From nucleon polarisability to pion threshold production process

Reporter: Zhaolong Zhang

Based on:

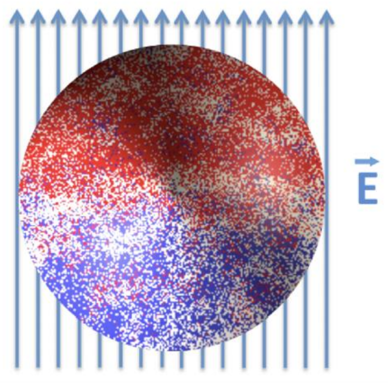
Phys. Rev. Lett. 134 (2025) 17,171904, Yu-Sheng Gao, Z-L Z, Xu Feng, Lu-Chang Jin, Chuan Liu, Ulf-G.Meißner

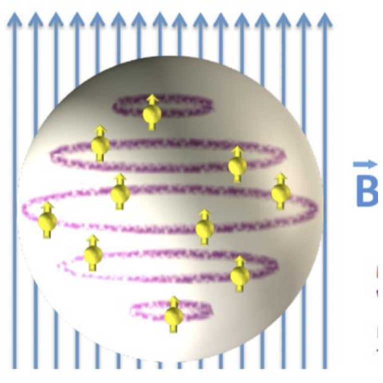
Phys. Rev. Lett. 133 (2024) 14, 141901, Xuan-He Wang, Z-L Z, Xiong-Hui Cao, Cong-Ling Fan, Xu Feng, Yu-Sheng Gao, Lu-Chang Jin, Chuan Liu,

2025.10.12

Nucleon electric polarisability: Background

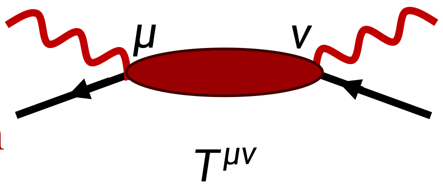
- Polarizabilities offer insights into the distribution of charge and magnetism within nucleons, revealing their response to external electromagnetic fields





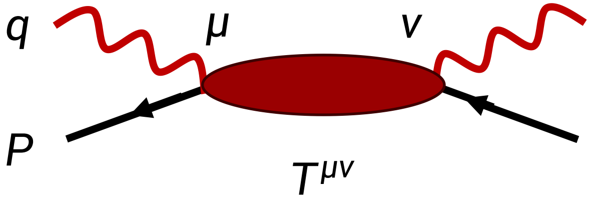
$$H^{(2)}_{eff} = -\frac{4\pi}{2}\alpha_E E^2 - \frac{4\pi}{2}\beta_M B^2$$

- Experimental determination of polarizabilities relies on Compton scattering, wherein external E&M fields polarize the target nucleon or deuteron



Nucleon electric polarisability: Background

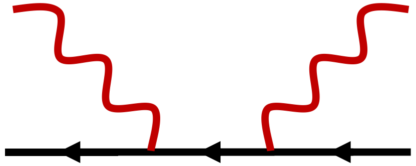
- Nucleon E&M polarizability are most central quantities relevant for Compton scattering



- Unpolarized doubly virtual Compton scattering

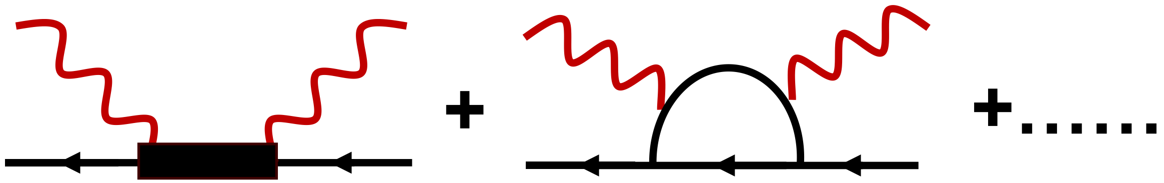
$$T^{\mu\nu}(P,q) = T^{\mu\nu}_{Born} + \frac{8\pi M}{e^2}\left[-(\beta_M + O(q))K_1^{\mu\nu} + (\alpha_E + \beta_M + O(q))K_2^{\mu\nu}\right]$$

