Lattice Parton Collaboration

# **Hybrid Renormalization Scheme** for the Lattice Calculation of **Baryon LCDAs**


报告人：白颢阳 高能所

ICW：张沐华，华俊，王伟，张家璐 ……


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*LPC, arXiv:2508.08971 (2025)*

# OUTLINE

1. ## Renormalizations
   Why Renormalization; Schemes
2. ## Baryon quasi-DAs on Lattice
   Set up; Results; Divergence; Ratio scheme
3. ## Self renormalization
   Self parameterization & Two step fitting
4. ## Hybrid scheme
   Regions; Hybrid renormalization
5. ## Summary
   Quais-DA of Lambda & Proton

Part 1

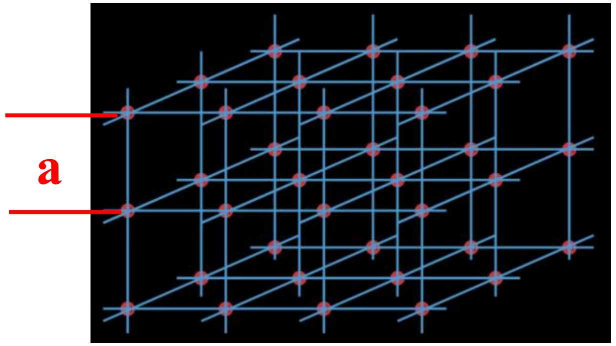
## Why Renormalization

# Why Renormalization



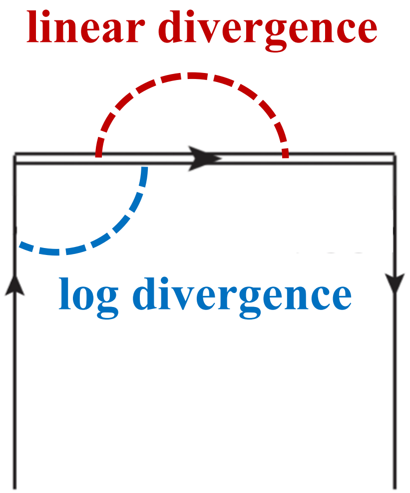
- **Scheme Conversion**

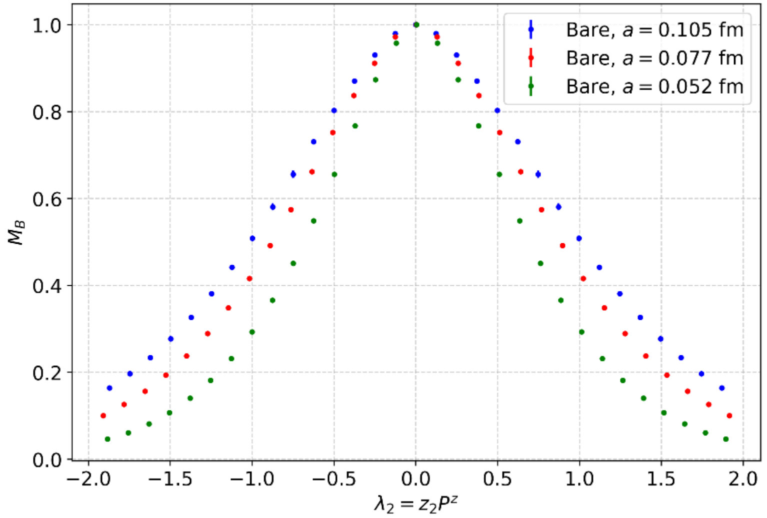
$$\frac{1}{a}\,cut \;\rightarrow\; \text{certain scheme}$$



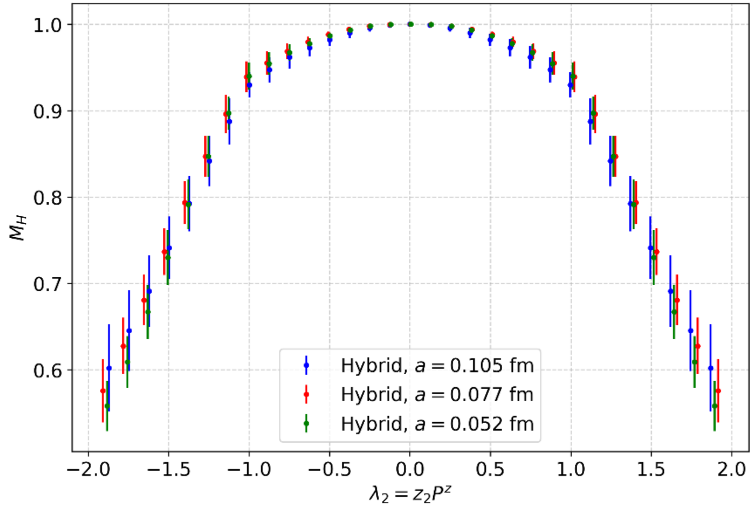

Renorm $\Longrightarrow$ $\overline{\text{MS}}$

- **Divergence Subtraction**




Renorm



# What is Renormalization



## Renormalization: **Bridging** the lattice scheme with the physics

Lattice scheme results $\xrightarrow{\text{Renormalization}}$ A well-defined intermediate scheme $\xrightarrow{\text{Matching}}$ Physical results

(Eliminating the lattice divergences)

RI-MOM, Ratio, Self, Hybrid, ···

- 1) Renormalization requires the ability to eliminate the divergence caused by the lattice effect.
- 2) The converted scheme is well-defined for effective matching or running …

# Comparison of Different Renormalization

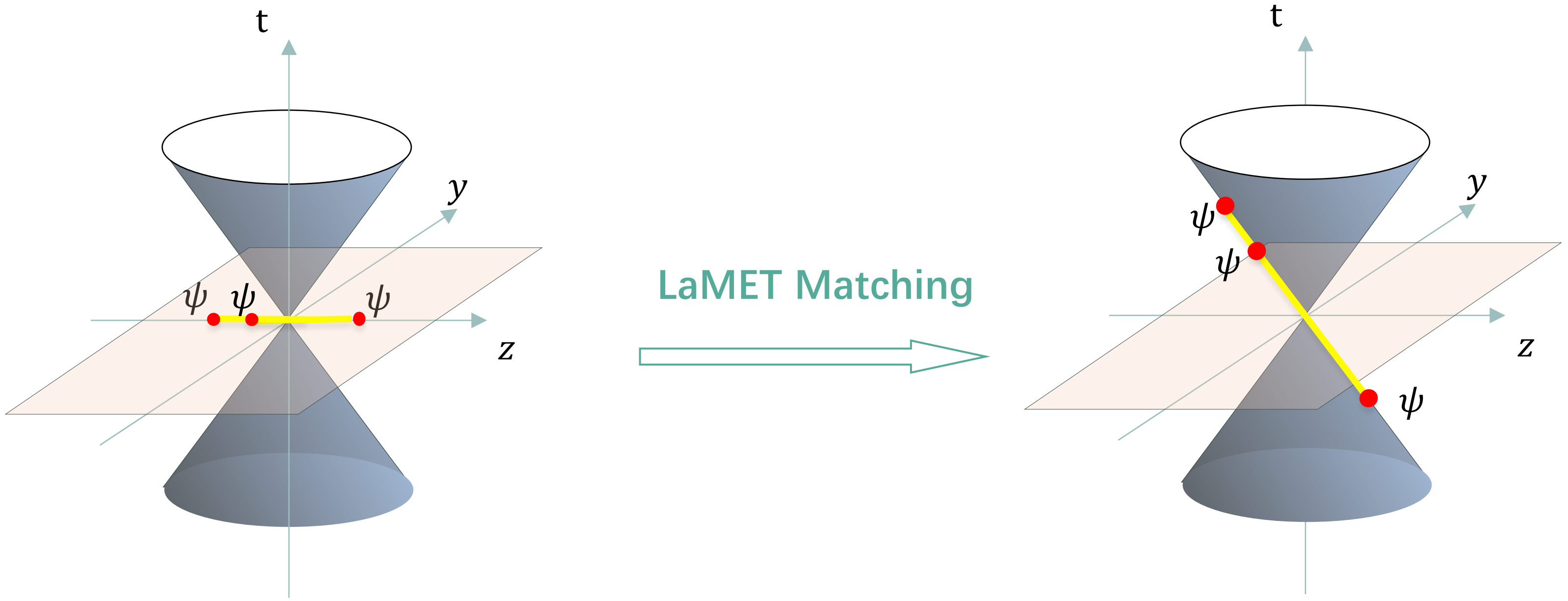


| Scheme | Pros | Cons |
| --- | --- | --- |
| **RI/MOM RI/SMOM Scheme**<br>*Martinelli et.al, NPB (1994);*<br>*Alexandrou et.al, NPB (2017); Stewart, Zhao, PRD (2018)*<br>*He et.al, PRD (2022); Bi et.al, PRD (2023)* | Strict subtraction<br>Well established<br>Widely applied<br>… | Not suitable for non-local<br>(e.g. Gauge dependence,<br>Signal problem …) |
| **Ratio Scheme**<br>*Radyushkin et.al, PRD (2017)* | Easy to use<br>Well subtraction<br>at short-distance | Extra IR behavior<br>at large-distance |
| **Self Renormalization and Hybrid Scheme**<br>*Ji et.al, NPB (2021); Huo et.al, NPB (2021)* | —— **The best match for the LaMET formalism** | |

Part 2

# Baryon quasi-DAs on Lattice

## Large Momentum Effective Theory (LaMET)




- With the LaMET, instead of taking light cone calcuation, one can perform an expansion for large but finite $P^z$.
- For LCDA:

LCDA　　Matching kernel　　Quasi-DA

$$\phi(x_1,x_2;\mu) = \int \mathrm{d}y_1\mathrm{d}y_2\ \mathcal{C}(x_1,x_2;y_1,y_2;P^z,\mu)\boxed{\tilde{\phi}(y_1,y_2;P^z,\mu)}$$

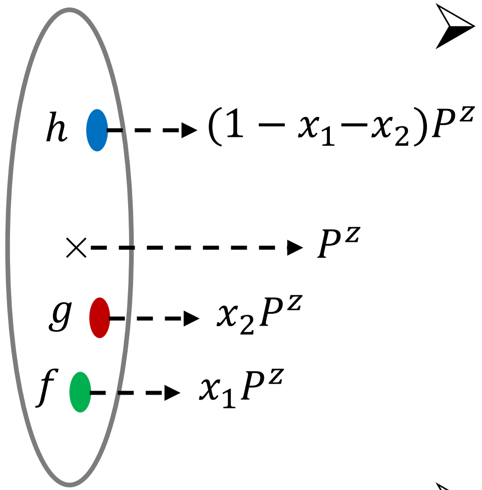
$$+\,\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(x_1P^z)^2},\frac{\Lambda_{\rm QCD}^2}{(x_2P^z)^2},\frac{\Lambda_{\rm QCD}^2}{[(1-x_1-x_2)P^z]^2}\right)$$

High power correction

*X.Ji PRL110 262002 (2013)*

# Baryon Quasi-DAs

Part ②




## ➢ Definition of Baryon LCDAs:

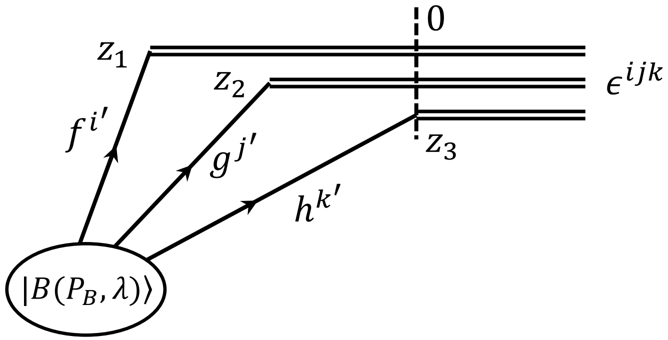
$$M_L\left(\xi_1,\xi_2;P\right)=\langle 0|\epsilon^{ijk}f_\alpha^{i'}\left(\xi_1 n\right)W_{i'i}\left(\xi_1,\xi_0\right)g_\beta^{j'}\left(\xi_2 n\right)W_{j'j}\left(\xi_2,\xi_0\right)h_\gamma^{k}\left(\xi_0 n\right)|B(P,\lambda)\rangle$$

$$\Phi\left(x_1,x_2,\mu\right)=\int\frac{\mathrm{d}P^+\xi_1}{2\pi}\frac{\mathrm{d}P^+\xi_2}{2\pi}\mathrm{e}^{\mathrm{i}x_1P^+\xi_1+\mathrm{i}x_2P^+\xi_2}\frac{M_L\left(\xi_1,\xi_2;P,\mu\right)}{M_L(0,0;P,\mu)}$$

*V.L.C & I.R.Z NPB 24652 (1984)*
*G.R.Farrar et.al. NPB 311585 (1989)*

## ➢ Corresponding quasi-DAs on Euclidean lattice:

*C.Han et.al. JHEP 12044 (2023)*
*LPC, PRD 111, 034510 (2025)*




$$f_M(p_1,p_2)\tilde{\Phi}^0(x_1,x_2)=\int\frac{p_{z1}dz_1}{2\pi}\frac{p_{z2}dz_2}{2\pi}e^{-i(x_1p_{z1}z_1+x_2p_{z2}z_2)}\langle 0|\hat{O}\left(z_1,z_2,\tilde{\Gamma}\right)|P^z\rangle$$

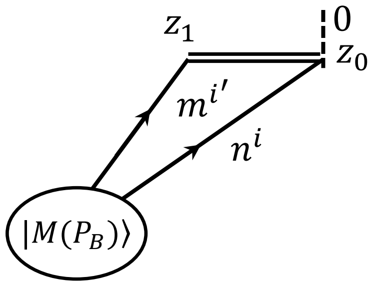
$$\begin{aligned}\hat{O}_\gamma(\vec{x},t;z_1,z_2)=\ &\epsilon^{ijk}W^{ii'}(\infty,\vec{x}+z_1n_z)f_\alpha^{i'}(\vec{x}+z_1n_z,t)\\ &\times\tilde{\Gamma}_{\alpha\beta}W^{jj'}(\infty,\vec{x}+z_2n_z,t)g_\beta^{j'}(\vec{x}+z_2n_z)\times W^{kk'}(\infty,\vec{x})h_\gamma^{k'}(\vec{x},t)\end{aligned}$$

| Octet | $n$ | $p$ | $\Lambda$ |
|---|---|---|---|
| $fgh$ | d d u | u u d | u d s |

# Meson and Baryon Quasi-DAs

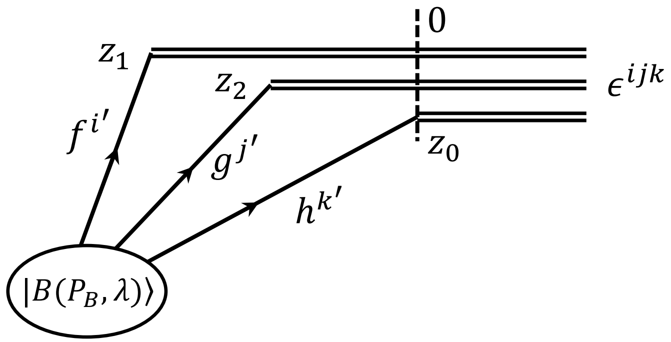
Part ②

- **Definition of Meson Quasi-DAs:**



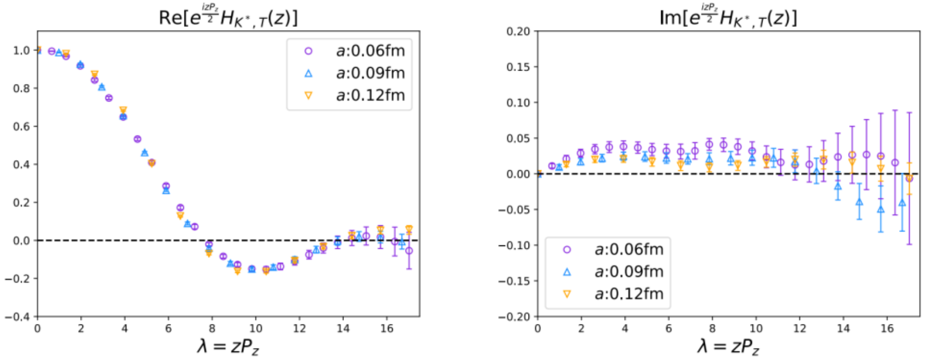

**1D** → **2D**

- **Definition of Baryon Quasi-DAs:**

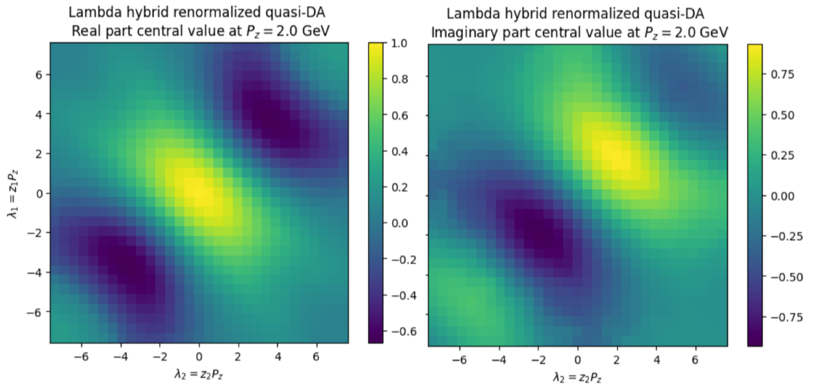



$$\hat{O}(\vec{x},t;z) \quad = m_\alpha^{i'}(\vec{x}+zn_z,t)\tilde{\Gamma}_{\alpha\beta}W^{i'i}(\vec{x}+zn_z,\vec{x},t)n_\beta^i(\vec{x},t)$$

$$\hat{O}_\gamma(\vec{x},t;z_1,z_2) = \epsilon^{ijk}W^{ii'}(\infty,\vec{x}+z_1n_z)f_\alpha^{i'}(\vec{x}+z_1n_z,t) \times \tilde{\Gamma}_{\alpha\beta}W^{jj'}(\infty,\vec{x}+z_2n_z,t)g_\beta^{j'}(\vec{x}+z_2n_z) \times W^{kk'}(\infty,\vec{x})h_\gamma^{k'}(\vec{x},t)$$




*LPC, PRL 127, 062002 (2021)*




*LPC, arXiv:2508.08971 (2025)*

# Lattice Setup

Part ②

- **Based on CLQCD Ensembles**

*CLQCD, PRD 109, 054507*

  - Three different lattice spacing

| Ensemble | Volume | Lattice spacing | $\pi$ mass | measurement | $P^z$ |
|---|---|---|---|---|---|
| C24P29 | $24^3 \times 72$ | 0.105 fm | 293 MeV | 864*4 | 1.96, 2.45, 2.94, 3.43 GeV |
| F32P30 | $32^3 \times 96$ | 0.077 fm | 303 MeV | 777*4 | 1.99, 2.50, 2.99, 3.49 GeV |
| H48P32 | $48^3 \times 144$ | 0.052 fm | 317 MeV | 550*6 | 1.99, 2.48, 2.98, 3.48GeV |

