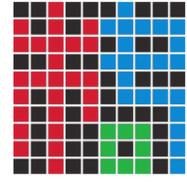




中国科学院大学  
University of Chinese Academy of Sciences



# KN散射的格点计算

calculate of KN scattering

汇报人：贾宇航 中国科学院大学

合作者：吴佳俊 王政力 中国科学院大学

孙鹏 刘柳明 近代物理实验所

时间：2025.10.12



# 目录

CONTENTS

---

01

背景介绍

02

格点计算

03

能级分析

04

小结与展望





中国科学院大学  
University of Chinese Academy of Sciences

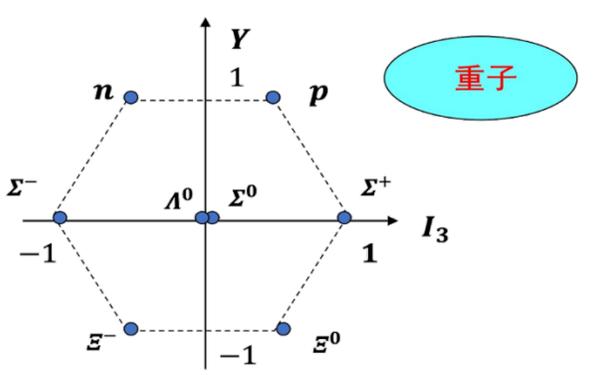
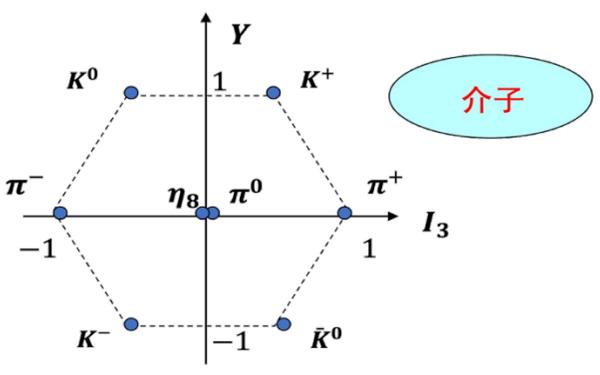
01

# 背景介绍



## 实验中的KN散射

K介子重子散射



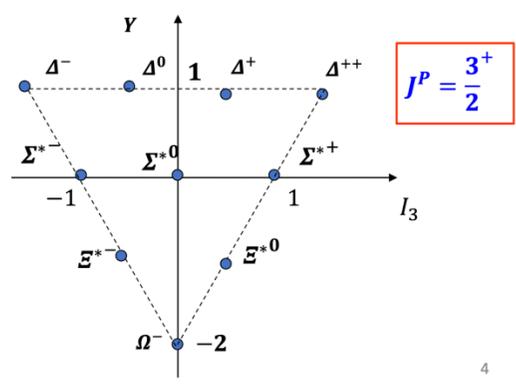
$\bar{K}N$

三夸克激发态

直接相互作用

$K^+N$

直接相互作用



陈莹老师《粒子物理基础课件》

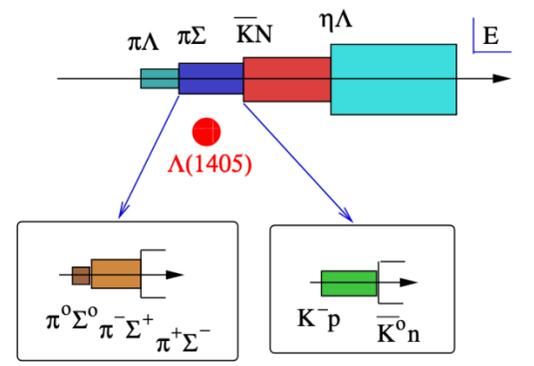


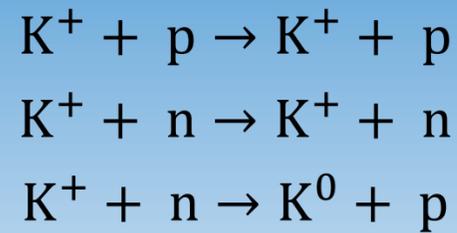
Figure 2: Complex energy plane in the vicinity of the  $\Lambda(1405)$ .

Two-pole structures in QCD: Facts, not fantasy! Ulf-G. Meißner

1. 目前无直接证据五夸克激发态
2. 即使有，由于产生阈值较高，在低能时可以认为不会发生



## 三个反应



由于同位旋对称性，上述三个反应只有两个线性独立

现有理论无法完美拟合三个过程

KN scattering amplitude revisited in a chiral unitary approach and a possible broad resonance in S = +1 channel Kenji Aoki<sup>1,2,\*</sup> and Daisuke Jido<sup>2,1</sup>  
(上方为solution1, 下方为solution2)

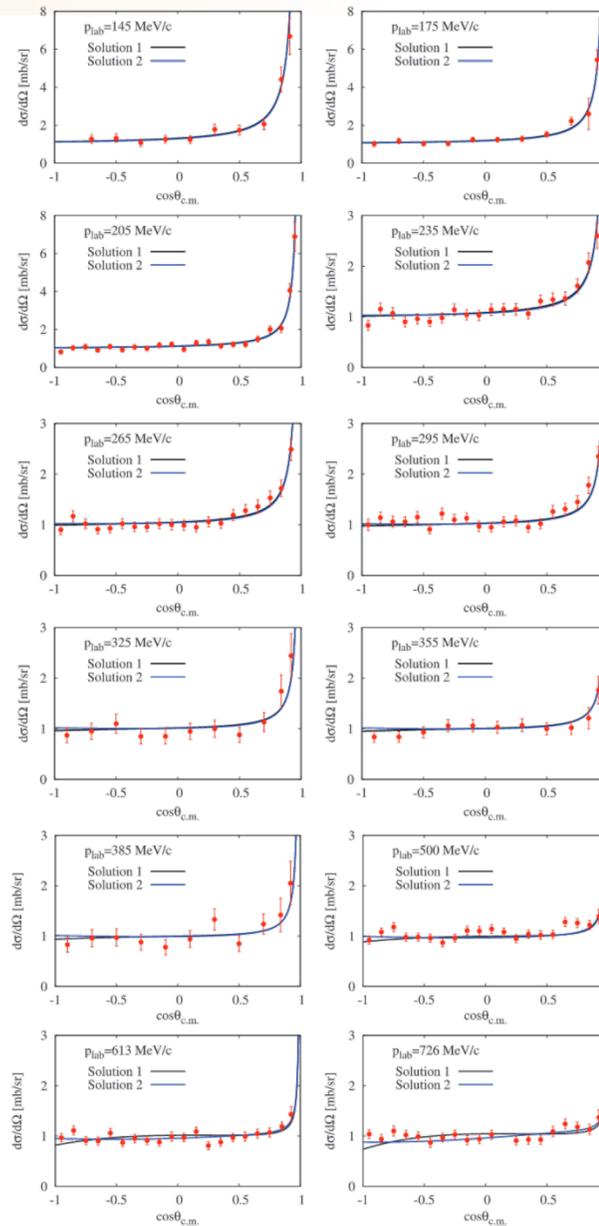


Fig. 2. The calculated differential cross sections of the K<sup>+</sup>p elastic scattering using Solutions 1 and 2 in comparison with the experimental data of Ref. [4].

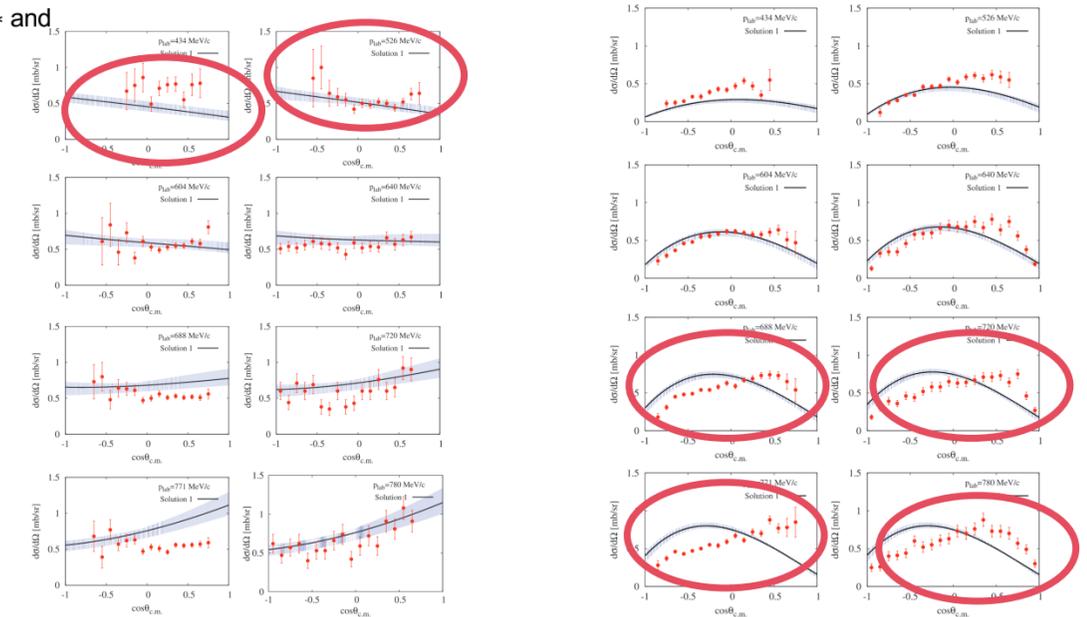


Fig. 4. The differential cross sections of K<sup>+</sup>n elastic scattering using Solution 1 in comparison with the experimental data of Refs. [18,44]. The momenta at p<sub>lab</sub> = 640, 720, and 780 MeV/c are from Ref. [18]; the others are from Ref. [44].  
Fig. 6. The differential cross sections of K<sup>+</sup>n charge exchange scattering using Solution 1 in comparison with the experimental data of Refs. [16,44]. The momenta at the p<sub>lab</sub> = 640, 720, and 780 MeV/c are from Ref. [16]; the others are from Ref. [44].

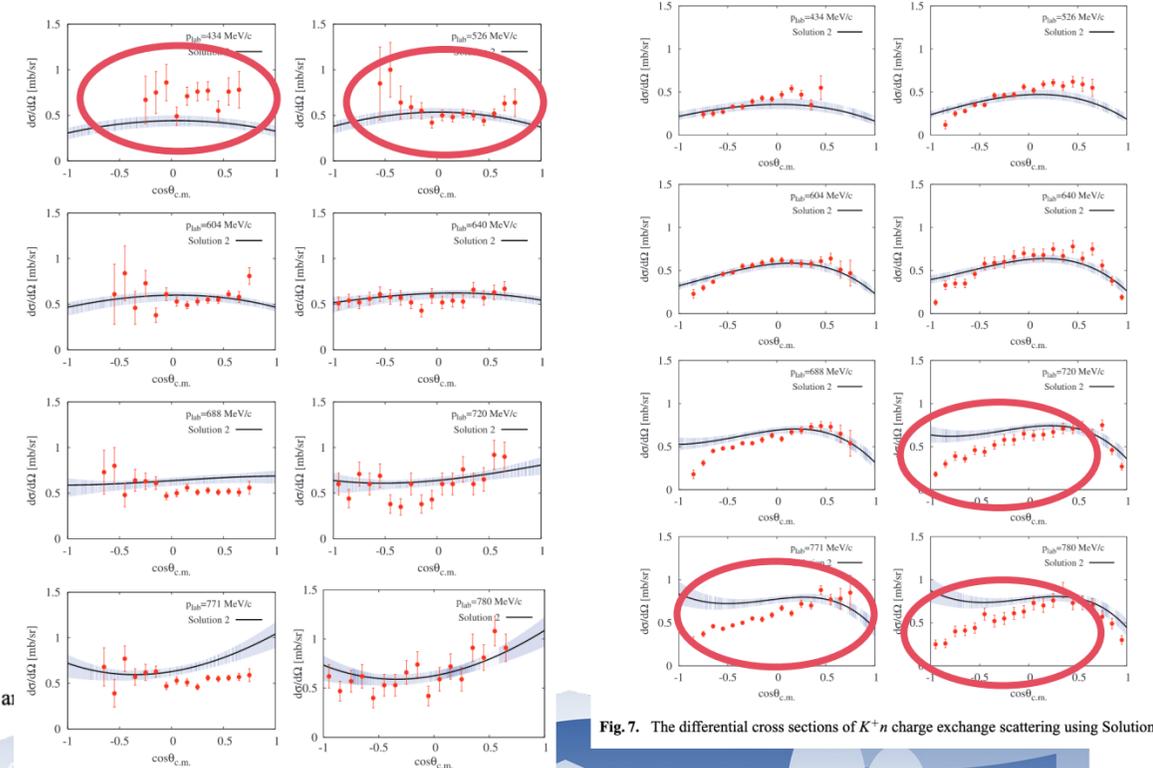


Fig. 5. The differential cross sections of K<sup>+</sup>n elastic scattering using Solution 2.

Fig. 7. The differential cross sections of K<sup>+</sup>n charge exchange scattering using Solution 2.

## 实验中抽取中子散射

### 实验中的中子散射

没有纯的中子靶

实验从介子氘核散射中抽取

介子中子散射信息

<https://www.hepdata.net/search/?q=KN+scattering>

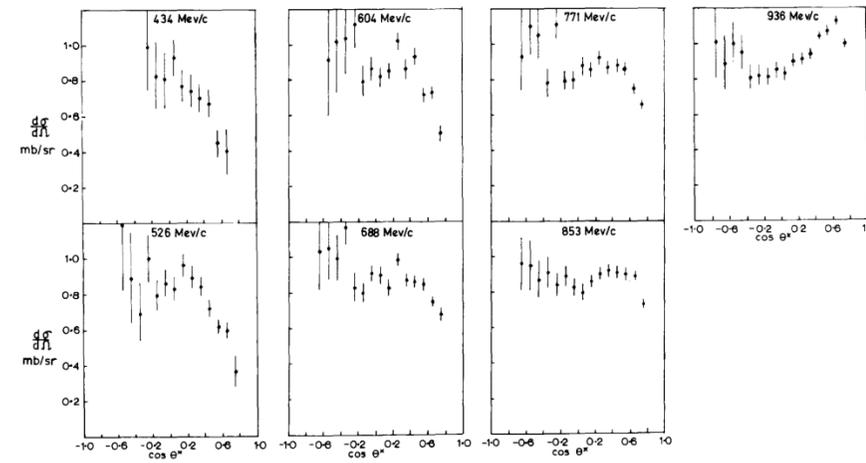


Fig. 11. Differential cross sections for the reaction  $K^+d \rightarrow K^+n(p)$  as measured in this experiment.