- Born term



Intermeidate states: N

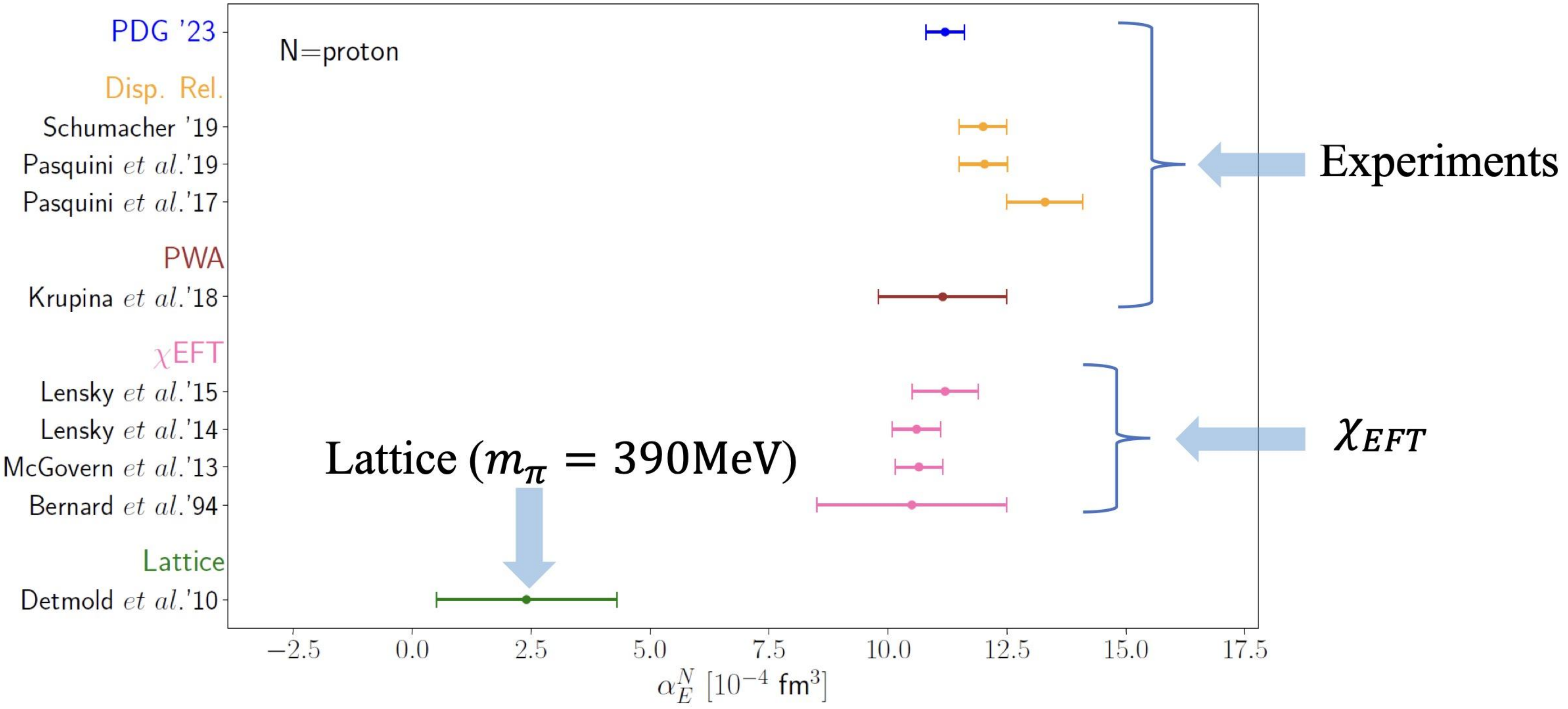
- Polarizabilities



Intermeidate states: $N\pi, N^*, \Delta, \cdots$

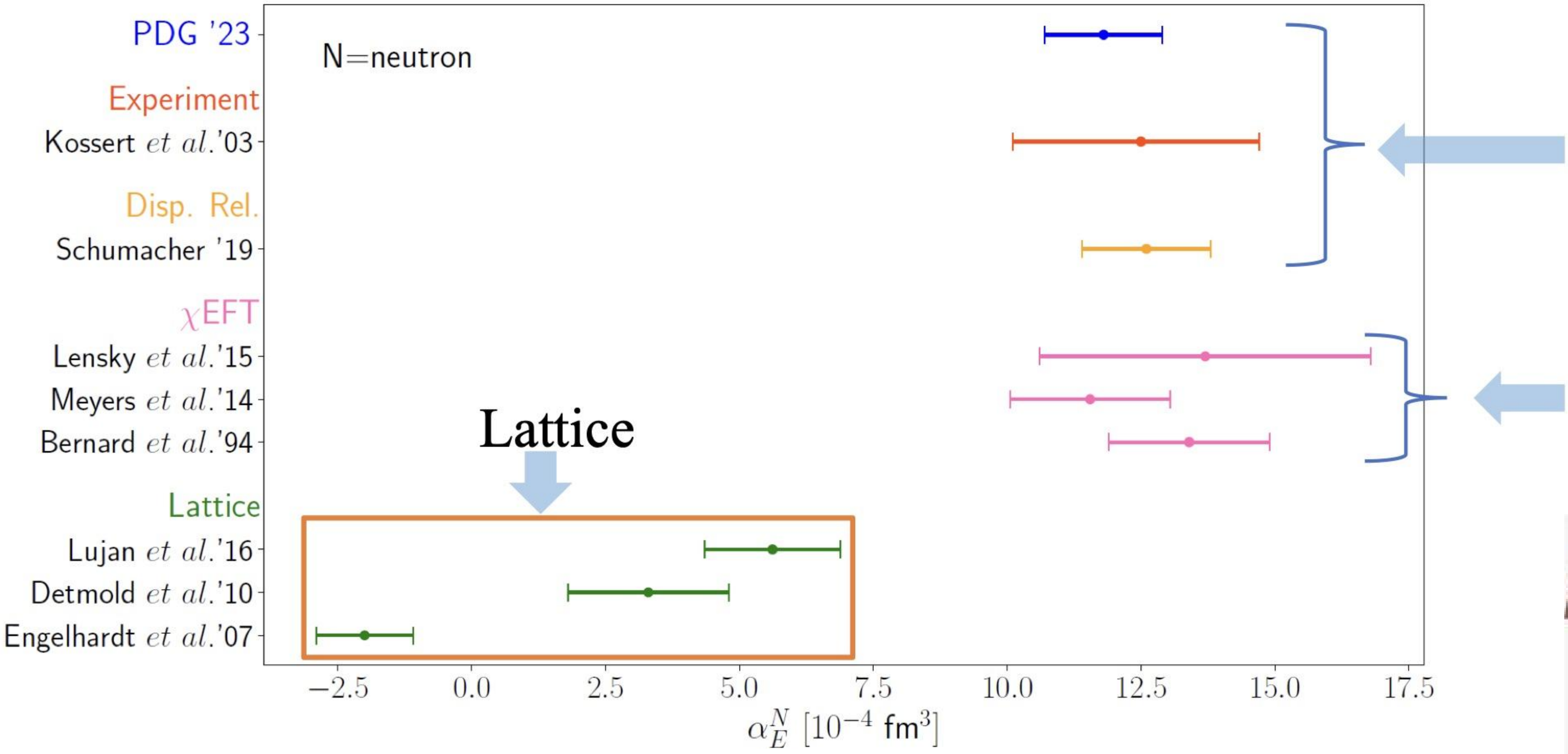
Research status of nucleon electric polarisability

- Review of the calculations of **proton** electric polarisability



Research status of nucleon electric polarisability

- Review of the calculations of **neutron** electric polarisability



Why Lattice results fall far below the experiments and Chiral EFT ?

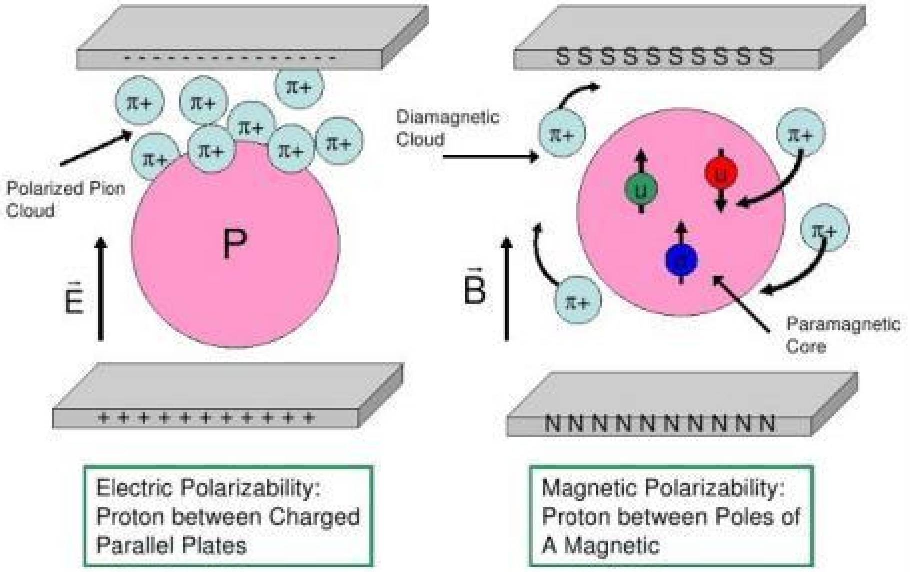
Research status of nucleon electric polarisability

- Two obvious drawbacks in previous lattice QCD calculations:
  - Use **Unphysical pion mass** ($m_\pi = 227 \sim 759\text{MeV}$)

Leading order ChPT：

$$\alpha_E = \frac{5}{96}\left(\frac{g_A}{f_\pi}\right)^2 \frac{\alpha_{em}}{M_\pi}$$

2~7 times discrepancy with physical α_E



- Use **Background field approach**: reduce computational resource, however difficult to justify the contribution(and their systematics) from various intermediate states.

Two crucial ingredients *to resolve the deficiency of* α_E *in lattice calculation !*

Lattice methodology for nucleon polarisability

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- For unphysical pion mass problem, we directly simulate the Compton tensor using **nucleon four point functions at physical pion mass**.

Lattice data input: $H^{\mu\nu}(x)$

$$T^{\mu\nu}(P,q) = \int d^4x\, e^{iqx} \langle N|J^{\mu}(x)J^{\nu}(0)|N\rangle = K_1^{\mu\nu}(T_1^B + T_1^{NB}) + K_2^{\mu\nu}(T_2^B + T_2^{NB})$$

Our choice: $\boxed{\alpha_E = -\frac{\alpha_{em}}{12M}\int d^4x\, t^2 \underline{H^{ii}(x)} + \underline{\alpha_E^r}}$

Compton tensor with $\vec{q} = 0$
Ground state vanishes. No need for subtraction!
Improve signal !

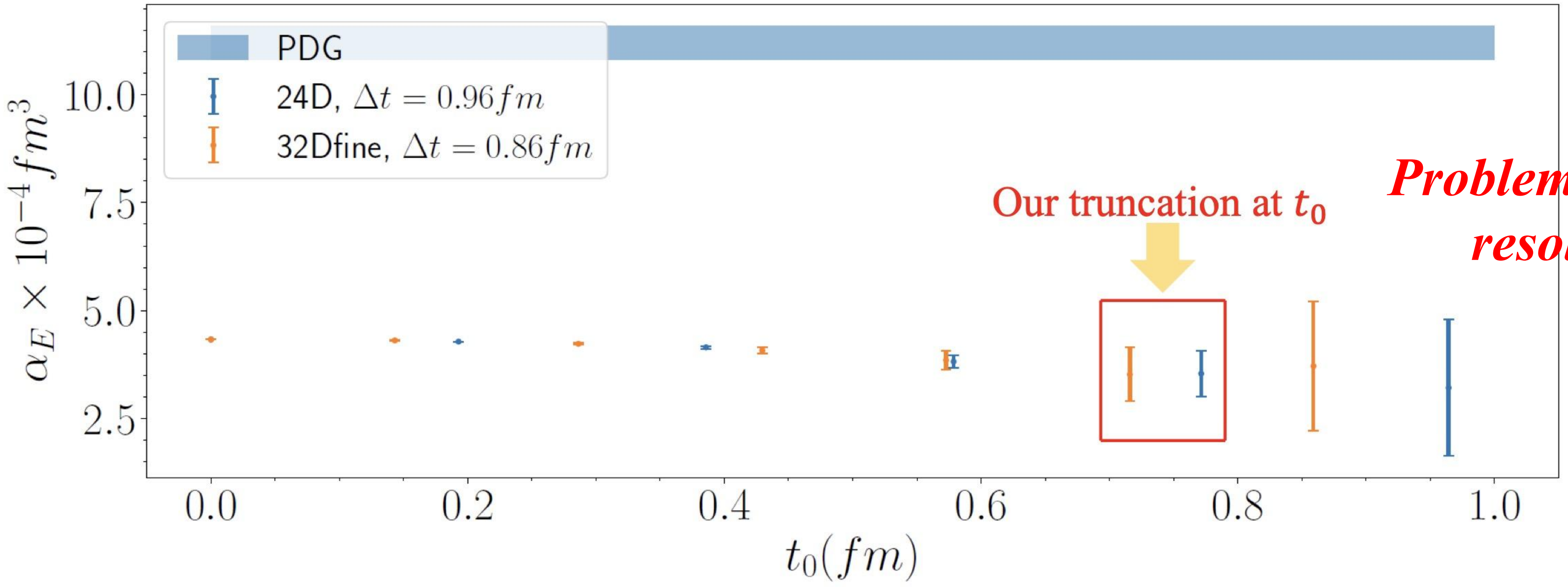
Analytical known residue coefficient

Lattice methodology for nucleon polarisability

- Numerical results:

Two Domain Wall Fermion ensembles:

Ensembles	m_π[MeV]	L/a	T/a	a[fm]	N_{conf}
24D	142.6(3)	24	64	0.1929	207
32Dfine	143.6(9)	32	64	0.1432	69

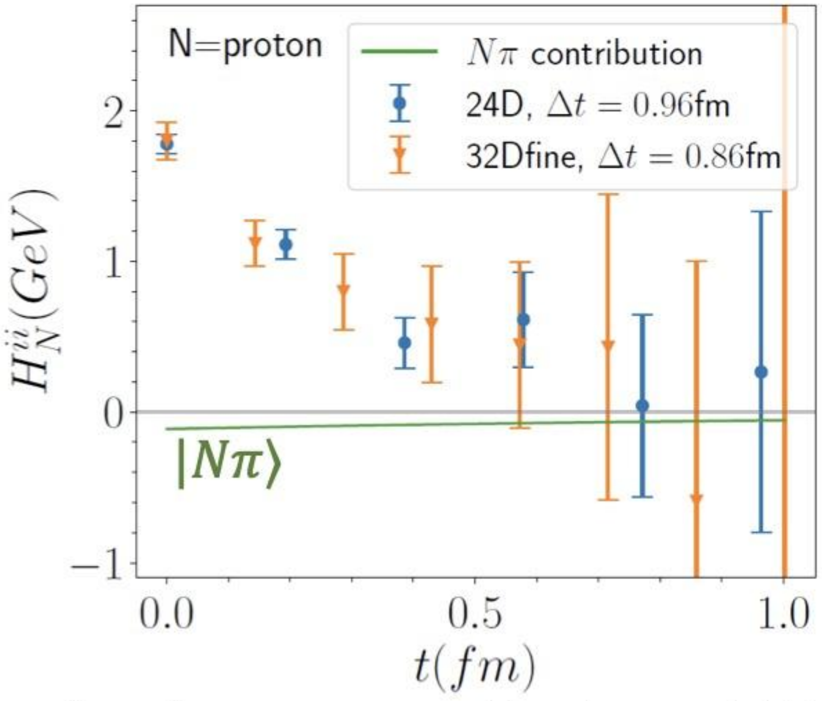


Problem still not resolved!