# Bare quasi-DA results

Part ②

**Normed Bare quasi-DA $\tilde{\psi}$:** $\tilde{\psi}(z_1, z_2, P^z) = \psi(z_1, z_2, P^z)/\psi(z_1 = z_2 = 0\,, P^z)$

- z-dependence of normed bare $\tilde{\psi}(z_1, z_2, P^z)$ on different lattice spacing

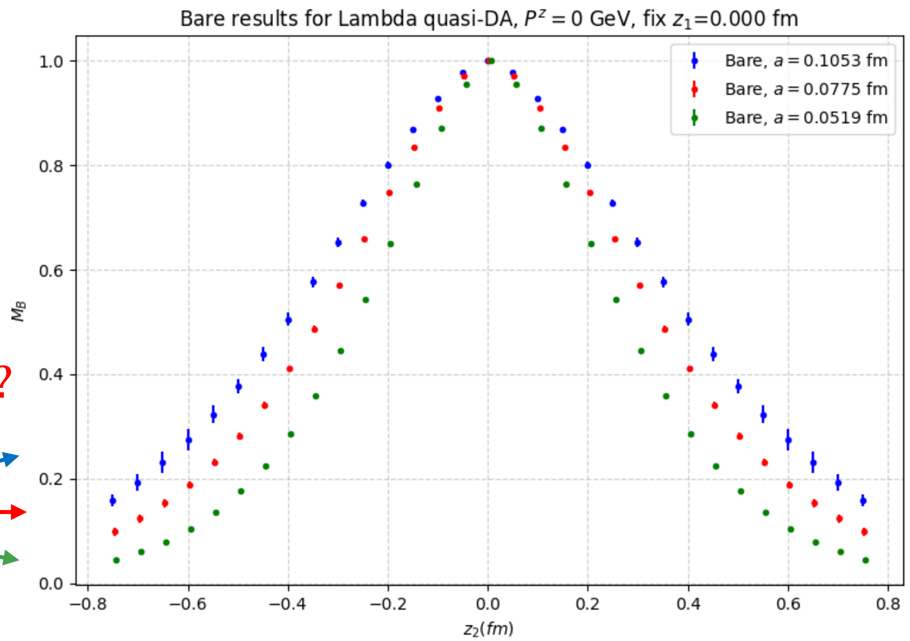
Lambda axial term

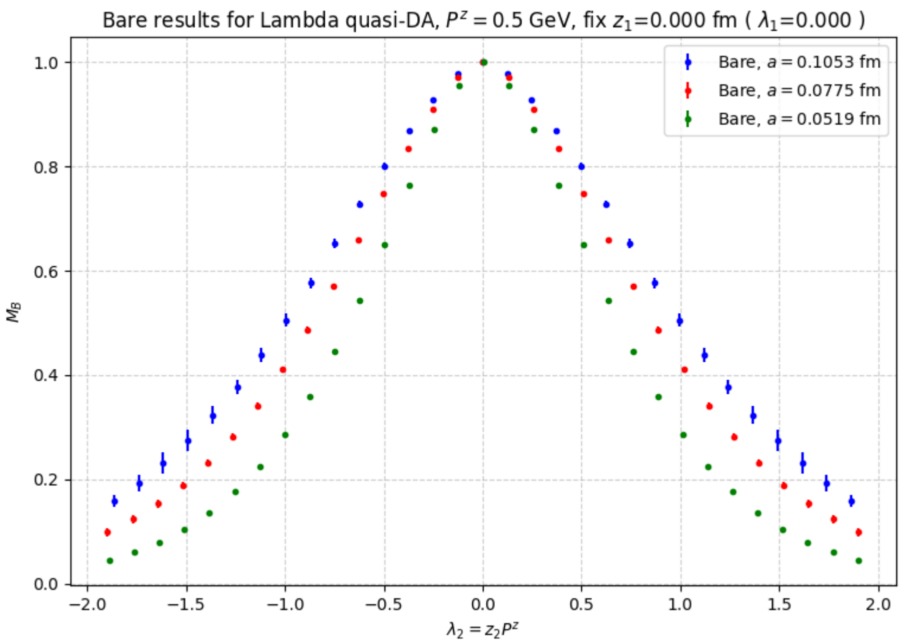
fix $z_1$=0 fm

$P^z$=0 GeV

$P^z$=0.5 GeV

Divergence?



# Bare quasi-DA results: Linear divergence    Part ②

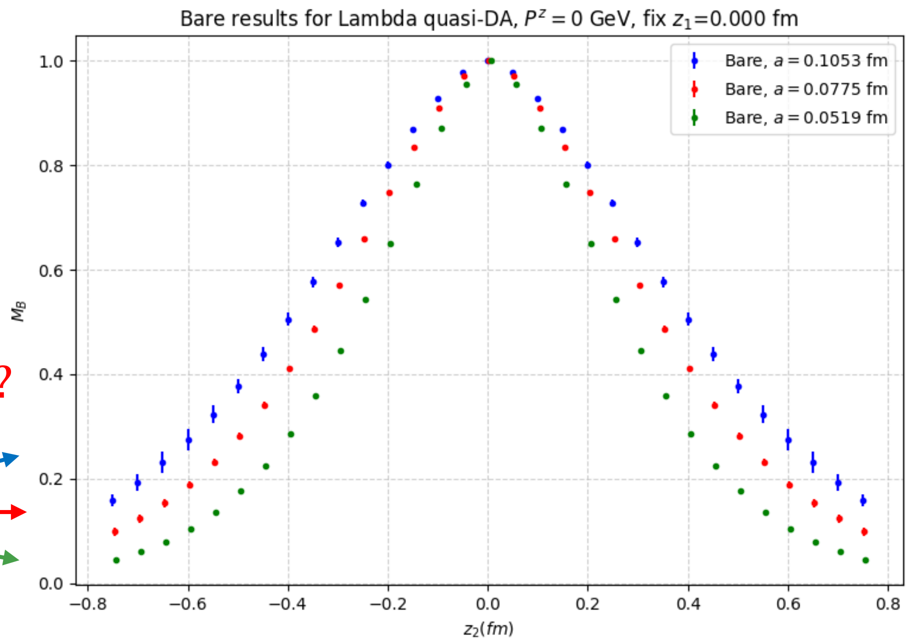
**Linear divergence form for Normed Bare quasi-DA** $\tilde{\psi}$ **:**

$$\tilde{z} = \begin{cases} |z_1 - z_2| & z_1 z_2 < 0 \\ \max(z_1, z_2) & z_1 z_2 \geq 0 \end{cases}$$

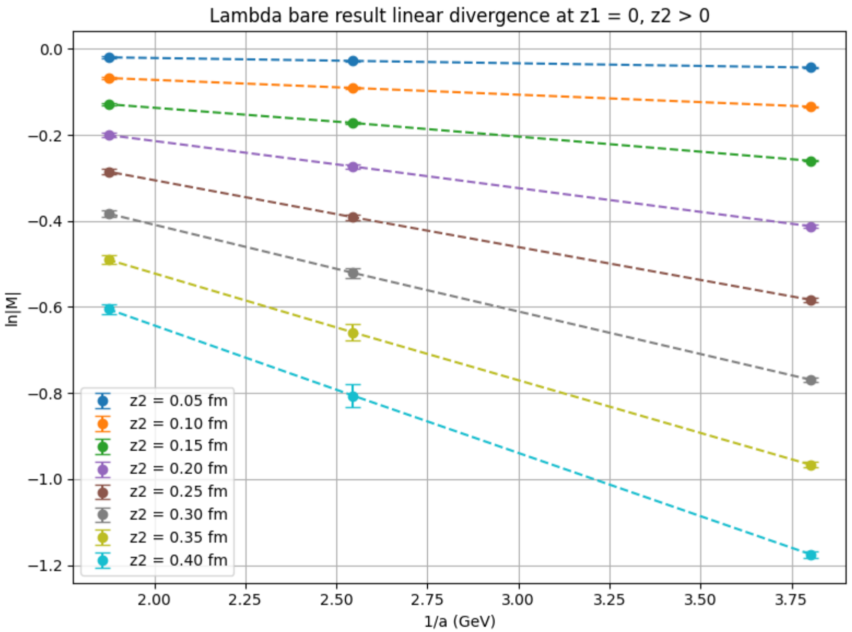
$$M(z_1, z_2; P_z; a) = exp\left[ \left( \frac{k}{a \ln(a\Lambda_{\rm QCD})} + m_0 \right) \tilde{z} \right] \times m(z_1, z_2; P_z; a)$$

$P^z$=0 GeV

Divergence?




