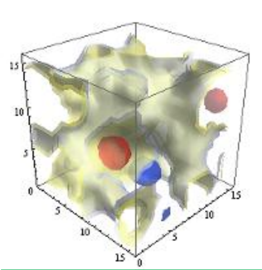
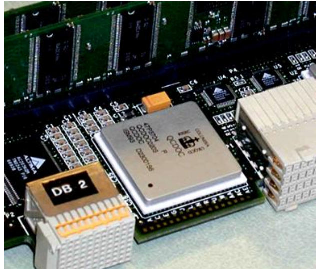
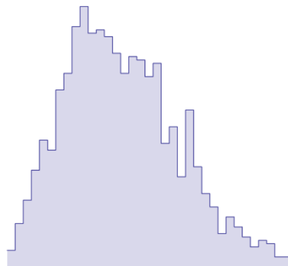
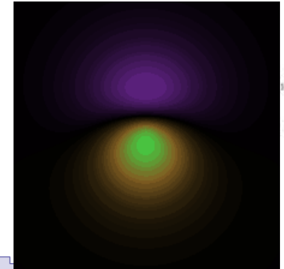
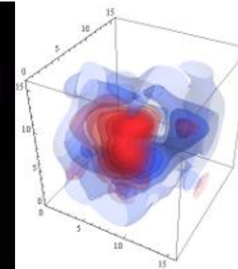
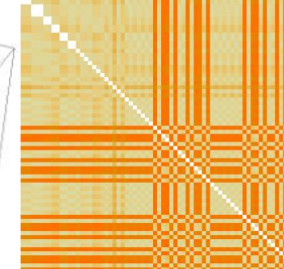
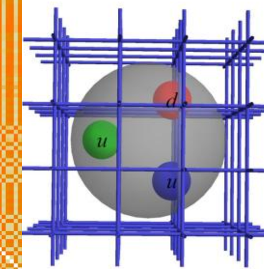




# Advancing Nucleon and Pion Parton Distributions with Lattice QCD: Insights and Impact on Global Analyses



HUEY-WEN LIN



This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award


@LinQCD

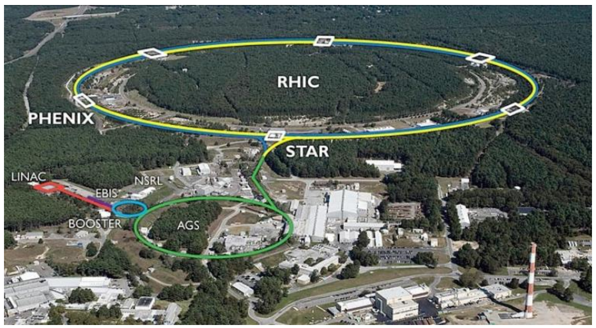
# Outline

- § Lattice QCD and Parton Distribution Functions
- § Selected *x*-Dependent Parton Distributions
  - Flavor non-singlet PDFs
  - Gluon PDFs
  - Generalized Parton Distributions (GPDs)
- § Impact of Lattice-QCD PDFs on Global Fits

# Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, EIcC, LHeC, …)





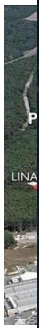
Imaging of the proton

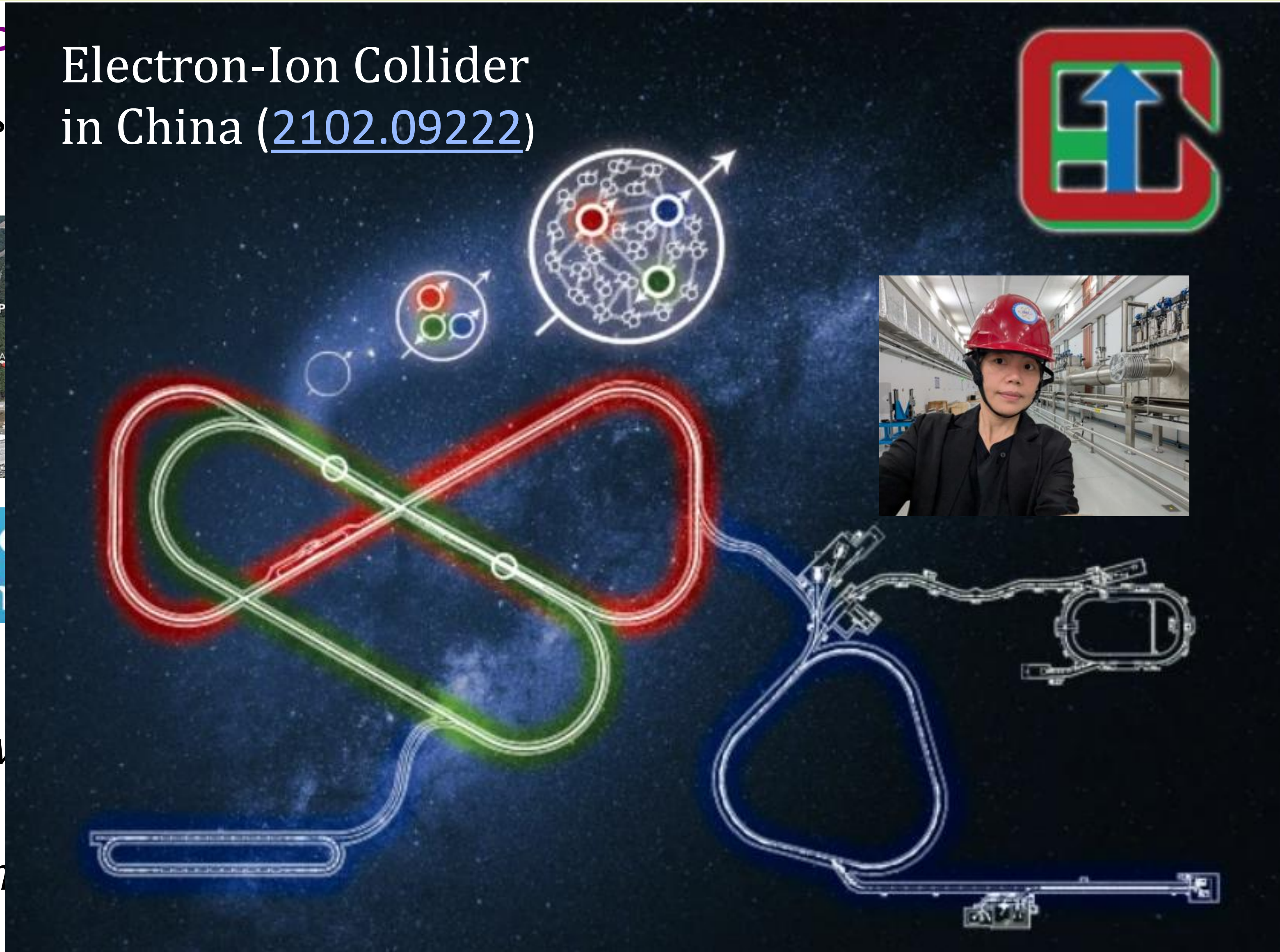
*How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?*

EIC White Paper, 1212.1701; The Present and Future of QCD (2303.02579)

# Parton Distribution Functions

§ P … on






El
Th

*How*
*and*
*mon*
EIC

# Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

**Theory Input** → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$$

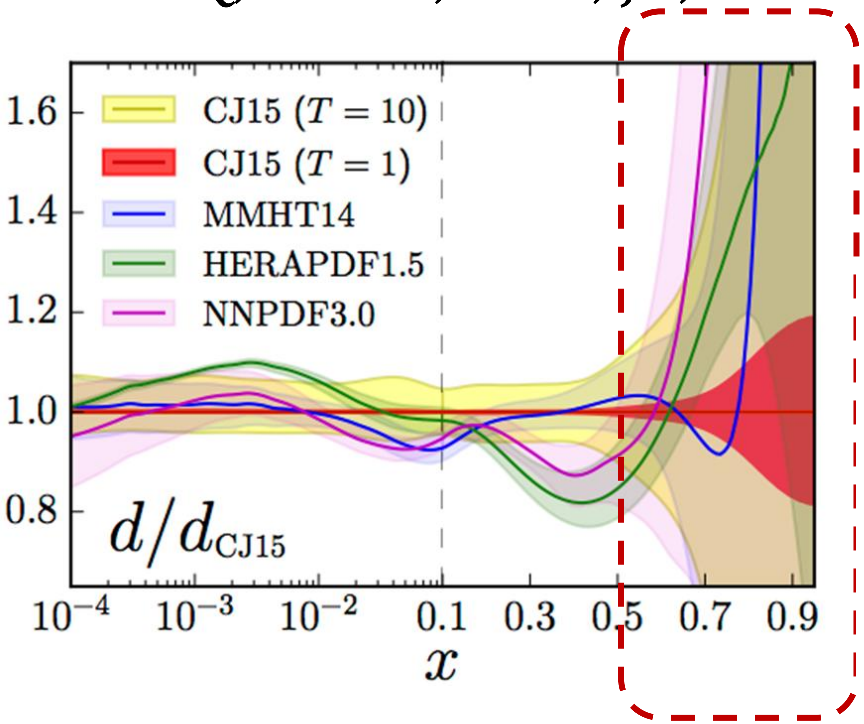
- Assumptions imposed

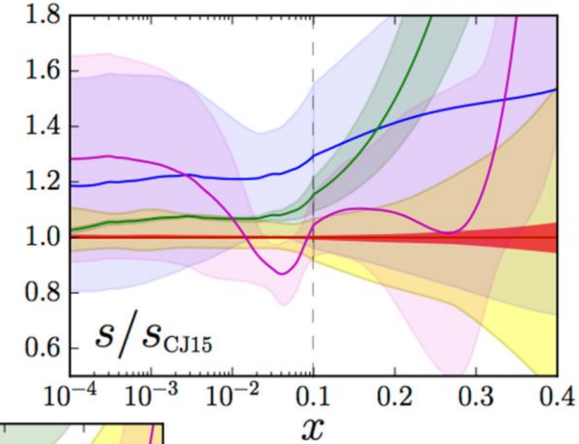
  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

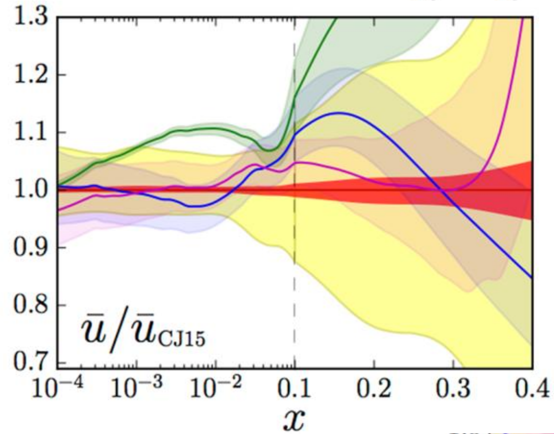
$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

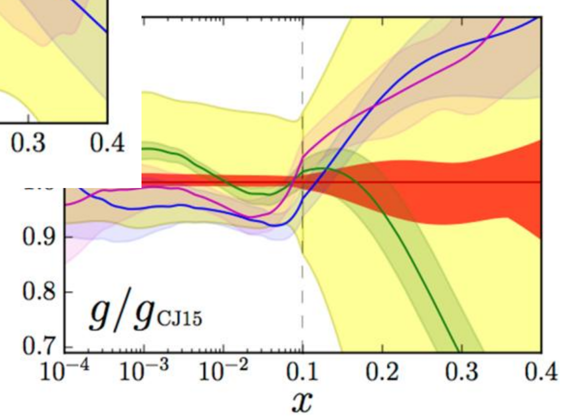
# Global Analysis

- Discrepancies appear when data is scarce
- Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.



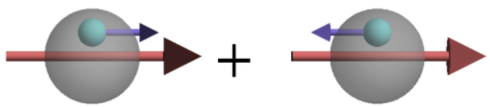




CTEQ-JLAB
https://www.jlab.org/theory/cj/

# PDFs on the Lattice

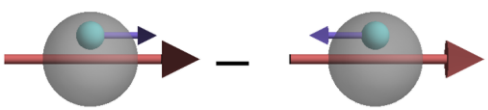
§ Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



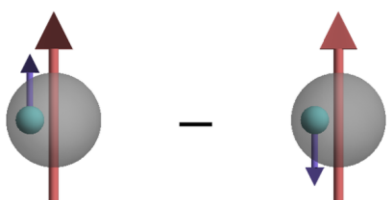
spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



spin-dependent
transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

§ True distribution can only be recovered with all moments

# Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$


LatticePDF Report, 1711.07916, 2006.08636

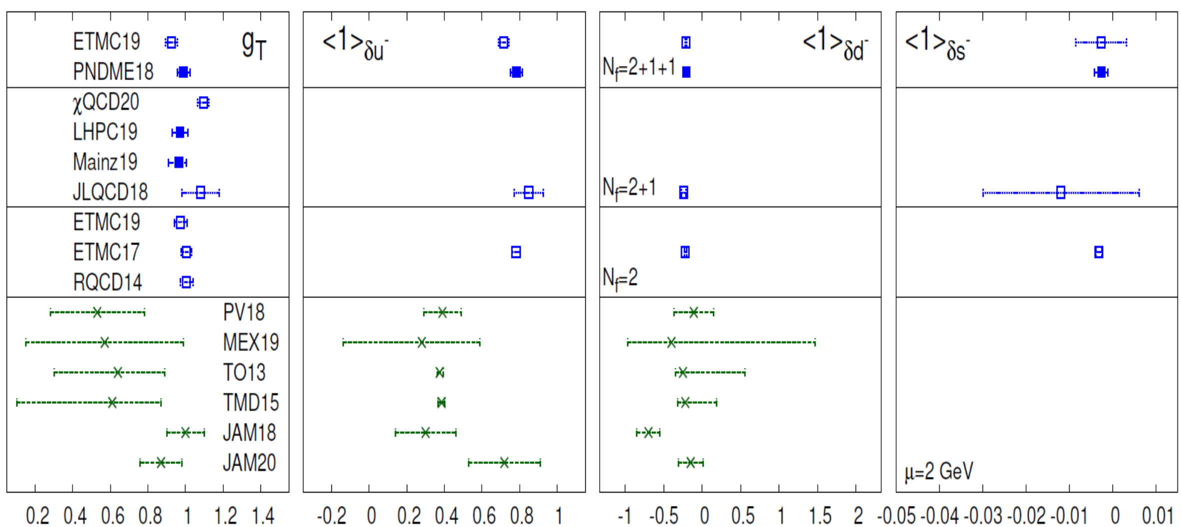
| Moment | Collaboration | Reference | $N_f$ | DE | CE | FV | RE | ES | | Value | Global Fit |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $g_T$ | ETMC 19 | (Alexandrou *et al.*, 2019b) | 2+1+1 | ■ | ★ | ○ | ★ | ★ | ** | 0.926(32) | 0.10 — 1.1 |
| | PNDME 18 | (Gupta *et al.*, 2018) | 2+1+1 | ★ | ★ | ★ | ★ | ★ | * | 0.989(32)(10) | |
| | $\chi$QCD 20 | (Horkel *et al.*, 2020) | 2+1 | ■ | ★ | ○ | ★ | ★ | † | 1.096(30) | |
| | LHPC 19 | (Hasan *et al.*, 2019) | 2+1 | ○ | ★ | ○ | ★ | ★ | * | 0.972(41) | |
| | Mainz 19 | (Harris *et al.*, 2019) | 2+1 | ★ | ○ | ★ | ★ | ★ | | $0.965(38)(^{+13}_{-41})$ | |
| | JLQCD 18 | (Yamanaka *et al.*, 2018) | 2+1 | ■ | ○ | ○ | ★ | ★ | | 1.08(3)(3)(9) | |
| | ETMC 19 | (Alexandrou *et al.*, 2019b) | 2 | ■ | ★ | ○ | ★ | ★ | ** | 0.974(33) | |
| | ETMC 17 | (Alexandrou *et al.*, 2017d) | 2 | ■ | ★ | ■ | ★ | ★ | | 1.004(21)(02)(19) | |
| | RQCD 14 | (Bali *et al.*, 2015) | 2 | ○ | ★ | ★ | ★ | ■ | | 1.005(17)(29) | |
| $\langle 1\rangle_{\delta u^-}$ | ETMC 19 | (Alexandrou *et al.*, 2019b) | 2+1+1 | ■ | ★ | ○ | ★ | ★ | ** | 0.716(28) | -0.14 — 0.91 |
| | PNDME 18 | (Gupta *et al.*, 2018) | 2+1+1 | ★ | ★ | ★ | ★ | ★ | * | 0.784(28)(10) | |
| | JLQCD 18 | (Yamanaka *et al.*, 2018) | 2+1 | ■ | ○ | ○ | ★ | ★ | | 0.85(3)(2)(7) | |
| | ETMC 17 | (Alexandrou *et al.*, 2017d) | 2 | ■ | ★ | ■ | ★ | ★ | | 0.782(16)(2)(13) | |
| $\langle 1\rangle_{\delta d^-}$ | ETMC 19 | (Alexandrou *et al.*, 2019b) | 2+1+1 | ■ | ★ | ○ | ★ | ★ | ** | -0.210(11) | -0.97 — 0.47 |
| | PNDME 18 | (Gupta *et al.*, 2018) | 2+1+1 | ★ | ★ | ★ | ★ | ★ | * | -0.204(11)(10) | |
| | JLQCD 18 | (Yamanaka *et al.*, 2018) | 2+1 | ■ | ○ | ○ | ★ | ★ | | -0.24(2)(0)(2) | |
| | ETMC 17 | (Alexandrou *et al.*, 2017d) | 2 | ■ | ★ | ■ | ★ | ★ | | -0.219(10)(2)(13) | |
| $\langle 1\rangle_{\delta s^-}$ | ETMC 19 | (Alexandrou *et al.*, 2019b) | 2+1+1 | ■ | ★ | ○ | ★ | ★ | ** | -0.0027(58) | N/A |
| | PNDME 18 | (Gupta *et al.*, 2018) | 2+1+1 | ★ | ★ | ★ | ★ | ★ | * | -0.0027(16) | |
| | JLQCD 18 | (Yamanaka *et al.*, 2018) | 2+1 | ■ | ○ | ○ | ★ | ★ | | -0.012(16)(8) | |
| | ETMC 17 | (Alexandrou *et al.*, 2017d) | 2 | ■ | ★ | ■ | ★ | ★ | | -0.00319(69)(2)(22) | |

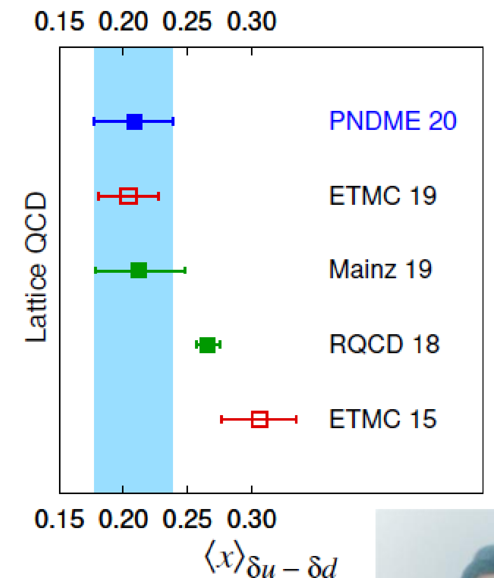
# Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$


LatticePDF Reports, 1711.07916, 2006.08636

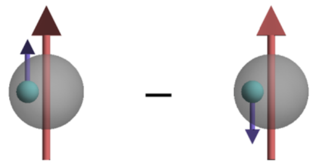


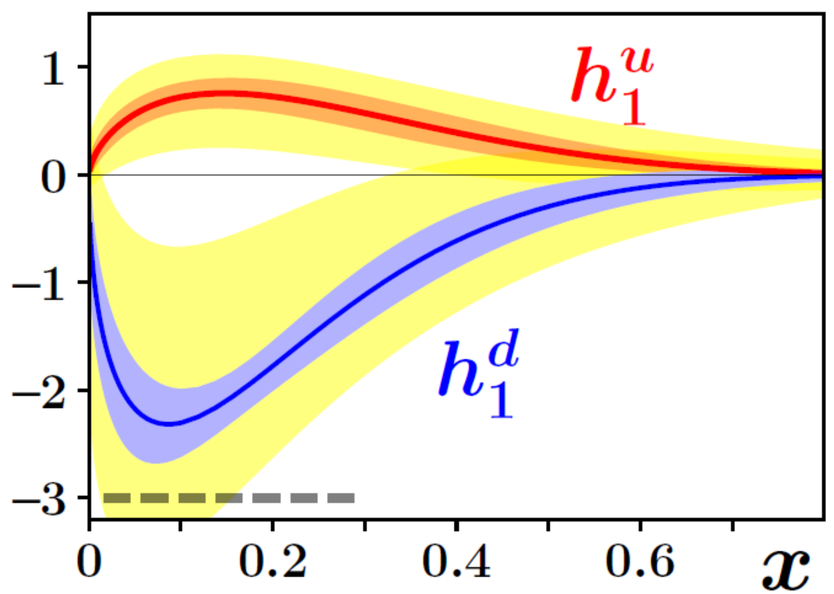

S. Mondal et al 2005.13779

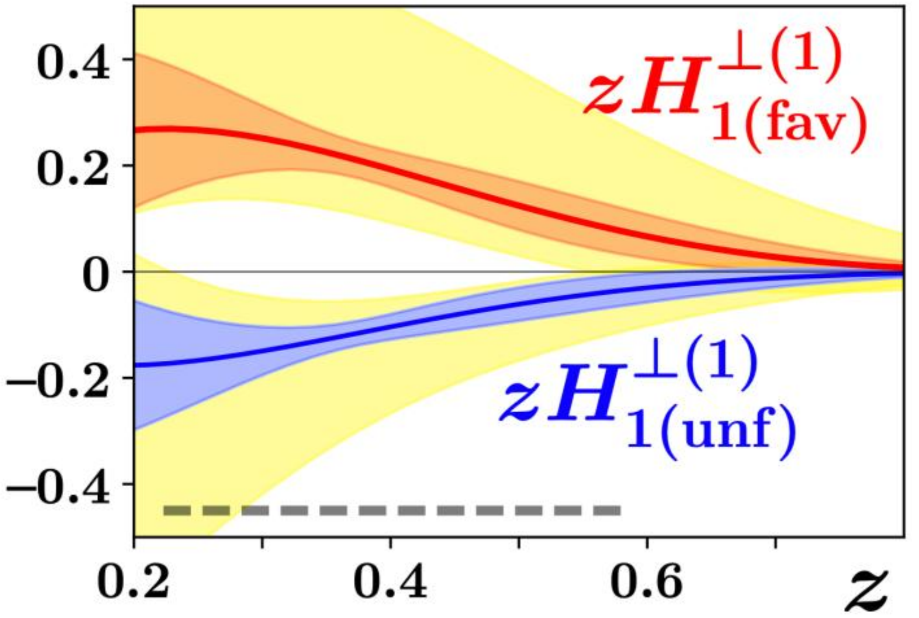
# From Charges to PDFs

- Improved **transversity** distribution with LQCD tensor charge, $g_T = \int_{-1}^{1} dx\,(\,h_1^u(x) - h_1^d(x)\,)$



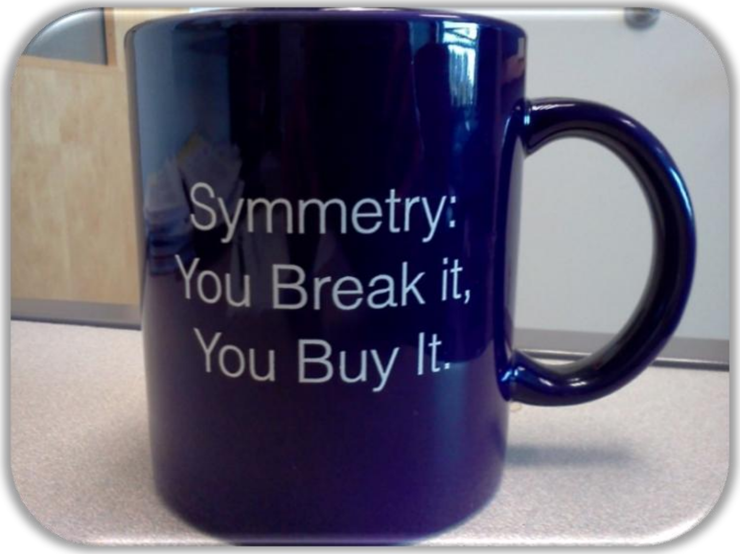

  - Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
  - Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets




Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

# Lattice Structure Limitation




§ Lattice calculations rely on operator product expansion, only provide moments

$$\left\langle x^{n-1}\right\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

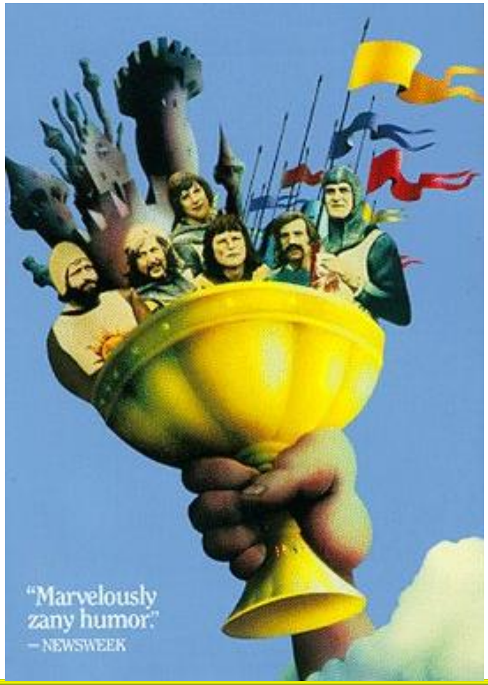
§ Longstanding obstacle!