Lattice methodology for nucleon polarisability

- Question: does the truncation REALLY saturate all the contribution?
- Answer: Not really, but not obvious.

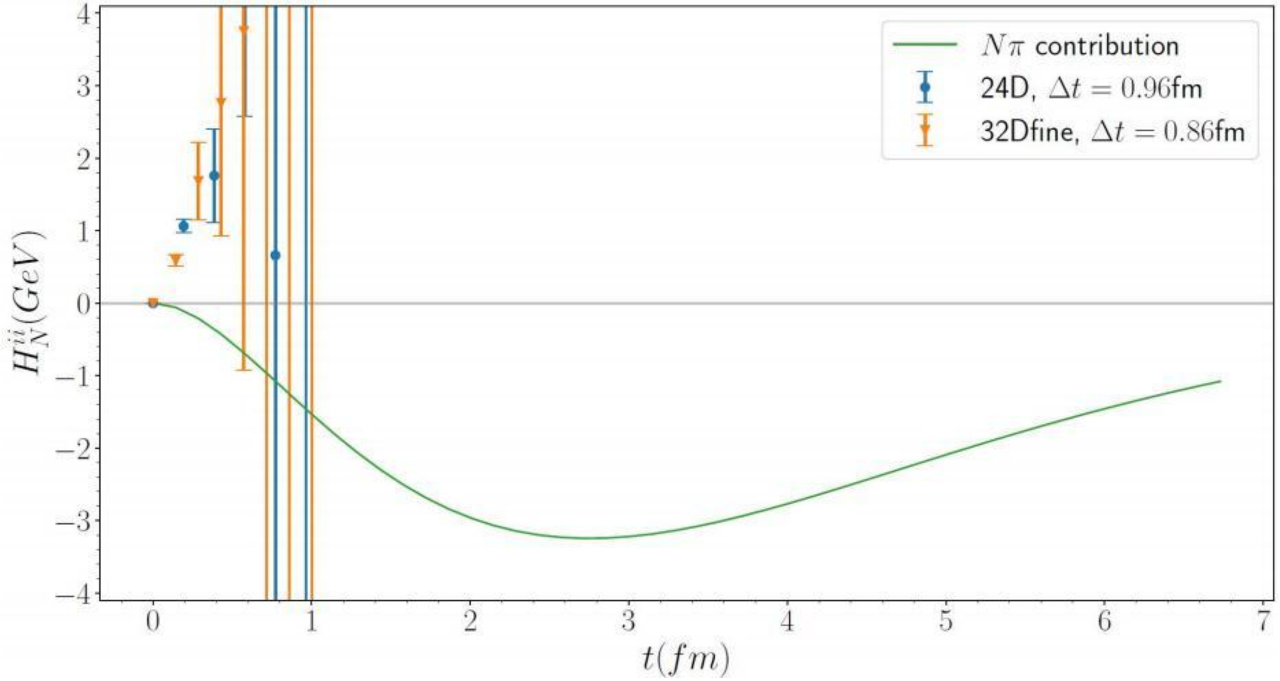
Original Compton tensor
$N\pi$ negligible



t^2

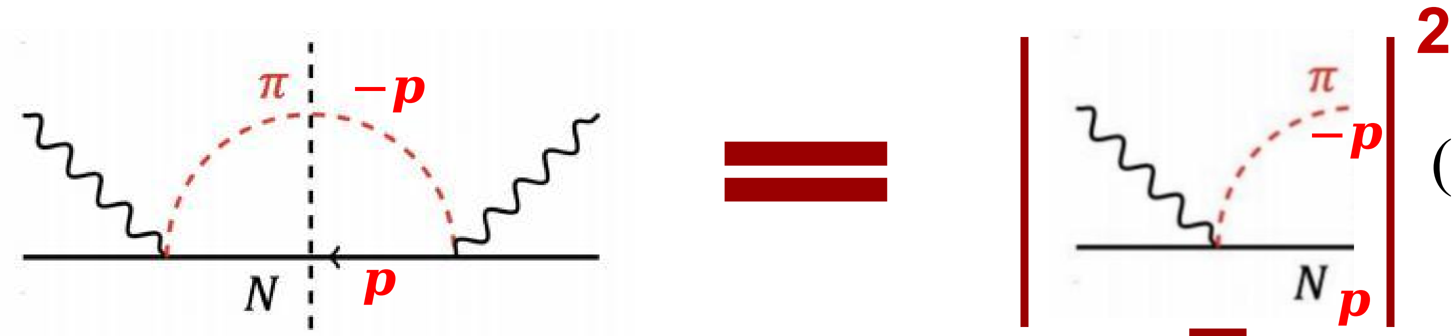
Weight function

t^2 weighted Compton tensor
long $N\pi$ tail



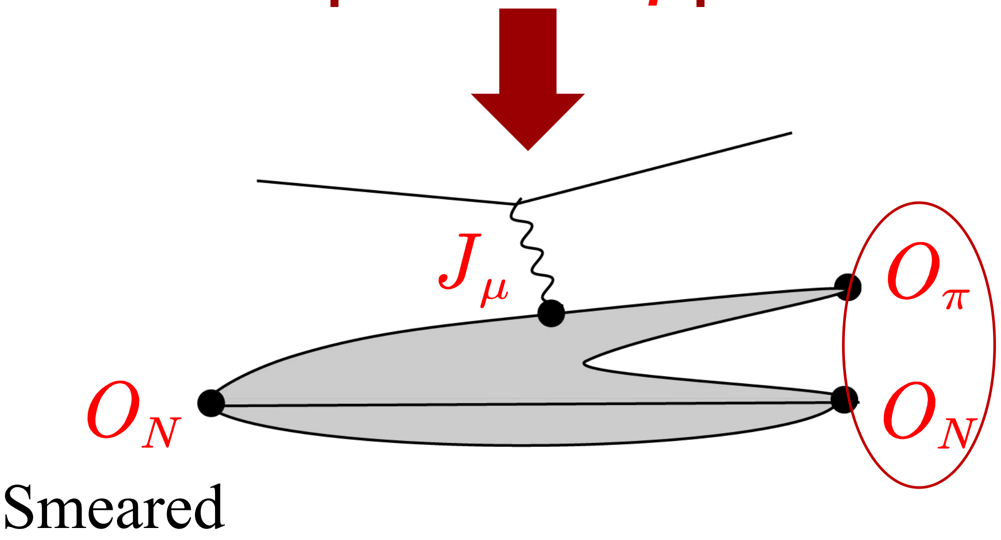
Lattice methodology for nucleon polarisability

- Need to evaluate $N\pi$ contribution on lattice.



$(\ |\langle N|J_\mu|N(\vec{p})\pi(-\vec{p})\rangle|^2 \ )$

Computation strategy for $\langle N|J_\mu|N(\vec{p})\pi(-\vec{p})\rangle$:

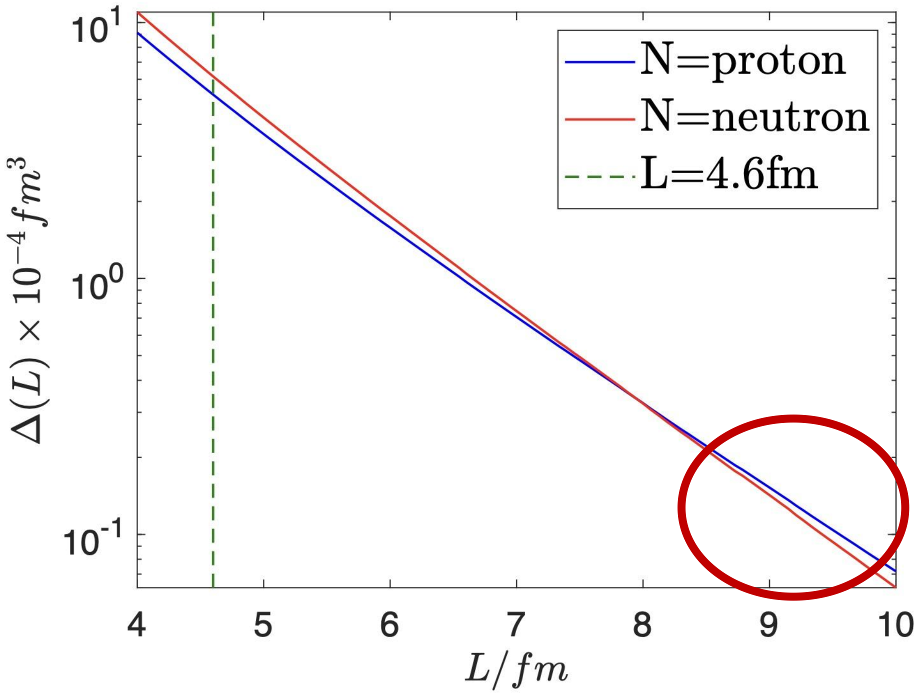


Smeared multi-hadrons operators

Leading 27 momentum modes $|\vec{p}|^2/(2\pi/L)^2 = 0, 1, 2, 3$ are included

Lattice methodology for nucleon polarisability

- Why do we need varies momentum modes?
- Control large finite volume effects due to long-distance pion-induced inteactions.



