Nuclear Science
Computing Center at CCNU

Chiral Properties of (2+1)-Flavor QCD in Magnetic Fields at Zero Temperature

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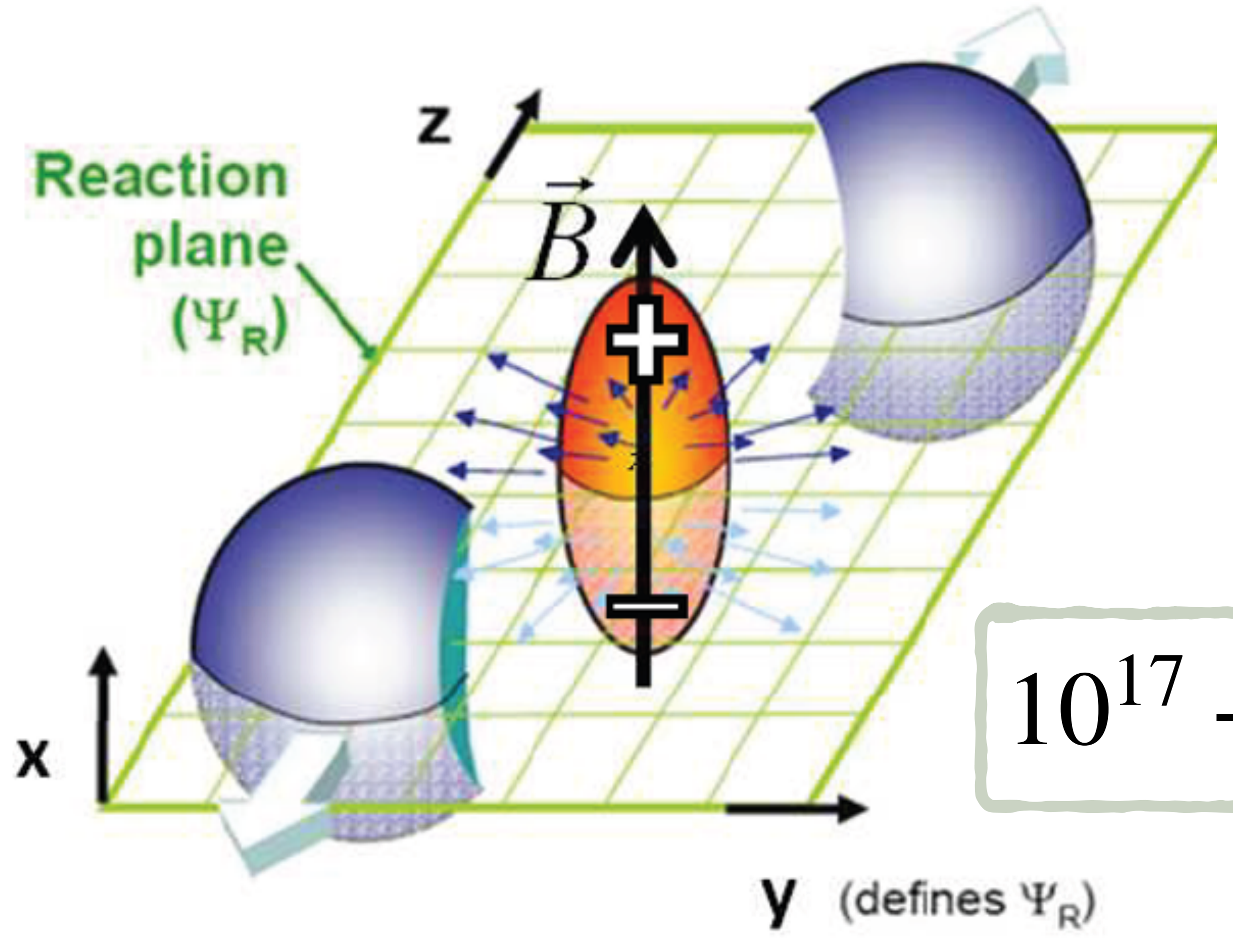
10.11, 2025@ 惠州

Strong Magnetic Fields: Strength Comparable to Λ_{QCD}^2

Heavy Ion-Collision



$10^{17} - 10^{18}$ G

RHIC $\sim 5m_\pi^2$ LHC $\sim 70m_\pi^2$

V. Skokov, *et al*., IJMPA 24, 5925 (2009)
W.-T. Deng, X.-G. Huang PRC 85, 044907 (2012)

Magnetars

surface $\sim 10^{14} - 10^{15}$ G

Kernel $\sim 10^{18}$ G

R.Turolla *et al*., **arXiv:1507.02924 [astro-ph.HE]**

Chiral Properties in Magnetic Fields

How do chiral properties change in comparable external magnetic fields?