Log scale



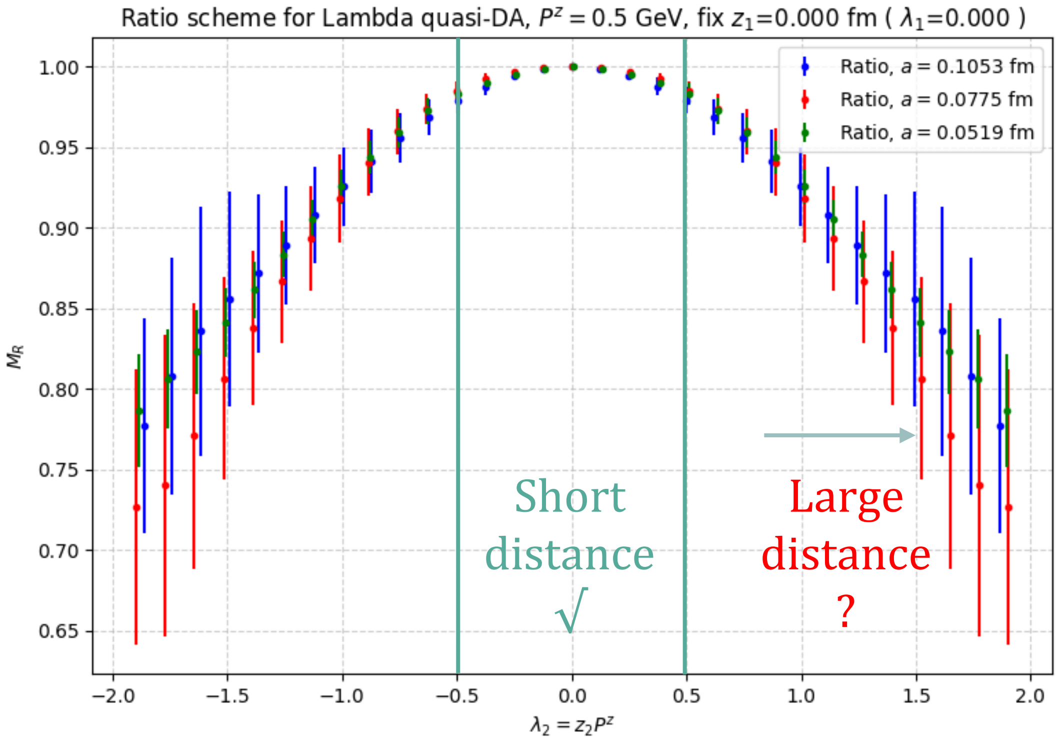
# Ratio scheme quasi-DA results

Part ②

**Ratio scheme quasi-DA** $\tilde{\psi}^{\text{ratio}}$ **:** $\tilde{\psi}^{\text{ratio}}(z_1, z_2, P^z) = \tilde{\psi}(z_1, z_2, P^z)/\tilde{\psi}(z_1, z_2, P^z = 0)$

- z-dependence of ratio scheme quasi-DA $\tilde{\psi}^{\text{ratio}}(z_1, z_2, P^z)$ on different lattice spacing

Lambda axial term $P^z$=0.5 GeV fix $z_1$=0 fm




$\tilde{\psi}(z_1, z_2, P^z)/\tilde{\psi}(z_1, z_2, P^z = 0)$

gives extra IR behavior at large distance

Part 3

# Self renormalization

# Self-renormalization scheme　Part ③

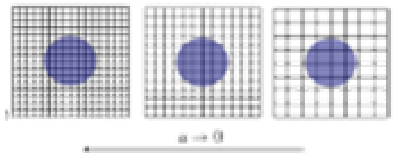
## Self renorm: Use Lattice data to **renorm them·selves**

- 1) Lattice perturbation theory gives a parametrization of the Lattice divergences

$$\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu) = \frac{\hat{M}(z_1, z_2, 0, P^z, a)}{Z_R(z_1, z_2, a, \mu)} \qquad \tilde{z} = \begin{cases} |z_1 - z_2| & z_1 z_2 < 0 \\ \max(z_1, z_2) & z_1 z_2 \geq 0 \end{cases}$$

$$Z_R(z_1, z_2, a, \mu) = \exp\left[\left(\frac{k}{\boxed{a}\ln[a\Lambda_{\mathrm{QCD}}]} - \underline{m_0}\right)\tilde{z} + \frac{\gamma_0}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{\mathrm{QCD}})]}{\ln[\mu/\Lambda_{\mathrm{MS}}]}\right] + \ln\left[1 + \frac{d}{\underline{\ln(a\Lambda_{\mathrm{QCD}})}}\right] + f(z_1, z_2)\boxed{a^2}\right]$$

- 2) With data from different lattice ensembles we can fit the terms with lattice spacing dependence



| Ensemble | Volume | Lattice spacing |
|---|---|---|
| C24P29 | $24^3 \times 72$ | 0.1052 fm |
| F32P30 | $32^3 \times 96$ | 0.0775 fm |
| H48P32 | $48^3 \times 144$ | 0.0520 fm |

- 3) Remaining terms can be obtained by matching with the perturbative result at the short-distance region.

# Self-renormalization: 2-step fitting

$$\tilde{z} = \begin{cases} |z_1 - z_2| & z_1 z_2 < 0 \\ \max(z_1, z_2) & z_1 z_2 \geq 0 \end{cases}$$

**Part ③**

- **Step-1** (fitting the lattice spacing dependence)

b0

$$\ln M(z_1, z_2; P_z = 0; a) = \frac{\underline{k}}{a \ln(a\Lambda_{\mathrm{QCD}})}\tilde{z} + g(z_1, z_2) + \underline{f(z_1, z_2)}a^2 + \frac{\gamma_0}{b_0}\ln\frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\overline{\mathrm{MS}}})} + \boxed{\ln\left[1 + \frac{d}{\ln(a\Lambda_{\mathrm{QCD}})}\right]}$$

fix $\Lambda_{\mathrm{QCD}} = 0.2$ GeV

- **Step-2** (matching the perturbative result)

$$g(z_1, z_2) - \ln Z_{\overline{\mathrm{MS}}}(z_1, z_2; \mu, \Lambda_{\overline{\mathrm{MS}}}) = \underline{m_0}\tilde{z} + \underline{b_0}$$

$$Z_{\overline{\mathrm{MS}}}(z_1, z_2; \mu, \Lambda_{\overline{\mathrm{MS}}}) = 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{7}{8}\ln\frac{z_1^2\mu^2 e^{2\gamma_E}}{4} + \frac{7}{8}\ln\frac{z_2^2\mu^2 e^{2\gamma_E}}{4} + \frac{3}{4}\ln\frac{(z_1 - z_2)^2\mu^2 e^{2\gamma_E}}{4} + 4\right]$$

- Re- matrix element & renorm factor

$$M_R(z_1, z_2; P_z = 0) = \exp\left(g(z_1, z_2) - m_0\tilde{z} - b_0\right)$$

$$Z_R(z_1, z_2; a) = \exp\left[\frac{k}{a\ln(a\Lambda_{\mathrm{QCD}})}\tilde{z} + m_0\tilde{z} + f(z_1, z_2)a^2 + \frac{\gamma_0}{b_0}\ln\frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\overline{\mathrm{MS}}})} + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\mathrm{QCD}})}\right]\right]$$

# Self fitting step-1

Part ③

$$\ln M(z_1, z_2; P_z = 0; a) = \frac{\boxed{k}}{a \ln(a\Lambda_{\mathrm{QCD}})} \tilde{z} + g(z_1, z_2) + f(z_1, z_2) a^2 + \frac{\gamma_0}{b_0} \ln \frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\overline{\mathrm{MS}}})} + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\mathrm{QCD}})}\right]$$