L=9~10 fm for percent level finite volume effects when we **direct summed** over $N\pi$ modes

Suppressed finite volume effects

Our choice: **isolate the singular pion pole behaviour** to acheive a smooth fit over momentum modes

$$\frac{\sum_s a_s (p^2)^s}{(2E)(2E_\pi)(E+E_\pi-M)^2}$$

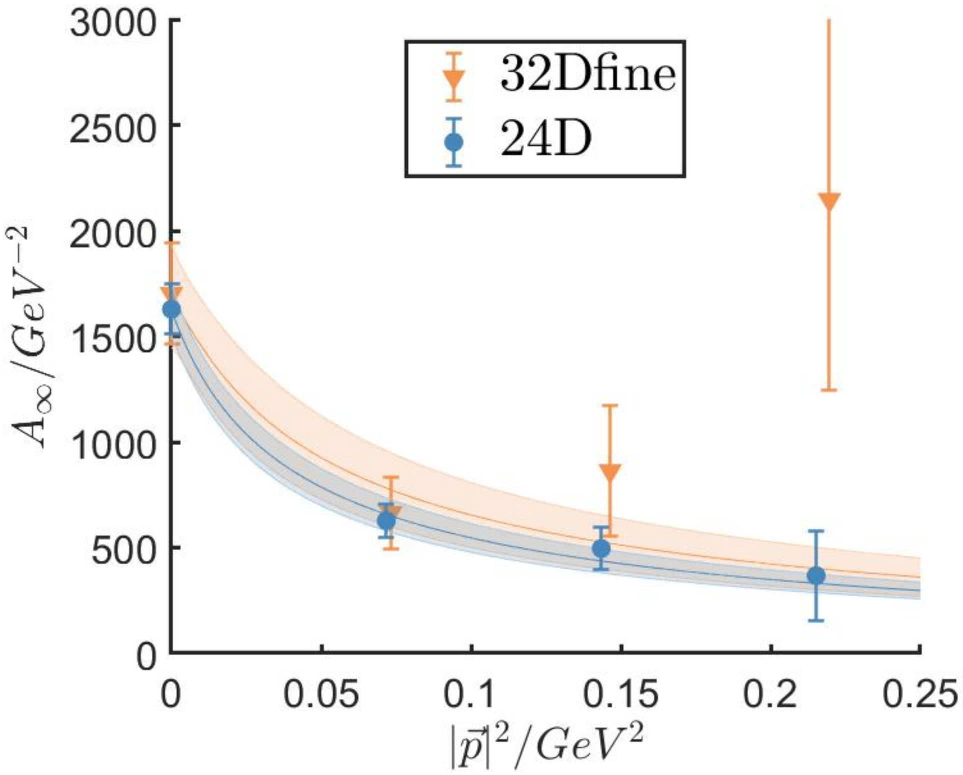
What we fit to

pion pole

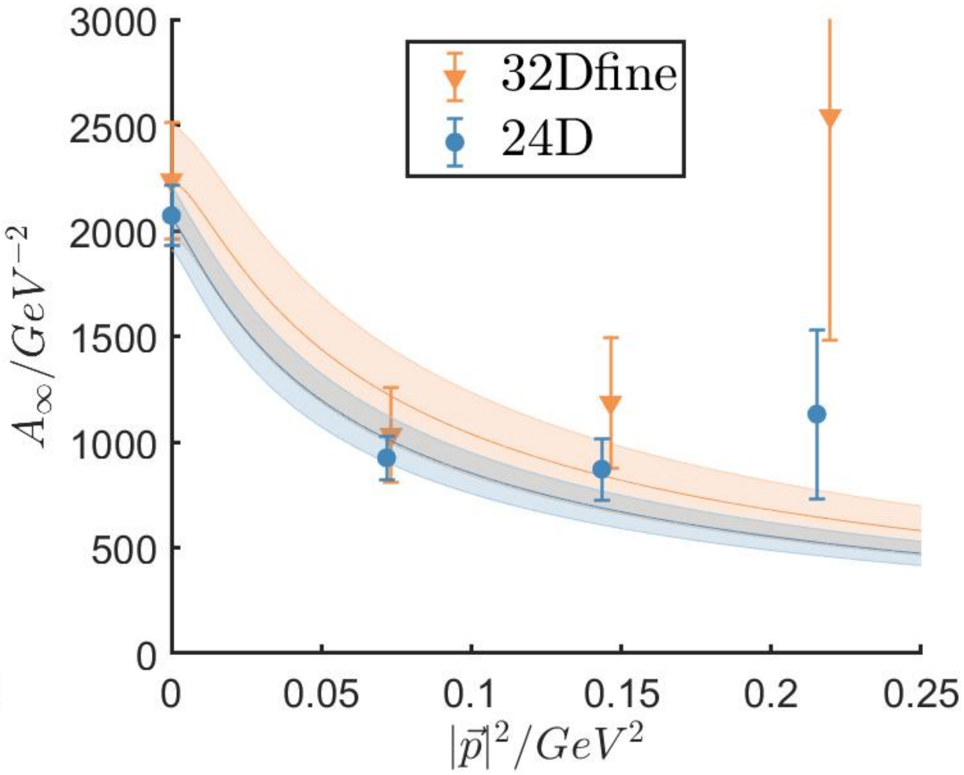
Lattice methodology for nucleon polarisability

- Numerical result of $N\pi$ contribution given by $\sum_i |\langle N|J_i|N(\vec{p})\pi(-\vec{p})\rangle|^2$

proton case



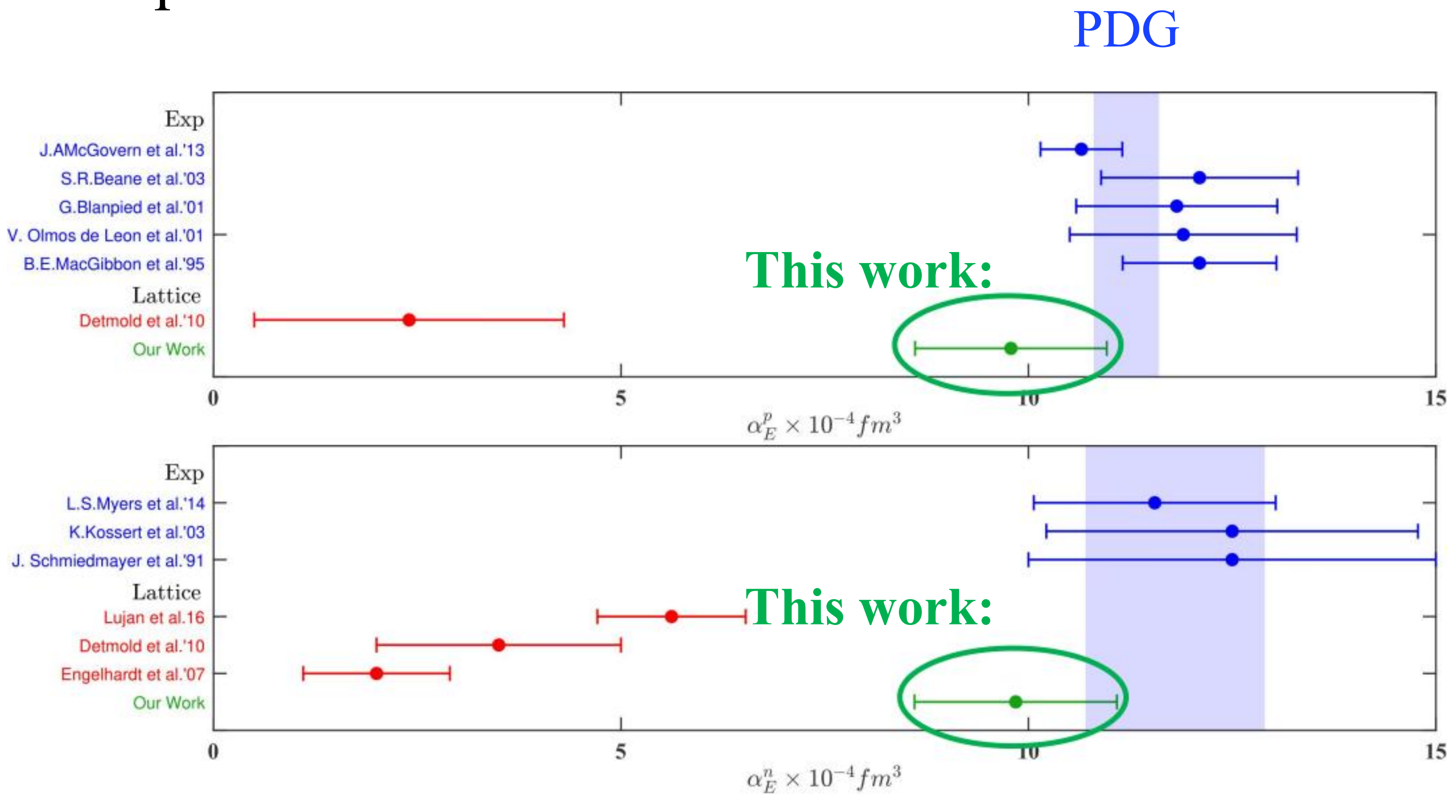
neutron case



Lattice methodology for nucleon polarisability

- Numerical result, and contribution decomposition.