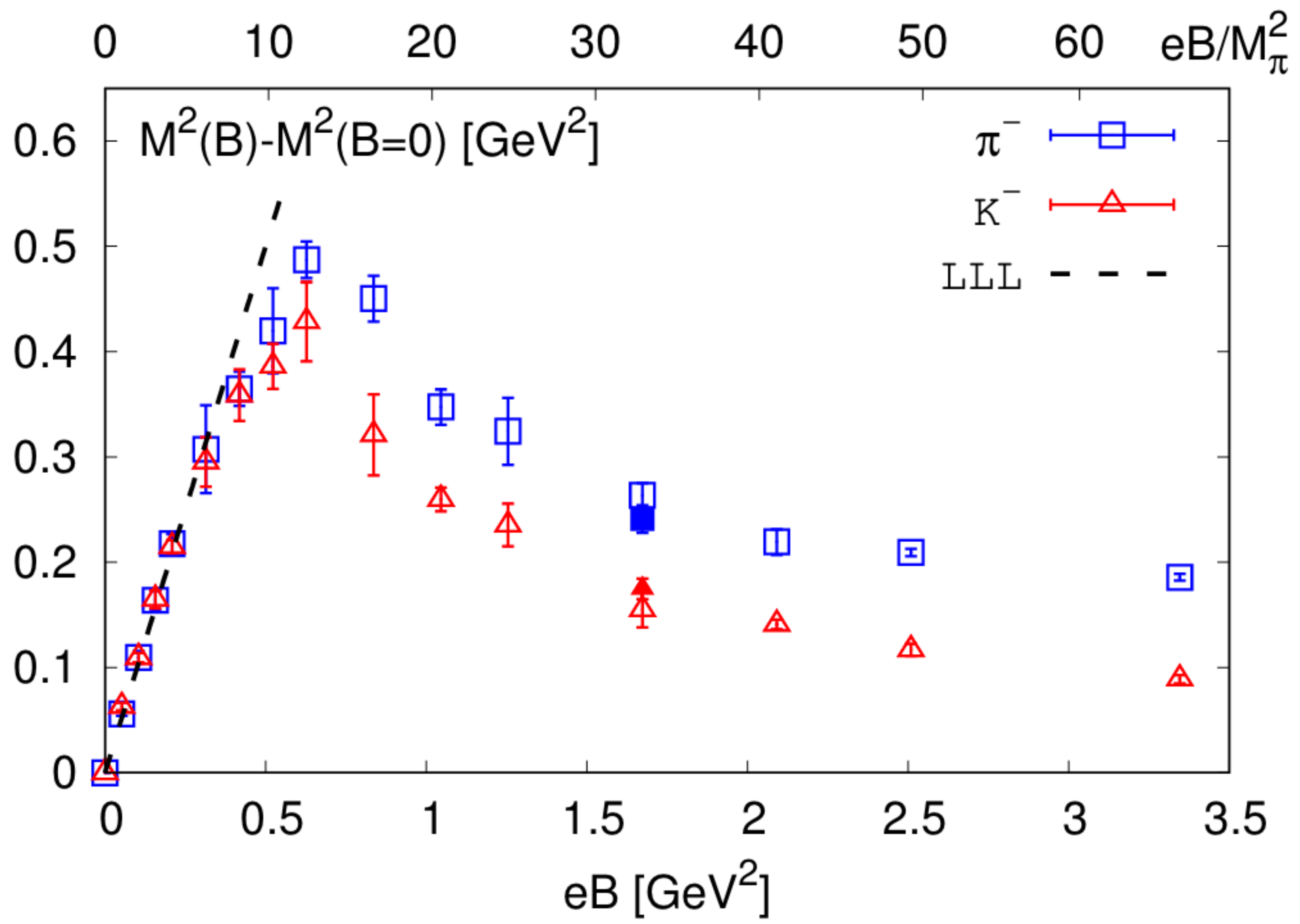
❖ (Inverse) Magnetic catalysis

- Magnetic catalysis (MC): Chiral condensate ↑ with *eB*
- Chiral condensate: Order parameter for spontaneous chiral symmetry breaking (SSB).
- $T = 0$: No thermal fluctuations

G. S. Bali *et al*., Phys. Rev. D 86, 071502 (2012)
I. A. Shovkovy, Lect. Notes Phys. 871, 13 (2013)
G. S. Bali *et al*., JHEP 02, 044 (2012)

❖ Masses and decay constants of mesons

- Goldstone bosons arising from SSB
- The previous work at $M_\pi \approx 220\ MeV$ at a finite lattice cutoff H.-T. Ding *et al*., Phys. Rev. D 104, 014505 (2021)
  - Opposite mass behavior of neutral mesons and charged mesons in magnetic fields



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G. S. Bali *et al*., Phys. Rev. D 86, 07150
I. A. Shovkovy, Lect. Notes Phys. 871, 1
G. S. Bali *et al*., JHEP 02, 044 (2012)

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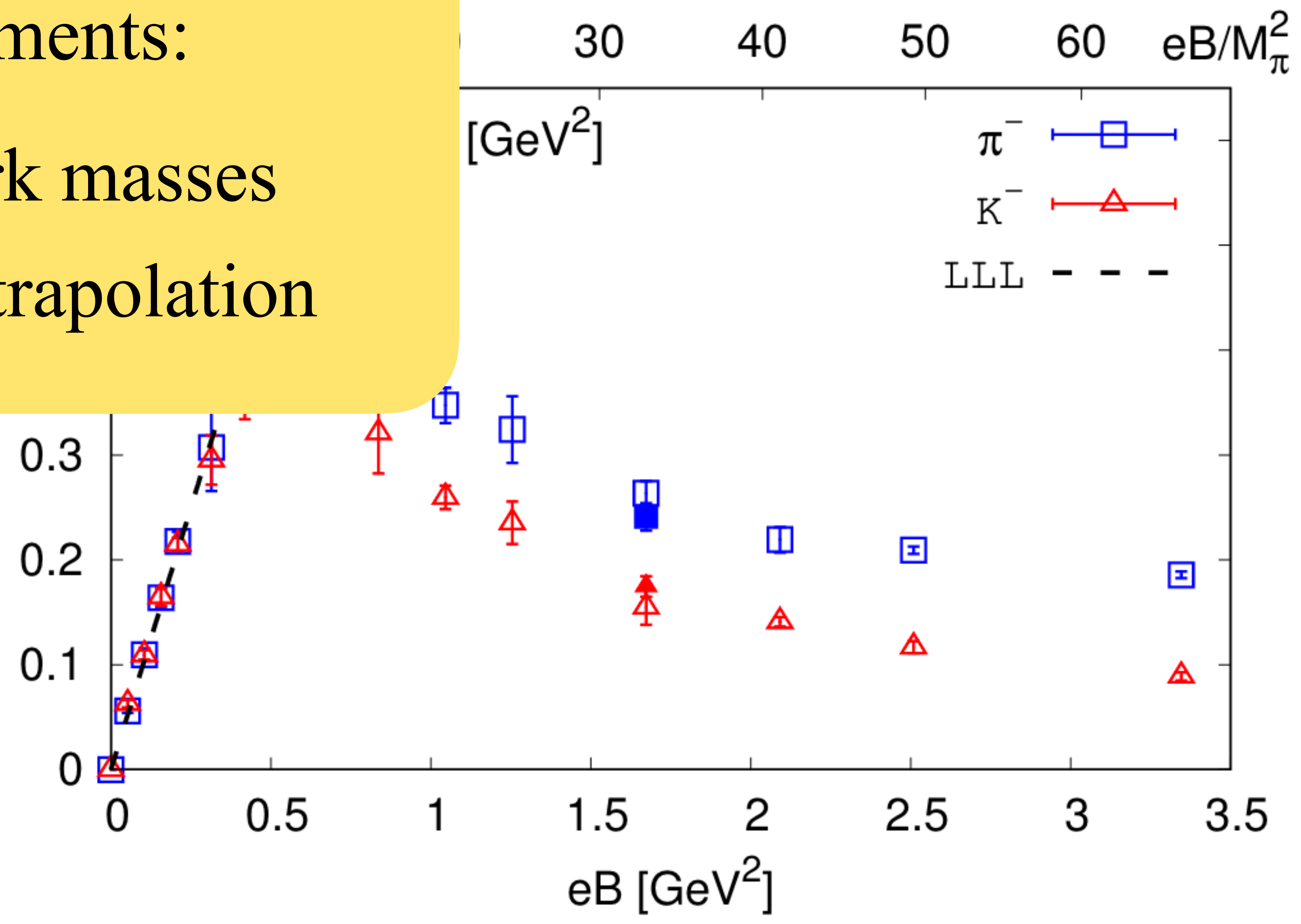
Our Improvements:

✓ Physical quark masses

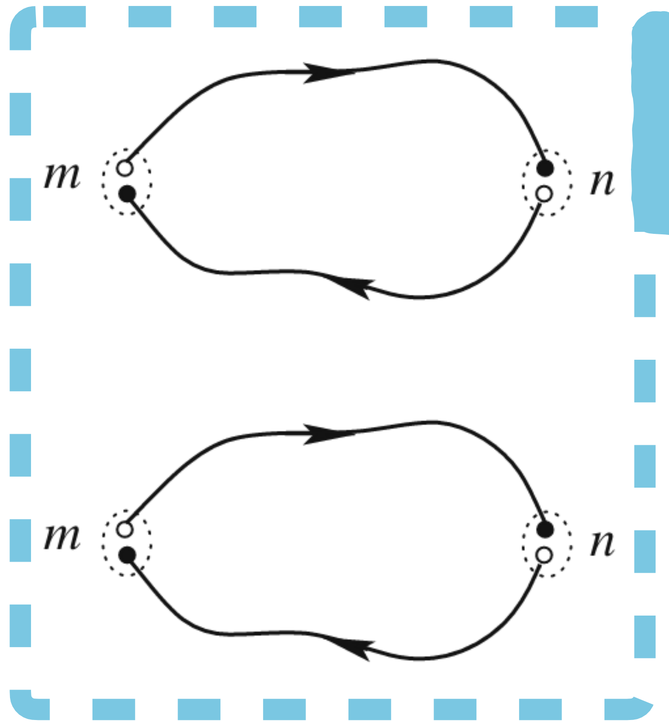
✓ Continuum extrapolation



Mesonic Two Point Correlation Functions

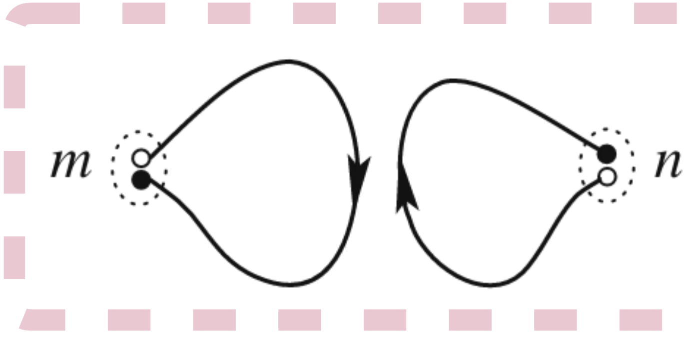
$\pi^{\pm}$ $\left\langle O_{\pi^{\pm}}(n)\overline{O}_{\pi^{\pm}}(m)\right\rangle_F$

π^{0} $\left\langle O_{\pi^{0}}(n)\overline{O}_{\pi^{0}}(m)\right\rangle_F$



connected part

+



disconnected part

- The connected part of the correlation function,

$$\mathrm{G}(\tau) = -\sum_{x,y,z} \zeta(\vec{n})\mathrm{Tr}\left[\left(M^{-1}(\vec{x},\tau;\vec{0},0)\right)^{\dagger} M^{-1}(\vec{x},\tau;\vec{0},0)\right],$$

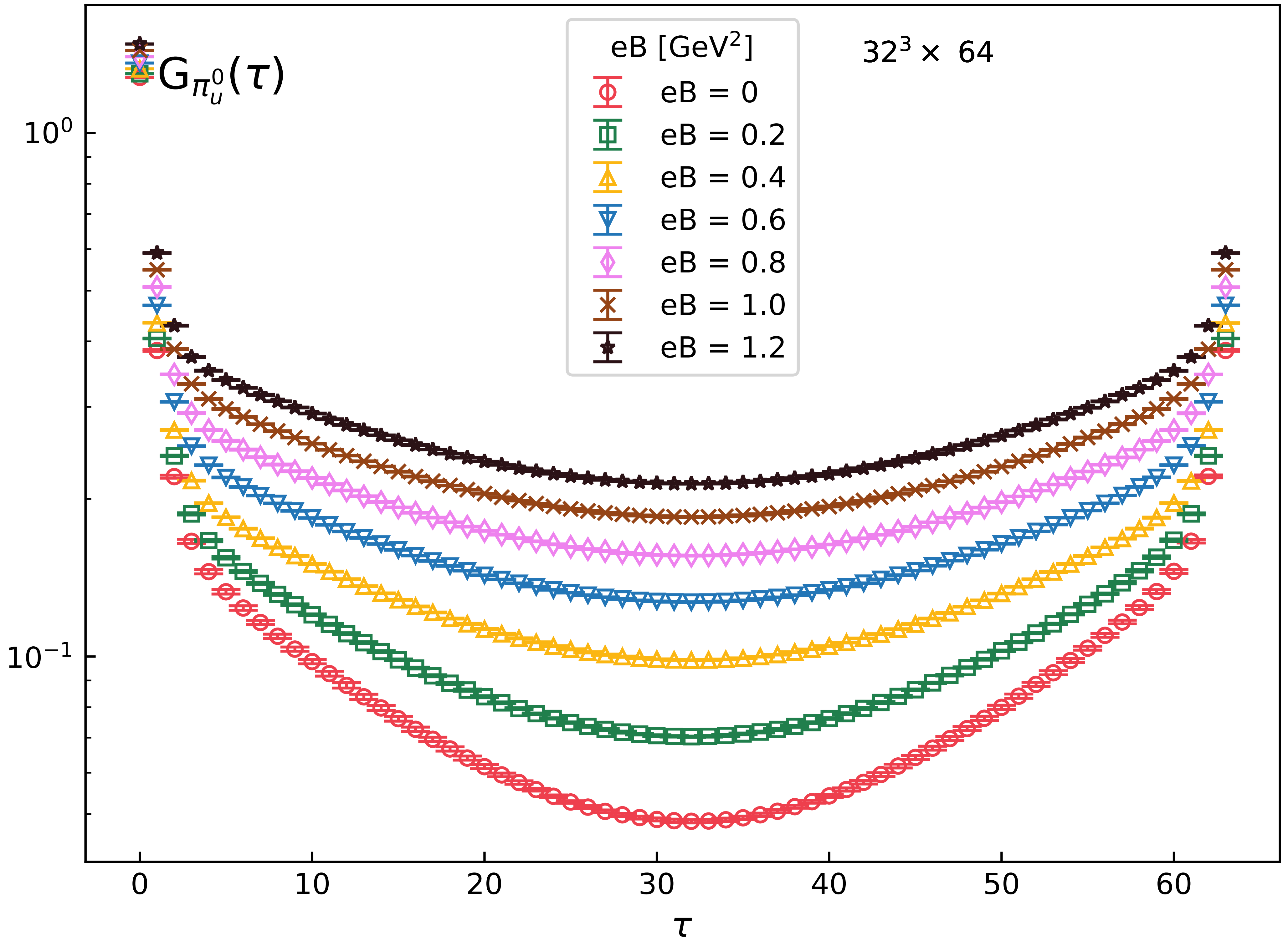
$M^{-1}(\vec{x},\tau;\vec{0},0)$: Staggered fermion propagator from $(\vec{0},0)$ to $(\vec{x},\tau)$