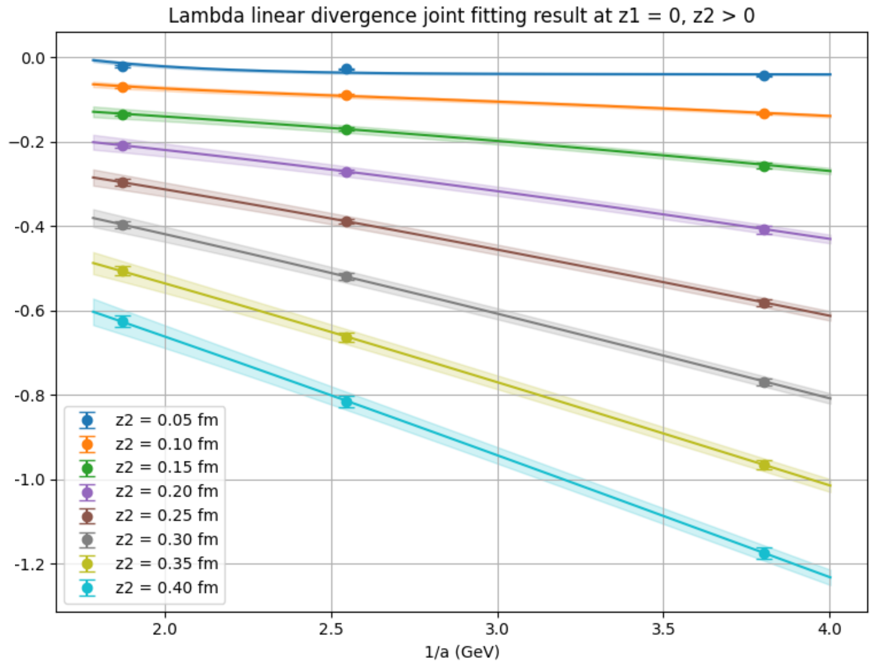
- Step-1 fit result

Interpolate to a0 = 0.05 fm

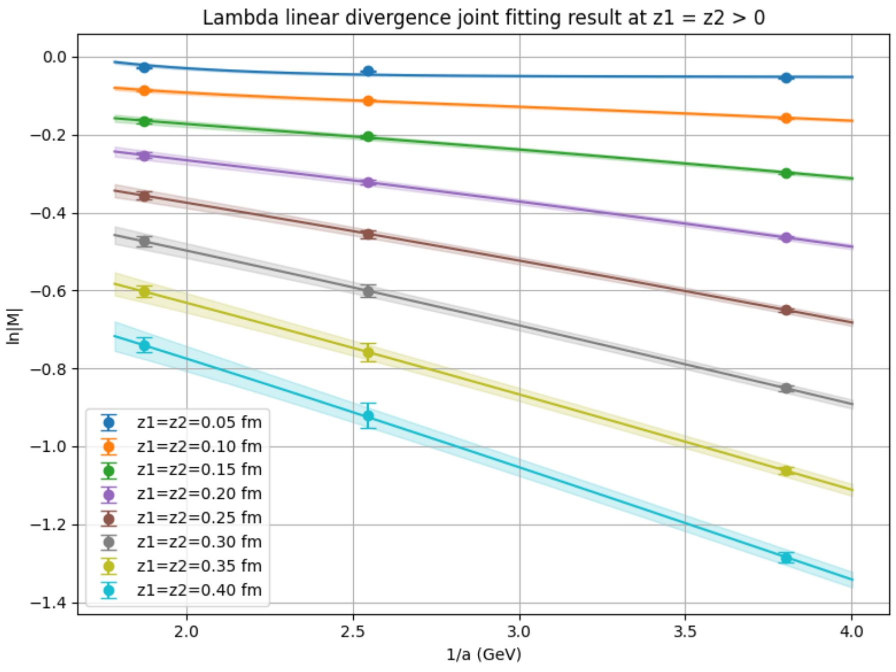
z1, z2: 0.15~ 0.70 fm

```
Parameters:
          k   0.7792 (27)     [     0 ± 10 ]
   g_-14_-10   1.020 (39)     [     0 ± 10 ]
   f_-14_-10    1.2 (5.9)     [     0 ± 10 ]
    g_-14_-9   1.066 (40)     [     0 ± 10 ]
    f_-14_-9    0.4 (5.5)     [     0 ± 10 ]
    g_-14_-8   1.100 (43)     [     0 ± 10 ]
    f_-14_-8   -0.3 (5.4)     [     0 ± 10 ]
    g_-14_-7   1.124 (46)     [     0 ± 10 ]
    f_-14_-7   -0.7 (5.4)     [     0 ± 10 ]
    g_-14_-6   1.135 (47)     [     0 ± 10 ]
    f_-14_-6   -0.6 (5.4)     [     0 ± 10 ]
    g_-14_-5   1.133 (46)     [     0 ± 10 ]
    f_-14_-5   -0.2 (5.3)     [     0 ± 10 ]
    g_-14_-4   1.119 (44)     [     0 ± 10 ]
    f_-14_-4    0.5 (5.1)     [     0 ± 10 ]
     g_-14_4   1.209 (43)     [     0 ± 10 ]
     f_-14_4    6.0 (6.2)     [     0 ± 10 ]
     g_-14_5   1.253 (49)     [     0 ± 10 ]
...
```

Fit result z1=0 (log scale)




Fit result z1=z2 (log scale)



# Self fitting step-2

Part ③

$$g(z_1,z_2) - \ln Z_{\overline{\rm MS}}(z_1,z_2;\mu,\Lambda_{\overline{\rm MS}}) = \underline{m_0}\tilde{z} + \underline{b_0}$$

$$Z_{\overline{\rm MS}}(z_1,z_2;\mu,\Lambda_{\overline{\rm MS}}) = 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{7}{8}\ln\frac{\boxed{z_1^2}\mu^2 e^{2\gamma_E}}{4} + \frac{7}{8}\ln\frac{\boxed{z_2^2}\mu^2 e^{2\gamma_E}}{4} + \frac{3}{4}\ln\frac{\boxed{(z_1-z_2)^2}\mu^2 e^{2\gamma_E}}{4} + 4\right]$$

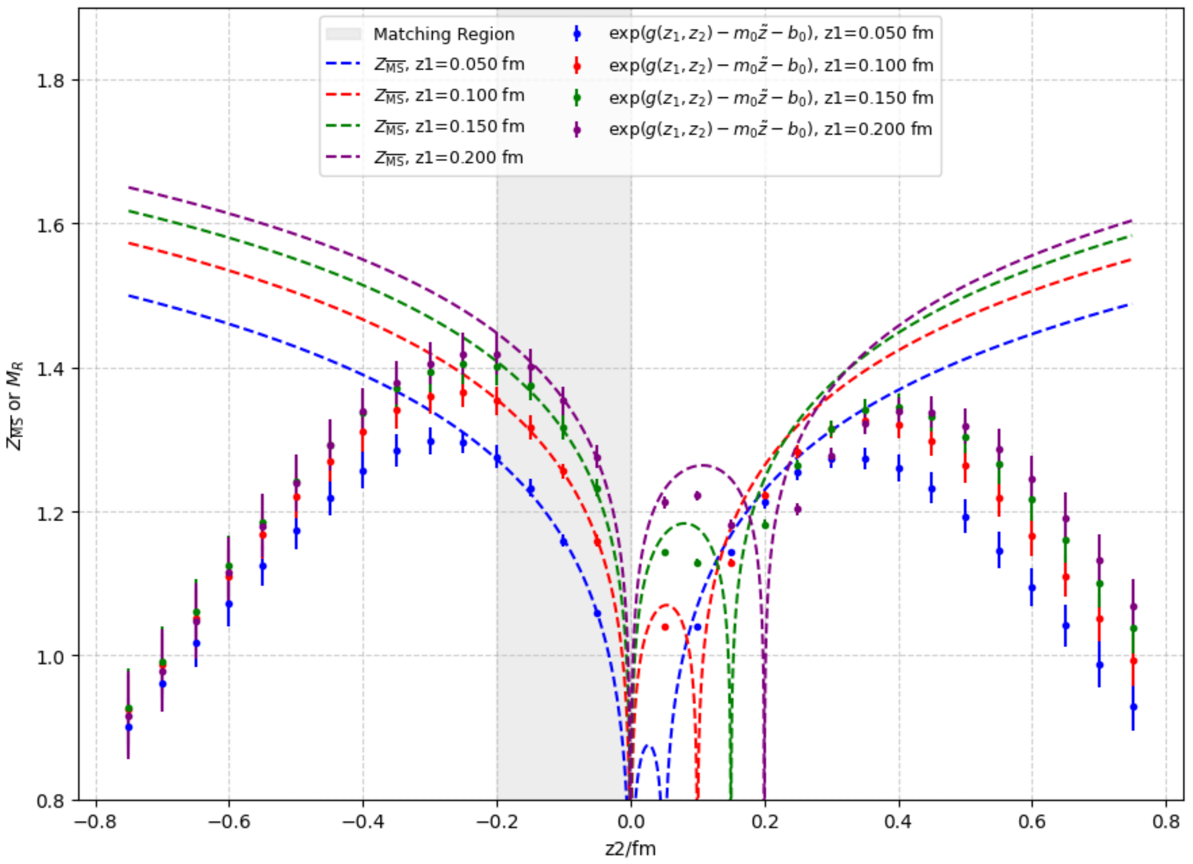
- Step-2 fit result

z1-, z2-: 0.05 ~ 0.20 fm

```
Parameters:
          m0     1.254 (11)     [     0 ± 10 ]
          b0    0.1093 (23)     [     0 ± 10 ]

Fit:
      key           y[key]       f(p)[key]
-----------------------------------------
    g_-4_1     0.6632 (58)     0.6636 (11)
    g_-4_2     0.7873 (73)     0.7897 (15)
    g_-4_3     0.8843 (88)     0.8905 (20)
    g_-4_4      0.960 (10)     0.9808 (25)  *
    g_-3_1     0.5688 (45)    0.56330 (87)  *
    g_-3_2     0.7001 (60)     0.6943 (11)
    g_-3_3     0.8029 (74)     0.7981 (15)
    g_-3_4     0.8843 (88)     0.8905 (20)
    g_-2_1     0.4438 (39)    0.44705 (95)
    g_-2_2     0.5897 (45)    0.58605 (87)
    g_-2_3     0.7001 (60)     0.6943 (11)
    g_-2_4     0.7873 (73)     0.7897 (15)
    g_-1_1     0.2911 (23)     0.2924 (13)
```

Lambda result of matching the lattice matrix elements to the MSbar scheme perturbative expression



# Self scheme quasi-DA $\tilde{\psi}^{\text{self}}$



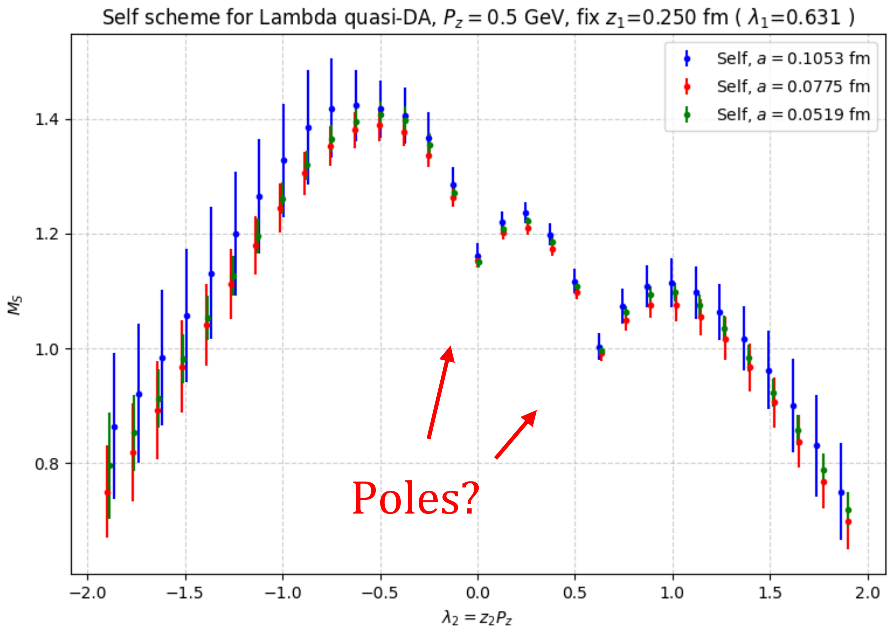
**Self scheme quasi-DA $\tilde{\psi}^{\text{self}}$ :**