		24D	32Dfine	PDG
$N\pi$:	$\alpha_E^{ii,N\pi}$	5.65(53)	6.5(1.2)	-
Other states:	$\alpha_E^{ii,es}$	−1.42(53)	−1.58(61)	-
proton	$\alpha_E^{ii,disc}$	1.4(1.2)	0.1(1.8)	-
	α_E^{r}	4.333(3)	4.333(3)	-
	α_E	10.0(1.3)	9.3(2.2)	11.2(4)
$N\pi$:	$\alpha_E^{ii,N\pi}$	8.33(75)	9.8(1.5)	-
Other states:	$\alpha_E^{ii,es}$	−0.60(40)	−0.40(47)	-
neutron	$\alpha_E^{ii,disc}$	1.4(1.2)	0.1(1.8)	-
	α_E^{r}	0.618(1)	0.618(1)	-
	α_E	9.7(1.4)	10.1(2.4)	11.8(1.1)



$N\pi$ contribution:
proton: **60% in total** α_E
neutron: **90% in total** α_E

Lattice consistent with PDG for the first time!

Step forward to pion threshold production

- Future ν oscillation experiment need high precision electro(weak) production data.

- DUNE designed uncertainty, 1) signal: 1%, 2) background: 5%

R. Acciarri *et al.* (DUNE), (2015)

$$\nu + N \to \nu/\mu/e + X$$
$$(X = N\pi, \Delta(1232), N\pi\pi)$$

Background	Normalization Uncertainty	Correlations
For $\nu_e/\bar{\nu}_e$ appearance:		
Beam ν_e	5%	Uncorrelated in ν_e and $\bar{\nu}_e$ samples
NC	5%	Correlated in ν_e and $\bar{\nu}_e$ samples
ν_μ CC	5%	Correlated to NC
ν_τ CC	20%	Correlated in ν_e and $\bar{\nu}_e$ samples
For $\nu_\mu/\bar{\nu}_\mu$ disappearance:		
NC	5%	Uncorrelated to $\nu_e/\bar{\nu}_e$ NC background
ν_τ	20%	Correlated to $\nu_e/\bar{\nu}_e$ ν_τ background

- **Reducing these background error need precise theoretical input!**

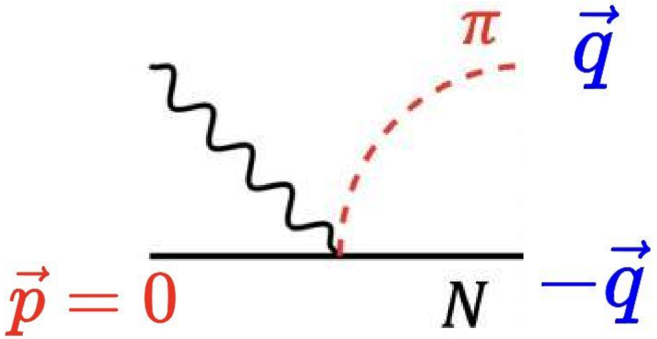
Step forward to pion threshold production

- From lattice point of view: pion threshold production serves as the **simplest starting point** for the exploratory calculation of those electro(weak)-production process.

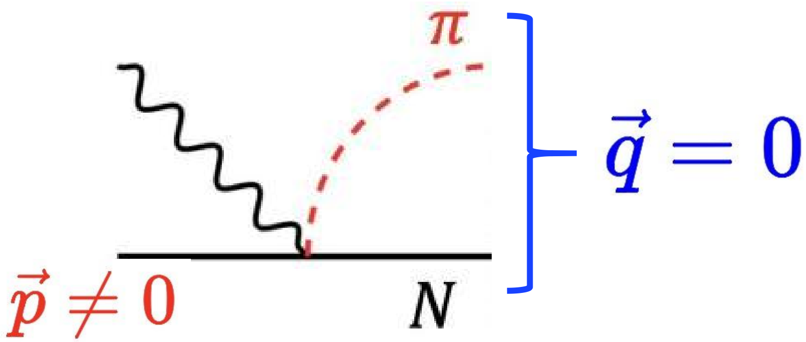
$$\gamma^*/W^*/Z^* + N \to N\pi$$

- Naturally extention from analysis of $N\pi$ contribution in polarisability:

$N\pi$ contribution for Compton tensor



pion threshold production



Key difference:

- **Static nucleon** $\xrightarrow{\text{More excited states}}$ **Moving nucleon**
- **$N\pi$ in cubic irreps** $\xrightarrow{\text{Finite volume errors}}$ **Scattering $N\pi$ state**

Lattice methodology for pion threshold production

- The experiment measured quantity are multipole amplitudes:

$$\mathcal{J}_\mu^{em} = \langle N\pi | \underline{J_\mu^{em}} (0) | N \rangle, \quad \mathcal{J}_\mu^{W(Z),A} = \langle N\pi | \underline{J_\mu^{W(Z),A}} (0) | N \rangle$$

Electromagnetic current

Weak current

Extracting S-wave multipole amplitudes

$$L_{0+}, \quad E_{0+}$$

(reduce one D.O.F. due to ward identity)

$$L_{0+}^W, \quad H_{0+}, \quad M_{0+}$$

$$[\mathcal{J}_0^{em}]_{s',s} = \alpha_m \frac{|\vec{k}|}{k_0} \xi_{s'}^\dagger (\vec{\sigma} \cdot \hat{k}) \xi_s \, L_{0+}$$

$$[\vec{\mathcal{J}}^{em}]_{s',s} = \alpha_m \xi_{s'}^\dagger \left\{ L_{0+} \hat{k} (\vec{\sigma} \cdot \hat{k}) + E_{0+} \left[\vec{\sigma} - \hat{k} (\vec{\sigma} \cdot \hat{k}) \right] \right\} \xi_s$$

$$[\mathcal{J}_0^{W,A}]_{s',s} = \alpha_m \xi_{s'}^\dagger \xi_s \left(L_{0+}^{(W)} + \frac{k_0}{m} H_{0+} \right)$$

$$[\vec{\mathcal{J}}^{W,A}]_{s',s} = \alpha_m \xi_{s'}^\dagger \left(\frac{\vec{k}}{m} H_{0+} - i (\vec{\sigma} \times \hat{k}) M_{0+} \right) \xi_s$$

Lattice methodology for pion threshold production

- **Analyzing excited states' contamination effects:**

- Consider 4 excited states of static $N\pi$, and 3 excited states of moving nucleon:

$$O_{N(0)\pi(0)} \longrightarrow |N(p)\pi(-p)\rangle,\ p \in \left\{0,\ \frac{2\pi}{L},\ \sqrt{2}\frac{2\pi}{L},\ \sqrt{3}\frac{2\pi}{L}\right\}$$

$$O_{N(p)} \longrightarrow |N(p)\rangle,\ |N(p)\pi(0)\rangle,\ |N(0)\pi(p)\rangle$$

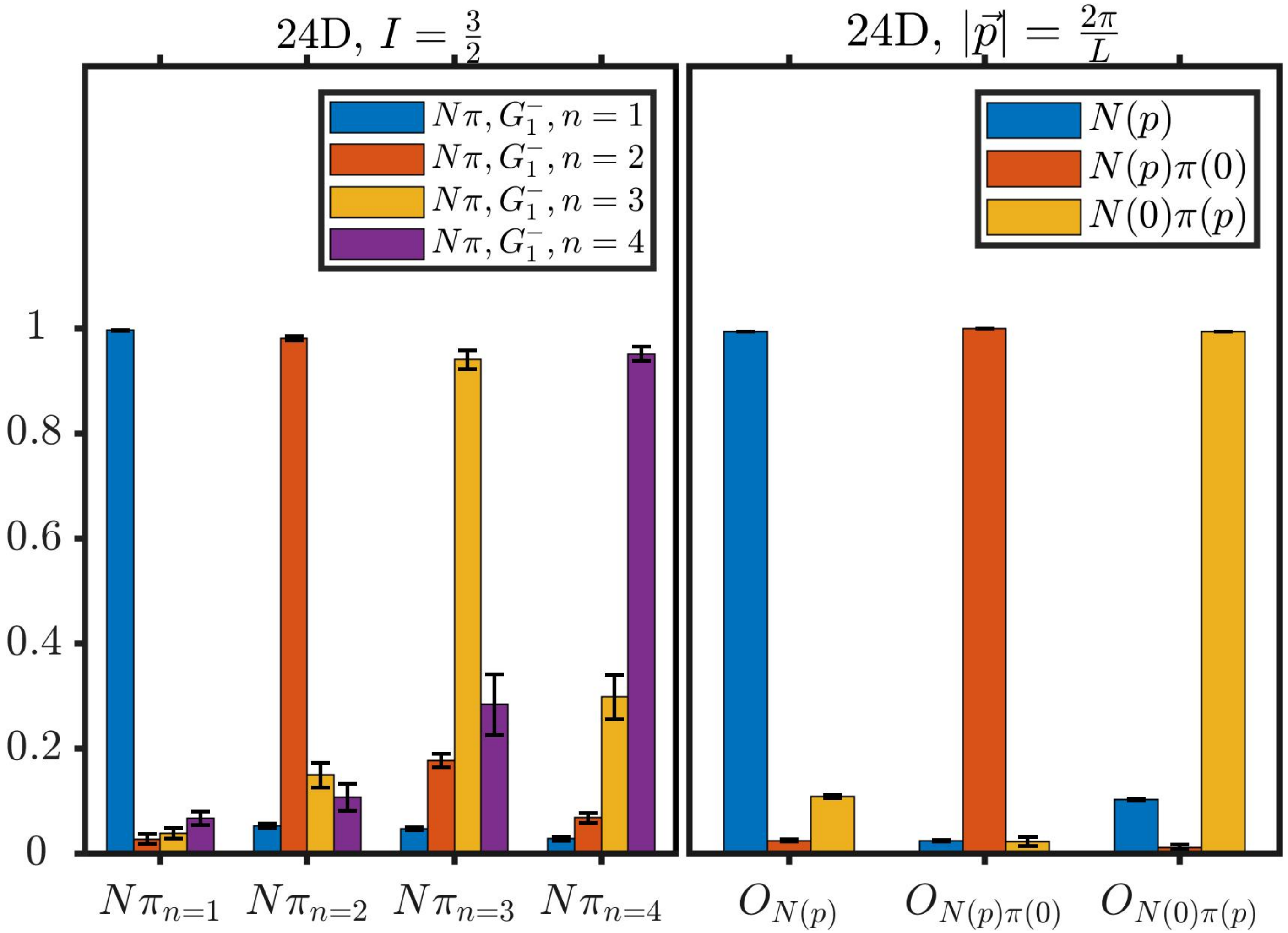
- Calculate the correlation function matrix, and employ standard GEVP method.

