Exotic hadrons with heavy quarks:

A ~~short~~ review and the interplay with Lattice QCD

personally biased selected

Meng-Lin Du (杜孟林)

@

University of Electronic Science and Technology of China (UESTC)

电子科技大学

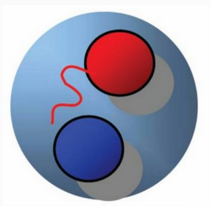
第五届中国格点量子色动力学研讨会 @ 惠州 2025.10.9-12

Outline

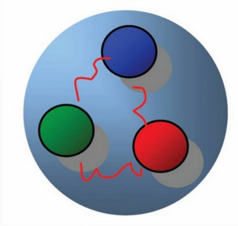
- Charmed mesons [$D_0^*(2300)$, $D_{s0}(2317)$]
- Hidden-charm tetraquarks [$Z_c(3900)$]
- Hidden-charm pentaquarks [P_c]
- Doubly-charmed tetraquarks [$T_{cc}(3875)$]
- Summary an Propects

Conventional hadrons vs. Exotic hadrons

Conv. Hadrons



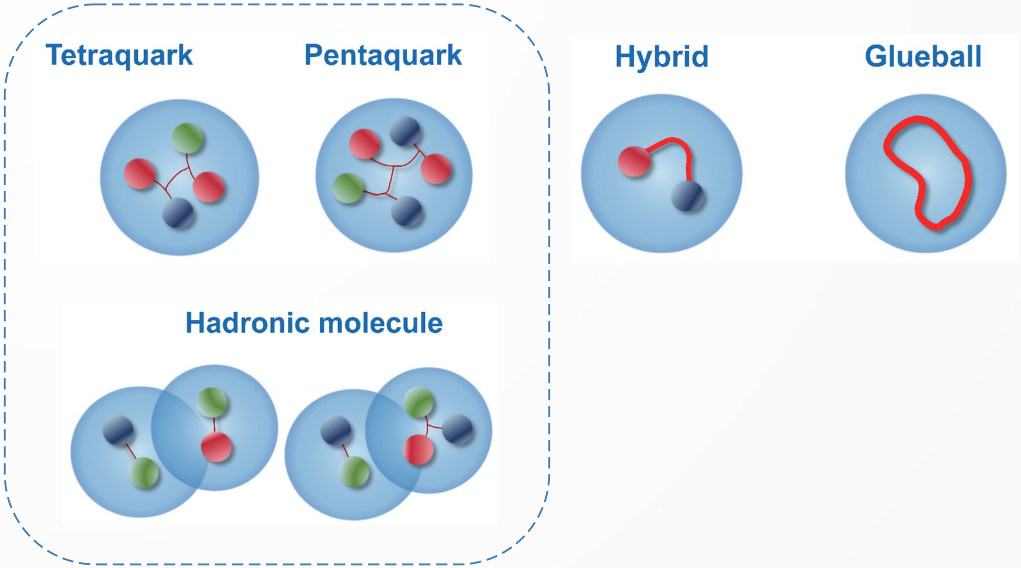
meson



baryon

Hundreds of hadrons can be described as $q\bar{q}$ and qqq states

Exotic Hadrons



Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

...

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while

21 February 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING II *)

G. Zweig
CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

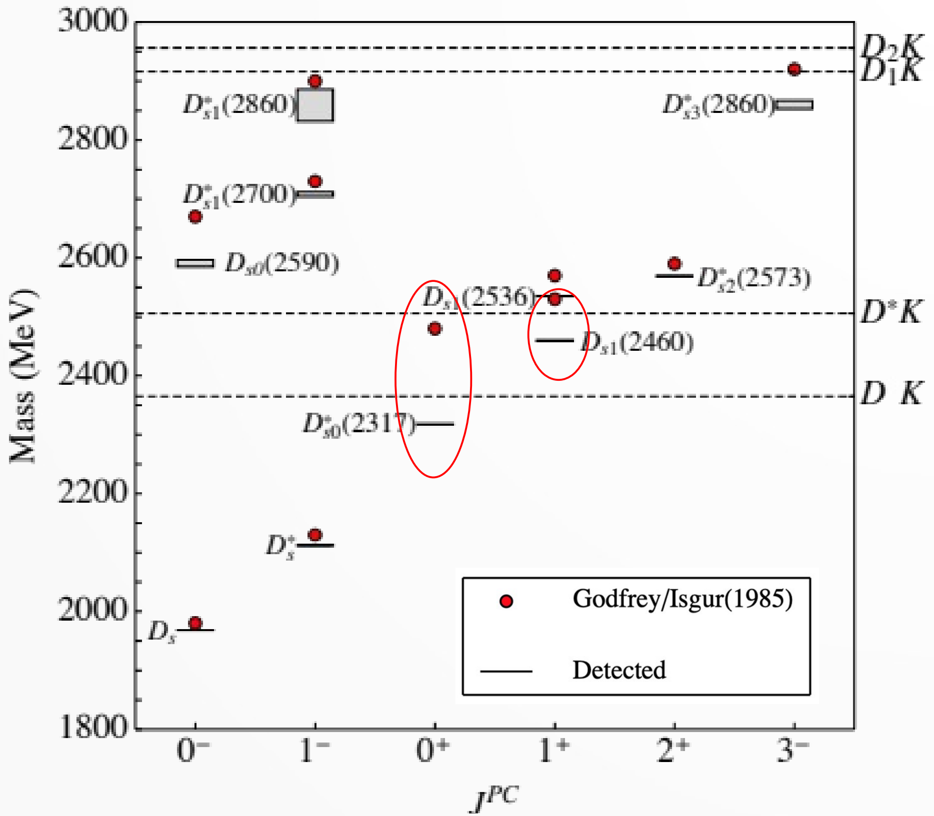
...

6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA, that is, "deuces and treys".

The multiquark states were predicted at the birth of Quark Model.

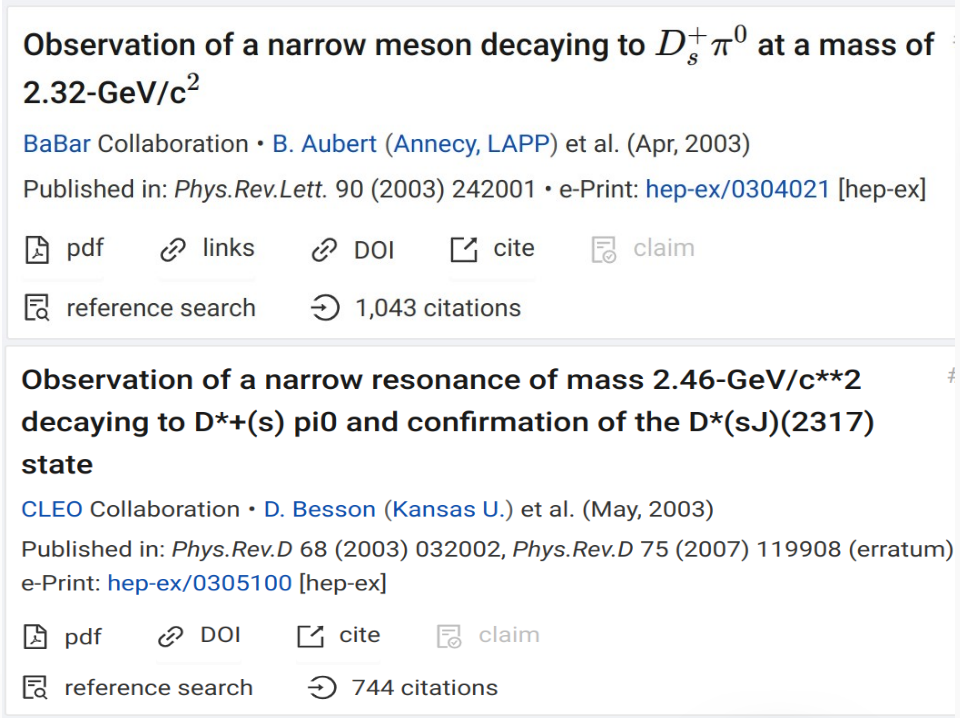
The experimental and theoretical study of such states is a key to understand QCD.

Ds mesons



The relativized quark model:
Godfrey, Isgur, Phys. Rev. D 32, 189(1985)

Figure taken from Zhi Yang.



Observation of a narrow meson decaying to $D_s^+\pi^0$ at a mass of 2.32-GeV/c^2

BaBar Collaboration • B. Aubert (Annecy, LAPP) et al. (Apr, 2003)

Published in: *Phys.Rev.Lett.* 90 (2003) 242001 • e-Print: hep-ex/0304021 [hep-ex]

pdf | links | DOI | cite | claim

reference search | 1,043 citations

Observation of a narrow resonance of mass 2.46-GeV/c2 decaying to D*+(s) pi0 and confirmation of the D*(sJ)(2317) state**

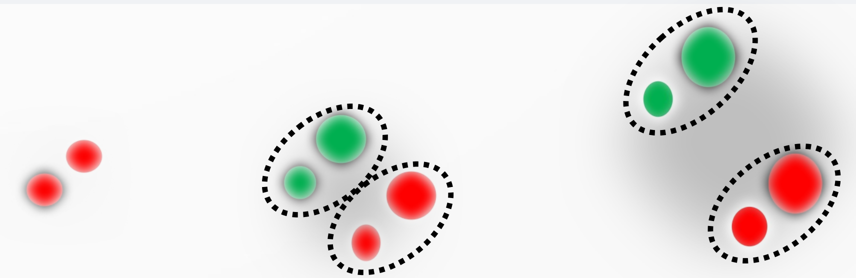
CLEO Collaboration • D. Besson (Kansas U.) et al. (May, 2003)

Published in: *Phys.Rev.D* 68 (2003) 032002, *Phys.Rev.D* 75 (2007) 119908 (erratum)

e-Print: hep-ex/0305100 [hep-ex]

pdf | DOI | cite | claim

reference search | 744 citations

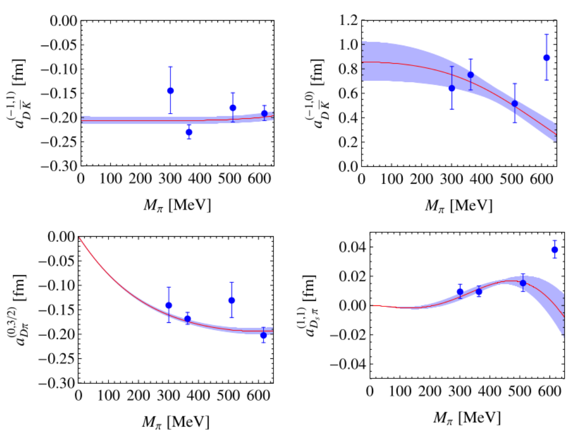


Lattice studies on the charmed strange meson

- Early studies using only $c\bar{s}$-type interpolators
  $\hookrightarrow$ give mass significantly larger than $D^*_{s0}(2317)$ Bali (2003); UKQCD (2003); ...
- $c\bar{s} + DK$ interpolators: right mass $M_\pi \approx 156$ MeV Mohler *et al.*, PRL111(2013)222001

- New calculation: $M_\pi = 150$ MeV Bali et al. [RQCD Col.], PRD96(2017)074501

m_{0+}	$2348(4)(+6)$	$2317.7(0.6)(2.0)$
m_{1+}	$2451(4)(+1)$	$2459.5(0.6)(2.0)$



Liu, Orginos, Guo, Hanhart, UGM (2013)

ChPT + unitiarization:

$(S, I) = (1, 0)$ channel: 2315^{+18}_{-28} MeV

$$M_{D^\star_{s0}(2317)} = (2317.7 \pm 0.6)\ \text{MeV}$$

Ds mesons: Exp. + EFT + Lattice

- Hadronic molecular model: $D_{s0}^*(2317)[DK]$, $D_{s1}(2460)[D^*K]$

  Barnes, Close, Lipkin(2003); van Beveren, Rupp(2003); Kolomeitsev, Lutz(2004); …

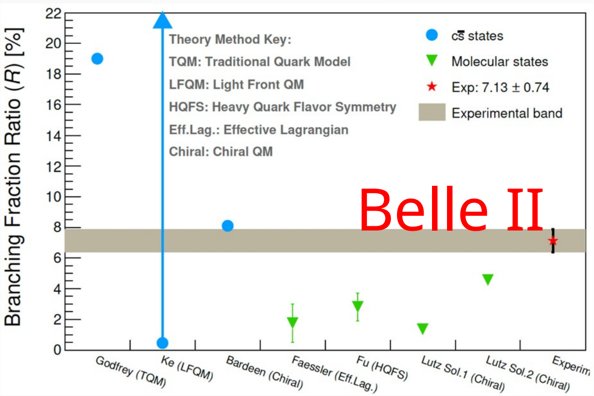
- Weinberg compositeness $X = 1 - Z$

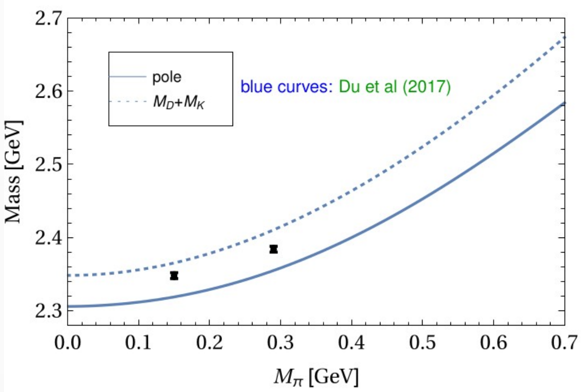
  $\hookrightarrow$ From the lattice energy levels in C. Lang et al., PRD90(2014)034510

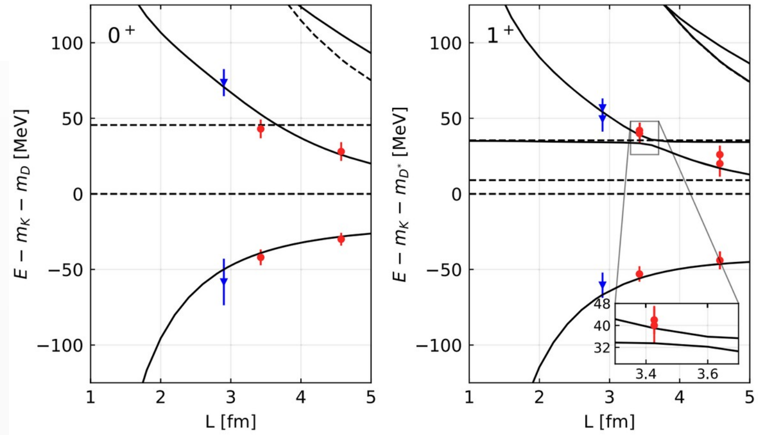
  $D_{s0}^*(2317)$ contains $\sim 70\%$ DK Martínez Torres, Oset, Prelovsek, Ramos, JHEP1505,053