$\zeta(\vec{n})$: Phase factor, in the pseudoscalar channel $\zeta(\vec{n}) = 1$

- Energy Eigenstate Expansion:

$$G(\tau) = \sum_{n} |\langle 0|\mathscr{M}(0)|n\rangle|^2 e^{-E_n\tau}$$

- At large distances τ: $\lim_{\tau\to\infty} G(\tau) \sim Ae^{-m_{\Gamma}\tau}$



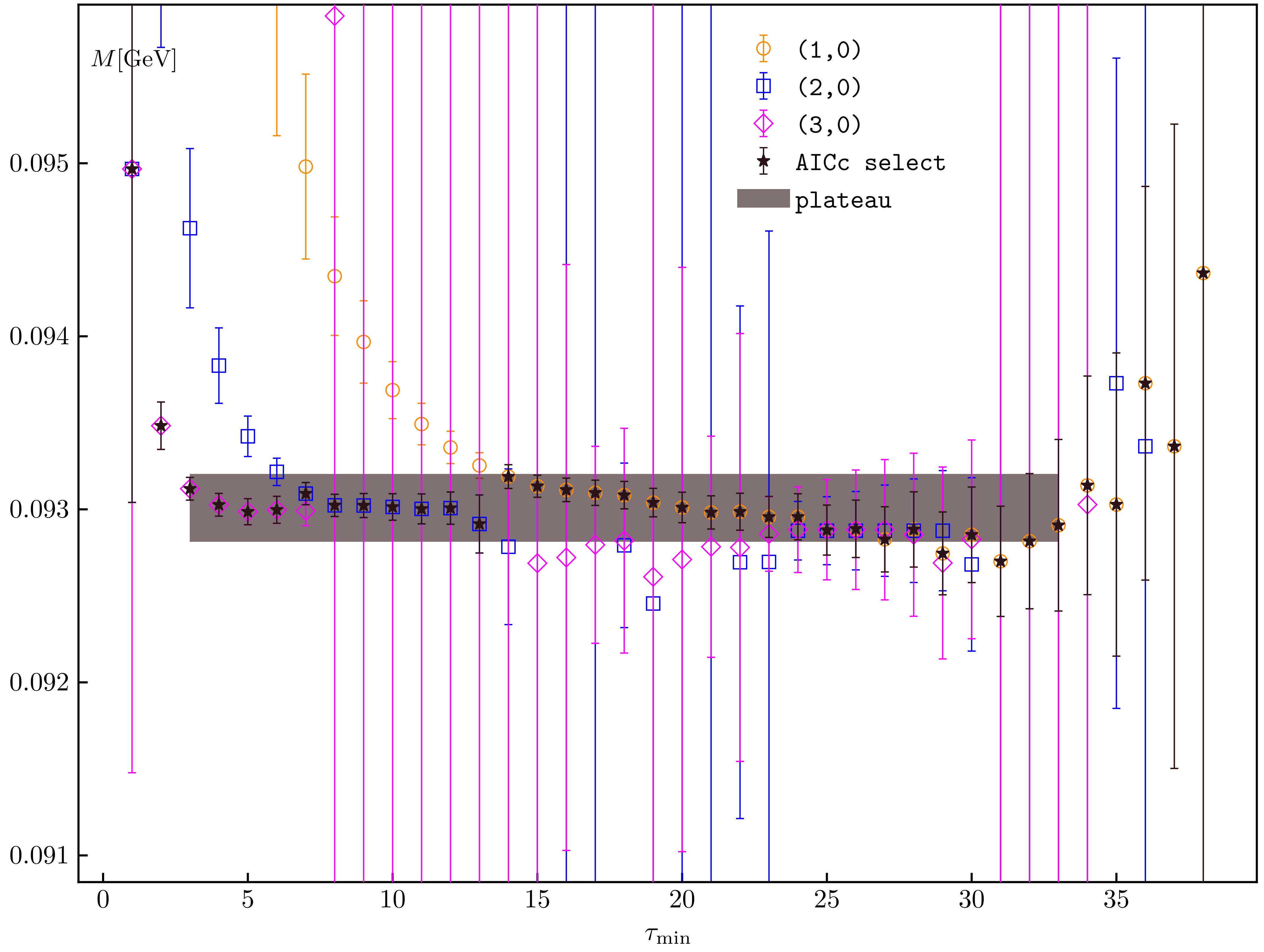
Mass & Decay Constant Extraction: Correlated Fitting

- Correlated fits: Account for the full covariance matrix in the fit.

- Fitting ansatz for different number of states:

$$G(n_\tau) = \sum_{i=1}^{N_{\mathrm{nosc}}} A_{\mathrm{nosc,i}} e^{-M_{\mathrm{nosc,i}} n_\tau} - (-1)^{n_\tau} \sum_{i=0}^{N_{\mathrm{osc}}} A_{\mathrm{osc,i}} e^{-M_{\mathrm{osc,i}} n_\tau}$$

  Multi-exponential fits: $(N_{\mathrm{nosc}}, N_{\mathrm{osc}})$
  Fit windows: $\tau \in [\tau_{\mathrm{min}}, N_\tau/2]$
  The physical meson mass is given by $M_{\mathrm{nosc,1}}$
  The amplitude $A_{\mathrm{nosc,1}} \propto f_\Gamma$ (meson's decay constant)



- Select the fit with lowest corrected Akaike Information Criterion (AICc):

$$\mathrm{AICc} = 2k - 2\ln \hat{L} + \frac{2k(k+1)}{n-k-1},$$

- Use Gaussian bootstrapping to estimate final value and error

- We also use the Oblique Lanczos Method to cross-check the extracted energies.

Lattice Setup

- **Flavors:** $N_f = 2 + 1$
- **Action:** Highly Improved Staggered Quarks (HISQ) and Tree-level Improved Symanzik action
- **Quark masses:** Physical values $(m_u = m_d, m_s)$; $m_s/m_u \approx 27$
- **Lattice sizes:** $24^3 \times 48\,(0.112\,\mathrm{fm}),\ 32^3 \times 64\,(0.084\,\mathrm{fm}),\ 40^3 \times 80\,(0.067\,\mathrm{fm}),\ 48^3 \times 96\,(0.056\,\mathrm{fm})$
- **eB ranges:** $0 \le \mathrm{eB} \le 1.22\ \mathrm{GeV}^2\,(0 \sim 66 m_\pi^2)$,
  - **Quantized:** $\mathrm{eB} = \dfrac{6\pi N_b}{N_x N_y} a^{-2}, N_b = 0, 2, 4, 6, 8, 10, 12$ (magnetic fluxes)

$N_\sigma^3 \times N_\tau$	a [fm]	β	am_s	am_l	# of conf at eB [GeV^2]						
					0.0	0.2	0.41	0.61	0.81	1.02	1.22
$24^3 \times 48$	0.112	6.722	0.0484	0.001793	600	701	1401	1350	1350	1350	1350
$32^3 \times 64$	0.084	7.016	0.0353	0.001307	600	800	800	800	800	800	800
$40^3 \times 80$	0.067	7.254	0.0274	0.001015	475	555	555	555	555	321	316
$48^3 \times 96$	0.056	7.455	0.0222	0.000822	150	266	250	250	220	306	215