$$M_R(z_1, z_2; P_z) = \frac{M(z_1, z_2; P_z; a)}{Z_R(z_1, z_2; a)}$$

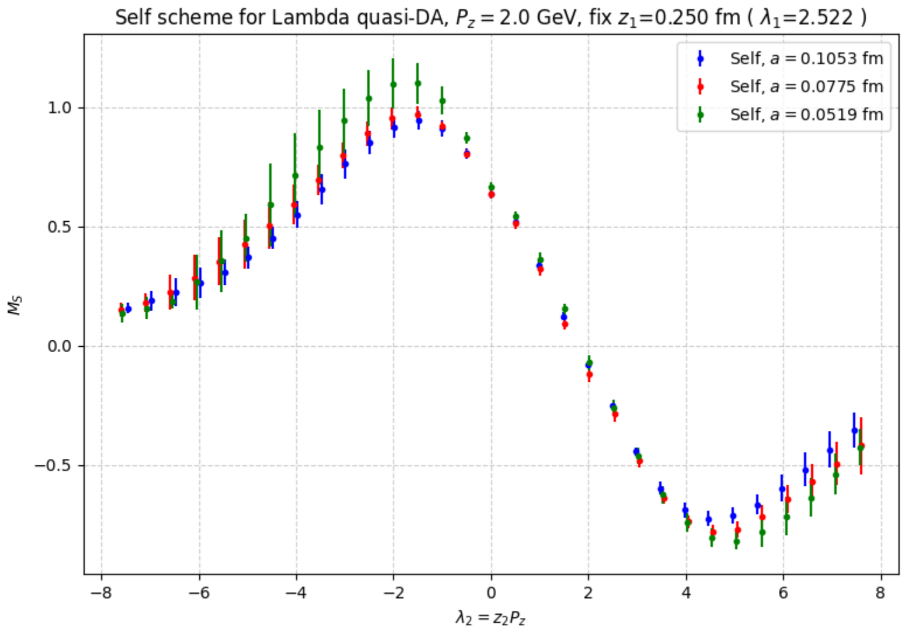
- z-dependence of quasi-DA $\tilde{\psi}^{\text{self}}(z_1, z_2, P^z)$ on different lattice spacing

Lambda Axial term    fix $z_1$=0.250 fm

$P^z$=0.5 GeV




$P^z$=2.0 GeV

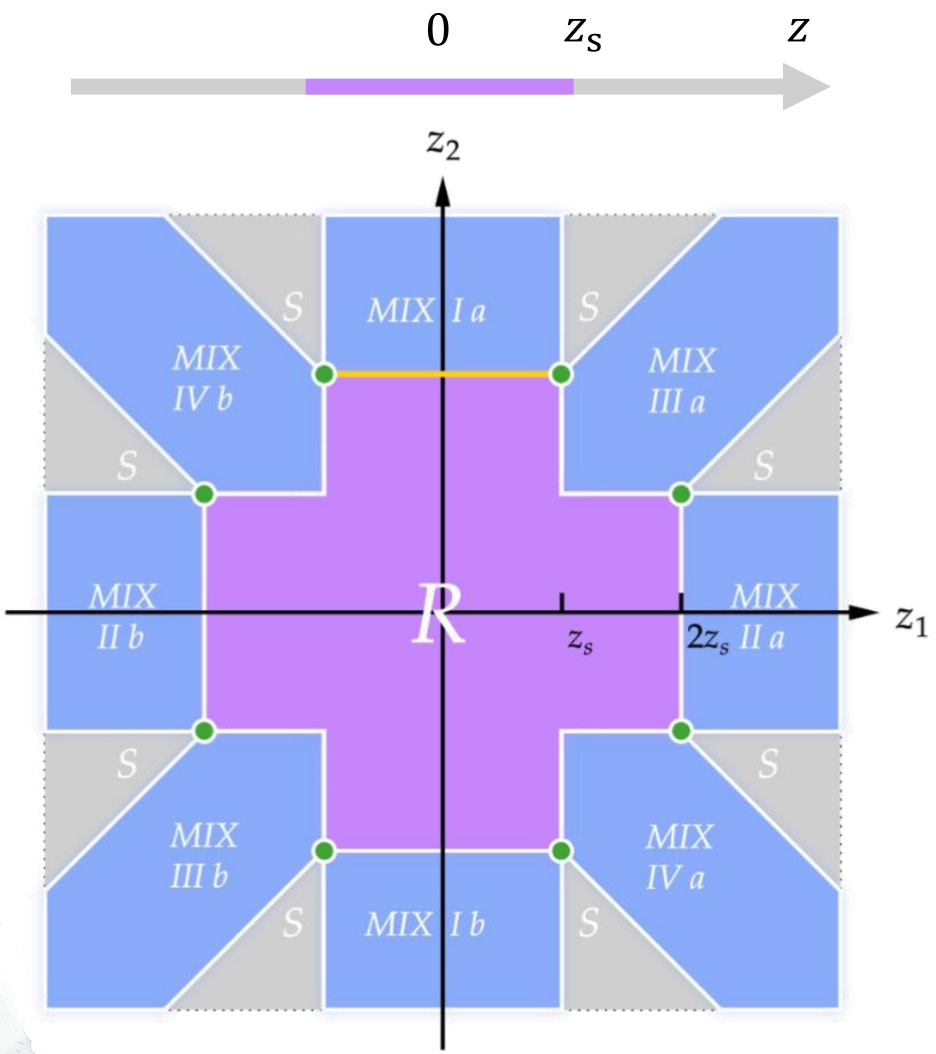



$\overline{MS}$: $\ln(\mu^2 z_1^2)$, $\ln(\mu^2 z_2^2)$, and $\ln(\mu^2 (z_1 - z_2)^2)$

Part 4

## Hybrid scheme

## Hybrid scheme: Use suitable schemes for different regions




- 1) **Short-distance region**

$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, 0, \mu)} \cdot S_{\mathrm{short}}(z_1, z_2)$$

- 2) **Long-distance region**

$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(\mathrm{sign}(z_1)z_s, \mathrm{sign}(z_2)2z_s, 0, 0, \mu)} \cdot S_{\mathrm{long}}(z_1, z_2)$$

- 3) **Mixing regions**

$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(z_1, \mathrm{sign}(z_2)2z_s, 0, 0, \mu)} \cdot \theta(z_s - |z_1|)\theta(|z_2| - 2z_s)$$

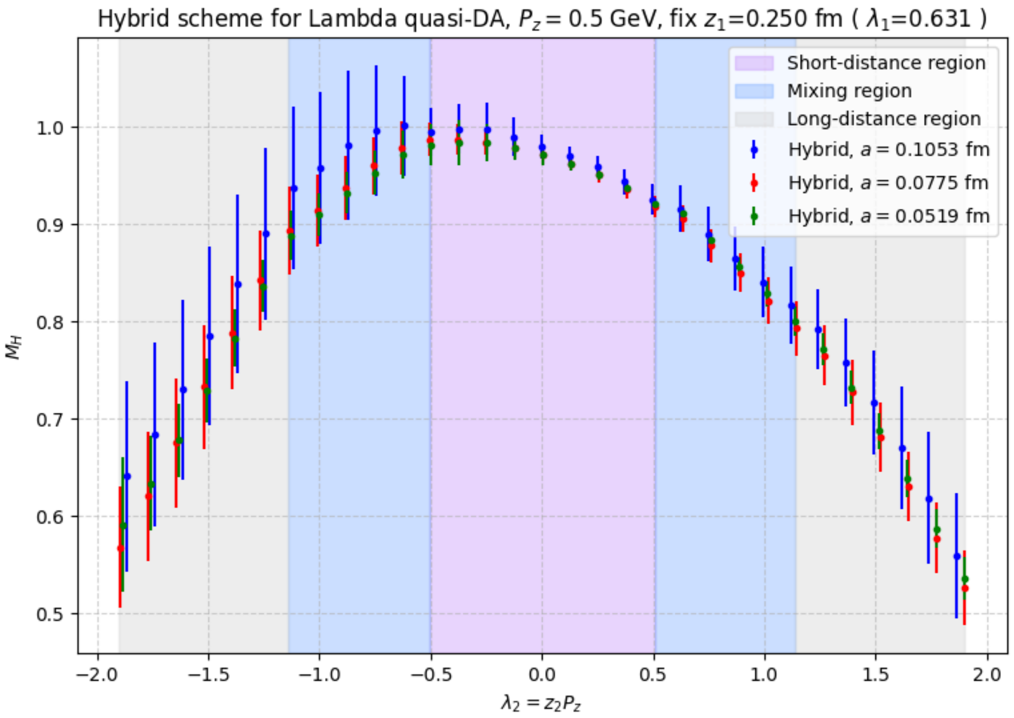
# Hybrid scheme quasi-DA $\tilde{\psi}^{\text{hybrid}}$ Part ④