- Holy grail of structure calculations

§ Applies to many structure quantities:

- Parton distribution functions (PDF)
- Generalized parton distributions (GPD)
- Transverse-momentum distributions (TMD)

⋮



# A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

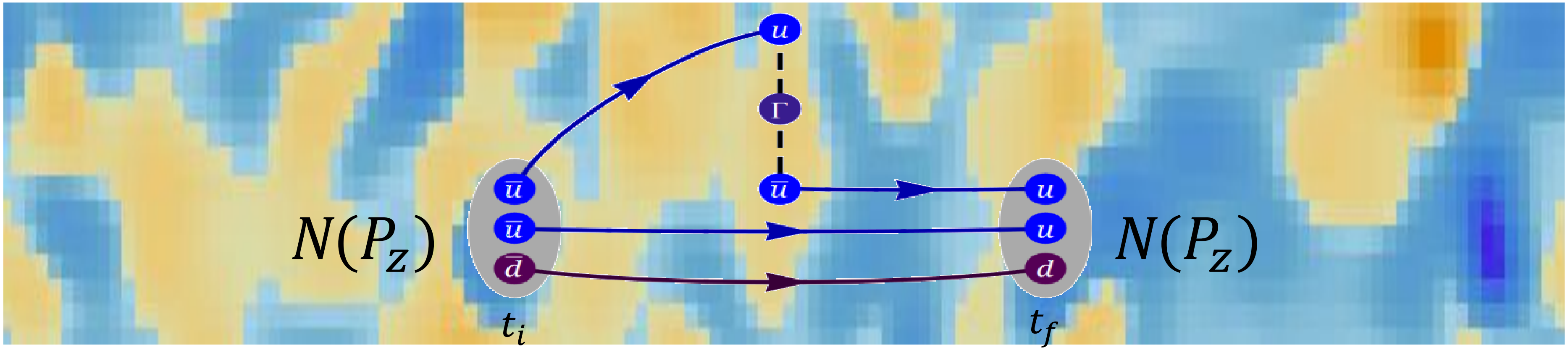
A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

# Lattice Parton Method

- Large-momentum effective theory (LaMET)/quasi-PDF
  (X. Ji, 2013; See 2004.03543 for review)




- Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0)\right| P \right\rangle$$

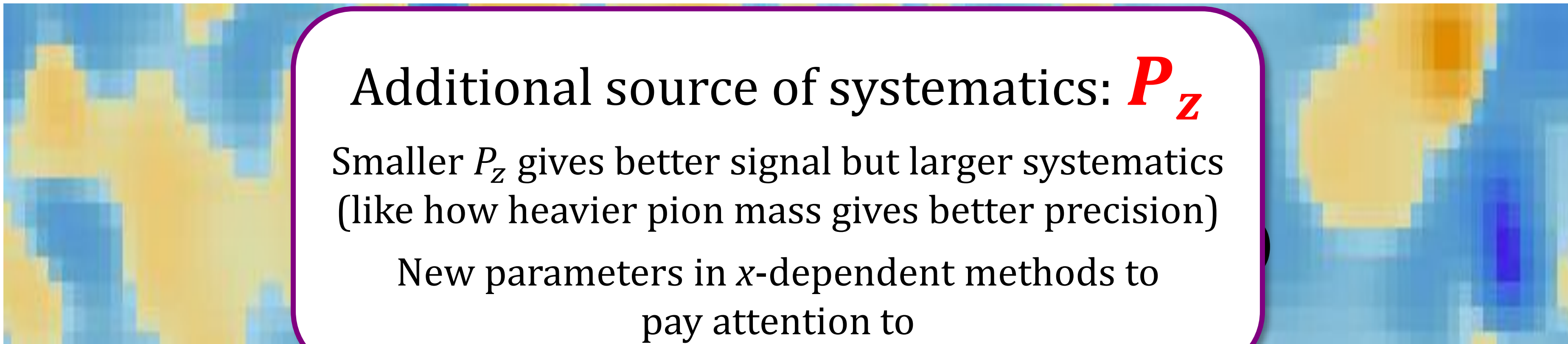
- Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(\frac{M_N^2}{\boldsymbol{P_z^2}},\frac{\Lambda_{\rm QCD}^2}{(x\boldsymbol{P_z})^2},\frac{\Lambda_{\rm QCD}^2}{((1-x)\boldsymbol{P_z})^2}\right)$$

X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

# Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF
(X. Ji, 2013; See 2004.03543 for review)



**Additional source of systematics: $\boldsymbol{P_z}$**

Smaller $P_z$ gives better signal but larger systematics
(like how heavier pion mass gives better precision)

New parameters in $x$-dependent methods to
pay attention to

§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z)=\int\frac{dz}{4\pi}e^{-izk_z}\left\langle P\left|\bar{\psi}(z)\Gamma\exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0)\right|P\right\rangle$$

§ Recover true distribution (take $P_z\rightarrow\infty$ limit)

$$\tilde{q}(x,\mu,P_z)=\int_{-\infty}^{\infty}\frac{dy}{|y|}C\left(\frac{x}{y},\frac{\mu}{P_z}\right)\boldsymbol{q(y,\mu)}+\mathcal{O}\left(\frac{M_N^2}{\boldsymbol{P_z^2}},\frac{\Lambda_{\mathrm{QCD}}^2}{(x\boldsymbol{P_z})^2},\frac{\Lambda_{\mathrm{QCD}}^2}{\left((1-x)\boldsymbol{P_z}\right)^2}\right)$$

X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

# Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today $= \sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

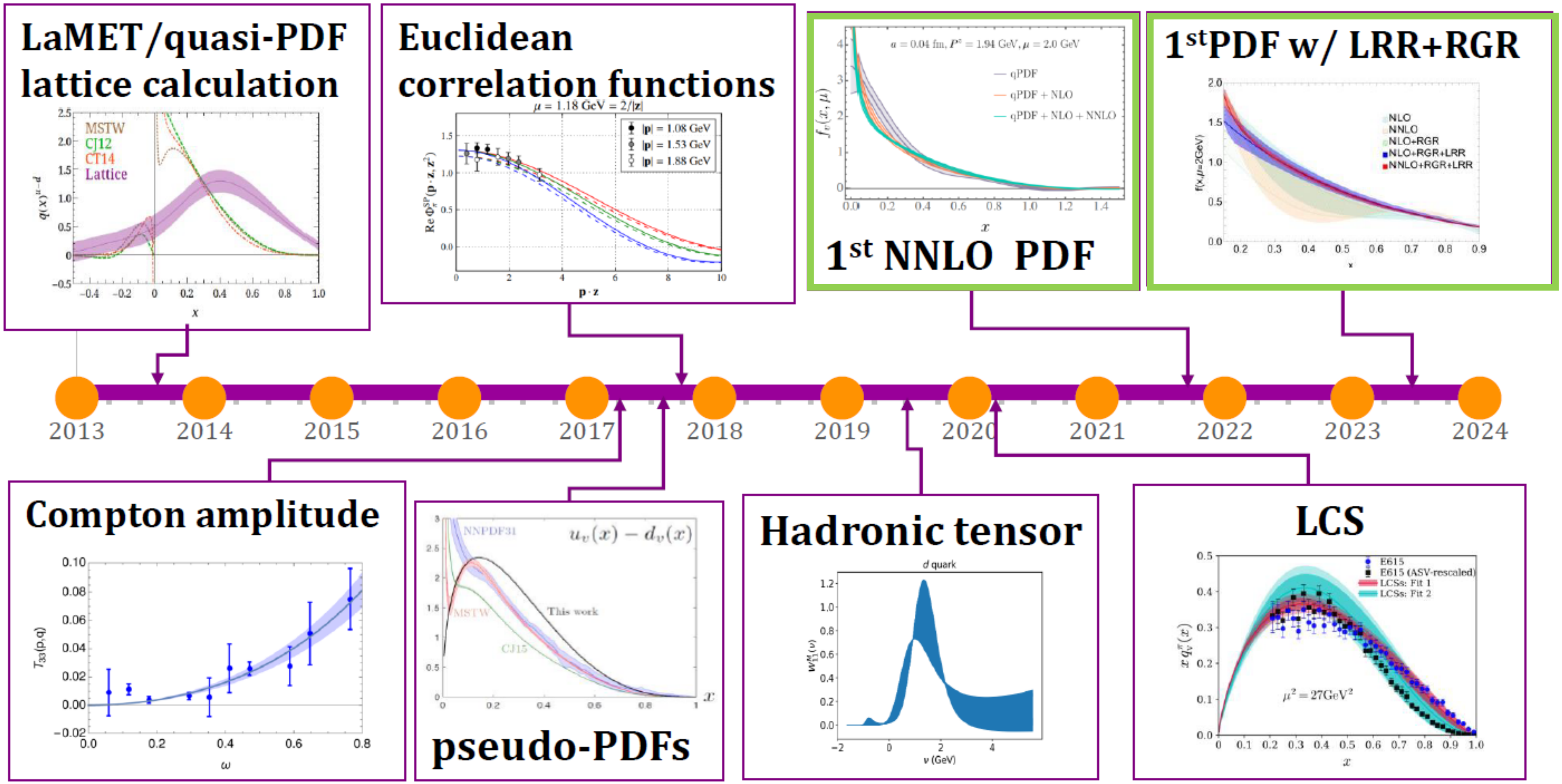
- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Compton amplitude method (A.J. Chambers et al., 1703.01153)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)

⋮

# Lattice Parton Calculations

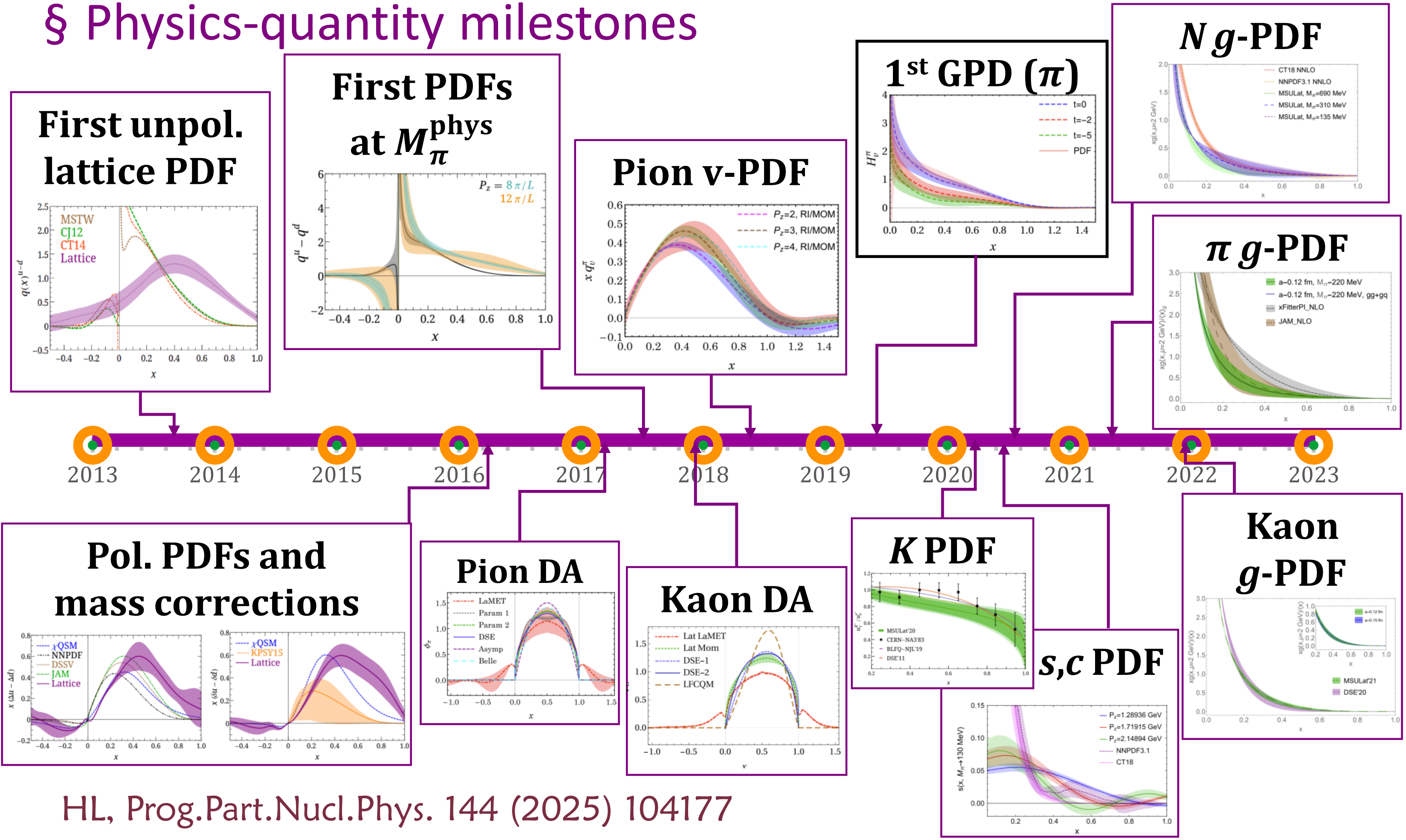
## § Rapid developments!

HL, Prog.Part.Nucl.Phys. 144 (2025)



# Lattice Parton Calculations

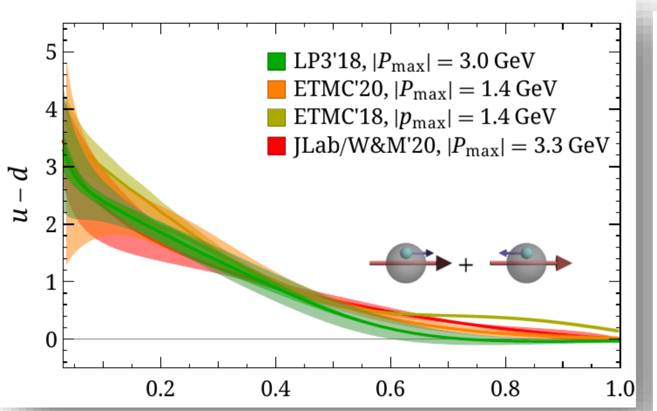
## § Physics-quantity milestones

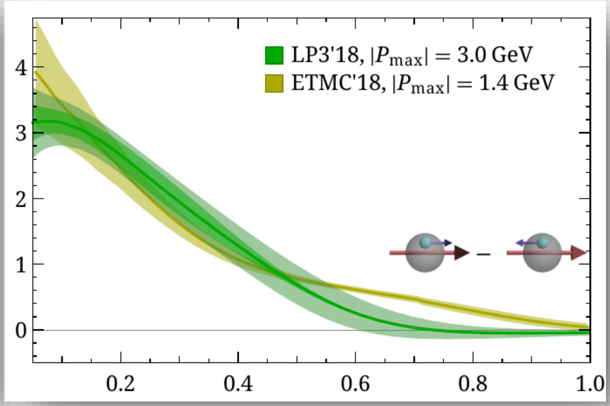


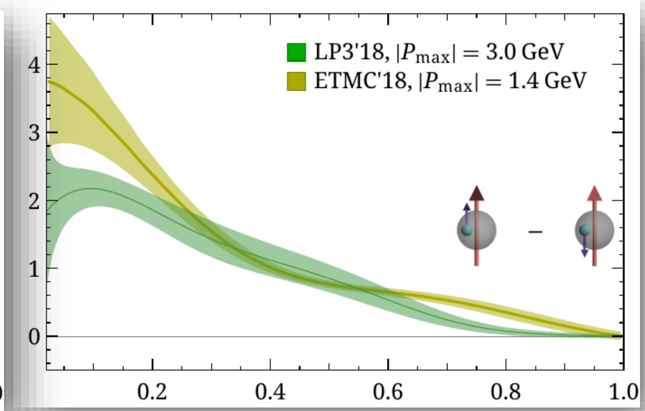

HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

# Lattice Example Results

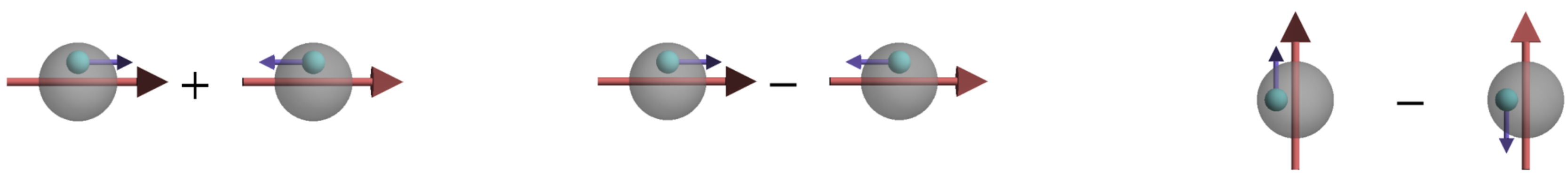
## § Summary of physical pion mass results: 2006.08636 (PDFLattice2019)



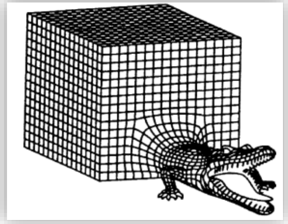




unpolarized | longitudinally polarized | transversely polarized



$u(x) - d(x)$ | $\Delta u(x) - \Delta d(x)$ | $\delta u(x) - \delta d(x)$



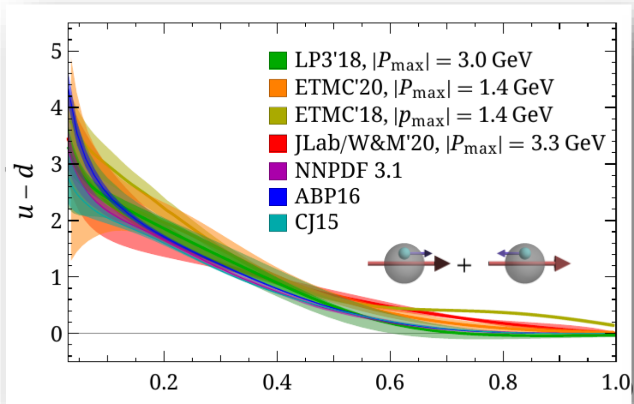
Finite volume,
Discretization,
...

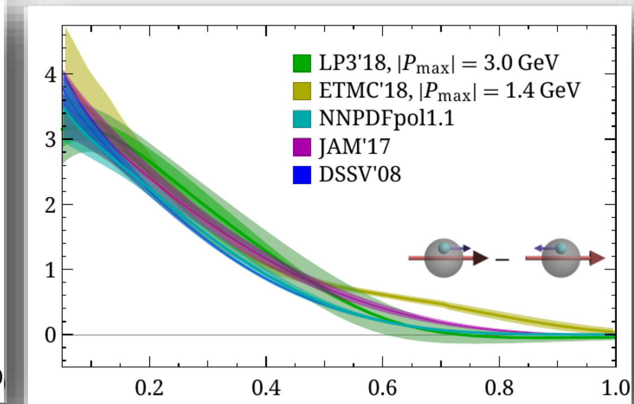
2006.08636 (PDFLattice2019)

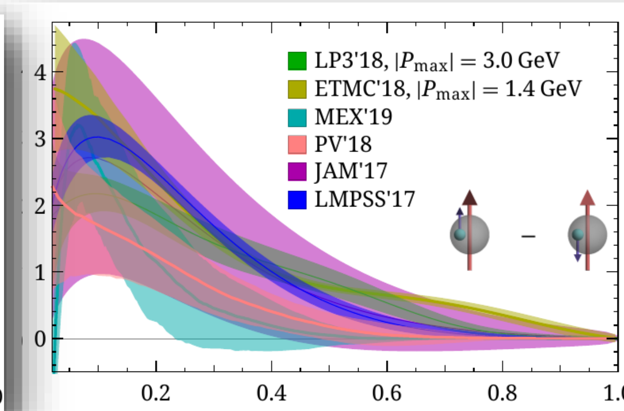
# Lattice Example Results

## § Summary of physical pion mass results:

2006.08636 (PDFLattice2019)



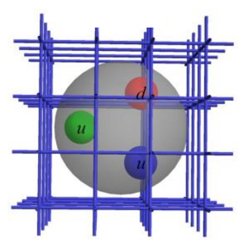



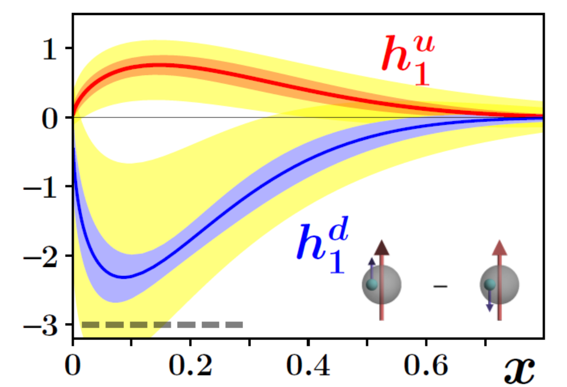

## § Complementary Lattice inputs for best PDFs

**Theory Input** → **Global Analysis of PDFs** ← **Exp't Input**

**+**



Works have been developed in this direction




Phys. Rev. Lett. 120, 152502 (2018)

# Isovector PDFs Update

§ Nucleon isovector PDF calculated directly at **physical pion mass**

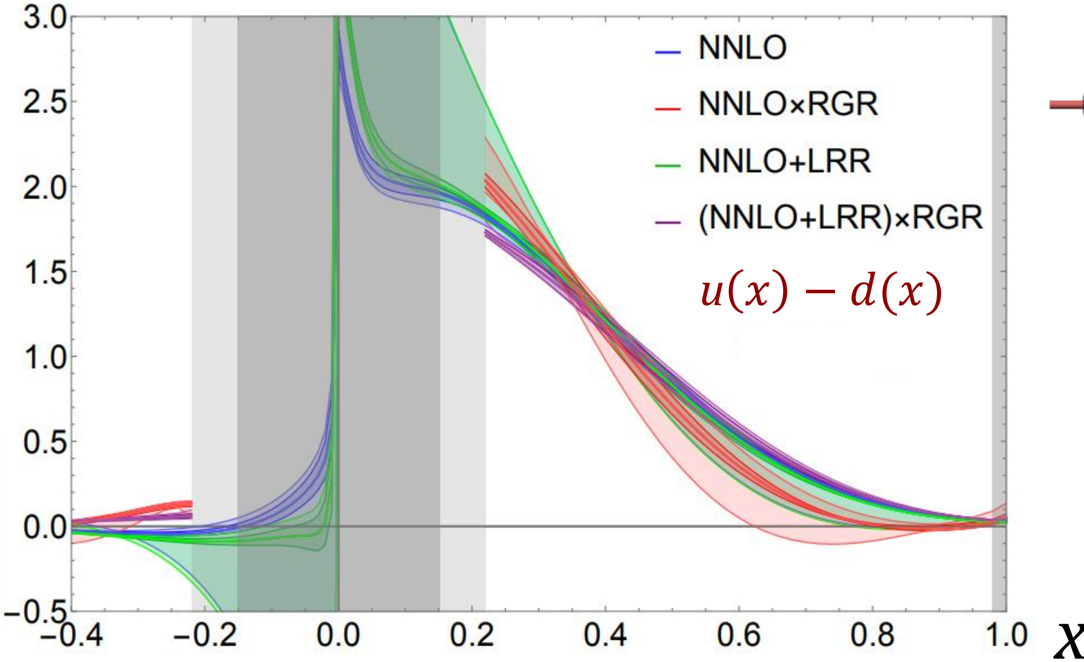
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

