$\Lambda\Lambda - N\Xi$ Interaction in $N_f = 2 + 1$ Lattice QCD

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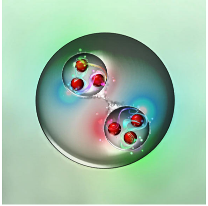
Outline

Introduction

Motivation

The H-dibaryon is a spin and isospin singlet, positive parity state composed of six quarks (*uuddss*). It was first proposed by Jaffe[1] using MIT bag model as a deeply bound state.



Related threshold:

$$\begin{aligned} 2\times m_{\Lambda}^{phy} &\approx 2231 \text{ MeV} \\ m_{\Xi^-}^{phy} + m_{p}^{phy} &\approx 2260 \text{ MeV} \\ 2\times m_{\Sigma}^{phy} &\approx 2387 \text{ MeV} \end{aligned} \tag{1}$$

[1] R. L. Jaffe (1977). DOI: 10.1103/PhysRevLett.38.195

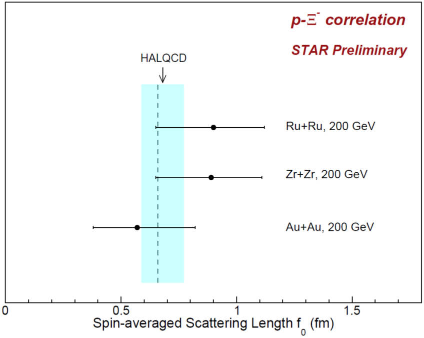
H-dibaryon on the Lattice

Author	n_f	a(fm)	m_π(MeV)	B_H(MeV)
Mackenzie, 1985[2]	0	0.13	600 – 900	< 0
...	0	...	...	...
HALQCD, 2011[3]	3	0.121	673 – 1015	30 – 40
NPLQCD, 2011[4]	2 + 1	0.123	389	$13.2 \pm 1.8 \pm 4.0$
HALQCD, 2020[5]	2 + 1	0.085	146	$B_{\Xi N} > B_{\Lambda\Lambda} \approx 0$
Mainz, 2021[6]	3	0.04 – 0.1	420	$4.56 \pm 1.13 \pm 0.63$*

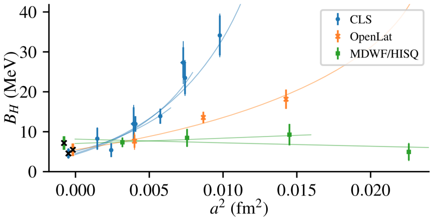
Table: Recent LQCD results of H-dibaryon calculation.

[2] Paul B. Mackenzie and H. B. Thacker (1985). DOI: 10.1103/PhysRevLett.55.2539
[3] HALQCD Collaboration et al. (2011). DOI: 10.1103/PhysRevLett.106.162002
[4] NPLQCD Collaboration et al. (2011). DOI: 10.1103/PhysRevLett.106.162001
[5] Kenji Sasaki et al. (2020). DOI: 10.1016/j.nuclphysa.2020.121737
[6] Jeremy R. Green et al. (2021). DOI: 10.1103/PhysRevLett.127.242003

- Recent experimental results from the STAR Collaboration are in good agreement with the HAL QCD results.

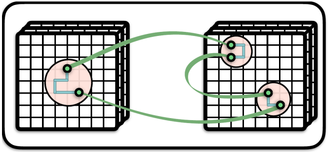


- It is observed that the lattice spacing can have a significant impact on the interaction strength.

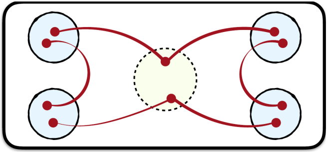


Methodology

Lüscher finite volume method



finite volume spectrum



scattering amplitude

Lüscher quantization condition:

$$\det[F^{-1}(E, \mathbf{P}, L) + \mathcal{M}(E)] = 0 \tag{2}$$

For a given total momentum $\mathbf{P}$ and a set of scattering amplitudes $\mathcal{M}$ (which might be matrices in the space of kinematically open channels), in an $L \times L \times L$ volume, the discrete spectrum of states having a specified quantum number $E_n(\mathbf{P}, L)$ is given by all energies for which the determinant evaluates to zero[7].