  $\hookrightarrow$ $1 - Z = 1.04(0.08)(0.30)$, G. Bali et al., PRD96(2017)074501

  $\hookrightarrow$ Bare $c\bar{s}$+DK: 68%, Z. Yang al., PRL128(2022)112001

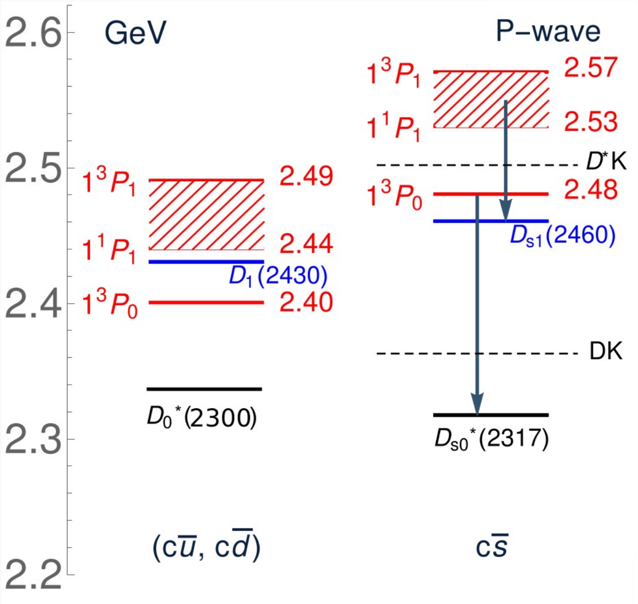






bare+DK Yang PRL128(2022)112001

Positive parity charmed meson [D₀*(2300)]



Godfrey and Isgur, PRD **32**, 189 (1985)

BaBar (2003), CLEO (2003); Belle (2004)

SU(3) partners $D_0^*(2300),\ D_1(2430)$
$D_{s0}^*(2317),\ D_{s1}(2460)$

$$M_{D_0^*(2300)} \gtrsim M_{D_{s0}^*(2317)}$$

$(S, I) = (0, 1/2)$:

- $c\bar{q} + D\pi$ interpolator   Mohler *et al.*, PRD87(2013)034501

$M_\pi \approx 266$ MeV, $M_D \approx 1558$ MeV, $M_{D^*} \approx 1690$ MeV

	Mohler et al.	PDG
$M_{D_0^*} - \frac{1}{4}(M_D + 3M_{D^*})$	(351 ± 21) MeV	369 MeV
$M_{D_1} - \frac{1}{4}(M_D + 3M_{D^*})$	(380 ± 21) MeV	439 MeV

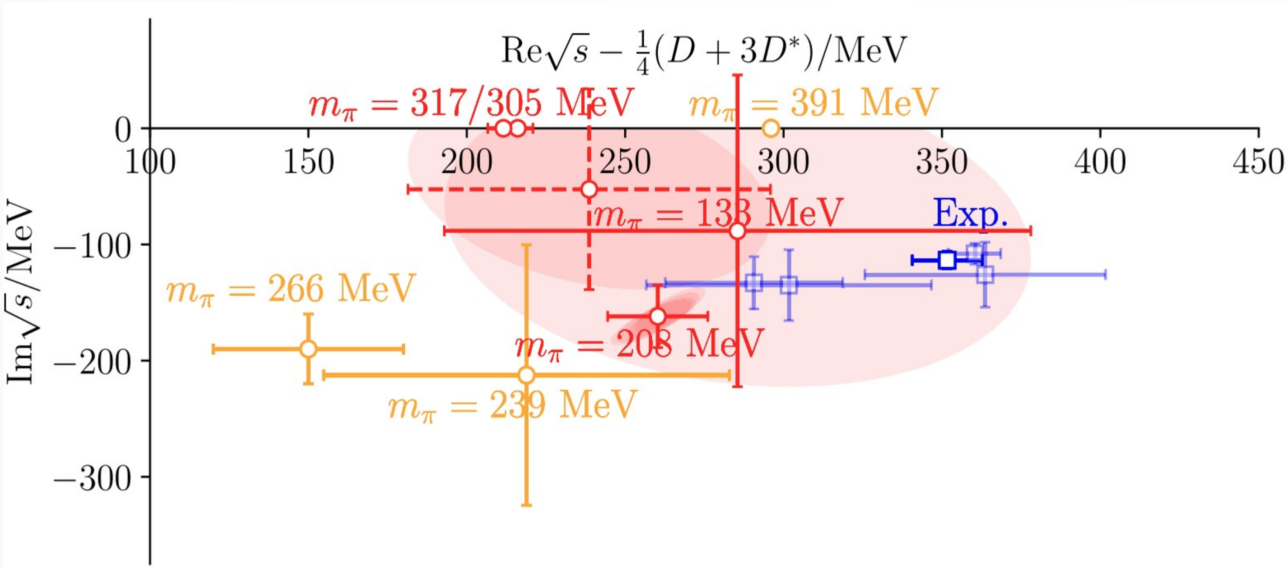
- Coupled-channel: Moir *et al.* [Hadron Spectrum Col.], JHEP1610(2016)011
  $\hookrightarrow c\bar{q} + D\pi + D\eta + D_s\bar{K}$
  $\hookrightarrow M_\pi \approx 391$ MeV, $M_D \approx 1885$ MeV: $D\pi$ threshold (2276.4 ± 0.9) MeV
  K-matrix: a pole below threshold is found: 2275.9$\pm$0.9 MeV ? $D_0^*(2300)$

- $m_\pi \approx 239$ MeV Gayer et al. [Hadron Spectrum], JHEP07(2021)123
  $m \approx 2200$ MeV $\Gamma \approx 400$ MeV

Lattice Calc.

$\boxed{m_\pi \approx 133,\ 208,\ 305,\ \text{and}\ 317\ \text{MeV}}$

Yan et al. PRD111(2025)014503



$$K_l(s) = \frac{g_l^2}{m_l^2 - s} + \sum_n \gamma_l^{(n)} s^n$$

$$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2$$

Du et al., PRD99(2019)114002

$$F_0(s) \propto \frac{E_\pi}{s - m_0^2 + i m_0 \Gamma}$$

$$\Delta = \frac{\Gamma^2 E_D}{4 m_0 E_\pi - \Gamma^2}$$

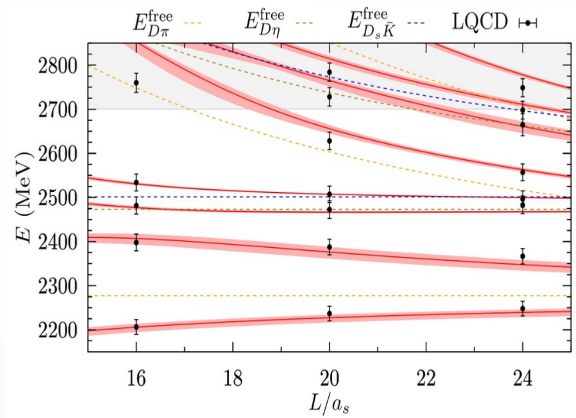
Goldstone bosons: energy-dependent interactions

The standard Breit-Wigner: constant coupling. ~~chiral symmetry~~

Two-pole structure of $D_0^*(2300)$

SU(3) + chiral symmetry ⟹ Two-pole structure

- Two poles in $I=1/2$ sector were found in Kolomeitsev, Lutz (2004); Guo, Shen, Chiang, Ping, Zou (2006); F.-K. Guo, Hanhart, Meißner (2009); Z.-H. Guo, Meißner, Yao (2015); X.-Y. Guo, Heo, Lutz (2018);...
- Postdicted finite volume energy levels in G. Moir et al. [Hadron Spectrum Collaboration], JEHP1610(2016)



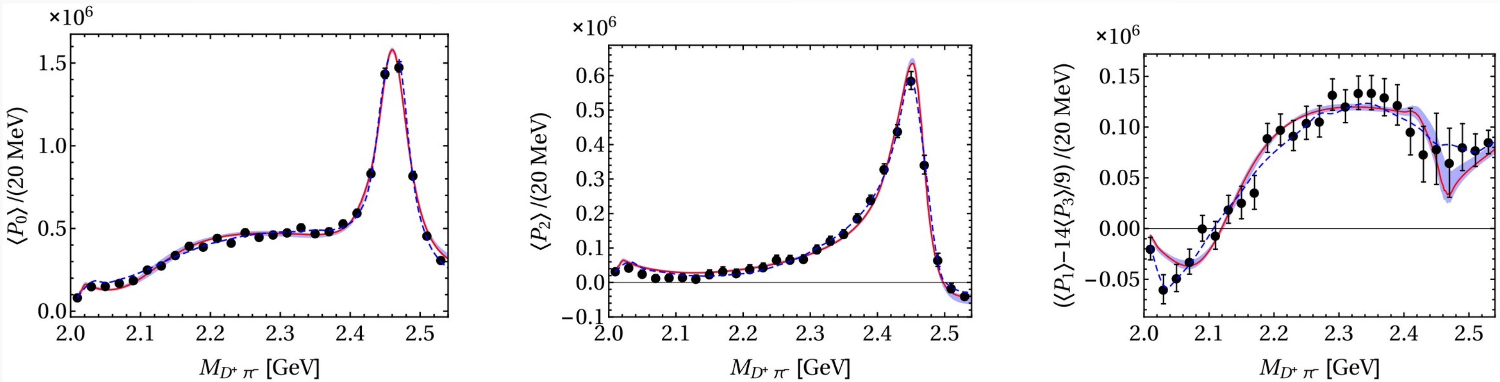
Alabladejo et al., PLB(2017)465

	lower pole	higher pole
D_0^*	$\left(2105^{+6}_{-8}, 102^{+10}_{-11}\right)$	$\left(2451^{+35}_{-26}, 134^{+7}_{-8}\right)$
D_1	$\left(2247^{+5}_{-6}, 107^{+11}_{-10}\right)$	$\left(2555^{+47}_{-30}, 203^{+8}_{-9}\right)$

RPP
$(2343 \pm 10, 115 \pm 8)$
$(2412 \pm 9, 157 \pm 15)$

why $M_{D_0^\star(2300)} \gtrsim M_{D_{s0}^\star(2317)}$ ✓

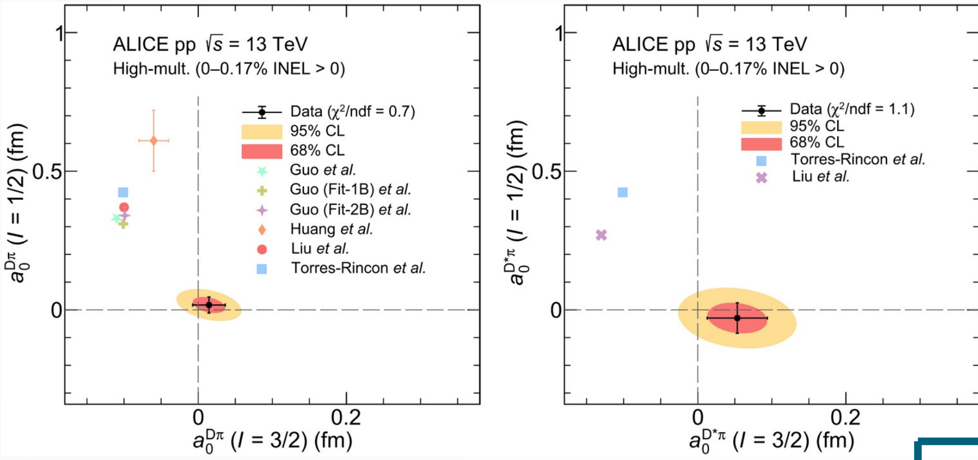
$\boxed{B^- \to D^+\pi^-\pi^-}$ LHCb, PRD94(2016)072001



$$\langle P_0\rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2$$
$$\langle P_2\rangle \propto \tfrac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt5}|\mathcal{A}_0||\mathcal{A}_2|\cos(\delta_2-\delta_0)$$
$$\langle P_{13}\rangle = \langle P_1\rangle - \frac{14}{9}\langle P_3\rangle \propto \frac{2}{\sqrt3}|\mathcal{A}_0||\mathcal{A}_1|\cos(\delta_1-\delta_0)$$

Du et al., PRD98(2018)094018

$D\pi$ scattering length



ALICE, PRD110(2024)032004

determined by fitting the experimental correlation functions with a model that employs a Gaussian potential.

The extracted values are small and compatible with zero.