$$C_{ij} = \left\langle O_i(t)\,\overline{O}_j(0)\right\rangle,\ i,j \in \left\{N(p)\pi(-p)|\ p = 0,\ \frac{2\pi}{L},\ \sqrt{2}\frac{2\pi}{L},\ \sqrt{3}\frac{2\pi}{L}\right\}$$

$$C_{ij} = \left\langle O_i(t)\,\overline{O}_j(0)\right\rangle,\ i,j \in \{N(p),\ N(p)\pi(0),\ N(0)\pi(p)\}$$

Lattice methodology for pion threshold production

- Operator mixing to different excited states:

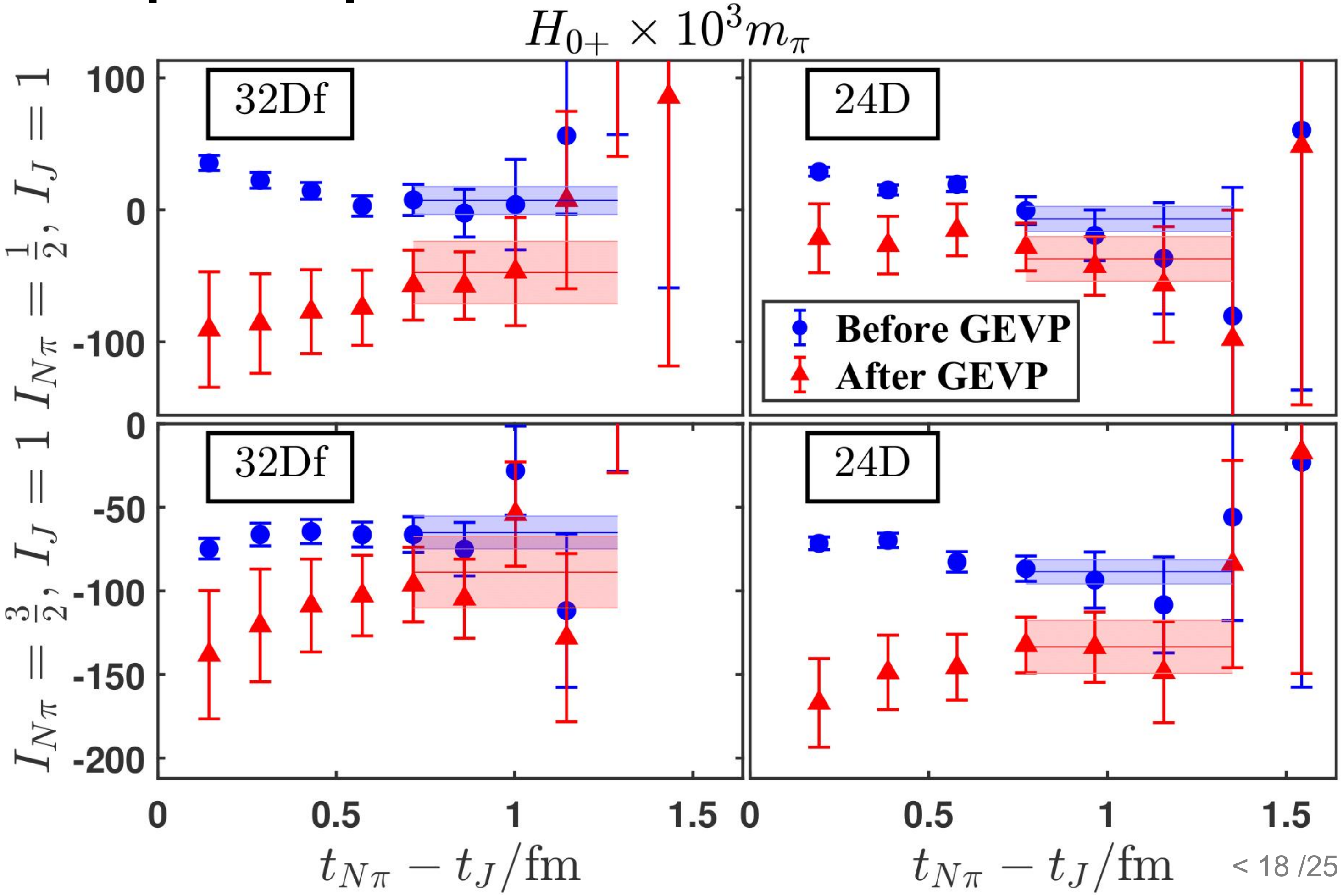


Lattice methodology for pion threshold production

- Though the operator mixing is basically under 10% level, it would **cause significant deviation to the extracted multipole amplitudes.**

- Removing excited states of $N(0)\pi(0)$

- **the results shift $1\sim3\sigma$**



Lattice methodology for pion threshold production

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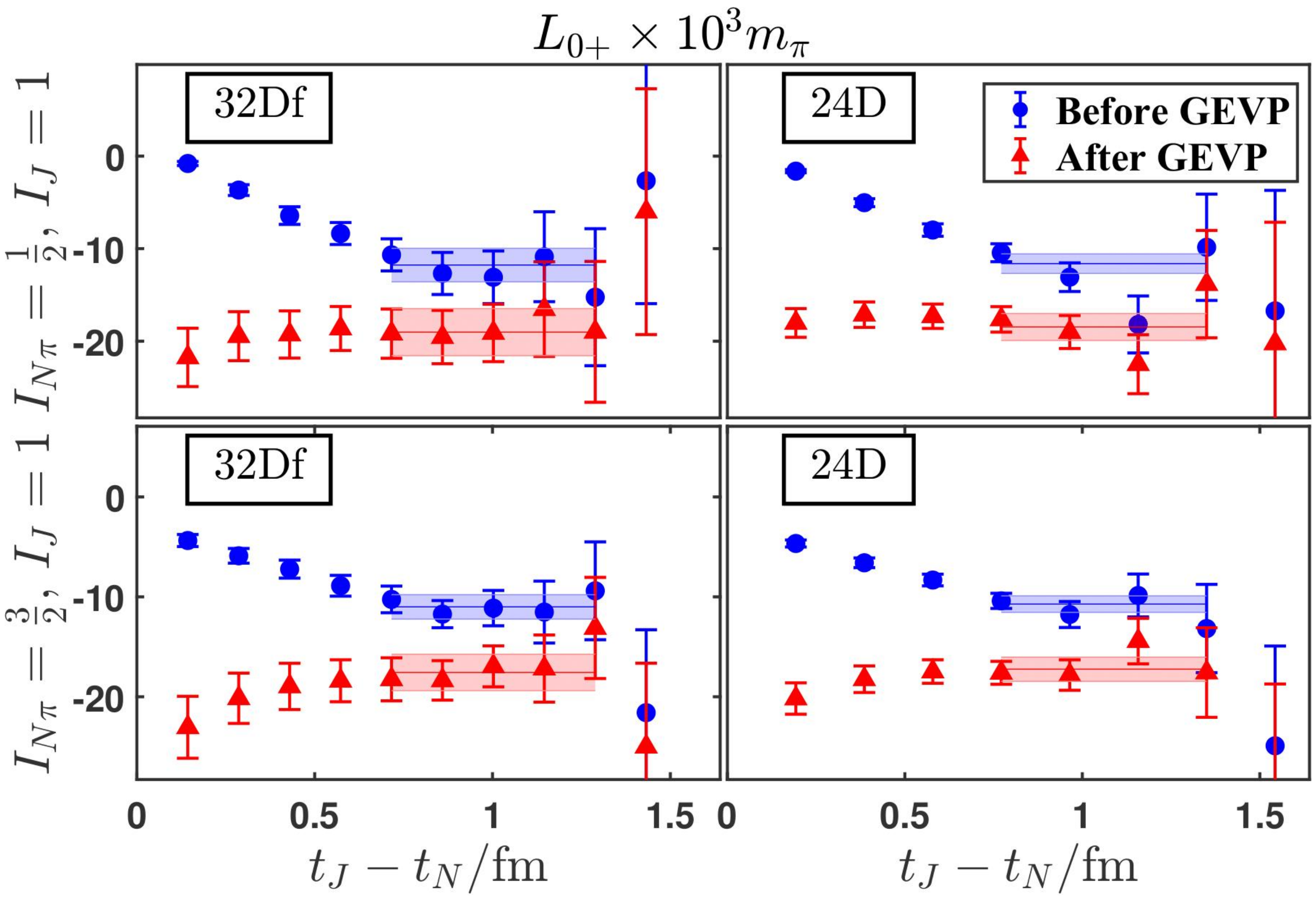
- Removing excited states of $N(p)$
- **the results shift $3 \sim 6\sigma$**



- **Lesson: One should be especially careful for the excited states' effect for a moving nucleon**

Lattice methodology for pion threshold production

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- **Analyzing two-particles state's finite volume error**

- Luscher formalism: the infinite volume $N\pi$ scattering state could be related to finite volume bound state by LL-factor