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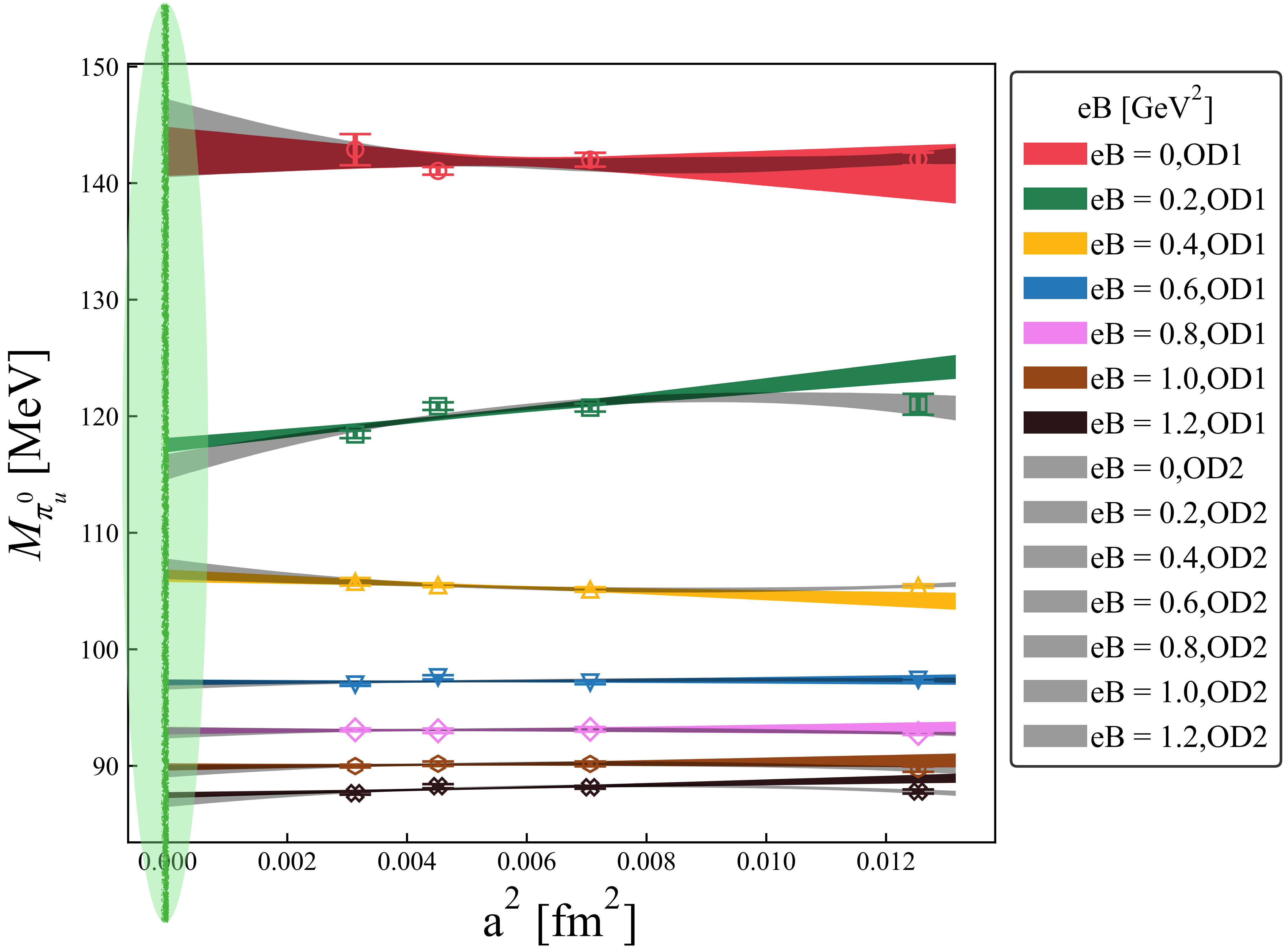
Continuum Extrapolation

- Obtain physical results by taking the **continuum limit** ($a \to 0$)
- Fit lattice observable $O(a^2)$ using different ansätze:
  - Linear: $O(a^2) = O_0^{\text{linear}} + a^2 A$

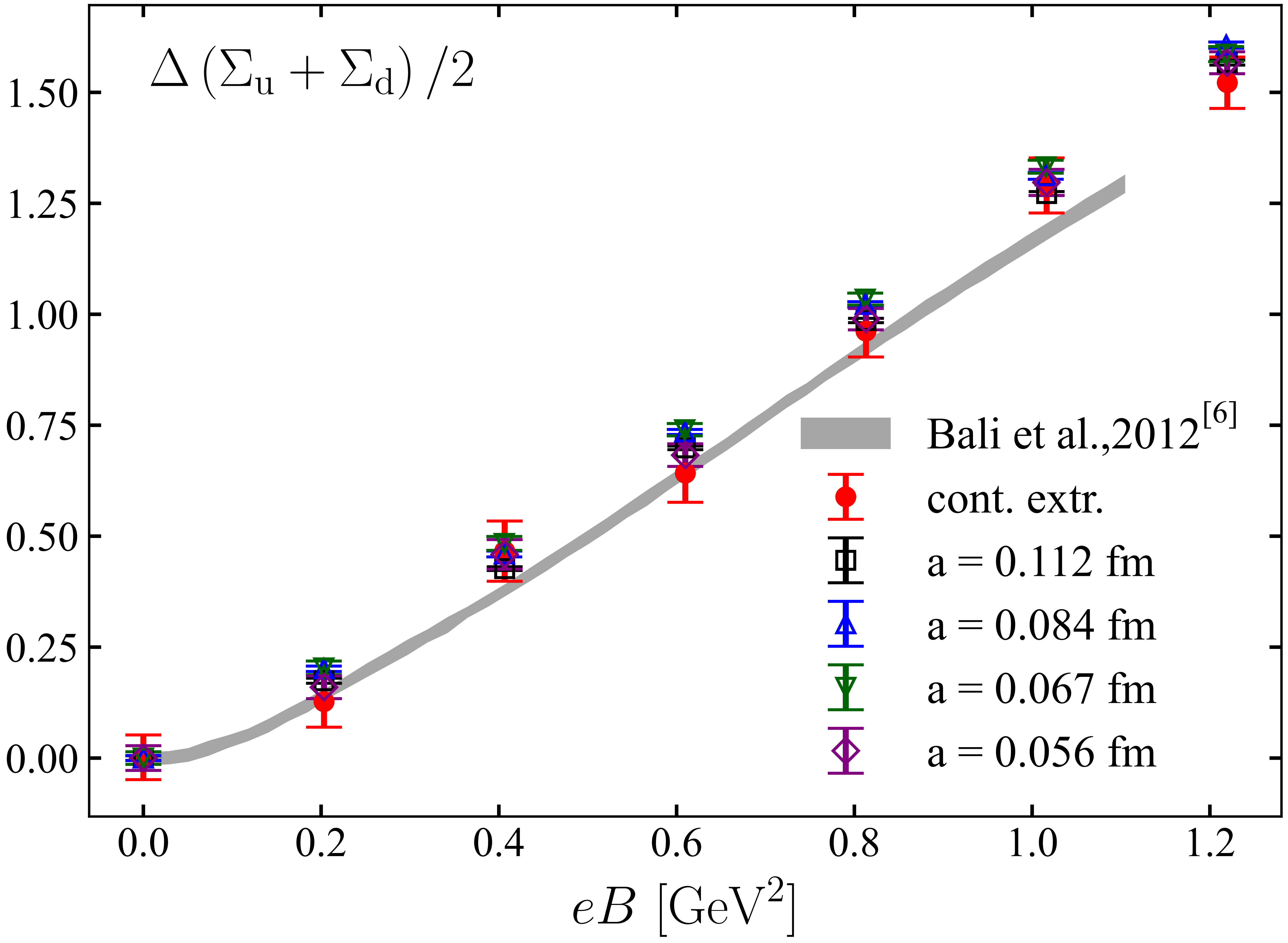
    Applied to the three finest lattices.
  - Quadratic: $O(a^2) = O_0^{\text{quad}} + a^2 A + a^4 B$