- Koonin-Pratt (KP) formula for the correlation function $C(\mathbf{k})$ in the CMS:

$$\boxed{C(\mathbf{k}) = \int d\mathbf{r}\, S_{12}(\mathbf{r})\, |\Psi(\mathbf{r},\, \mathbf{k})|^2}$$

Questioned in

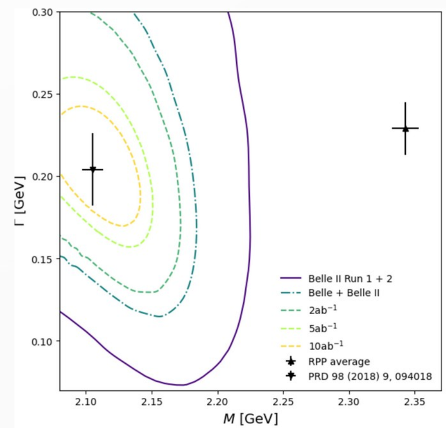
Epelbaum et al., 2504.08631

- Fundamental flaw of the KP formula:

  Combined with the universality assumption for the source function $S_{12}(\mathbf{r})$, it implies the measurability of hadronic wave functions and thus also of the corresponding interaction potentials

$B \to D\pi\ell\nu$

Du et al., 2509.12133

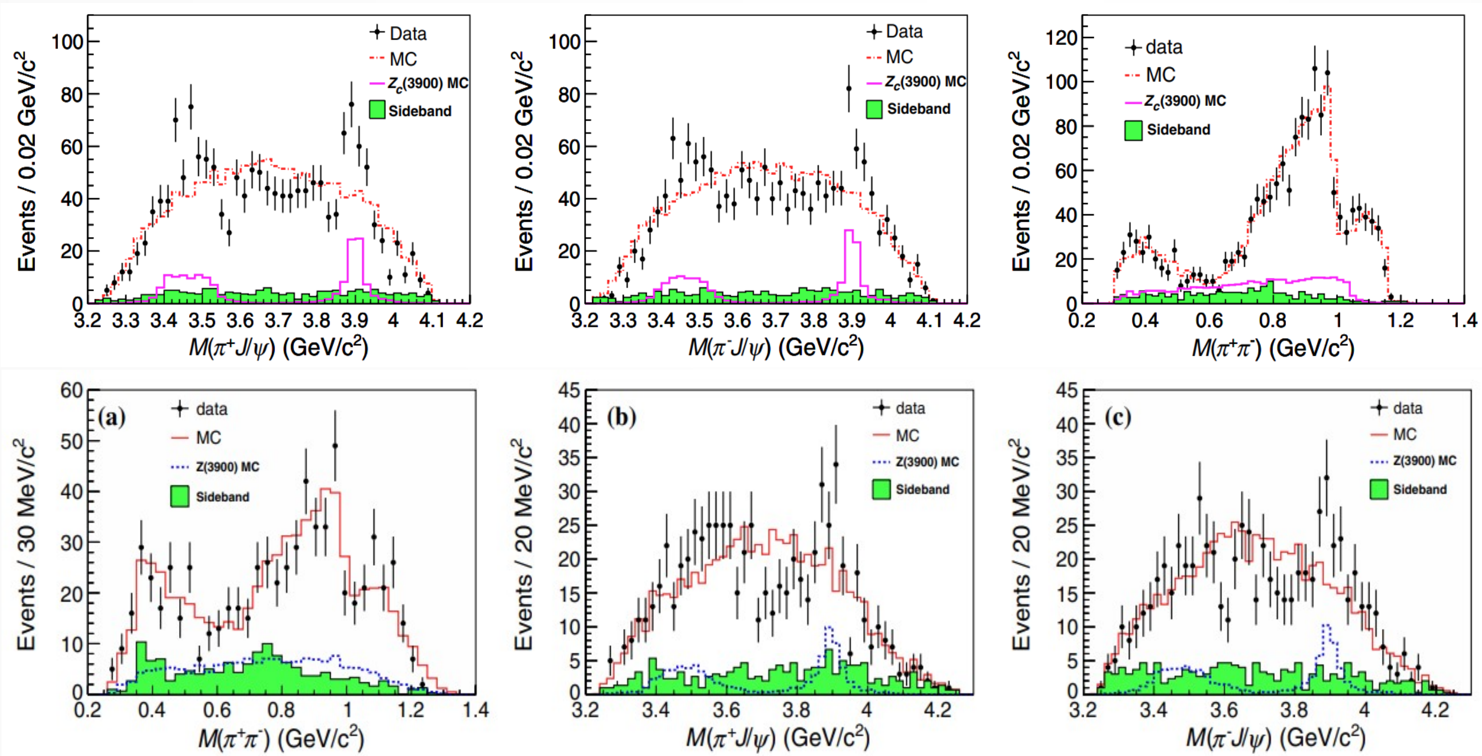


Chiral symmetry

$$K(s) = \frac{E_\pi (g_0 + g_1 s)^2}{s - M_R^2} + E_\pi g_2$$

$$q_1 \cot \delta_0^{(1/2)} = \frac{c_{-1}}{E_\pi} + c_0 + c_1 E_\pi + \mathcal{O}(E_\pi^2)$$

Zc(3900)



Observation of a Charged Charmoniumlike Structure in $e^+e^- \to \pi^+\pi^- J/\psi$ at $\sqrt{s}$ =4.26 GeV #1

BESIII Collaboration • M. Ablikim (Beijing, Inst. High Energy Phys.) et al. (Mar 24, 2013)

Published in: *Phys.Rev.Lett.* 110 (2013) 252001 • e-Print: 1303.5949 [hep-ex]

pdf links DOI cite claim

reference search 1,220 citations

Study of $e^+e^- \to \pi^+\pi^- J/\psi$ and Observation of a Charged Charmoniumlike State at Belle #1

Belle Collaboration • Z.Q. Liu (Beijing, Inst. High Energy Phys.) et al. (Mar 30, 2013)

Published in: *Phys.Rev.Lett.* 110 (2013) 252002, *Phys.Rev.Lett.* 111 (2013) 019901 (erratum) • e-Print: 1304.0121 [hep-ex]

pdf links DOI cite datasets claim

reference search 990 citations

Molecule:

Wang et al., PRL111(2013)
Guo et al., PRD88(2013)
He et al., EPJC73(2013)
Gong et al., PRD94(2016)
....

Compact:

Braaten, PRL111(2013)
Dias et al., PRD88(2013)
Qiao et al., EPJC74(2014)
Deng et al., PRD90(2014)
....

Cusp w/o pole:

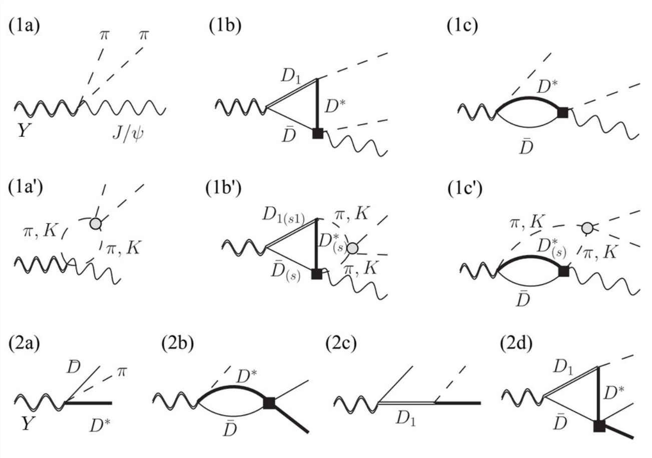
Chen, et al., PRL110(2013)
Swanson, PRD91(2015)
Ikeda et al., PRL117(2016)
....

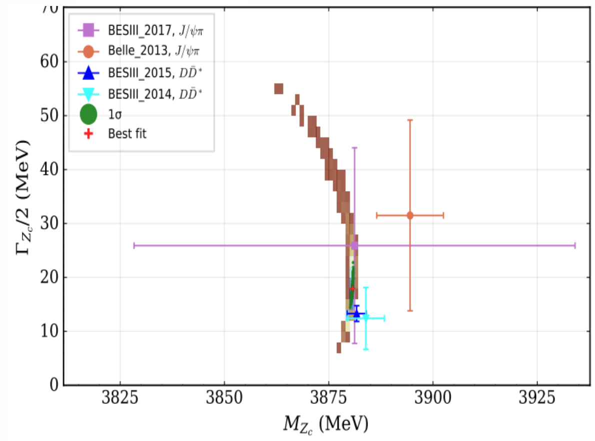
Triangle Singularity:

Wang et al., PRL111(2013)
Szczepaniak, PLB747(2015)
Albaladejo et al., PLB(2016)
Pilloni et al., PLB772(2017)
....

Review: Chen et al., PR639(2016), Liu et al., PPNP107(2019), Brambilla et al., PR873(2020), Guo et al., PPNP112(2020)...

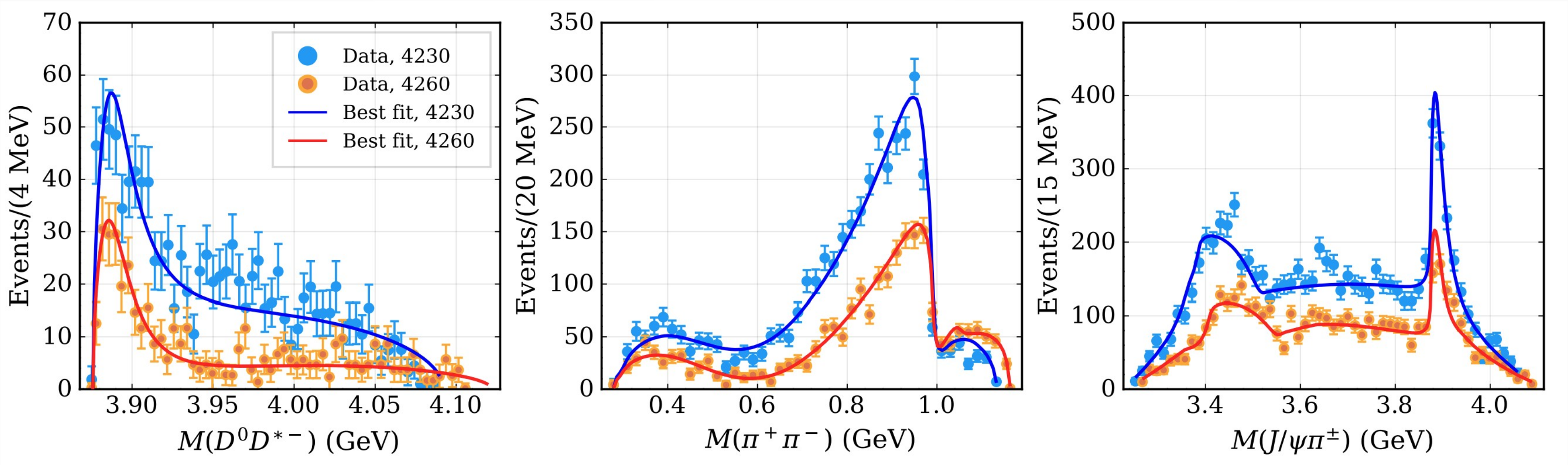
Zc(3900): TS + CS + FSI





$$\begin{aligned} a_0 &= \big(1.11 \pm 0.16 - i(0.01 \pm 0.01)\big)\ \text{fm}\,, \\ r_0 &= \big(-1.88 \pm 0.36 + i(0.01 \pm 0.01)\big)\ \text{fm}\,, \\ \bar{X}_A &= 0.48 \pm 0.02\,, \end{aligned}$$

Chen et al., Sci. China Phys. Mech. Astron. 67 (2024)291011



Data: BESIII, PRD92(2015)092006

Data: BESIII, PRL119(2015)072001

Zc(3900): Lattice

Prelovsek et al., PLB737(2013)172

$m_\pi = 266(4)$ MeV $\quad \pi J/\psi$ and $D\bar{D}^*$ meson-meson interpolators

do not find a candidate for $Z_c^+(3900)$ in the channel with $J^{PC} = 1^{+-}$ and $I = 1$

Prelovsek et al., PRD91(2015)014504

+ more meson-meson + quark-diquark (22 interpolating operators)

no exotic state below 4.2 GeV.

Chen et al., PRD89(2014)094506

$m_\pi = 485, 420, 300 MeV$ $\quad$ meson-meson interpolators

weakly repulsive!

Liu et al., PRD101(2020)054502

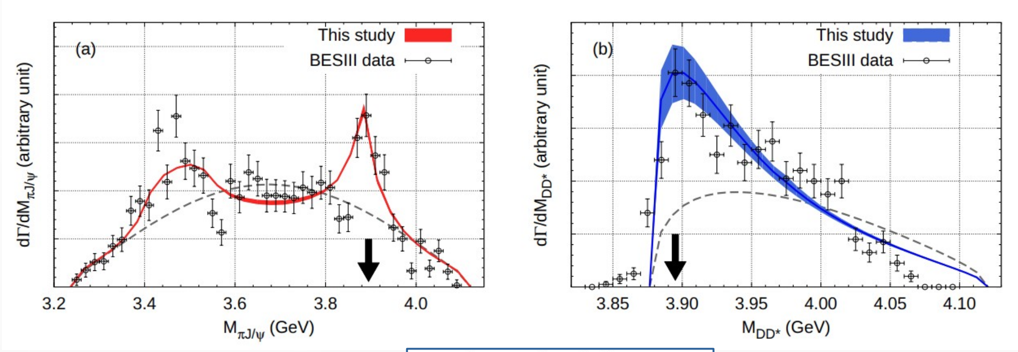
Three-channel Ross-Shaw effective range theory
Multi-channel Luescher formula

HAL QCD, PRL117(2016)242001

HAL QCD, JPG45(2018)024002

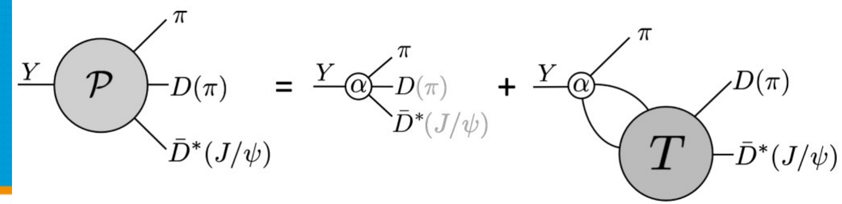
$m_\pi = 410\text{–}700$ MeV

are dominated by the off-diagonal $\pi J/\psi$-$\bar{D}D^*$ and $\rho\eta_c$-$\bar{D}D^*$