R. Zhang, et. al.
PLB 844, 138081 (2023)

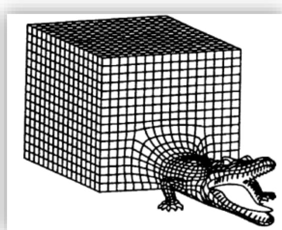
- $N_f = 2+1+1$ clover/HISQ, $a \approx 0.09$ fm, $P_z \approx 2$ GeV

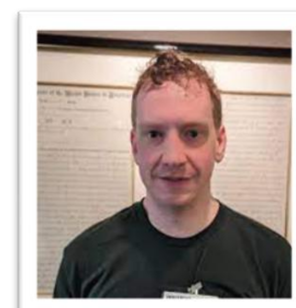
J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…





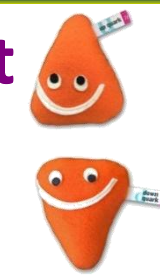


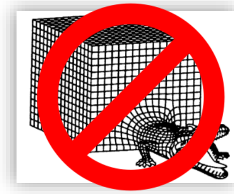


P: Jack Holligan

# Continuum PDF

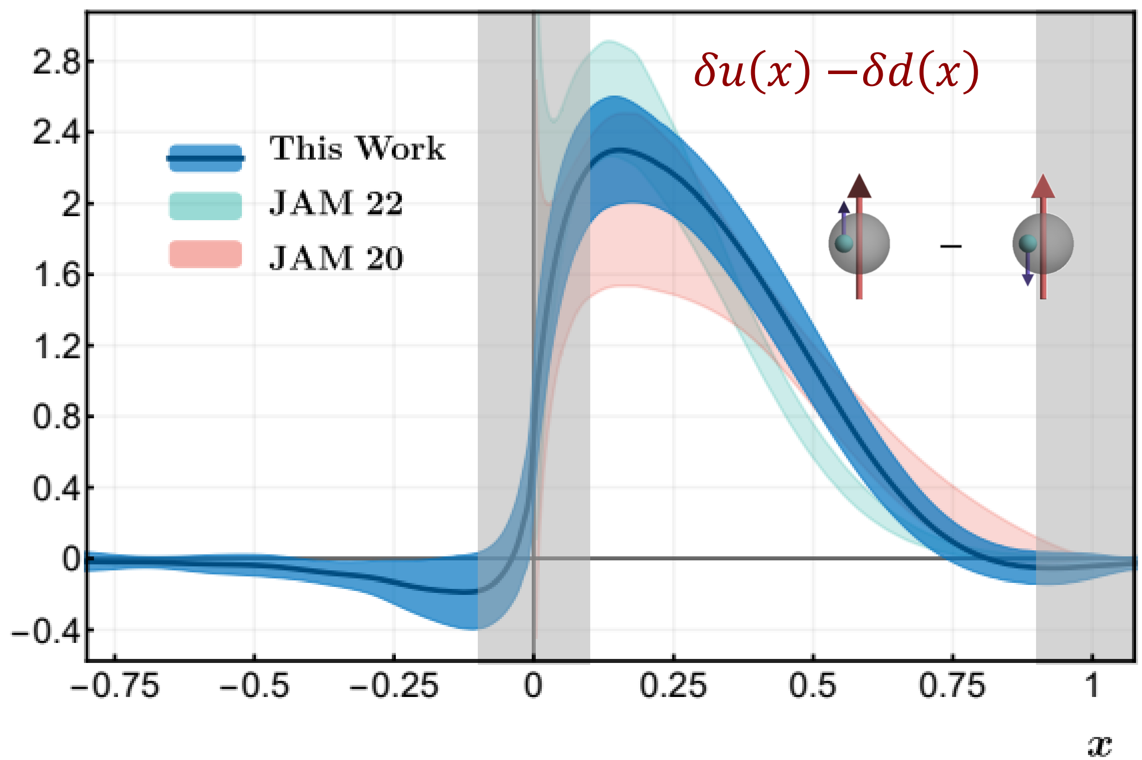
## § Nucleon PDFs using quasi-PDFs in the **continuum limit**





- Lattice details: clover/2+1 clover (LPC)
  - $a \approx \{0.49, 0.64, 0.85, 0.98\}$ fm,
  - $M_\pi \in [222, 354]$-MeV pion,
  - $M_\pi L \in [3.9, 8.1]$
  - $P_z \in [1.8, 2.8]$

F. Yao et al (LPC), 2208.08008



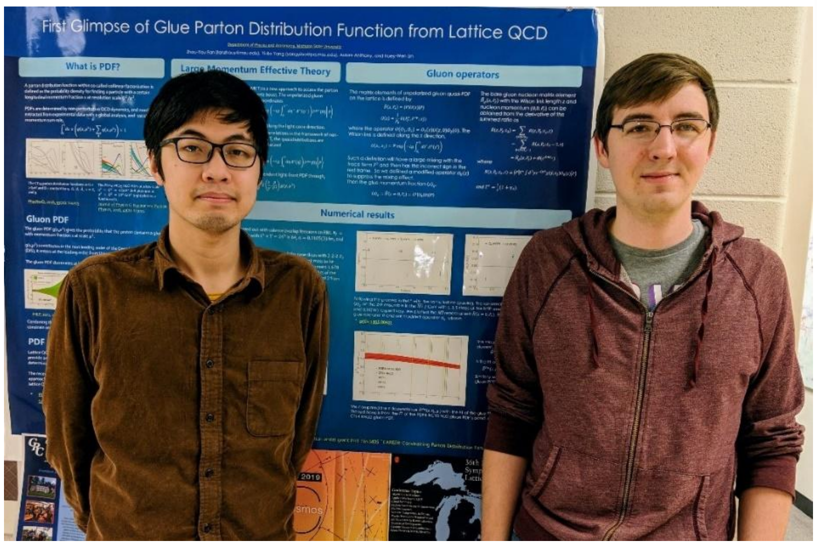

Wanted PDFs, GPDs, etc…

# Nucleon Gluon PDF (2018)

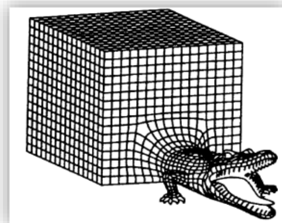
## § Pioneering first glimpse into gluon PDF using LaMET

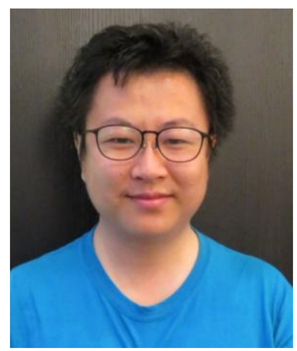
- Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- Promising results using coordinate-space comparison, but signal does not go far in $z$
- Hard numerical problem to be solved

Fan et al, Phys.Rev.Lett. 121, 242001 (2018)

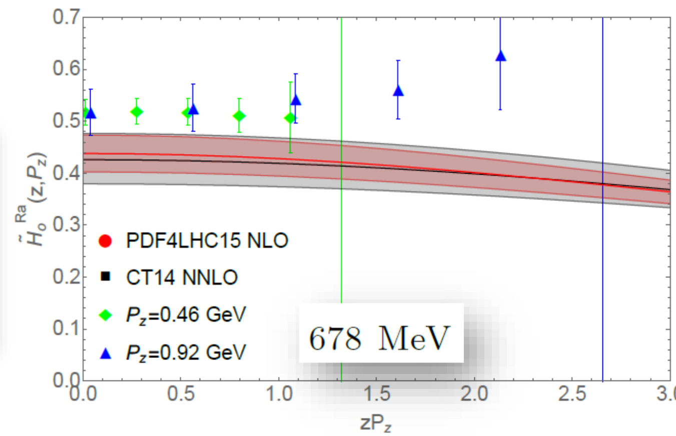


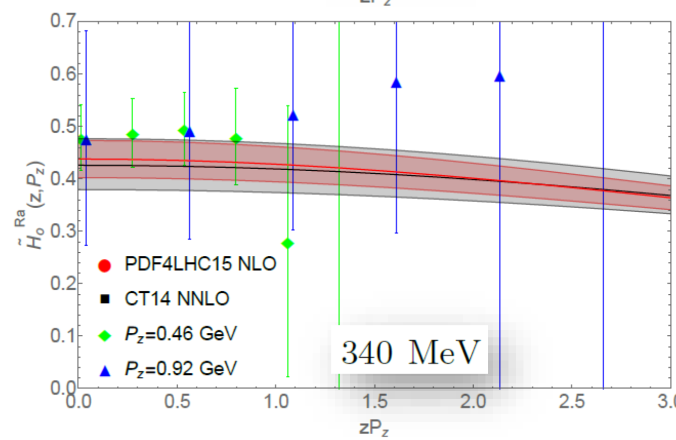

G: Zhouyou Fan G: Adam Antony





P: Yi-Bo Yang



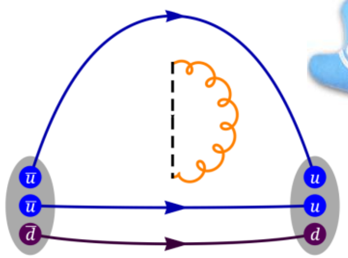

iCER@MSU is crucial for earlier code development and completion of this work

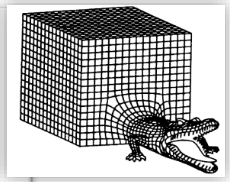
# Nucleon Gluon PDF (2020)

## § Gluon PDF using pseudo-PDF

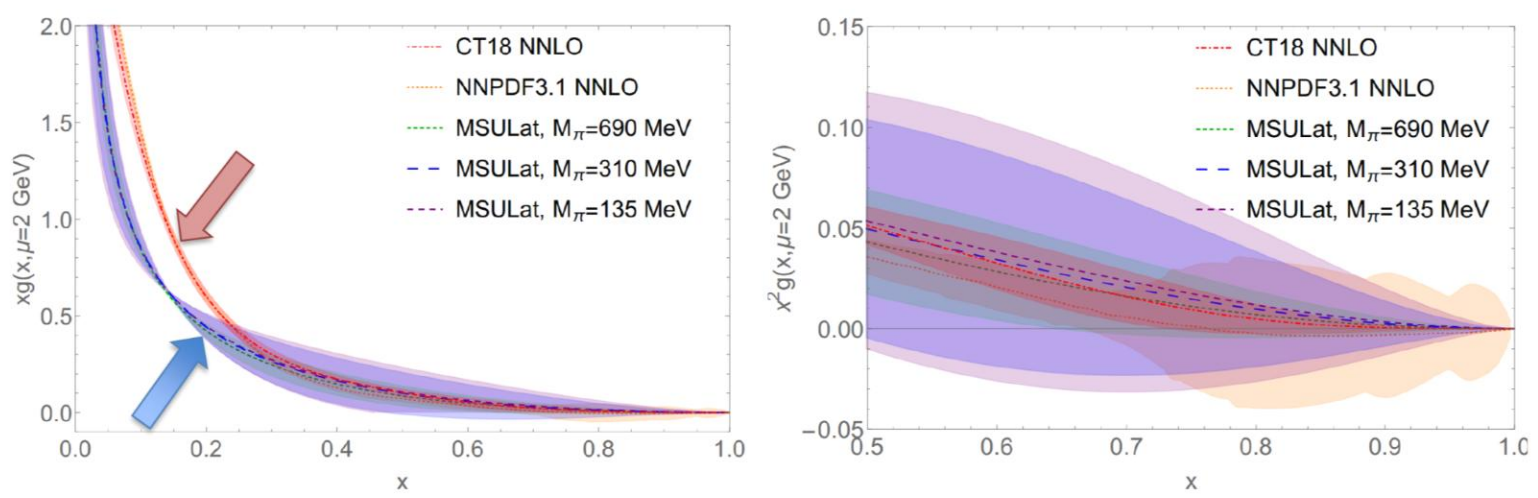
- Lattice details: clover/2+1+1 HISQ 0.12 fm, 310-MeV sea pion
- Study strange/light-quark

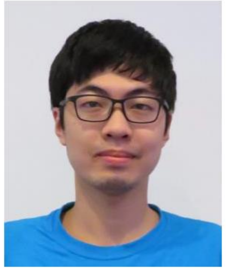
Z. Fan. et al (MSULat), 2007.16113



The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{\mathrm{MS}}$ scheme.



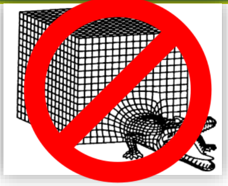



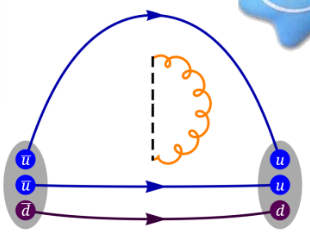
Slide by Zhouyou Fan@DNP2020

FG: Zhouyou Fan

# Gluon PDF in Nucleon

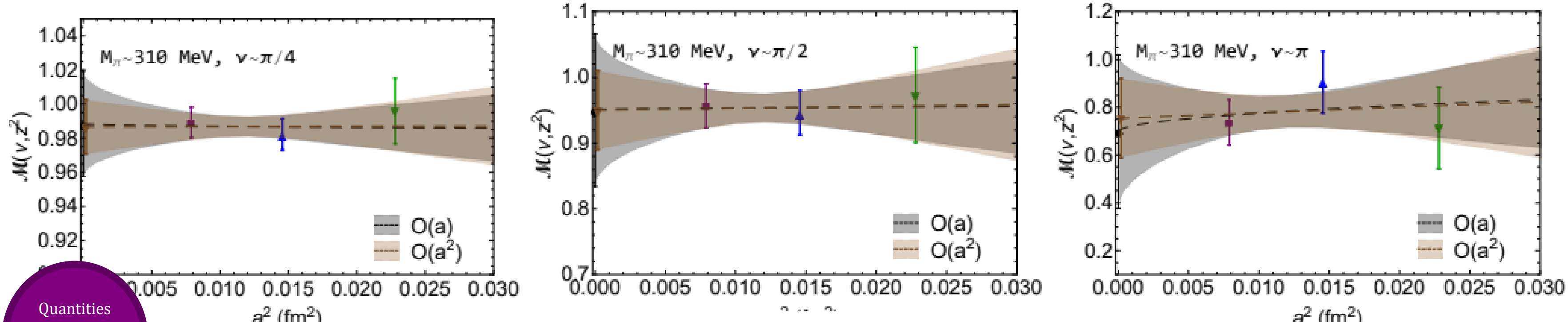
## § Continuum Gluon PDF w/ pseudo-PDF



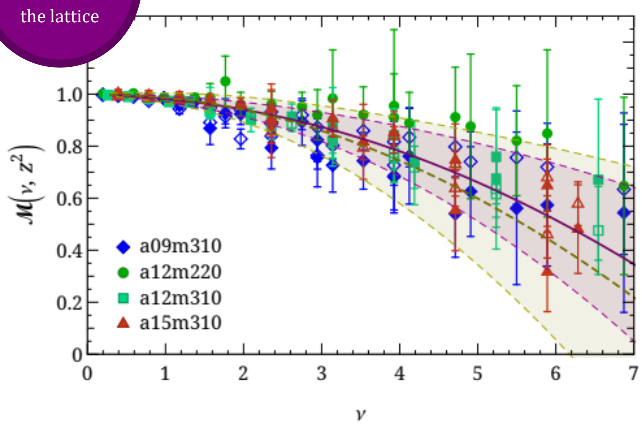


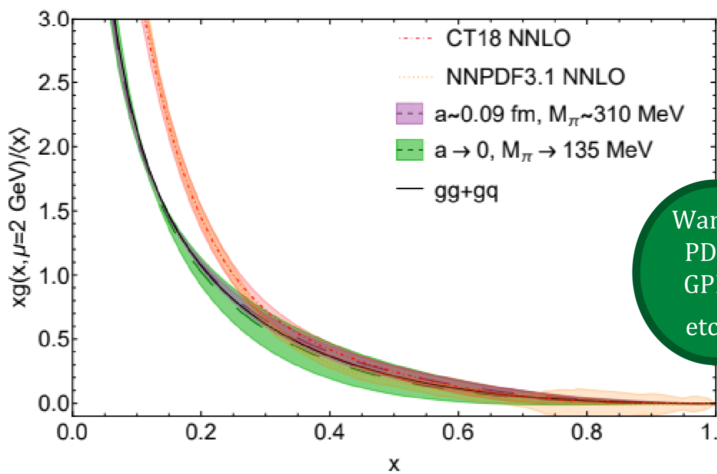

- 2+1+1 HISQ {0.09, 0.12, 0.15} fm
  [220, 310, 700]-MeV pion, $10^5$–$10^6$ statistics

Z. Fan et al (MSULat), 2210.09985




Quantities that can be calculated on the lattice



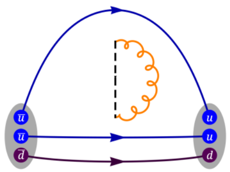

Wanted PDFs, GPDs, etc…

G: Bill Good

# First LaMET Gluon PDF

## § Hybrid-renormalized Gluon PDF w/ quasi-PDFs

- 2+1+1 clover/HISQ 0.12 fm, {310,690}-MeV pion, $10^6$ statistics
  Good et al (MSULat), 2505.13321
- PQCD calculated Wilson coefficient and matching kernal

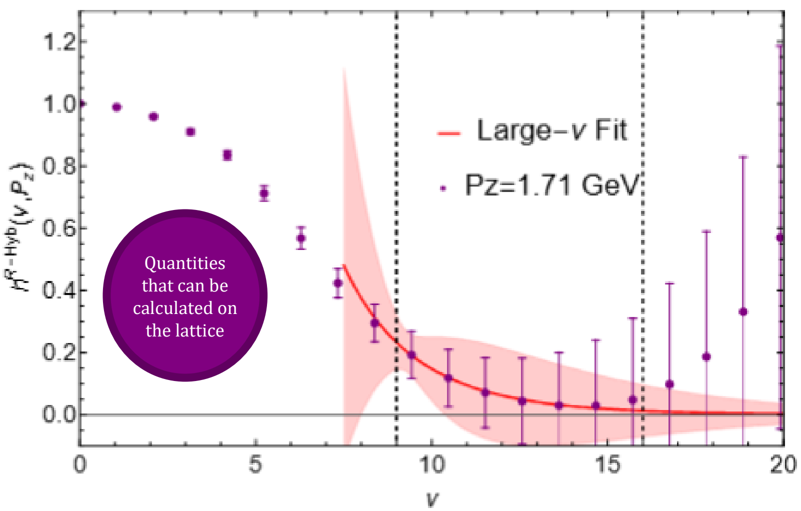


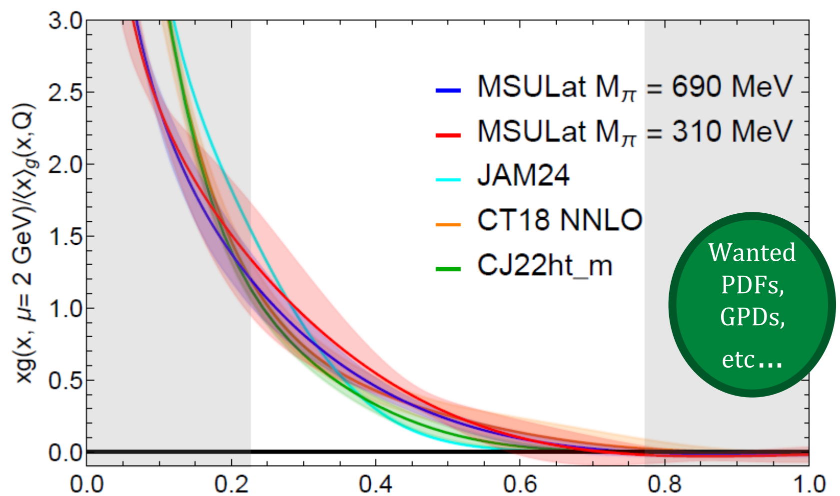
$$C_0 = 1 + \frac{\alpha_s C_A}{2\pi}\left(\frac{5}{6}\ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) + \frac{3}{2}\right)$$

$$C^{\text{hyb.}}\left(\xi, \frac{\mu}{p^z}\right) = C^{\text{ratio}}\left(\xi, \frac{\mu}{p^z}\right) + \frac{\alpha_s C_A}{2\pi}\frac{5}{6}\left[-\frac{1}{|1-\xi|} + \frac{2\mathrm{Si}((1-\xi)z_s p^z)}{\pi(1-\xi)}\right]_+$$

Formula by Fei Yao (BNL)

- First LaMET result  Plots by Bill Good (MSU)

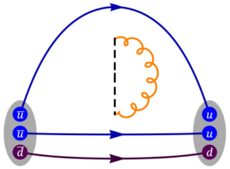


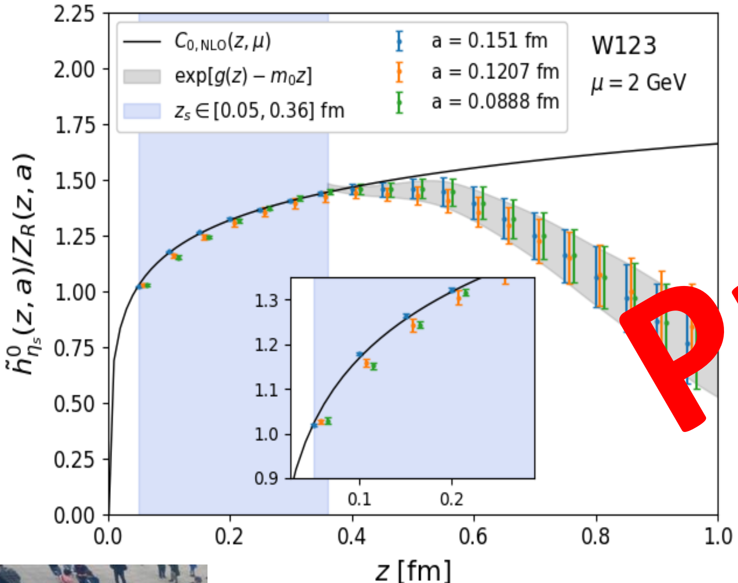



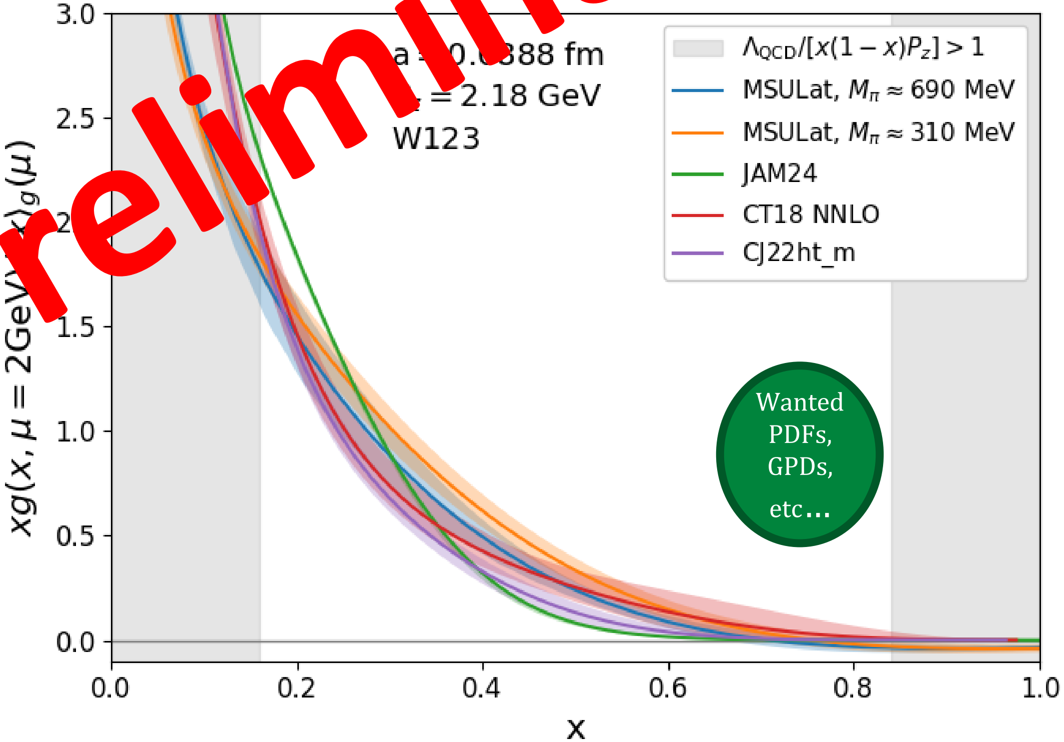
# Self-renormalized Gluon PDF

## § Self-renormalized gluon PDF w/ quasi-PDFs

- 2+1+1 clover/HISQ {0.15, 0.12, 0.09} fm, {310,690}-MeV pion, $10^6$ statistics   NieMiera et al (MSULat), 2510.XXXX
- Preliminary result





