results

ensembles	models	Fit parameters	pole position s_0	χ^2/dof
F48P30	BW	$g = 0.404(38)$, $m = 0.35105(87)$	$0.12315(63) - i0.00097(15)$	$2.06/11 = 0.19$
	ERE	$1/a_1 = 0.0052(15)$, $r_1/2 = -1.26(32)$	$0.12322(55) - i0.00090(14)$	$1.44/11 = 0.13$
F32P30	PR	$z = 0.12318(61) - i0.00096(17)$	$0.12322(55) - i0.00090(14)$	$2.03/11 = 0.18$
F48P21	BW	$g = 0.442(24)$, $m = 0.33578(77)$	$0.11229(52) - i0.00345(36)$	$6.28/14 = 0.49$
	ERE	$1/a_1 = 0.00827(91)$, $r_1/2 = -0.99(11)$	$0.11248(55) - i0.00326(34)$	$5.78/14 = 0.41$
F32P21	PR	$z = 0.11229(52) - i0.00353(38)$	$0.11229(52) - i0.00353(38)$	$7.02/14 = 0.50$
C48P23	BW	$g = 0.43761(79)$, $m = 0.390(21)$	$0.19122(67) - i0.00309(35)$	$10.52/10 = 1.05$
	ERE	$1/a_1 = 0.0176(19)$, $r_1/2 = -1.63(18)$	$0.19107(82) - i0.00295(34)$	$12.73/10 = 1.27$
	PR	$z = 0.19125(67) - i0.00312(36)$	$0.19125(67) - i0.00312(36)$	$10.04/10 = 1.00$


F48P21
F48P30
C48P23
CFD
Aston
Estabrooks
175
150
125
100
75
50
25
0
δ°
700
750
800
850
900
950
1000
W[MeV]


819.9(5.0) - 6.63(75)
853.8(2.8) - 12.4(1.7)
894.4(2.9) - 3.33(65)
893.0(1.0) - 28.0(2.0)
Phy.
F48P21
F48P30
C48P23
Im[W](MeV)
Re[W](MeV)
$g_{KK^*\pi}$
m_π(MeV)
Bali et al.
Brett et al.
Fu et al.
HadSpec
Lang et al.
Rendon et al.
This work

1. Introduction
2. Method
3. Lüscher analysis
4. Conclusions

Conclusions

- We have performed a lattice calculation on 5 ensembles and 3 pion masses.

Conclusions

- We have performed a lattice calculation on 5 ensembles and 3 pion masses.
- 3 different parameterizations for the P-wave amplitude in the Lüscher analyses.

Conclusions

- We have performed a lattice calculation on 5 ensembles and 3 pion masses.
- 3 different parameterizations for the P-wave amplitude in the Lüscher analyses.
- The coupling $g_{K^*K\pi}$ is independent of the pion mass, which is consistent with other results.

Thanks for your listening!