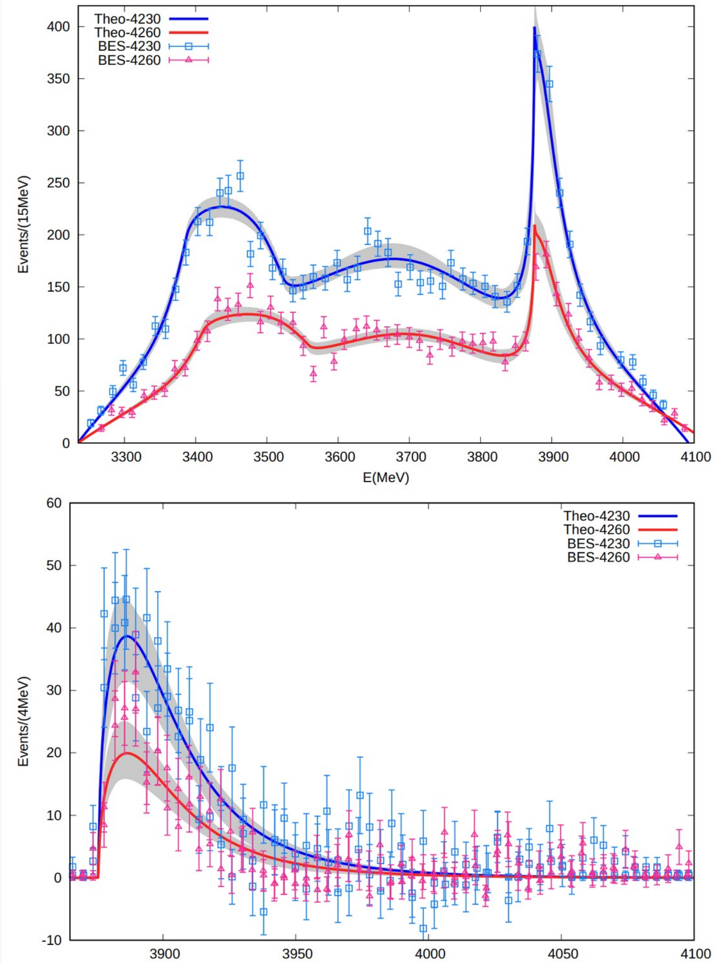


a threshold cusp

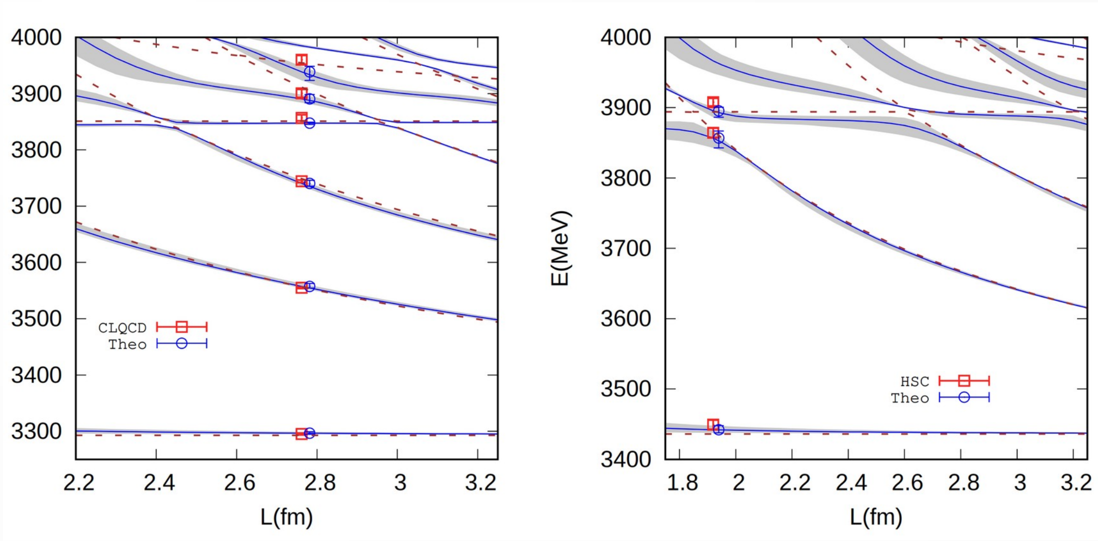
Zc(3900): Lattice + EFT



No TS.



Yan et al., PRD109(2024)014026



RS	M_R (MeV)	$\Gamma_R/2$ (MeV)	$\lvert\gamma_1\rvert$ (GeV)	$\lvert\gamma_2\rvert$ (GeV)
III	$3874.8^{+3.7}_{-4.0}$	$32.7^{+1.6}_{-1.9}$	$4.3^{+0.3}_{-0.3}$	$8.7^{+0.8}_{-0.7}$
II	$3902.7^{+1.3}_{-1.3}$	$3.0^{+2.4}_{-2.4}$	$4.9^{+0.2}_{-0.2}$	$8.3^{+0.3}_{-0.3}$
IV	$3902.4^{+1.2}_{-2.2}$	$3.3^{+4.4}_{-2.6}$	$4.6^{+0.3}_{-0.4}$	$8.6^{+0.4}_{-0.3}$

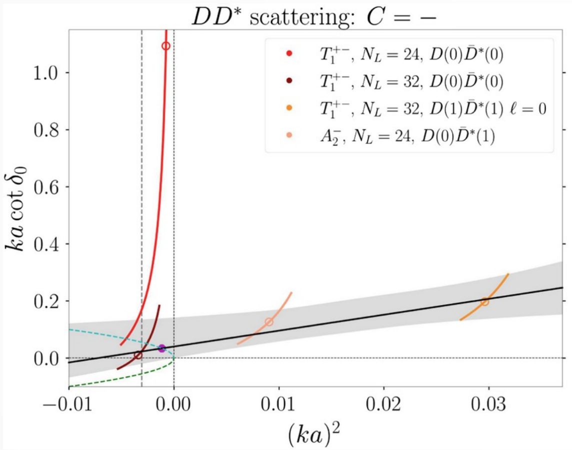
$$\boxed{\bar{X}_2 = \hat{X}_2 = 0.36^{+0.06}_{-0.06}}$$

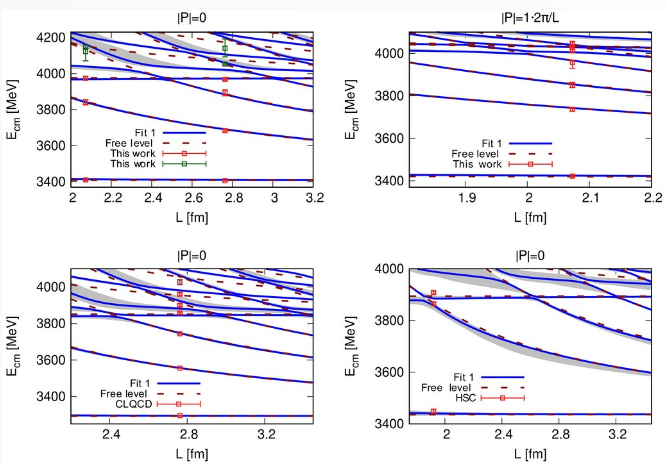
$$a_{\bar{D}D^*} = \left(-0.56^{+0.11}_{-0.13} - i0.28^{+0.04}_{-0.04}\right)\,\text{fm}\,, \qquad r_{\bar{D}D^*} = \left(-2.03^{+0.51}_{-0.68} - i0.77^{+0.24}_{-0.13}\right)\,\text{fm}\,.$$

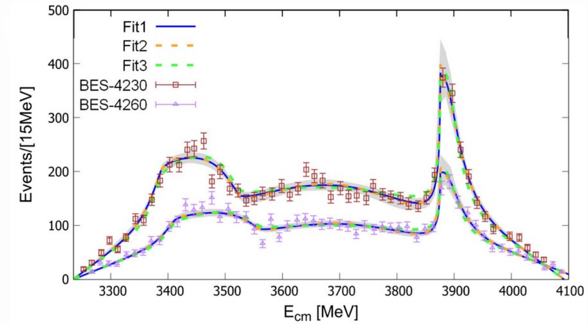
Zc(3900): Lattice + EFT

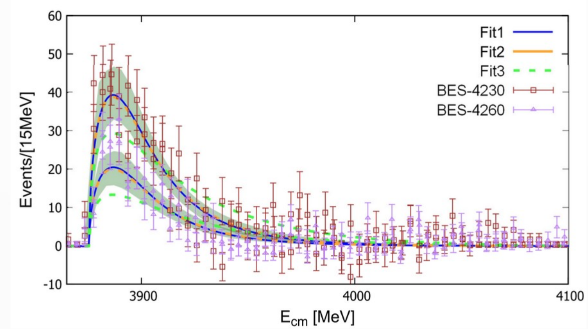
$m_\pi \simeq 280$ MeV   More than one volume + moving frame

Sadl et al., PRD111(2025)054513

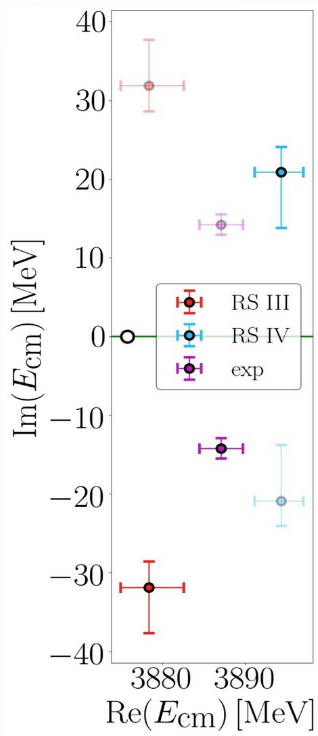


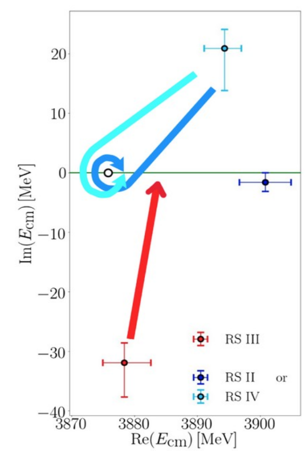






$$X_2^{\rm III} = 0.65^{+0.05}_{-0.15}, \quad X_2^{\rm IV} = 0.59^{+0.04}_{-0.09} \quad ({\rm Fit1}),$$
$$X_2^{\rm III} = 0.48^{+0.10}_{-0.07}, \quad X_2^{\rm IV} = 0.46^{+0.05}_{-0.04} \quad ({\rm Fit2}),$$
$$X_2^{\rm III} = 0.43^{+0.09}_{-0.06}, \quad X_2^{\rm IV} = 0.42^{+0.05}_{-0.02} \quad ({\rm Fit4}).$$

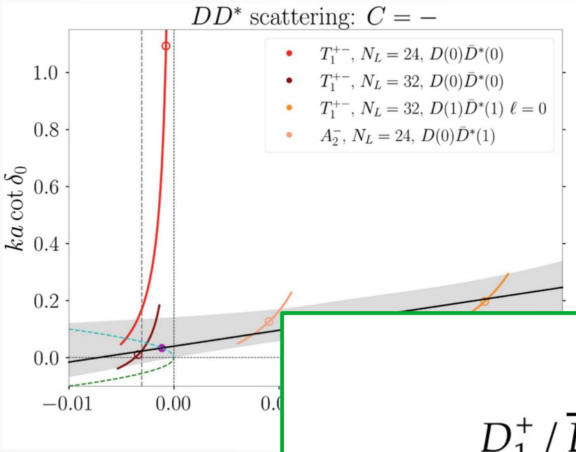


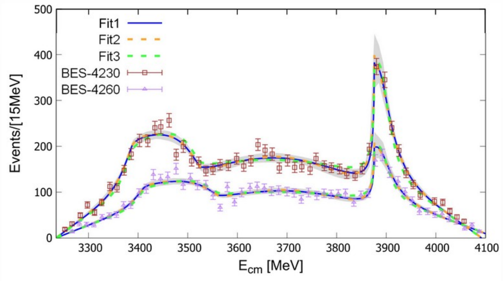


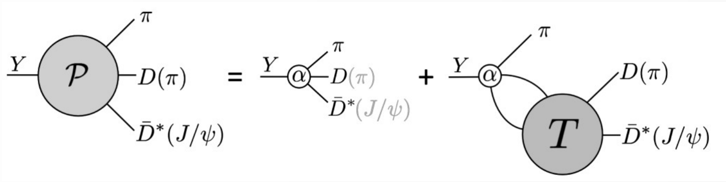
Zc(3900): Lattice + EFT

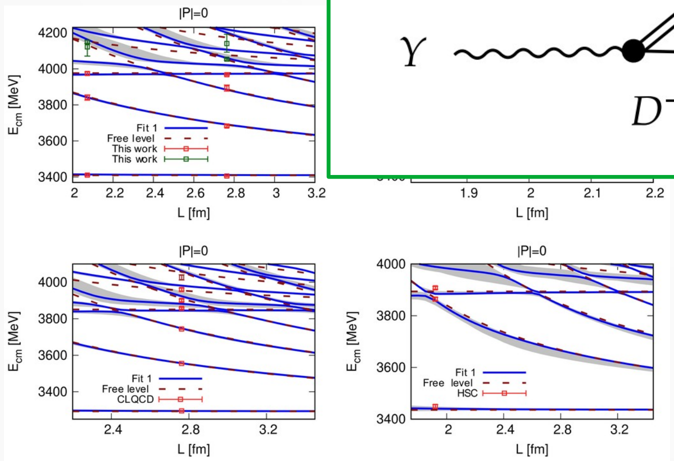
$m_\pi \simeq 280$ MeV More than one volume + moving frame