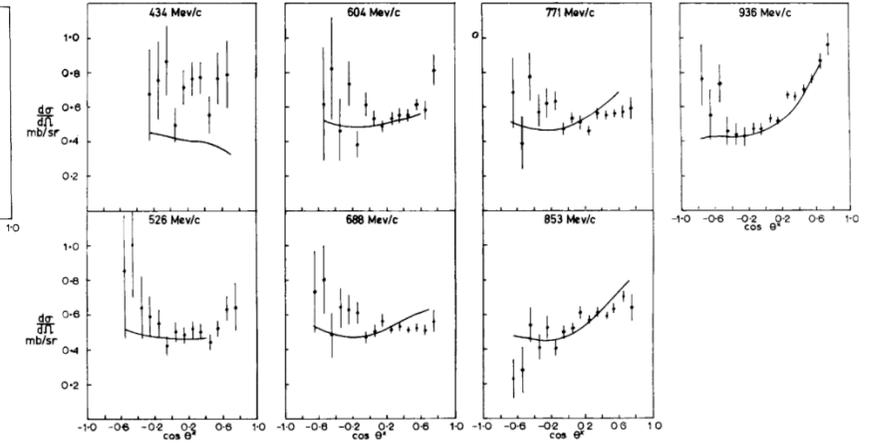


Fig. 10. Differential cross sections for the reaction  $K^+d \rightarrow K^+n(p)$  as measured in this experiment. The solid lines are the fits from solution 1 of ref. [15].

K+n ELASTIC AND CHARGE-EXCHANGE SCATTERING BETWEEN 430 AND 940 MeV/c  
C.J.S. DAMERELL, M.J. HOTCHKISS and F. WICKENS

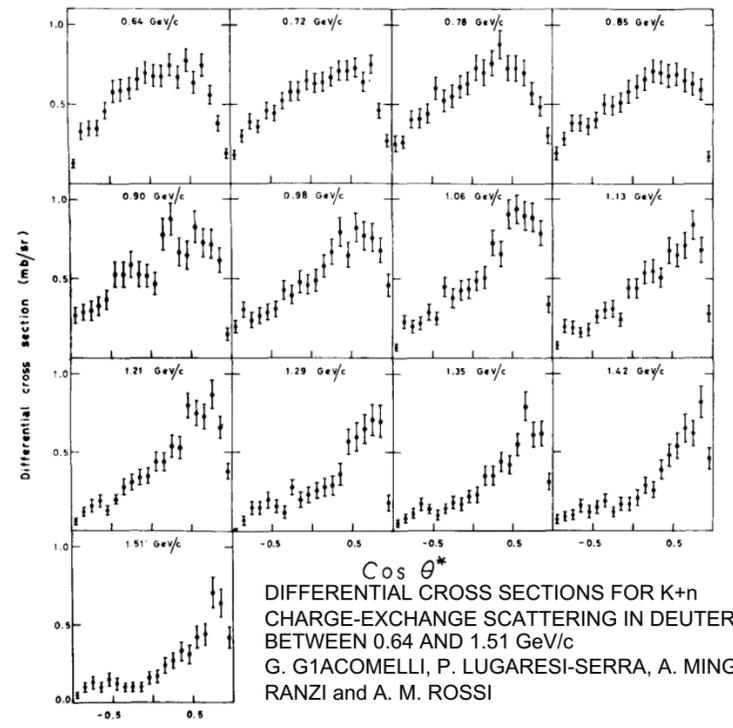


Fig. 1. Charge-exchange differential cross sections.

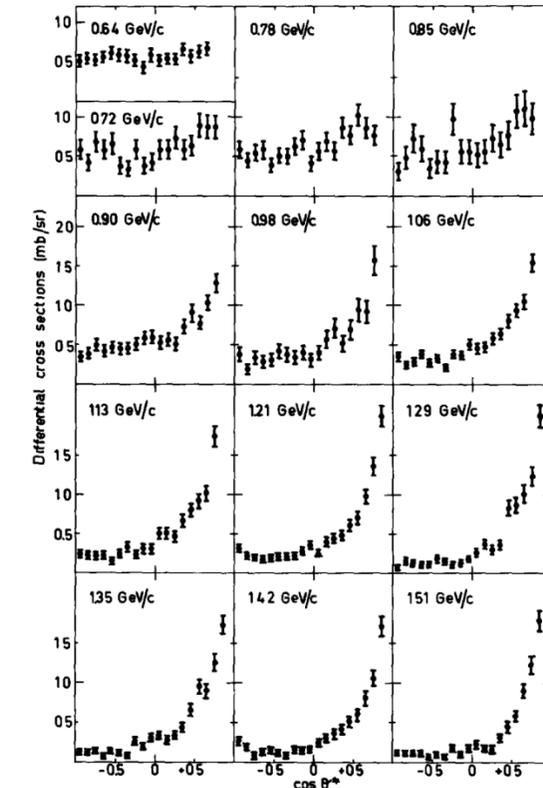
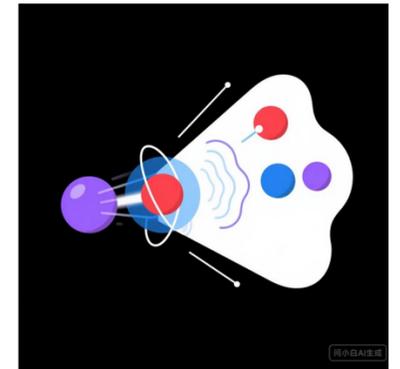


Fig. 2.  $K^+n$  elastic differential cross sections at the 13 momenta measured in this experiment. These cross sections have not been divided by  $I_0$ .



DIFFERENTIAL CROSS SECTIONS FOR K+n ELASTIC SCATTERING BETWEEN 0.64 and 1.51 GeV/c  
BGR T Collaboration

## 实验流程

01

If OK

02

03

### 格点-实验匹配

通过格点QCD计算 $I=1$ 的KN散射，  
对比  
 $K^+ + p \rightarrow K^+ + p$  反应的实验数据。

### 格点计算

再通过格点计算 $I=0, 1$ 的KN散  
射，抽取出  
 $K^+ + n \rightarrow K^+ + n$   
 $K^+ + n \rightarrow K^0 + p$   
散射长度等信息。

### 数据分析

将格点计算的结果与实验数据  
比较，分析实验中Kd散射，  
检验之前实验组从介子氘核散  
射中抽取介子中子散射长度方  
法的有效性。

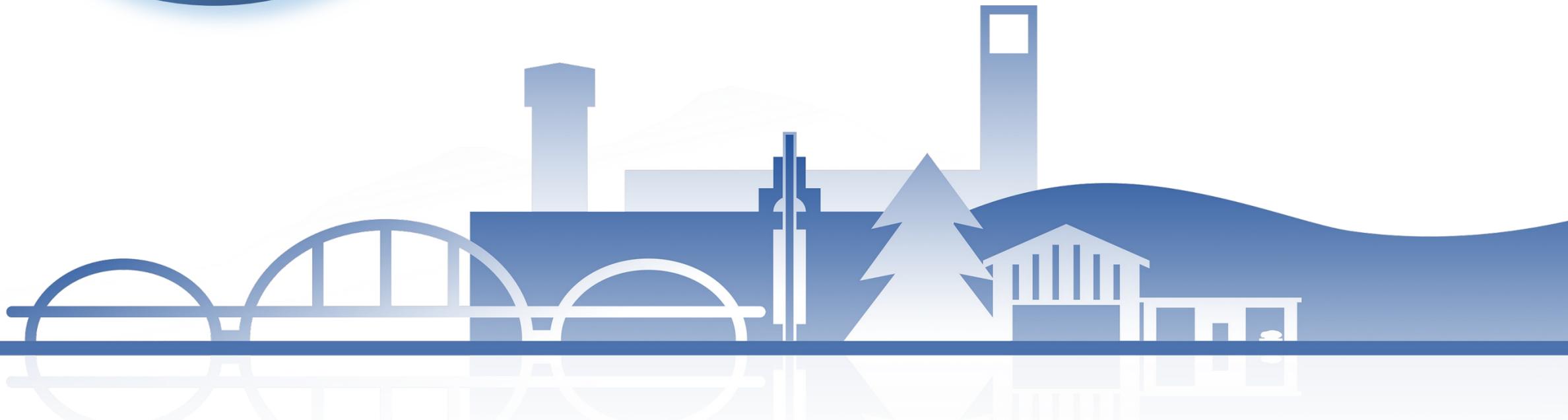




中国科学院大学  
University of Chinese Academy of Sciences

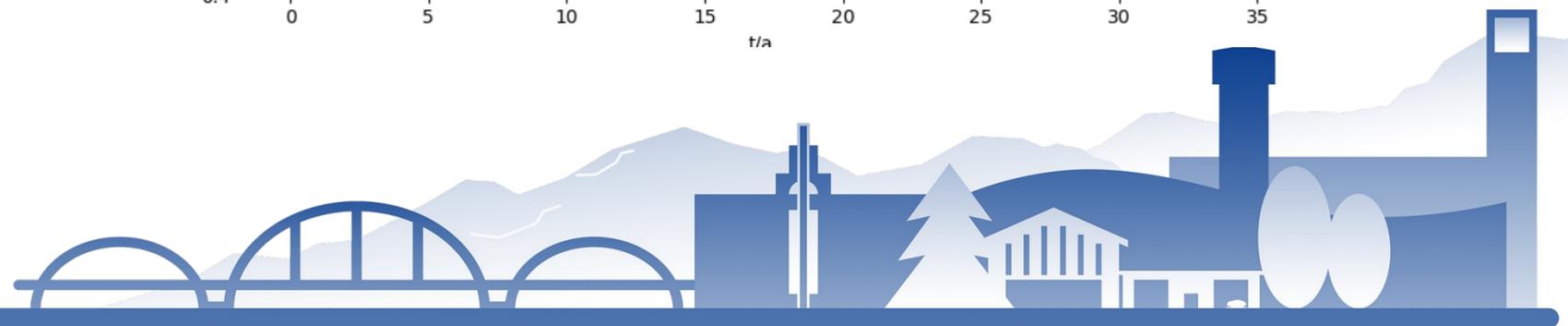
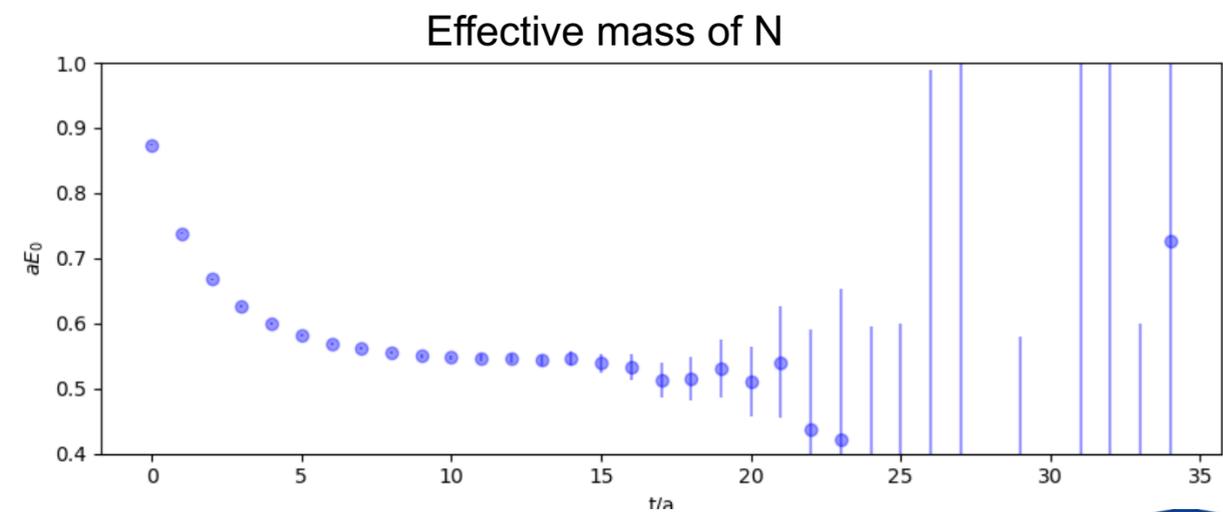
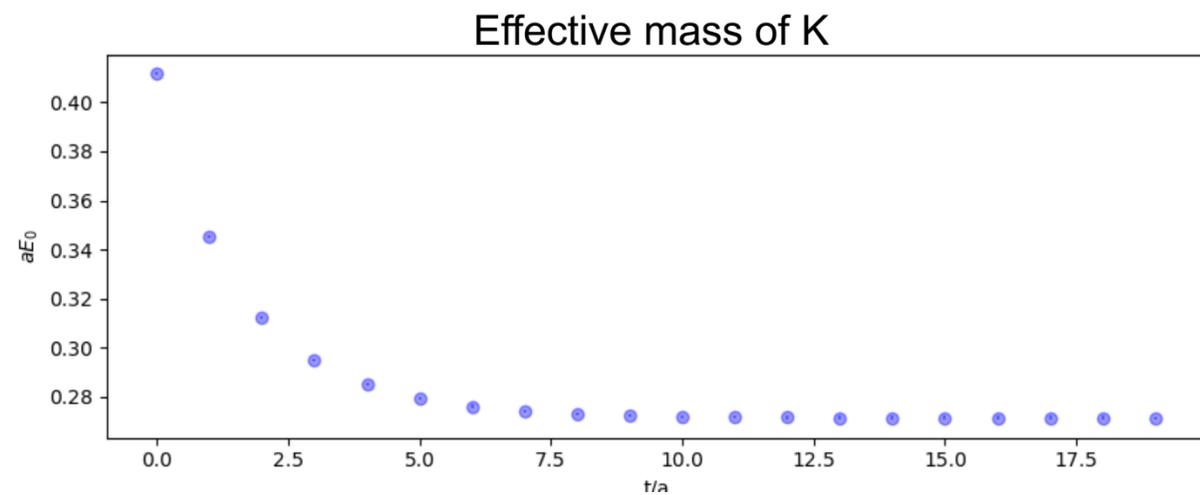
02