[7] Raul A. Briceno, Jozef J. Dudek, and Ross D. Young (2018). DOI: 10.1103/RevModPhys.90.025001

Smearing & Laplacian

Define gauge covariant Laplacian Δ:

$$\begin{aligned}\Delta_{\boldsymbol{m},\boldsymbol{n}} &= \sum_{\hat{i}=1}^{3}\left[\delta_{\boldsymbol{m}+\hat{i},\boldsymbol{n}}U_i(\boldsymbol{m}) + \delta_{\boldsymbol{m}+\hat{i},\boldsymbol{n}}U_i(\boldsymbol{m}-\hat{i}) - 2\delta_{\boldsymbol{m},\boldsymbol{n}}\right] \\ &= \sum_{\hat{i}=\pm1,\pm2,\pm3}\left[\delta_{\boldsymbol{m}+\hat{i},\boldsymbol{n}}U_i(\boldsymbol{m}) - \delta_{\boldsymbol{m},\boldsymbol{n}}\right], \end{aligned} \tag{3}$$

Δ commutes with lattice translations, rotations & reflection
$\rightarrow$ interpolating operator built from $\psi_s = f(\Delta)\psi$ hase same symmetry properties as operator built from original quark field ψ.

Example : Gaussian smearing:

$$S_G(\boldsymbol{n}|\boldsymbol{m}) = e^{\sigma^2\Delta_{\boldsymbol{m},\boldsymbol{n}}} \simeq (1 + \frac{\sigma}{n_\sigma}\Delta_{\boldsymbol{m},\boldsymbol{n}})^{n_\sigma}, \tag{4}$$

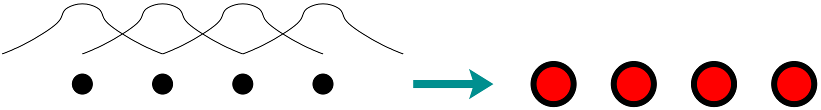


Figure: Schematic diagram of Gaussian smearing method.

Distillation(LapH) Method

Perform eigendecomposition to the Laplacian Δ:

$$\Delta = V\boldsymbol{\lambda}V^{\dagger}, \tag{5}$$

- $\boldsymbol{\lambda}$: diagonal matrix whose diagonal elements are the eigenvalues of the Laplacian.
- V is the matrix composed of all eigenvectors of the Laplace operator, satisfying the relation: $VV^{\dagger} = I$.

Generalize the smearing operator to be an arbitrary function of the eigenvalues of the Laplacian:

$$\mathcal{S}_D(\boldsymbol{n}|\boldsymbol{m}) = VfV^{\dagger}, \tag{6}$$

Let f be the Heaviside function of the Laplacian's eigenvalues λ_i[8]:

$$f = \Theta(\sigma_d + \lambda_i), \tag{7}$$

[8] Hadron Spectrum Collaboration et al. (2009). DOI: 10.1103/PhysRevD.80.054506.

Distillation(LapH) Method

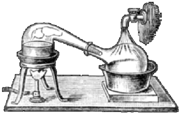
Apply smearing operator $\mathcal{S}$ to quark field :

$$\mathcal{S}\psi_q = V\Theta V^{\dagger}\psi_q \tag{8}$$

The smeared quark propagator can be decomposed into a combination of perambulator and eigenvector:

$$\tilde{Q} = V(t')V^{\dagger}(t')Q(t',t)V(t)V^{\dagger}(t) \tag{9}$$

- Provide an efficient way of computing all-to-all quark propagators.
- Suppress the contamination from unwanted high-lying modes.