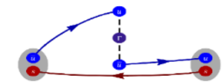



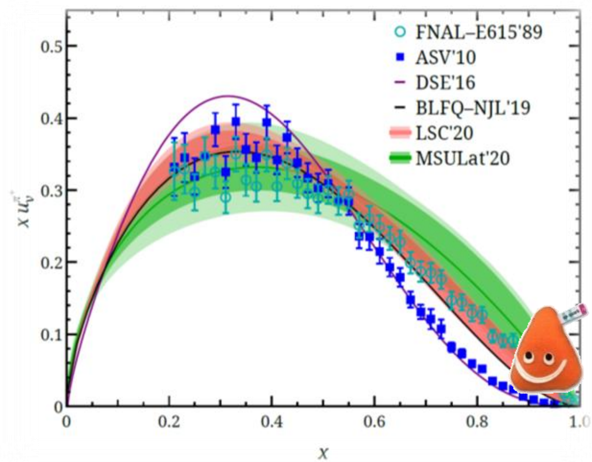

Wanted PDFs, GPDs, etc…

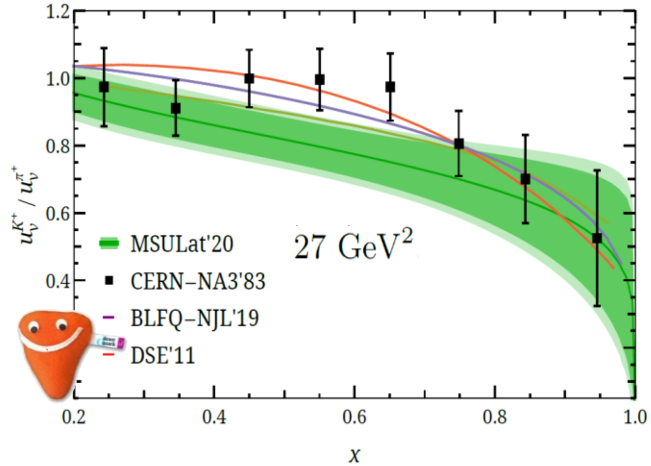
Plots by
Alex NieMiera (MSU)

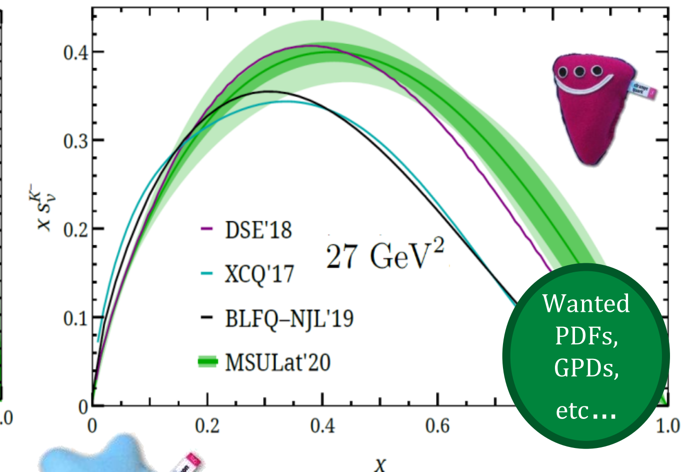
# Meson PDFs

§ Pion/Kaon valence-quark PDFs using quasi-PDF in the continuum limit MSULat. 2003.14128



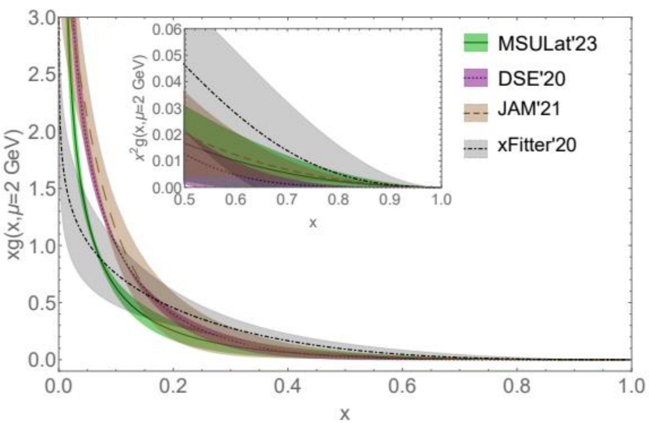


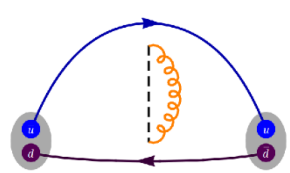



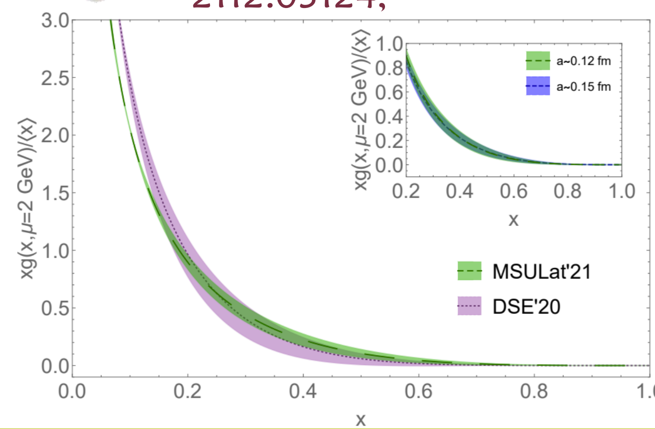

§ Pion/Kaon Gluon PDF w/ pseudo-PDF

Good et al. (MSULat), 2310.12034,



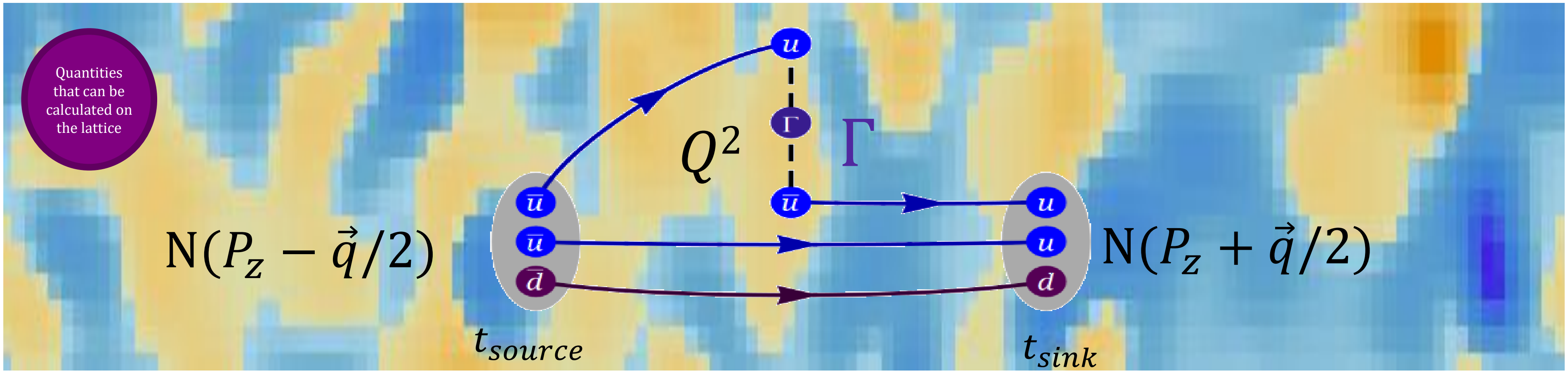
Salas-Chavira et al. (MSULat), 2112.03124,



# Isovector Nucleon GPDs

## § Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results



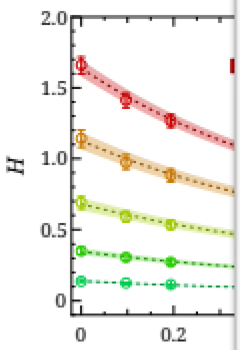

$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(H(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + E(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

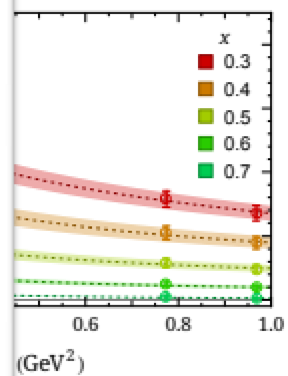
$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

# Tomography @ Physical Pion Mass

## § Lots of … borations



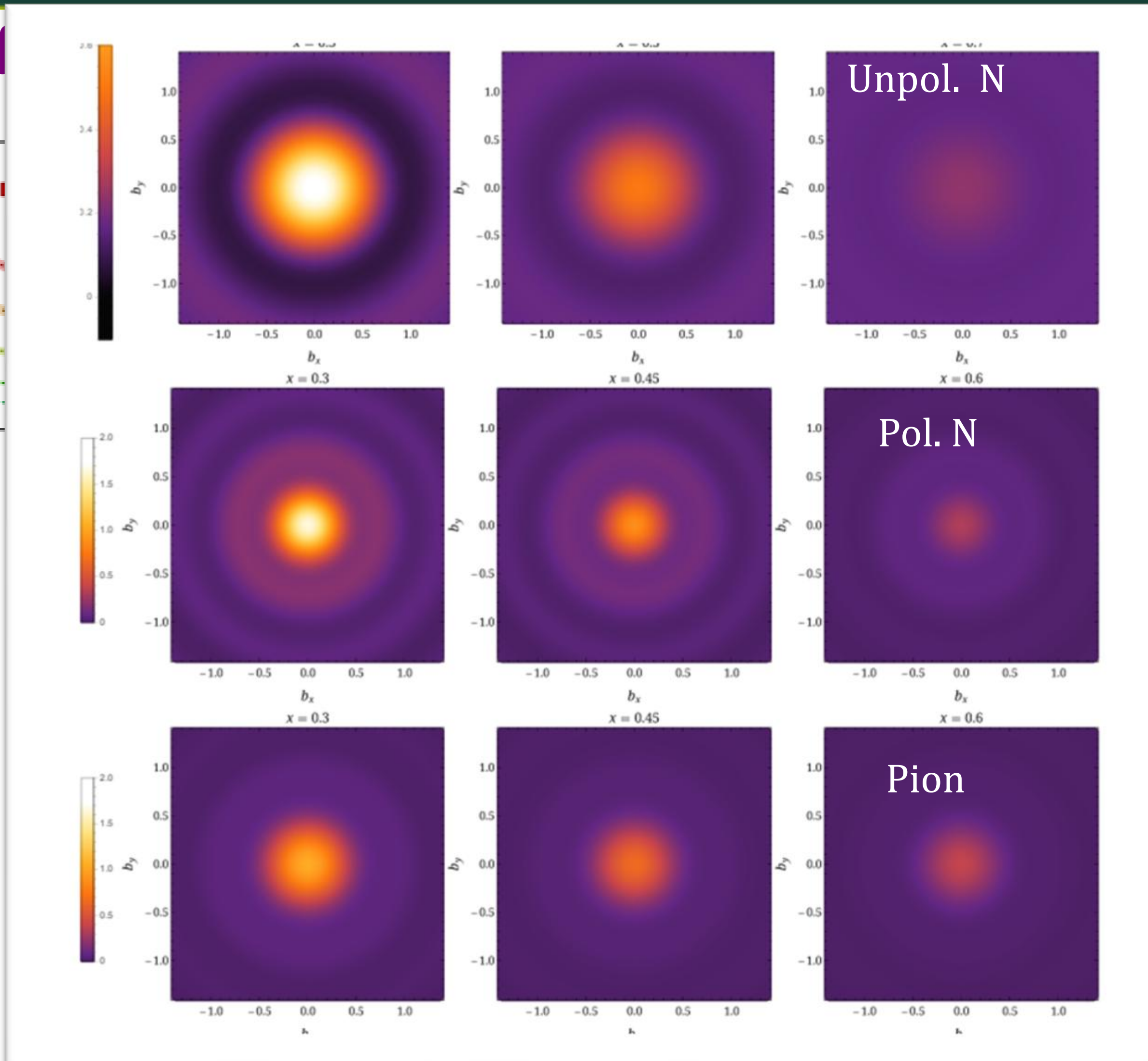




Figure 4.1: … selected $x$ values.

HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

# GPD Systematic Update

## § Nucleon isovector GPDs & pion valence-quark GPDs

- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

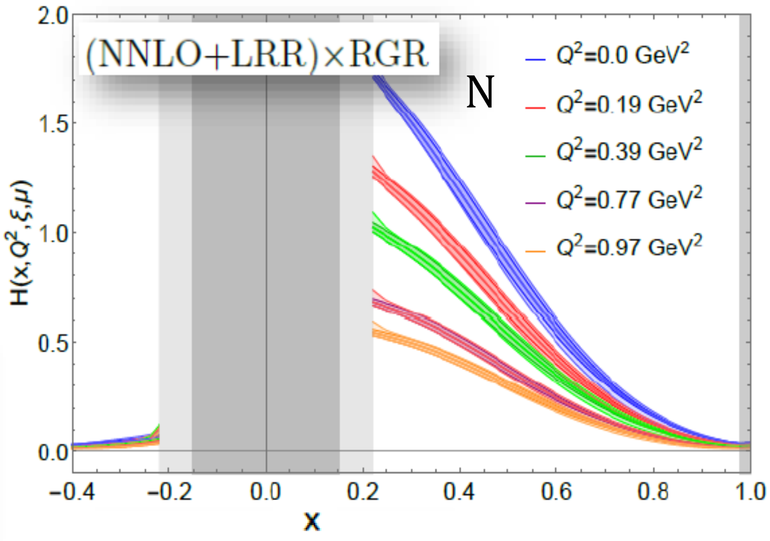
R. Zhang, et. al.
PLB 844, 138081 (2023)

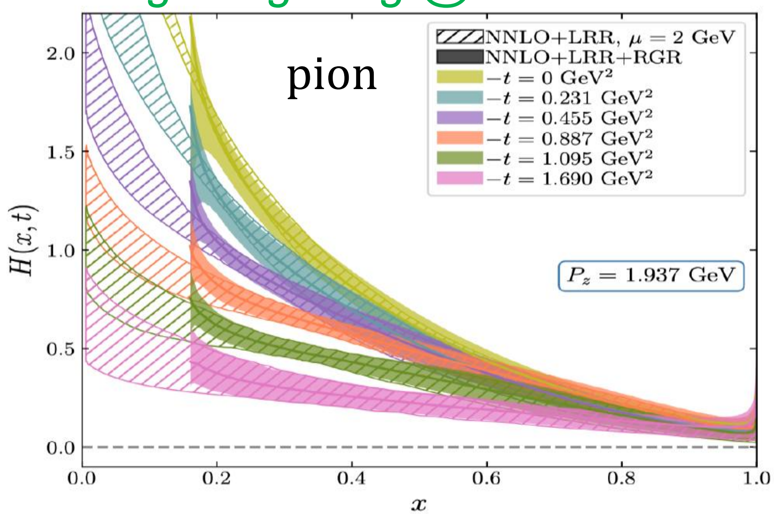
- $N_f$= 2+1+1 clover/HISQ, $a \approx 0.09$ fm, 135-MeV pion, $P_z \approx 2$ GeV
  J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

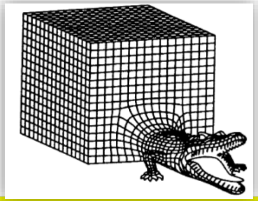
- $N_f$= 2+1 clover/HISQ, $a \approx 0.09$ fm, 300-MeV pion, $P_z \approx 2$ GeV
  H. Ding et al (ANL/BNL/Wuhan), 2407.03516 [hep-lat]

*Heng-Tong Ding @ Tue. 9:30 AM*

Wanted PDFs, GPDs, etc…



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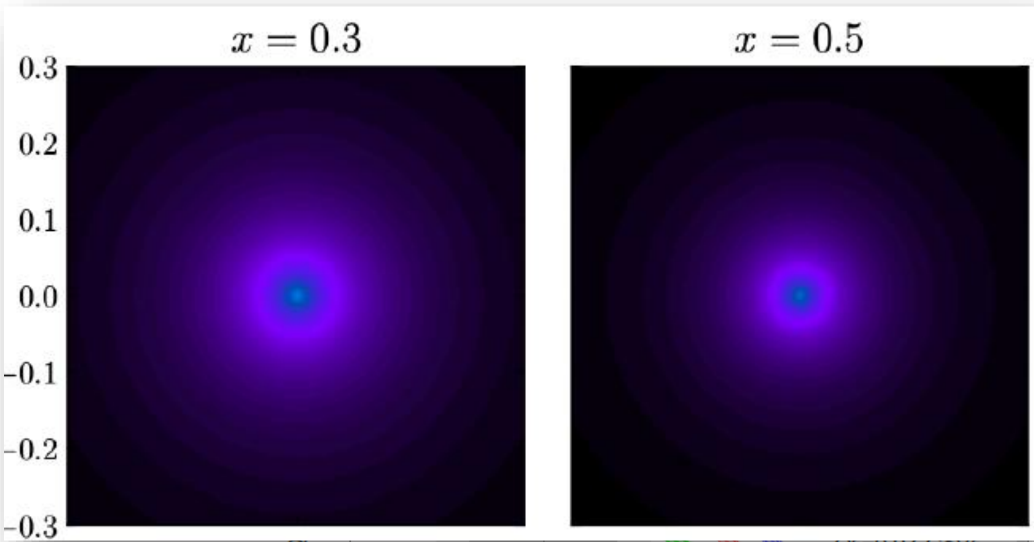


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H. Ding et al (ANL/BNL/Wuhan), 2407.03516 [hep-lat]

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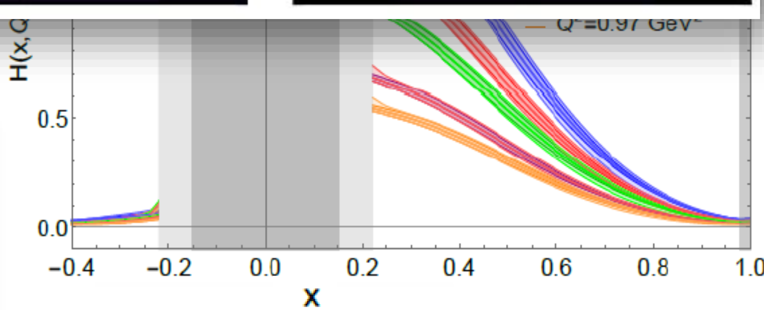
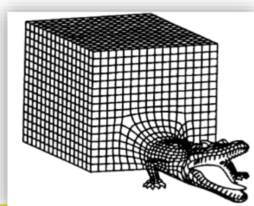
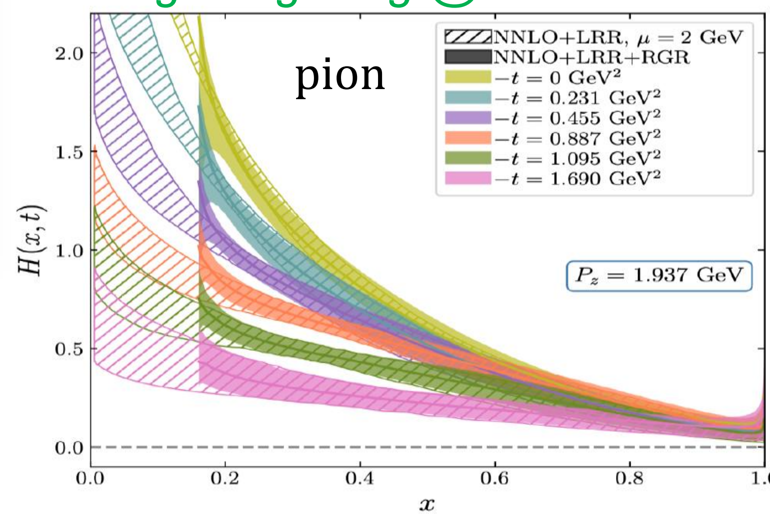




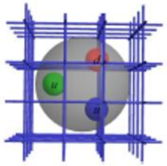
# *Impact of Lattice-QCD PDFs on Global Fits*

Theory Input

Exp't Input

Global Analysis of PDFs

+



CTEQ

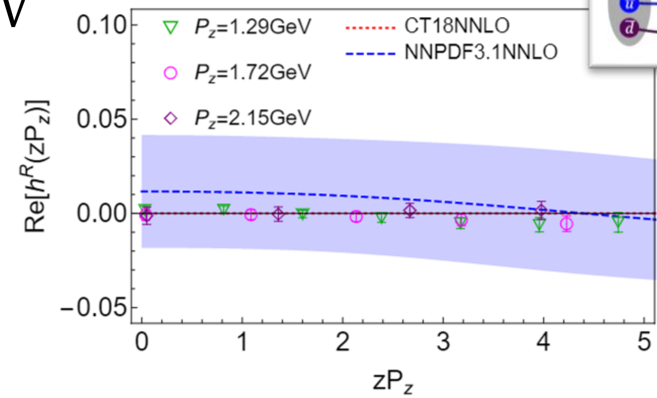
# First Lattice Strange PDF

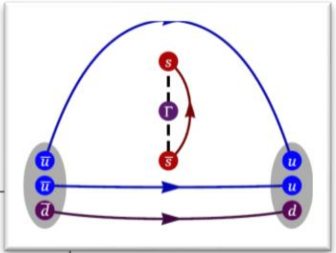
## § Results by MSULat/quasi-PDF method

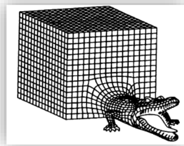
- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV

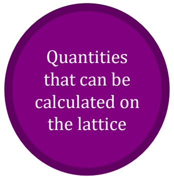
R. Zhang et al (MSULat), 2005.01124

$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x) - \bar{s}(x)) \cos(xzP_z)$$