    Applied to all four lattices.
- Final Continuum Value:

$$O_{\text{cont}} = \frac{1}{2}\left(O_0^{\text{linear}} + O_0^{\text{quad}}\right)$$



Light Quark Chiral Condensates in Magnetic Fields



*G. S. Bali *et al.*, Phys. Rev. D **86**, 071502 (2012)

Chiral Condensate

$$\langle\bar{\psi}\psi\rangle_q(B,T) \equiv \frac{T}{V}\frac{\partial \ln \mathcal{Z}(B,T)}{\partial m_f}$$

$\mathcal{Z}$: Partition function
V: Spatial volume
$T = 1/(aN_\tau)$
$q = u, d$

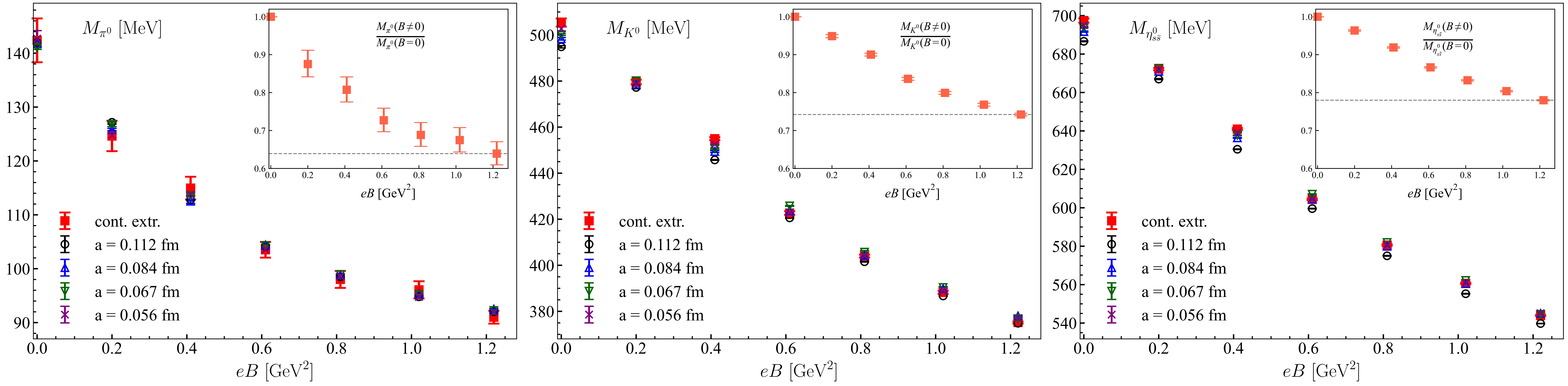
To remove additive and multiplicative divergences

Renormalized light quark chiral condensate

$$\Delta\Sigma_q(B) = \frac{2m_q}{m_\pi^2 F_\pi^2}\left(\langle\bar{\psi}\psi\rangle_q(B,T=0) - \langle\bar{\psi}\psi\rangle_q(B=0,T=0)\right)$$

☑ Magnetic Catalysis: the light quark condensates increase with the magnetic field at $T = 0$

Masses of Neutral Mesons: $\pi^0, k^0, \eta^0_{s\bar{s}}$



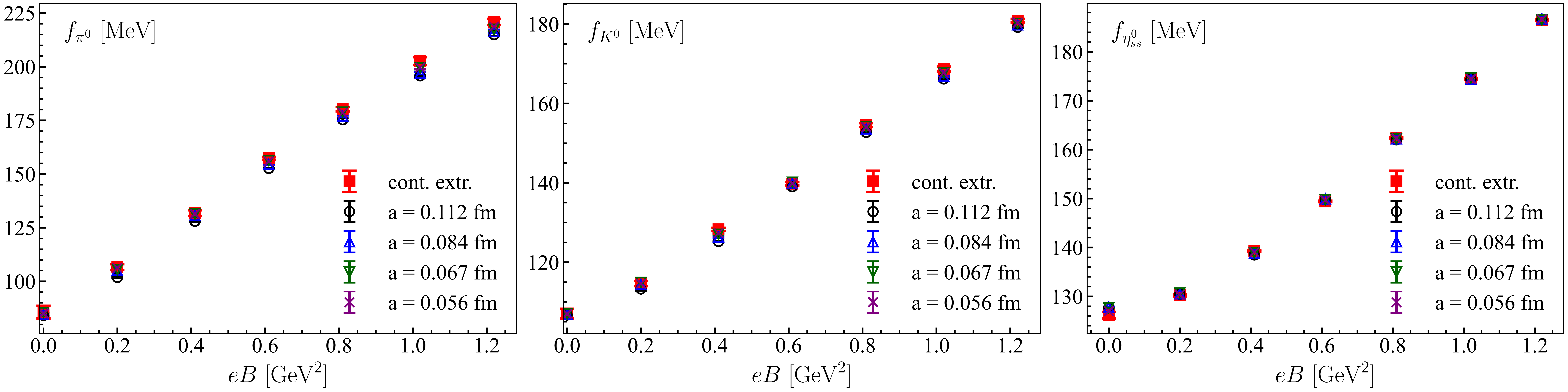
M_{π^0} using $G_{\pi^0} = (G_{\pi^0_u} + G_{\pi^0_d})/2$;

$M_{K^0(s\bar{d})}$

$M_{\eta^0_{s\bar{s}}}$

- The masses of neutral mesons decrease as eB increases, with the neutral pion mass reaching about 65% of its original value at $eB \sim 1.2$ GeV^2.
- The heavier the neutral meson, the less it is affected by the magnetic field.