Example : Pion correlation function with distillation

The pion interpolating operator with definite momentum $\boldsymbol{p}$ can be expressed as:

$$\mathcal{O}_\pi(\boldsymbol{p}, t_1) : \sum_{\boldsymbol{n}} \bar{d}(n)_{\substack{\alpha_1 \\ c_1}} \gamma_{5\,\alpha_1\beta_1} u(n)_{\substack{\beta_1 \\ c_1}} e^{-i\boldsymbol{n}\cdot\boldsymbol{p}}, \tag{10}$$

Thus pion 2pt correlation function can be expressed as:

$$C_\pi(\boldsymbol{p}) = \sum_{\boldsymbol{m},\boldsymbol{n}} \gamma_{5\,\alpha_1\beta_1} \gamma_{5\,\alpha_2\beta_2} \mathcal{G}_u(n|m)_{\substack{\beta 1\alpha_2 \\ c_1 c_2}} \mathcal{G}_d(m|n)_{\substack{\beta 2\alpha_1 \\ c_2 c_1}} e^{-i\boldsymbol{n}\cdot\boldsymbol{p}} e^{i\boldsymbol{m}\cdot\boldsymbol{p}}, \tag{11}$$

Replace the all-to-all propagator in the above equation with the distillation smeared propagator:

$$\mathcal{G}_u(n|m)_{\substack{\beta\alpha \\ cd}} = V(t_1)_{\boldsymbol{n}c;b} \mathcal{P}_u(t_1|t_0)_{\substack{\beta\alpha \\ bc}} V^\dagger(t)_{c;\boldsymbol{m}d}, \tag{12}$$

Therefore, with distillation smearing the pion correlation function can be rewritten as:

$$C_\pi(\boldsymbol{p}) = \left[V^\dagger V\right]_{eb}(\boldsymbol{p}, t_1) \mathcal{P}_u(t_1|t_0)_{\substack{\beta_1\alpha_2 \\ bc}} \mathcal{P}^*_d(t_1|t_0)_{\substack{\delta_1\delta_2 \\ ed}} \left[V^\dagger V\right]_{cd}(\boldsymbol{p}, t_0). \tag{13}$$

CLQCD Configurations

- $N_f = 2 + 1$ Wilson clover configurations
- $a = 0.052 - 0.105$ fm, $m_\pi = 135 - 320$ MeV

ID	a(fm)	m_π(MeV)	m_K(MeV)	$m_\pi L$	N_{ev}	N_{conf}
C24P29	0.10530(18)	292.7	509.4	3.75	100	872
C32P29	0.10530(18)	292.4	509.0	5.00	100	984
C32P23	0.10530(18)	228.0	484.1	3.89	100	450
C48P23	0.10530(18)	225.6	484.1	5.78	200	365
C48P14	0.10530(18)	135.5	510.0	3.47	200	259
F32P30	0.07746(18)	303.2	524.6	3.81	100	776
F48P30	0.07746(18)	303.4	523.6	5.72	100	201
F48P21	0.07746(18)	207.2	493.0	3.90	100	222
H48P32	0.05187(26)	317.2	536.1	4.00	200	274

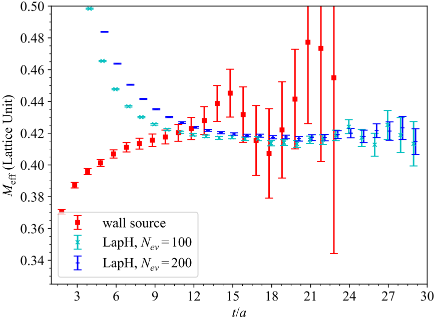


Figure: Comparison of nucleon effective mass using different sources.

- Light/strange/charm quark perambulator of CLQCD configurations are generated on HPC cluster of Southern Nuclear Science Computing Center (**SNSC**) and the Dongjiang Yuan Intelligent Computing Center.
- LapH in its exact form: $N_{ev} = 100$ for $Nx \leq 32$ & $N_{ev} = 200$ for $Nx = 48$.