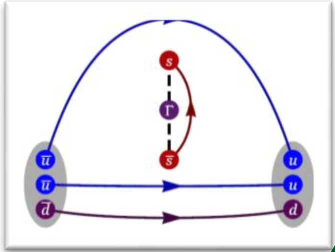



FG: Rui Zhang

# Lattice Strangeness Asymmetry Impact

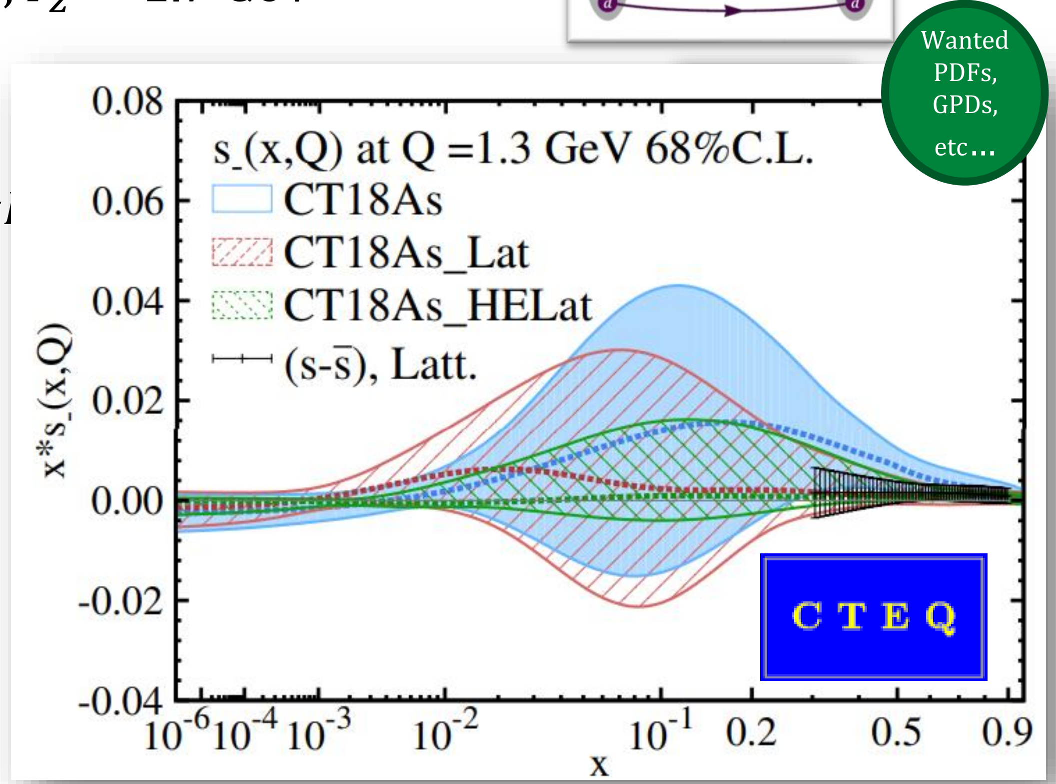
## § Results by MSULat/quasi-PDF method

- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV, $P_z \approx 1.7$ GeV



R. Zhang et al (MSULat), 2005.01124

$$\mathrm{Re}[h(z)] \propto \int dx\, \big(s(x) - \bar{s}(x)\big) \cos(xzP_z)$$




## § From quasi-PDF to PDF

$$\tilde{f}_q(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} f_q(y) C_{q/q}(x, y, P_z, \mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(1-x)^2 P_z^2}\right)$$

T. Hou, HL, M. Yan, C. Yuan, 2211.11064

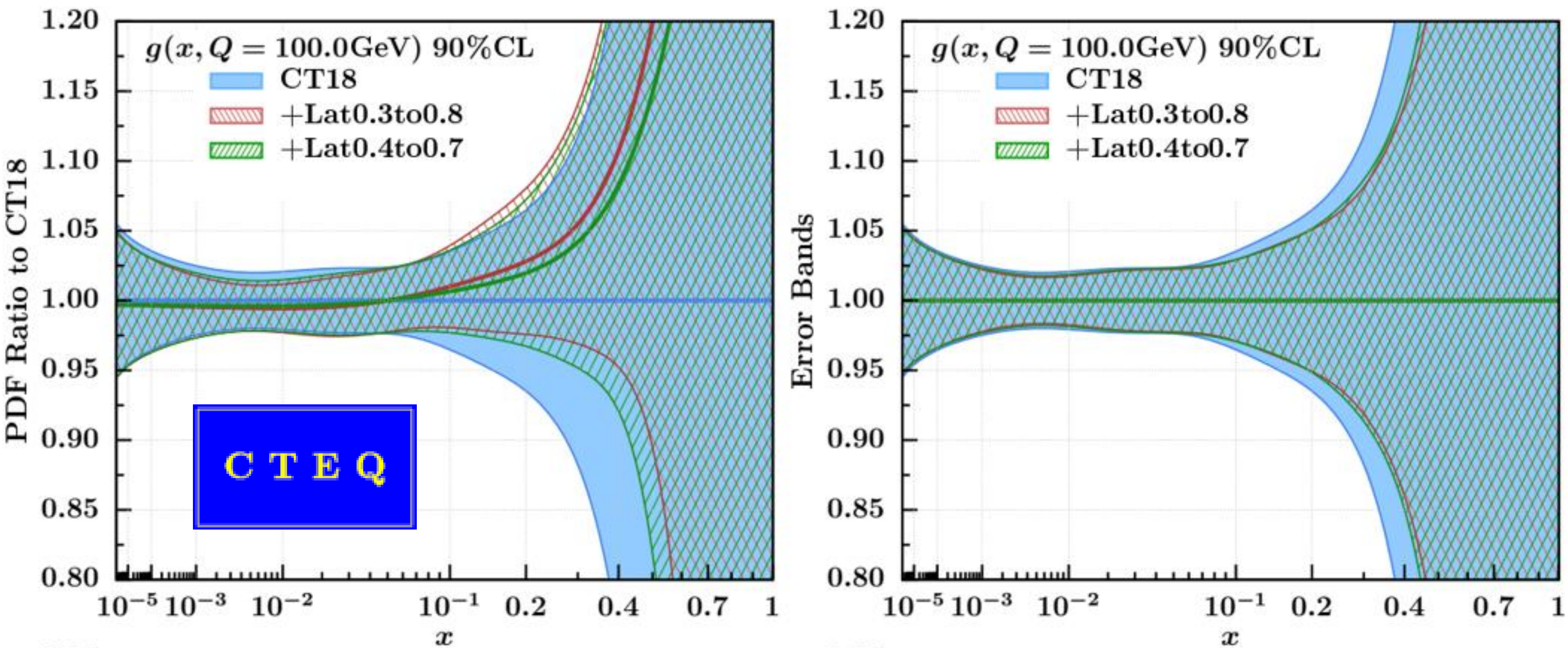
## § The strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

# Nucleon Gluon PDF Impact

## § Impact study with CTEQ-TEA analysis

- Take lattice inputs in the region where no strong experimental data constraints, $x \in [0.3, 0.8]$

Plots by Alim Ablat (Xinjiang U.); 2502.10630



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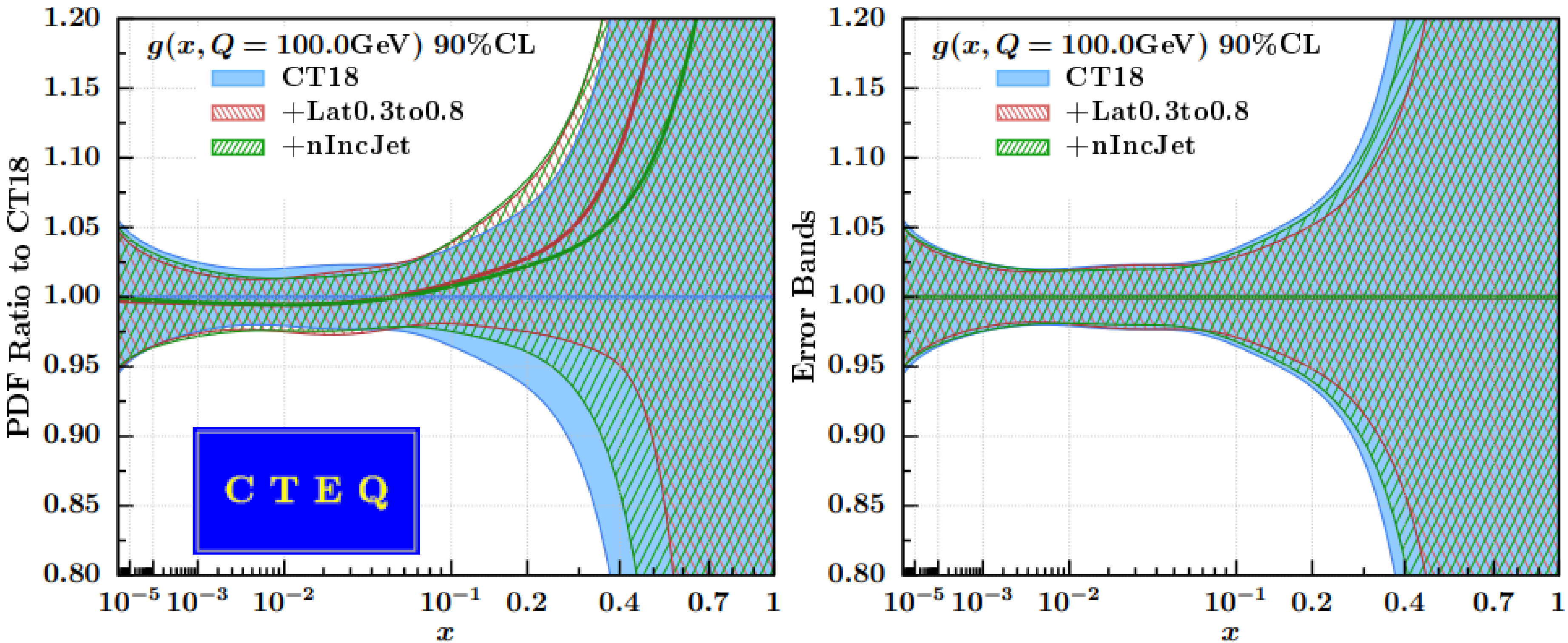

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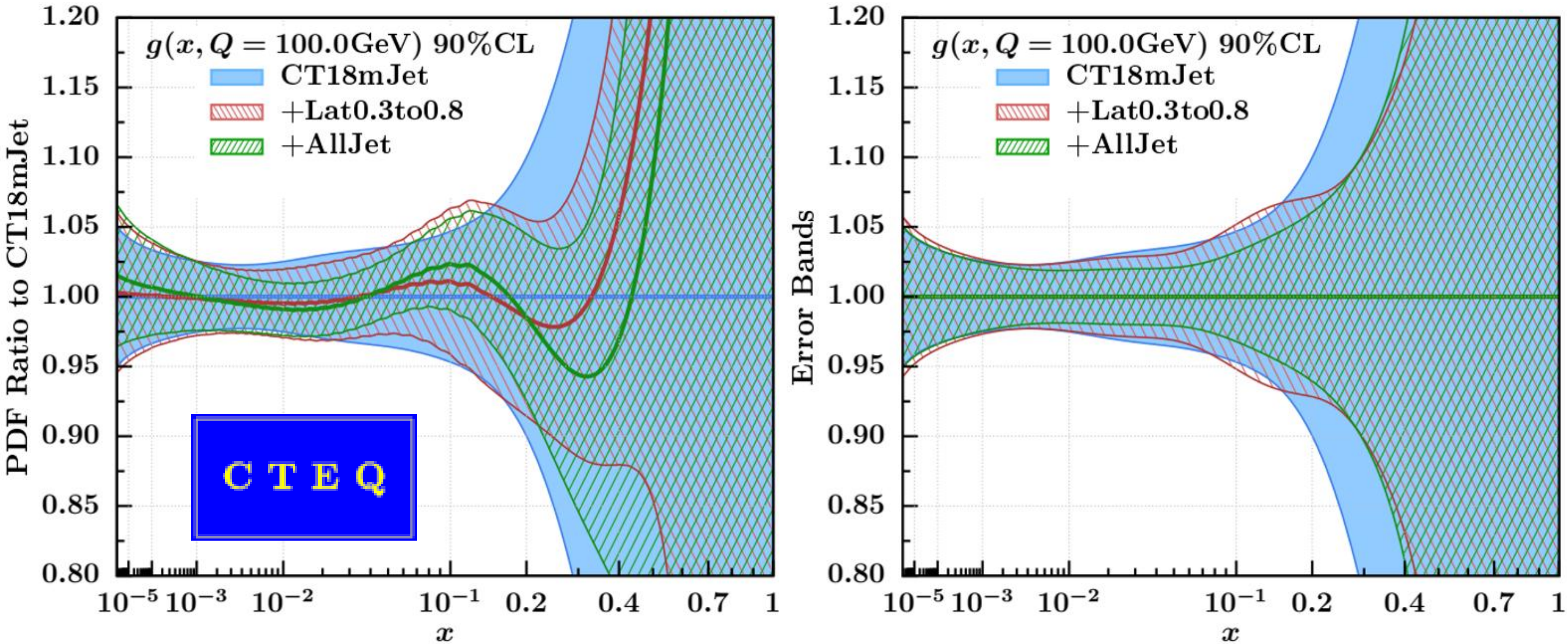
- Take lattice inputs in the region where no strong experimental data constraints, $x \in [0.3, 0.8]$

Plots by Alim Ablat (Xinjiang U.); 2502.10630

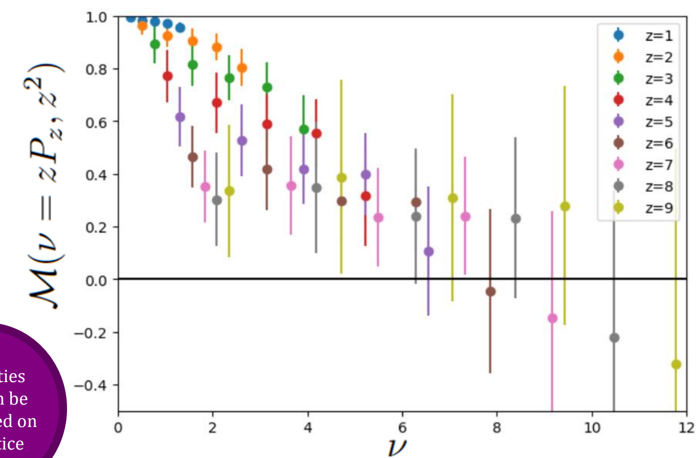


# Pion Gluon PDF Impact

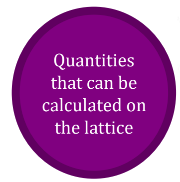
## § Study with JAM analysis

Plots by Bill Good (MSU); 2507.22730

- Treat lattice reduced pseudo-Ioffe time distribution (RpITD) as "cross-section" inputs in the JAM global fit




W. Good et al, 2409.02750 [hep-lat]


$$\boxed{\mathcal{M}(\nu, z^2)} = \int_0^1 dx \boxed{\frac{xg(x)}{\langle x_g \rangle}} \left[ \boxed{R_{gg}(x\nu, z^2\mu^2) + R_r(x\nu, z^2\mu^2)} \boxed{\frac{\langle x_S \rangle}{\langle x_g \rangle}} \right] + \int_0^1 dx \boxed{\frac{x f_S(x)}{\langle x_g \rangle}} \boxed{R_{gq}(x\nu, z^2\mu^2)} + \boxed{\mathcal{O}(z^2 m^2, z^2 \Lambda_{\mathrm{QCD}})}$$

Lattice data

Non-perturbative PDFs and moments

Perturbative matching kernels

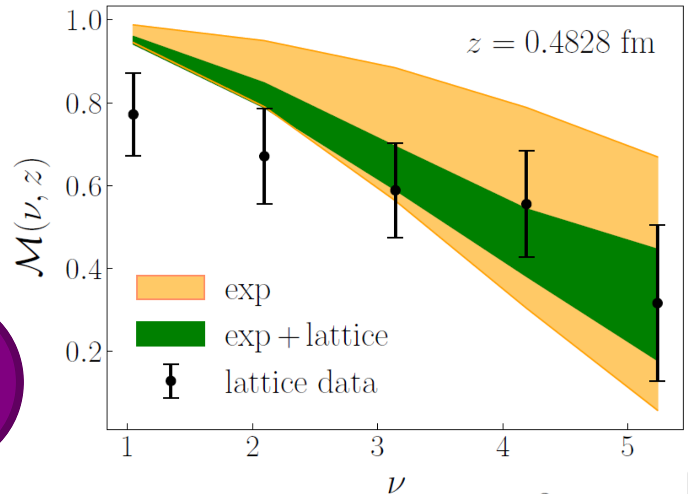
Residual systematic error

A. V. Radyushkin, PRD96, 034025 (2017);
Balitsky, et al., PLB808, 135621 (2020)

# Pion Gluon PDF Impact

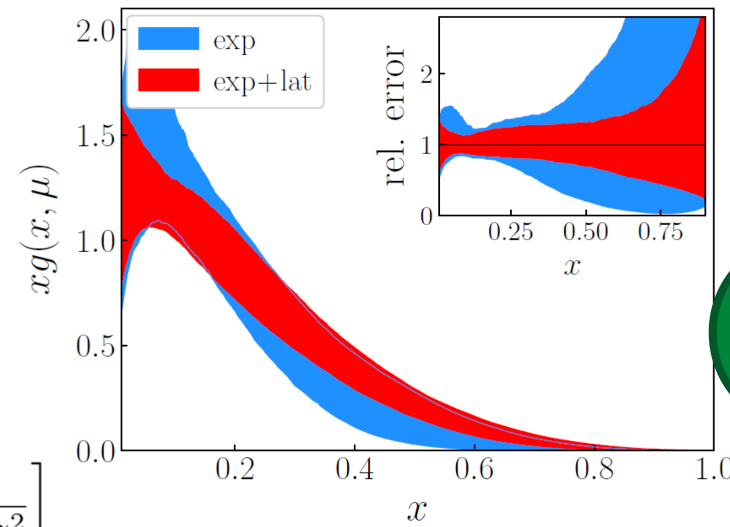
## § Study with JAM analysis

Plots by Bill Good (MSU); 2507.22730



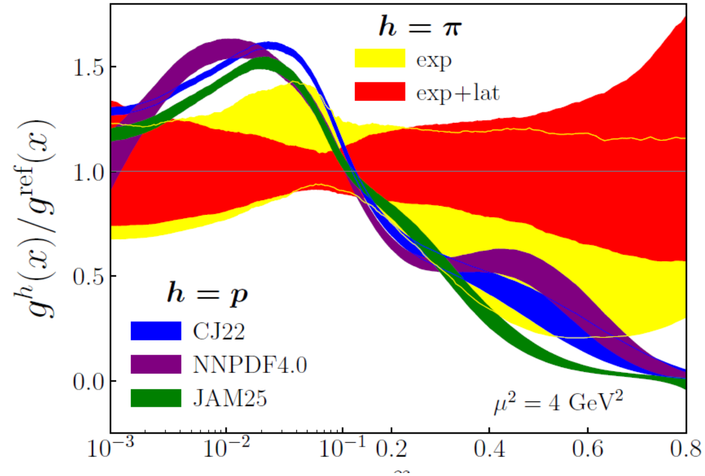

Quantities that can be calculated on the lattice

$$\mu^2 = \max\left[m_c^2, \frac{4}{e^{2\gamma_E} z^2}\right]$$




Wanted PDFs, GPDs, etc…

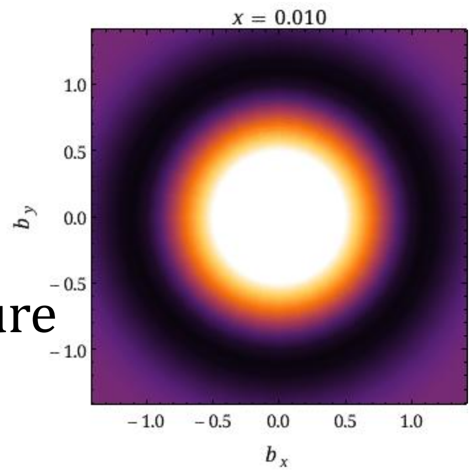

also in Collaboration with Wally and Nobuo



# Summary and Outlook

§ Exciting era using LQCD to study *x*-dependent parton distributions

- Bjorken-*x* dependence of parton distributions now widely studied
- More study of systematics planned for the near future



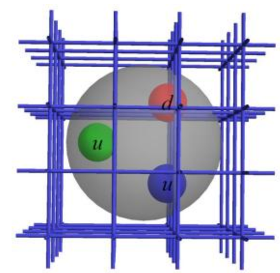

§ Lattice strange and gluon PDFs can have impacts

- Treat lattice matrix elements as expt inputs in the future
- Computational resources are needed for precision calculations (community support! )

§ Precision and progress are limited by resources

**Theory Input**

**Exp't Input** +




**Global Analysis of PDFs/GPDs**

EXCLAIM


Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
This work is partially sponsored by grants NSF PHY 1653405 & 1653405, DOE DE-SC0024053 & RCSA Cottrell Scholar Award

# *Backup Slides*

# Lattice Progress & Challenges

## § Beyond the standard twist-2 collinear PDFs

- Generalized parton distributions (GPDs) for the pion and unpolarized/polarized nucleon
- Transverse-momentum– dependent distributions (TMDs)
  - Collins-Soper kernel, soft function and wavefunctions
- Twist-3 PDFs and GPDs

For more details and references, refer to 2202.07193

## § Challenges ahead for precision PDFs (C3NT can help)

- Large momentum is essential
  - With sufficient statistics nucleons may reach 5 GeV
- Methods for signal-to-noise improvement
  - Gluonic observables, new ideas for large momentum
- Access small-x physics; some methods have inverse problem in PDF extraction, more computational resources, etc.

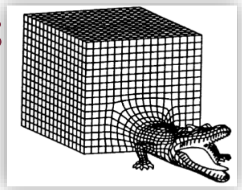
# First Lattice GPDs

## § First glimpse into pion GPD using Quasi-PDF/LaMET

- Lattice details: clover/HISQ, **0.12fm**, **310-MeV** pion mass

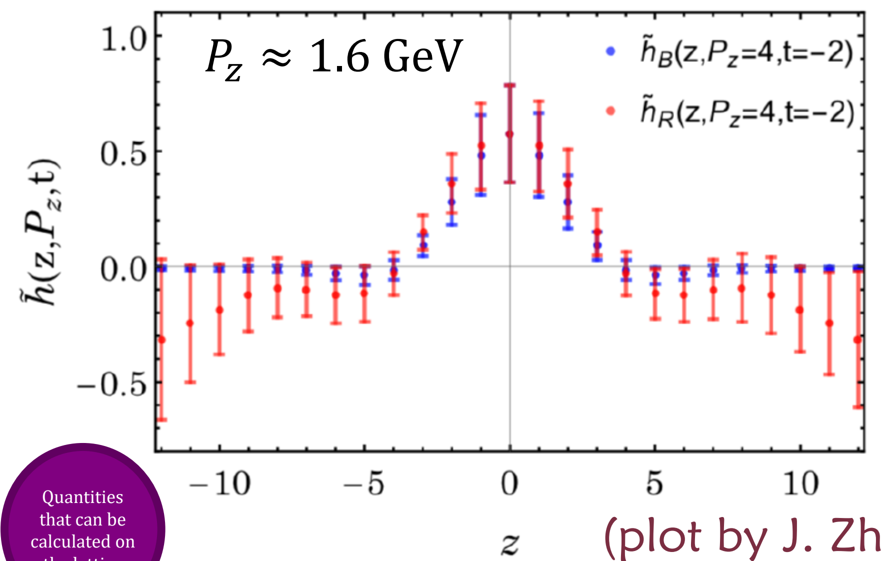
$P_z \approx 1.3, 1.6$ GeV

MILC, Phys. Rev. D, 82 (2010), 074501; Phys. Rev. D, 87 (2013), 0545056



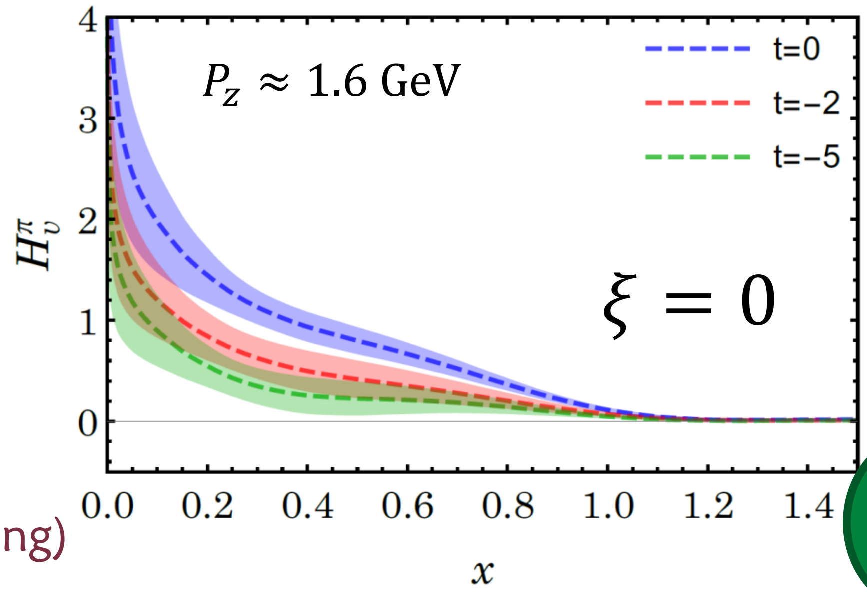
J. Chen, HL, J. Zhang, 1904.1237;