# 格点计算



## 组态信息

Name	Volume	Spacing/fm	$m_{\pi}/\text{Mev}$	$m_K/\text{Mev}$	$m_N/\text{Mev}$
C24P29	$24^3 \times 72$	0.1053(2)	~290	~508	~1021



## $O_h^{(2)}$ 群不可约表示

- 计算了G1-表示, 对应S波
- 使用了五个能级对应的内插场算符
- $p: P_+ u(u^T C \gamma_5 d)$
- $N: -P_+ d(d^T C \gamma_5 u)$
- $K^+: \bar{s} \gamma_5 u; K^0: \bar{s} \gamma_5 d$
- 宇称投影+反周期边界条件  $\rightarrow$  exp拟合

$$\begin{aligned}
 & P_2[\mathbf{e}_x] N_{1\frac{1}{2}}[\mathbf{e}_{-x}] + P_2[\mathbf{e}_{-x}] N_{1\frac{1}{2}}[\mathbf{e}_x] + P_2[\mathbf{e}_y] N_{1\frac{1}{2}}[\mathbf{e}_{-y}] + P_2[\mathbf{e}_{-y}] N_{1\frac{1}{2}}[\mathbf{e}_y] + P_2[\mathbf{e}_z] N_{1\frac{1}{2}}[\mathbf{e}_{-z}] + P_2[\mathbf{e}_{-z}] N_{1\frac{1}{2}}[\mathbf{e}_z] \\
 & P_2[\mathbf{e}_{x,y,z}] N_{1\frac{1}{2}}[\mathbf{e}_{-x,-y,-z}] + P_2[\mathbf{e}_{x,y,-z}] N_{1\frac{1}{2}}[\mathbf{e}_{-x,-y,z}] + P_2[\mathbf{e}_{x,-y,z}] N_{1\frac{1}{2}}[\mathbf{e}_{-x,y,-z}] + P_2[\mathbf{e}_{x,-y,-z}] N_{1\frac{1}{2}}[\mathbf{e}_{-x,y,z}] + \\
 & P_2[\mathbf{e}_{-x,y,z}] N_{1\frac{1}{2}}[\mathbf{e}_{x,-y,-z}] + P_2[\mathbf{e}_{-x,y,-z}] N_{1\frac{1}{2}}[\mathbf{e}_{x,-y,z}] + P_2[\mathbf{e}_{-x,-y,z}] N_{1\frac{1}{2}}[\mathbf{e}_{x,y,-z}] + P_2[\mathbf{e}_{-x,-y,-z}] N_{1\frac{1}{2}}[\mathbf{e}_{x,y,z}] \\
 & P_2[\mathbf{e}_{x,y}] N_{1\frac{1}{2}}[\mathbf{e}_{-x,-y}] + P_2[\mathbf{e}_{x,-y}] N_{1\frac{1}{2}}[\mathbf{e}_{-x,y}] + P_2[\mathbf{e}_{x,z}] N_{1\frac{1}{2}}[\mathbf{e}_{-x,-z}] + P_2[\mathbf{e}_{x,-z}] N_{1\frac{1}{2}}[\mathbf{e}_{-x,z}] + P_2[\mathbf{e}_{-x,y}] N_{1\frac{1}{2}}[\mathbf{e}_{x,-y}] + P_2[\mathbf{e}_{-x,-y}] N_{1\frac{1}{2}}[\mathbf{e}_{x,y}] + \\
 & P_2[\mathbf{e}_{-x,z}] N_{1\frac{1}{2}}[\mathbf{e}_{x,-z}] + P_2[\mathbf{e}_{-x,-z}] N_{1\frac{1}{2}}[\mathbf{e}_{x,z}] + P_2[\mathbf{e}_y,z] N_{1\frac{1}{2}}[\mathbf{e}_{-y,-z}] + P_2[\mathbf{e}_y,-z] N_{1\frac{1}{2}}[\mathbf{e}_{-y,z}] + P_2[\mathbf{e}_{-y,z}] N_{1\frac{1}{2}}[\mathbf{e}_y,-z] + P_2[\mathbf{e}_{-y,-z}] N_{1\frac{1}{2}}[\mathbf{e}_y,z] \\
 & P_2[\mathbf{e}_x] N_{1\frac{1}{2}}[-\mathbf{e}_x] + P_2[-\mathbf{e}_x] N_{1\frac{1}{2}}[\mathbf{e}_x] + P_2[\mathbf{e}_y] N_{1\frac{1}{2}}[-\mathbf{e}_y] + P_2[-\mathbf{e}_y] N_{1\frac{1}{2}}[\mathbf{e}_y] + P_2[\mathbf{e}_z] N_{1\frac{1}{2}}[-\mathbf{e}_z] + P_2[-\mathbf{e}_z] N_{1\frac{1}{2}}[\mathbf{e}_z] \\
 & P_2[0] N_{1\frac{1}{2}}[0]
 \end{aligned}$$

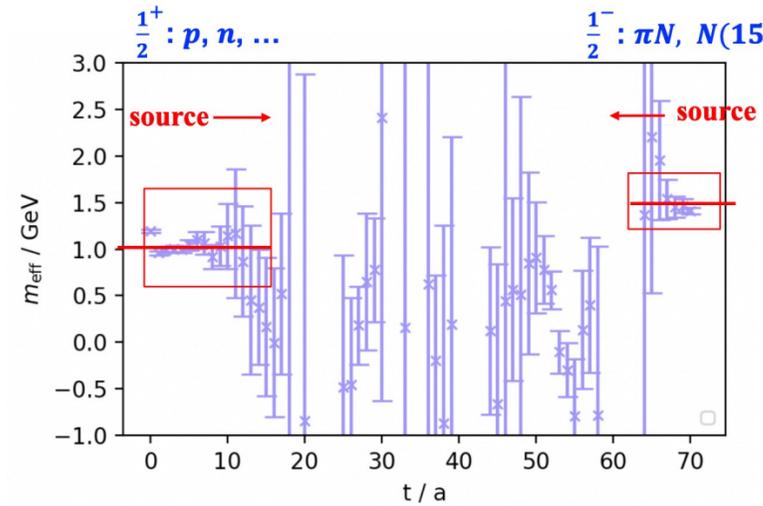
<< Option`

**Option: package for Operator construction in lattice QCD.**

Authors: Haobo Yan and Liuming Liu

Contact: haobo@stu.pku.edu.cn

- Effective mass of proton:
  - Parity: forward propagating  $\Rightarrow +$
  - backward propagating  $\Rightarrow -$



格点QCD中的强子谱学研究, 张其安, Jul. 18, 2023

So we can extract the effective mass of proton from the first window, or .....

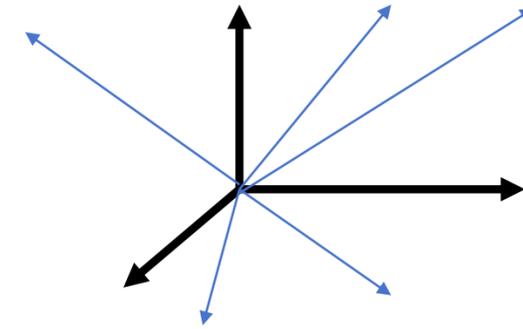
J	$\Gamma$ (dim $_{\Gamma}$ )
0	$A_1(1)$
$\frac{1}{2}$	$G_1(2)$
1	$T_1(3)$
$\frac{3}{2}$	$H(4)$
2	$E(2) \oplus T_2(3)$
$\frac{5}{2}$	$H(4) \oplus G_2(2)$
3	$A_2(1) \oplus T_1(3) \oplus T_2(3)$

**Table 1.** Continuum spins  $J$ , corresponding lattice irreps  $\Gamma$  and their dimension  $\dim_{\Gamma}$  for group  $O^{(2)}$ .

Lattice operators for scattering of particles with spin S. Prelovsek,a,b,c U. Skerbisb and C.B. Langd

## GEVP & jackknife重采样

- GEVP (广义本征值分解):
  - $[H, L^2] = 0; [H, L_z] = 0$
  - H 本征态为上述五个算符的线形组合 (忽略更高激发态的贡献)
- Jackknife重采样
  - 基本思想: 移除一个点看影响
  - 核心操作: 每次移除一个数据点
  - 关键特点: 移除一个点, 每次只用n-1个点, 计算量小。并且只需n次计算。可以显示哪些点对结果影响大

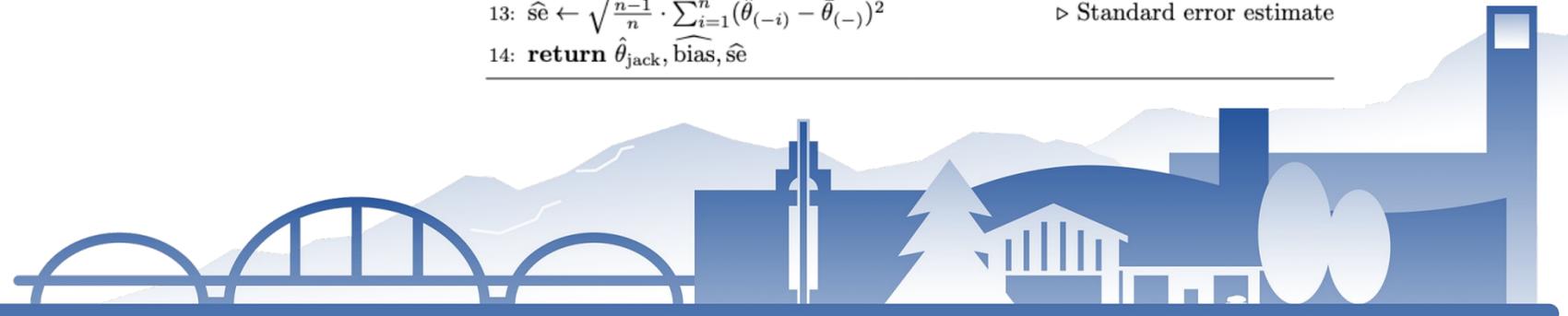


### Algorithm 1 Jackknife Resampling Algorithm

**Require:** Dataset  $D = \{x_1, x_2, \dots, x_n\}$ , statistic function  $T(\cdot)$

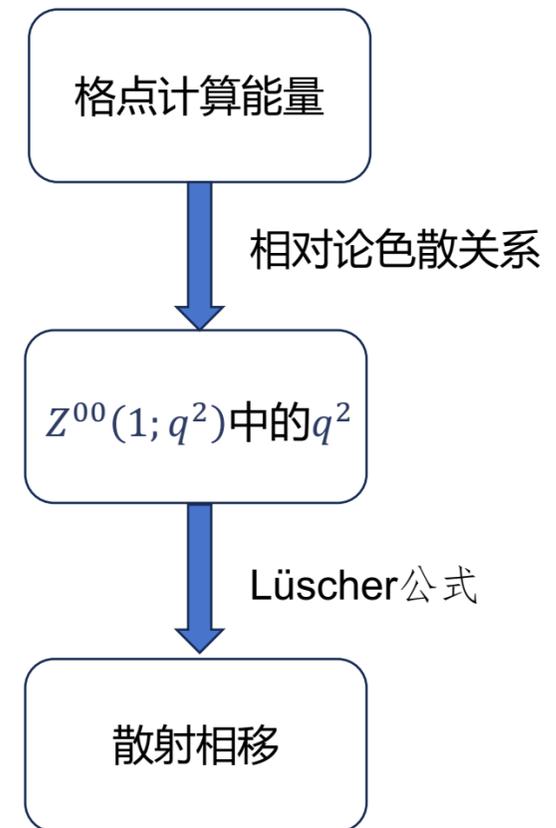
**Ensure:** Jackknife estimate  $\hat{\theta}_{\text{jack}}$ , bias estimate  $\widehat{\text{bias}}$ , standard error estimate  $\widehat{\text{se}}$

```
1:  $n \leftarrow |D|$  ▷ Sample size
2:  $\hat{\theta}_{\text{full}} \leftarrow T(D)$  ▷ Full sample statistic
3: Initialize empty array  $\hat{\theta}_{(-)}$  ▷ Statistics with one observation deleted
4: for  $i = 1$  to  $n$  do
5:    $D_{(-i)} \leftarrow D \setminus \{x_i\}$  ▷ Remove the  $i$ -th observation
6:    $\hat{\theta}_{(-i)} \leftarrow T(D_{(-i)})$  ▷ Compute statistic for jackknife sample
7:   Append  $\hat{\theta}_{(-i)}$  to  $\hat{\theta}_{(-)}$ 
8: end for
9: Compute pseudo-values:  $\text{pseudo}_i = n \cdot \hat{\theta}_{\text{full}} - (n-1) \cdot \hat{\theta}_{(-i)}$ ,  $i = 1, \dots, n$ 
10:  $\hat{\theta}_{\text{jack}} \leftarrow \text{mean}(\text{pseudo}_i)$  ▷ Jackknife estimate
11:  $\bar{\theta}_{(-)} \leftarrow \text{mean}(\hat{\theta}_{(-)})$  ▷ Mean of jackknife statistics
12:  $\widehat{\text{bias}} \leftarrow (n-1) \cdot (\bar{\theta}_{(-)} - \hat{\theta}_{\text{full}})$  ▷ Bias estimate
13:  $\widehat{\text{se}} \leftarrow \sqrt{\frac{n-1}{n} \cdot \sum_{i=1}^n (\hat{\theta}_{(-i)} - \bar{\theta}_{(-)})^2}$  ▷ Standard error estimate
14: return  $\hat{\theta}_{\text{jack}}, \widehat{\text{bias}}, \widehat{\text{se}}$ 
```