Decay Constants of $\pi^0, K^0, \eta^0_{s\bar{s}}$



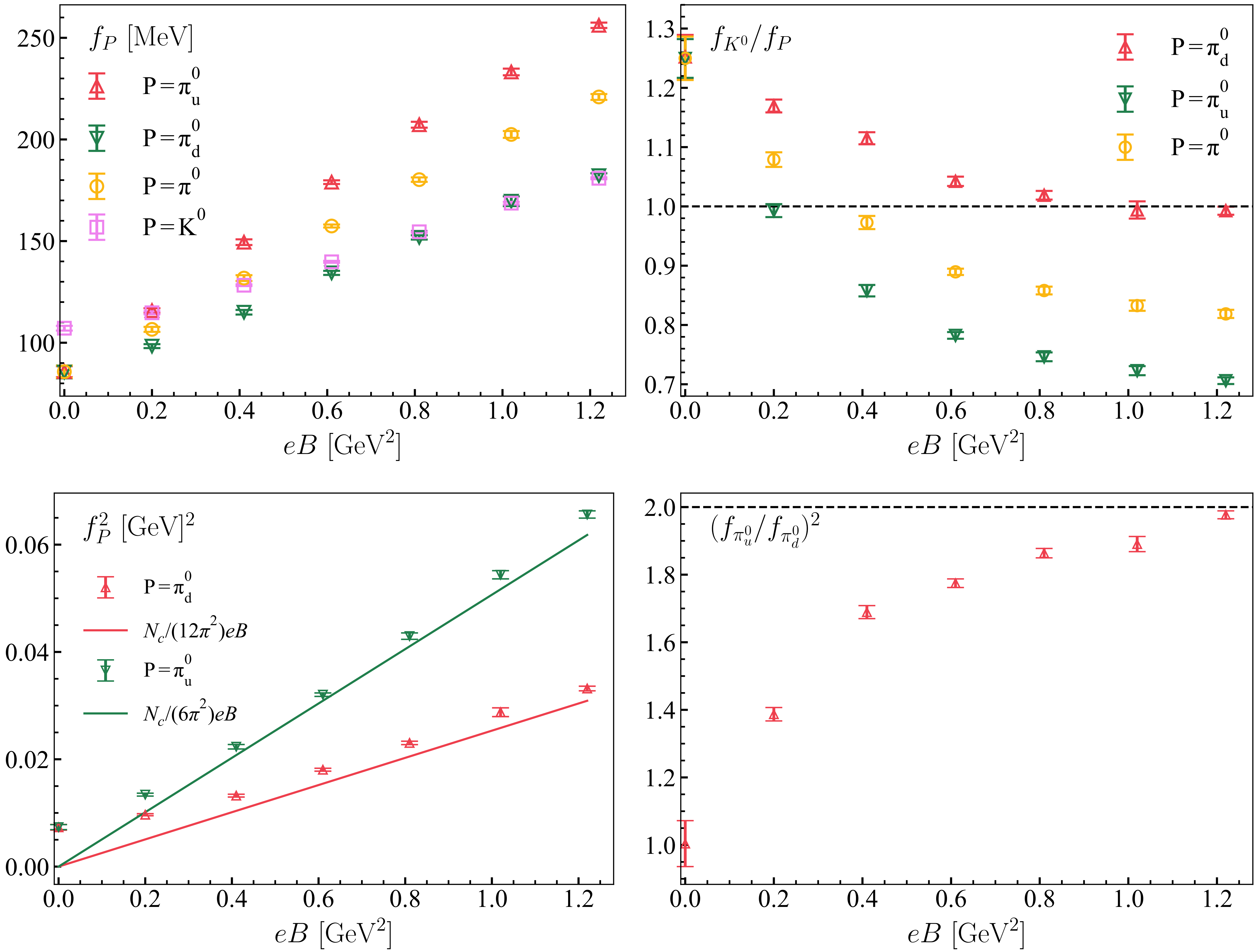
$$f_{\pi^0} = 2m_l\sqrt{\frac{A_{\pi^0;\mathrm{nosc},1}}{4M^3_{\pi^0}}}$$

$$f_{K^0} = (m_l + m_s)\sqrt{\frac{A_{K^0;\mathrm{nosc},1}}{4M^3_{K^0}}}$$

$$f_{\eta_{s\bar{s}}} = 2m_s\sqrt{\frac{A_{\eta_{s\bar{s}};\mathrm{nosc},1}}{4M^3_{\eta_{s\bar{s}}}}}$$

☑ All three decay constants increase with the magnetic field strength.

Decay Constants of $\pi^0, K^0, \eta^0_{s\bar{s}}$

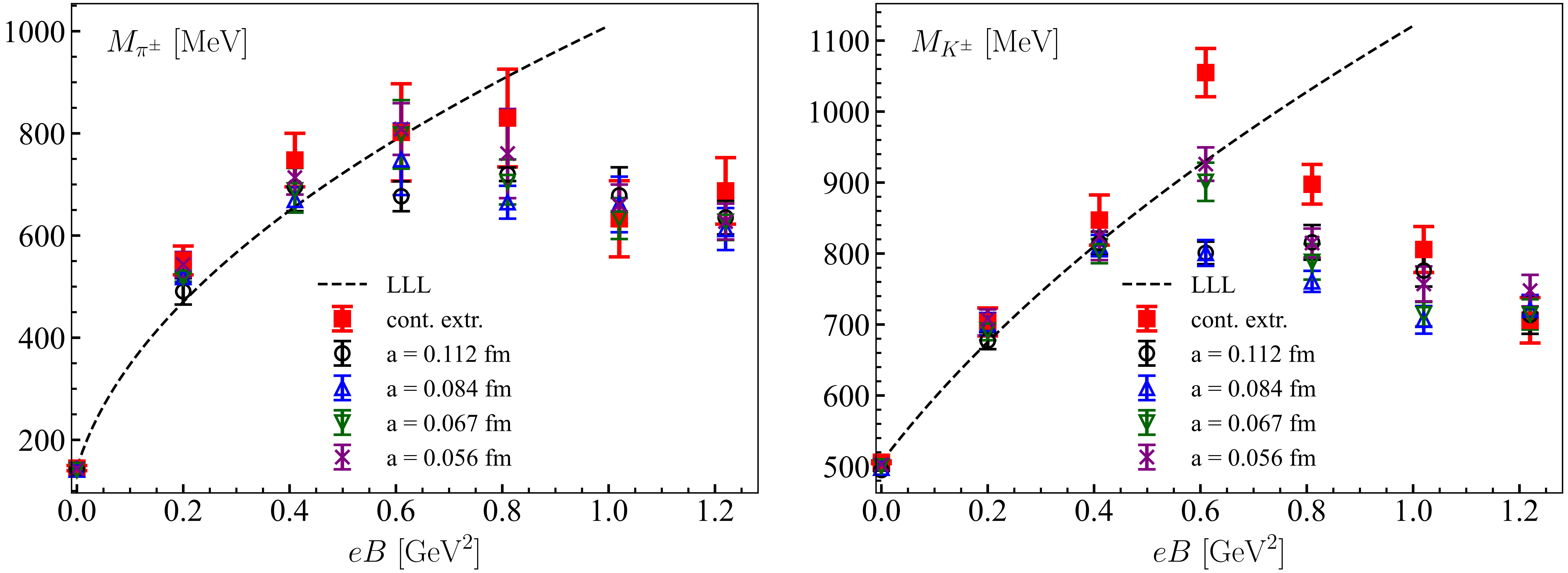


- ☑ $eB\uparrow$, $f_{\pi^0_u}$ increase fastest, f_{K^0} increase slowest
- ☑ f_{K^0}/f_π decreases with eB, especially for the up-quark component

$$\tilde{f}^2 = \frac{(N_d f_u + N_u f_d)^2}{N_f^2} = \frac{(\sqrt{2}N_d + N_u)^2 N_c}{12\pi^2 N_f^2}|eB|.$$