$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right)\gamma^+\Gamma\left(\frac{\eta^-}{2},-\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$




Quantities that can be calculated on the lattice

(plot by J. Zhang)

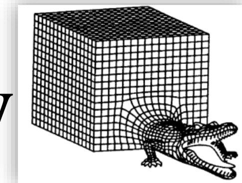



Wanted PDFs, GPDs, etc…

# Valence-Quark Pion GPD

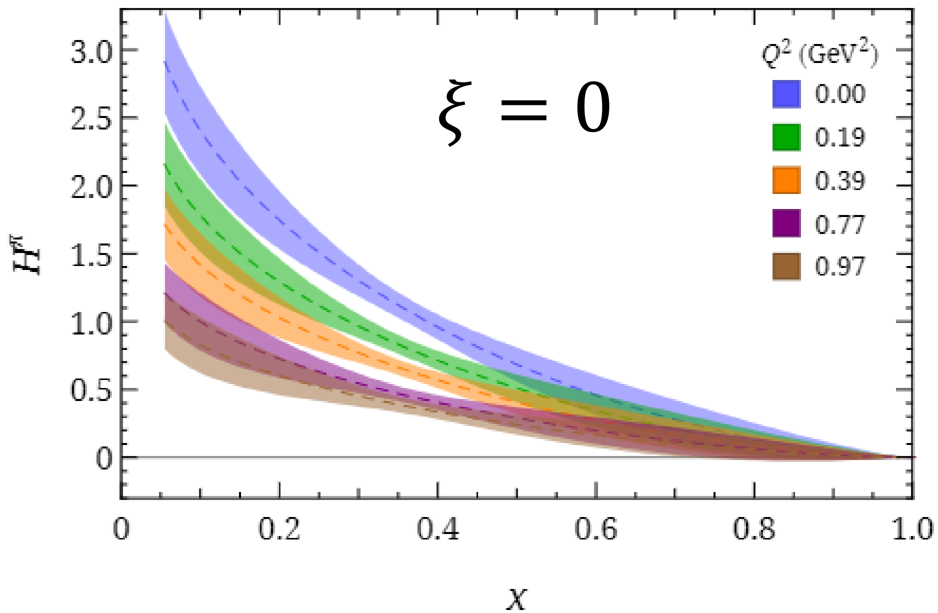
## § Pion GPD ($H^{\pi}$) using quasi-PDFs at physical pion mass

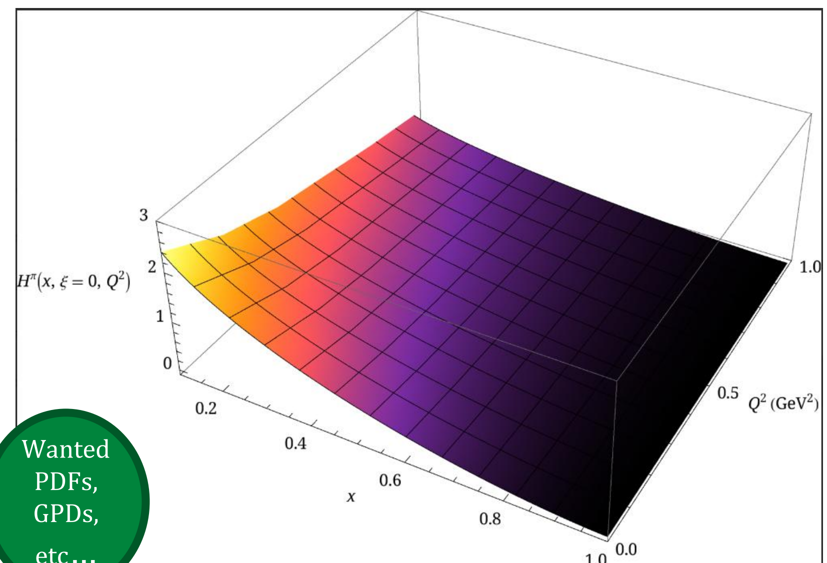
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV
- $\xi = 0$ valence-quark Pion GPD results



finite-volume, discretization,


HL (MSULat), Phys. Lett. B 846 (2023) 138181

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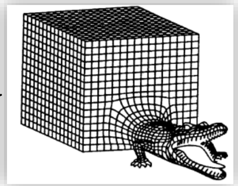
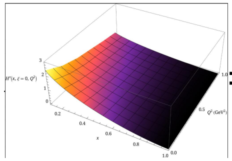
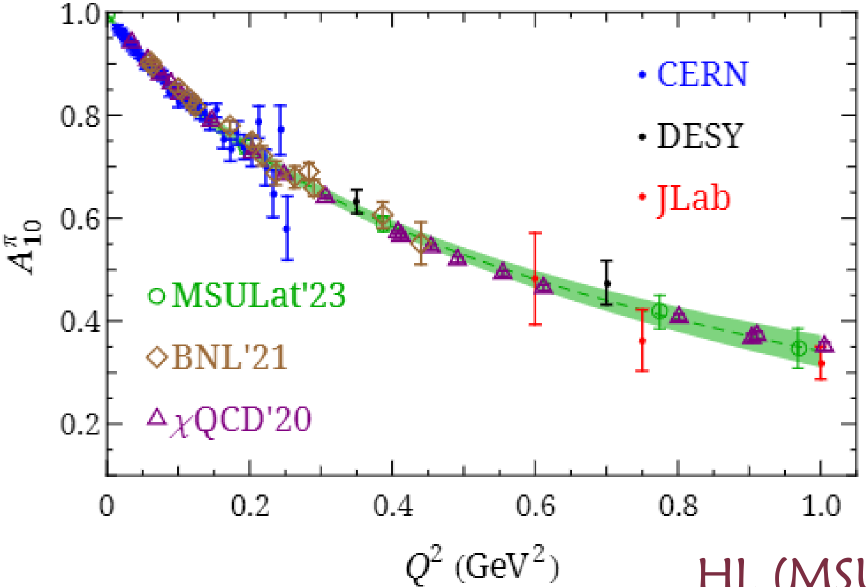


finite-volume,
discretization,


- $\xi = 0$ valence-quark Pion GPD results

$$\int_{-1}^{+1} dx\, x^{n-1}$$

$$= A^{\pi}_{ni}(t)$$

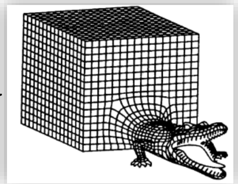



HL (MSULat), Phys. Lett. B 846 (2023) 138181

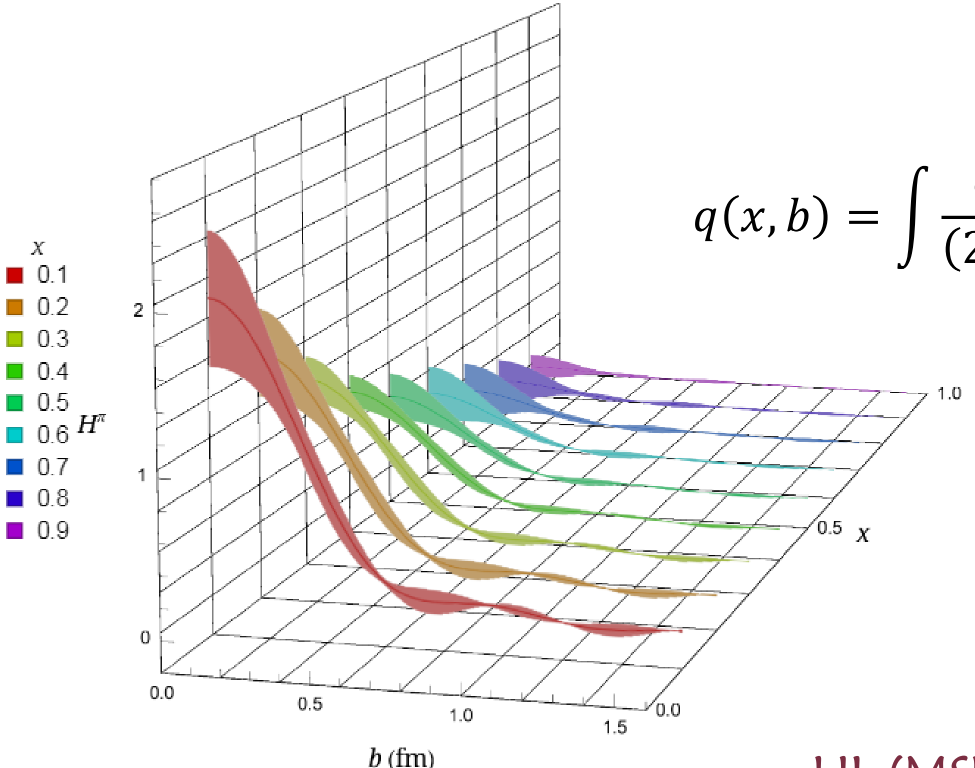
# Pion Tomography

## § Nucleon GPD using quasi-PDFs at physical pion mass

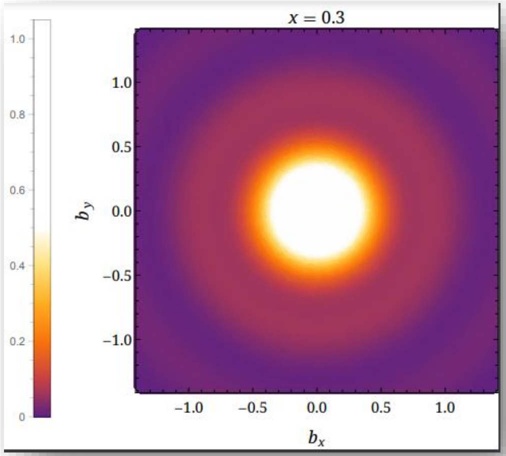
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV
- $\xi = 0$ valence-quark Pion GPD results



finite-volume,
discretization,


$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H\left(x, \xi = 0, t = -\vec{q}^{\,2}\right) e^{i\vec{q}\cdot\vec{b}}$$



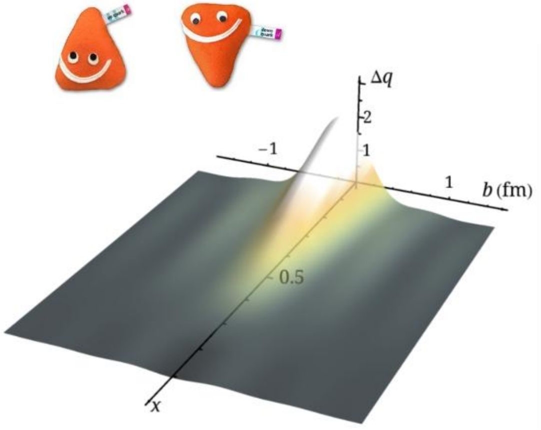

HL (MSULat), Phys. Lett. B 846 (2023) 138181

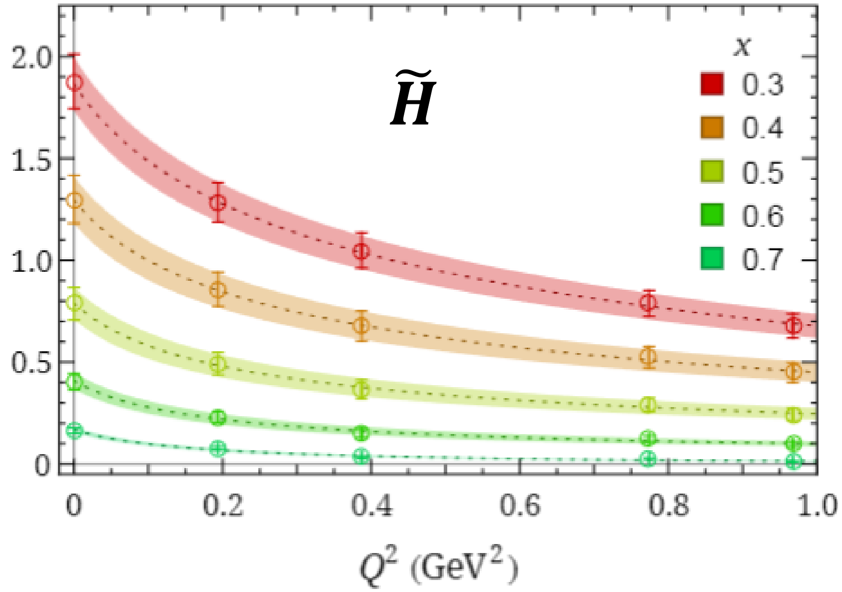
# Nucleon Polarized GPDs

## § Helicity GPD ($\widetilde{H}$ )using quasi-PDFs at **physical pion mass**

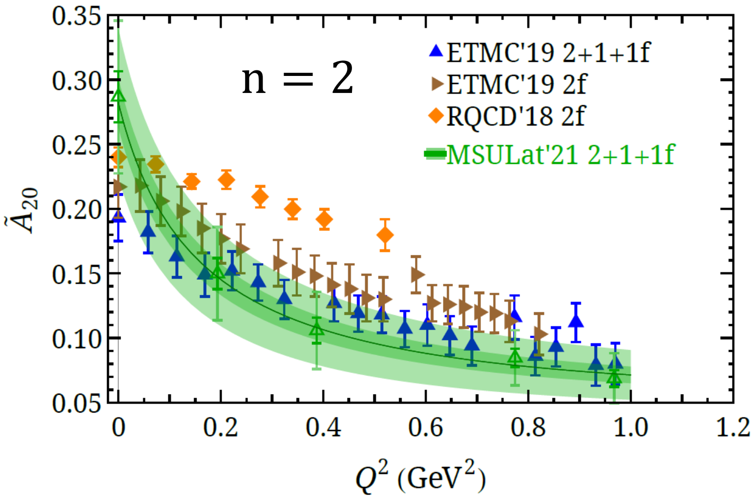
- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon (quasi-)GPD results

HL (MSULat), Phys.Lett.B 824 (2022) 136821



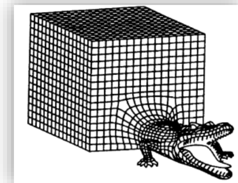

- Take the integral to form moments



# Nucleon Tomography

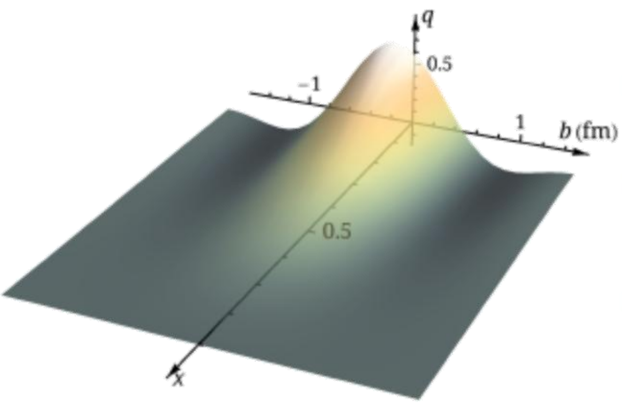
## § Nucleon GPD using quasi-PDFs at physical pion mass

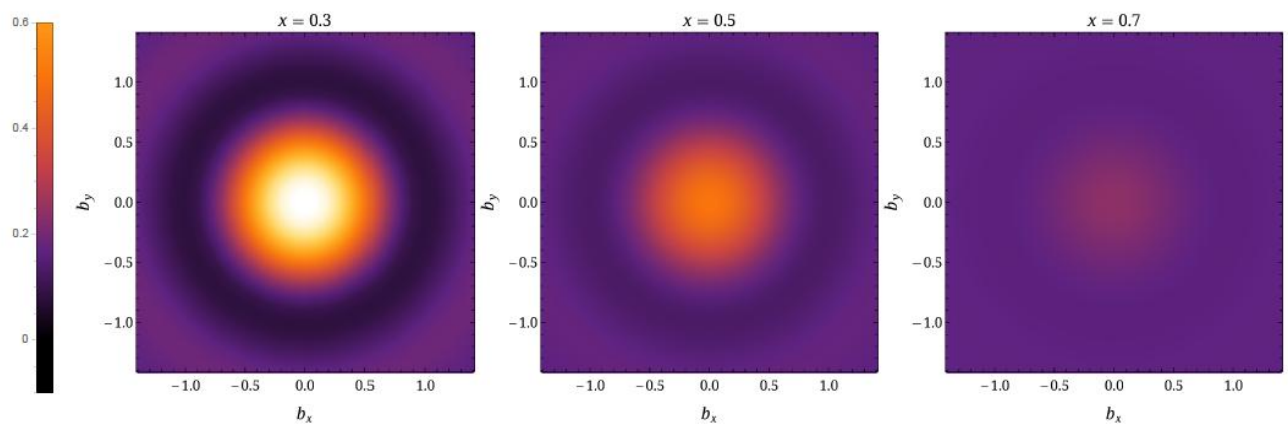
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results



finite-volume,
discretization,


$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x,\xi=0,t=-\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$




HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Also see work done by ANL/BNL/ETMC, [2209.05373](2209.05373) ,[2310.13114](2310.13114)

# GPD Systematic Update

§ Nucleon isovector GPDs calculated directly at physical pion mass

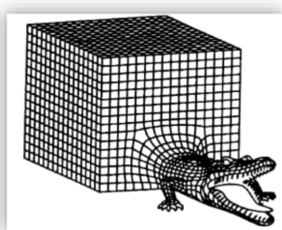
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

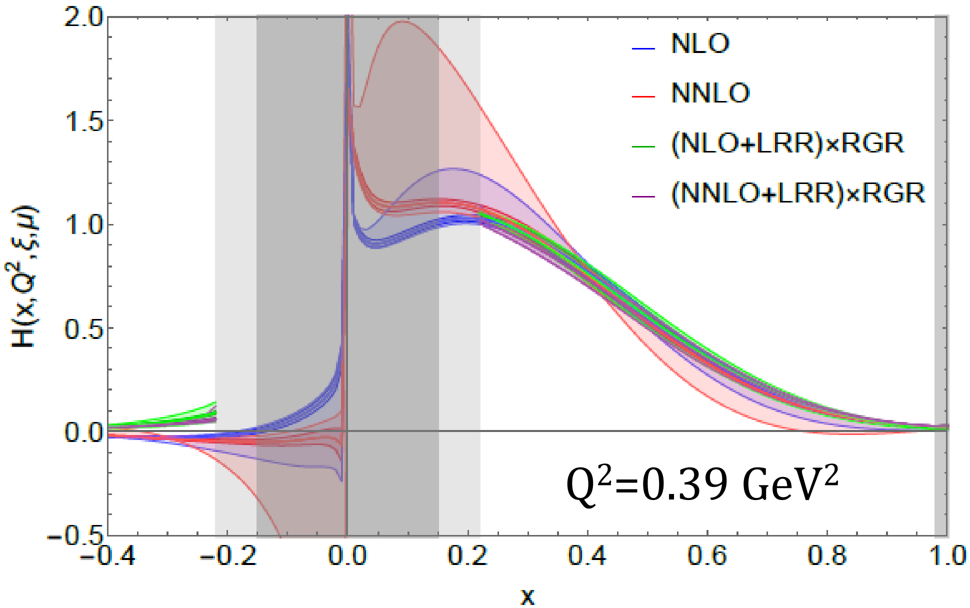
R. Zhang, et. al.
PLB 844, 138081 (2023)

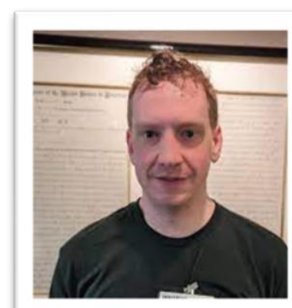
- $N_f = 2+1+1$ clover/HISQ, $a \approx 0.09$ fm, 135-MeV pion, $P_z \approx 2$ GeV

J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…





P: Jack Holligan

# GPD Systematic Update

## § Nucleon isovector GPDs & pion valence-quark GPDs

- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
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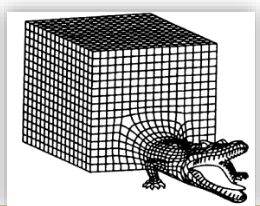
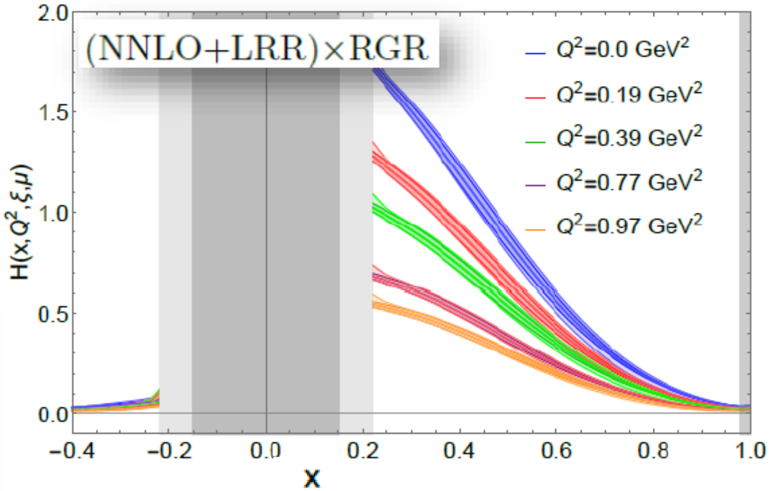
R. Zhang, et. al.
PLB 844, 138081 (2023)

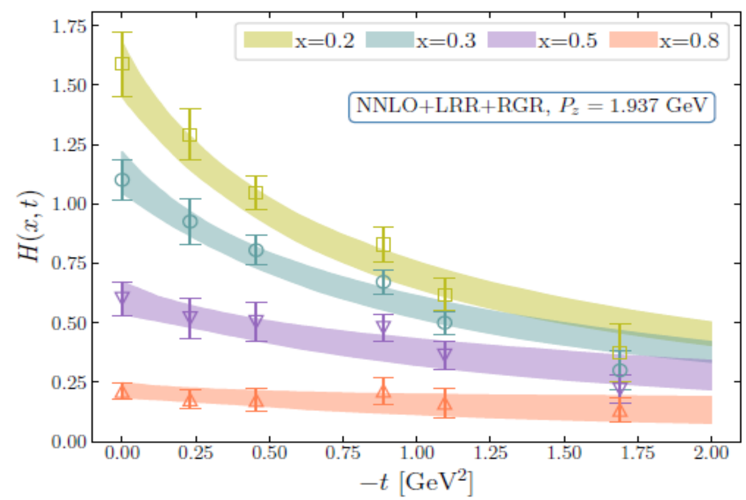
- $N_f$= 2+1+1 clover/HISQ, $a \approx 0.09$ fm, 135-MeV pion, $P_z \approx 2$ GeV
  J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

- $N_f$= 2+1 clover/HISQ, $a \approx 0.09$ fm, 300-MeV pion, $P_z \approx 2$ GeV
  H. Ding et al (ANL/BNL/Wuhan), 2407.03516 [hep-lat]

Wanted PDFs, GPDs, etc…





# Valence-quark PDFs Update

## § Pion PDFs calculated directly at physical pion mass

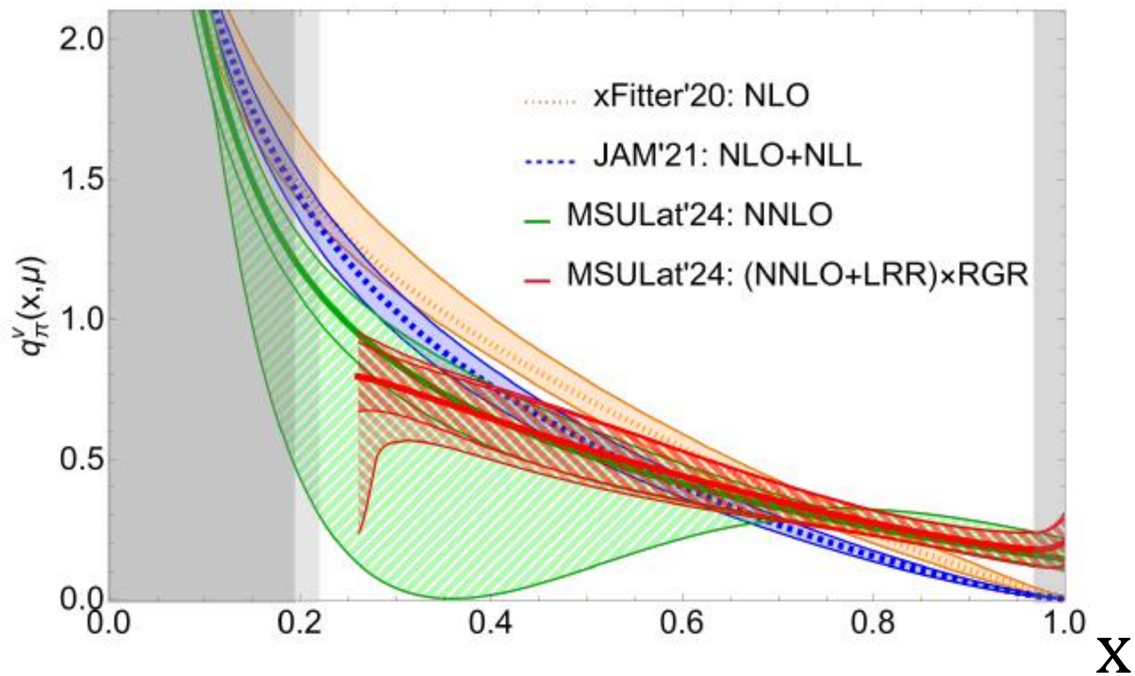
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