## Lüscher公式 (S波)

- 勾连LQCD计算的能级与物理世界的散射相移
- $k * \cot(\delta^0(k)) = (2/\sqrt{\pi}) * (1/L) * Z^{00}(1; q^2)$ 
  - $k$ : 质心系相对动量
  - $\delta_0(k)$ : S 波散射相移
  - $L$ : 空间格点尺寸
  - $Z^{00}(1; q^2)$ : Lüscher Z 函数

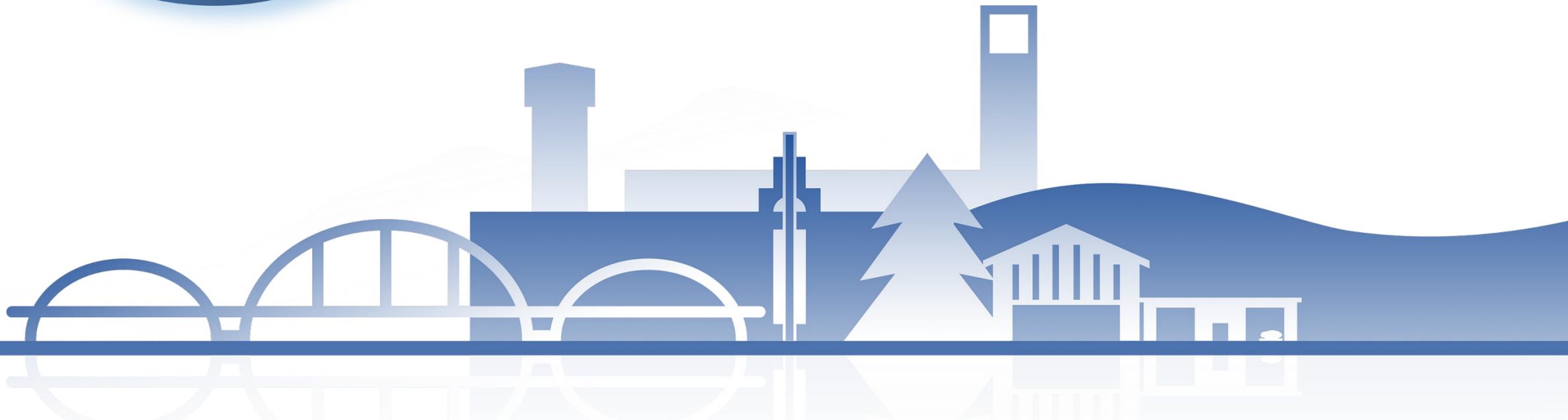




中国科学院大学  
University of Chinese Academy of Sciences

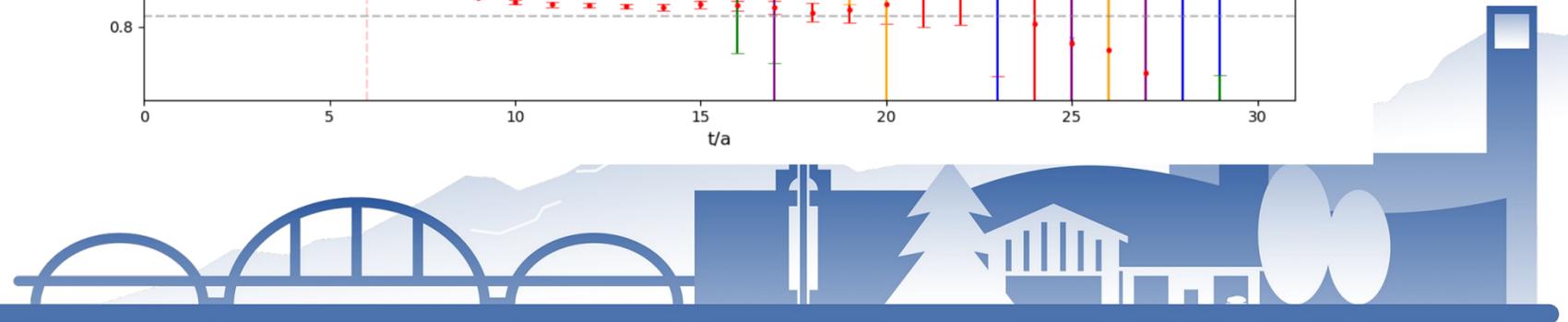
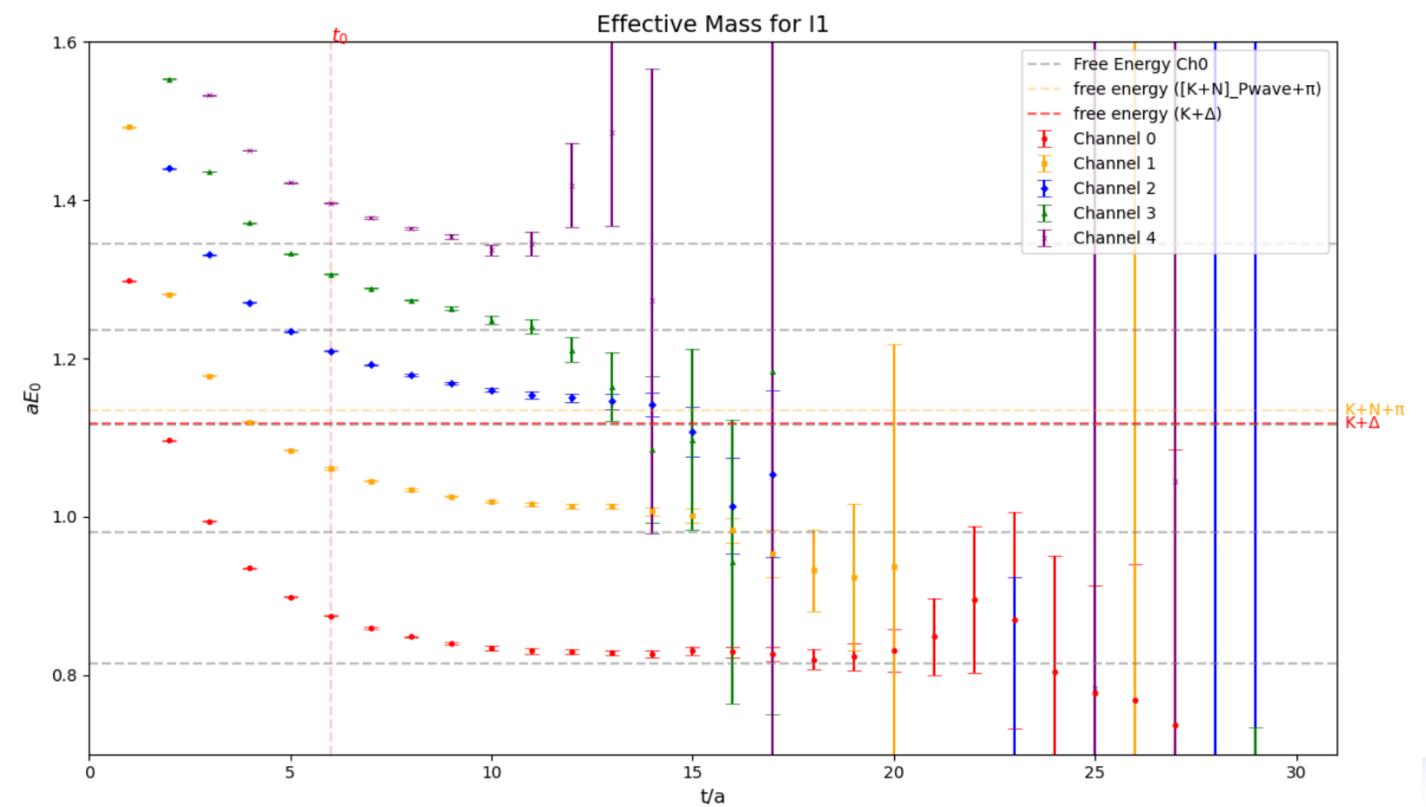
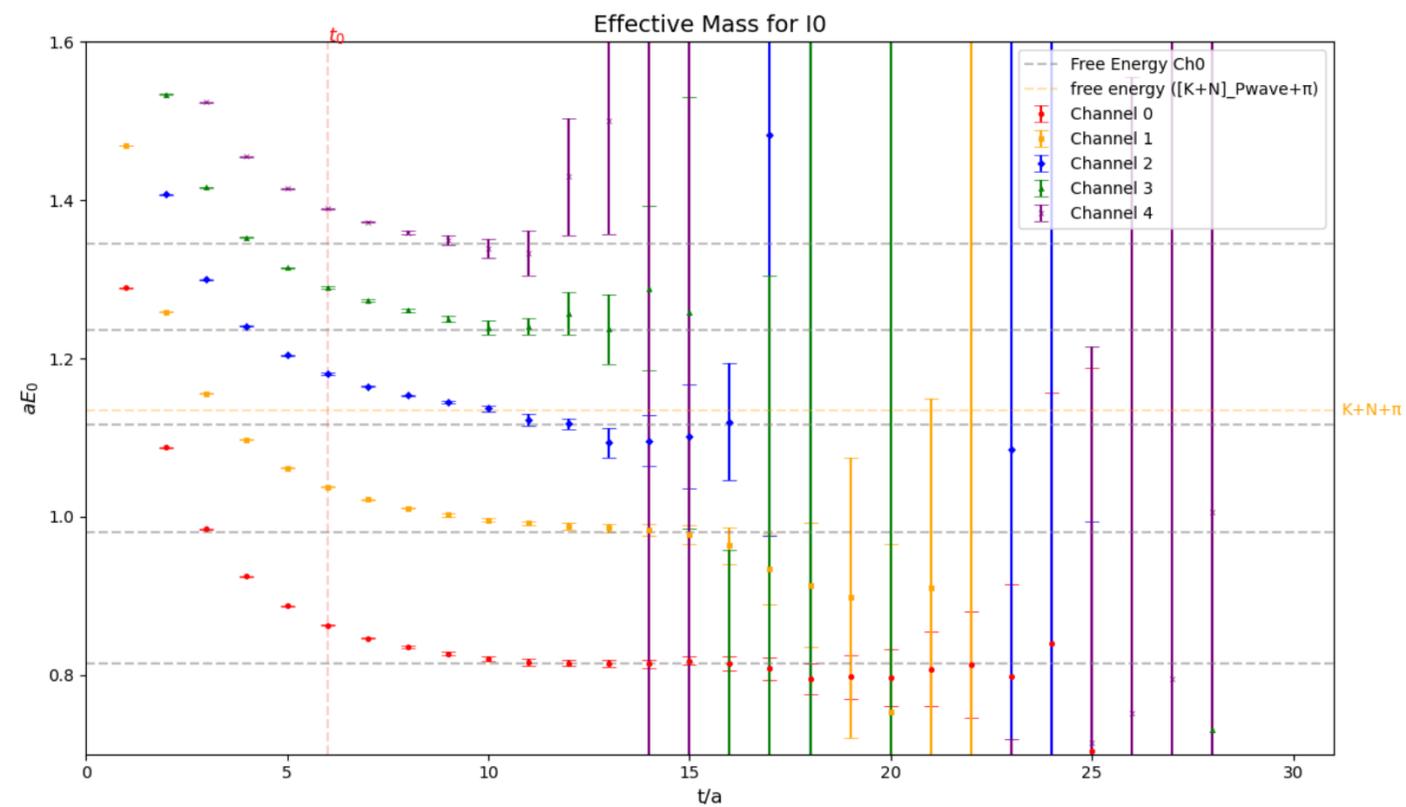
03