GEVP

Goal: Find the optimal combination of operators $\Omega = \sum v_i O_i$ that maximizes the overlap with the energy levels:

$$min\{\langle 0|\Omega_n(t)\Omega_n(0)|0\rangle\} \rightarrow min\{\sum_{ij} v_i^\dagger C_{ij} v_j\} \tag{14}$$

By imposing the orthonormality condition $v_m^\dagger C(t_0) v_n = \delta_{mn}$ and solving the Generalized Eigenvalue Problem (GEVP):

$$C(t)v_n(t) = \lambda_n(t) C(t_0) v_n(t) \tag{15}$$

The overlap factor of the n-th GEVP eigenvector with the i-th operator in the correlator can be defined as:

$$\langle n|O_i|0\rangle = \sqrt{2m_n} v_j^{n*} C_{ji}(t_0), \tag{16}$$

Sorting the eigenvalues in descending order: $\lambda_0 \geq \lambda_1 \geq ... \geq \lambda_{r-1}$, we have:

$$\lim_{t\rightarrow\infty} \lambda_n(t) = c_n e^{-E_n t}\left(1 + O(e^{-\Delta E_n t})\right), \tag{17}$$

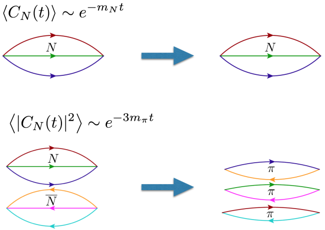
Signal-to-Noise Ratio

The signal-to-noise ratio for a baryon correlator can be written as:

$$\frac{S}{N} = \frac{\langle \mathcal{O} \rangle}{\sqrt{\mathrm{Var}(\mathcal{O})}}, \tag{18}$$

where the variance of the observable $\mathcal{O}$ can be expressed as:

$$\mathrm{Var}(\mathcal{O}) = \frac{\langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2}{N_{stat}}, \tag{19}$$



Thus, the behavior of the signal-to-noise ratio as $t \longrightarrow \infty$ can be expressed as:

$$\frac{S}{N} \underset{t \to \infty}{\longrightarrow} \sqrt{N_{stat}}\, e^{-A\left(m_N - \frac{3}{2} m_\pi\right) t}. \tag{20}$$

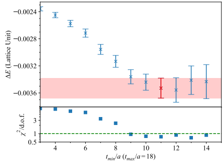
Ratio Correlator

Construct the ratio correlator $R(t)$:

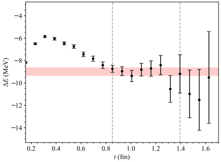
$$R(t) = \frac{C_{12}^{n}(t)}{C_1(\mathbf{p_1}, t)C_2(\mathbf{p_2}, t)} \approx C_0 e^{-\Delta E_{12}^{n} t}. \tag{21}$$

In the process of extracting energy levels, we used the lsqfit program for fitting, adopting the following criteria:

1. $\chi^2/d.o.f.$ is close to 1.
2. The chosen initial time slice t_{min} should be at least close to 1 fm.
3. The energy level obtained from the optimal t_{min} should remain consistent when choosing a larger t_{min}.



(a)



(b)

Figure: The t_{min} fitting window and effective mass for the lowest energy level of the ΞN channel in the F48P30 ensemble.

Numerical Results

Operator Construction & Parameterization

Single particle interpolating operators involved in studying the H-dibaryon system can be written as:

$$\begin{aligned} \mathcal{O}_N &: p = [ud]u,\ n = [ud]d, \\ \mathcal{O}_\Lambda &: \Lambda^0 = [ud]s, \end{aligned} \tag{22}$$

The interacting-system operators used to study the H-dibaryon system (with coefficients $c_{\alpha\beta,\mathbf{p_1},\mathbf{p_2}}$ computed using the OpTion package) are:

$$\begin{aligned} \mathcal{O}_{\Lambda\Lambda}(\mathbf{P},t) &: c_{\alpha\beta,\mathbf{p_1},\mathbf{p_2}}\Lambda^0_\alpha(\mathbf{p_1},t)\Lambda^0_\beta(\mathbf{p_2},t), \\ \mathcal{O}_{\Xi N}(\mathbf{P},t) &: c_{\alpha\beta,\mathbf{p_1},\mathbf{p_2}}\left(\Xi^0_\alpha(\mathbf{p_1},t)n_\beta(\mathbf{p_2},t) - \Xi^-_\alpha(\mathbf{p_1},t)p_\beta(\mathbf{p_2},t)\right), \end{aligned} \tag{23}$$