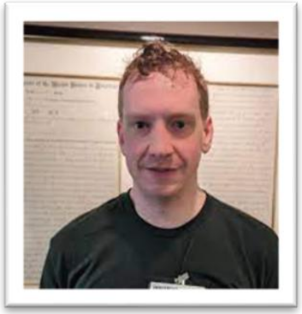
R. Zhang, et. al.
PLB 844, 138081 (2023)

- $N_f$=2+1+1 clover/HISQ, a~0.09 fm

J. Holligan, HL (MSULat), 10.1088/1361-6471/ad3162

Wanted PDFs, GPDs, etc…

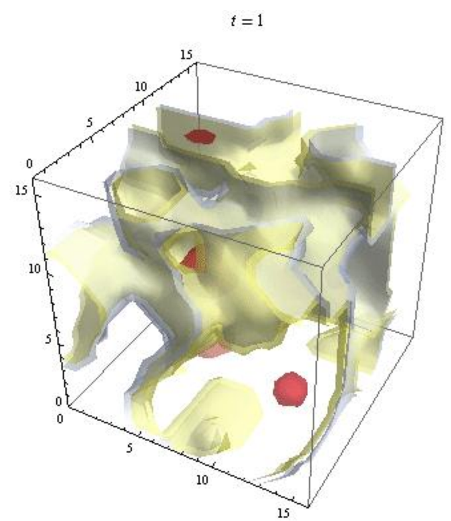


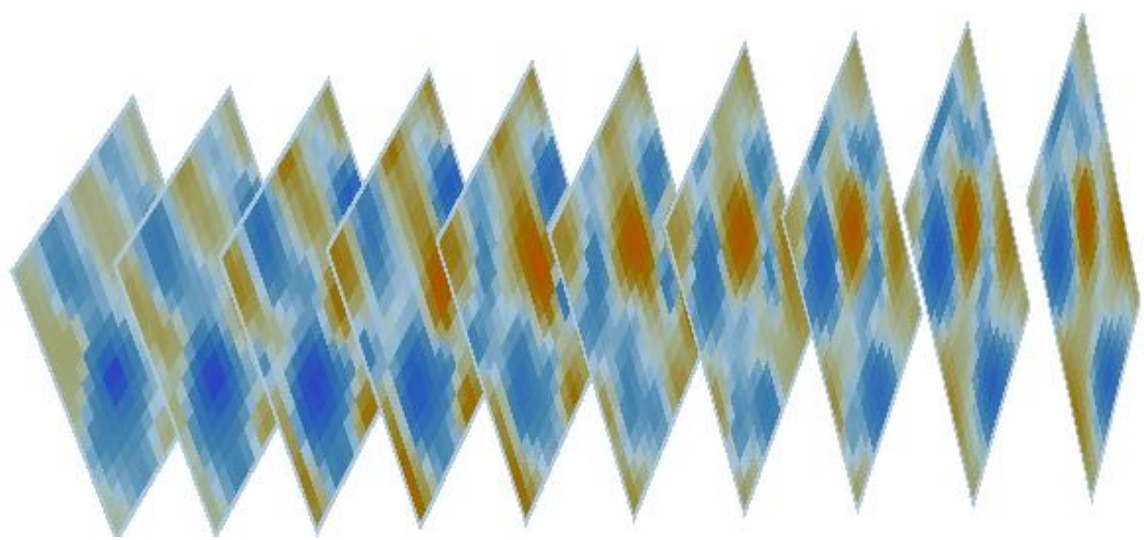



P: Jack Holligan

# Anatomy of a Lattice Calculation

## 1. Start with QCD Vacuum (gauge configurations)



# 2025 INT Program

## § Precision QCD with the Electron-Ion Collider

- Week 1: (May 12–16)
  - Precision theory for hard scattering at the EIC
  - Factorization and resummation
- Week 2: (May 19–23)
  - Parton distributions and the interplay of EIC and LHC
  - Lattice QCD meets phenomenology
- Week 3: (May 26–30)
  - Small-*x* physics in the EIC era
- Week 4: Workshop (June 2–6)
  - Bridging theory and experiment
  - Organizers: A. Bacchetta, F. Ringer, A. Stasto, W. Cosyn
- Week 5: (June 9–13)
  - Artificial intelligence and enhanced design
- Week 6: (June 16–20)
  - Jets and semi-inclusive reactions
  - Nucleon and nuclear tomography

- Organizers
  - Renee Fatemi
  - Huey-Wen Lin
  - Werner Vogelsang

# Students Wanted

LGT4HEP website: https://lgt4hep.github.io/

LGT4HEP

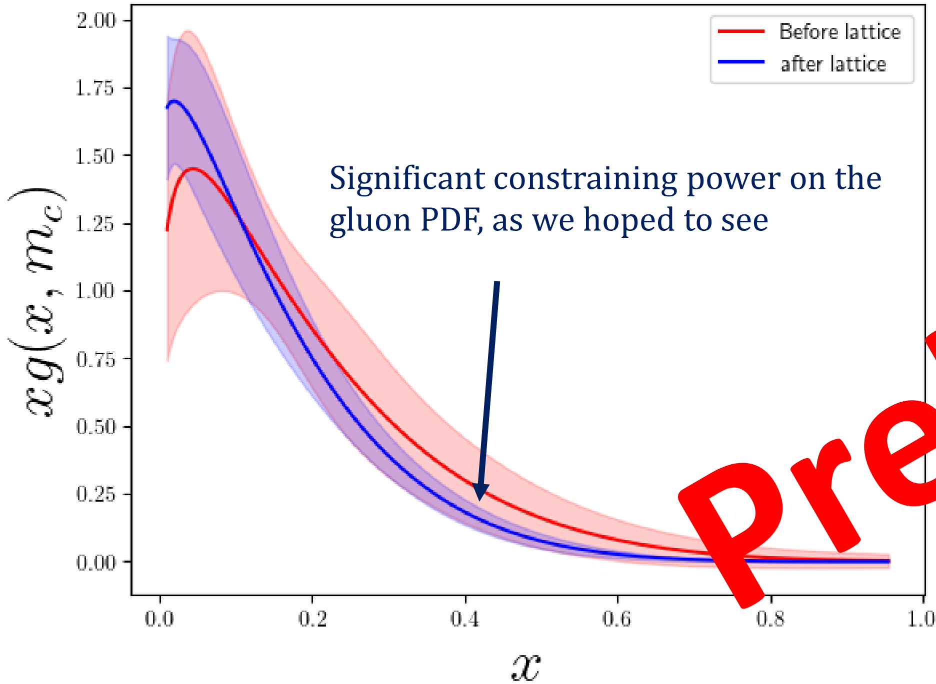
**High Energy Physics Computing Traineeship for Lattice Gauge Theory**

**Apply now:**

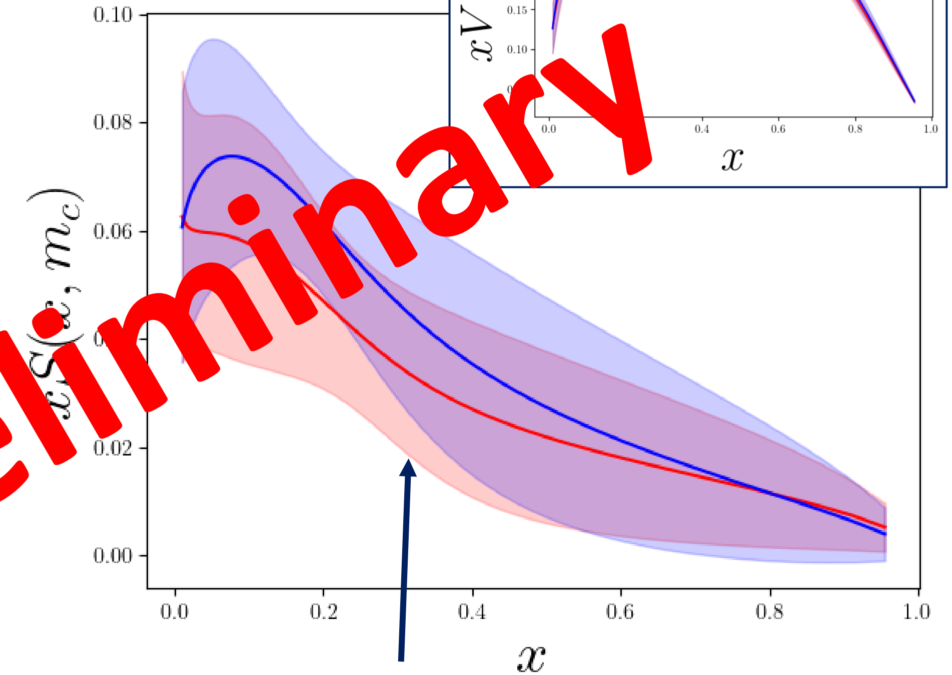
Visit lgt4hep.github.io to learn more and where to apply for the traineeship graduate school program.

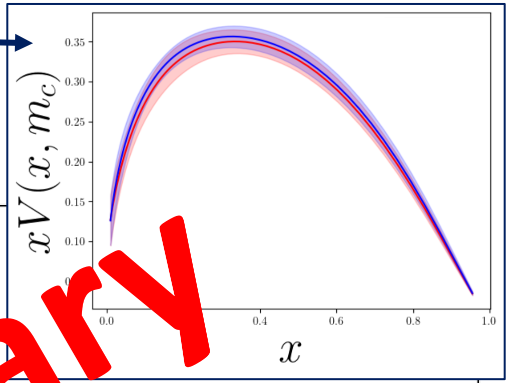
# More Pion PDF Updates

Not much effect on the valence quarks (as expected)




Significant constraining power on the gluon PDF, as we hoped to see



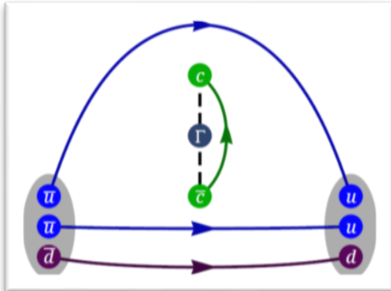

**Overall compatibility with "before" lattice PDFs**

Some shape change in the sea quark PDF, need to look at relative errors

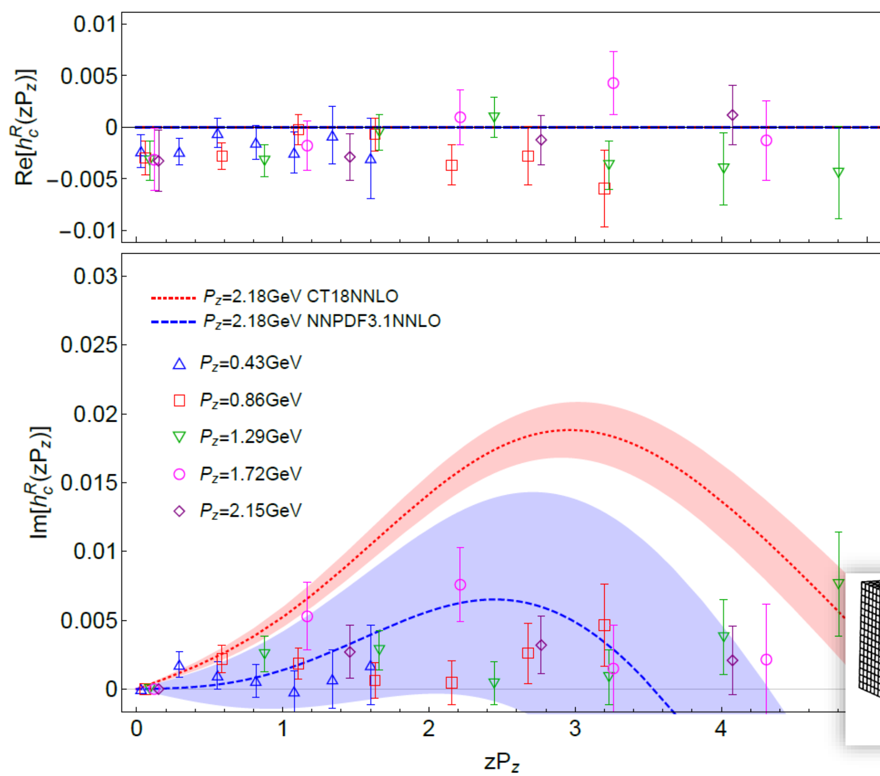
# First Lattice Charm PDF

- Large uncertainties in global PDFs
- Results by MSULat/quasi-PDF method
  - Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

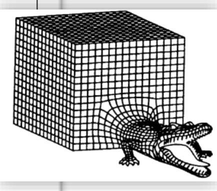
2005.01124, R. Zhang et al (MSULat)




- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF




Quantities that can be calculated on the lattice



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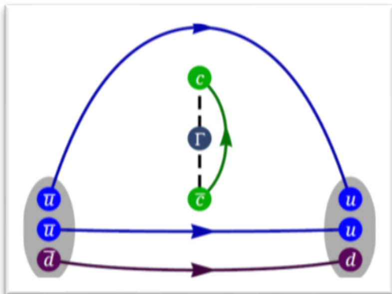
2005.01124, R. Zhang et al (MSULat)

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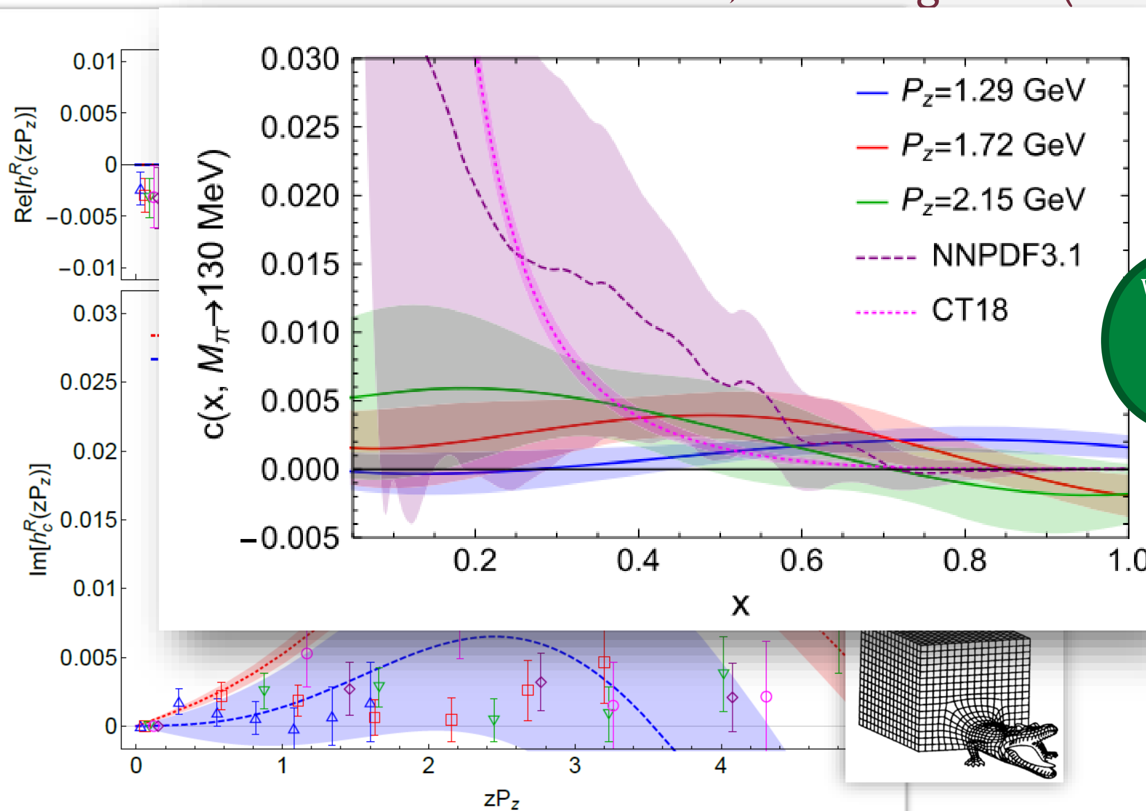


Wanted PDFs, GPDs, etc...

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  - Leading-renormalon resummation (LRR)
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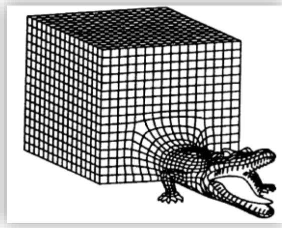
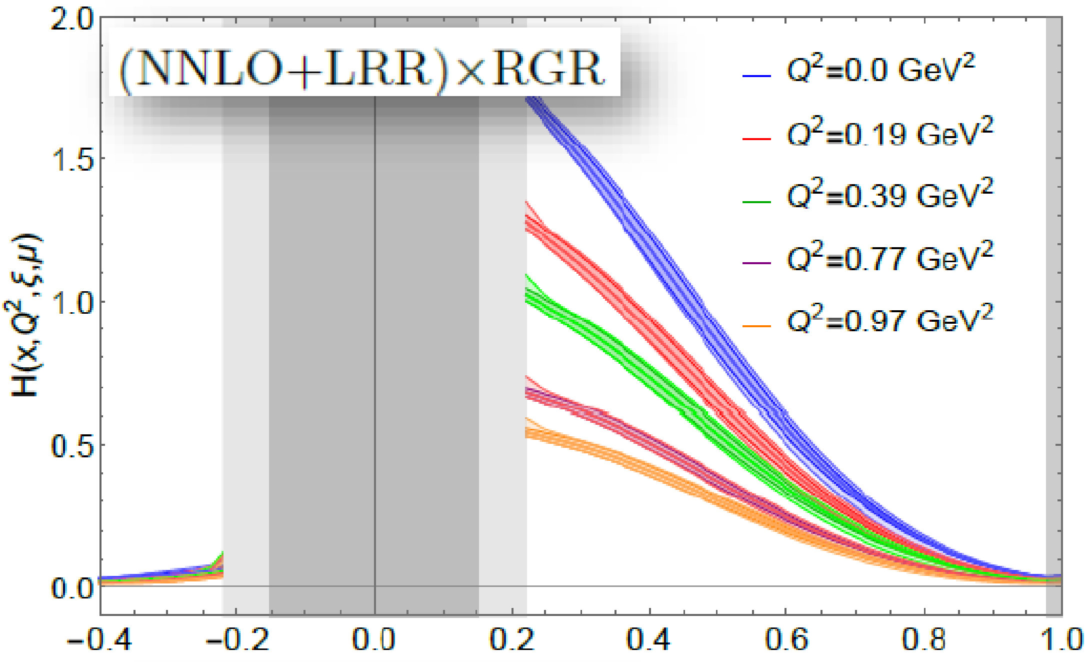
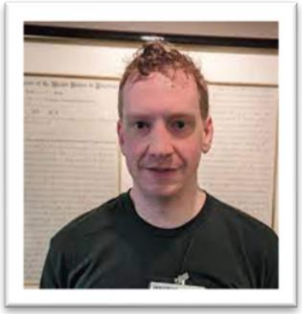
R. Zhang, et. al.
PLB 844, 138081 (2023)

- $N_f$=2+1+1 clover/HISQ, $a \approx 0.09$ fm, 135-MeV pion, $P_z \approx 2$ GeV

J. Holligan, HL (MSULat), 2312.10829 [hep-lat]



P: Jack Holligan

# GPD Systematic Update

## § Nucleon isovector GPDs calculated directly at physical pion mass

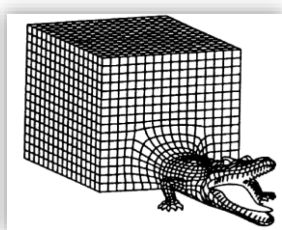
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

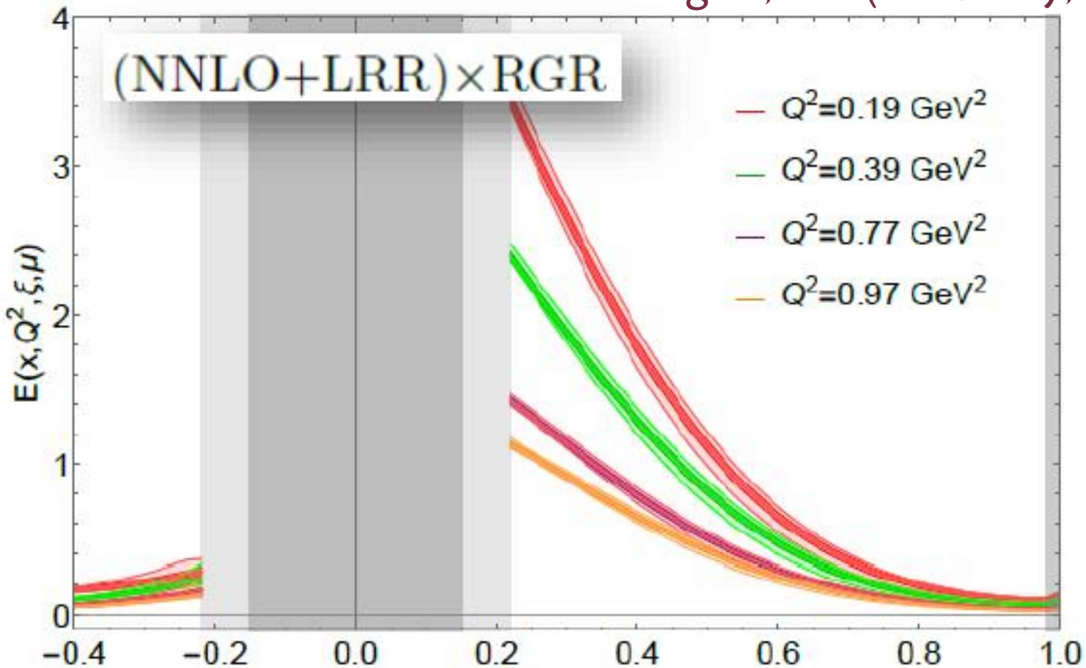
R. Zhang, et. al.
PLB 844, 138081 (2023)

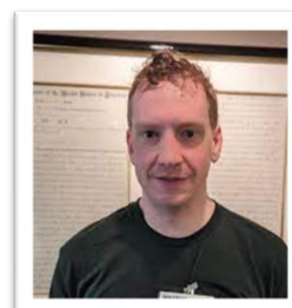
- $N_f$= 2+1+1 clover/HISQ, $a \approx 0.09$ fm, 135-MeV pion, $P_z \approx 2$ GeV