# 能级分析

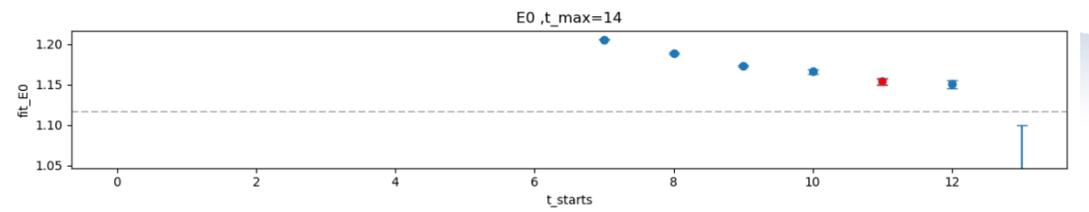
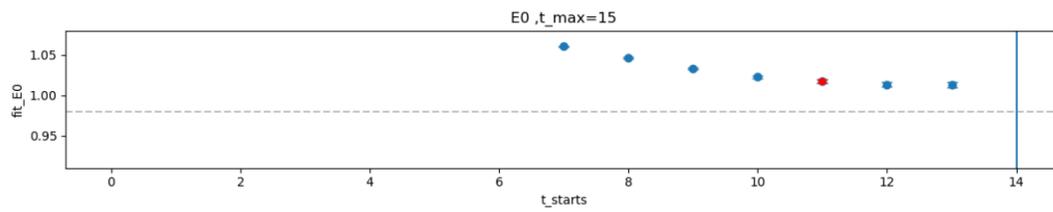
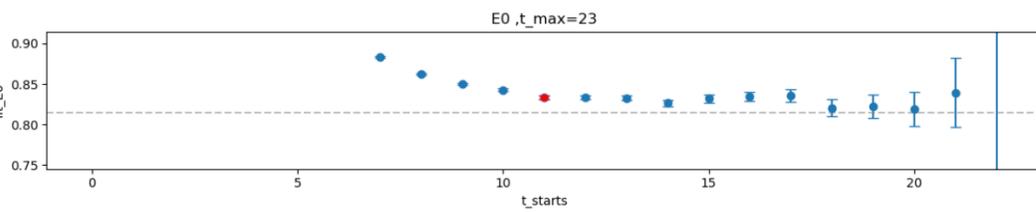
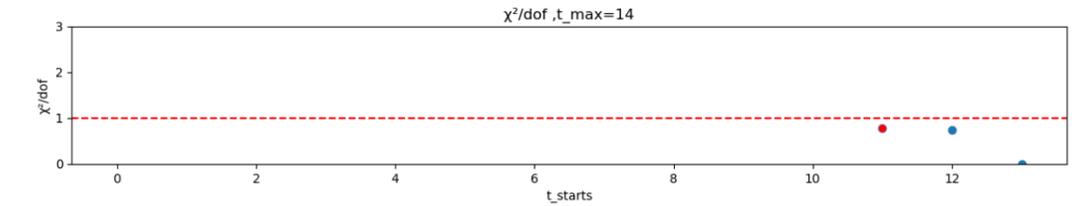
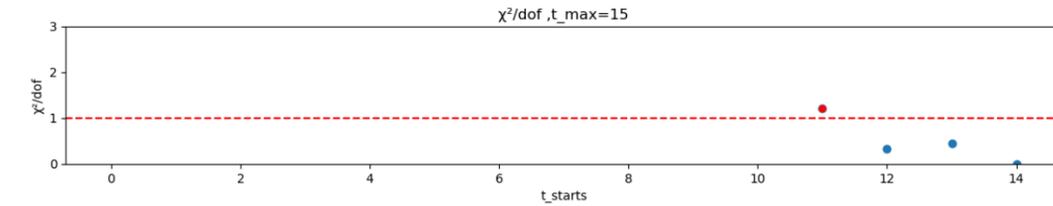
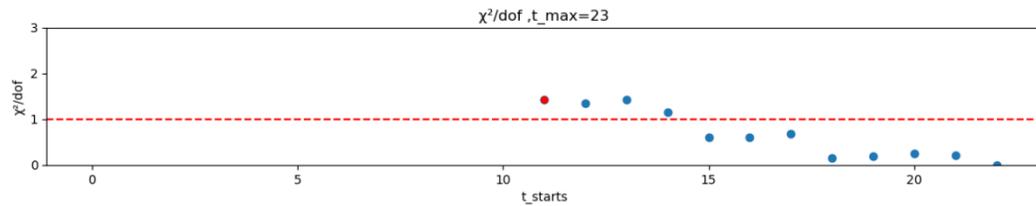
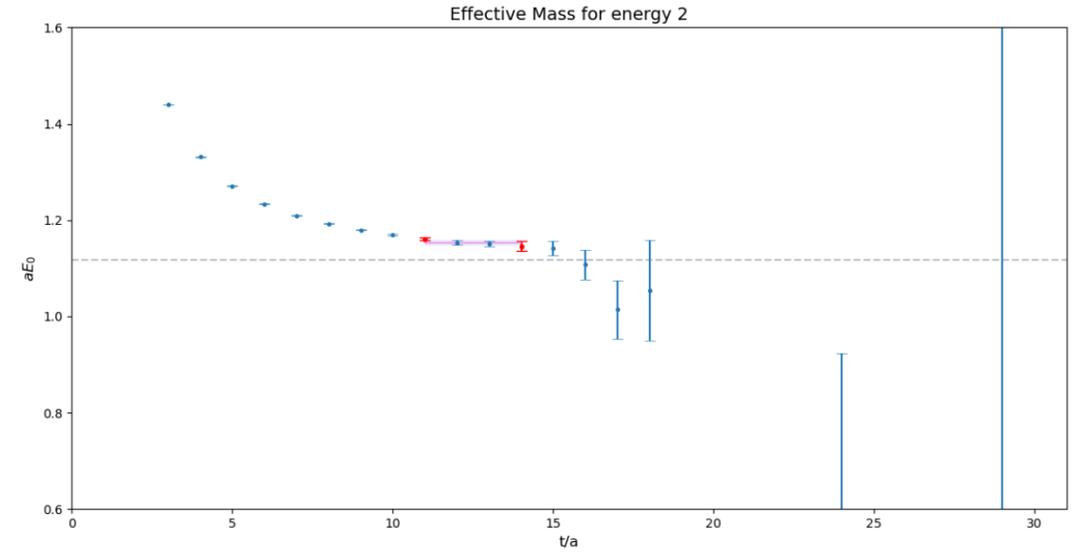
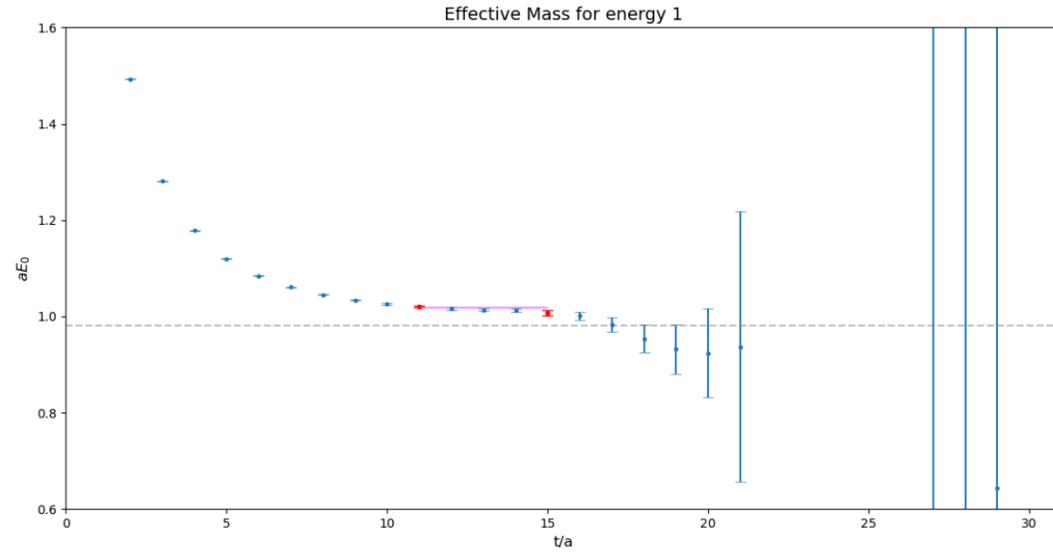
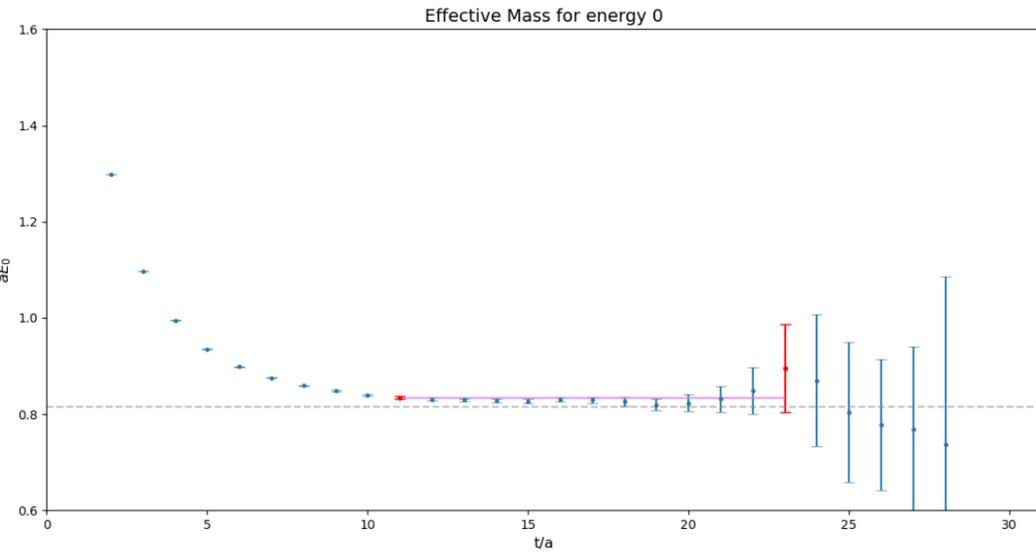
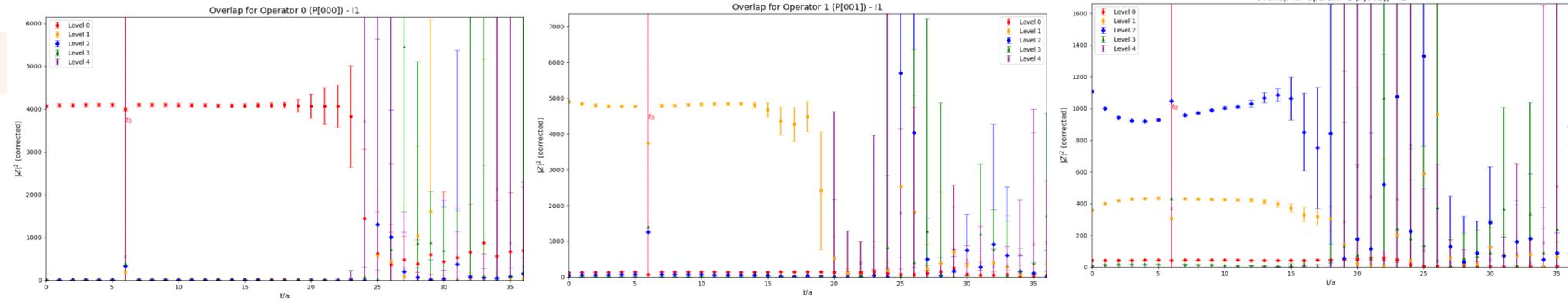