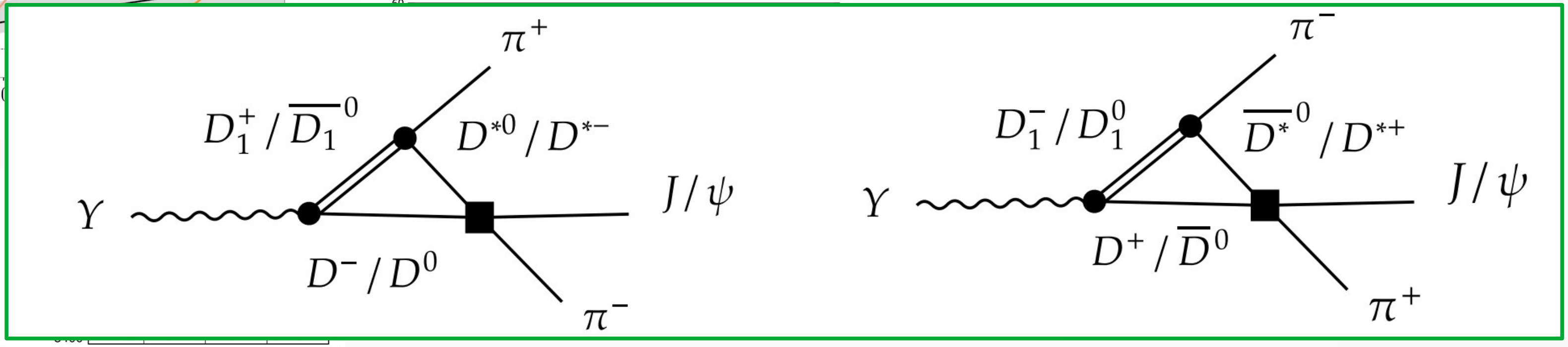
Sadl et al., PRD111(2025)054513







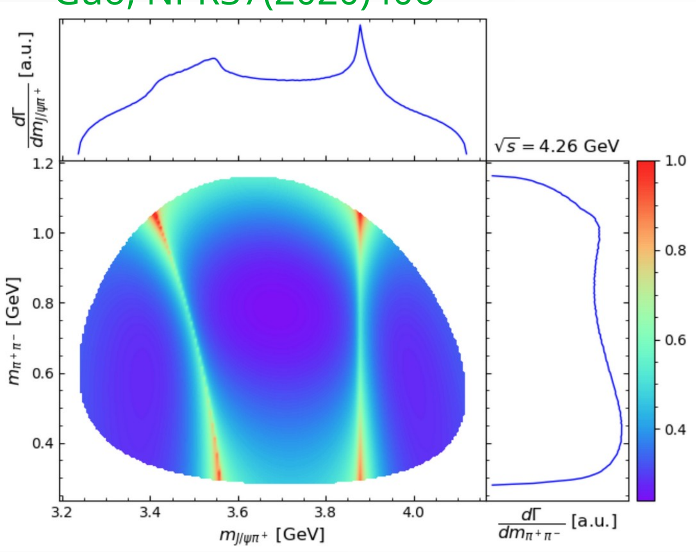


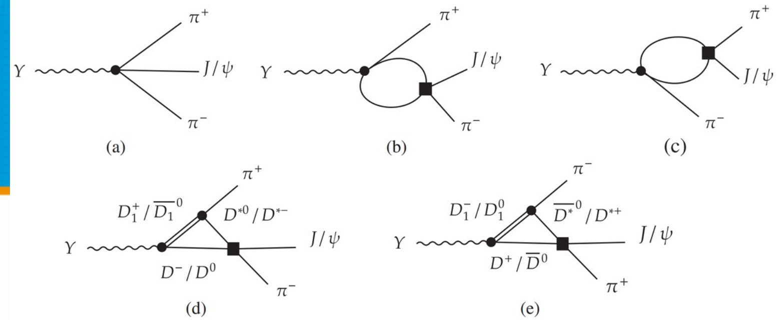


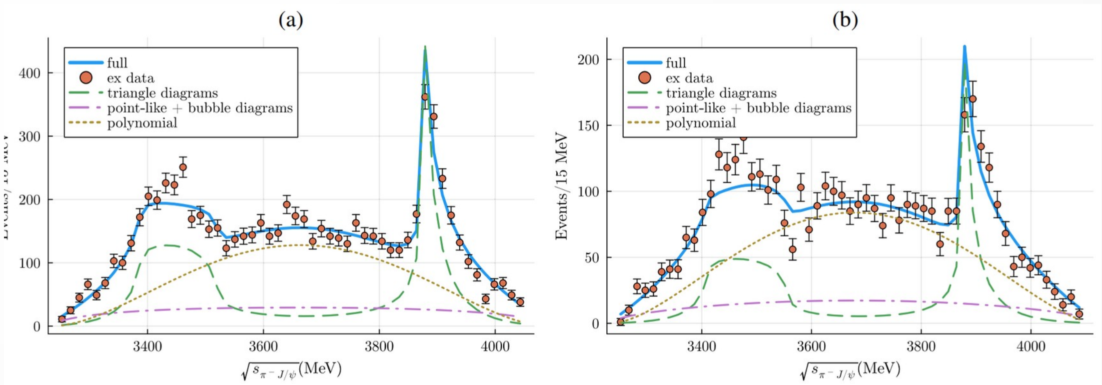
No triangle diagrams!

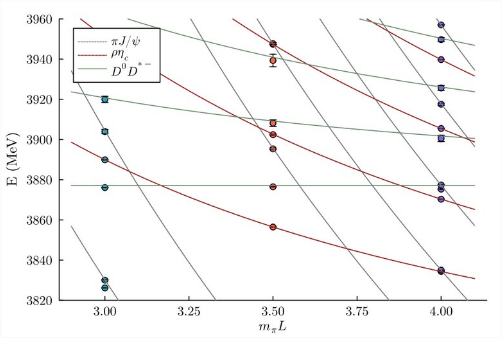
Zc(3900): Lattice + EFT

Guo, NPR37(2020)406







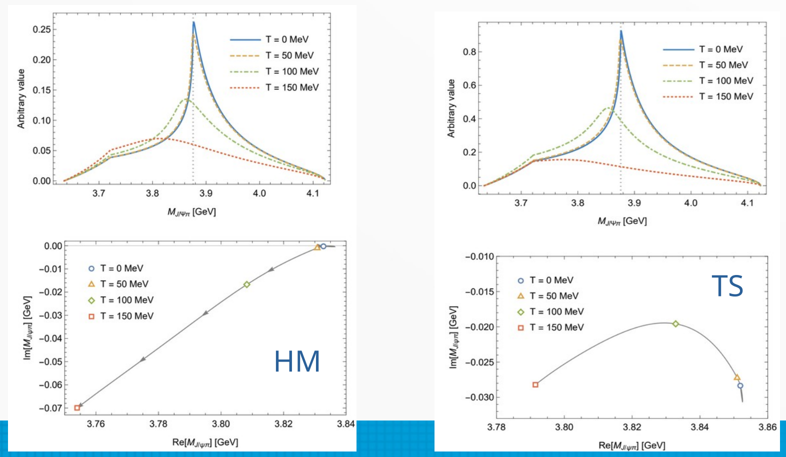


$m_\pi = 139$ MeV prediction

Yu et al., PRD110(2024)114029

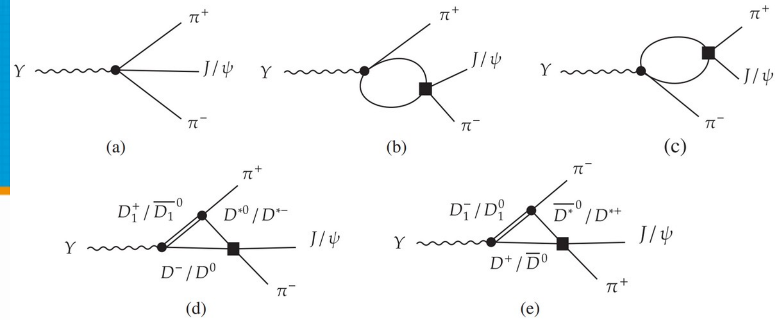
	Pole position	Type	Scheme ($\Lambda_{\pi J/\psi}$)
This work	$3798.72 - 1.10i$	Virtual	1(1.3 GeV)
	$3798.46 - 1.71i$		1(1.5 GeV)
	$3798.12 - 2.26i$		1(1.7 GeV)
	$3798.27 - 2.02i$		2(1.5 GeV)
	$3797.80 - 2.64i$		2(1.7 GeV)
Reference [25]	3798^{+25}_{-31}	Virtual	
	$3902(6) - 38(9)i$	Resonance	
Reference [19]	3831^{+27}_{-38}	Virtual	
	$3894(6) - 30(13)i$	Resonance	
Reference [21]	3870	Virtual	
Reference [20]	3879	Virtual	-
Reference [22]	3872	Virtual	
Reference [26]	$3880(3) - 13(1)i$	Resonance	
Reference [32]	$3884 - 22i$	Resonance	
Reference [27]	3840	Virtual	
Reference [62]	3839(11)	Virtual	

Finite temperature

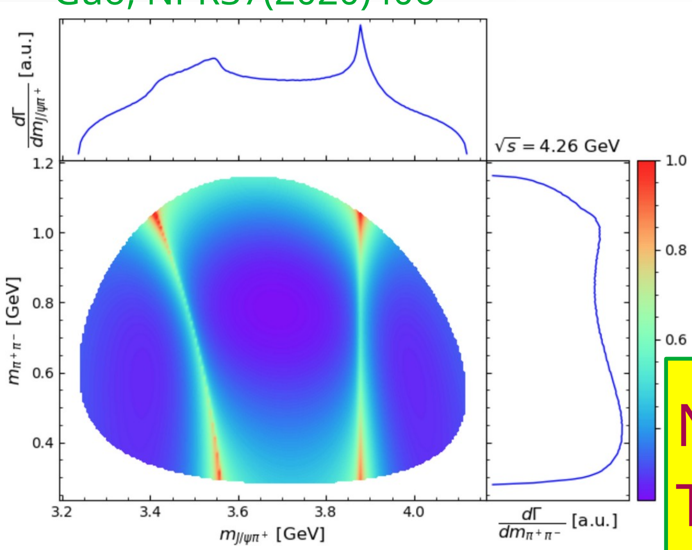


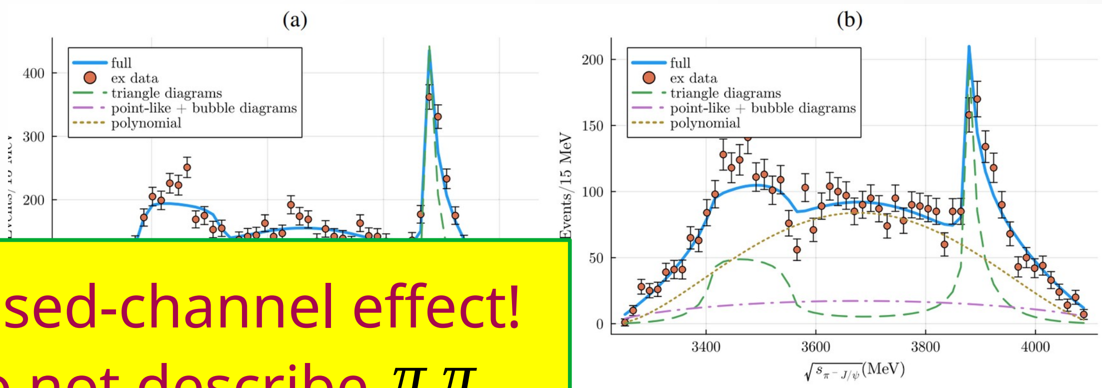
Zhang et al., PRD112(2025)016014

Zc(3900): Lattice + EFT



Guo, NPR37(2020)406





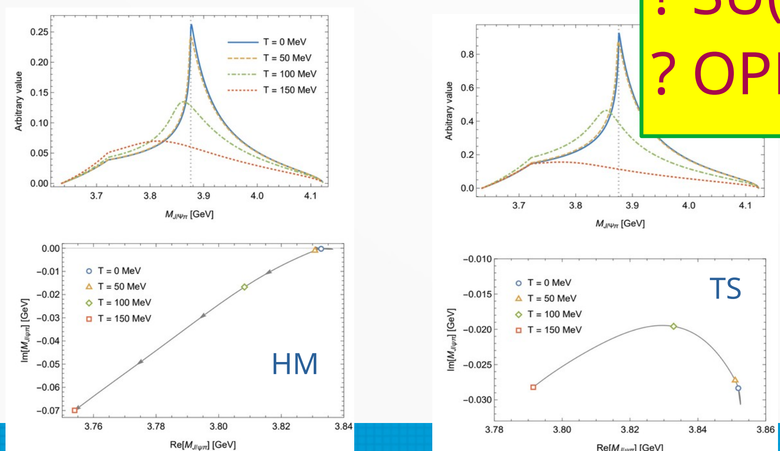
No crossed-channel effect!

Thus do not describe $\pi\pi$

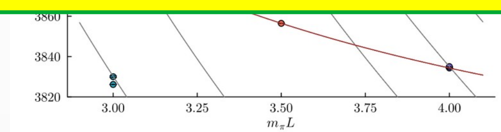
? SU(3) Zcs(3985)

? OPE

Finite temperature



Zhang et al., PRD112(2025)016014



$m_\pi = 139$ MeV prediction

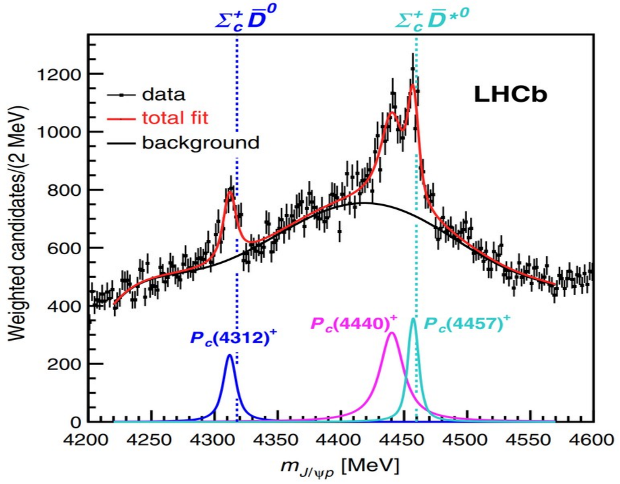
Yu et al., PRD110(2024)114029

	Pole position	Type	Scheme ($\Lambda_{\pi J/\psi}$)
...s work	3798.72 − 1.10i	Virtual	1(1.3 GeV)
	3798.46 − 1.71i		1(1.5 GeV)
	3798.12 − 2.26i		1(1.7 GeV)
	3798.27 − 2.02i		2(1.5 GeV)
	3797.80 − 2.64i		2(1.7 GeV)
...erence [25]	3798^{+25}_{-31}	Virtual	
	3902(6) − 38(9)i	Resonance	
Reference [19]	3831^{+27}_{-38}	Virtual	
	3894(6) − 30(13)i	Resonance	
Reference [21]	3870	Virtual	
Reference [20]	3879	Virtual	
Reference [22]	3872	Virtual	-
Reference [26]	3880(3) − 13(1)i	Resonance	
Reference [32]	3884 − 22i	Resonance	
Reference [27]	3840	Virtual	
Reference [62]	3839(11)	Virtual	

Hidden-charm pentaquarks

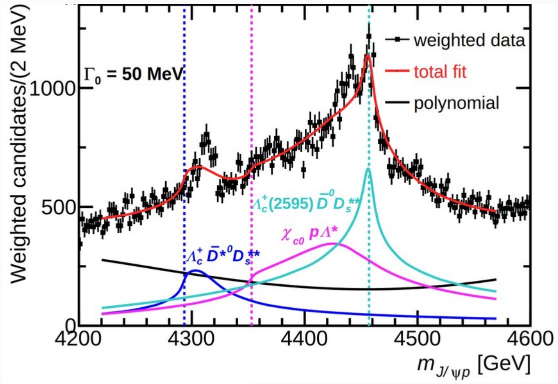
Observation of exotic structures (P_c) in $\Lambda_b^0 \to J/\psi p K^-$

LHCb, PRL 115, 072001 (2015)



LHCb, PRL 122, 222001 (2019)

State	M [MeV]	Γ [MeV]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$



Compact Pentaquark

Cheng and Liu, PRD100, 054002(2019)
Ali et al., JHEP10, 256 (2019)
Deng, PRD105, 116021(2022)
Zhu et al., PLB797, 134869(2019)
Giron et al., JHEP05, 061 (2019)
……

Hadronic Molecule

Chen et al., PRD100, 051501 (2019)
Chen et al., PRD100, 011502 (2019)
Guo et al., PRD99, 091501 (2019)
Liu et al., PRL122, 242001 (2019)
Guo et al., PLB793, 144 (2019)
Meng et al., PRD100, 014031(2019)
Wu et al., PRC100, 035026 (2019)
……