## Hybrid scheme quasi-DA $\tilde{\psi}^{hybrid}$

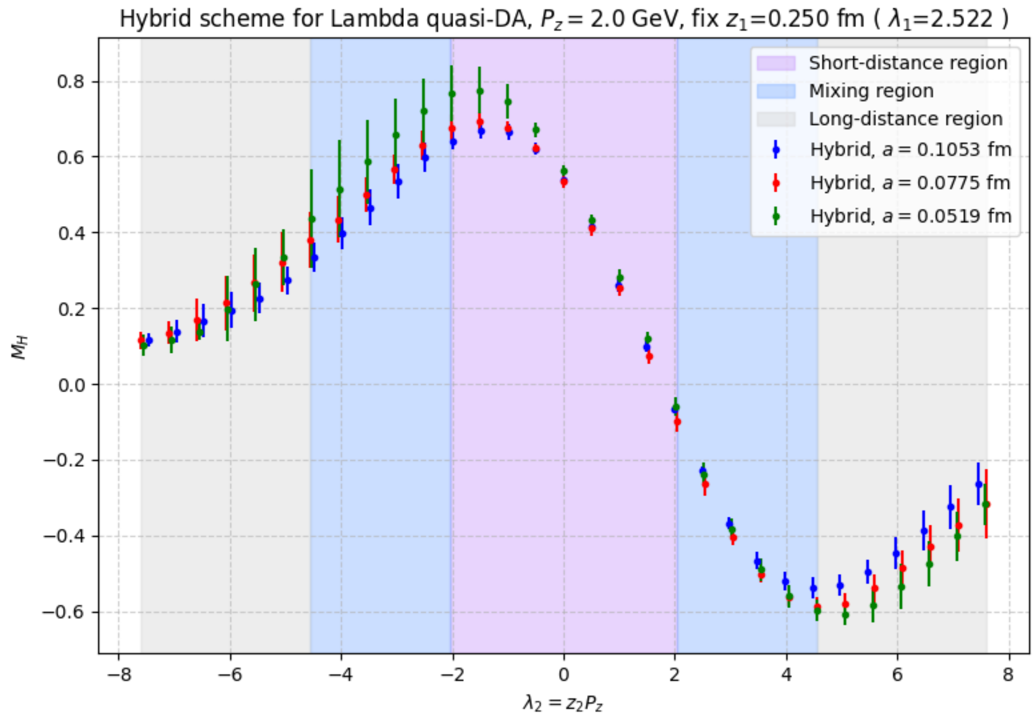
- z-dependence of quasi-DA $\tilde{\psi}^{hybrid}(z_1, z_2, P^z)$ on different lattice spacing

Lambda A term fix $z_1$=0.250 fm

$P^z$=0.5 GeV




$P^z$=2.0 GeV

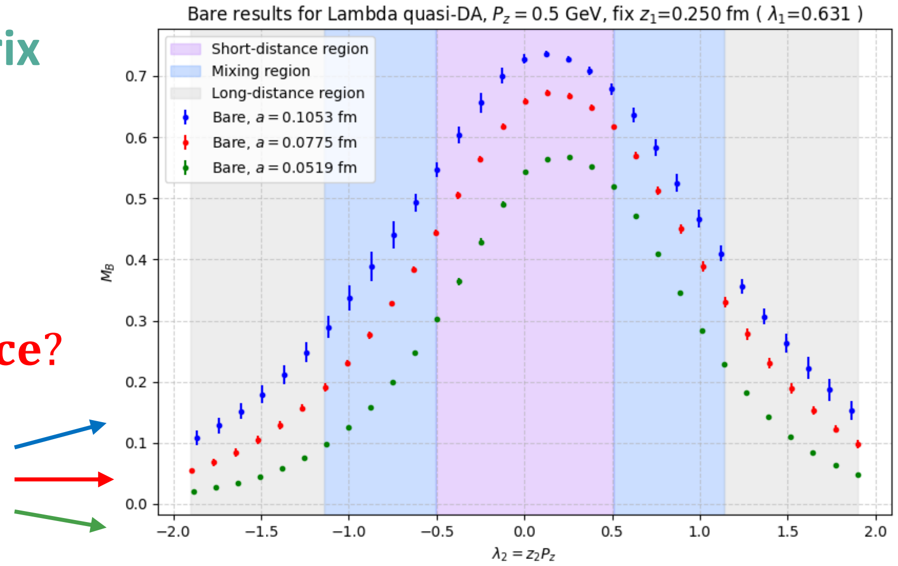


# Schemes Comparison (Hybrid)

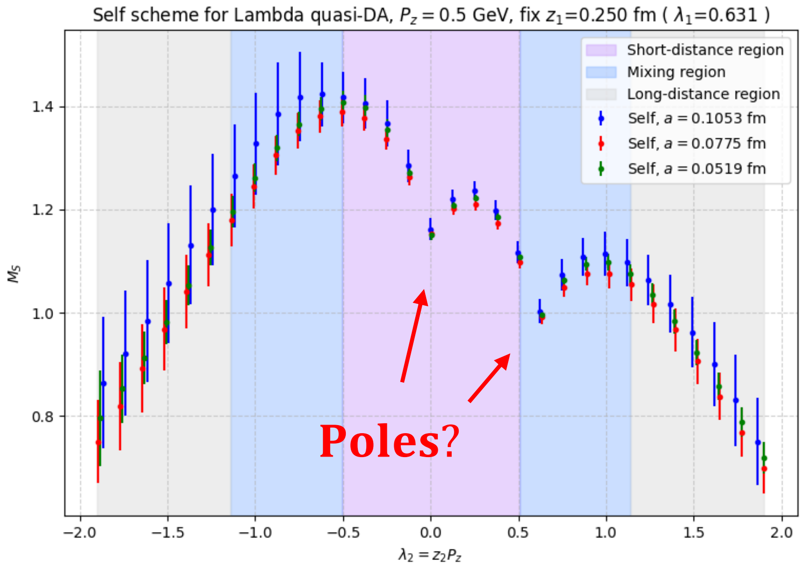
Part ④

Bare Matrix Element

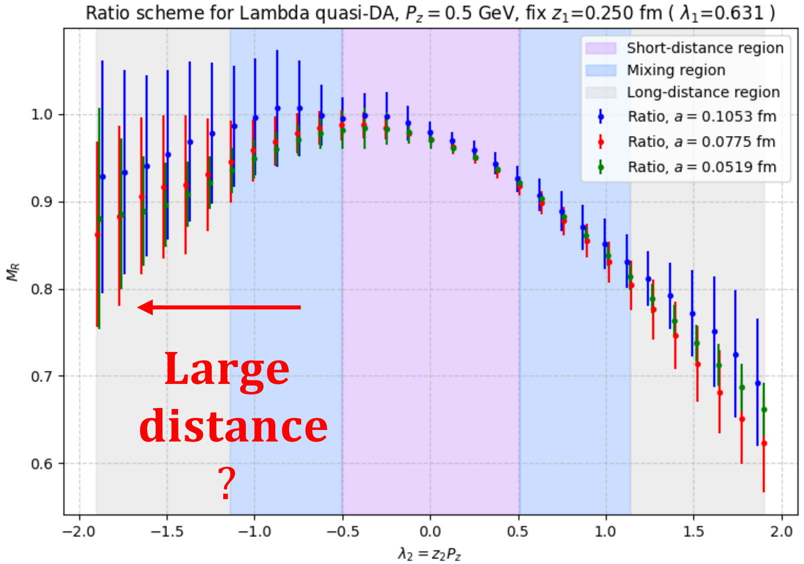
Divergence?



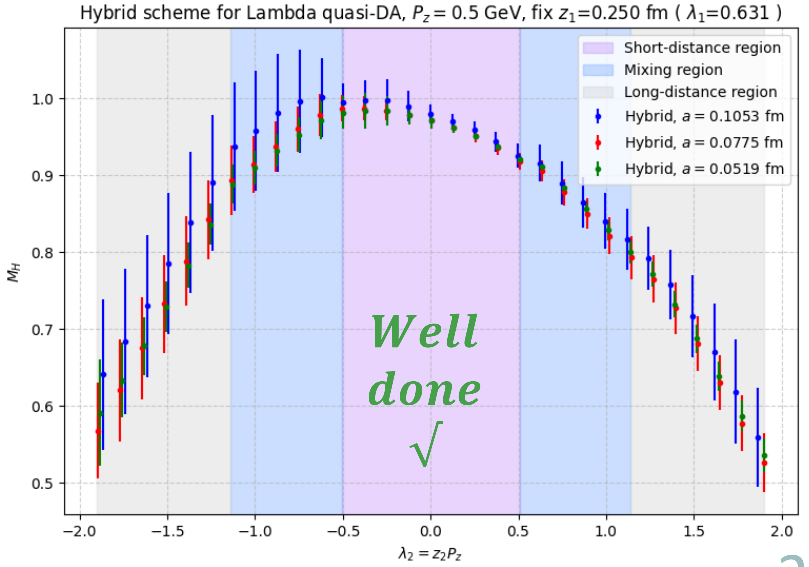

Self scheme




Ratio scheme




Hybrid scheme

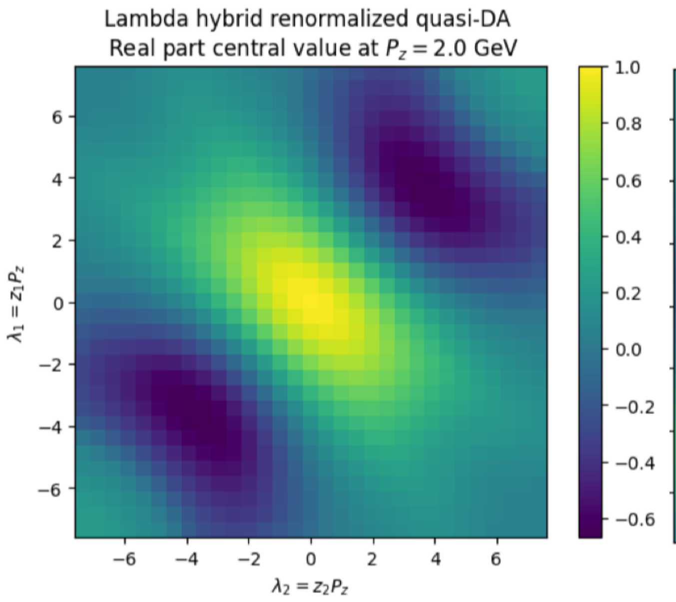


# Quasi-DA results under Hybrid renormalization Part ④

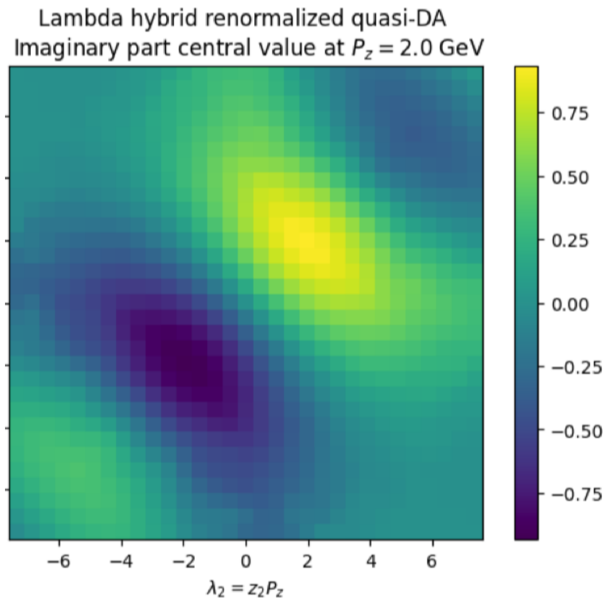
- Hybrid renormalized large momentium baryon quasi-DA（$P^z = 2.0\text{GeV}$）