L. Lellouch and M. Lüscher, **Commun. Math. Phys**, (2001)

$$|N\pi\rangle_{\infty} = \left(\frac{dn}{dE}\right)^{1/2} |N\pi\rangle_{V}$$

$$\frac{dn}{dE} = \frac{1}{\pi}\frac{d(\delta_1 + \phi^{\Gamma}_{\mathbf{P}})}{dE} = \frac{E}{4\pi k^2}\left(\underbrace{k\frac{\partial \delta_1}{\partial k}}_{\text{Derivative of phase shift}} + \underbrace{q\frac{\partial \phi^{\Gamma}_{\mathbf{P}}}{\partial q}}_{\text{Analytical known zeta function}}\right)$$

Derivative of phase shift

Analytical known zeta function

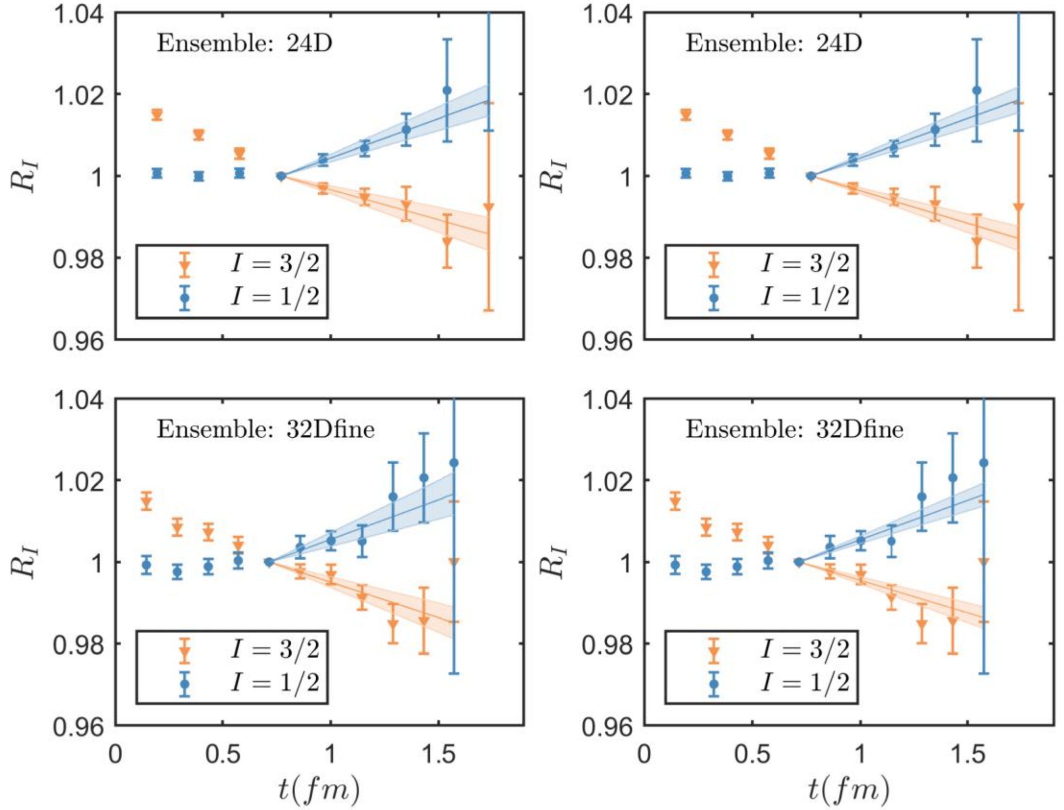
Need scattering length as input!

Lattice methodology for pion threshold production

M. Lüscher, **Commun. Math. Phys.** (1986)

- Calculate the S-wave scattering length by Lüsher method, at $\vec{p}=0$.

$$R^I(t)=\frac{C^I_{N\pi}(t_f,t_i)}{C_N(t_f,t_i)C_\pi(t_f,t_i)}\approx A_I(1-\delta E_I\,t)\longrightarrow \delta E_I \xrightarrow{\text{Lüsher method}} a_0^I$$



	Ensemble	24D	32Dfine	Combined fit
$I=\frac{1}{2}$	χ^2/dof	0.26	0.47	0.34
	δE_I [MeV]	−3.78(79)	−3.9(1.2)	−3.80(66)
	$M_\pi a_0^I$	0.157(37)	0.157(56)	0.157(31)
$I=\frac{3}{2}$	χ^2/dof	0.26	0.51	0.38
	δE_I [MeV]	2.91(80)	3.40(90)	3.13(60)
	$M_\pi a_0^I$	−0.098(25)	−0.111(27)	−0.104(18)

Only one previous LQCD at physical point: $M_\pi a^{3/2}=-0.13(4)$

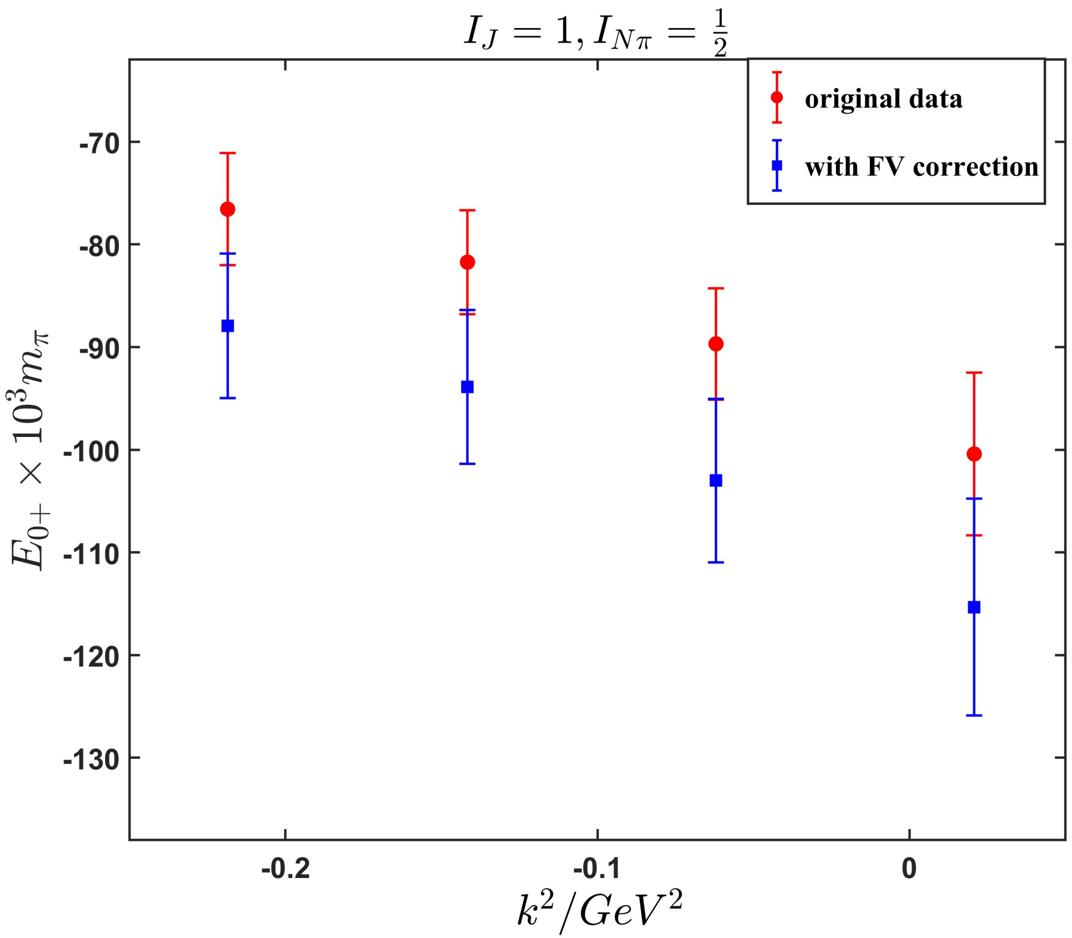
Data driven analysis: $M_\pi a^{3/2}=-0.087(2)$, $M_\pi a^{1/2}=0.170(2)$

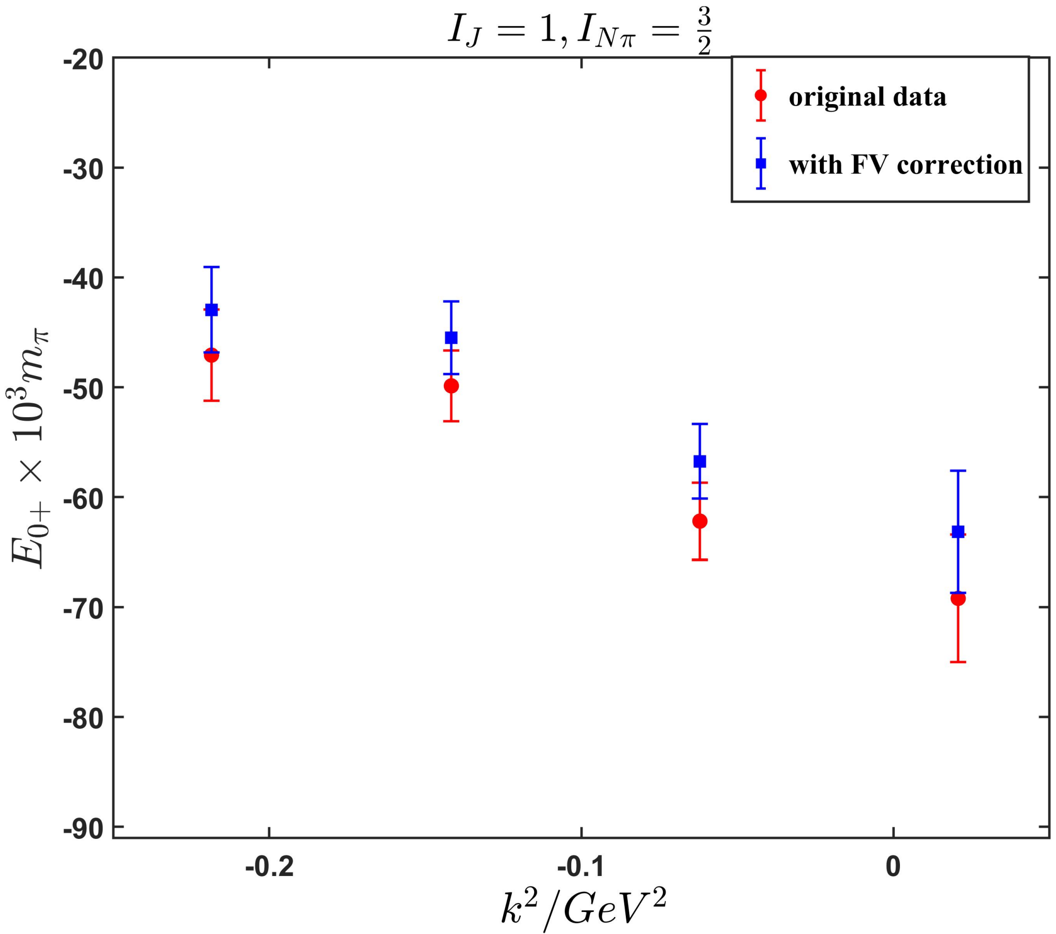
M. Hoferichter et al, **PLB** (2023)