J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…





P: Jack Holligan

# GPD Systematic Update

## § Pion valence-quark GPDs

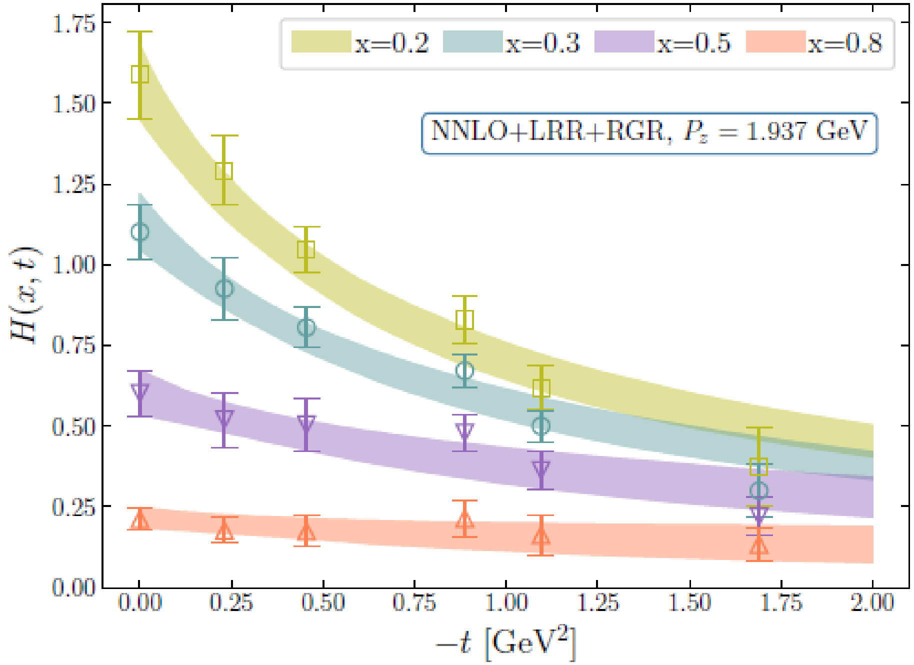
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

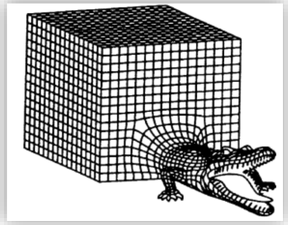
R. Zhang, et. al.
PLB 844, 138081 (2023)

- $N_f$= 2+1 clover/HISQ, $a \approx 0.09$ fm, 300-MeV pion, $P_z \approx 2$ GeV

Heng-Tong Ding et al (ANL/BNL/Wuhan), 2407.03516 [hep-lat]

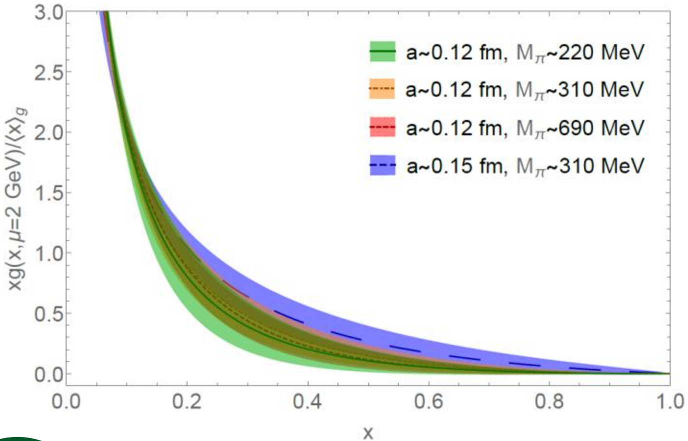
Wanted PDFs, GPDs, etc…

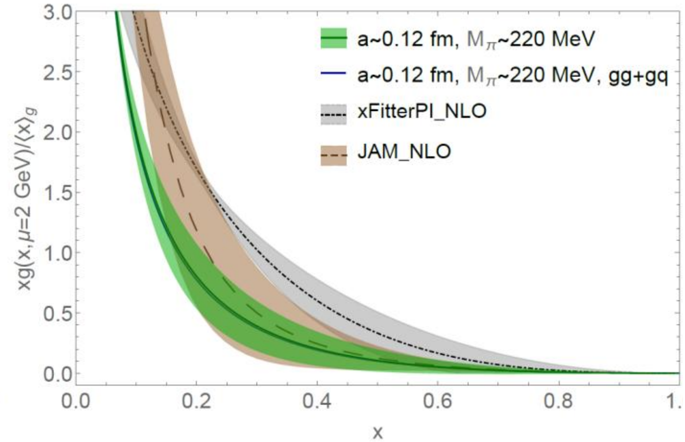


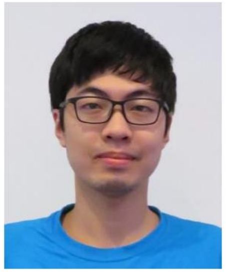



# Meson Gluon PDFs

## § First pion and kaon gluon PDFs $g(x)/\langle x\rangle$ using pseudo-PDF



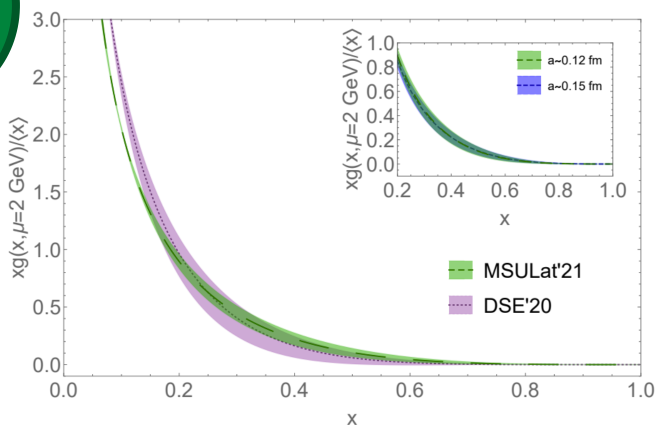



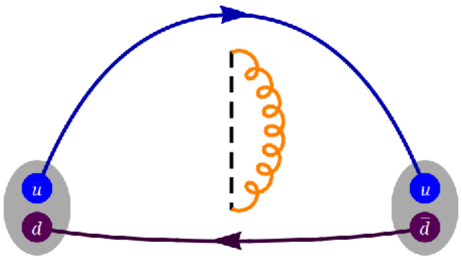
G: Zhouyou Fan

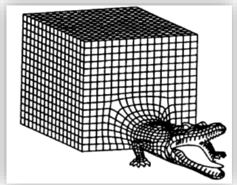
Wanted PDFs, GPDs, etc…

2104.06372, Fan et al. (MSULat); 2112.03124, Salas-Chavira et al. (MSULat)




G: Alejandro Salas-Chavira



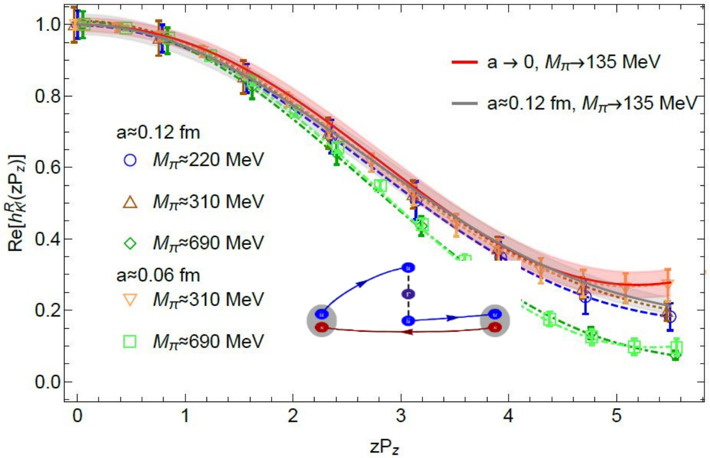



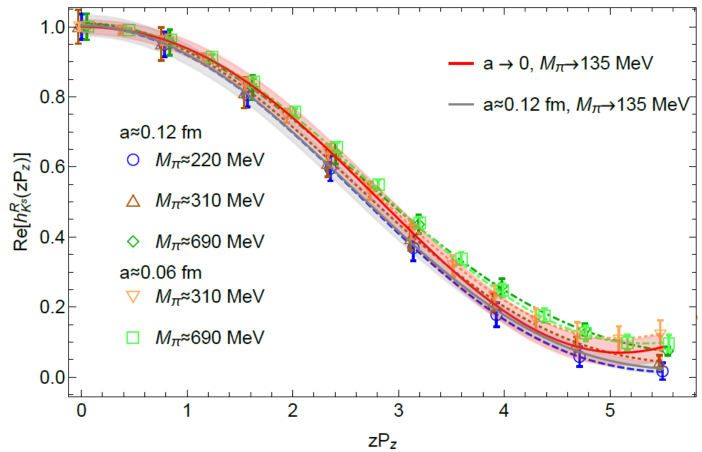
finite-volume, discretization, heavy quark mass, …

## § What does lattice QCD say about $g(x)$?

# Meson Valence-quark PDFs

## § Pion/Kaon PDFs using quasi-PDF in the continuum limit

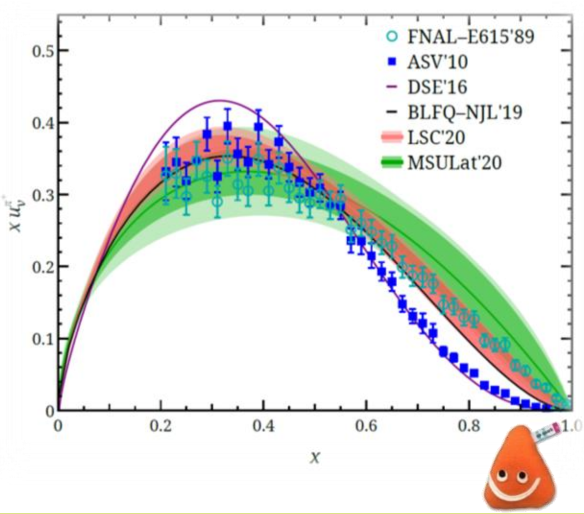


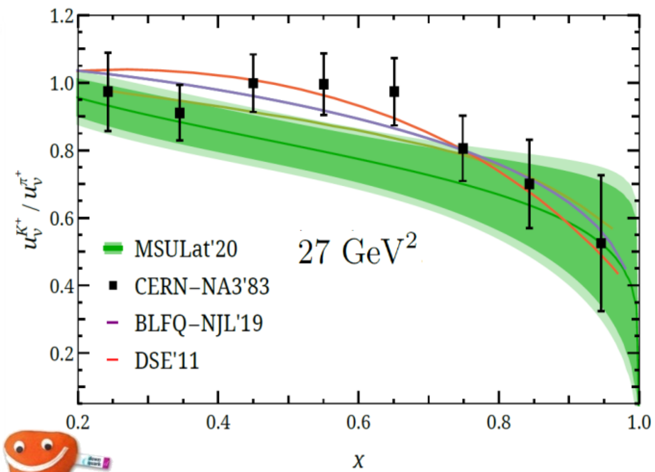



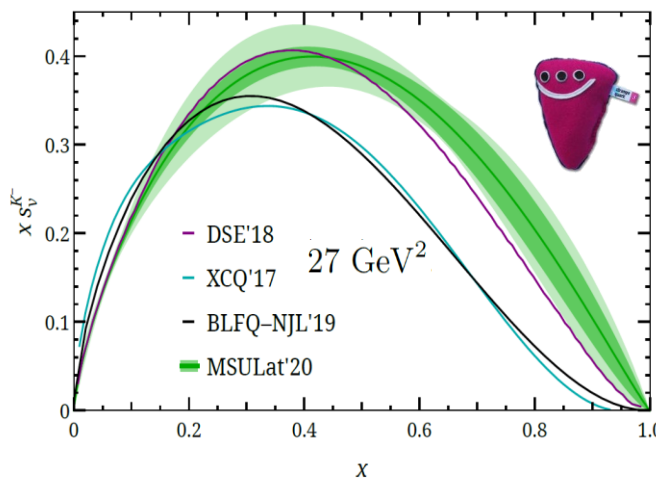

Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

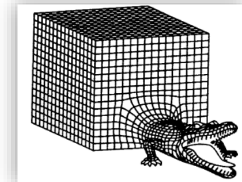
MSULat, 2003.14128



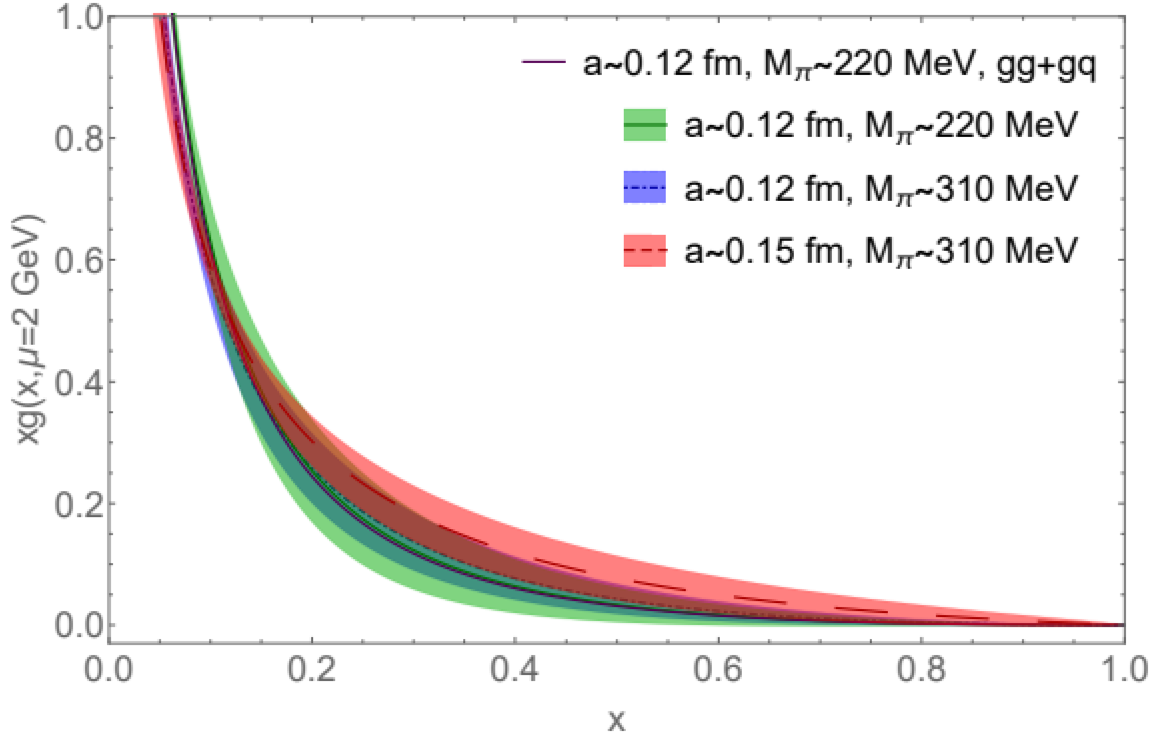



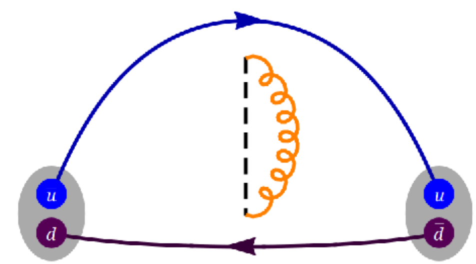



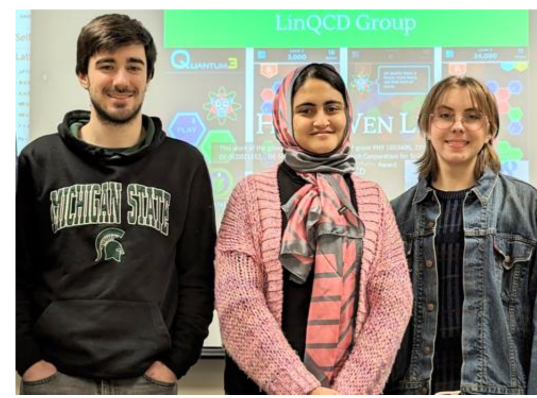
## § Back to Pion gluon PDF $g(x)$



- Update previous calculated $g(x)/\langle x\rangle$ in 2021



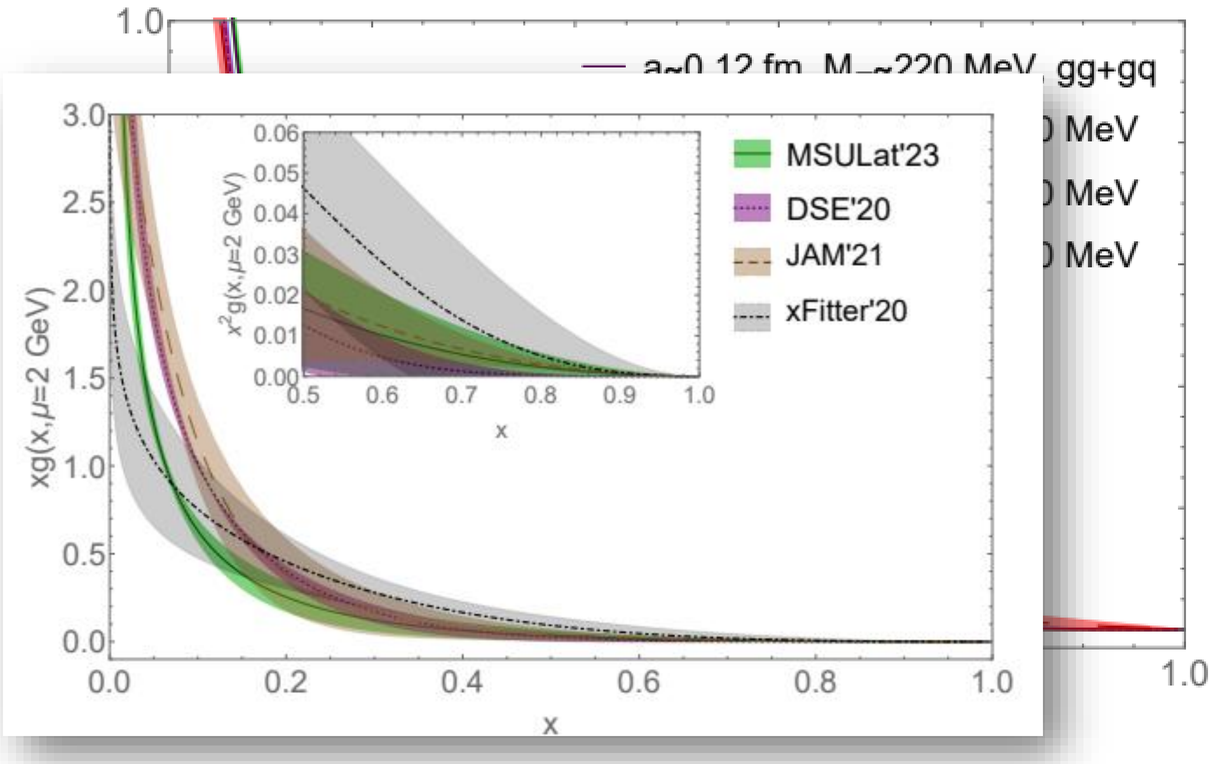



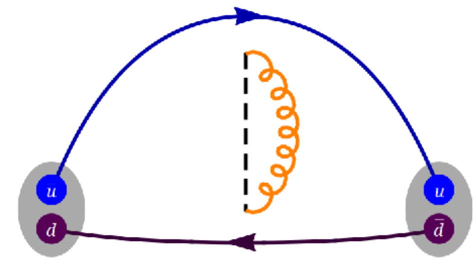

2310.12034, Good et al. (MSULat)

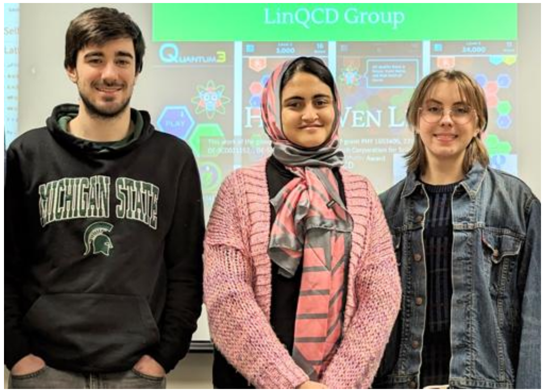
## § Back to Pion gluon PDF $g(x)$

- Update previous calculated $g(x)/\langle x\rangle$ in 2021




2310.12034, Good et al. (MSULat)

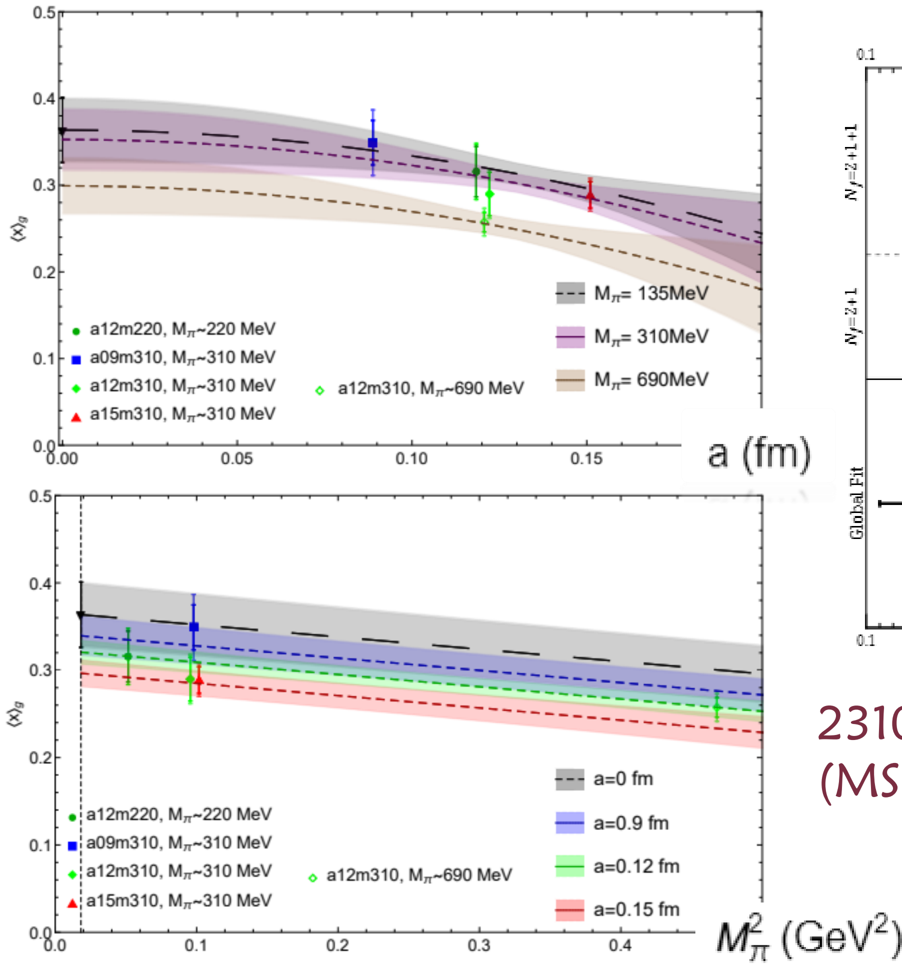


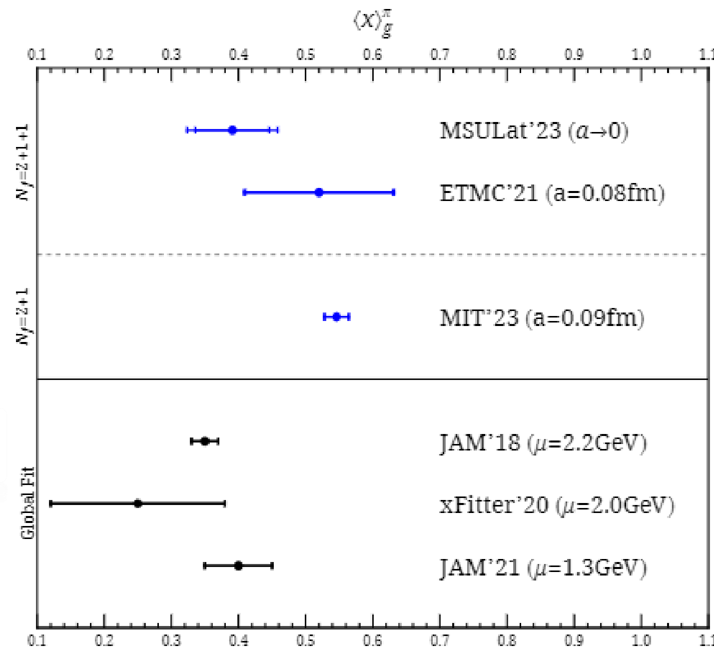



# Pion Gluon PDF Update

## § Study discretization systematic in $\langle x \rangle_{\{\pi,g\}}$

- Lattice details: clover/HISQ, HISQ, a ~{0,15, 0.12, 0.09} fm




2310.12034, Good et al. (MSULat)