**Λ quasi-DA**

**Re**

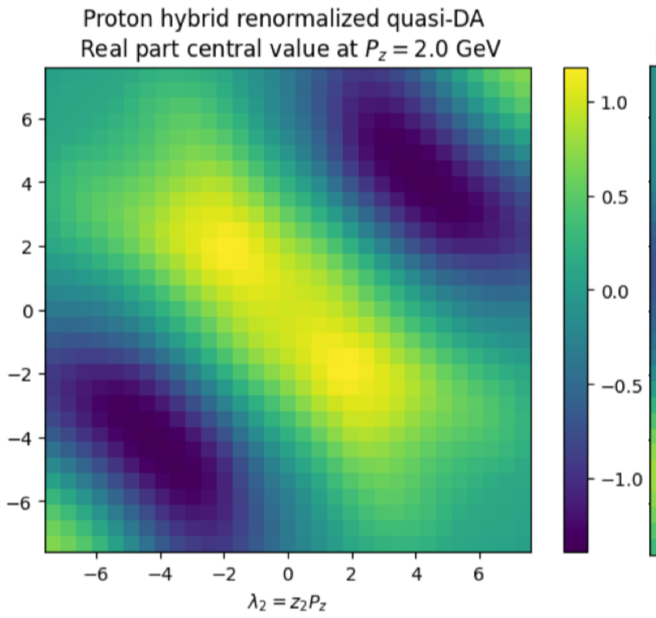



**Im**

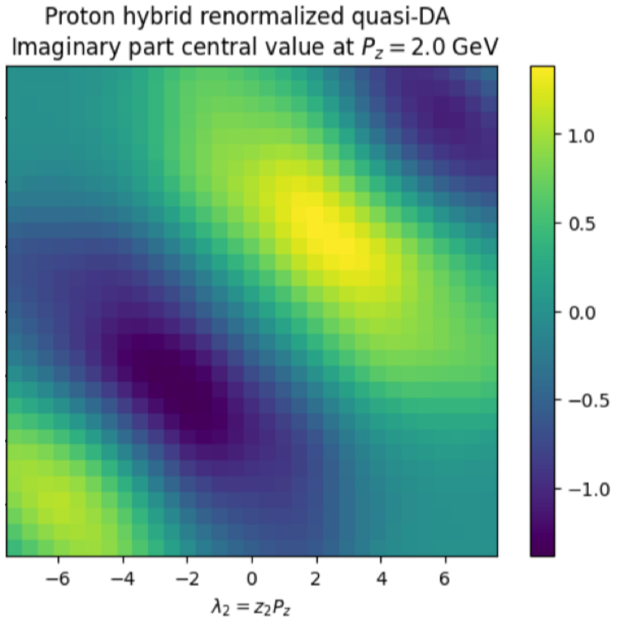



**Proton quasi-DA**

**Re**




**Im**




**well-defined** and **smooth** in all regions

Part 5

## Summary

## Towards Reliable Light Baryon LCDA on Lattice:

Lattice scheme results → Renormalization → A well-defined scheme → LaMET Matching → Light cone results

(Calculating the LaMET matching kernel under self scheme) Done!

| Ensemble | Volume | Lattice spacing |
|---|---|---|
| C24P29 | $24^3 \times 72$ | 0.105 fm | New
| F32P30 | $32^3 \times 96$ | 0.077 fm |
| G36P29 | $36^3\text{x}108$ | 0.068 fm | New
| H48P32 | $48^3 \times 144$ | 0.052 fm | New

Ratio scheme

Self renormalization New

Hybrid scheme New

*LPC, PRD 111, 034510 (2025)*

*LPC, arXiv:2508.08971 (2025)*

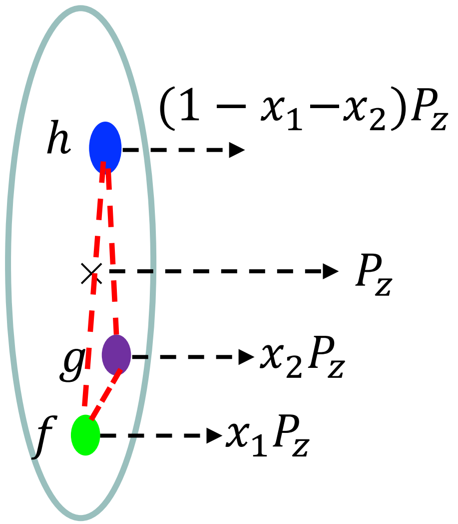
Thanks For Your Attention!

Part 6

# Backup

# Baryon LCDAs

Part ①




- **Definition of baryon LCDA:**

$$M_L(\xi_1,\xi_2;P) = \langle 0|\epsilon^{ijk} f_\alpha^{i'}(\xi_1 n) W_{i'i}(\xi_1,\xi_0) g_\beta^{j'}(\xi_2 n) W_{j'j}(\xi_2,\xi_0) h_\gamma^k(\xi_0 n)|B(P,\lambda)\rangle$$

$$\Phi(x_1,x_2,\mu) = \int \frac{\mathrm{d}P^+\xi_1}{2\pi}\frac{\mathrm{d}P^+\xi_2}{2\pi} \mathrm{e}^{\mathrm{i}x_1P^+\xi_1+\mathrm{i}x_2P^+\xi_2} \frac{M_L(\xi_1,\xi_2;P,\mu)}{M_L(0,0;P,\mu)}$$

*C.Han et.al. JHEP 07019 (2024);*
*V.L.C & I.R.Z NPB 24652(1984); G.R.Farrar et.al. NPB 311585(1989)*

- **Leading twist octet baryon LCDA:**

| Octet | $n$ | $p$ | $\Lambda$ |
|---|---|---|---|
| $fgh$ | d d u | u u d | u d s |

$$\begin{aligned}&\langle 0|f_\alpha(z_1n)g_\beta(z_2n)h_\gamma(z_3n)|B(P_B,\lambda)\rangle\\ &= \frac{1}{4}f_V\left[(\not{P}_BC)_{\alpha\beta}(\gamma_5u_B)_\gamma V^B(z_in\cdot P_B)+(\not{P}_B\gamma_5C)_{\alpha\beta}(u_B)_\gamma A^B(z_in\cdot P_B)\right]\\ &+\frac{1}{4}f_T(i\sigma_{\mu\nu}P_B^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5u_B)_\gamma T^B(z_in\cdot P_B),\end{aligned}$$

# Lattice Setup

Part ⑥

## ➢ Definition of the Operators on Lattice

**Matrix element**

$$C_2(z_a, z_2; t, \vec{P}) = \int \mathrm{d}^3x \mathrm{e}^{-\mathrm{i}\vec{P}\cdot\vec{x}} \langle 0|\mathcal{O}_{\mathrm{Sink}}(\vec{x}, t; z_1, z_2)\bar{\mathcal{O}}_{\mathrm{Src}}(0,0;0,0)T|0\rangle$$

**Source-side Operator**

$$\bar{O}^P_{\mathrm{mod}} = (u^T C\gamma_5\gamma^t d)u,$$

$$\bar{O}^\Lambda_{\mathrm{mod}} = \frac{2(u^T C\gamma_5\gamma^t d)s + (u^T C\gamma_5\gamma^t s)d + (s^T C\gamma_5\gamma^t d)u}{\sqrt{6}}.$$

**Projection Operator**

$$T = \gamma_t + \gamma_z$$

**Sink-side Operator**

$$O^V_P = [u^T(z_1 n_z)(C\gamma^t)u(z_2 n_z)]\gamma_5 d(z_3 n_z)$$

$$O^A_P = [u^T(z_1 n_z)(C\gamma_5\gamma^t)u(z_2 n_z)]d(z_3 n_z)$$

$$O^T_P = [u^T(z_1 n_z)\frac{1}{2}C[\gamma^t, \gamma^\mu]u(z_2 n_z)]\gamma_5\gamma_\mu d(z_3 n_z)$$

$$O^V_\Lambda = [u^T(z_1 n_z)(C\gamma^t)d(z_2 n_z)]\gamma_5 s(z_3 n_z)$$

$$O^A_\Lambda = [u^T(z_1 n_z)(C\gamma_5\gamma^t)d(z_2 n_z)]s(z_3 n_z)$$

$$O^T_\Lambda = [u^T(z_1 n_z)\frac{1}{2}C[\gamma^t, \gamma^\mu]d(z_2 n_z)]\gamma_5\gamma_\mu s(z_3 n_z)$$