C. Alexandrou et al, **PRD** (2024)

Lattice methodology for pion threshold production

- The influence of finite volume correction is around 10%
- Shift the central value about $1 \sim 2\sigma$





Lattice methodology for pion threshold production

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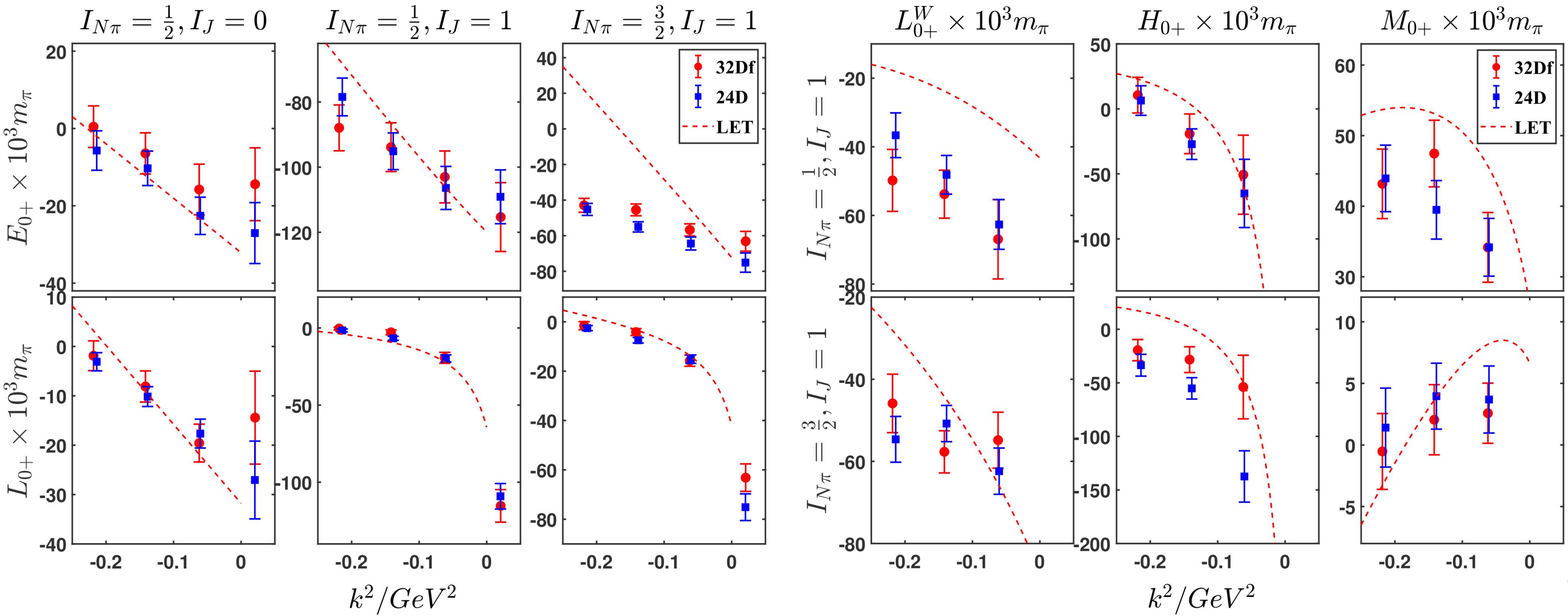
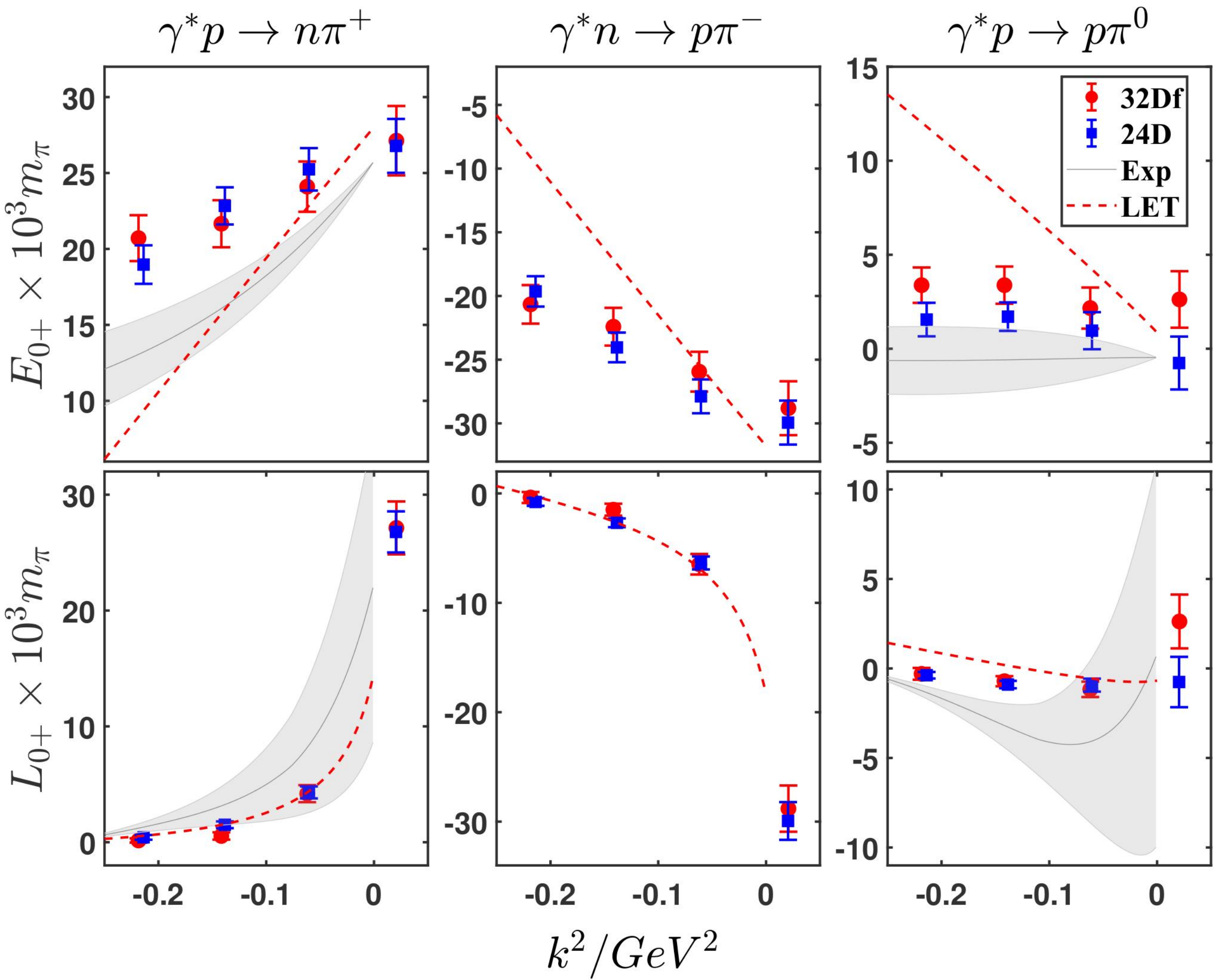
- we present the **first LQCD calculation** of pion threshold production process at physical pion mass. V.Bernard,N.Kaiser,T.S.H.Lee, andU.-G.Meißner, **Phys.Rept.** (1994)
- isospin channel, comparison to low energy theorem.

Vector multipoles **Axial multipoles**



Lattice methodology for pion threshold production

- Physical Channel in comparison with experiments and low energy theorem.



V.Bernard,N.Kaiser,T.S.H.Lee, andU.-G.Meißner **Phys.Rept.** (1994)

M. Mai, J. Hergenrather, M. Döring, T. Mart, U. G. Meißner, D. Rönchen, and R. Workman, **EPJA**, (2023)

Conclusion and future prospect

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- We present a calculation of nucleon electric polarisabilty:
  - at physical pion mass in the first time.
  - after careful analyzing long-distance $N\pi$ effects, the defliction of polarisability in LQCD is resolved.
- We further extend the study into pion threshold production, as the first step toward calculating neutrino-induced hadron production process

Remain to be explored:

- Extend the formalism to generalized polarisabilities, spin polarisabilities, color polarisabilities
- Electro(weak) production of $\Delta(1232)$ resonance, $N\pi\pi$ state ?
- ……

Thank you for listening!