S-wave Lüscher quantization condition:

$$p\cot\delta_0(p) = \frac{2}{L\sqrt{\pi}}\mathcal{Z}_{00}(q^2,\mathbf{P}), \tag{24}$$

Parameterization in ERE expansion:

$$p\cot\delta(p) = -\frac{1}{a_0} + \frac{1}{2}r_0p^2. \tag{25}$$

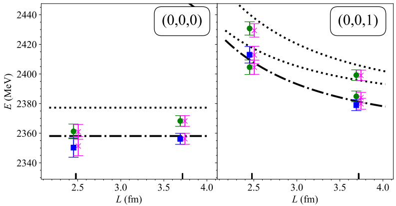


Figure: Comparison of $\Xi N - \Lambda\Lambda$ couple channel energy levels and single channel energy levels in configuration F32P30&F48P30.

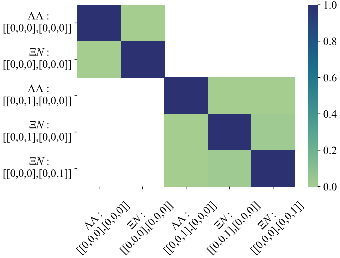
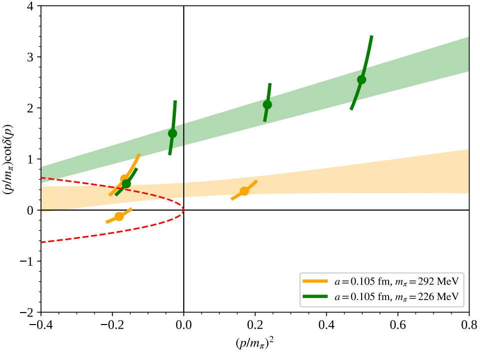


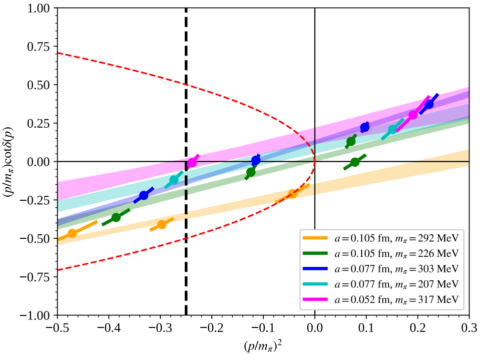
Figure: Normalized $\Xi N - \Lambda\Lambda$ couple channel correlation matrix of configuration F48P30.

Preliminary Results of ΛΛ channel



ID	$1/a_0(\mathrm{fm}^{-1})$	$r_0(\mathrm{fm})$	χ^2d.o.f.
CP29	−0.58(21)	0.62(70)	3.58
CP23	−1.70(24)	3.42(31)	2.81

Preliminary Results of ΞN channel



ID	$1/a_0(\mathrm{fm}^{-1})$	$r_0(\mathrm{fm})$	χ^2d.o.f.
CP29	0.268(52)	0.930(91)	3.82
CP23	−0.069(31)	1.76(22)	7.01
FP30	−0.193(24)	1.359(54)	0.18
F48P21	−0.100(44)	1.45(10)	–
H48P32	−0.268(86)	0.90(23)	–

Summary & Outlook

Summary & Outlook

- Our preliminary result indicates interaction in $\Lambda\Lambda$ channel is weak and a possible virtual state could exist in ΞNchannel.
- More difficulties lie ahead: couple channels, left-hand cut, and a/m_π extrapolation, etc.
- More "baseline" computations in the dibaryon sector would help enable direct comparisons between different methodologies.

Thank you!