Kinematic effects

Kuang et al., EPJC80,433 (2020)
Nakamura, PRD103, L111503(2021)
Burns et al., PRD106,054029(2022)
……

Hadron-Charmonium

Compact $(\bar{Q}Q)$ surrounded by light quarks

Eides et al., MPLA 35, 2050151 (2020)

Heavy Quark Spin Symmetry (HQSS)

- For a heavy-quark Q ($c$harm, $b$ottom) with $m_Q \gg \Lambda_{\mathrm{QCD}}$
  - ☞ chromomag. interaction $\propto \dfrac{\sigma \cdot \mathbf{B}}{m_Q}$ $\to 0$: independent of heavy-quark spin
- For a hadron containing a heavy-quark Q: $J = s_Q + j_\ell$

S-wave baryon-meson system
(ground states)

$s_Q = \frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

$j_\ell = \underline{\frac{1}{2} \oplus \frac{3}{2}}$ $\longrightarrow$ $\boxed{\text{LO Contact Interaction: Only 2 LECs}}$

	Molecule	J^P	M (MeV)		Molecule	J^P	M (MeV)
A	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	$4311.8 - 4313.0$	B	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	$4306.3 - 4307.7$
A	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	$\boxed{4376.1 - 4377.0}$	B	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	$\boxed{4370.5 - 4371.7}$
A	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	4440.3^*	B	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	4457.3^*
A	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	4457.3^*	B	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	4440.3^*
A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	$4500.2 - 4501.0$	B	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	$4523.2 - 4523.6$
A	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	$4510.6 - 4510.8$	B	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	$4516.5 - 4516.6$
A	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	$4523.3 - 4523.6$	B	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	$4500.2 - 4501.0$

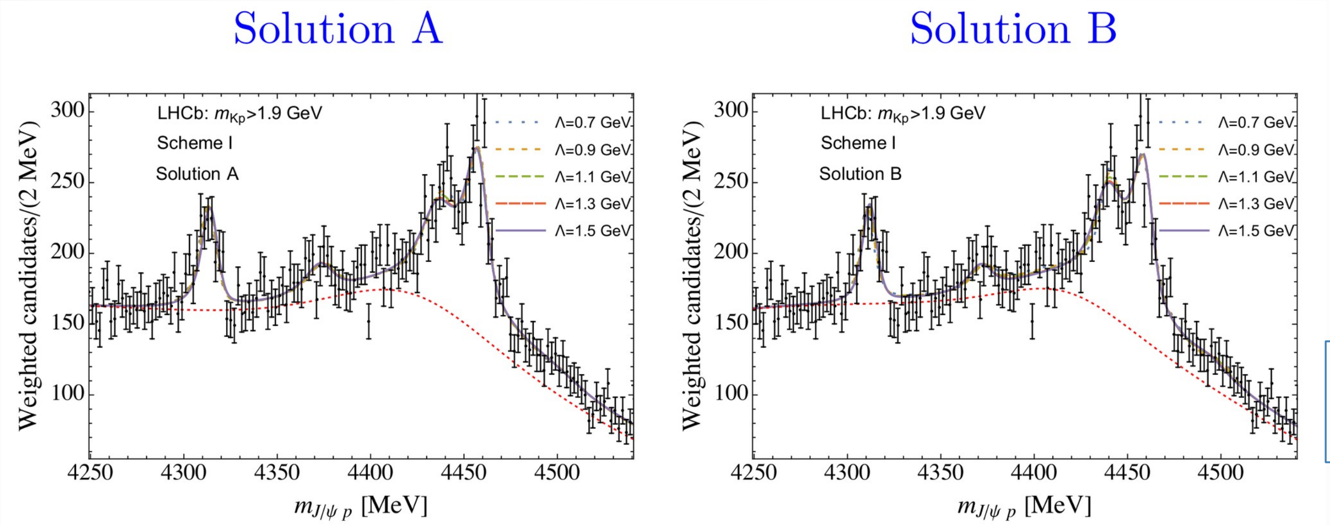
Liu et al., PRL122,242001 (2019)

7 Pc states!

Xiao et al., PRD88, 56012 (2013)

LHCb pentaquarks: EFT w/o pion

MLD et al., PRL124,072001(2020)
MLD et al., JHEP08,157 (2021)



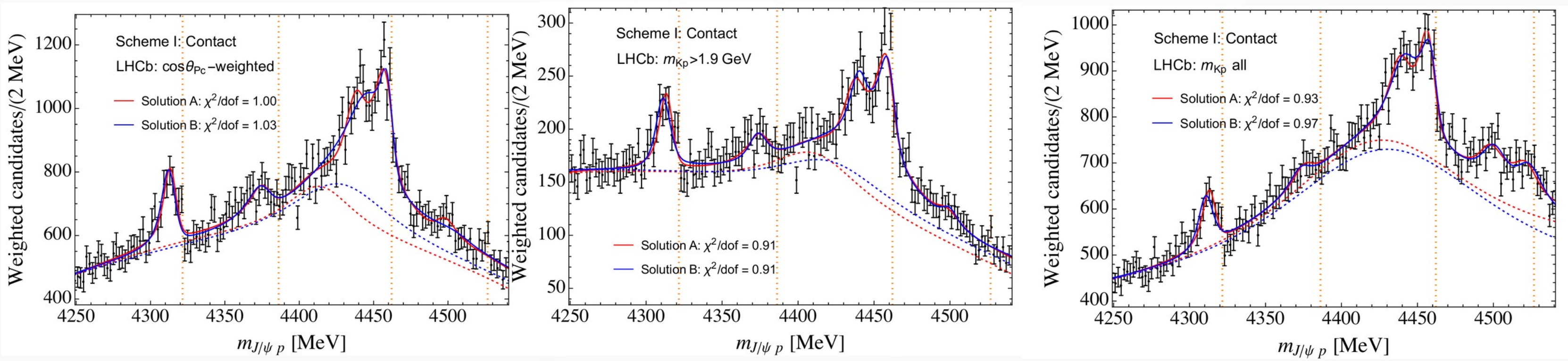
☞ $\Lambda > \Lambda_{\rm soft} \sim \sqrt{2\mu\delta} \sim 0.7$ GeV

☞ Cutoff-independent for both solution A and B

$\Sigma_c \bar{D}^*$	$P_c(4440)$	$P_c(4457)$
Fit A	$\frac{1}{2}^-$	$\frac{3}{2}^-$
Fit B	$\frac{3}{2}^-$	$\frac{1}{2}^-$

If $\Sigma_c^{(*)}\bar{D}^{(*)} \to \Lambda_c \bar{D}^{(*)}$ is included, one more LEC

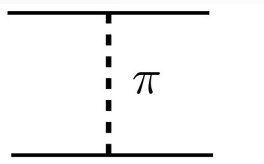
☞ No need for $\Lambda_c \bar{D}^{(*)}$ overdetermined



LHCb pentaquarks: EFT w/ pion

MLD et al., PRL124,072001(2020)
MLD et al., JHEP08,157 (2021)

$\boxed{\text{contact} + \text{OPE}}$



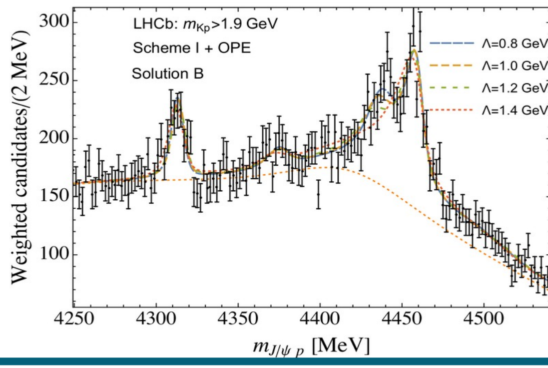
Long range: OPE

Determined by exp. data

☞ No solution A

☞ Solution B:
Cut-off dependent

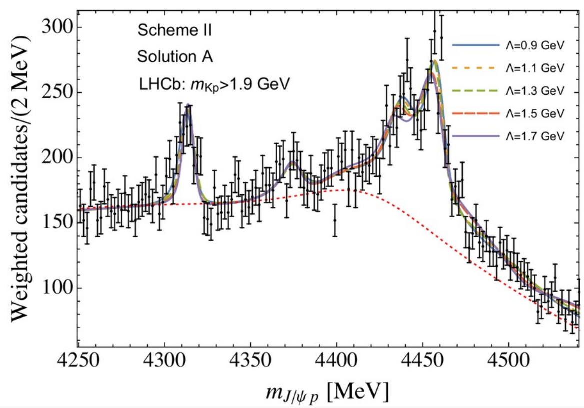
☞ $\Lambda_{\text{soft}} \sim 700$ MeV



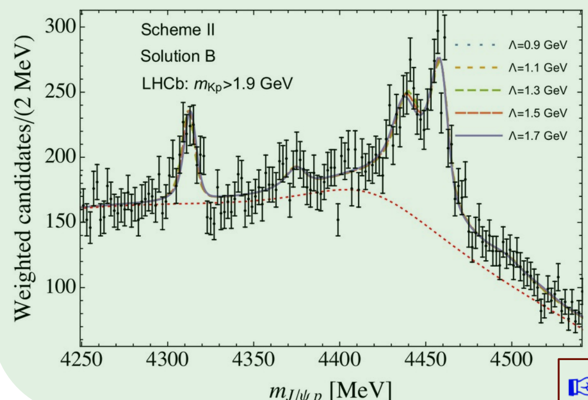
$\boxed{+\,\Lambda_c \bar{D}^{(*)}}$

$\boxed{\text{contact} + \text{OPE} + \text{S-D}}$

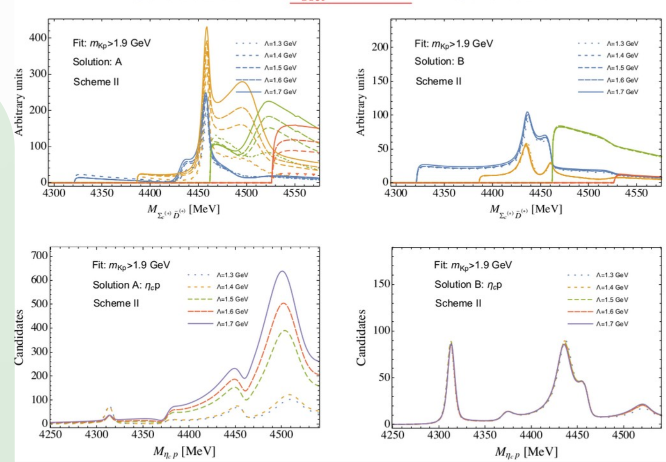
Solution A



Solution B



Solution A $\Lambda_{\text{soft}} \sim 0.7$ GeV Solution B



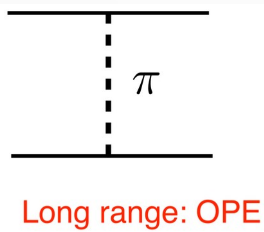
☞ Overdetermined. w/ $\Lambda_c \bar{D}^{(*)}$ $\Lambda_{\text{soft}} \sim 0.9$ GeV

☞ Cutoff-independent only for solution B

LHCb pentaquarks: EFT w/ pion

MLD et al., PRL124,072001(2020)
MLD et al., JHEP08,157 (2021)

$\boxed{\text{contact} + \text{OPE}}$



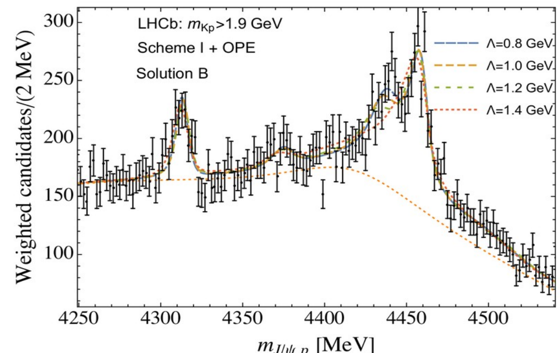
Long range: OPE

Determined by exp. data

☞ No solution A

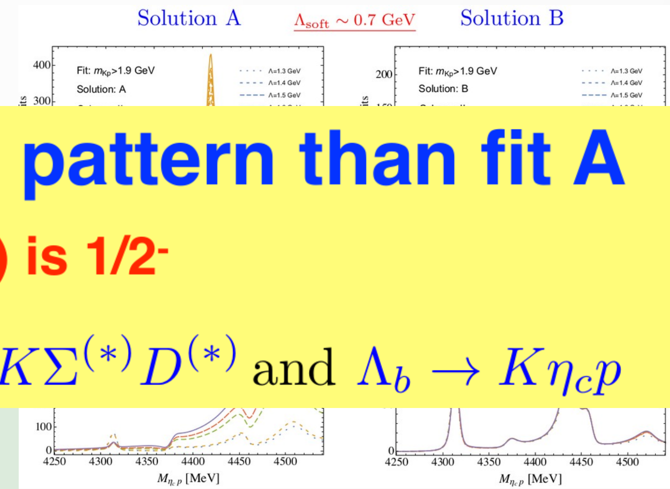
☞ Solution B:
Cut-off dependent

☞ $\Lambda_{\text{soft}} \sim 700$ MeV



$\boxed{+\Lambda_c \bar{D}^{(*)}}$

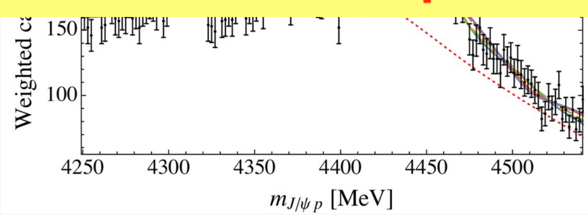
$\boxed{\text{contact} + \text{OPE} + \text{S-D}}$

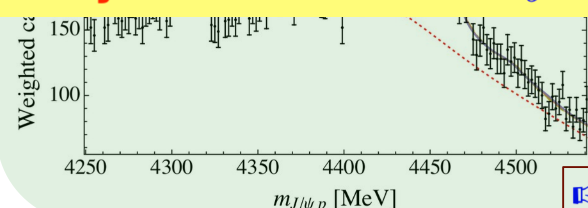


Fit B shows a much more natural pattern than fit A

⇒ $P_c(4440)$ is $3/2^-$ and $P_c(4457)$ is $1/2^-$

⇒ experimentally testable via $\Lambda_b \to K\Sigma^{(*)} D^{(*)}$ and $\Lambda_b \to K \eta_c p$