V. A. Miransky, I. A. Shovkovy, Phys.Rev. D66, 045006(2002)

The Masses of Charged Meson $\pi^{\pm}, K^{\pm}$



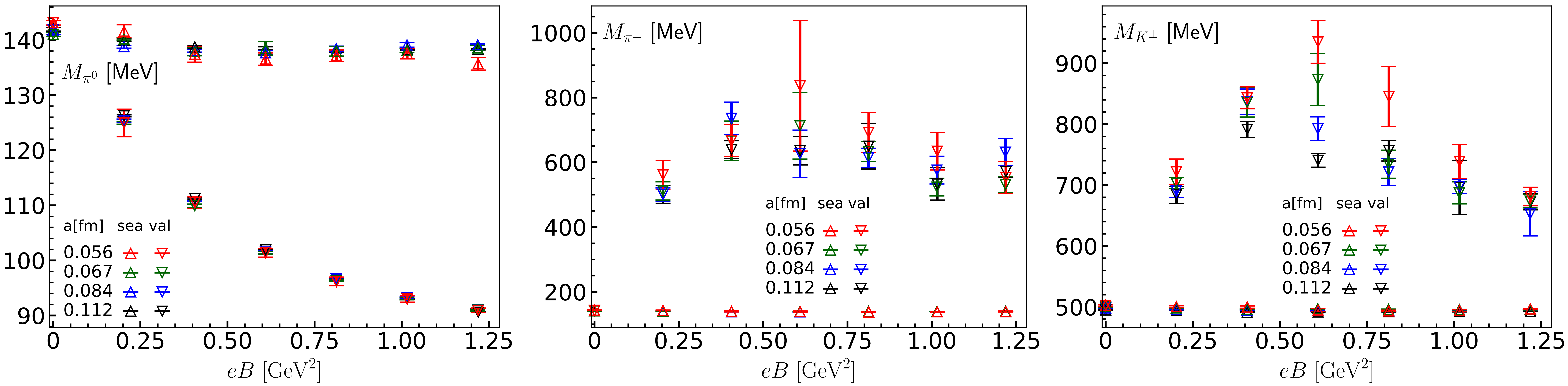
- The Lowest Landua Level (LLL),

$$M^{\pm}_{\rm ps}(B) = \sqrt{\left(M^{\pm}_{\rm ps}(B=0)\right)^2 + |eB|}\,.$$

☑ $M_{\pi^\pm}, M_{K^0}$

- Initially increase with eB,
- Around $eB \approx 0.6\,\mathrm{GeV}^2$, plateau or decrease slightly, deviate from the lowest Landau level.

Masses From Sea and Valence Quark Contributions



- ☑ Sea quark: weak eB dependence, change < 3%
- ☑ Valence quark: follows total mass behavior vs eB

Summary

- Magnetic catalysis observed up to $eB \sim 1.2\ \mathrm{GeV}^2$.

From Lattice correlator calculations in pseudoscalar channel at zero temperature

- Neutral meson masses $\downarrow$ as $eB \uparrow$, with heavier mesons less affected.
- Decay constants of neutral pseudoscalar mesons $\uparrow$ with eB.
- The masses of charged mesons deviate from Lowest Landau Levels at $eB \approx 0.6\,\mathrm{GeV}^2$, with no further increase.
- Valence quarks drive the eB dependence of meson masses; sea-quark effects are minor.

Thank You for your attention!

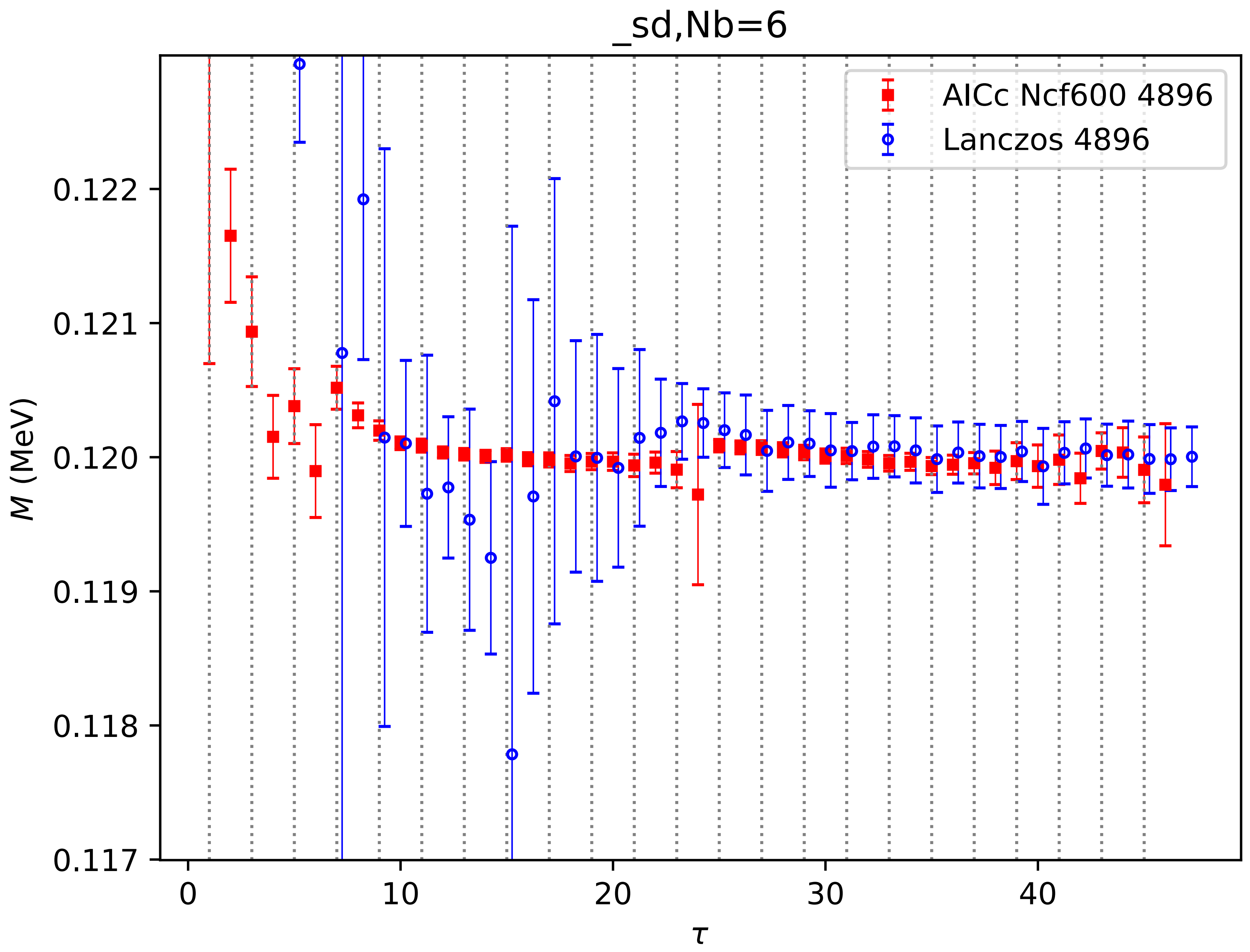
Backup

Oblique Lanczos Method: Fit - Independent Energy Extraction

- Why use Oblique Lanczos?
  - **Cross-check** multi-state fit analysis
  - Provide a **fit-independent** estimate of ground-state mass

- How it works:
  - Direct application to transfer matrix $T = e^{-aH}$
  - Extracts energy levels from Krylov-space projection
  - Energy levels: $E_k = -(1/a)\ln \lambda_k$

    λ_k: Eigenvalues of effective tridiagonal matrix T_m



Ground-state mass for K^0 via Oblique Lanczos (bule points) and Correlated Fit (red point) on a $48^3 \times 96$ lattice at $eB = 0.6\,\mathrm{GeV}^2$

☑ **Agreement** with AICc-preferred fit results