## 质心系

- 组态: C24P29
- $N_x=24$ ;  $N_t=72$ ;  $a=0.1052$
- G1-表示, 对应S波
- 样本数: 572

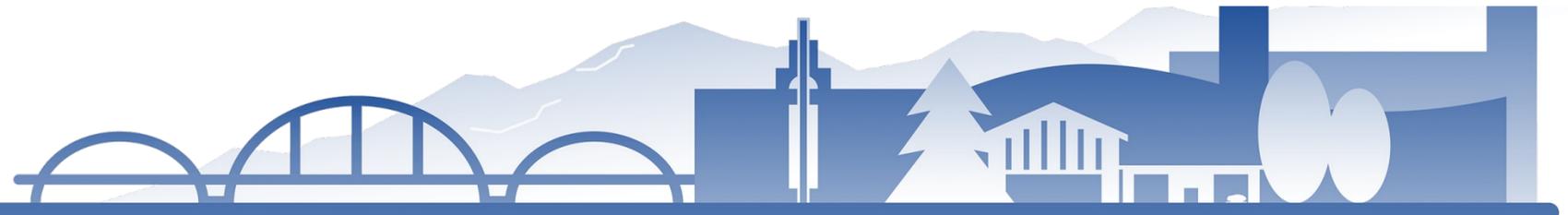
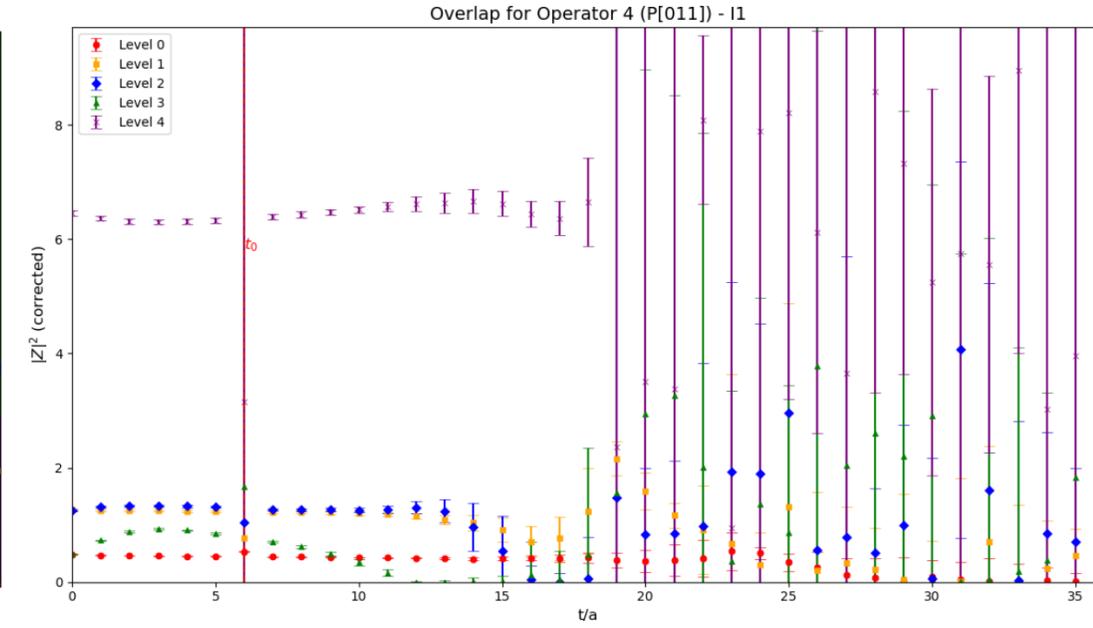
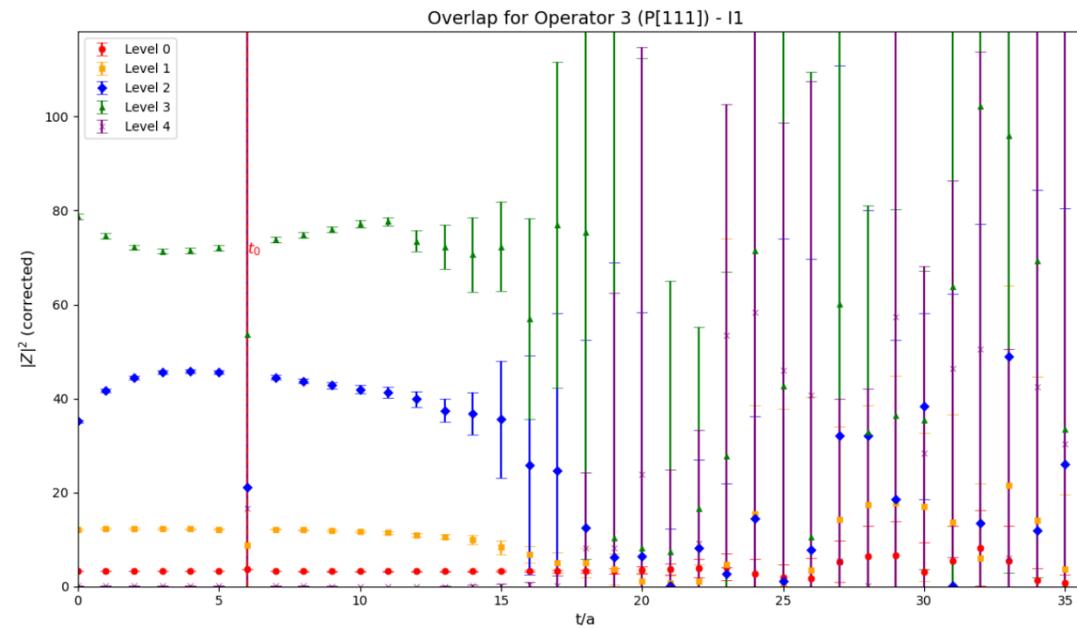
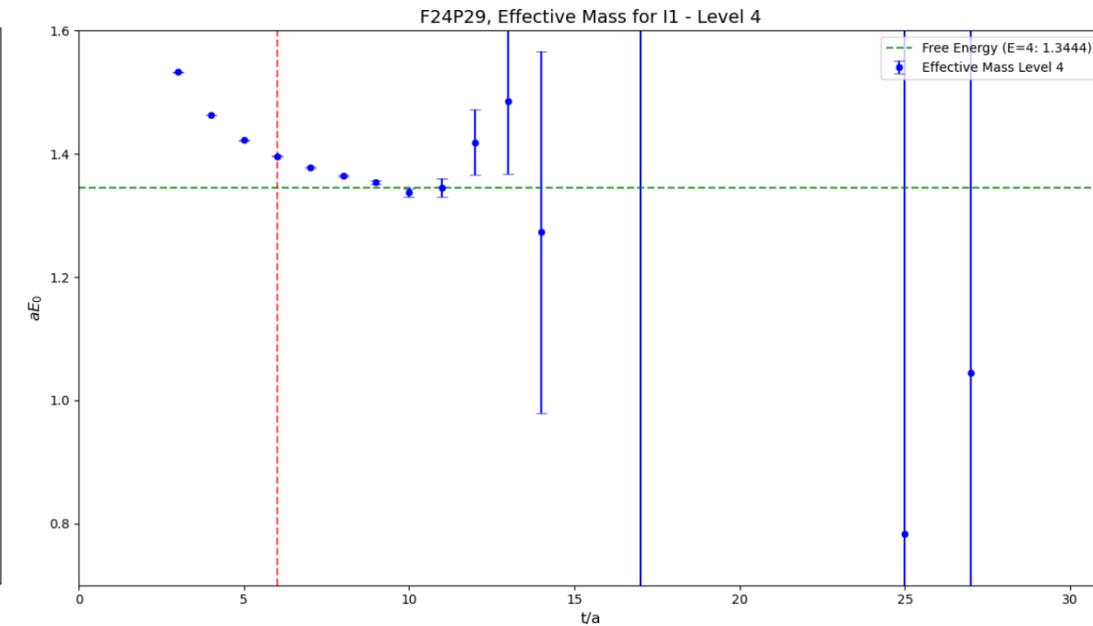
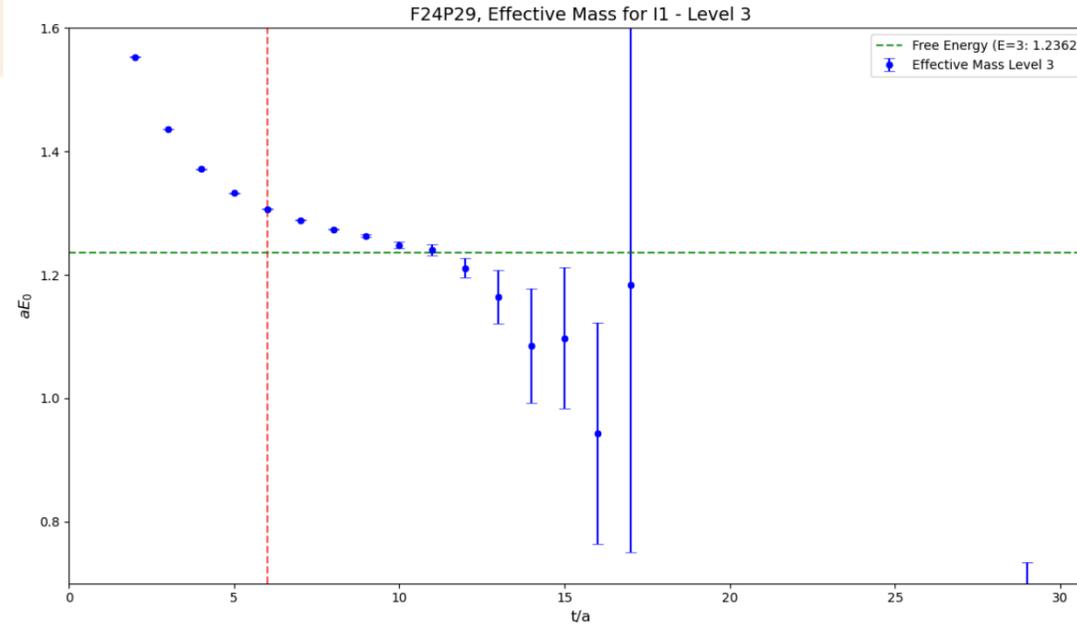


## 同位旋为1 ( $K^+p$ 散射)



## 高能级耦合

高能级的算符会和 $K\Delta$ （同位旋为1）以及产生 $\pi$ 介子的 $KN\pi$ 系统耦合，导致该阈值向上的数据无法与纯的KN弹性散射匹配。

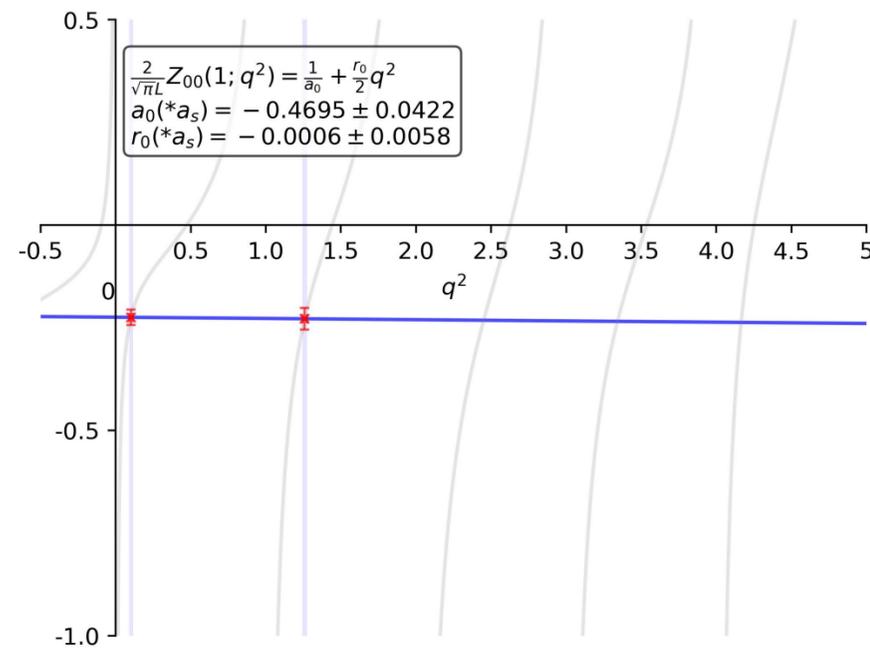


## 初步结果

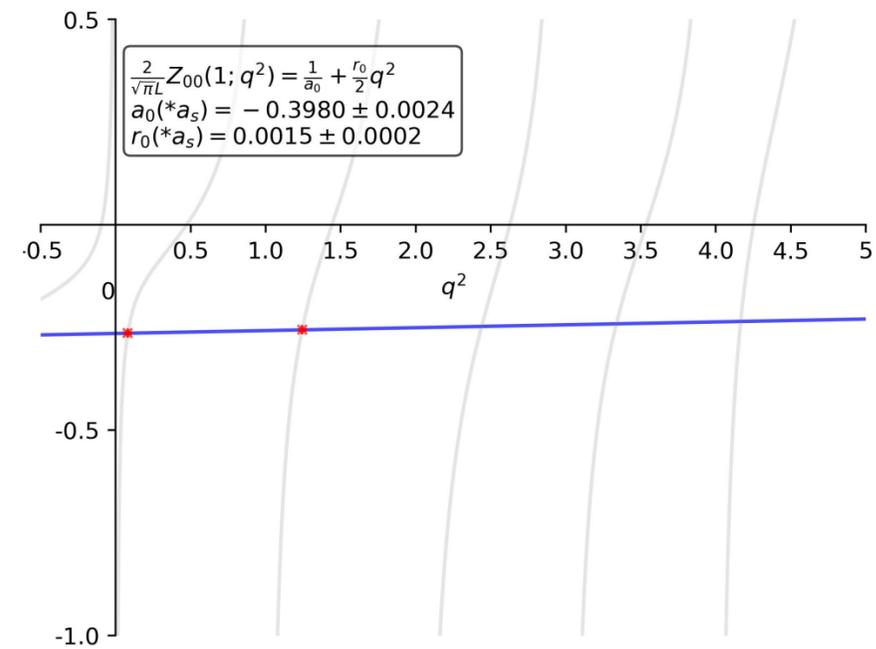
### 同位旋为1

- G1-表示
- 使用S波Lüscher公式
- 最小二乘法线性拟合
- 采用两种方法分析误差

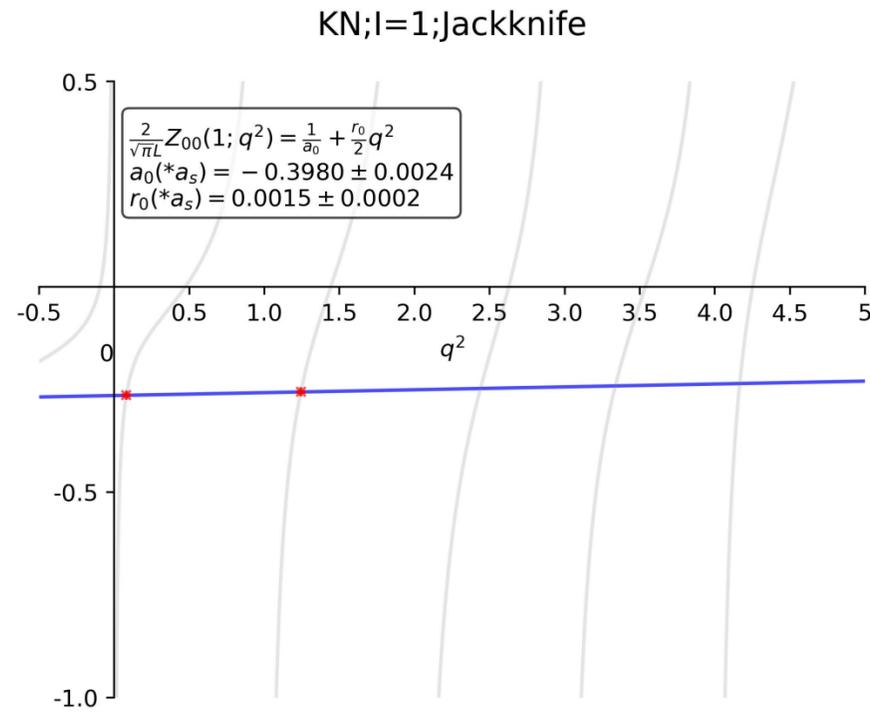
KN;l=1;Error Propagation



KN;l=1;Jackknife



## 实验比较



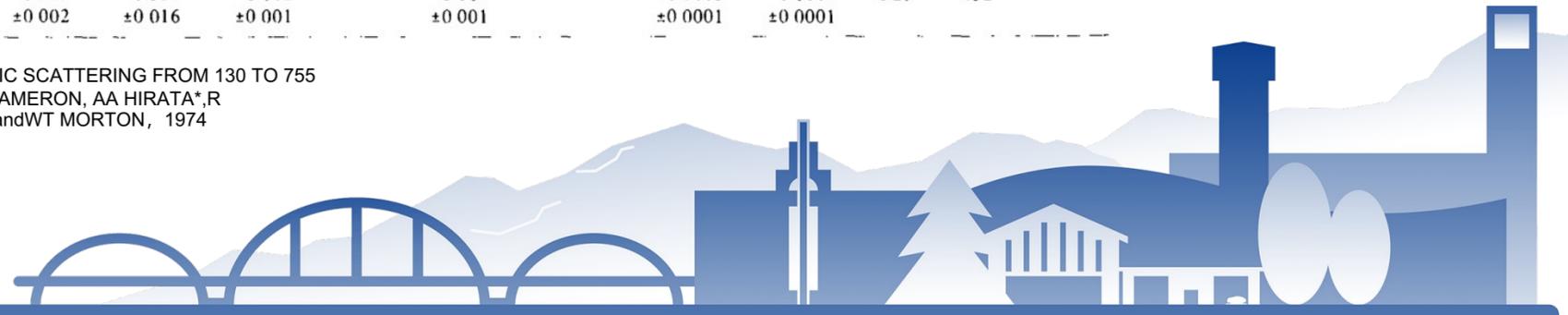
$a_{S\frac{1}{2}}$	$r_{S\frac{1}{2}}$	reference
$-0.292 \pm 0.006$	$0.35 \pm 0.09$	Culkosky 1970, PRD 2547
$-0.32 \pm 0.01$		Albrow, Nucl Phys B30(1971)273
$-0.29 \pm 0.02$		B.R. Martin, Nuclear Physics B
$-0.309 \pm 0.002$	$0.32 \pm 0.02$	W. Cameron, Nuclear Physics B
$-0.3980 \pm 0.0024$	$0.015 \pm 0.0002$	Present work, $m_\pi \approx 288 \text{ Mev}$