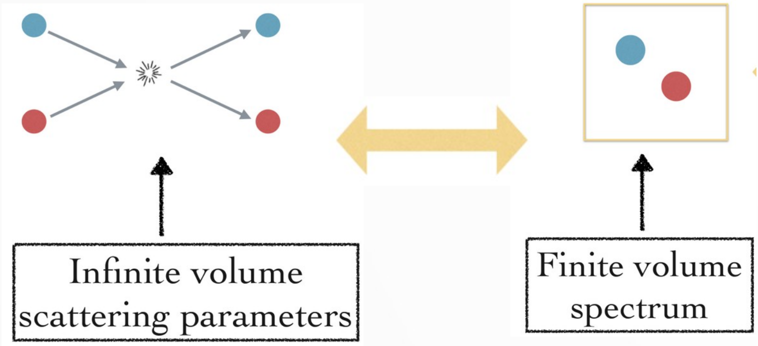


☞ Overdetermined. w/ $\Lambda_c \bar{D}^{(*)}$ $\Lambda_{\text{soft}} \sim 0.9$ GeV

☞ Cutoff-independent only for solution B

$\Sigma_c^{(*)}\bar{D}^{(*)} - \Lambda_c(2595)\bar{D}$ Yalikun et al., PRD104,094039(2021)

Lattice QCD study on the Pc states



$\Sigma_c\bar{D}$ and $\Sigma_c\bar{D}^*$ scattering$(J^P = \frac{1}{2}^-)$

✦ Five operators:

$\mathcal{O}_1 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (|\mathbf{p}| = 0)$

$\mathcal{O}_2 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (|\mathbf{p}| = 1)$

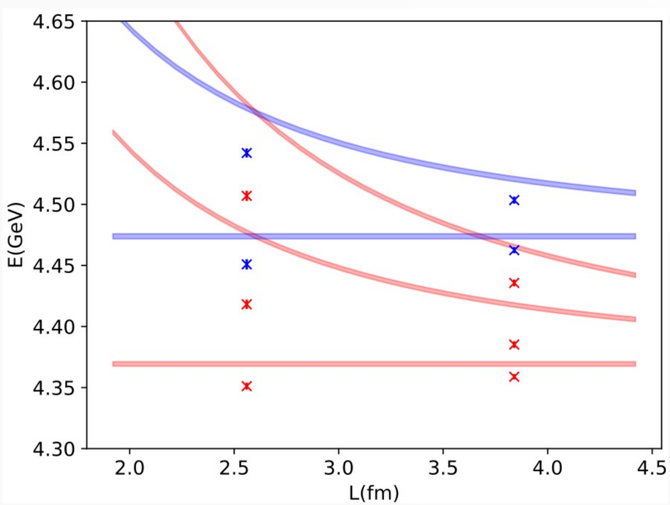
$\mathcal{O}_3 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (|\mathbf{p}| = \sqrt{2})$

$\mathcal{O}_4 = \Sigma_c(\mathbf{p})\bar{D}^*(-\mathbf{p})\ (|\mathbf{p}| = 0)$

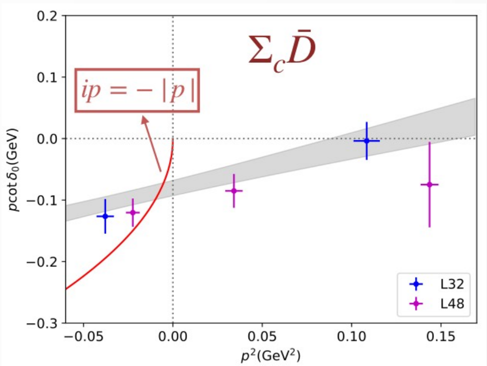
$\mathcal{O}_5 = \Sigma_c(\mathbf{p})\bar{D}^*(-\mathbf{p})\ (|\mathbf{p}| = 1)$

HY Xing et al., 2210.08555

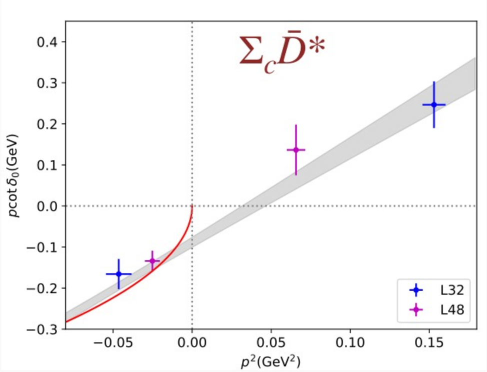
Single-Channel



$\Sigma_c\bar{D}$: $P_c(4312)$?
$a_0 = -2.0(3)(5)$fm
$E_B = 6(2)(2)$MeV



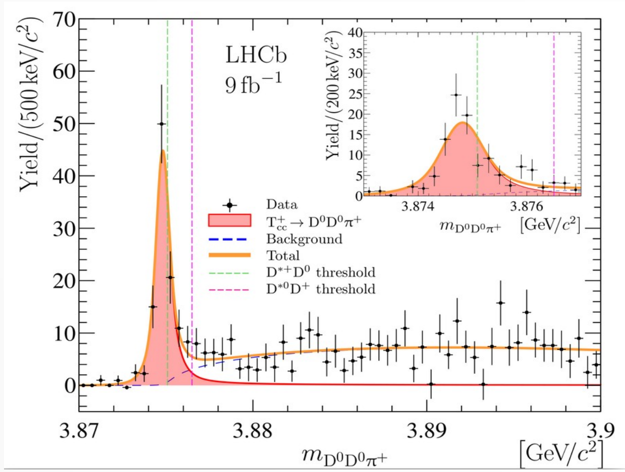
$\Sigma_c\bar{D}^*$: $P_c(4440)/P_c(4457)$?
$a_0 = -2.3(5)(1)$fm
$E_B = 7(3)(1)$MeV

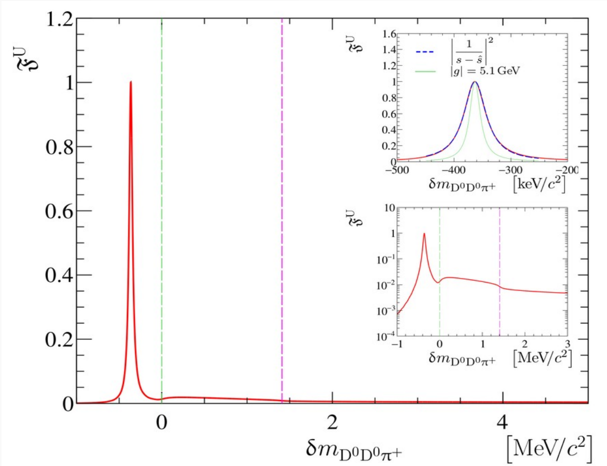


✦ The finite-volume energies lie below the free energies, indicating rather strong attractive interactions.

Coupled-Channel still going!

Doubly charm tetraquark state





Breit-Wigner fit

LHCb, Nature Phys. 18, (2022) 751

Parameter	Value
N	117 ± 16
δm_{BW}	-273 ± 61 keV
Γ_{BW}	410 ± 165 keV

☞ $\mathfrak{R} \sim 400$ keV.

Unitarized and analytical

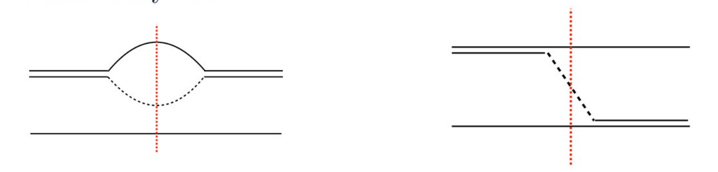
LHCb, Nature Commun. 13 (2022), 3351

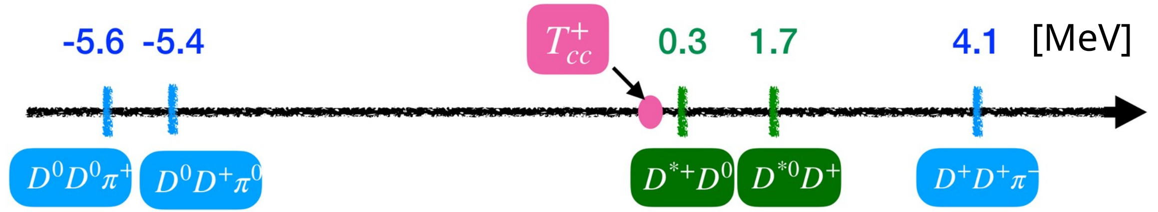
$\delta m = m_{T_{cc}^{+}} - m_{D^{*+}} - m_{D^0}$

$$\delta m_{\mathrm{pole}} = -360 \pm 40^{+4}_{-0} \text{ keV}$$

$$\Gamma_{\mathrm{pole}} = 48 \pm 2^{+0}_{-14} \text{ keV}$$

☞ Three-body cuts





Doubly charm tetraquark on the Lattice

$m_\pi \approx 350$ MeV Chen et al., PLB833(2022)137391

attractive ⟺ ρ meson exchange

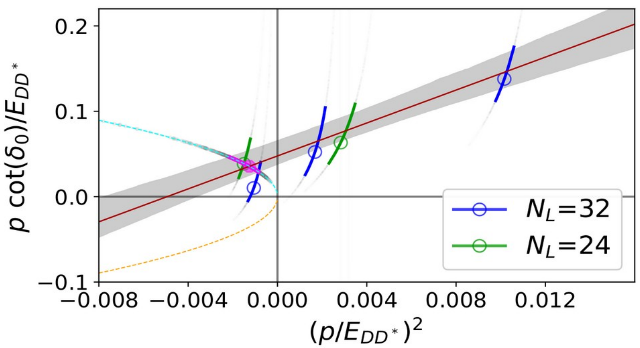
$$m_\pi = 146\ \text{MeV}$$
$$E_{\text{pole}} = -59\,(^{+53}_{-99})\,(^{+2}_{-67})\ \text{keV}$$

Lyu [HALQCD], PRL131(2023)161901

Virtual state

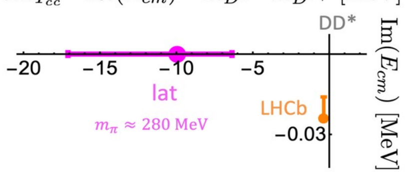
Padmanath *et al*, PRL129,032002(2022)

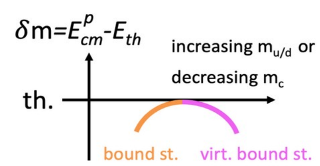
	m_D (MeV)	m_{D^*} (MeV)	M_{av} (MeV)	$a_{l=0}^{(J=1)}$ (fm)	$r_{l=0}^{(J=1)}$ (fm)	$\delta m_{T_{cc}}$ (MeV)	T_{cc}
Lattice ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)	1.04(29)	$0.96(^{+0.18}_{-0.20})$	$-9.9^{+3.6}_{-7.2}$	Virtual bound st.
Lattice ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)	0.86(0.22)	$0.92(^{+0.17}_{-0.19})$	$-15.0(^{+4.6}_{-9.3})$	Virtual bound st.
Experiment [2,41]	1864.85(5)	2010.26(5)	3068.6(1)	−7.15(51)	[−11.9(16.9), 0]	−0.36(4)	Bound st.



$$t = \frac{E_{\text{cm}}}{2}\frac{1}{p\cot\delta - ip},$$
$$p\cot\delta = \frac{1}{a_0} + \frac{1}{2}r_0 p^2,$$

$$\delta m_{T_{cc}} = \text{Re}(E_{cm}) - m_{D^0} - m_{D^{*+}}\ [\text{MeV}]$$



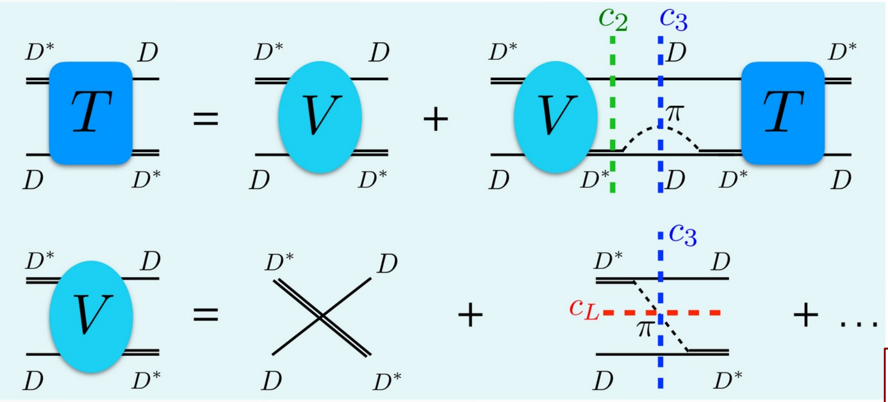


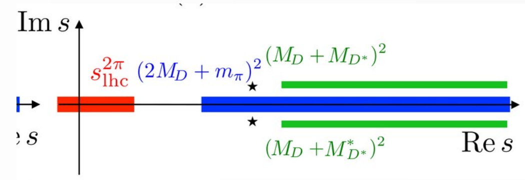
Nagatsuka et al., 2507.20712

Twisted boundary conditions.

attractive
virtual

The left-hand cut vs three-body cut



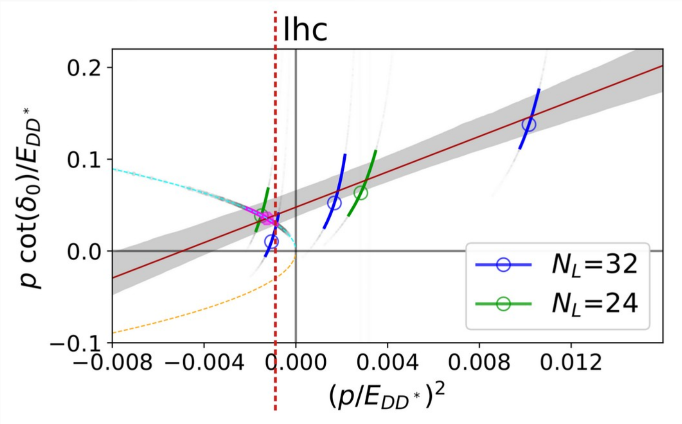


☞ three-body cut

$$E > M_D + M_D + M_\pi$$

☞ left-hand cut

$$\int_{-1}^{1} d\cos\theta G_\pi(E,p,p)$$



$$G_\pi^{-1}(E,\mathrm{k},\mathrm{k}') \xrightarrow[\text{on shell: } k=k'=p]{\cos\theta=\pm1} E_{D*}(p^2) - E_D(p^2) - \omega_\pi(4p^2/0) = 0$$