Table 5

Scattering lengths and effective ranges from the energy dependent analyses  $P_{\text{max}}$  is the lab momentum up to which data were used for each analysis

$P_{\text{max}}$ (MeV/c)	$a_{S\frac{1}{2}}$ (fm)	$r_{S\frac{1}{2}}$ (fm)	$a_{P\frac{1}{2}}$ (fm <sup>3</sup> )	$r_{P\frac{1}{2}}$ (fm <sup>-1</sup> )	$a_{D\frac{3}{2}}$ (fm <sup>3</sup> )	$r_{D\frac{3}{2}}$ (fm <sup>-1</sup> )	$a_{D\frac{5}{2}}$ (fm <sup>5</sup> )	$\chi^2/\text{ND}$	No. of data points
400	$-0.317 \pm 0.002$							0.79	138
400	$-0.309 \pm 0.005$	$0.256 \pm 0.126$						0.76	138
400	$-0.316 \pm 0.003$		$-0.036 \pm 0.010$		$0.014 \pm 0.008$			0.76	138
755	$-0.305 \pm 0.002$	$0.389 \pm 0.022$						0.89	198
755			$1.3 \times 10^6 \pm 1.0 \times 10^6$	$17.6 \pm 0.4$	$3.7 \times 10^7 \pm 3.0 \times 10^7$	$-9.0 \pm 0.1$		0.95	198
870	$-0.304 \pm 0.002$	$0.342 \pm 0.010$	$-0.006 \pm 0.001$		$0.004 \pm 0.001$			2.44	434
870	$-0.310 \pm 0.002$	$0.283 \pm 0.020$	$-0.017 \pm 0.001$		$0.009 \pm 0.001$		$-0.0005 \pm 0.0001$ $-0.0008 \pm 0.0001$	1.47	452
870	$-0.309 \pm 0.002$	$0.318 \pm 0.016$	$-0.012 \pm 0.001$		$0.007 \pm 0.001$		$-0.0003 \pm 0.0001$ $-0.0007 \pm 0.0001$	1.29	452

K+P ELASTIC SCATTERING FROM 130 TO 755 MeV/c W. CAMERON, AA HIRATA\*, R JENNINGS and WT MORTON, 1974



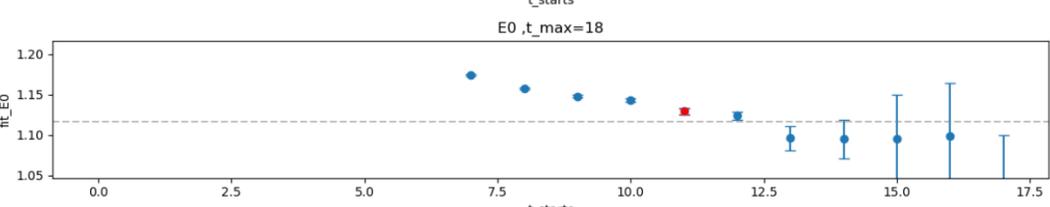
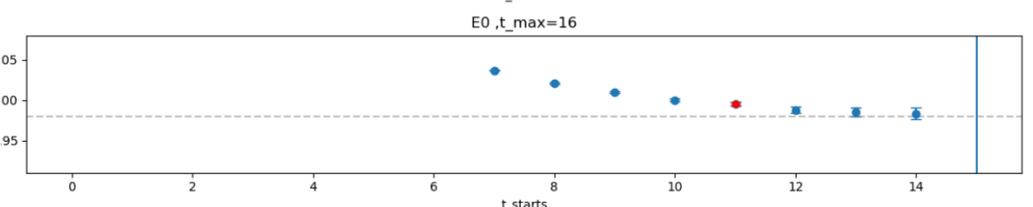
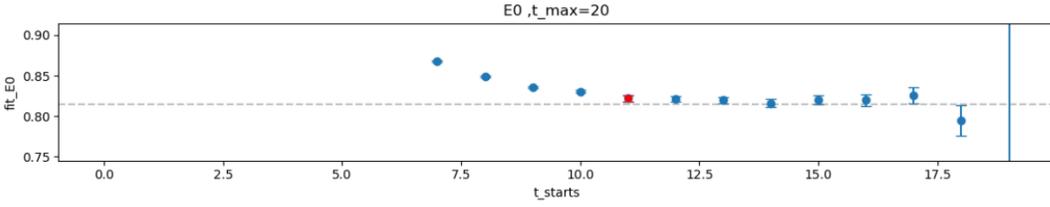
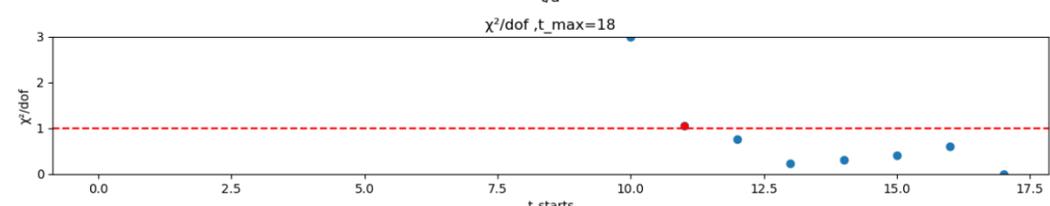
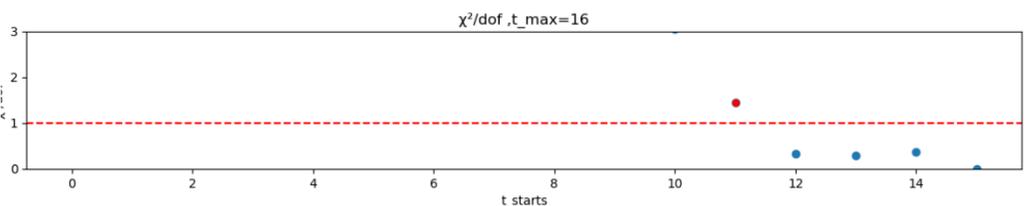
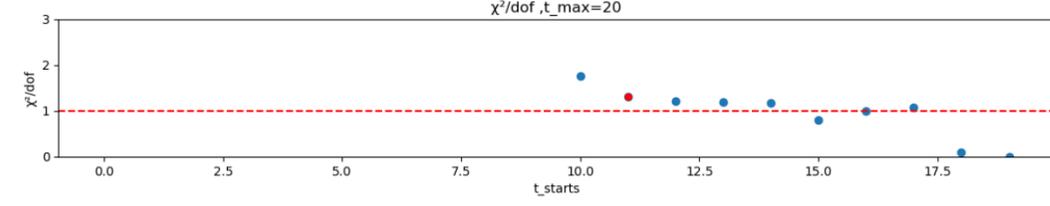
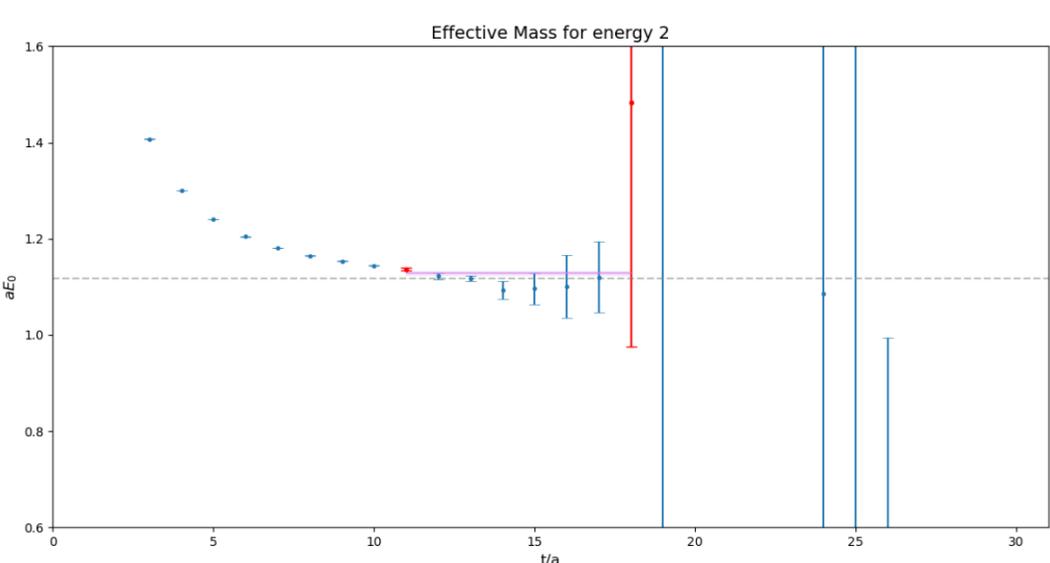
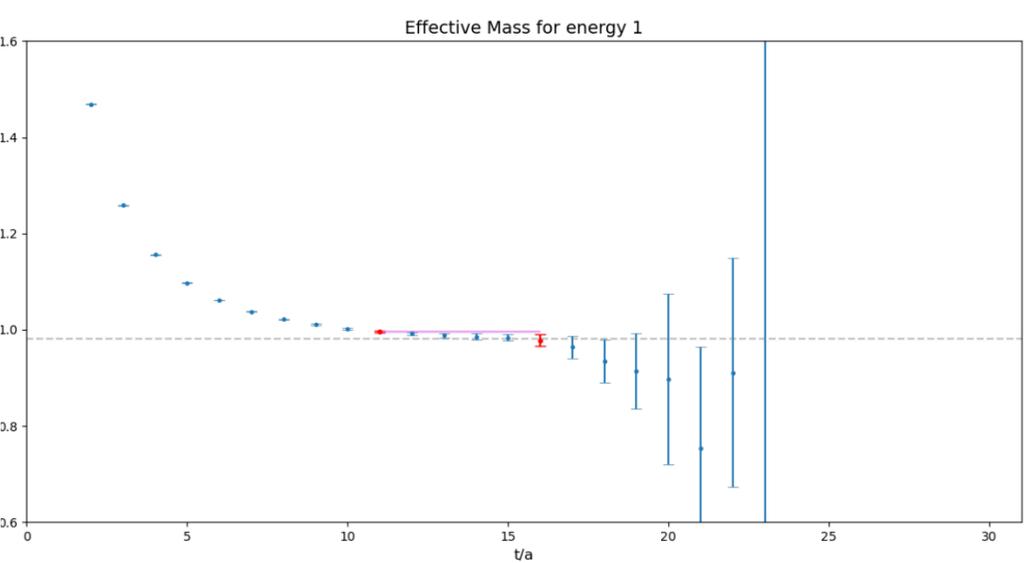
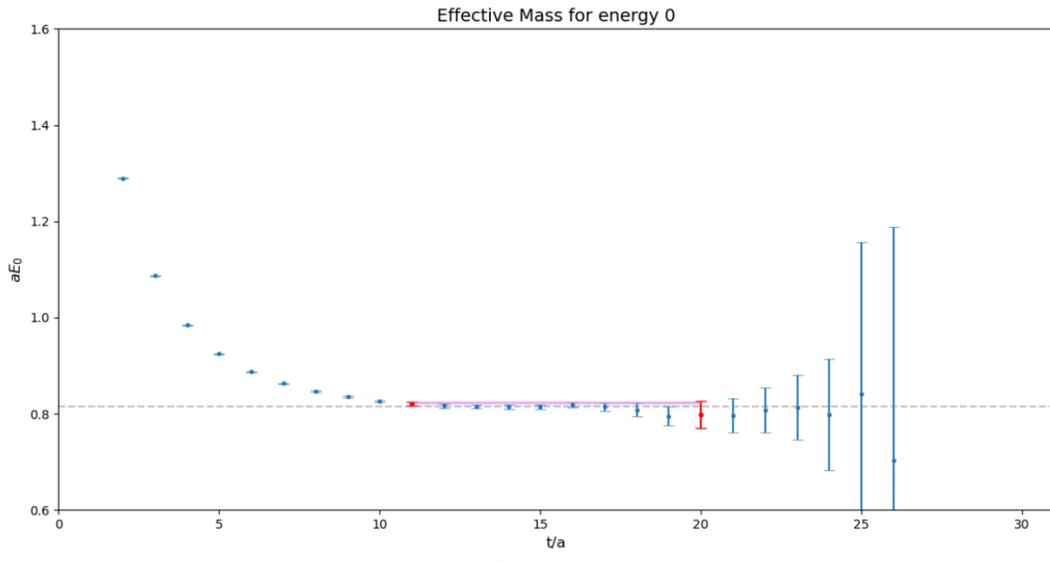
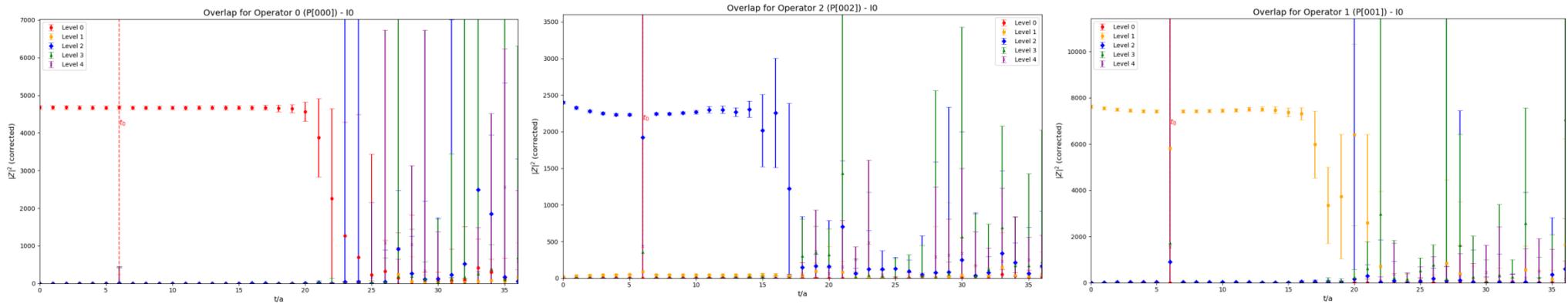