$m_\pi = 280$ MeV

☞ two-body branch point:

$E = M_D + M_{D^*}$

$\Longrightarrow p^2_{\rm rhc_2} = 0$

☞ three-body branch point:

$E = M_D + M_D + m_\pi$

$\Longrightarrow \left(\frac{p_{\rm rhc_3}}{E_{DD^*}}\right)^2 = +0.019$

☞ left-hand cut branch point:

$\Longrightarrow \left(\frac{p^{1\pi}_{\rm lhc}}{E_{DD^*}}\right)^2 = -0.001$

$\left(\frac{\tilde{p}^{1\pi}_{\rm lhc}}{E_{DD^*}}\right)^2 = -0.190$

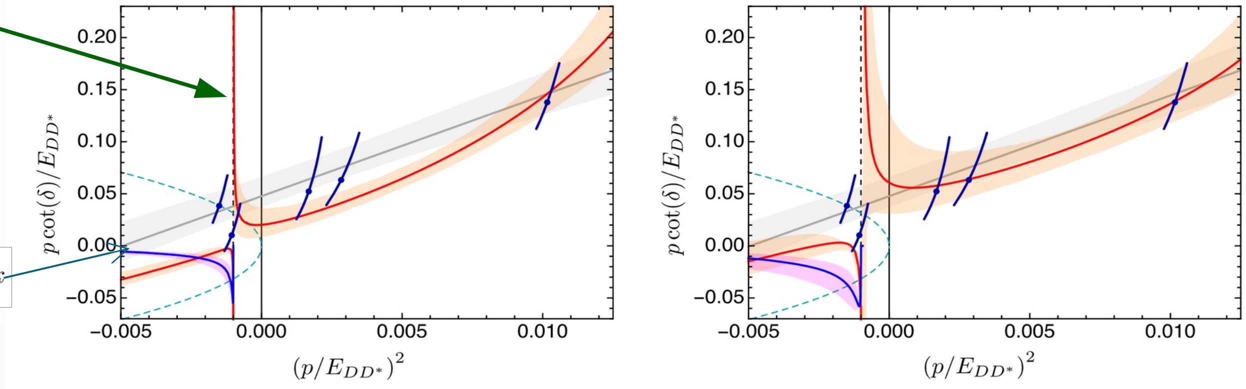
The phase shift with a left-hand cut

Du et al., PRL131(2023)131903

$M_D = 1927$ MeV, $M_{D^*} = 2049$ MeV, $m_\pi = 280$ MeV

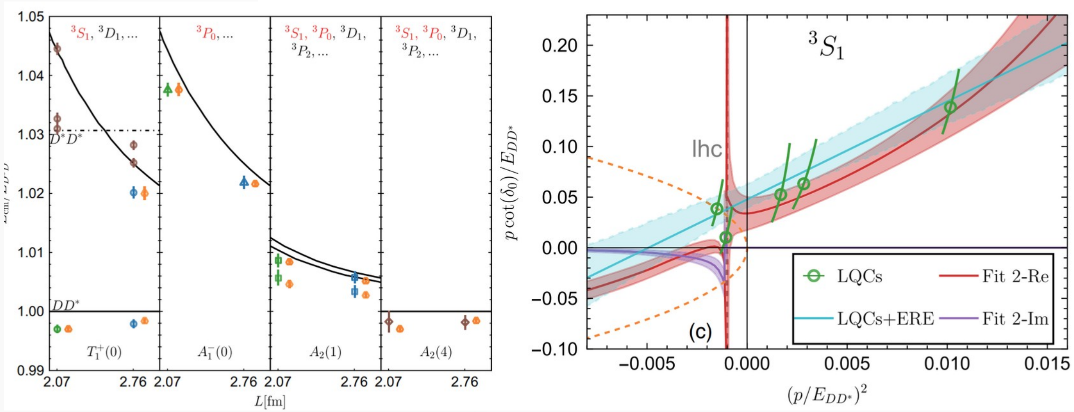
Limit ERE

$$p\cot\delta = -\frac{2\pi}{\mu}\frac{1}{T} + ik$$

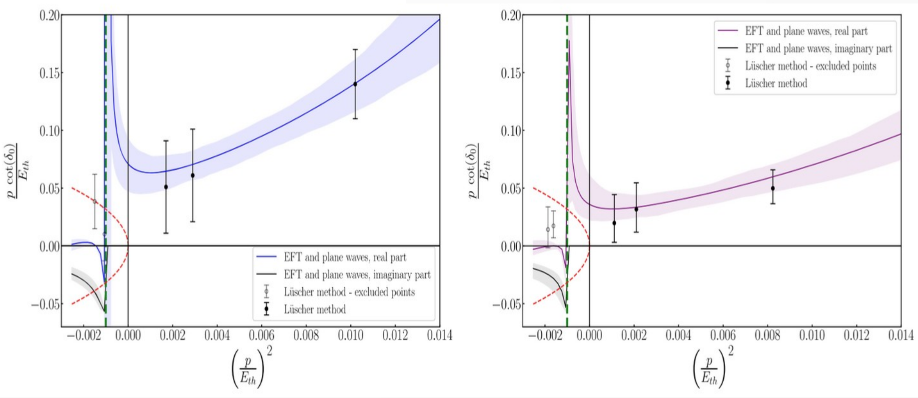


$$\frac{1}{T_\ell^{\mathrm{II}}} = \frac{1}{T_\ell} + 2i\rho$$

$$T^{\mathrm{II}} = \frac{1}{\frac{D}{N} + 2i\rho} = \frac{N}{D + 2i\rho N}$$



Meng et al., PRD111(2025)034509



meson-meson

+ quark-antidiquark

Prelovsek et al., PRD112(2025)014507

Effective-range expansion (ERE) with the left-hand cut

Du et al., PRL135(2025)011903

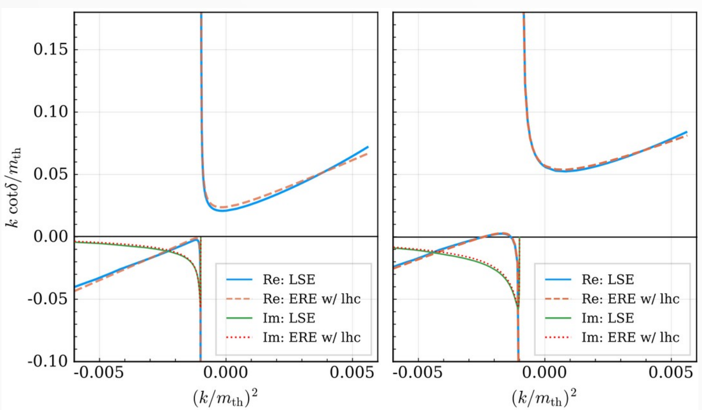
$$d_u^{\mathrm{R}}(k^2) = \frac{i}{4k}\left(\log\frac{\mu_+/2+ik}{\mu_+/2-ik} - \log\frac{\mu_+/2+i\eta k}{\mu_+/2-i\eta k}\right)$$

$$L(k^2) = -\frac{1}{4k^2}\log\frac{\mu_+^2/4+k^2}{\mu_+^2/4+\eta^2k^2}$$

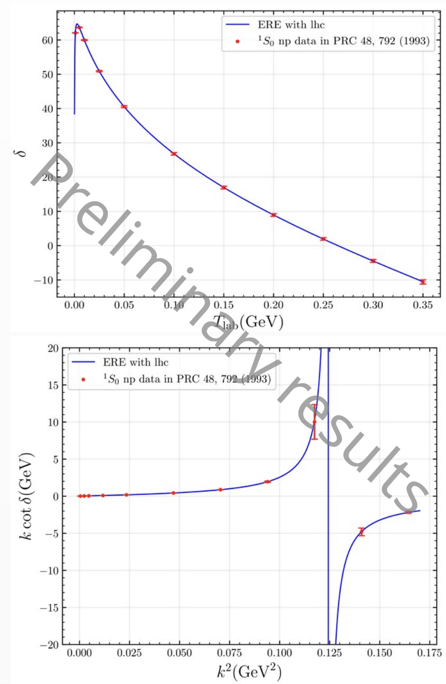
$$\boxed{f_{[0,1]}(k^2) = \left[\frac{\tilde{d}_0+\tilde{d}_1k^2-\tilde{g}d^{\mathrm{R}}(k^2)}{1+\tilde{g}(L(k^2)-L_0)} - ik\right]^{-1}}$$

$\tilde{g}\to 0$

$$\frac{1}{f(k^2)} = \frac{1}{a} + \frac{1}{2}rk^2 - ik$$



DD* scattering amplitude



$$\boxed{\frac{1}{f_{[m,n]}(k^2)} = \frac{\sum_{i=0}^{n}\tilde{d}_ik^{2i}-\tilde{g}d^{\mathrm{R}}(k^2)}{1+\sum_{j=1}^{m}\tilde{n}_jk^{2j}+\tilde{g}(L(k^2)-L_0)} - ik}$$

ERE w/lhc	$g^2_{\pi NN}/4\pi$	$E_{\rm pole}-m_{\rm thr}$
d_0, d_1, d_2, n_1, g_0	$12.4^{+2.2}_{-2.3}$	-66.4 keV
d_0, d_1, n_1, n_2, g_0	$11.0^{+2.6}_{-2.8}$	-66.6 keV
$d_0, d_1, d_2, n_1, n_2, g_0$	$13.0^{+3.0}_{-4.1}$	-66.3 keV

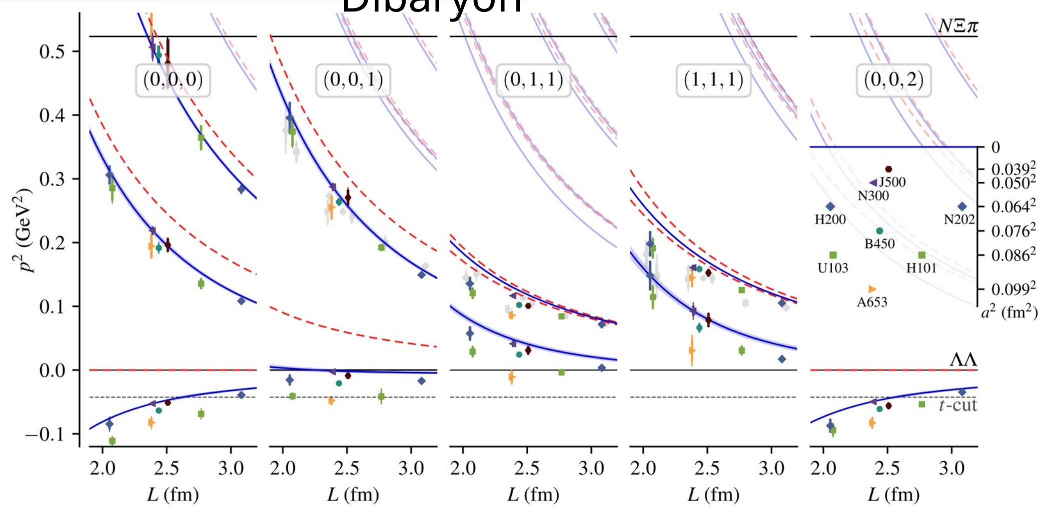
TABLE I. Some deduced values for the πNN coupling constant. The quoted uncertainty are those quoted by the authors and usually do not include systematic uncertainties.

Source	Year	System	$g^2_{\pi NN}/4\pi$
Karlsruhe-Helsinki [3]	1980	πp	14.28(18)[a]
Kroll [4]	1981	pp	14.52(40)[a]
Nijmegen [6]	1993	pp,np	13.58 (5)[a]
VPI [8]	1994	pp,np	13.70
Nijmegen [7]	1997	pp,np	13.54 (5)[a]
Timmermans [12]	1997	π^+p	13.45(14)[a]
VPI [9]	1994	GMO, πp	13.75(15)[a]
Uppsala [2]	1998	$np\to pn$	14.52(26)
Pavan *et al.* [11]	1999	πp	13.73 (9)
Schröder *et al.*, corrected [14,10]	1999	GMO, $\pi^\pm p$	13.77(18)
Present work	2001	GMO, $\pi^\pm p$	14.11(20)

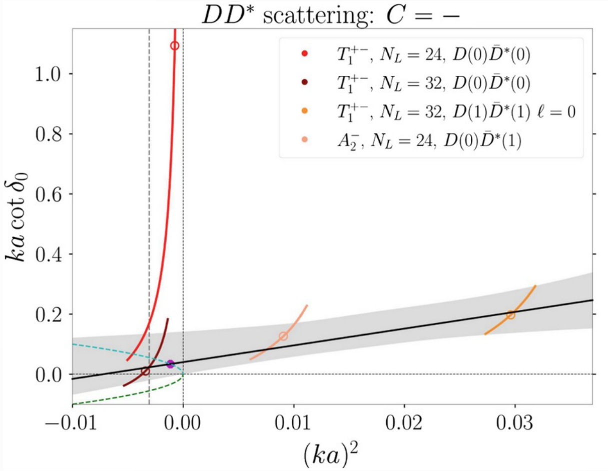
[a]Statistical uncertainty only.

Long-range force on the Lattice

Dibaryon



Green et al., PRL127(2021)242003



Sadl et al., PRD111(2025)054513

Related recent works on FV w/ LHC...

Plane-wave basis to treat long-range interactions	Meng and Epelbaum, JHEP (2021) Meng et al., PRD (2024)
Project to irep. of the cubic to avoid the lhc associated to the partial wave projection	Mai and Döring, EPJA (2017), PRL (2019)
Generalization of the Lüscher + *K*-matrix	Hansen and Raposo, JHEP (2024)
Three-body framework (automatically includes lhc)	Dawid et al., PRD (2023) Hansen et al., PRD (2024)
Modify the Lüscher formula via “modified effective range expansion”	Bubna et al., JHEP (2024)

Integral equations!

quantization conditionfor the ERE w/ the lhc?

Summary and Propects

- **Charmed mesons (chiral symmetry + SU(3))**
- **Hidden charm tetraquark (OPE + SU(3) + crossed-channel + triangle singularity)**
- **Hidden charm pentaquark (production mechanism, lattice: coupled-channel)**
- **Doubly charm tetraquark (the long-range force, in hidden charm tetraquark also)**

Thank you very much for your attention!

Thank you very much for your attention!