中国科学院大学  
University of Chinese Academy of Sciences

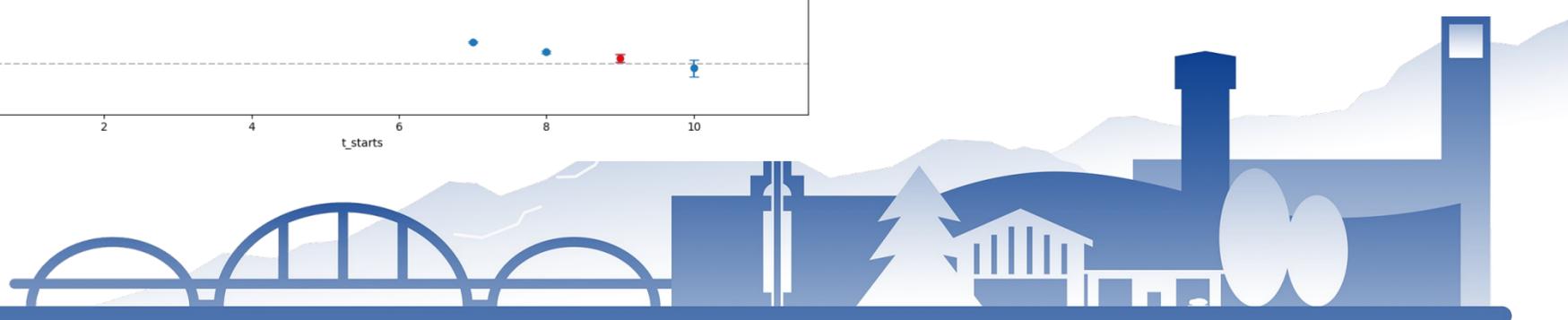
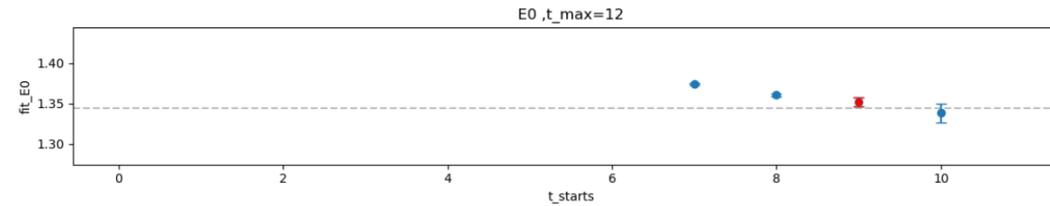
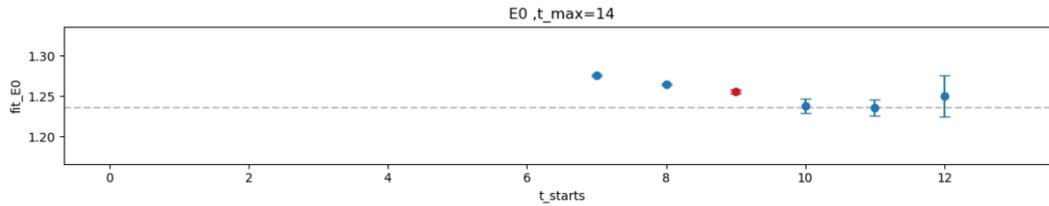
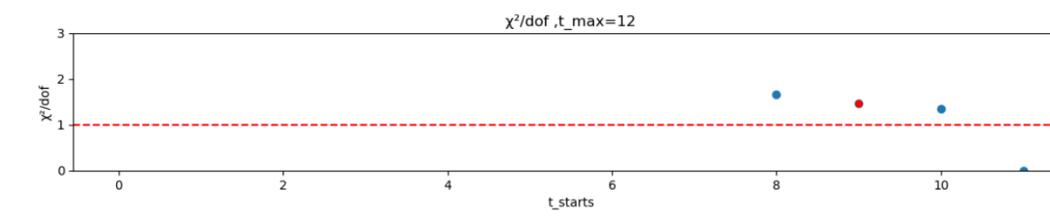
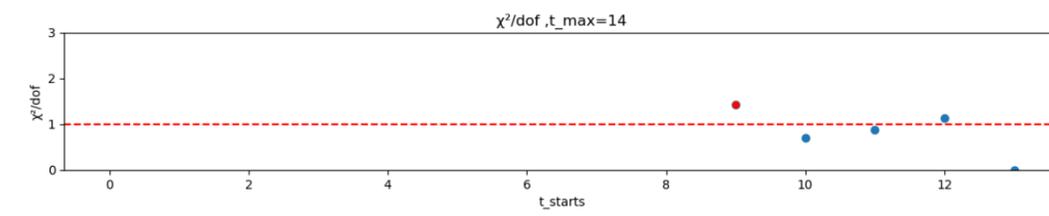
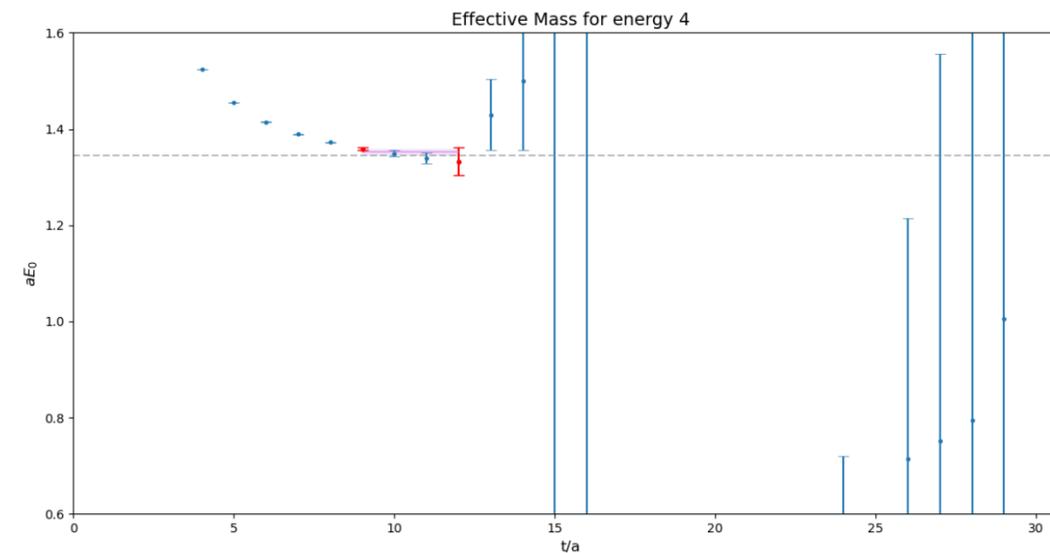
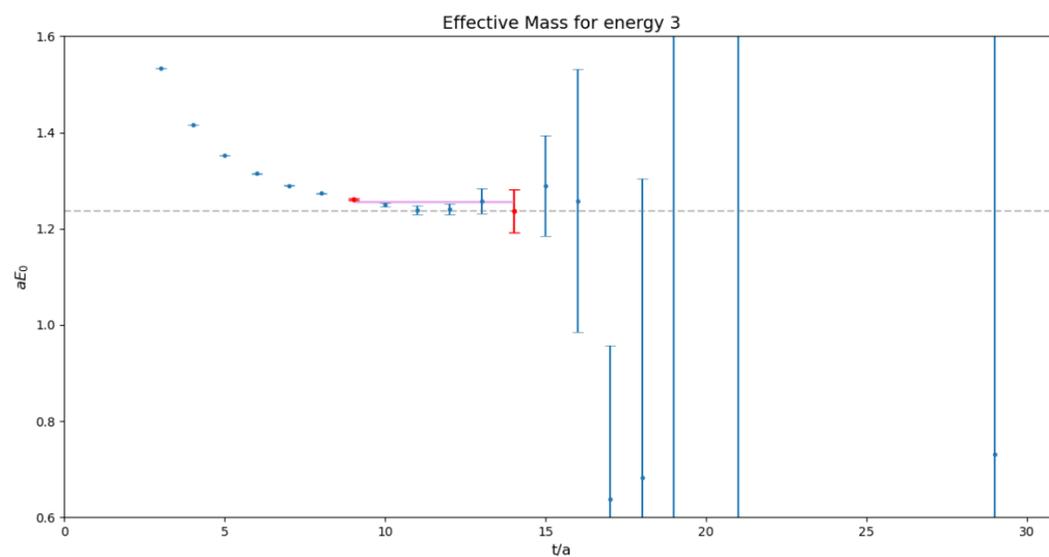
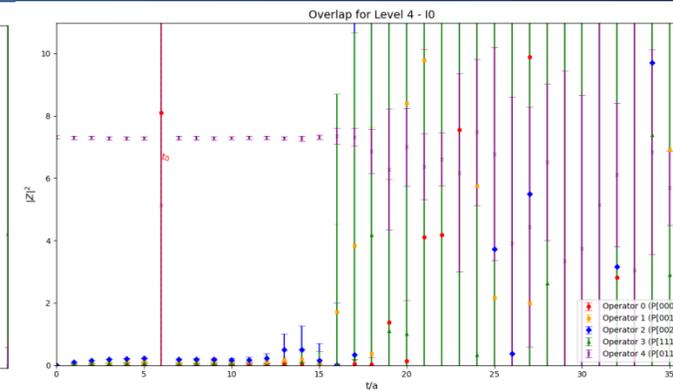
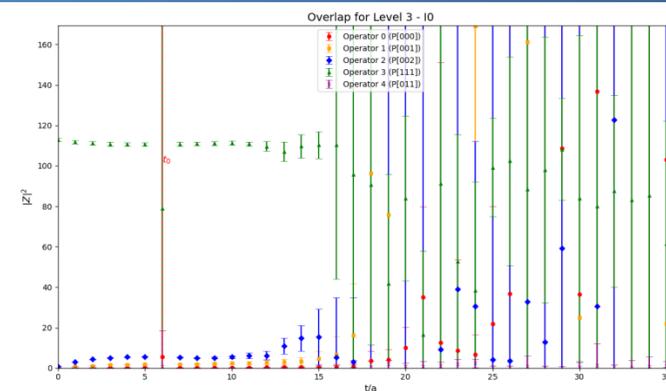
04

小结&展望





## 同位旋为0



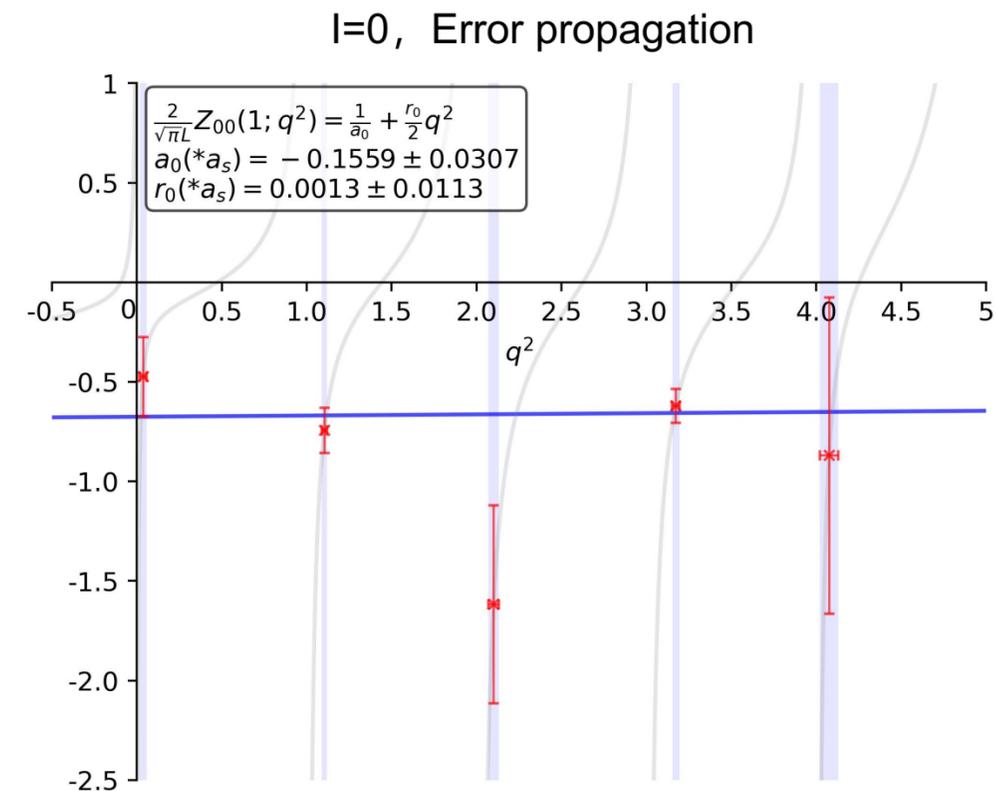
## 结论与不足

### 初步结论

- $K^+N$  散射在低能的时候格点有效质量接近于自由能级，与之前推断的  $K^+N$  散射低能时相互作用较弱相吻合。
- 根据 Lüscher 公式分析散射长度以及有效力程。

### 不足

- 由于近阈以及统计量的问题，误差较大，需要增加统计量。
- 只有质心系，没有运动系
- 只计算了一个组态
- 未来需要计算不同不可约表示



## 未来计划

01

### 增加统计量

以C24P29为例，未来样本数增加到880个

02

### 不同参考系

未来计算 $K\Delta$ 以及 $KN\pi$ 阈值以下的不同运动系的能级，为lucher分析增加数据点。

$P_{total}$ : [0, 0, 1]; [0, 1, 1]; [1, 1, 1]

03

### 不同组态

在不同基本信息的组态上讲该过程计算，以便未来将结果外推到物理极限。





中国科学院大学  
University of Chinese Academy of Sciences

# 感谢观看

Thank you for watching

